

Electroweak properties of light nuclei

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PRC78(2008)064002 - PRC80(2009)034004 - PRC84(2011)024001 - PRC87(2013)014006 - PRC87(2013)035503 - PRL111(2013)062502 -

PRC90(2014)024321 - JPhysG41(2014)123002 - PRC(2016)015501

Key Questions

3. **Nuclei** continue to be used as **laboratories for precision tests of the standard model** of particle physics and in searches for physics that may exist beyond the standard model. ...
 - What are the present **uncertainties** in **nuclear matrix elements** relevant for dark matter searches and **double beta decay**, and how can they be improved?
 - How do **nuclei** interact with **neutrinos** in the GeV energy regime and how can calculations of these interaction **cross sections** be improved?

Question # 1.

- What are the present **uncertainties** in **nuclear matrix elements** relevant for dark matter searches and **double beta decay**, and how can they be improved?
 - * EM transitions as test case
 - * Single beta-decay in $A \leq 10$ nuclei: g_A quenching and the role of correlations *Preliminary results*
 - * Ab-initio calculations of 2β -decay m.e.'s in light nuclei and the role of correlations *Preliminary results*

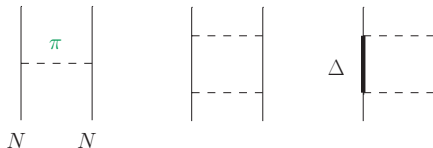
The Basic Model: Nuclear Potentials

- ▶ The nucleus is a system made of A non-relativistic interacting nucleons, its energy is given by

$$H = T + V = \sum_{i=1}^A t_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

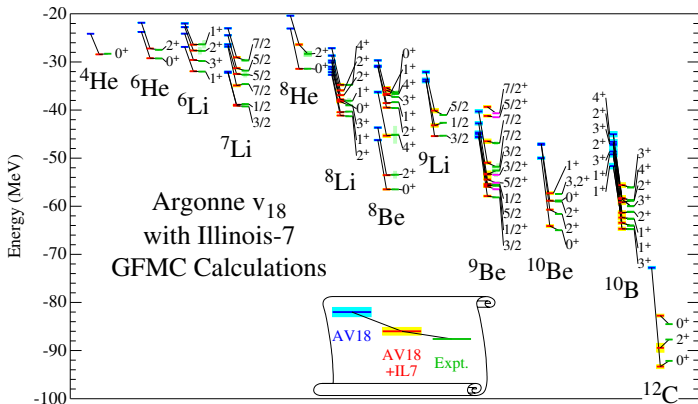
where v_{ij} and V_{ijk} are 2- and 3-nucleon interaction operators

- ▶ Realistic v_{ij} and V_{ijk} interactions are based on EXPT data fitting and fitted parameters subsume underlying QCD
- ▶ Realistic potentials at large inter-particle distances are described in terms of one-pion-exchange, range $\sim 1/m_\pi$. Other mechanisms are, *e.g.*, two-pion exchange, range $\sim 1/2m_\pi$; Δ -excitations ...



- ▶ Potentials utilized in these sets of calculations to generate nuclear wave functions $|\Psi_i\rangle$ solving $H|\Psi_i\rangle = E_i|\Psi_i\rangle$ are:
[AV18+UIX], [AV18+IL7], [NN(N3LO)+3N(N2LO)]

AV18+IL7



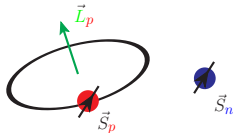
Carlson *et al.* - [Rev.Mod.Phys.87\(2015\)1067](https://doi.org/10.1103/RevModPhys.87.1067)

The Basic Model: Nuclear Electromagnetic Currents - One-body component

- ▶ Current and charge operators describe the interaction of nuclei with external fields. They are expanded as a sum of 1-, 2-, ... nucleon operators:

$$\rho = \sum_{i=1}^A \rho_i + \sum_{i<j} \rho_{ij} + \dots, \quad \mathbf{j} = \sum_{i=1}^A \mathbf{j}_i + \sum_{i<j} \mathbf{j}_{ij} + \dots$$

- ▶ In Impulse Approximation **IA** nuclear EM currents are expressed in terms of those associated with individual protons and nucleons, *i.e.*, ρ_i and \mathbf{j}_i

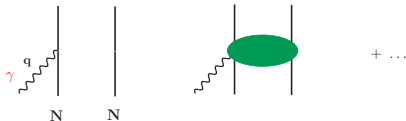


- ▶ IA picture is however incomplete; Historical evidence is the 10% underestimate of the np radiative capture ‘fixed’ by incorporating corrections from two-body meson-exchange EM currents - Riska&Brown 1972

The Basic Model: Nuclear Electromagnetic Currents

- ▶ Current and charge operators describe the interaction of nuclei with external fields. They are expanded as a sum of 1-, 2-, ... nucleon operators:

$$\rho = \sum_{i=1}^A \rho_i + \sum_{i<j} \rho_{ij} + \dots, \quad \mathbf{j} = \sum_{i=1}^A \mathbf{j}_i + \sum_{i<j} \mathbf{j}_{ij} + \dots$$



- ▶ Longitudinal EM current operator \mathbf{j} linked to the nuclear Hamiltonian via continuity eq. (\mathbf{q} momentum carried by the external EM probe γ)

$$\mathbf{q} \cdot \mathbf{j} = [H, \rho] = [t_i + \mathbf{v}_{ij} + V_{ijk}, \rho]$$

- * Meson-exchange currents **MEC** follow once meson-exchange mechanisms are implemented to describe nuclear forces - Villars&Miyazawa 40ies

These days we have:

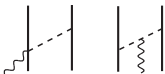
- ▶ Highly sophisticated MEC projected out realistic potentials
- ▶ EM currents derived from χ EFTs

EM current up to $n = 1$ (or up to N3LO)

LO : $j^{(-2)} \sim eQ^{-2}$



NLO : $j^{(-1)} \sim eQ^{-1}$



N²LO : $j^{(0)} \sim eQ^0$



* Two-body charge operators enter at N3LO and vanish in the static limit *

- ▶ LO = IA
- N2LO = IA(relativistic- correction)
- ▶ NLO is purely isovector
- ▶ Strong contact LECs fixed from fits to np phases shifts—PRC68, 041001 (2003)
- ▶ No three-body EM currents at this order !

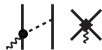
▶ 5 unknown EM LECs enter the N3LO contact and tree-level currents:

- ▶ 2 isovector LECs entering the tree-level current are fixed by Δ -saturation
- ▶ remaining 3 LECs fixed to reproduce, *e.g.*, $A = 2$ and 3 magnetic moments

N³LO: $j^{(1)} \sim eQ$



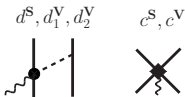
unknown LEC's →



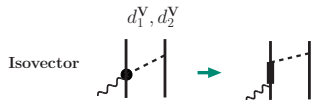
PRC78(2008)064002, PRC80(2009)034004, PRC84(2011)024001

* analogue expansion exists for the Axial nuclear current - Baroni et al. PRC93 (2016)015501 *

χ EFT EM currents at N3LO: fixing the EM LECs



Five LECs: d^S , d_1^V , and d_2^V could be determined by pion photo-production data on the nucleon



d_2^V and d_1^V are known assuming Δ -resonance saturation

Left with 3 LECs: Fixed in the $A = 2 - 3$ nucleons' sector

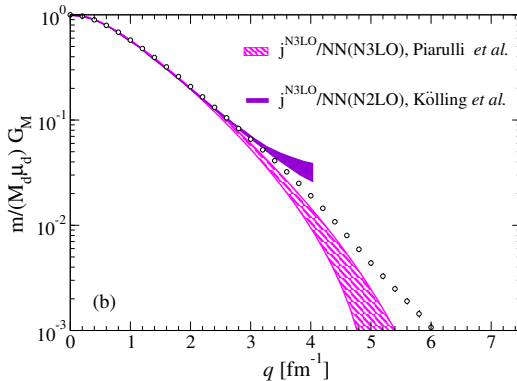
▶ Isoscalar sector:

* d^S and c^S from EXPT μ_d and $\mu_S(^3\text{H}/^3\text{He})$

▶ Isovector sector:

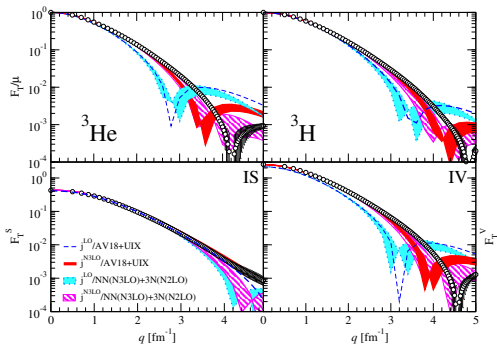
* c^V from EXPT $\mu_V(^3\text{H}/^3\text{He})$ m.m. ← our choice

Predictions with χ EFT EM Currents for the Deuteron Magnetic f.f.



PRC86(2012)047001 & PRC87(2013)014006

Predictions with χ EFT EM Currents for ${}^3\text{He}$ and ${}^3\text{H}$ Magnetic f.f.'s



LO/N3LO with AV18+UIX – LO/N3LO with χ -potentials NN(N3LO)+3N(N2LO)

- ▶ ${}^3\text{He}/{}^3\text{H}$ m.m.'s used to fix EM LECs; $\sim 15\%$ correction from two-body currents
- ▶ Two-body corrections crucial to improve agreement with EXPT data

Λ	${}^3\text{He} \langle r \rangle_{\text{EXP}} = 1.976 \pm 0.047 \text{ fm}$		${}^3\text{H} \langle r \rangle_{\text{EXP}} = 1.840 \pm 0.181 \text{ fm}$	
	500	600	500	600
LO	2.098 (2.092)	2.090 (2.092)	1.924 (1.918)	1.914 (1.918)
N3LO	1.927 (1.915)	1.913 (1.924)	1.808 (1.792)	1.794 (1.797)

PRC87(2013)014006

Calculations with EM Currents from χ EFT with π 's and N's

- ▶ Park, Min, and Rho *et al.* (1996)

applications to A=2–4 systems by Song, Lazauskas, Park *et al.* (2009-2011)
within the hybrid approach

.....

* Based on EM χ EFT currents from [NPA596\(1996\)515](#)

- ▶ Meissner and Walzl (2001);

Kölling, Epelbaum, Krebs, and Meissner (2009–2011)

applications to:

d and ^3He photodisintegration by Rozpedzik *et al.* (2011); e -scattering (2014);

d magnetic f.f. by Kölling, Epelbaum, Phillips (2012);

radiative $N - d$ capture by Skibinski *et al.* (2014)

.....

* Based on EM χ EFT currents from [PRC80\(2009\)045502](#) & [PRC84\(2011\)054008](#) and consistent χ EFT potentials from UT method

- ▶ Phillips (2003-2007)

applications to deuteron static properties and f.f.'s

.....

* Axial currents by *

Park *et al.* (1996), Baroni *et al.* (2016), Krebs *et al.* (2016), Klos *et al.* (2015) ...

Predictions with χ EFT EM currents for $A = 6-10$ systems:
Variational Monte Carlo (VMC)

Minimize expectation value of $H = T + AV18 + IL7$

$$E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \geq E_0$$

using trial function

$$|\Psi_V\rangle = \left[\mathcal{S} \prod_{i<j} (1 + U_{ij} + \sum_{k \neq i,j} U_{ijk}) \right] \left[\prod_{i<j} f_c(r_{ij}) \right] |\Phi_A(JMTT_3)\rangle$$

- ▶ single-particle $\Phi_A(JMTT_3)$ is fully antisymmetric and translationally invariant
- ▶ central pair correlations $f_c(r)$ keep nucleons at favorable pair separation
- ▶ pair correlation operators U_{ij} reflect influence of v_{ij} (AV18)
- ▶ triple correlation operator U_{ijk} added when V_{ijk} (IL7) is present
- ▶ Ψ_V are spin-isospin vectors in $3A$ dimensions with $\sim 2^A \binom{A}{Z}$ components

Lomnitz-Adler, Pandharipande, and Smith NPA361(1981)399
Wiringa, PRC43(1991)1585

Green's function Monte Carlo

A trial w.f. Ψ_V is obtained by minimizing the $H = T + \text{AV18} + \text{IL7}$ expectation value

$$E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \geq E_0$$

Ψ_V is further improved it by “filtering” out the remaining excited state contamination:

$$\Psi(\tau) = \exp[-(H - E_0)\tau] \Psi_V = \sum_n \exp[-(E_n - E_0)\tau] a_n \psi_n$$

$$\Psi(\tau \rightarrow \infty) = a_0 \psi_0$$

Evaluation of $\Psi(\tau)$ is done stochastically (Monte Carlo method) in small time steps $\Delta\tau$ using a Green's function formulation.

In practice, we evaluate a “mixed” estimates

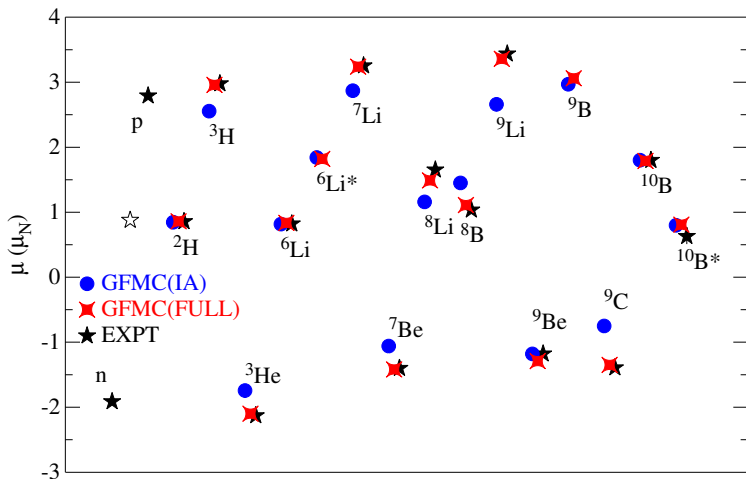
$$\langle O(\tau) \rangle = \frac{f \langle \Psi(\tau) | O | \Psi(\tau) \rangle_i}{\langle \Psi(\tau) | \Psi(\tau) \rangle} \approx \langle O(\tau) \rangle_{\text{Mixed}}^i + \langle O(\tau) \rangle_{\text{Mixed}}^f - \langle O \rangle_V$$

$$\langle O(\tau) \rangle_{\text{Mixed}}^i = \frac{f \langle \Psi_V | O | \Psi(\tau) \rangle_i}{f \langle \Psi_V | \Psi(\tau) \rangle_i} ; \quad \langle O(\tau) \rangle_{\text{Mixed}}^f = \frac{f \langle \Psi(\tau) | O | \Psi_V \rangle_i}{f \langle \Psi(\tau) | \Psi_V \rangle_i}$$

[Wiringa *et al.* PRC51(1995)38 + Piper *et al.* PRC64(2001)014001]

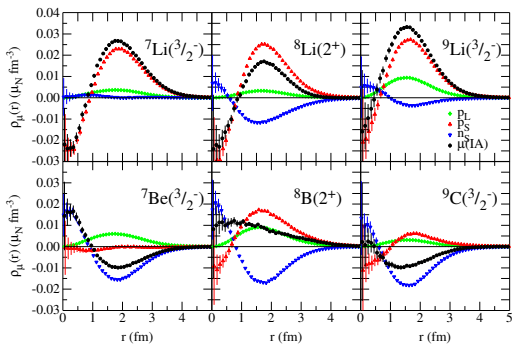
Magnetic Moments in $A \leq 10$ Nuclei

Predictions for $A > 3$ nuclei



- ▶ $\mu(\text{IA}) = \mu_N \sum_i [(L_i + g_p S_i)(1 + \tau_{i,z})/2 + g_n S_i(1 - \tau_{i,z})/2]$
- ▶ GFMC calculations based on $H = T + \text{AV18} + \text{IL7}$

One-body magnetic densities

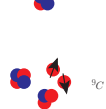
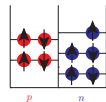
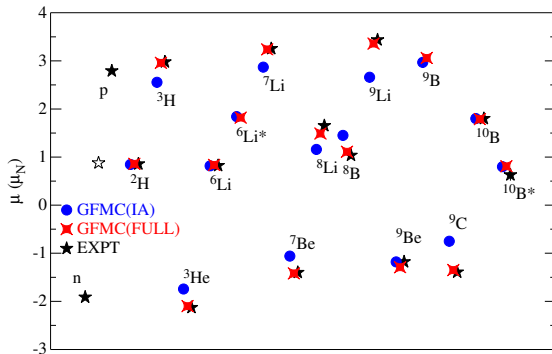


► IA magnetic moment operator

$$\mu(\text{IA}) = \mu_N \sum_i [(L_i + g_p S_i)(1 + \tau_{i,z})/2 + g_n S_i(1 - \tau_{i,z})/2]$$

Magnetic Moments in $A \leq 10$ Nuclei - bis

Predictions for $A > 3$ nuclei

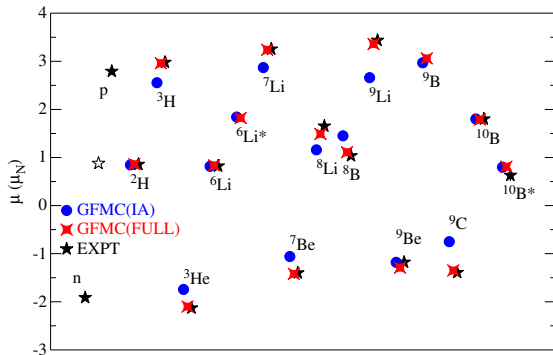


- ▶ $\mu_N(\text{IA}) = \sum_i [(L_i + g_p S_i)(1 + \tau_{i,z})/2 + g_n S_i(1 - \tau_{i,z})/2]$
- ▶ ${}^9\text{C}$ (${}^9\text{Li}$) dominant spatial symmetry [s.s.] = [432] = $[\alpha, {}^3\text{He}({}^3\text{H}), pp(nn)] \rightarrow$ Large MEC
- ▶ ${}^9\text{Be}$ (${}^9\text{B}$) dominant spatial symmetry [s.s.] = [441] = $[\alpha, \alpha, n(p)]$

PRC87(2013)035503

Magnetic Moments in $A \leq 10$ Nuclei - ter

Predictions for $A > 3$ nuclei



EE *et al.* error algorithm
Epelbaum, Krebs, and
Meissner EPJA51(2015)53

$$\delta^{\text{N3LO}} = \max \left[Q^4 |\mu^{\text{LO}} - \mu^{\text{NLO}}|, \right. \\ \left. Q^2 |\mu^{\text{NLO}} - \mu^{\text{N2LO}}|, \right. \\ \left. Q^1 |\mu^{\text{N2LO}} - \mu^{\text{N3LO}}| \right]$$

$$Q = \max \left[\frac{m_\pi}{\Lambda}, \frac{p}{\Lambda} \right]$$

m.m.	THEO	EXP
${}^9\text{C}$	-1.35(4)(7)	-1.3914(5)
${}^9\text{Li}$	3.36(4)(8)	3.4391(6)

* 'N3LO- Δ ' corrections can be 'large' *

* SNPA and χ EFT currents qualitatively in agreement, χ EFT isoscalar currents provide better description exp data *

PRC87(2013)035503

EM Transitions in $A \leq 9$ Nuclei

- ▶ Two-body EM currents bring the theory in a better agreement with the EXP
- ▶ Significant correction in $A = 9$, $T = 3/2$ systems. Up to $\sim 40\%$ correction found in ${}^9\text{C}$ m.m.
- ▶ Major correction ($\sim 60 - 70\%$ of total MEC) is due to the one-pion-exchange currents at NLO – purely isovector

One M1 prediction: ${}^9\text{Li}(1/2 \rightarrow 3/2)^*$

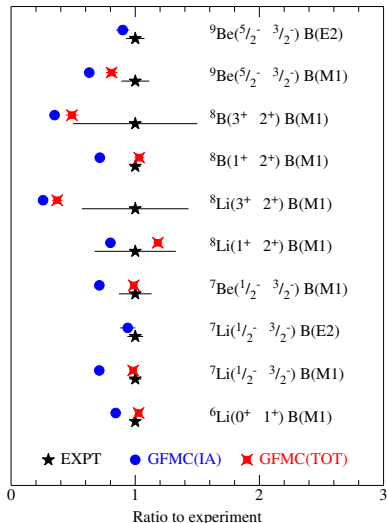
$$\Gamma(\text{IA}) = 0.59(2) \text{ eV}$$

$$\Gamma(\text{TOT}) = 0.79(3) \text{ eV}$$

+ a number of B(E2)s in IA

*Ricard-McCutchan *et al.* TRIUMF proposal 2014 - ongoing data analysis

PRC87(2013)035503

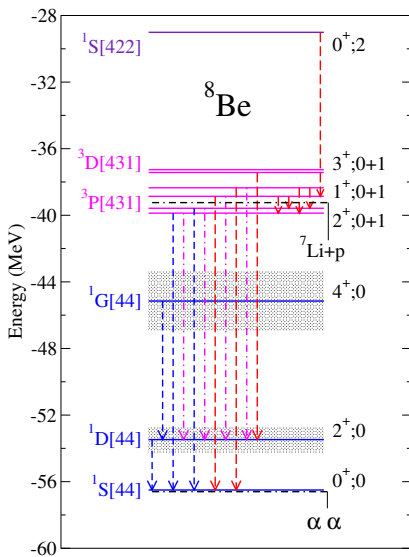


* M1 transtions in ${}^8\text{Be}$ reported in PRL111(2013)062502 & PRC90(2014)024321 MEC $\sim 20-30\%$ *

^8Be Energy Spectrum

- ▶ 2^+ and 4^+ broad states at ~ 3 MeV and ~ 11 MeV
- ▶ isospin-mixed states at ~ 16 MeV, ~ 17 MeV, ~ 19 MeV
- ▶ **M1** transitions
- ▶ **E2** transitions
- ▶ **E2 + M1** transitions

$J^\pi; T$	GFMC	Iso-mixed	Experiment
0^+	-56.3(1)		-56.50
2^+	+ 3.2(2)	+ 3.03(1)	+ 3.03(1)
4^+	+11.2(3)		+11.35(15)
$2^+; 0$	+16.8(2)	+16.746(3)	+16.626(3)
$2^+; 1$	+16.8(2)	+16.802(3)	+16.922(3)
$1^+; 1$	+17.5(2)	+17.67	+17.640(1)
$1^+; 0$	+18.0(2)	+18.12	+18.150(4)
$3^+; 1$	+19.4(2)	+19.10	+19.07(3)
$3^+; 0$	+19.9(2)	+19.21	+19.235(10)



PRL111(2013)062502 & PRC90(2014)024321

M1 transitions in ^8Be isospin-mixed states

- ▶ 2^+ , 1^+ , and 3^+ states are isospin mixed, with mixing coefficients $\alpha_J^2 + \beta_J^2 = 1$

$$\psi^a = \alpha_J \psi_{T=0} + \beta_J \psi_{T=1}$$

$$\psi^b = \beta_J \psi_{T=0} - \alpha_J \psi_{T=1}$$

- ▶ ($\alpha_2 \sim 0.77$, β_2) well known; while ($\alpha_1 \sim 0.21$, β_1) and ($\alpha_3 \sim 0.41$, β_3) involve multiple decay channels \rightarrow hard to extract them (Barker NP83(1966)418)

Using empirical mixings, we find:

- * MEC contribute $\sim 20\text{--}30\%$ of the total m.e.'s
- * m.e.'s are not stable against variations in the mixing angles

Questions on the '5th force' exp paper Krasznahorkay *et al.* PRL111(2016)042501:

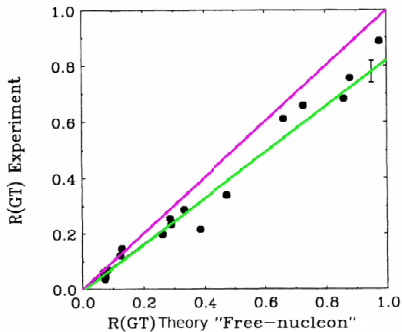
- * In the Ref. the ~ 17 (~ 18) MeV state is taken as purely isovector (isoscalar). Mixing angles do not affect M1 m.e.'s entering resonant capture
 $^7\text{Li} + p \rightarrow ^8\text{Be}^* \rightarrow ^8\text{Be} + e^+ + e^-$
- * Data are corrected by accounting for direct capture contaminations occurring via E1. Are higher multipoles negligible?

Beta-decay rates for $A \leq 10$ nuclei: M1 vs GT operators in IA

$$\text{M1}^{\text{IA}} = \sum_i (e_i \mathbf{L}_i + \mu_i \boldsymbol{\sigma}_i)$$

$$\text{GT}^{\text{IA}} = \sum_i \tau_{i,\pm} \boldsymbol{\sigma}_i$$

Theory vs Experiment: Quenching



$$3 \leq A \leq 18$$

Fig. from Chou *et al.* [PRC47\(1993\)163](#)

perfect agreement

theory > experiment

$$g_A^{\text{eff}} \simeq 0.70 g_A$$

Quenching origin: *i*) better w.f.'s and/or *ii*) many body currents are required

$\beta \pm - (J_i^\pi, T_i) \rightarrow (J_f^\pi, T_f)$	simple w.f.'s	IA	IA+MEC	Experiment
${}^3\text{H}(1/2^+, 1/2) \rightarrow {}^3\text{He}(1/2^+, 1/2)$	2.449	2.2765(1)		2.357(10)*
${}^6\text{He}(0^+, 1) \rightarrow {}^6\text{Li}(1+, 0)$	2.449	2.150	2.187	2.182*
${}^7\text{Be}(3/2^-, 1/2) \rightarrow {}^7\text{Li}(3/2^-, 1/2)$	2.582	2.292	2.395	2.290*
${}^{10}\text{C}(0^+, 1) \rightarrow {}^{10}\text{B}(1+, 0)$	2.449	2.024	2.076	1.862* - 2.344* (?)

Preliminary

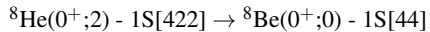
• in collaboration with **Bob Wiringa**, **Stefano Gandolfi**, **Rocco Schiavilla**, **Joe Carlson**

* data from TUNL compilations

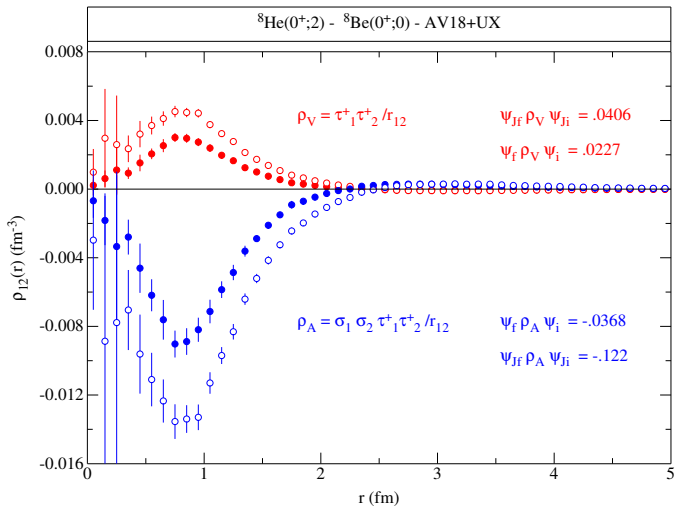
* data from Suzuki *et al.* [PRC67\(2003\)044302](#)

* data from Chou *et al.* [PRC47\(1993\)163](#)

Double beta-decay m.e.'s: a test case



Double beta-decay m.e.'s in ${}^8\text{He}(0^+;2) \rightarrow {}^8\text{Be}(0^+;0)$



Bob Wiringa et al. *Preliminary*

Summary # 1.

The microscopic description of nuclei successfully reproduces EXPT data provided that many-body effects in nuclear interactions and EM currents are accounted for.

J.Phys.G41(2014)123002 - S.Bacca&S.P.

- ▶ Two-body EM currents from χ EFT tested in $A \leq 10$ nuclei
- ▶ Two-body corrections can be sizable and improve on theory/EXPT agreement
- ▶ EM structure of $A = 2-3$ nuclei well reproduced with chiral charge and current operators for $q \lesssim 3m_\pi$
- ▶ $\sim 40\%$ two-body correction found in ${}^9\text{C}$'s m.m.
- ▶ $\sim 20-30\%$ corrections found in M1 transitions in low-lying states of ${}^8\text{Be}$
- ▶ Preliminary studies on beta decay m.e.'s in light nuclei indicate that correlations and MEC play an important role in explaining the origin of the g_A quenching
- ▶ Light nuclei are used in preliminary calculations on double-beta decay m.e.'s as test case to study the role of correlations and MEC

Outlook # 1.

The microscopic description of nuclei successfully reproduces EXPT data provided that many-body effects in nuclear interactions and EM currents are accounted for.

J.Phys.G41(2014)123002 - S.Bacca&S.P.

- * EM structure and dynamics of light nuclei
 - ▶ Charge and magnetic form factors of $A \leq 10$ systems
 - ▶ M1/E2 transitions in light nuclei
 - ▶ Radiative captures, photonuclear reactions . . .
 - ▶ Role of Δ -resonances in ‘MEC’ (EM current consistent with the chiral ‘ Δ -full’ NN potential developed by M. Piarulli et al. PRC91(2015)024003)
 - ▶ Fully consistent χ EFT calculations with ‘MEC’ for $A > 4$ (based on, *e.g.*, PRC91(2015)024003)

- * Electroweak structure and dynamics of light nuclei
 - ▶ Test axial currents (chiral and conventional) in light nuclei (A. Baroni et al. PRC93(2016)015501)

Question # 2.

- How do **nuclei** interact with **neutrinos** in the GeV energy regime and how can calculations of these interaction **cross sections** be improved?

i.e.

Towards a microscopic description of the ν -A inclusive cross section:
The Short-Time-Approximation

Inclusive (e, e') scattering in light nuclei

- ▶ ν/e inclusive xsecs are completely specified by the response functions
- ▶ 2 response functions for (e, e') inclusive xsec

$$R_{\alpha}(q, \omega) = \sum_f \delta(\omega + E_0 - E_f) |\langle f | O_{\alpha}(\mathbf{q}) | 0 \rangle|^2 \quad \alpha = L, T$$

Longitudinal response induced by $O_L = \rho$

Transverse response induced by $O_T = \mathbf{j}$

- ▶ Requires accurate knowledge of:
 - i) nuclear Hamiltonians and ii) associated EW currents, and
 - iii) final states $|f\rangle$ (which are hard to calculate)

* Sum Rules *

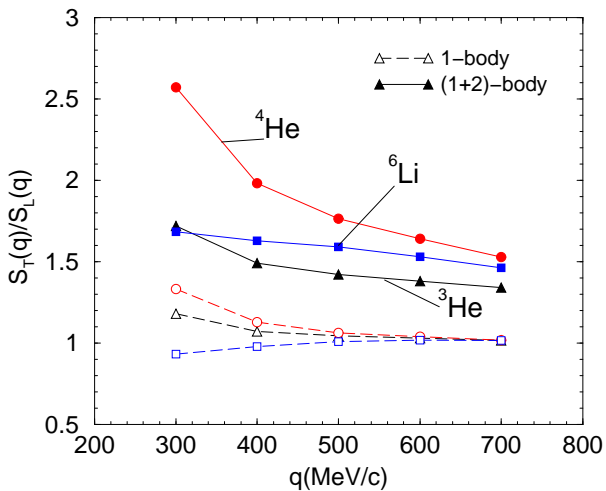
Exploit integral properties of the response functions + closure over the final states to avoid explicit calculation of the final states \rightarrow Sum Rules = ground state observables

$$S(q, \tau) = \int_0^{\infty} d\omega K(\tau, \omega) R_{\alpha}(q, \omega)$$

kernel $K(\tau, \omega)$, e.g., Laplace, Lorentz.

Coulomb Sum Rules = $S_{\alpha}(q) = \int_0^{\infty} d\omega R_{\alpha}(q, \omega) \propto \langle 0 | O_{\alpha}^{\dagger}(\mathbf{q}) O_{\alpha}(\mathbf{q}) | 0 \rangle$

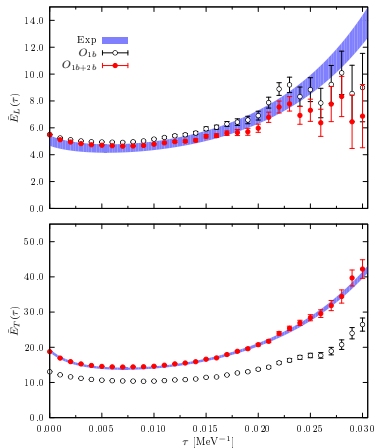
Sum Rules and MEC: Excess Transverse Strength



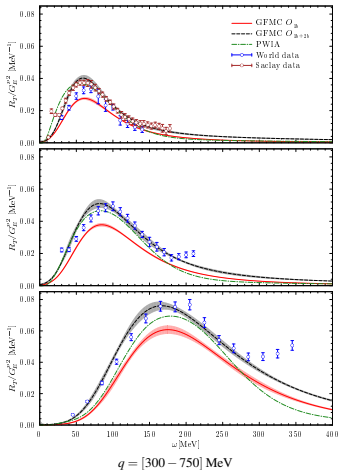
Carlson, Jourdan, Schiavilla, and Sick - [PRC65\(2002\)024002](#)

Euclidean Response Functions: Recent Developments on ^{12}C

$$\tilde{E}(q, \tau) = \int_{\omega_{th}}^{\infty} d\omega e^{-\tau(\omega-E_0)} R_\alpha(q, \omega) \propto \langle 0 | O_\alpha^\dagger(\mathbf{q}) e^{-\tau(H-E_0)} O_\alpha(\mathbf{q}) | 0 \rangle$$



~ 45 million core hours



~ 100 million core hours

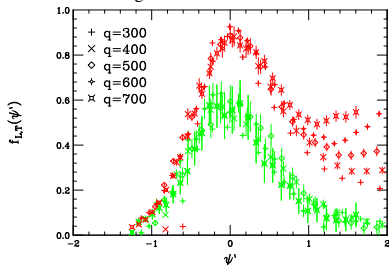
At present, an implementation for $A = 40$ is not feasible

Lovato, Gandolfi *et al.* - [PRC91\(2015\)062501 + arXiv:1605.00248](https://arxiv.org/abs/1605.00248)

Scaling properties of the Response Functions

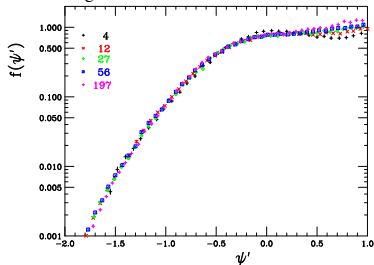
Inclusive xsec depends on a single (scaling) function of ω and q

Scaling of the 1st kind - ^4He



Carlson *et al.* - [PRC65\(2002\)024002](#)

Scaling of the 2nd kind - fixed kinematics



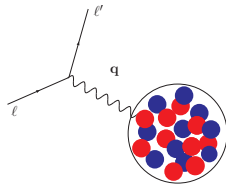
Donnelly and Sick - [PRC60\(1999\)065502](#)

- ▶ Scaling 1st kind: independence from the momentum transfer
 - ▶ Scaling 2nd kind: independence from A
- * Rely on observed scaling properties of inclusive xsecs, universal behavior of nucleon/A momentum distributions, and exhibited locality of nuclear properties to build approximate response functions for $A > 12$ nuclei *

Factorization I: PWIA

Plane Wave Impulse Approximation:

Response functions given by incoherent scattering off **single nucleons that propagate freely in the final state** (plane waves)

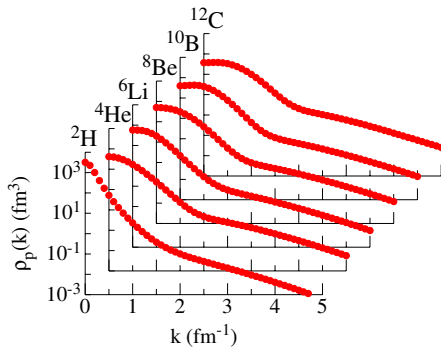


$$\begin{aligned}
 R_L^{\text{PWIA}}(q, \omega) &= \sum_f \delta(\omega + E_0 - E_f) \langle 0 | O_L^{(1)\dagger}(\mathbf{q}) | f \rangle \langle f | O_L^{(1)}(\mathbf{q}) | 0 \rangle \\
 &= \int dt \langle 0 | O_L^{(1)\dagger}(\mathbf{q}) e^{i(H-E_0)t} O_L^{(1)}(\mathbf{q}) | 0 \rangle e^{-i\omega t} \\
 O_L^{(1)}(\mathbf{q}) &= \sum_i e^{\frac{1+\tau_{z,i}}{2}} e^{i\mathbf{q}\cdot\mathbf{r}_i} \\
 E_f - E_0 &= \frac{(\mathbf{k} + \mathbf{q})^2}{2m_N} - \frac{\mathbf{k}^2}{2m_N}, \quad |f\rangle \sim e^{i(\mathbf{k} + \mathbf{q})\cdot\mathbf{r}}
 \end{aligned}$$

- ▶ PWIA L-Response in terms of the proton momentum distribution $n_p(\mathbf{k})$ as

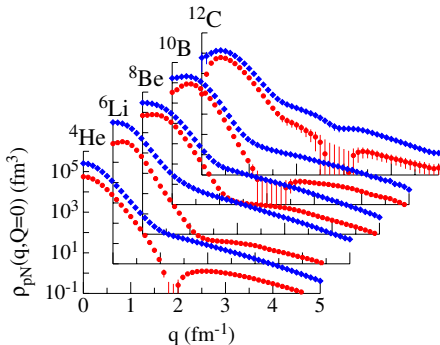
$$R_L^{\text{PWIA}}(q, \omega) = \int d\mathbf{k} n_p(\mathbf{k}) \delta\left(\omega - \frac{(\mathbf{k} + \mathbf{q})^2}{2m_N} + \frac{\mathbf{k}^2}{2m_N}\right)$$

Proton Momentum Distributions



Wiringa *et al.* - PRC89(2014)024305

Back-to-back np and pp Momentum Distributions



Wiringa *et al.* - PRC89(2014)024305

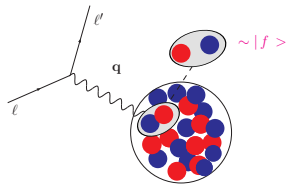
Nuclear properties are strongly affected by correlations!

Triple coincidence reactions $A(e, e' np \text{ or } pp)A - 2$ measurements @ JLab on ^{12}C indicate that 20% of the nucleons act in correlated pairs, and that at high values of relative momenta (400 – 500 MeV), $\sim 90\%$ of the pairs are in the form of np pairs and $\sim 5\%$ in pp pairs

Factorization II: Short-Time Approximation - STA

In STA:

Response functions are given by the scattering off **pairs of fully interacting nucleons** that propagate (for a short time) into a correlated pair of nucleons in the final state



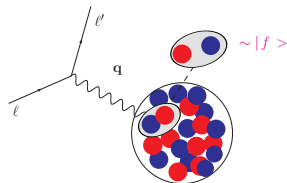
$$\begin{aligned}
 R_{\alpha}^{40}(q, \omega) &= \sum_f \delta(\omega + E_0 - E_f) \langle 0 | O_{\alpha}^{\dagger}(\mathbf{q}) | f \rangle \langle f | O_{\alpha}(\mathbf{q}) | 0 \rangle \\
 &= \int dt \langle 0 | O_{\alpha}^{\dagger}(\mathbf{q}) e^{i(H-E_0)t} O_{\alpha}(\mathbf{q}) | 0 \rangle e^{-i\omega t} \\
 O_{\alpha}(\mathbf{q}) &= O_{\alpha}^{(1)}(\mathbf{q}) + O_{\alpha}^{(2)}(\mathbf{q}) \\
 |f\rangle &\sim |\psi_{p,P,J,M,L,S,T,M_T}(r, R)\rangle
 \end{aligned}$$

- ▶ We keep $A - 2$ nucleons still, while the final states of the interacting two-nucleon system are generated by the AV18 two-nucleon potential

- * Interaction occurs via 1- and 2-body operators *
- * Fully interacting nucleon-pairs in the final states *
- * $R_{\alpha}^{40}(q, \omega)$ is a ground-state observable *

Benchmarking STA in ^4He : two tests

$$R_{\alpha}^{40}(q, \omega) = \sum_f \delta(\omega + E_0 - E_f) |\langle f | O_{\alpha}(\mathbf{q}) | 0 \rangle|^2$$



- ▶ $R_L^{40}(q, \omega)$ with

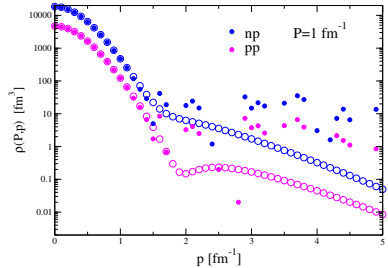
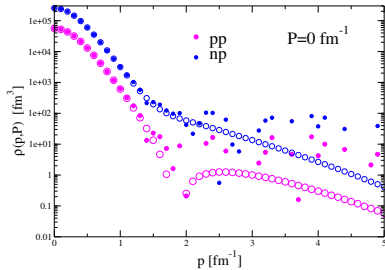
$$O_L^{(1)}(\mathbf{q}) = \sum_i e \frac{1 + \tau_{z,i}}{2} e^{i\mathbf{q} \cdot \mathbf{r}_i}, \quad |f\rangle \sim e^{i\mathbf{p} \cdot \mathbf{r}} e^{i\mathbf{P} \cdot \mathbf{R}}$$

- ▶ reduces to

$$R_L^{40}(q, \omega) = \int d\mathbf{p} \int d\mathbf{P} \rho_{NN}(\mathbf{p}, \mathbf{P}) \delta(\omega + E_0 - E_f) \equiv R_L^{\text{PWIA}}(q, \omega)$$

- ▶ We can benchmark ^4He *i*) nucleon-pair momentum distributions and *ii*) PWIA longitudinal response function

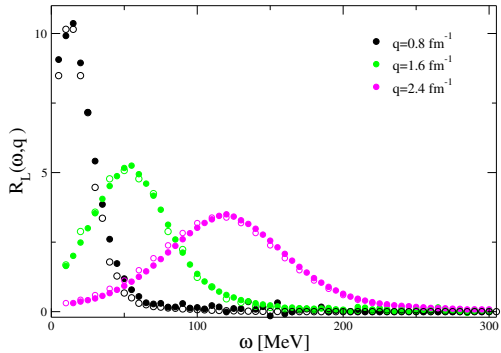
Benchmark on ${}^4\text{He}$: 2-body momentum distributions



Open circles from Wiringa *et al.* - [PRC89\(2014\)024305](#)

- ▶ $\sim 3\text{k}$ core hours
- ▶ Noise for $p > 1.5 \text{ fm}^{-1}$ can be reduced with more statistics

Benchmark on ${}^4\text{He}$: PWIA Longitudinal Response



Summary and Outlook # 2.

The microscopic description of nuclei successfully reproduces EXPT data provided that many-body effects in nuclear interactions and EM currents are accounted for.

J.Phys.G41(2014)123002 - S.Bacca&S.P.

- ▶ In STA the response functions are given by the scattering off pairs of fully interacting nucleons that propagate into a correlated pair of nucleons in the final state
- ▶ The interactions with the external probe occurs via one- and two-body current operators
- ▶ The responses in STA are ground-state observables

To do list:

- ▶ Implement two-body current operators
- ▶ Implement fully interacting two-nucleon propagators