

# From EFTs to Nuclei

Thomas Papenbrock

THE UNIVERSITY of TENNESSEE  KNOXVILLE

and

OAK RIDGE NATIONAL LABORATORY



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# Collaborators

@ ORNL / UTK:

**B. Acharya, A. Bansal, S. Binder, E. A. Coello Pérez, G. Hagen, D. Odell, L. Platter**

@ Chalmers:

**B. Carlsson, A. Ekström, C. Forssén, D. Säaf**

@ MPI-K Heidelberg:

H. A. Weidenmüller

@ Michigan State University:

S. K. Bogner

@ The Ohio State University:

R. J. Furnstahl, **S. König, S. More**

@ TU Darmstadt:

**K. A. Wendt**

# Key question of this KITP program

1. What is the best path forward to refine the nuclear forces, the structure and interactions of nuclei and provide fundamental insights into nuclear physics directly from QCD?
2. Direct and model-independent **method of Lüscher** has been tremendously successful in the two-body sector, connecting two-particle spectra in a finite volume to scattering amplitudes. Is it realistic to assume that (the generalizations of) such method will be practical and successful **in the three/four-body sectors** too?
3. Is matching the many-body calculations to LQCD energy levels the way forward? Is the **ab initio many-body community ready to implement their methods in a finite volume** and with arbitrary boundary conditions? What are the challenges and expectations in executing such program?
4. What are the **prospects of evaluating finite-volume matrix elements in many-body calculations**? How should we match them to lattice renormalized matrix elements?

Short answer: A finite harmonic oscillator basis corresponds to a finite volume, and we have Lüscher-like formulas for extrapolations of various bound-state expectation values and matrix elements even for many-body systems.

# Energy scales and relevant degrees of freedom

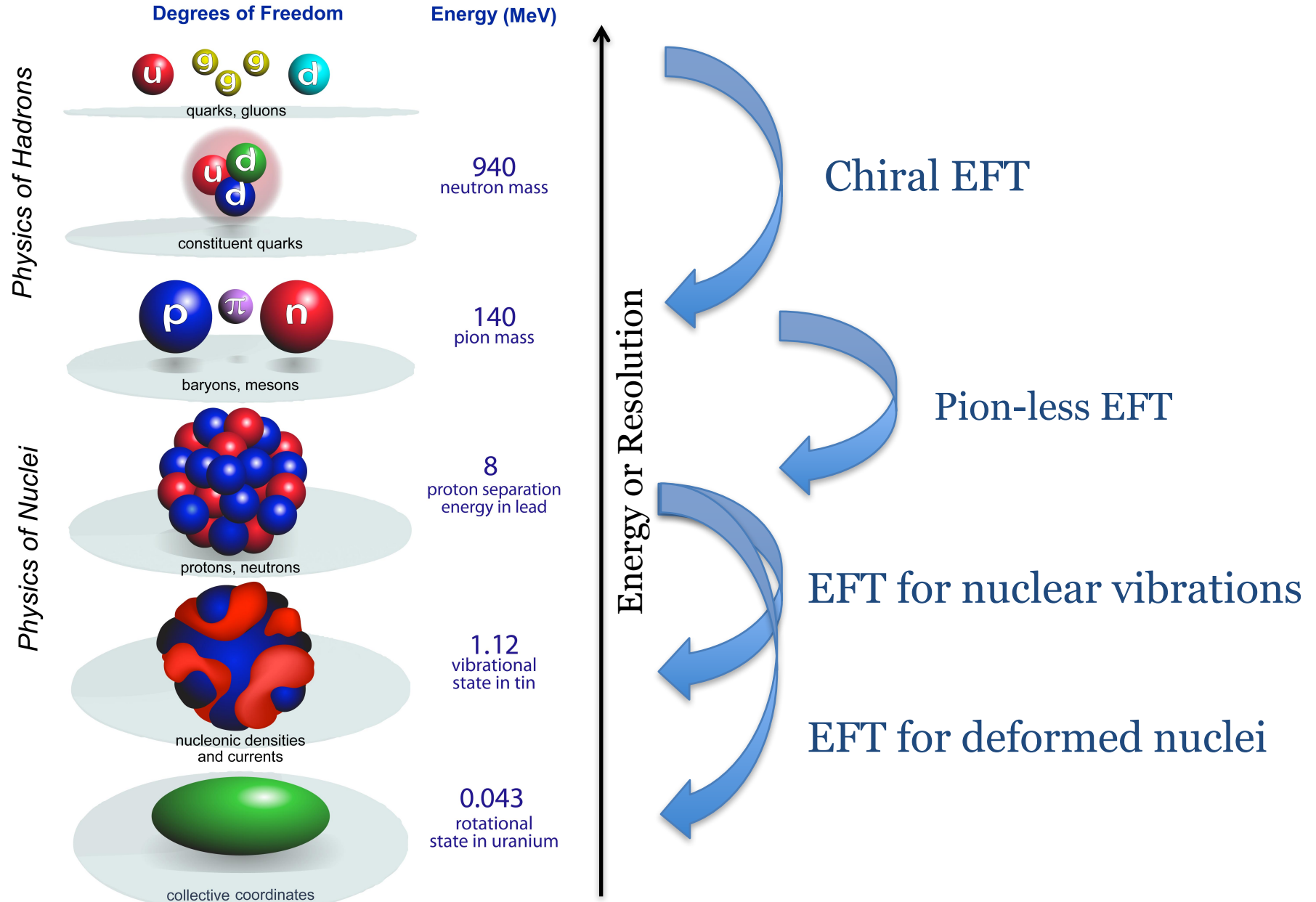


Fig.: Bertsch, Dean, Nazarewicz (2007)

# Recipe: calculations of atomic nuclei

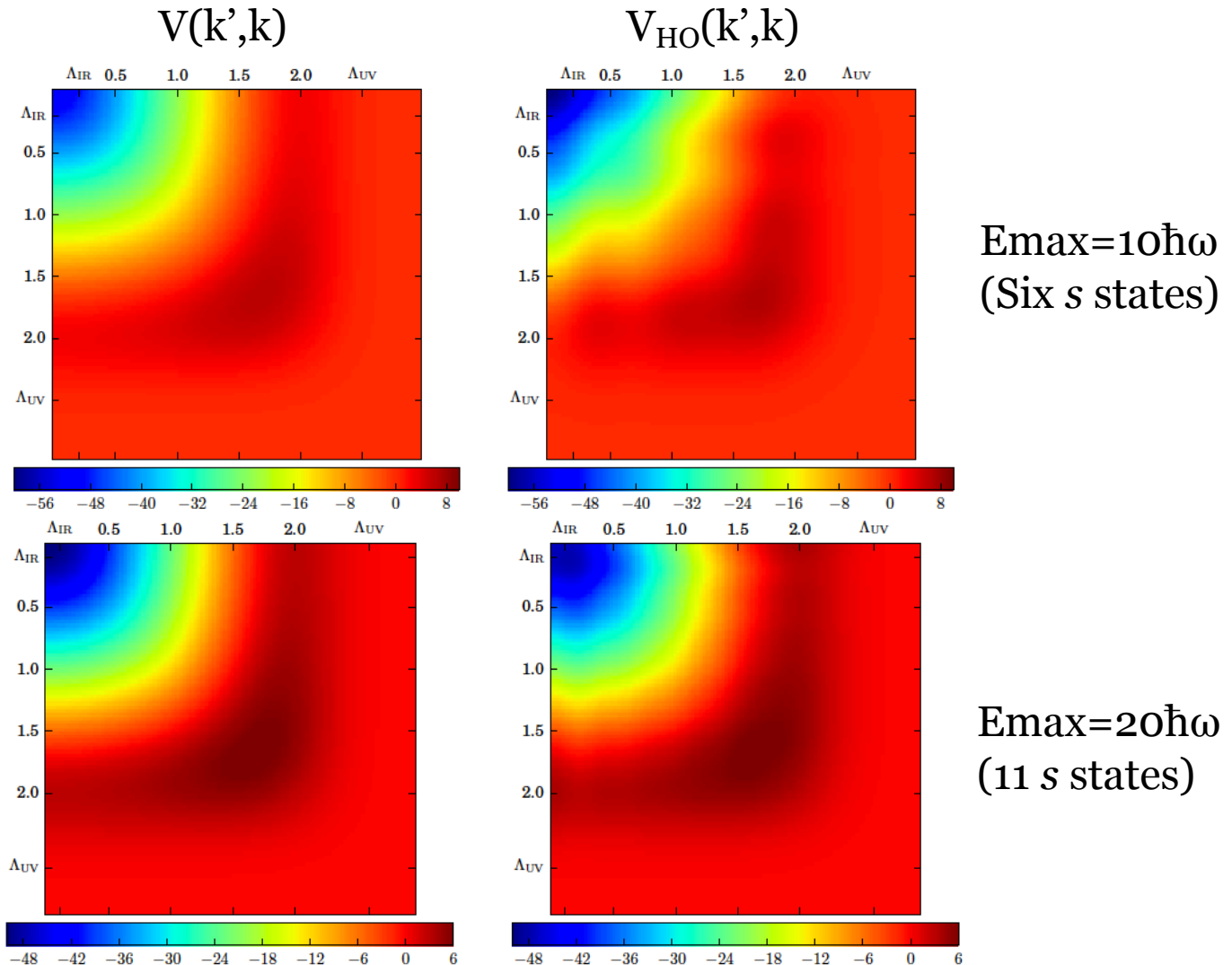
1. Interactions from chiral EFT (usually in momentum space)
  - a. Not perfect but best approach to date
2. Transform from momentum space to finite harmonic oscillator basis
  - a. Keeps rotational invariance, factors out center of mass
  - b. Spherical mean field reflects nuclear shell structure
  - c. (Nuclear Lattice EFT has advantages for  $\alpha$ -particle clusters)
3. Transform from center-of-mass system to laboratory frame
  - a. To deal with fermions
  - b. Center of mass: exact factorization in NCSM; still factors out for intrinsic Hamiltonians to a very good approximation for
4. Employ many-body method and solve nucleus; go to 2 and increase model space until convergence is reached

How good is the approximation in step 2 ? How can one extrapolate to infinite oscillator spaces? What are the IR and UV properties of the HO basis ?

Why not start directly at step 2 and construct an EFT in the oscillator basis?

# Matrix elements in a finite oscillator basis

$k$ -space matrix elements in  $^1S_0$  channel of NNLO<sub>sim</sub> ( $\Lambda_\chi = 400$  MeV)



Motivates direct construction of EFT in HO basis

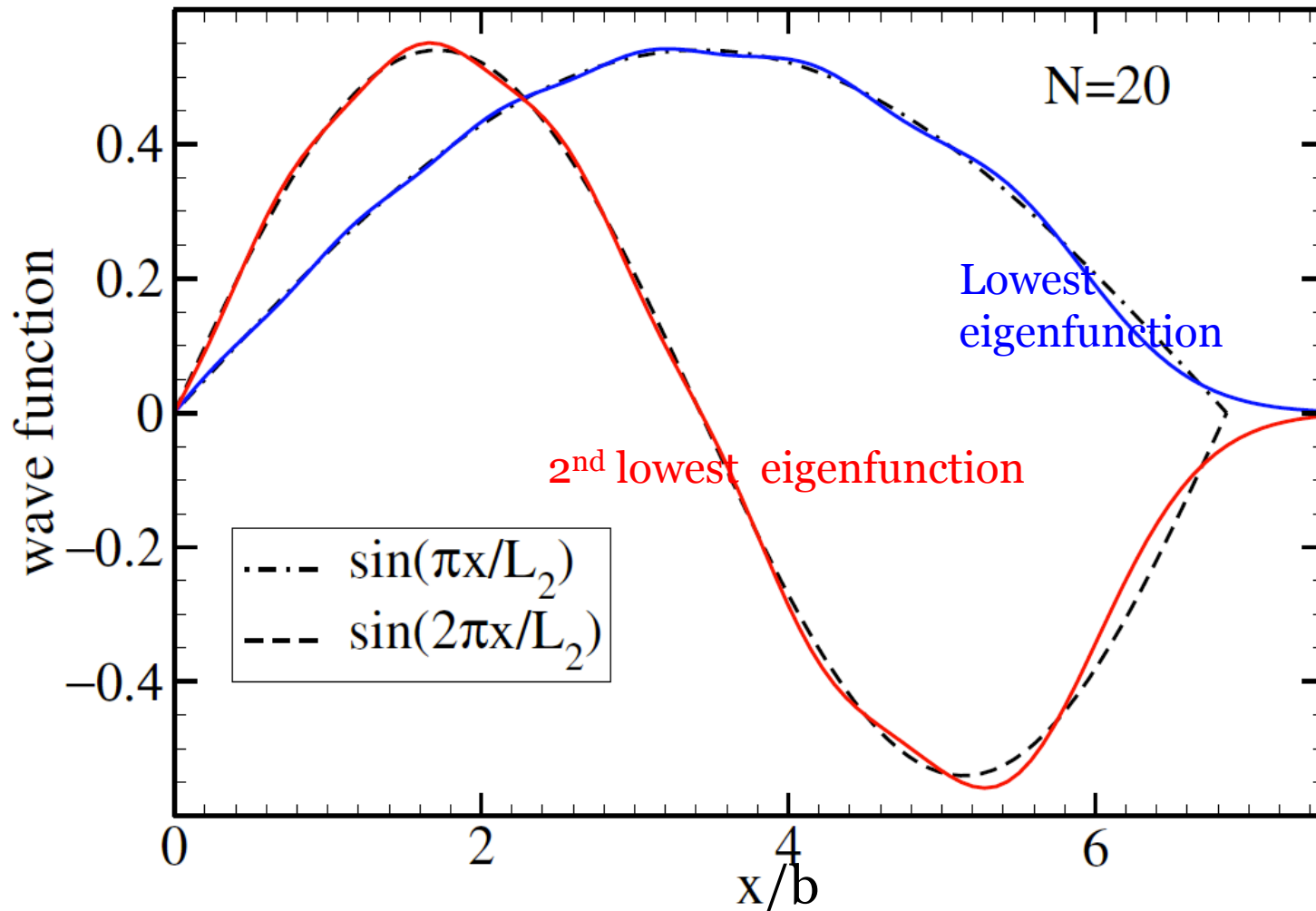
# EFT in harmonic oscillator

Motivation: optimize and generate interactions in basis of computation

- Formulate EFT directly in the oscillator basis [Haxton & Song (2000); Stetcu, Barrett & van Kolck (2007); Tölle, Hammer & Metsch (2011)]
- A finite harmonic oscillator basis exhibits IR and UV cutoffs [Stetcu, Barrett & van Kolck (2007); Coon *et al.* (2012); Furnstahl, Hagen & TP (2012)]
- Discrete momentum eigenstates from diagonalization of  $p^2$  for DVR in oscillator basis [Binder *et al.*, PRC 93, 044332 (2016)]

Useful tool: Computation of scattering phase shifts directly in the finite oscillator basis [Heller & Yamani (1974); Bang *et al.* (2000); Shirokov *et al.* (2004)]

# Eigenfunctions of $p^2$ with lowest eigenvalues in oscillator basis



Eigenfunctions look like those from a box of size  $L_2$ .

$$L_2 = \sqrt{2(N + 3/2 + 2)}b$$



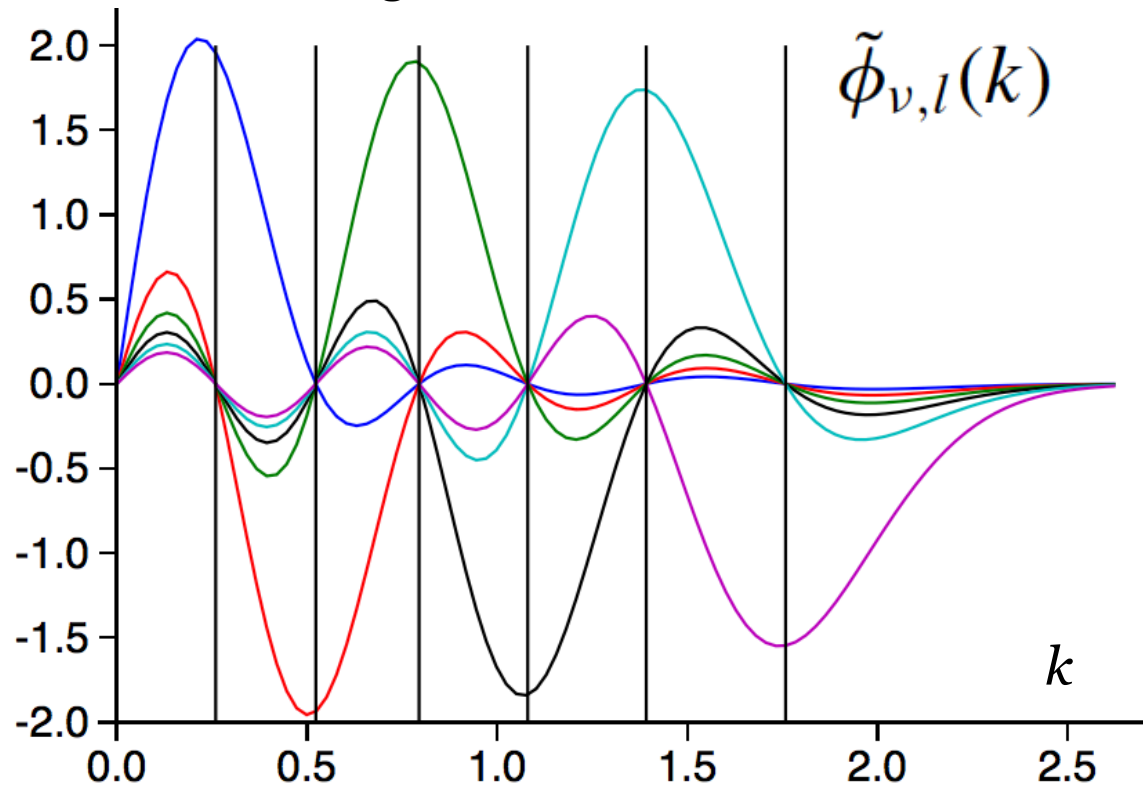
# Eigenfunctions and eigenvalues of $p^2$

For a partial wave  $l$  in a Hilbert space with energies up to  $(2N+l)\hbar\omega$ , the eigenvalues  $k_{\mu l}^2$  of  $p^2$  are the roots of the associated Laguerre polynomial  $L_{N+1}^{l+1/2}$ .

The eigenfunctions of  $p^2$  are a DVR (discrete variable representation).

DVRs see: [Harris, Engerholm, & Gwinn (1965); Light, Hamilton, & Lill (1985); Baye & Heenen (1986); Littlejohn *et al.* (2002); Bulgac & McNeil Forbes (2013).]

Eigenfunctions of  $p^2$   
for  $l=0$ ,  $N=10$ , and  
 $\hbar\omega = 10$  MeV.  
Vertical lines are  
eigenvalues  $k_{\mu l}$ .



# DVR in the oscillator basis

In the DVR, matrix elements are computed easily

$$\langle \phi_{\nu, l'} | \hat{V} | \phi_{\mu, l} \rangle = c_{\nu, l'} c_{\mu, l} \langle k_{\nu, l'}, l' | \hat{V} | k_{\mu, l}, l \rangle$$

This corresponds to an  $(N+1)$ -point Gauss Laguerre quadrature. The  $c_{\mu, l}$  are normalization constants and related to Gauss-Laguerre weights. [Binder *et al.*, PRC 93, 044332 (2016)]

Scalar products in DVR:  $(\tilde{\Psi}_f | \tilde{\Psi}_g) = \sum_{\mu=0}^N c_{\mu, l}^2 \tilde{\Psi}_f(k_{\mu, l}) \tilde{\Psi}_g(k_{\mu, l})$

Analytical formulas for lowest momentum eigenvalue sets IR length [More, Ekström, Furnstahl, Hagen & TP (2013)]

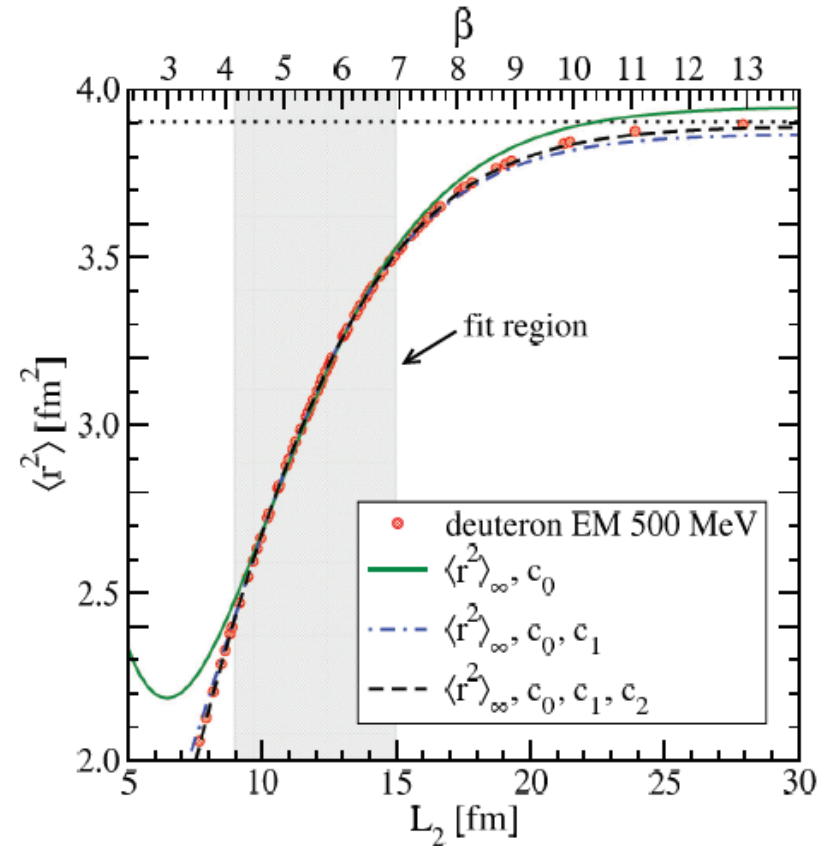
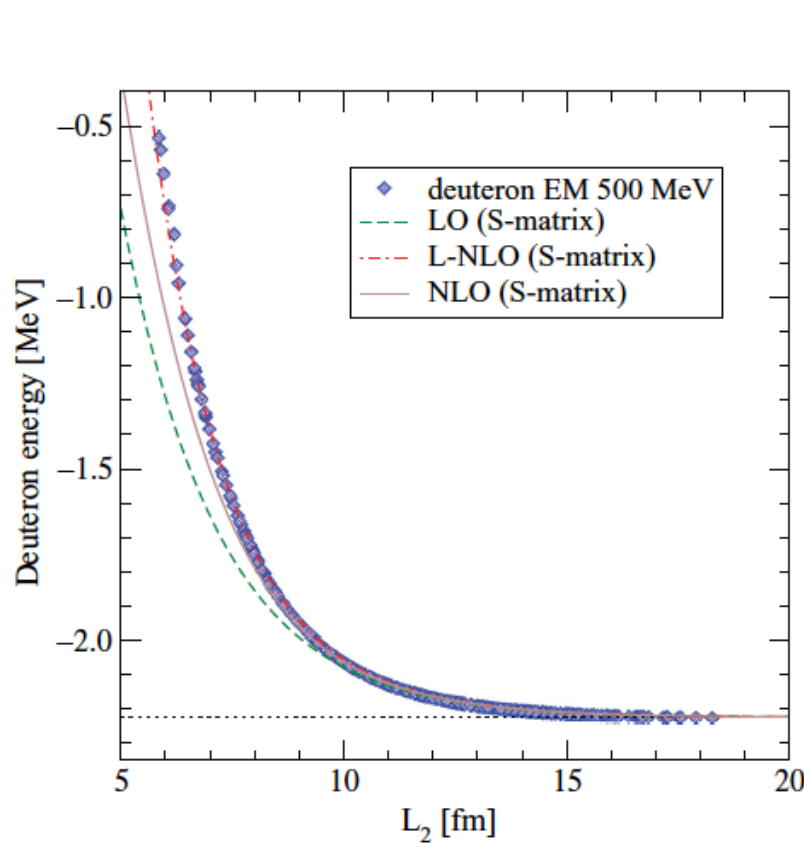
$$k_{00} \approx \pi/L_2 \quad \text{with} \quad L_2 = \sqrt{2(N + 3/2 + 2)}b$$

[See also Szegő *Orthogonal Polynomials* (1939)]

# Lüscher-like formulas for the oscillator basis

For long wave lengths, a finite HO basis resembles a spherical cavity. [Furnstahl, Hagen & TP (2012)]

$$u_E(r) \xrightarrow{r \gg R} A_E(e^{-k_E r} + \alpha_E e^{+k_E r})$$



$$E_L = E_\infty + a_0 e^{-2k_\infty L}$$

$$\langle r^2 \rangle_L \approx \langle r^2 \rangle_\infty [1 - (c_0 \beta^3 + c_1 \beta) e^{-\beta}]$$

$$\beta \equiv 2k_\infty L$$

# Lüscher formulas for many-body systems

**Key idea:** compute eigenvalues of kinetic energy and compare with *corresponding* (hyper)spherical cavity to find L.

What is the corresponding cavity?

Single particle	A particles (product space)	A particles in No-core shell model
Diagonalize $T_{\text{kin}}=p^2$	Diagonalize A-body $T_{\text{kin}}$	Diagonalize A-body $T_{\text{kin}}$
3D spherical cavity	A fermions in 3D cavity	3(A-1) hyper-radial cavity

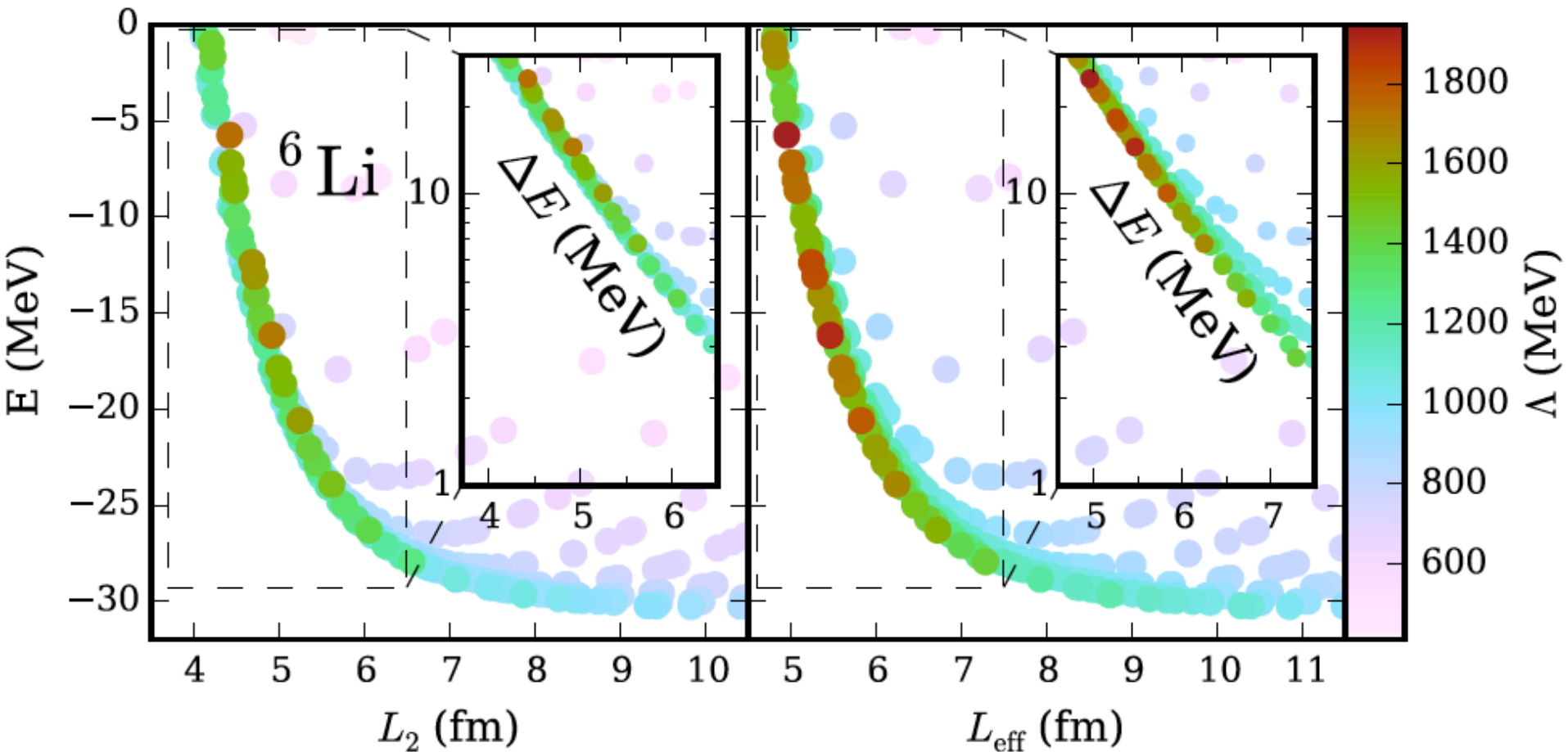
$$L_2 = \sqrt{2(N + 3/2 + 2)}b \quad L_{\text{eff}} = \left( \frac{\sum_{nl} \nu_{nl} a_{l,n}^2}{\sum_{nl} \nu_{nl} k_{l,n}^2} \right)^{1/2} \quad L_{\text{eff}} = b \frac{X_{1,\mathcal{L}}}{\sqrt{T_{1,\mathcal{L}}(N_{\text{max}}^{\text{tot}})}}$$

More, Ekström,  
Furnstahl, Hagen, TP,  
PRC 87, 044326 (2013)

Furnstahl, Hagen, TP,  
Wendt, J. Phys. G 42,  
034032 (2015)

Wendt, Forssén, TP, Sääf,  
PRC 91, 061301(R) (2015)

# IR length in NCSM spaces

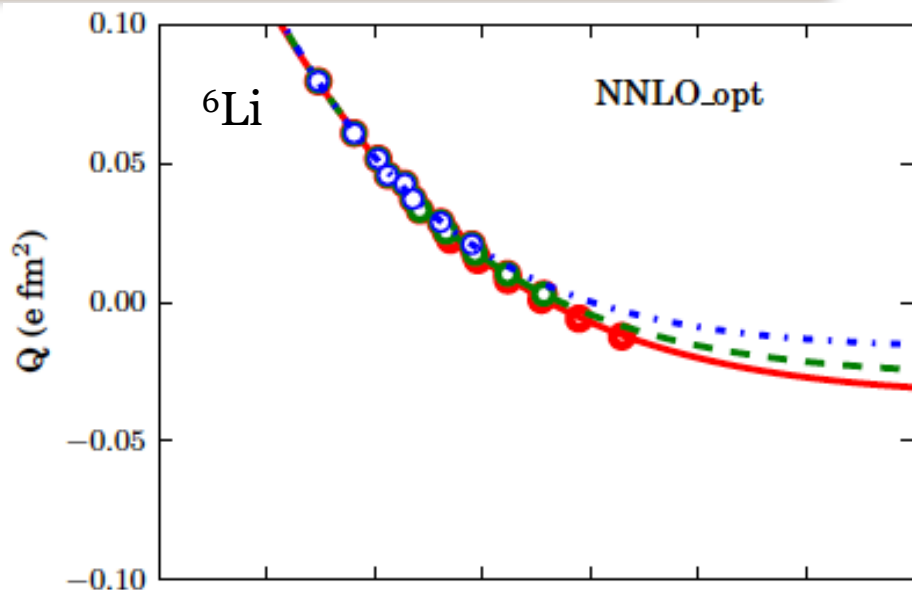


Diagonalize kinetic energy in  $3(A-1)$  dimensional harmonic oscillator; seek lowest antisymmetric state and equate to hyperspherical cavity with radius  $L_{\text{eff}}$ .

# Extrapolations in finite Hilbert spaces

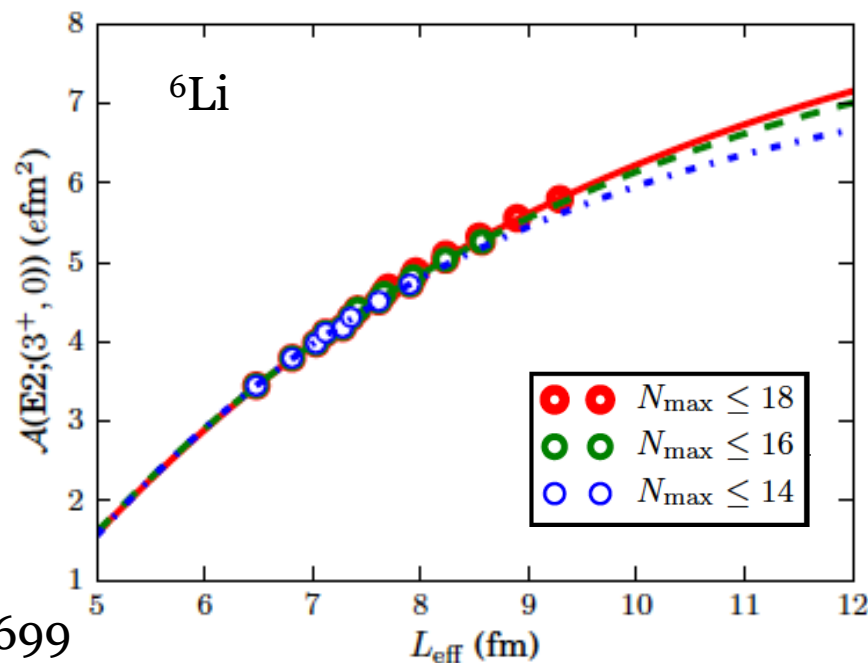
## Quadrupole moment

$$Q_L = Q_\infty - a(k_\infty L)^3 e^{-2k_\infty L}$$



## E2 transition amplitude

$$\mathcal{A}_L = \mathcal{A}_\infty + a_0 e^{-2k_\infty L}$$

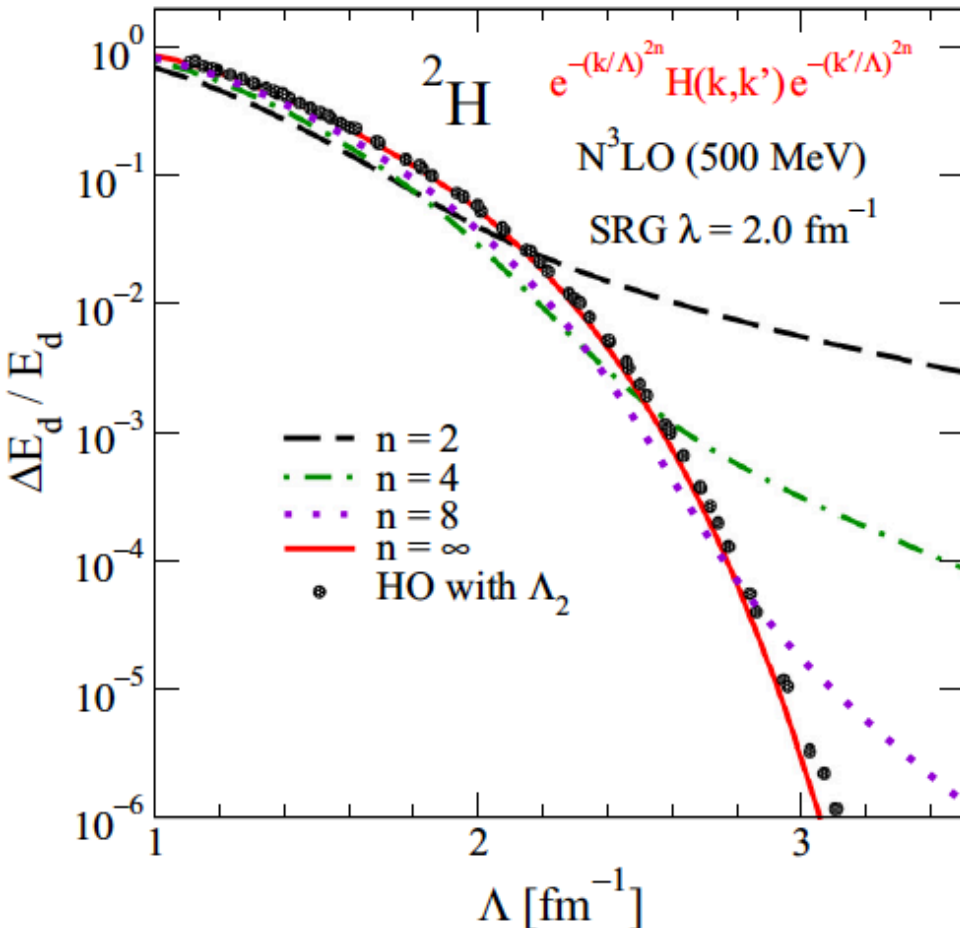


Derivation: Odell, TP, Platter, PRC (2016).

Application: Ik Jae Shin *et al.*, 1605.02819.

Radiative capture: Acharya *et al.*, arXiv:1608.04699

# UV cutoff



UV cutoff imposed by HO basis resembles a sharp cutoff; cutoff dual to IR length

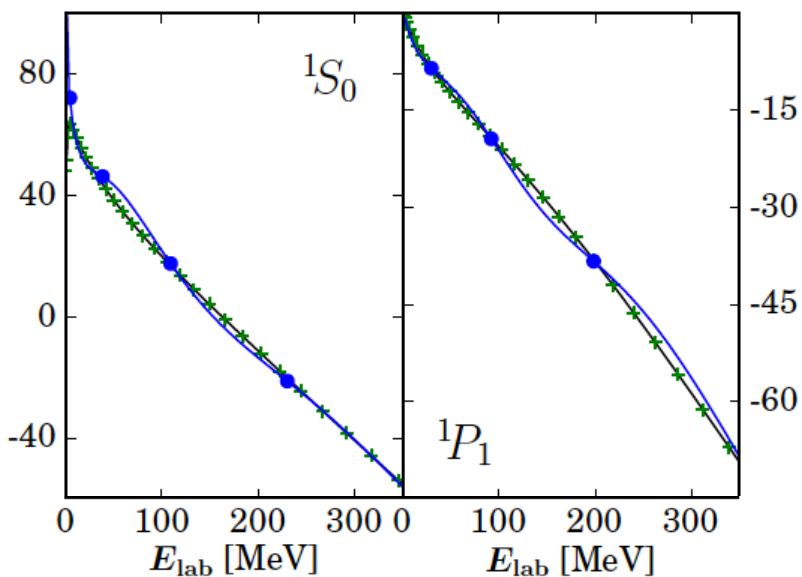
Analytical extrapolation formulas for separable potentials (or separable approximations of other potentials)

→ optimize interaction directly in HO basis:  
EFT in HO basis is DVR

# Chiral interaction at NLO in the oscillator basis

- Construct and optimize interaction in oscillator basis
- UV convergence by construction
- NLO interaction constructed with  $E_{\max}=10\hbar\omega$  at  $\hbar\omega=22$  MeV
- Rapid convergence of ground-state energies even for heavy nuclei

Phase shifts compared to  $NLO_{\text{sim}}$



Convergence of ground-state energies

$N_{\max}$	${}^4\text{He}$	${}^{16}\text{O}$	${}^{40}\text{Ca}$	${}^{90}\text{Zr}$	${}^{132}\text{Sn}$
	$E_{\text{CCSD}}$ [MeV]				
10	-31.57	-142.89	-402.0	-918.4	-1230.0
12	-31.57	-142.92	-402.4	-923.1	-1249.3
14	-31.57	-142.93	-402.5	-924.6	-1255.6
16	-31.57	-142.93	-402.5	-925.1	-1258.3
$\infty$	-31.57	-142.93	-402.5	-925.4	-1260.1
exp	-28.30	-127.62	-342.1	-783.9	-1102.9

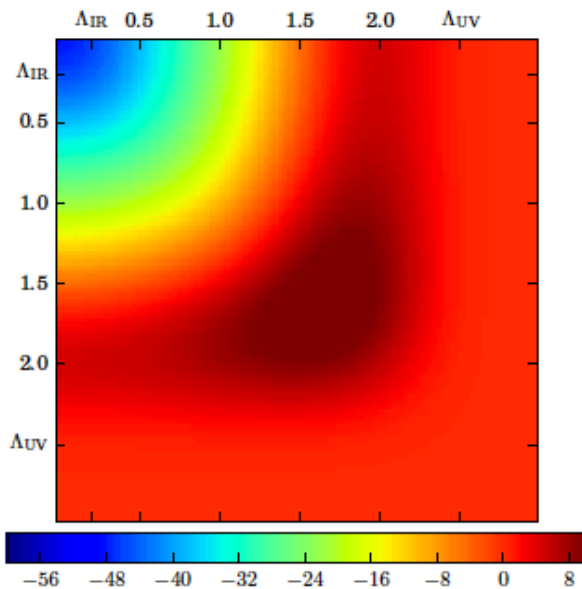


# Matrix elements in a finite oscillator basis

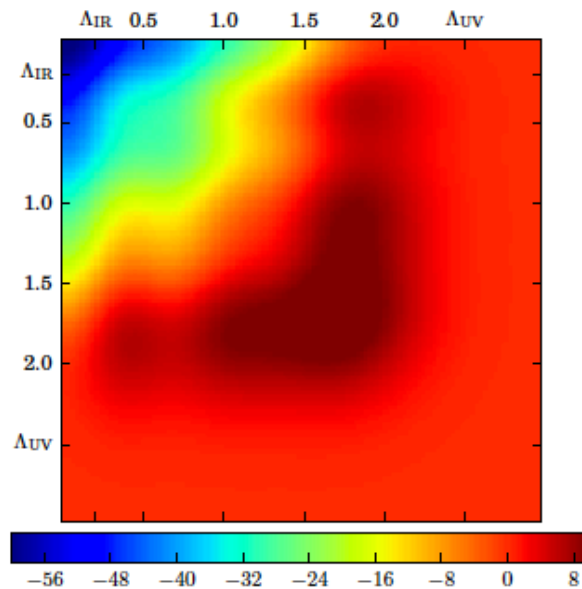
Momentum-space matrix elements

$^1S_0$  channel of  $\text{NNLO}_{\text{sim}}$  with  $\Lambda_\chi = 400 \text{ MeV}$   
 $E_{\text{max}} = 10\hbar\omega$  in harmonic oscillator (6 s states)

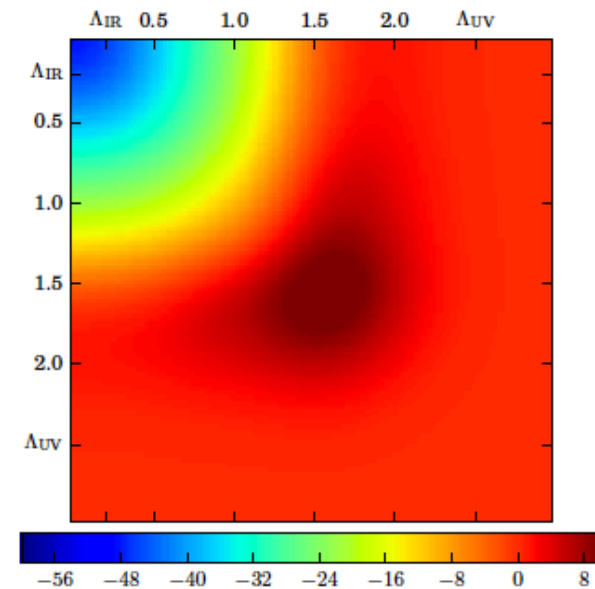
$V(k',k)$



DVR:  $V_{\text{HO}}(k',k)$



IR improved  $V_{\text{HO}}(k',k)$



IR improvement in oscillator EFT:

Modify matrix elements at high discrete momenta to improve low-momentum physics

# Energy scales and relevant degrees of freedom

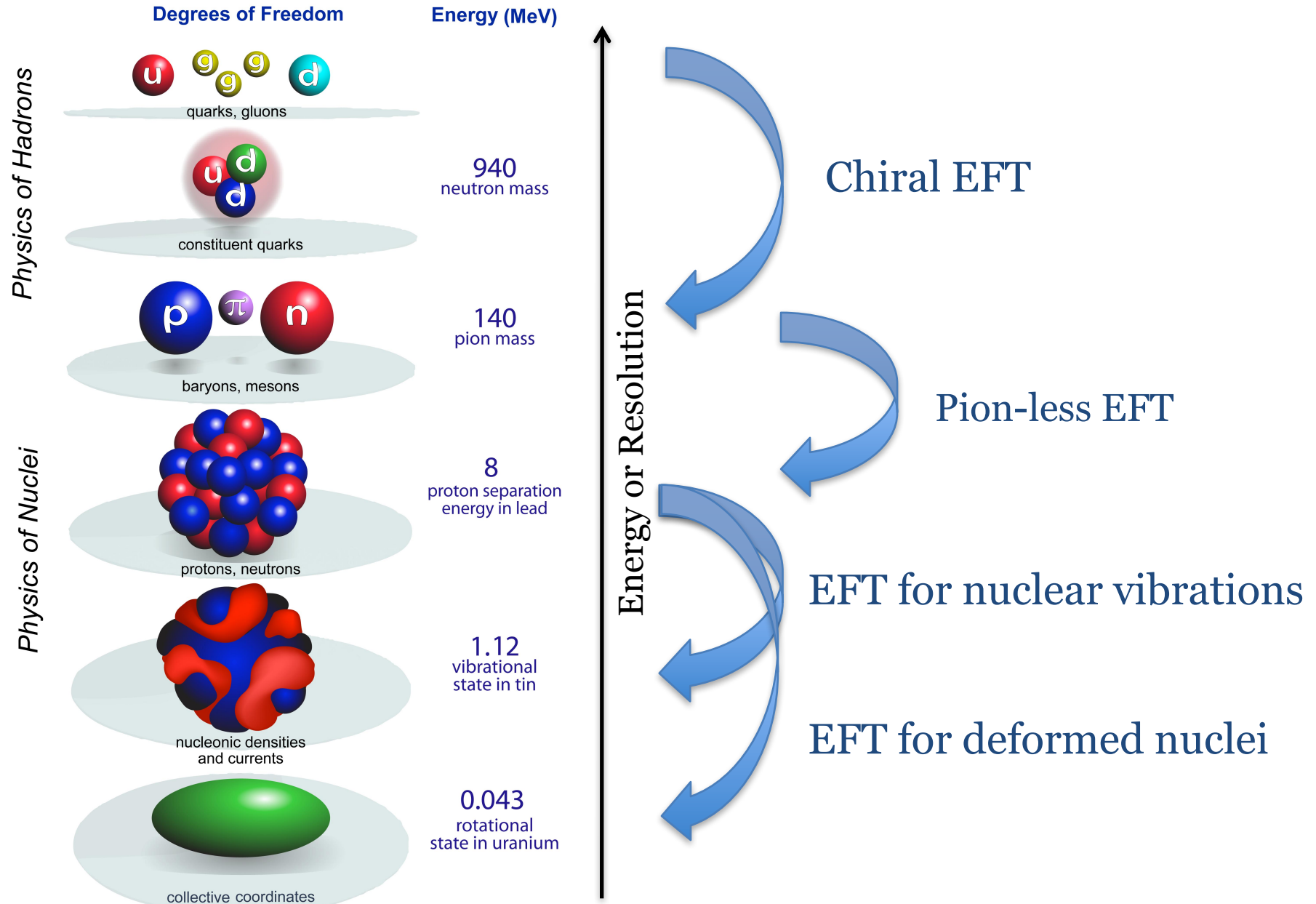
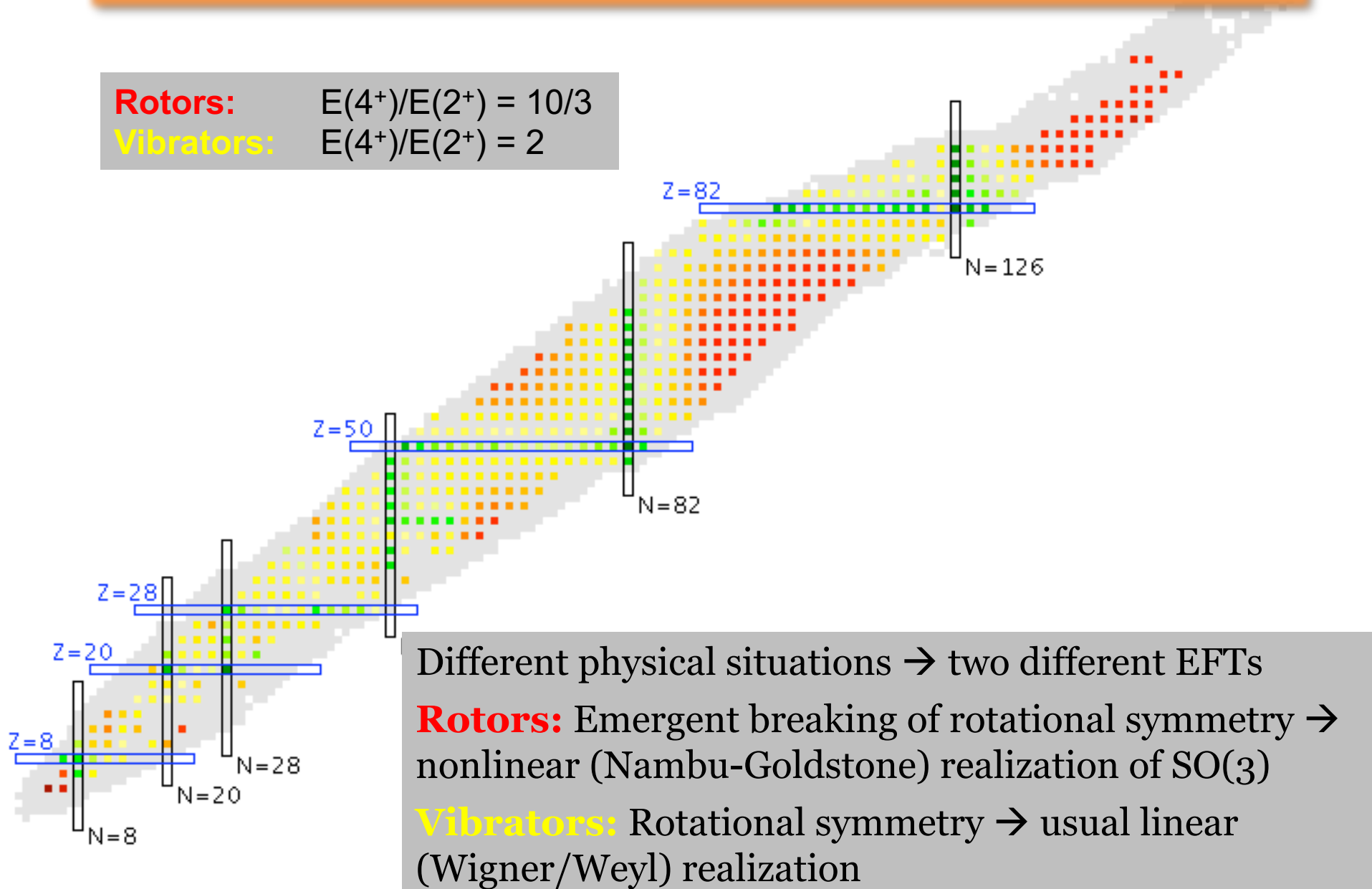


Fig.: Bertsch, Dean, Nazarewicz (2007)

# EFTs for heavy nuclei

**Rotors:**  $E(4^+)/E(2^+) = 10/3$

**Vibrators:**  $E(4^+)/E(2^+) = 2$



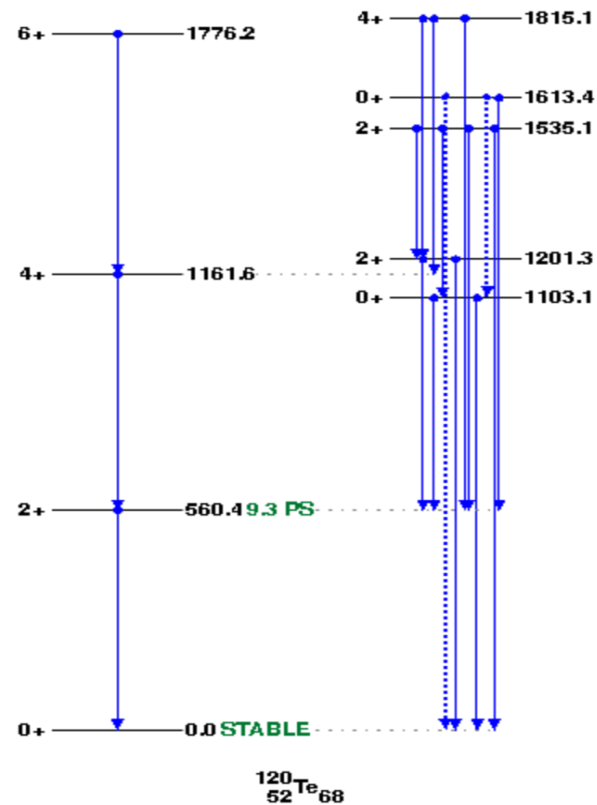
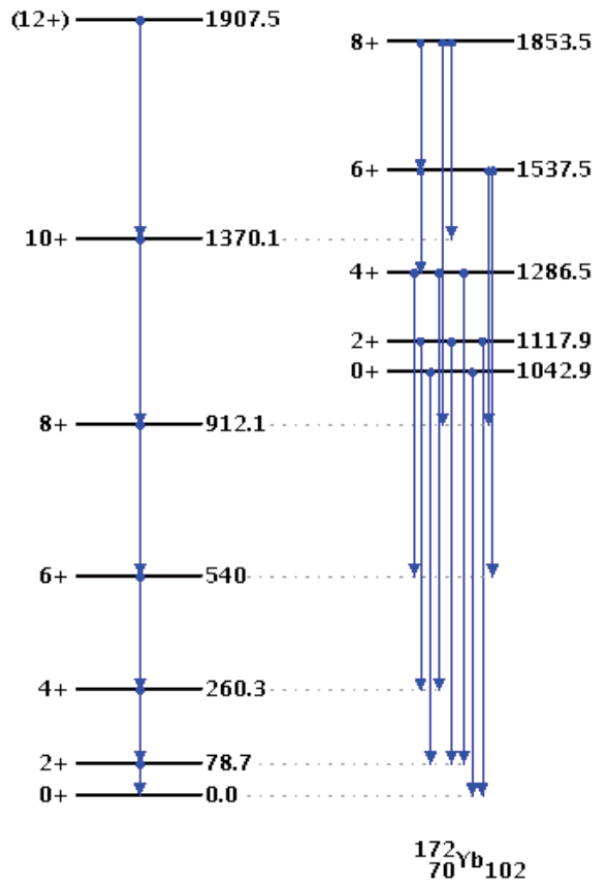
Different physical situations  $\rightarrow$  two different EFTs

**Rotors:** Emergent breaking of rotational symmetry  $\rightarrow$  nonlinear (Nambu-Goldstone) realization of  $SO(3)$

**Vibrators:** Rotational symmetry  $\rightarrow$  usual linear (Wigner/Weyl) realization

# Spectra of heavy nuclei

Quadrupole degrees of freedom describe spins and parity of low-energy spectra



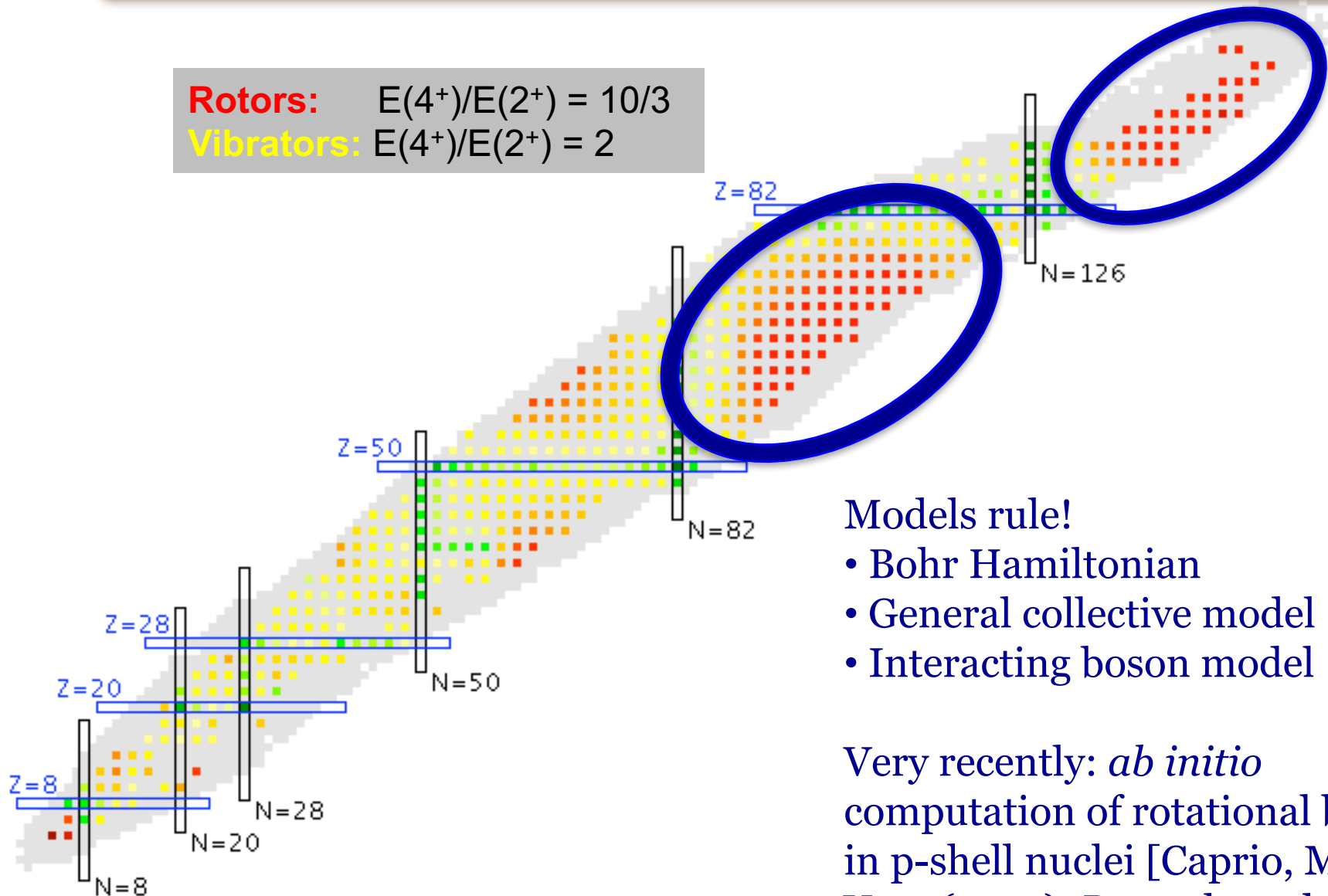
Nuclear rotation: emergent breaking of rotational symmetry of  $\text{SO}(3) \rightarrow \text{SO}(2)$ ; EFT based on nonlinear realization (Nambu-Goldstone) of  $\text{SO}(3)$

Nuclear vibration: EFT based on linear realization (Wigner / Weyl) of  $\text{SO}(3)$

# EFT for deformed nuclei

**Rotors:**  $E(4^+)/E(2^+) = 10/3$

**Vibrators:**  $E(4^+)/E(2^+) = 2$



Models rule!

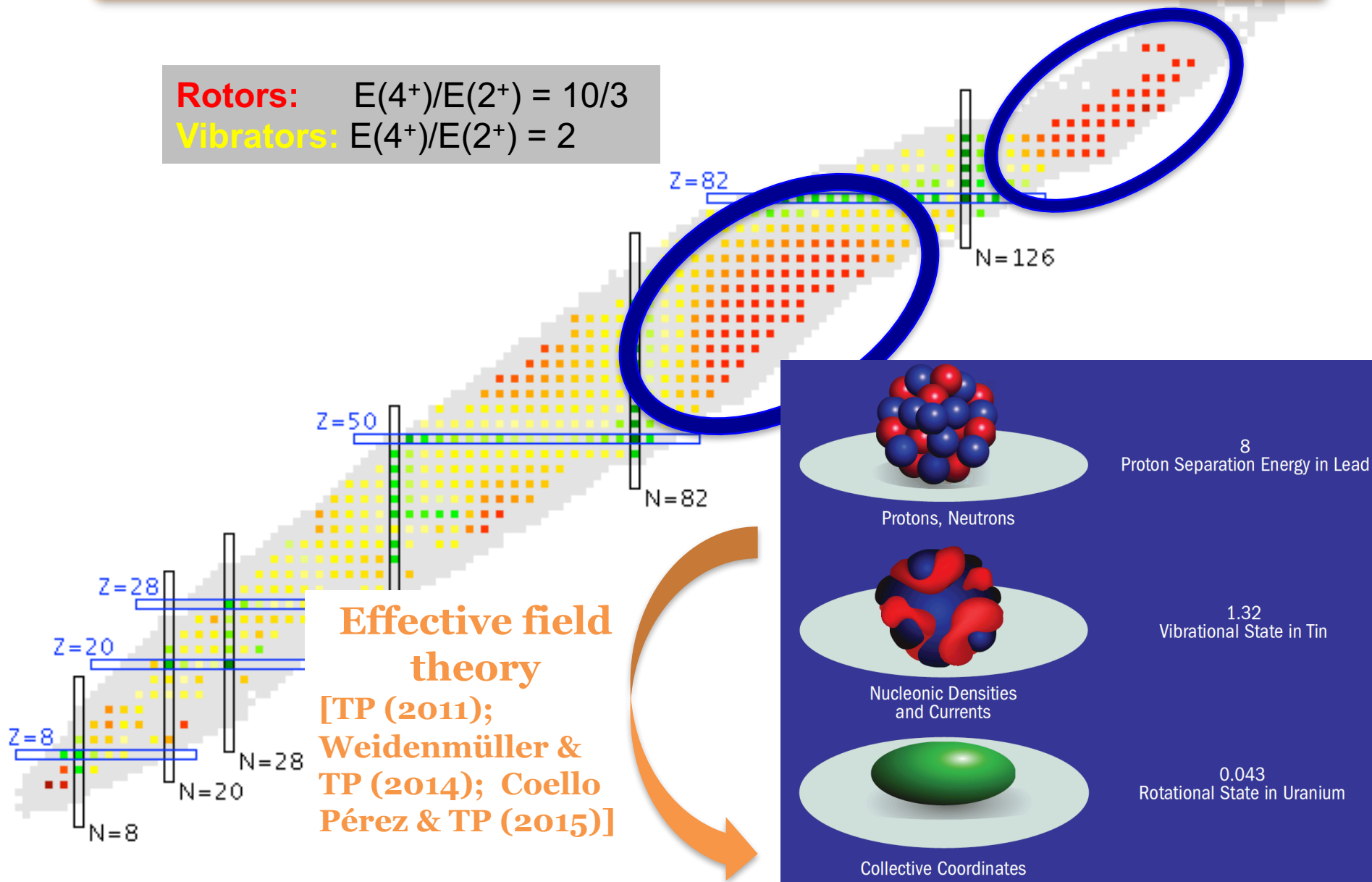
- Bohr Hamiltonian
- General collective model
- Interacting boson model

Very recently: *ab initio* computation of rotational bands in p-shell nuclei [Caprio, Maris, Vary (2013); Dytrych et al (2013)]

# EFT for deformed nuclei

**Rotors:**  $E(4^+)/E(2^+) = 10/3$

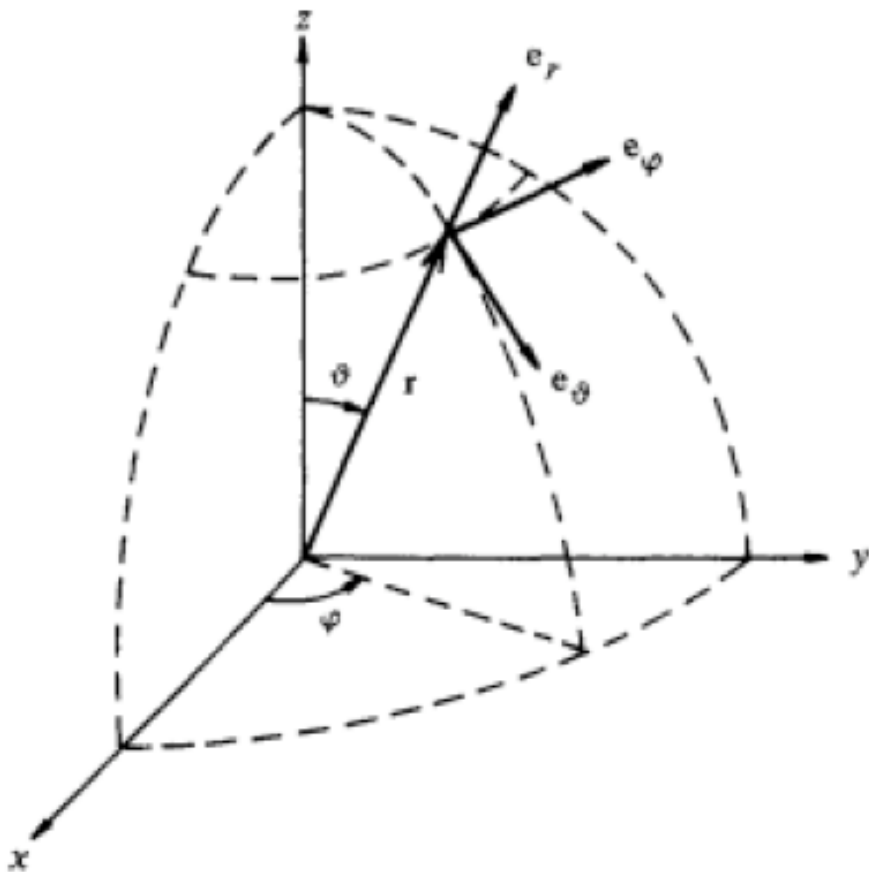
**Vibrators:**  $E(4^+)/E(2^+) = 2$



# Nonlinear realization of rotational symmetry

[Weinberg 1967; Coleman, Callan, Wess & Zumino 1969]

Spontaneous breaking of rotational symmetry: Nambu-Goldstone modes parameterize the coset  $SO(3)/SO(2) \sim S^2$ , i.e. the two sphere



$$\vec{n}(\theta, \phi) = \begin{pmatrix} \cos \phi \sin \theta \\ \sin \phi \sin \theta \\ \cos \theta \end{pmatrix}$$

Comments:

- Further degrees of freedom in the tangential plane can be added to the tangential plane
- Addition of monopole field yields nuclei with nonzero ground-state spins

# Emergent symmetry breaking

Finite system cannot exhibit spontaneous symmetry breaking

Instead: Emergent symmetry breaking [Yannouleas & Landman 2007]

Infinite system: Hilbert spaces with different orientations (of the nucleus) are inequivalent. No rotation, i.e. no unitary transformation can connect states in inequivalent Hilbert spaces

Finite system: Hilbert spaces with different orientations are connected by a rotation: Zero mode, i.e. purely time-dependent “Nambu-Goldstone field” has to be added; amplitudes of this mode can be large.

Low-lying modes in finite systems: [Gasser & Leutwyler 1988; Hasenfratz & Niedermayer 1993]

Field theory of (anti)ferromagnet as example of  $SO(3) \rightarrow SO(2)$ : [Leutwyler 1987; Roman & Soto 1999; Hofmann 1999; Bär, Imboden & Wiese 2004; Kämpfer, Moser & Wiese 2005]



# EFT for deformed nuclei

Spectrum of ground-state band

$$E(I) = \frac{I(I+1)}{2C_0} - \frac{C_2}{4C_0^4} [I(I+1)]^2$$

Strength of quadrupole transitions  $I_i \rightarrow I_i - 2$  in ground-state band  
(Clebsch-Gordan coefficient divided out)

$$Q_{if}^2 = Q_0^2 \left[ 1 + \frac{b}{a} I_i (I_i - 1) \right]$$

No surprises here: the EFT reproduces well known results from phenomenological models (e.g. Variable Moment of Inertia, Mikhailov theory...)

EFT provides us with insight in scale of parameters in expansion of observables

E. A. Coello Pérez and TP, Phys. Rev. C 92, 014323 (2015)

# EFT: expansion parameter & naturalness

Natural sizes as expected!

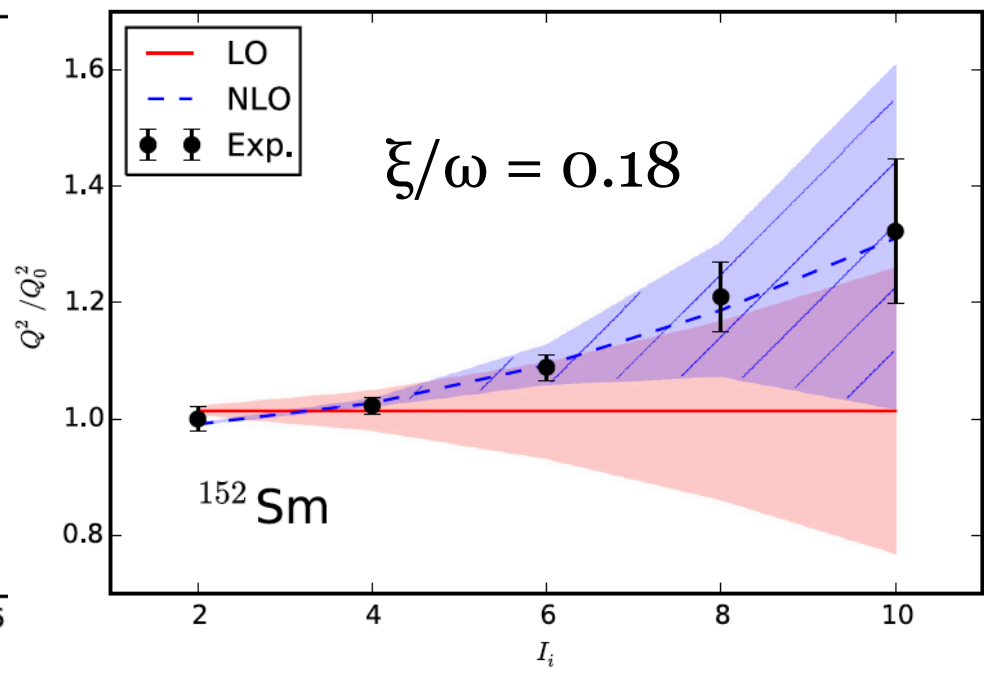
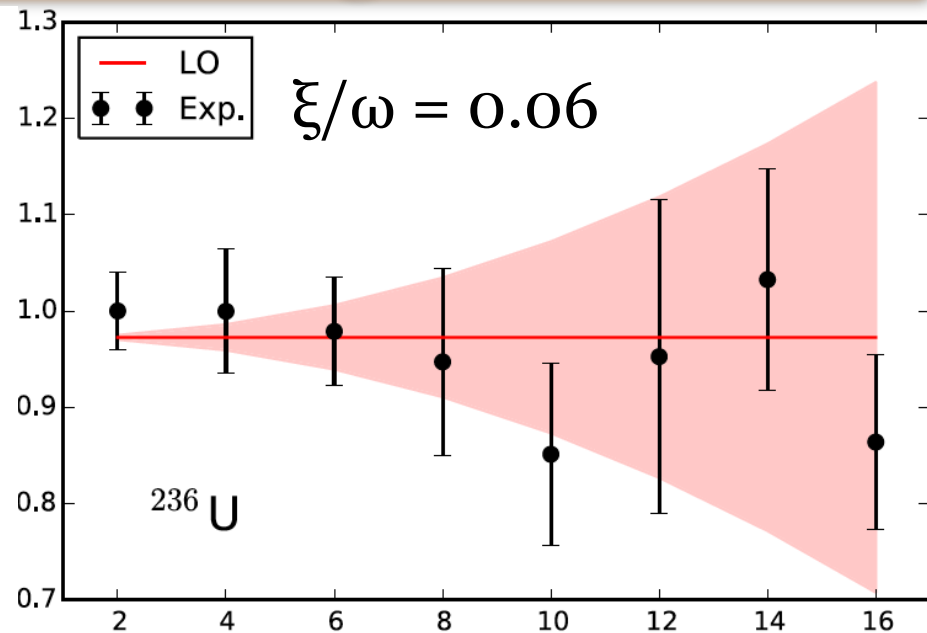
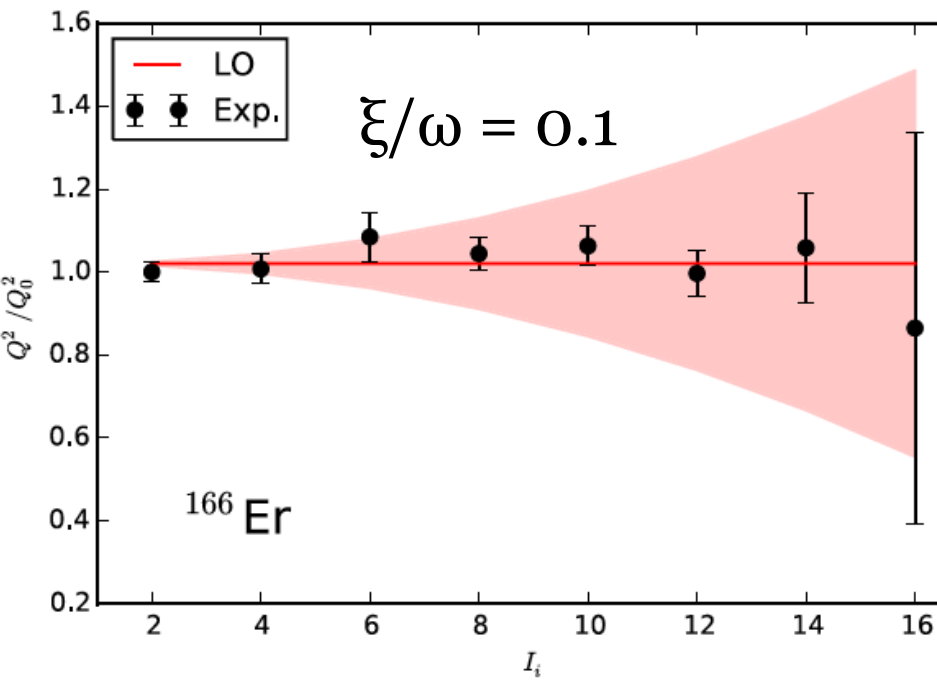
	Expansion parameter: $E_{\text{rot}} / E_{\text{vib}}$	Natural LECs: spectrum	Natural LECs: transitions	
System	$(\xi/\omega)^2$	$C_2/C_0^3$	$b/a$	
Molecules	N <sub>2</sub>	0.000 026	0.000 006	-0.000 011
	H <sub>2</sub>	0.0062	0.0015	0.0022
Rotational nuclei	<sup>236</sup> U	0.0043	0.0011	—
	<sup>174</sup> Yb	0.0026	0.0010	—
	<sup>168</sup> Er	0.0094	0.0010	—
	<sup>166</sup> Er	0.011	0.0020	—
	<sup>162</sup> Dy	0.0083	0.0017	—
	<sup>154</sup> Sm	0.0056	0.0033	—
Transitional nuclei	<sup>188</sup> Os	0.06	0.012	0.008
	<sup>154</sup> Gd	0.033	0.013	0.006
	<sup>152</sup> Sm	0.032	0.013	0.003
	<sup>150</sup> Nd	0.037	0.017	0.011

less rigid rotor

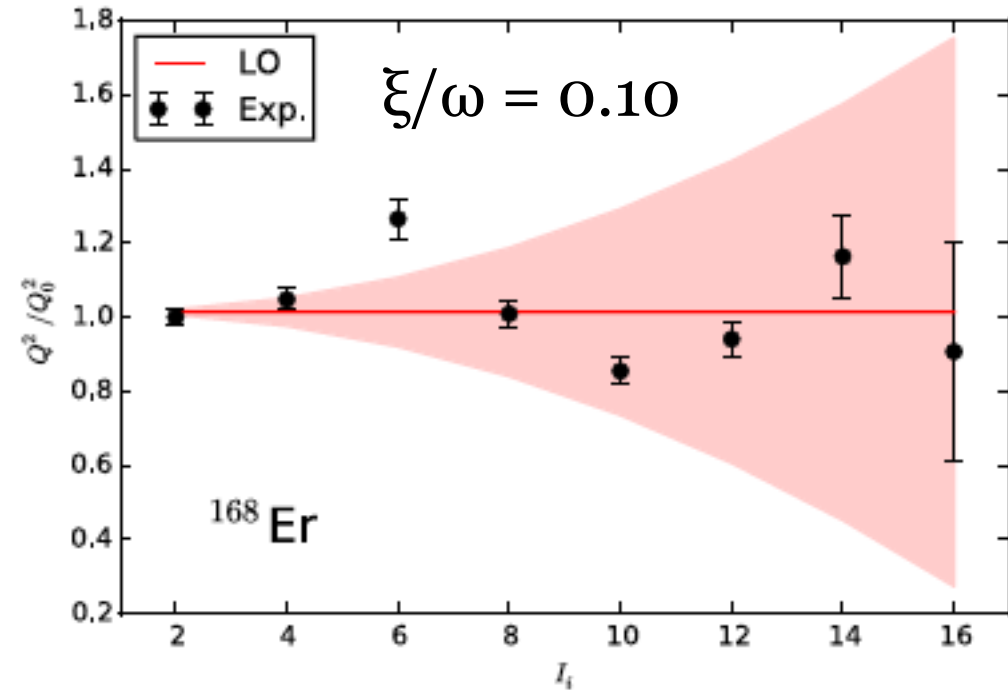
# EFT works well for a wide range of rotors

Bohr & Mottelson (1975):

“The accuracy of the present measurements of E2-matrix elements in the ground-state bands of even even nuclei is in most cases barely sufficient to detect deviations from the leading-order intensity relations.”



# EFT can not explain oscillatory patterns in supposedly “good” rotors $^{168}\text{Er}$ , $^{174}\text{Yb}$

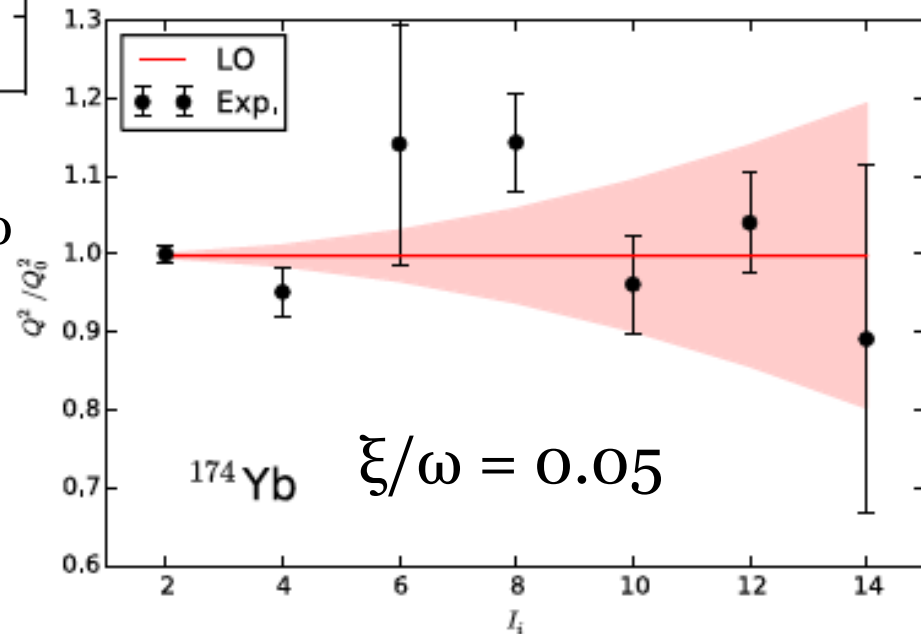


$^{168}\text{Er}$ :  $B(E2)$  for  $6^+ \rightarrow 4^+$  very difficult to understand.

$^{174}\text{Yb}$ :  $B(E2)$  for  $8^+ \rightarrow 6^+$  difficult to reconcile with  $4^+ \rightarrow 2^+$ .

Theoretical uncertainty estimates relevant.

Based on results for molecules, well-deformed nuclei, and transitional nuclei, EFT suggests that a few transitions in text-book rotors could merit re-measurement.



# EFT and weak interband transitions ( $^{154}\text{Sm}$ )

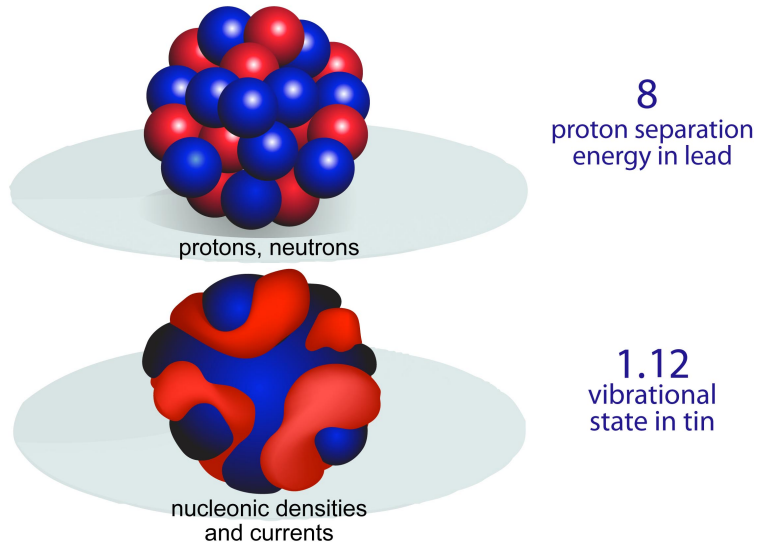
$i \rightarrow f$	$B(E2)_{\text{exp}}$	$B(E2)_{\text{ET}}$	$B(E2)_{\text{CBS}}$	$B(E2)_{\text{BH}}$
$2_g^+ \rightarrow 0_g^+$	0.863 (5)	0.863 <sup>a</sup>	0.853	0.863
$4_g^+ \rightarrow 2_g^+$	1.201 (29)	1.233 (9)	1.231	1.234
$6_g^+ \rightarrow 4_g^+$	1.417 (39)	1.358 (23)	1.378	1.355
$8_g^+ \rightarrow 6_g^+$	1.564 (83)	1.421 (43)	1.471	1.424
$2_\gamma^+ \rightarrow 0_g^+$	0.0093 (10)	0.0110 (28)		0.0492
$2_\gamma^+ \rightarrow 2_g^+$	0.0157 (15)	0.0157 <sup>a</sup>		0.0703
$2_\gamma^+ \rightarrow 4_g^+$	0.0018 (2)	0.0008 (2)		0.0050
$2_\beta^+ \rightarrow 0_g^+$	0.0016 (2)	0.0025 (6)	0.0024	0.0319
$2_\beta^+ \rightarrow 2_g^+$	0.0035 (4)	0.0035 <sup>a</sup>	0.0069	0.0456
$2_\beta^+ \rightarrow 4_g^+$	0.0065 (7)	0.0063 (16)	0.0348	0.0821

<sup>a</sup>Values employed to adjust the LECs of the effective theory.

In-band transitions [in  $e^2b^2$ ] are LO, inter-band transitions are NLO. Effective theory is more complicated than Bohr Hamiltonian both in Hamiltonian and E2 transition operator. EFT correctly predicts strengths of inter-band transitions with natural LECs.

[E. A. Coello Pérez and TP, Phys. Rev. C 92, 014323 (2015)]

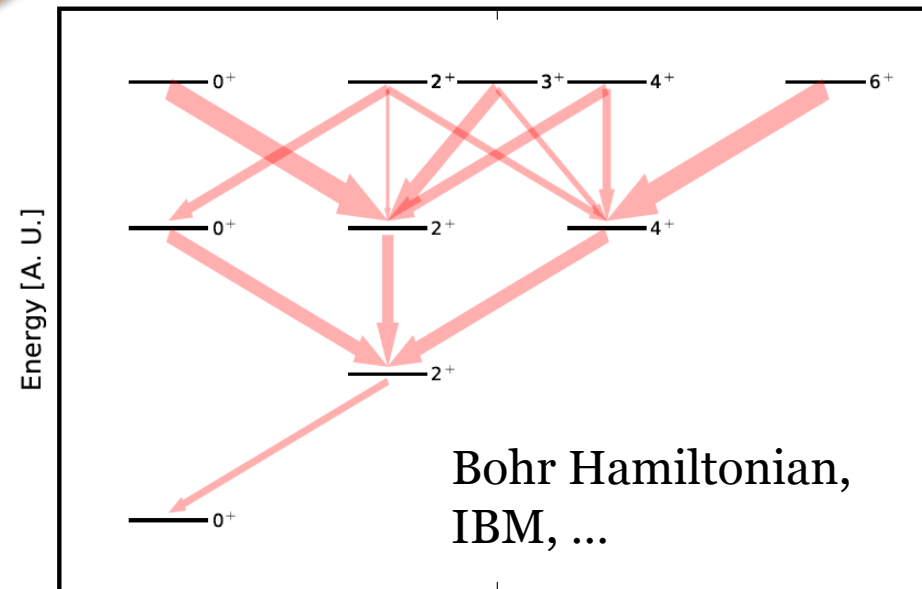
# EFT for nuclear vibrations



## EFT for nuclear vibrations [Coello Pérez & TP 2015, 2016]

While spectra of certain nuclei appear to be harmonic,  $B(E2)$  transitions do not.

Garrett & Wood (2010): “Where are the quadrupole vibrations in atomic nuclei?”

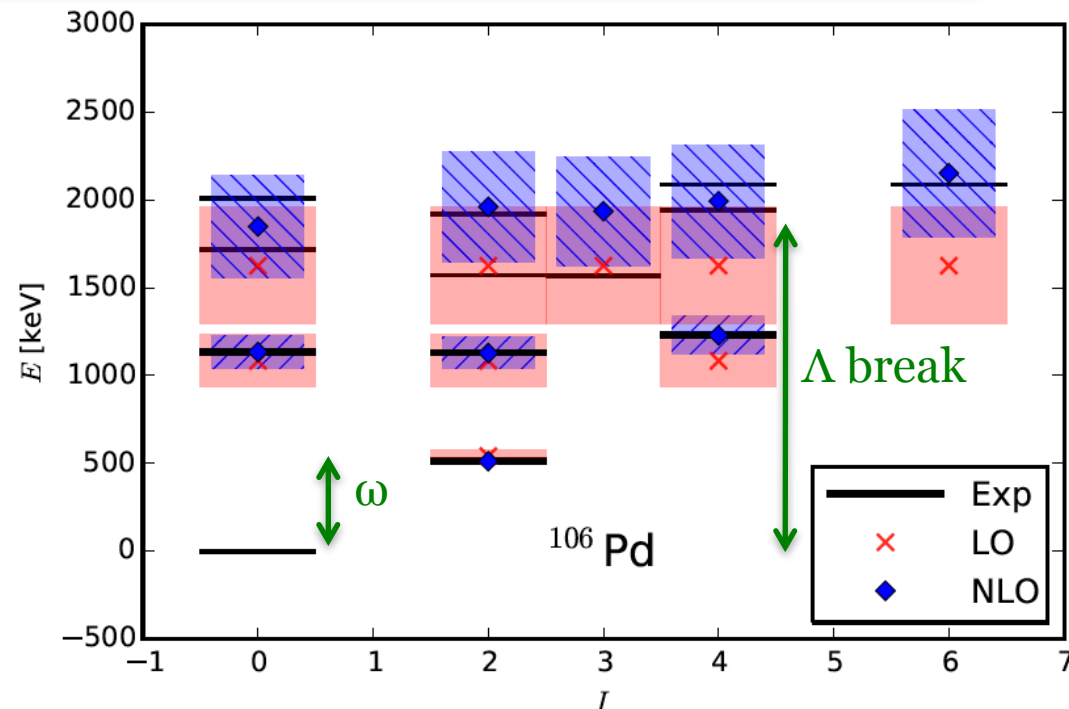


Spectrum and  $B(E2)$  transitions of the *harmonic* quadrupole oscillator

# EFT for nuclear vibrations

EFT ingredients:

- quadrupole degrees of freedom
- breakdown scale around three-phonon levels
- “small” expansion parameter: ratio of vibrational energy to breakdown scale:  $\omega/\Lambda \approx 1/3$



- Uncertainties show 68% DOB intervals from truncating higher EFT orders [Cacciari & Houdeau (2011); Bagnaschi et al (2015); Furnstahl, Klco, Phillips & Wesolowski (2015)]
  - Expand observables according to power counting
  - Employ “naturalness” assumptions as log-normal priors in Bayes’ theorem
  - Compute distribution function of uncertainties due to EFT truncation
  - Compute degree-of-believe (DOB) intervals.

# Hamiltonian

LO Hamiltonian  $\hat{H}_{\text{LO}} = \omega \hat{N}$

NLO correction  $\hat{h}_{\text{NLO}} = g_N \hat{N}^2 + g_v \hat{\Lambda}^2 + g_I \hat{I}^2$

with  $\hat{N}^2 = (d^\dagger \cdot \tilde{d})^2,$

$$\hat{\Lambda}^2 = -(d^\dagger \cdot d^\dagger)(\tilde{d} \cdot \tilde{d}) + \hat{N}^2 - 3\hat{N},$$

$$\hat{I}^2 = 10(d^\dagger \otimes \tilde{d})^{(1)} \cdot (d^\dagger \otimes \tilde{d})^{(1)}.$$

“Small” expansion parameter  $\varepsilon \equiv (N\omega/\Lambda)$



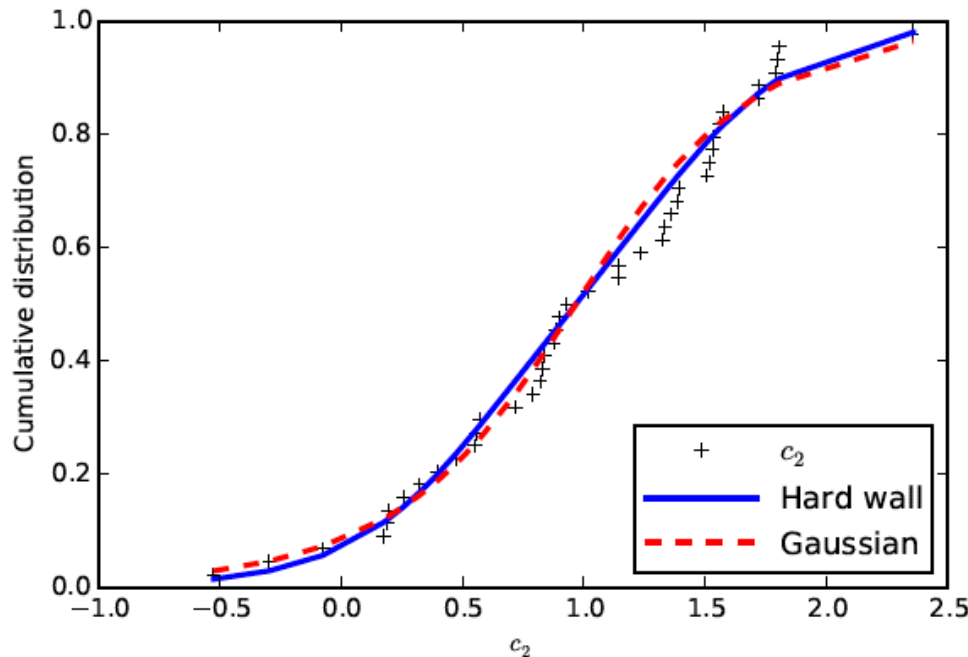
# Uncertainty quantification

$$E_{\text{NLO}} = \omega N + g_\omega N + g_N N^2 + g_v v(v+3) + g_I I(I+1)$$

$$c_2 \equiv c_2(N, v, I)$$

$$= \frac{g_\omega N + g_N N^2 + g_v v(v+3) + g_I I(I+1)}{\varepsilon^2 \omega}$$

$$X = X_0 \sum_{n=0}^{\infty} c_n \varepsilon^n$$



Linear combinations of LECs enter observables. LECs are random, but with EFT expectations, i.e. log-normal distributed. Making assumptions about these distributions then allows one to quantify uncertainties. The assumptions can be tested.

$$\Delta_k^{(M)} = \sum_{n=k+1}^{k+M} c_n \varepsilon^n$$

$$p_M(\Delta|c_0, \dots, c_k) = \frac{\int_0^\infty dc \text{pr}(c) p_M(\Delta|c) \prod_{m=0}^k \text{pr}(c_m|c)}{\int_0^\infty dc \text{pr}(c) \prod_{m=0}^k \text{pr}(c_m|c)}$$

# B(E2) transitions in vibrational nuclei

Nucleus	$2_1^+ \rightarrow 0_1^+$	EFT	$0_2^+ \rightarrow 2_1^+$	$2_2^+ \rightarrow 2_1^+$	$4_1^+ \rightarrow 2_1^+$	EFT
$^{62}\text{Ni}$	12.1(4)	11(4)	42(23)	14.9(42)	21(6)	21(7)
$^{98}\text{Ru}$	31(1)	28(9)		47(5)	57.6(40)	56(19)
$^{100}\text{Ru}$	35.6(4)	24(8)	35(5)	30.9(4)	51(4)	47(16)
$^{106}\text{Pd}$	44.3(15)	30(10)	35(8)	44(4)	76(11)	61(20)
$^{108}\text{Pd}$	49.5(13)	37(12)	52(5)	71(5)	73(8)	74(25)
$^{110}\text{Cd}$	27.0(8)	21(7)		30(5)	42(9)	42(14)
$^{112}\text{Cd}$	30.2(3)	23(8)	51(14)	15(3)	61(6)	46(15)
$^{114}\text{Cd}$	31.1(19)	22(7)	27.4(17)	22(6)	62(4)	43(15)
$^{120}\text{Te}$	31 (6)	31(10)				62(21)
$^{122}\text{Te}$	36.9(3)	41(14)		100(30)		81(27)

B(E2) transition strengths consistent with EFT expectations within the theoretical uncertainties (68% DOB intervals).

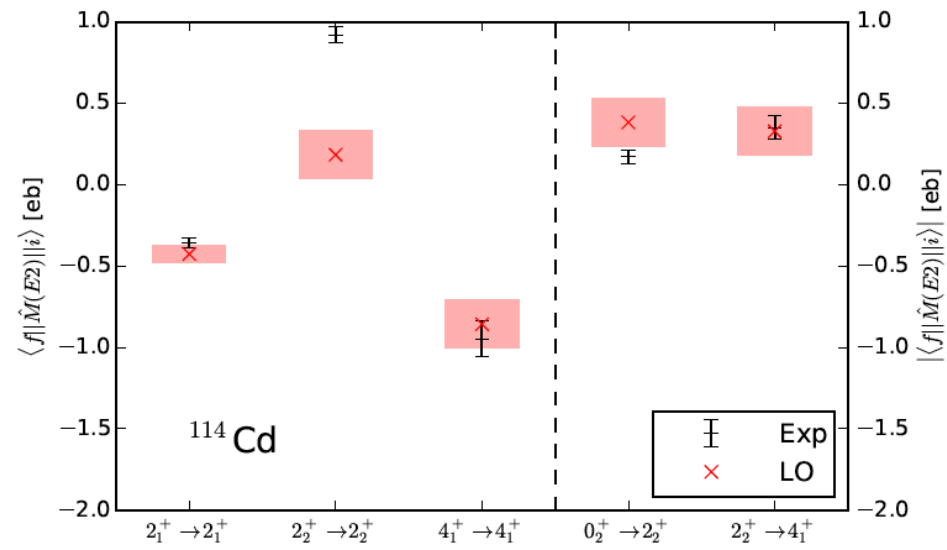
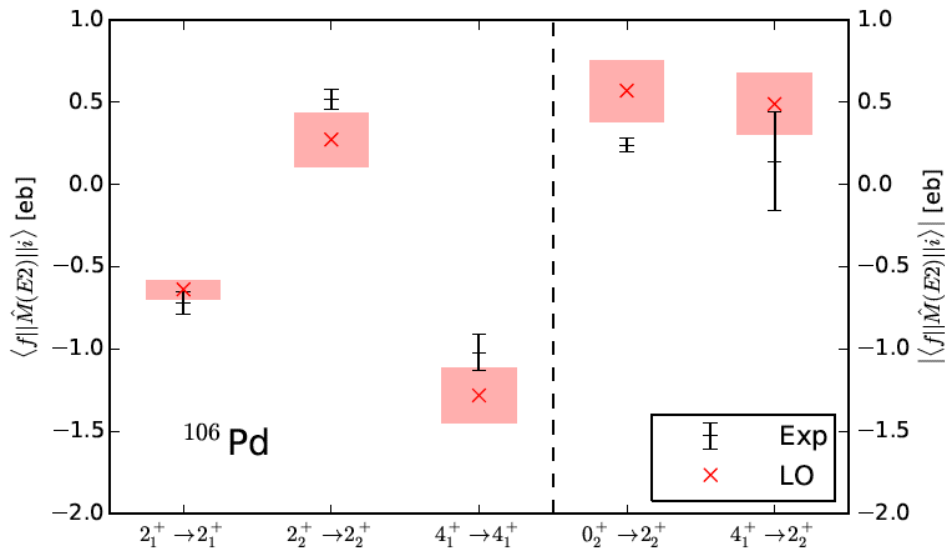
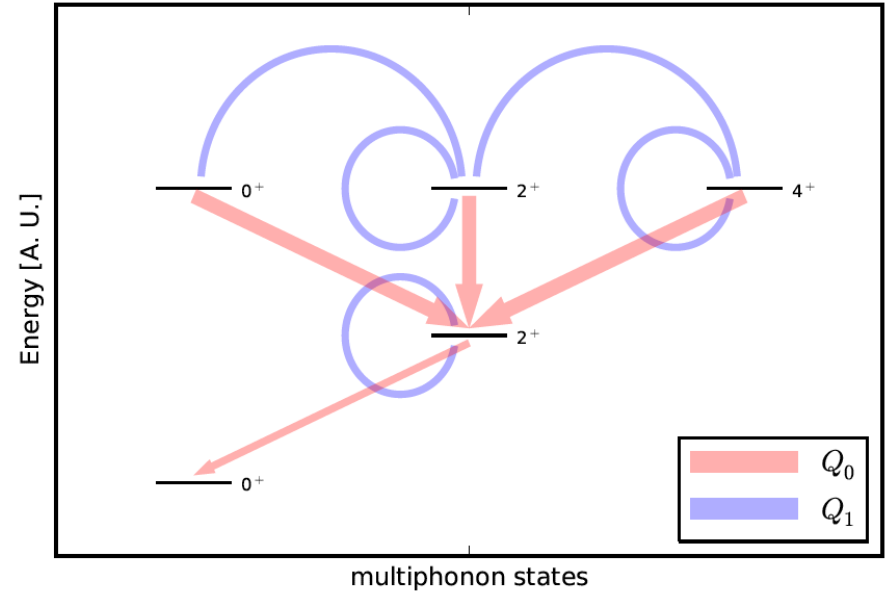
# EFT result: sizeable quadrupole matrix elements are natural in size

In the EFT, the quadrupole operator is also expanded:

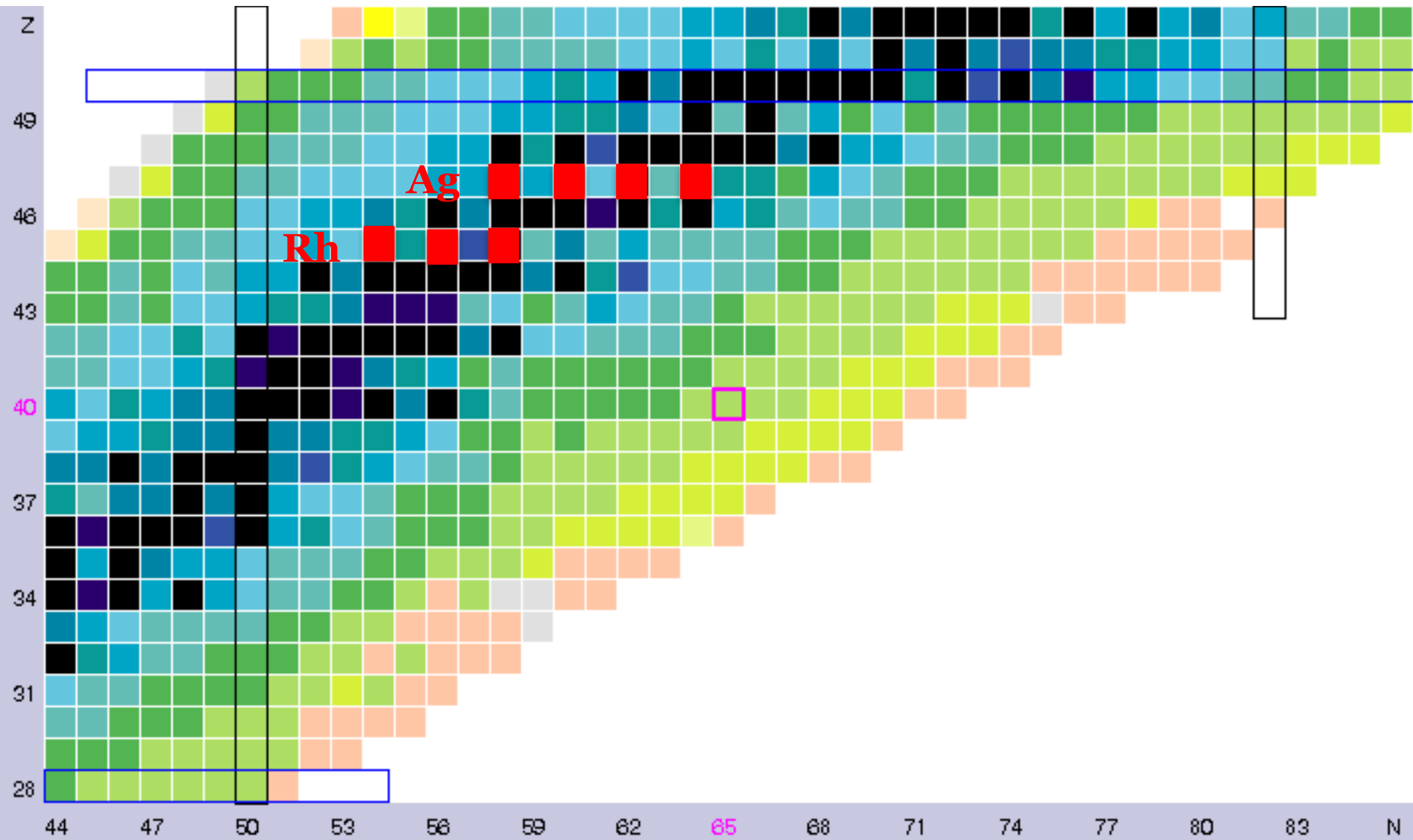
$$\hat{Q}_\mu = Q_0 (d_\mu^\dagger + \tilde{d}_\mu) + Q_1 (d^\dagger \times d^\dagger + \tilde{d} \times \tilde{d} + 2d^\dagger \times \tilde{d})_\mu^{(2)}$$

Subleading corrections are sizable:

$$Q_1 \sim \left(\frac{\omega}{\Lambda}\right)^{1/2} Q_0$$



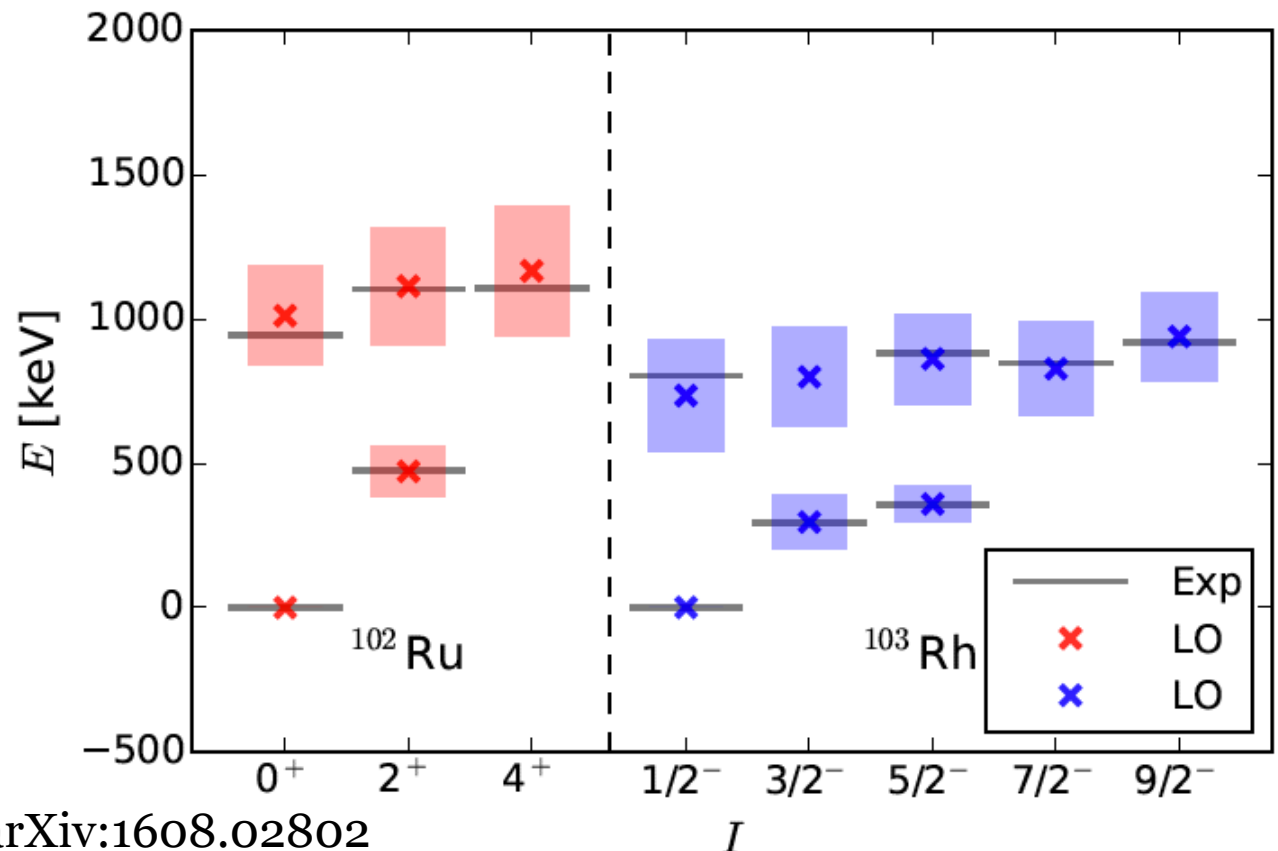
$^{99,101,103}\text{Rh}$  and  $^{105,107,109,111}\text{Ag}$  as a proton coupled to  $^{98,100,102}\text{Ru}$  and  $^{104,106,108,110}\text{Pd}$ , respectively, or  $^{107,109,111}\text{Ag}$  as a proton-hole in  $^{108,110,112}\text{Cd}$



# Fermion coupled to vibrating nucleus

Approach: Follow Halo EFT [Bertulani, Hammer, van Kolck (2002); Higa, Hammer, van Kolck (2008); Hammer & Phillips (2011); Ryberg et al. (2014)], and couple a fermion to describe odd-mass neighbors; particle-vibrator models [de Shalit (1961); Iachello & Scholten (1981); Vervier (1982);...]

Two new LECs enter at LO



# Static E2 moments (in eb)

Nucleus	$I_i^\pi$	$Q_{\text{exp}}$	$Q_{\text{EFT}}$	Nucleus	$I_i^\pi$	$Q_{\text{exp}}$	$Q_{\text{EFT}}$
$^{102}\text{Ru}$	$2_1^+$	-0.63(3)	-0.41(6)	$^{108}\text{Pd}$	$2_1^+$	-0.56(3)	-0.57(7)
	$2_2^+$		0.18(18)		$2_2^+$	0.73(9)	0.24(20)
	$4_1^+$		-0.82(14)		$4_1^+$	-0.78(11)	-1.14(17)
$^{103}\text{Rh}$	$\frac{3}{2}_1^-$	-0.3(2)	-0.29(7)	$^{109}\text{Ag}$	$\frac{3}{2}_1^-$	-0.7(3)	-0.40(8)
	$\frac{5}{2}_1^-$	-0.4(2)	-0.41(6)		$\frac{5}{2}_1^-$	-0.3(3)	-0.57(6)
$^{106}\text{Pd}$	$2_1^+$	-0.54(4)	-0.50(7)	$^{110}\text{Cd}$	$2_1^+$	-0.39(3)	-0.57(7)
	$2_2^+$	0.39(6)	0.21(20)		$2_2^+$		0.24(17)
	$4_1^+$	-0.79(11)	-1.00(17)		$4_1^+$		-1.12(14)
$^{107}\text{Ag}$	$\frac{3}{2}_1^-$		-0.35(8)	$^{109}\text{Ag}$	$\frac{3}{2}_1^-$	-0.7(3)	-0.39(6)
	$\frac{5}{2}_1^-$		-0.50(7)		$\frac{5}{2}_1^-$	-0.3(3)	-0.56(6)

Single LEC fit to all data with EFT weighting.

# Magnetic moments: Relations between even-even and even-odd nuclei

Nucleus	$I_i^\pi$	$\mu_{\text{exp}}(I_i^\pi)$	$\mu_{\text{EFT}}(I_i^\pi)$	Nucleus	$I_i^\pi$	$\mu_{\text{exp}}(I_i^\pi)$	$\mu_{\text{EFT}}(I_i^\pi)$
$^{102}\text{Ru}$	$2_1^+$	$0.85(3)^*$	$0.85(5)$	$^{106}\text{Pd}$	$2_1^+$	$0.79(2)^*$	$0.79(5)$
	$2_2^+$		$0.85(10)$		$2_2^+$	$0.71(10)$	$0.79(10)$
	$4_1^+$		$1.70(8)$		$4_1^+$	$1.8(4)$	$1.58(8)$
$^{103}\text{Rh}$	$\frac{1}{2}_1$	$-0.088^*$	$-0.088$	$^{107}\text{Ag}$	$\frac{1}{2}_1$	$-0.11^*$	$-0.11$
	$\frac{3}{2}_1$	$0.77(7)$	$0.81(5)$		$\frac{3}{2}_1$	$0.98(9)$	$0.78(5)$
	$\frac{5}{2}_1$	$1.08(4)$	$0.76(5)$		$\frac{5}{2}_1$	$1.02(9)$	$0.68(4)$
	$\frac{7}{2}_1$	$2.0(6)$	$1.7(1)$		$\frac{7}{2}_1$		$1.6(1)$
	$\frac{9}{2}_1$	$2.8(5)$	$1.6(1)$		$\frac{9}{2}_1$		$1.5(1)$

At LO, one new LEC enters to describe the magnetic moments in the odd-mass neighbor

# Summary

- UV and IR cutoffs of HO basis for single particle, NCSM, and many-body product spaces
  - Lüscher-like formulas for energies, radii, quadrupole moments, transitions, radiative capture reactions
- Construct EFT directly in HO basis
  - Based on discrete momentum eigenstates
  - DVR in momentum space; IR corrections of ME
- EFT for nuclear rotations
  - nonlinear realization of emergent symmetry breaking
  - correct description of weak inter-band transitions
- EFT for nuclear vibrations
  - Picture of anharmonic vibrations consistent with data within uncertainties