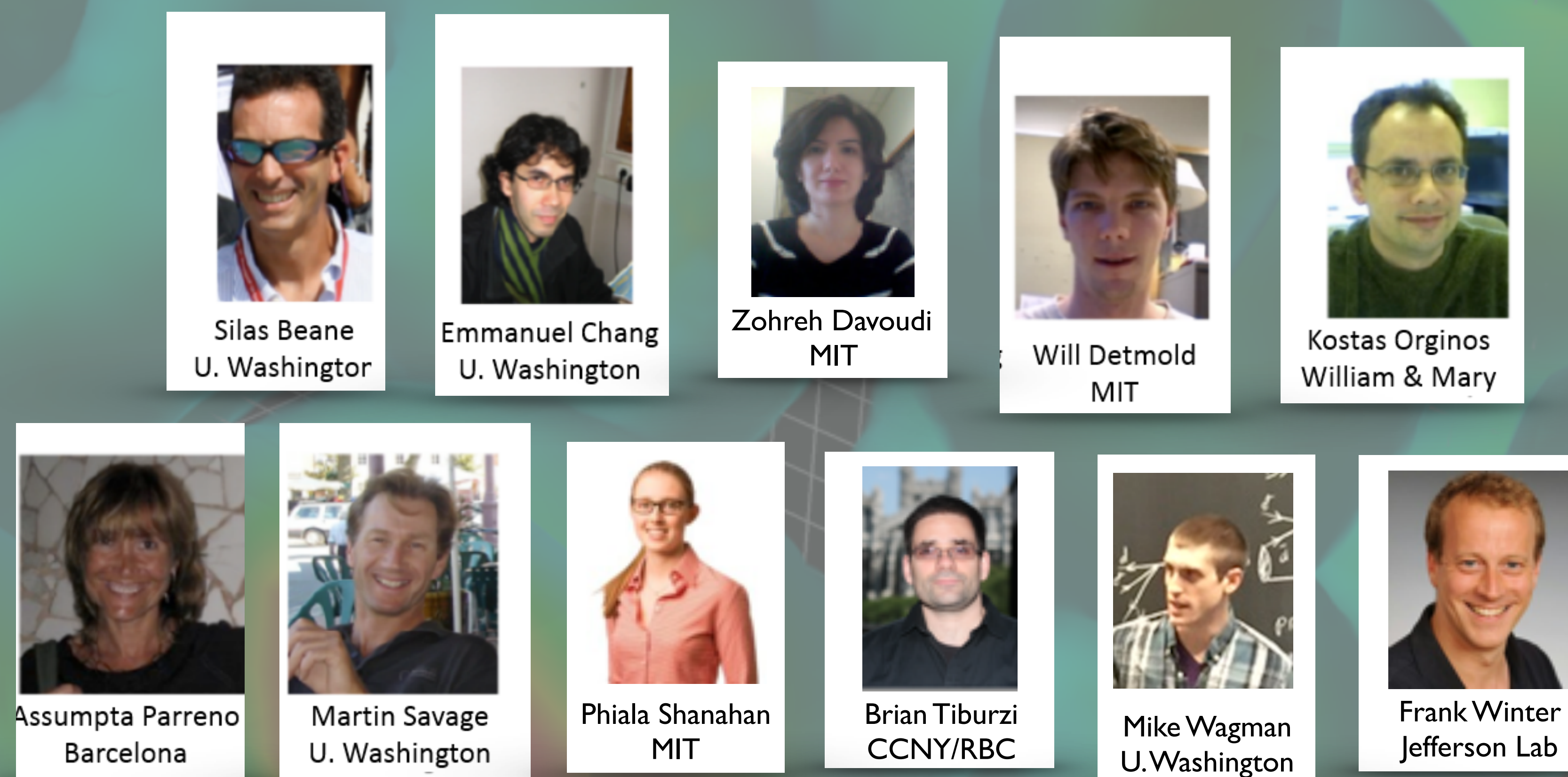
Proton-proton fusion and tritium β-decay from lattice QCD



Phiala Shanahan, MIT

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INT-PUB-16-033, JLAB-THY-16-2362, MIT-CTP-4844, NT@UW-16-12

NPLQCD

Proton-proton fusion and tritium β-decay from lattice quantum chromodynamics

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(Dated: October 17, 2016)

The nuclear matrix element determining the $pp \to de^+\nu$ fusion cross section and the Gamow-Teller matrix element contributing to tritium β-decay are calculated with lattice Quantum Chromodynamics (QCD) for the first time. Using a new implementation of the background field method, these quantities are calculated at the SU(3)-flavor-symmetric value of the quark masses, corresponding to a pion mass of $m_\pi \sim 806$ MeV. The Gamow-Teller matrix element in tritium is found to be 0.979(03)(10) at these quark masses, which is within 2σ of the experimental value. Assuming that the short-distance correlated two-nucleon contributions to the matrix element (meson-exchange currents) depend only mildly on the quark masses, as seen for the analogous magnetic interactions, the calculated $pp \to de^+\nu$ transition matrix element leads to a fusion cross section at the physical quark masses that is consistent with its currently accepted value. Moreover, the leading two-nucleon axial counterterm of pionless effective field theory is determined to be $L_{1,A} = 3.9(0.1)(1.0)(0.3)(0.9)$ fm^3 at a renormalization scale set by the physical pion mass, also in agreement with the accepted phenomenological range. This work concretely demonstrates that weak transition amplitudes in few-nucleon systems can be studied directly from the fundamental quark and gluon degrees of freedom and opens the way for subsequent investigations of many important quantities in nuclear physics.

PACS numbers: 11.15.Ha, 12.38.Gc, 13.40.Gp

Weak nuclear processes

- Matrix element determining $pp \to de^+\nu$ fusion cross-section $\Rightarrow L_{1,A},\ \ell_{1,A},\ \overline{L}_{1,A}, \ldots$

- Gamow-Teller matrix element in tritium

Proton-proton fusion

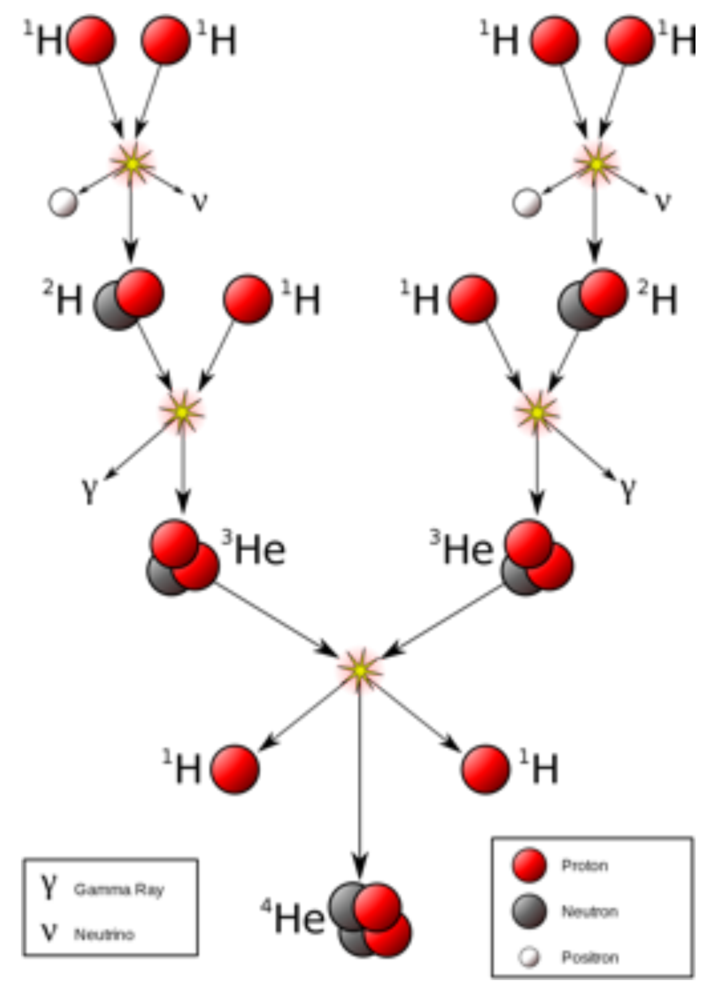
- Stars emit heat/light from conversion of H to He
- Sun + cooler stars: proton-proton fusion chain reaction

We calculate $\langle d;3|A_3^3|pp\rangle$

$pp \to de^+\nu$ cross-section

$\rightarrow$ $L_{1,A},\ \ell_{1,A},\ \overline{L}_{1,A},\ldots$

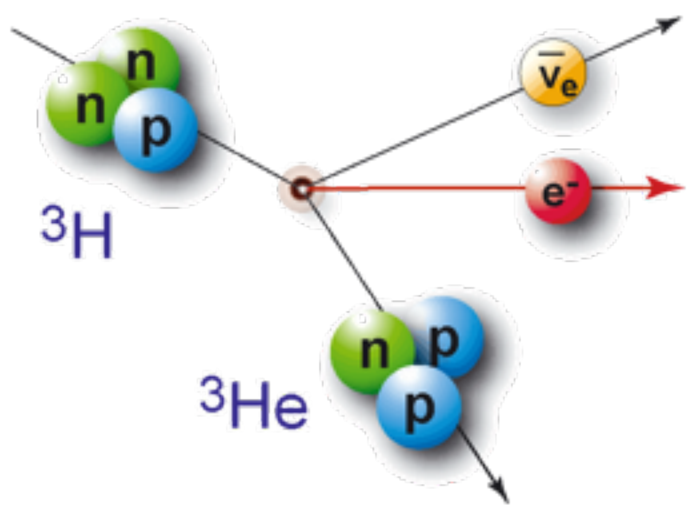
- Related to:
  - Neutrino breakup reaction (SNO)
  - Muon capture reaction (MuSun)



pp	
$p + p$	$\rightarrow {}^2\mathrm{H} + e^+ + \nu_e$
${}^2\mathrm{H} + p$	$\rightarrow {}^3\mathrm{He}$
${}^3\mathrm{He} + {}^3\mathrm{He}$	$\rightarrow {}^4\mathrm{He} + 2p$

Tritium β-decay

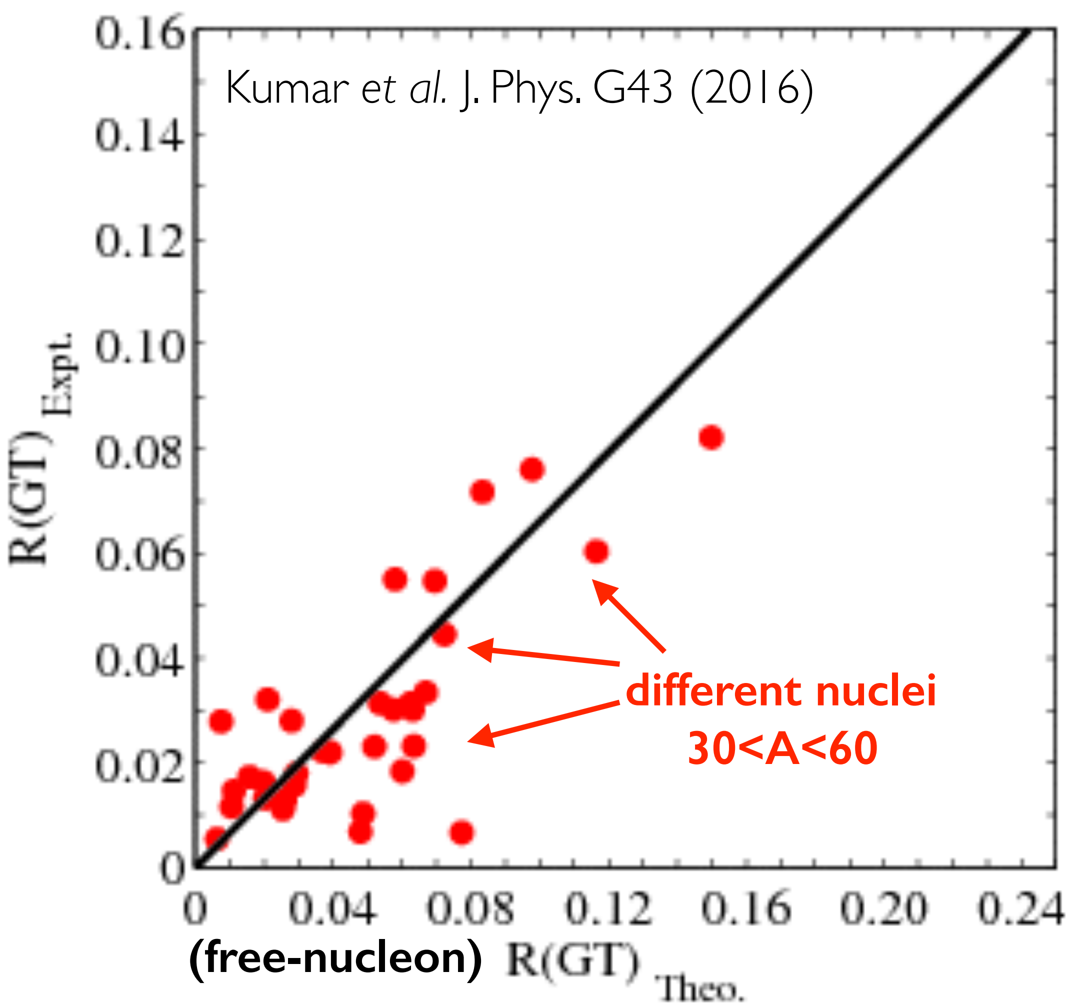
- Simplest semileptonic weak decay of a nuclear system



- Gamow-Teller (axial current) contribution to decays of nuclei not well-known from theory
- Understand multi-body contributions to $\langle \mathbf{GT} \rangle$ ➡ better predictions for decay rates of larger nuclei

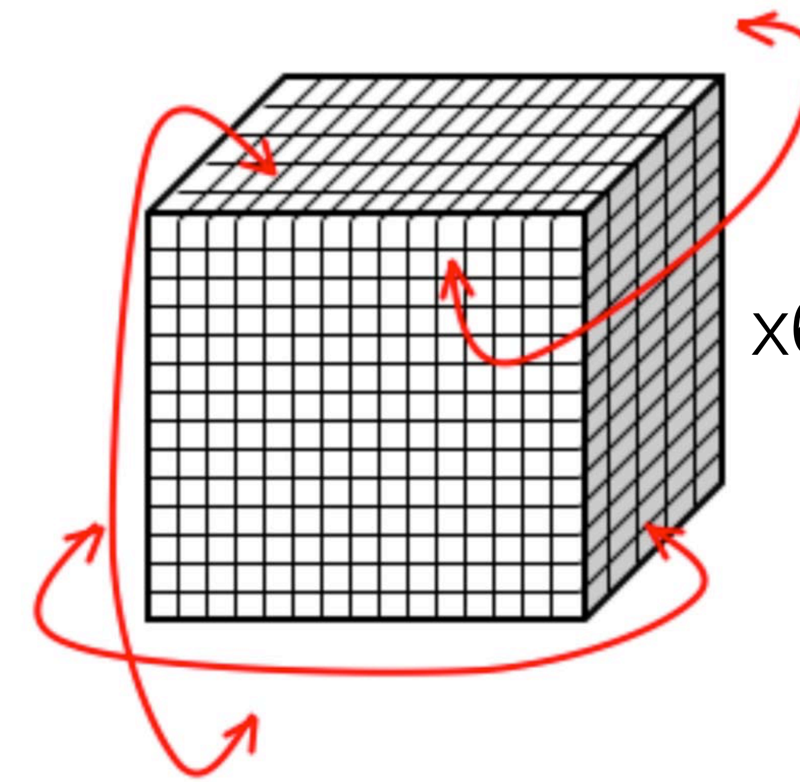
We calculate

$$g_A \langle \mathbf{GT} \rangle = \langle {}^3\mathrm{He} | \bar{\mathbf{q}} \gamma_\mathbf{k} \gamma_\mathbf{5} \tau^- \mathbf{q} | {}^3\mathrm{H} \rangle$$



Lattice QCD

- Numerical first-principles approach
- Euclidean space-time $t \to i\tau$
  - Finite lattice spacing a
  - Volume $L^3 \times T \approx 32^3 \times 64$
  - Boundary conditions
- Finite but large number of d.o.f (10^{12})



Approximate the QCD path integral by **Monte Carlo**

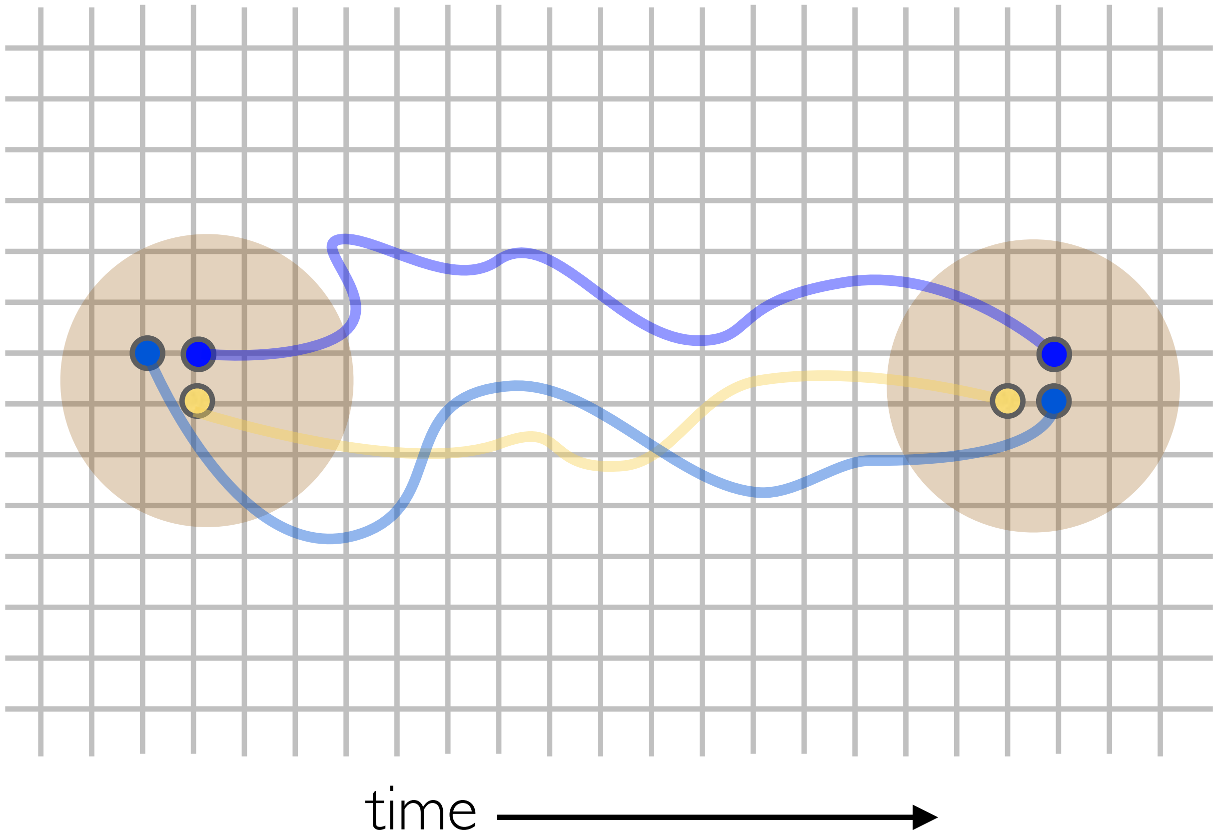
$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A \mathcal{D}\overline{\psi} \mathcal{D}\psi \mathcal{O}[A, \overline{\psi}\psi] e^{-S[A,\overline{\psi}\psi]} \quad \Rightarrow \quad \langle \mathcal{O} \rangle \simeq \frac{1}{N_{\text{conf}}} \sum_i^{N_{\text{conf}}} \mathcal{O}([U^i])$$

with field configurations U^i distributed according to $e^{-S[U]}$

Doing lattice QCD

How do we calculate the proton mass?

- Create three quarks (correct quantum numbers) at a source and annihilate the three quarks at sink far from source
- QCD adds all the quark anti-quark pairs and gluons automatically: only eigenstates with correct q#'s propagate

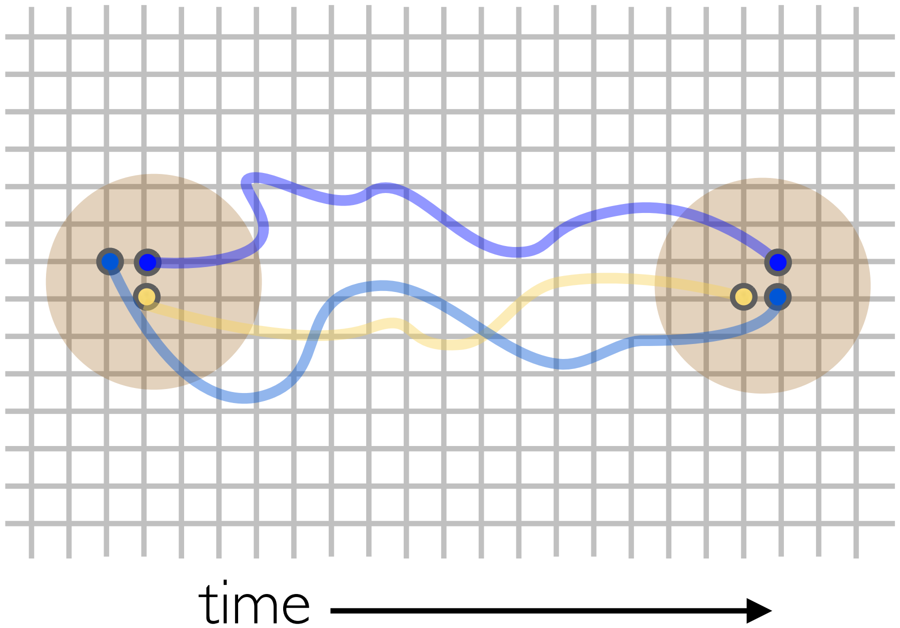


Doing lattice QCD

- Correlation decays exponentially with distance in time:

$$C(t) = \sum_n Z_n \exp(-E_n t)$$

n ← all eigenstates with q#'s of proton

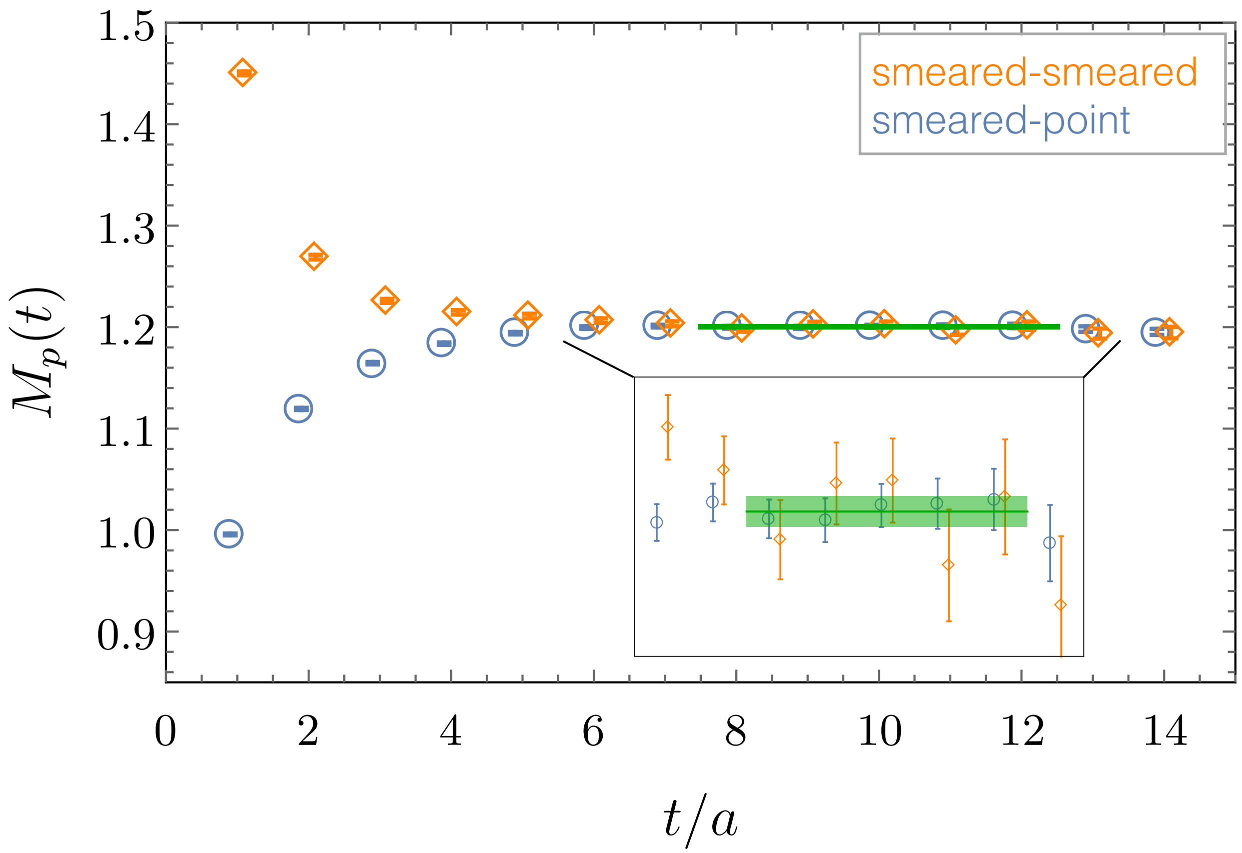


At late times:

$$\rightarrow Z_0 \exp(-E_0 t)$$

- Ground state mass revealed through "effective mass plot"

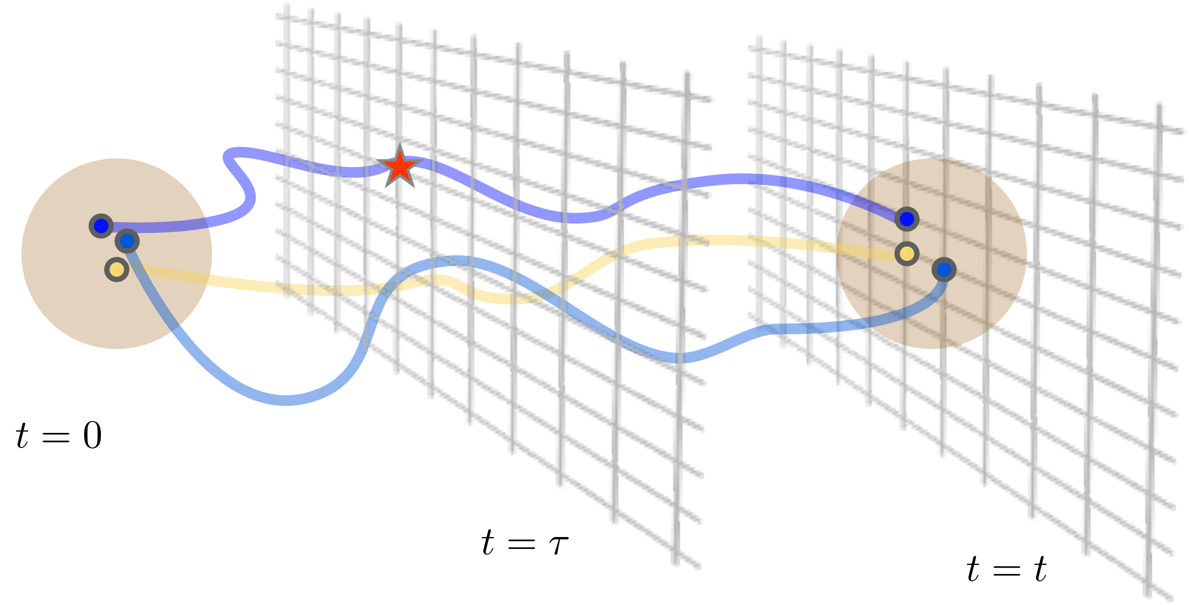
$$M(t) = \ln\left[\frac{C(t)}{C(t+1)}\right] \xrightarrow{t\to\infty} E_0$$



Doing lattice QCD

How do we calculate matrix elements?

- Create three quarks (correct quantum numbers) at a source and annihilate the three quarks at sink far from source
- Insert operator at intermediate timeslice

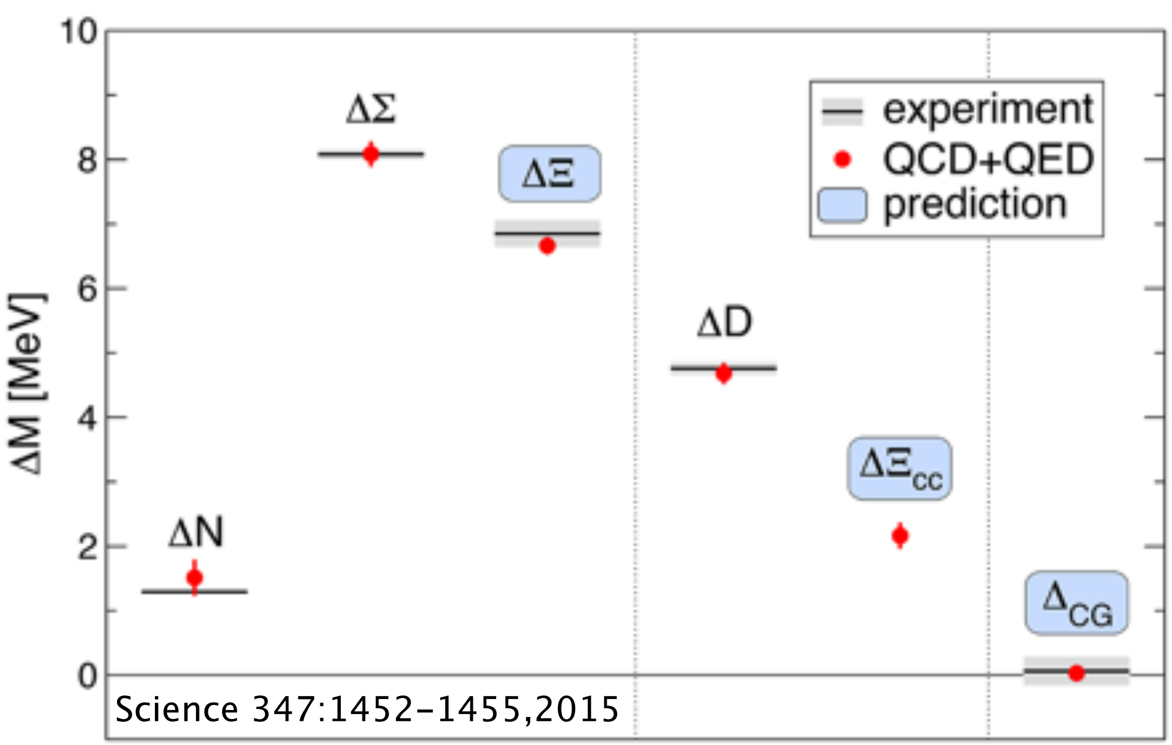


- Remove time-dependence by dividing out with two-point correlators:

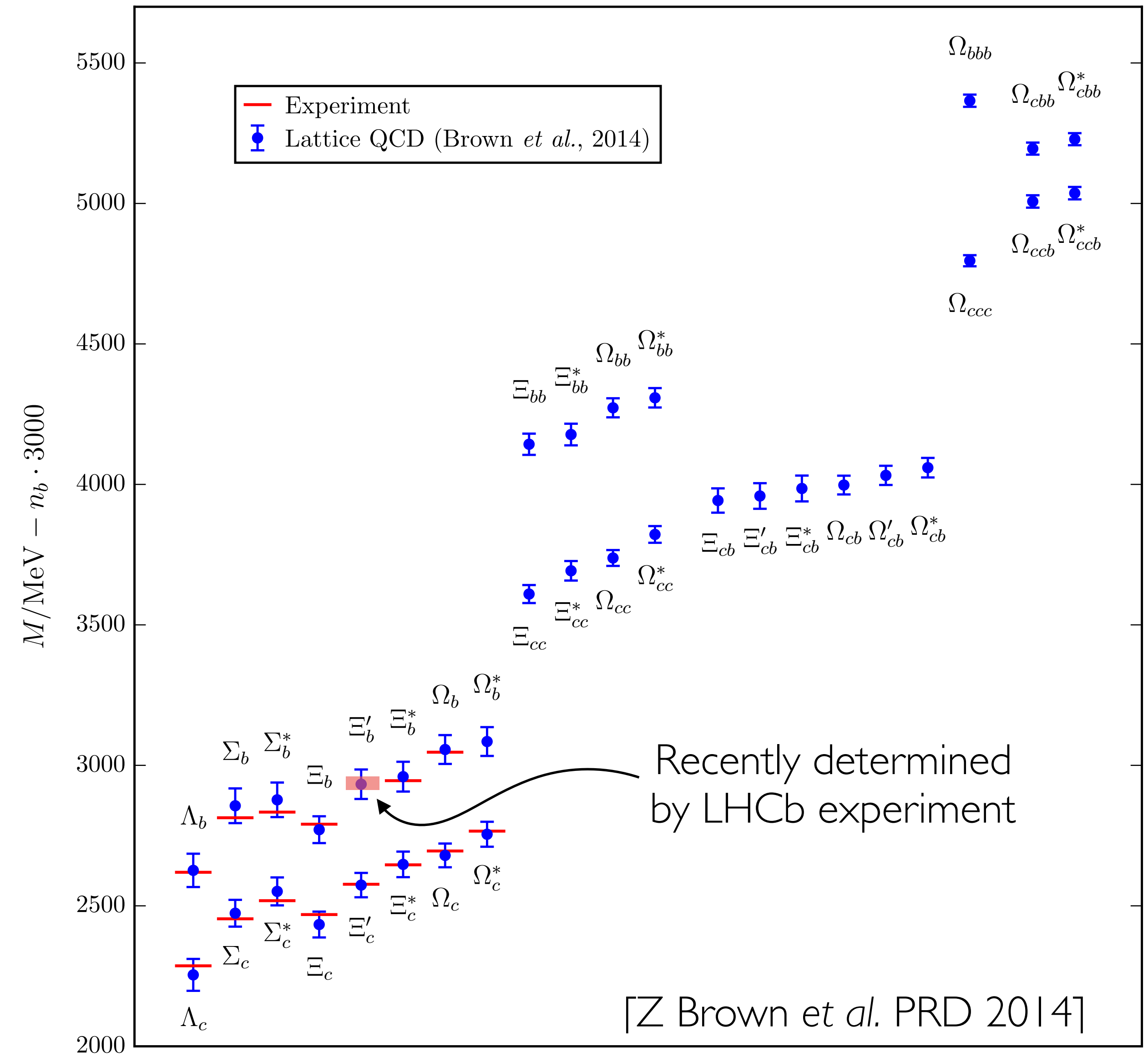
$$\frac{C_3(t, \tau, \vec{p'}, \vec{q})}{C_2(t-\tau, p')C_2(\tau, p)} \xrightarrow{t\to\infty} \langle N(p')|\mathcal{O}(q)|N(p)\rangle$$

Lattice QCD works

- Ground state hadron spectrum reproduced
- p-n mass splitting reproduced
- …



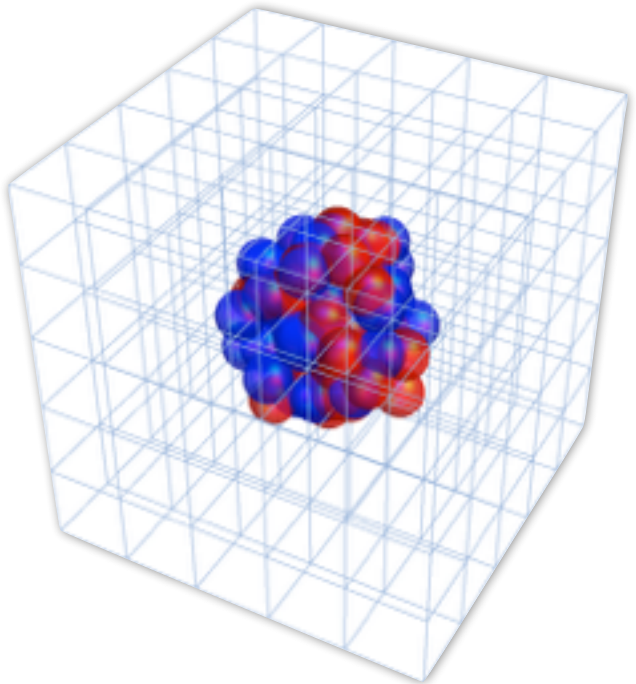
- Predictions for new states with controlled uncertainties

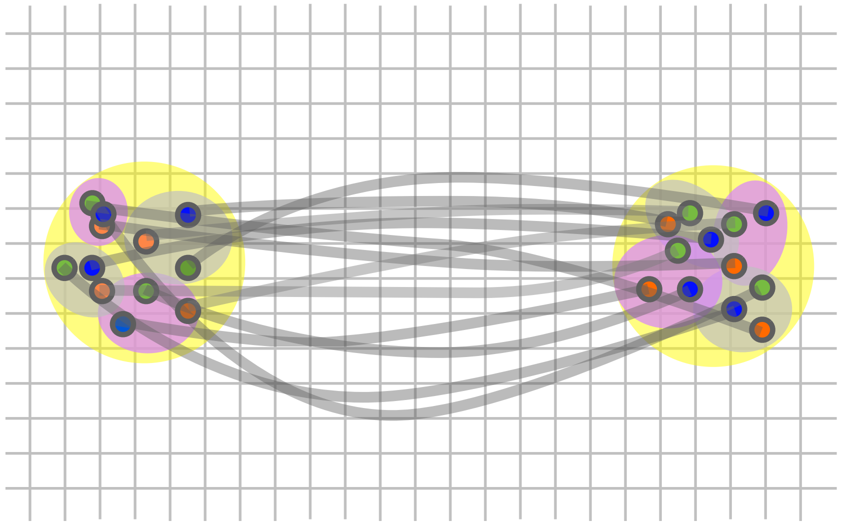


Nuclear physics

Nuclei on the lattice

- Hard problem
  - Noise:
    Statistical uncertainty grows exponentially with A
  - Complexity:
    Number of contractions grows factorially
    [Detmold & Savage, Detmold & Orginos; Doi & Endres]





Unphysical nuclei

NPLQCD collaboration

- QCD with unphysical quark masses

  m_π~800 MeV, m_N~1,600 MeV

  m_π~450 MeV, m_N~1,200 MeV

- Spectrum of light nuclei (A<5)

  [PRD **87** (2013), 034506]

- Nuclear structure: magnetic moments, polarisabilities (A<5)

  [PRL **113**, 252001 (2014), PRD 92, 114502 (2015)]

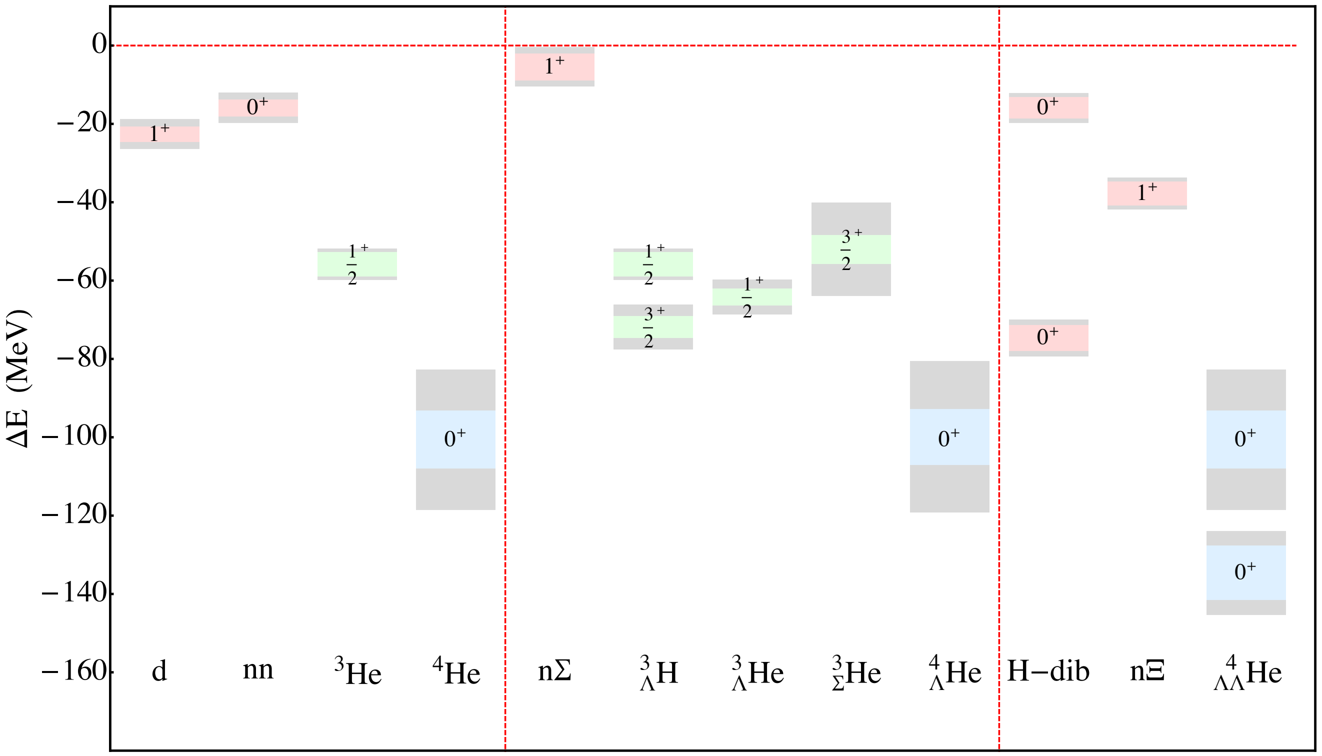
- First nuclear reaction: np→dγ

  [PRL **115**, 132001 (2015)]

Hot off the press

- Proton-proton fusion and tritium β-decay

  m_π~800 MeV, m_N~1,600 MeV



Background field method

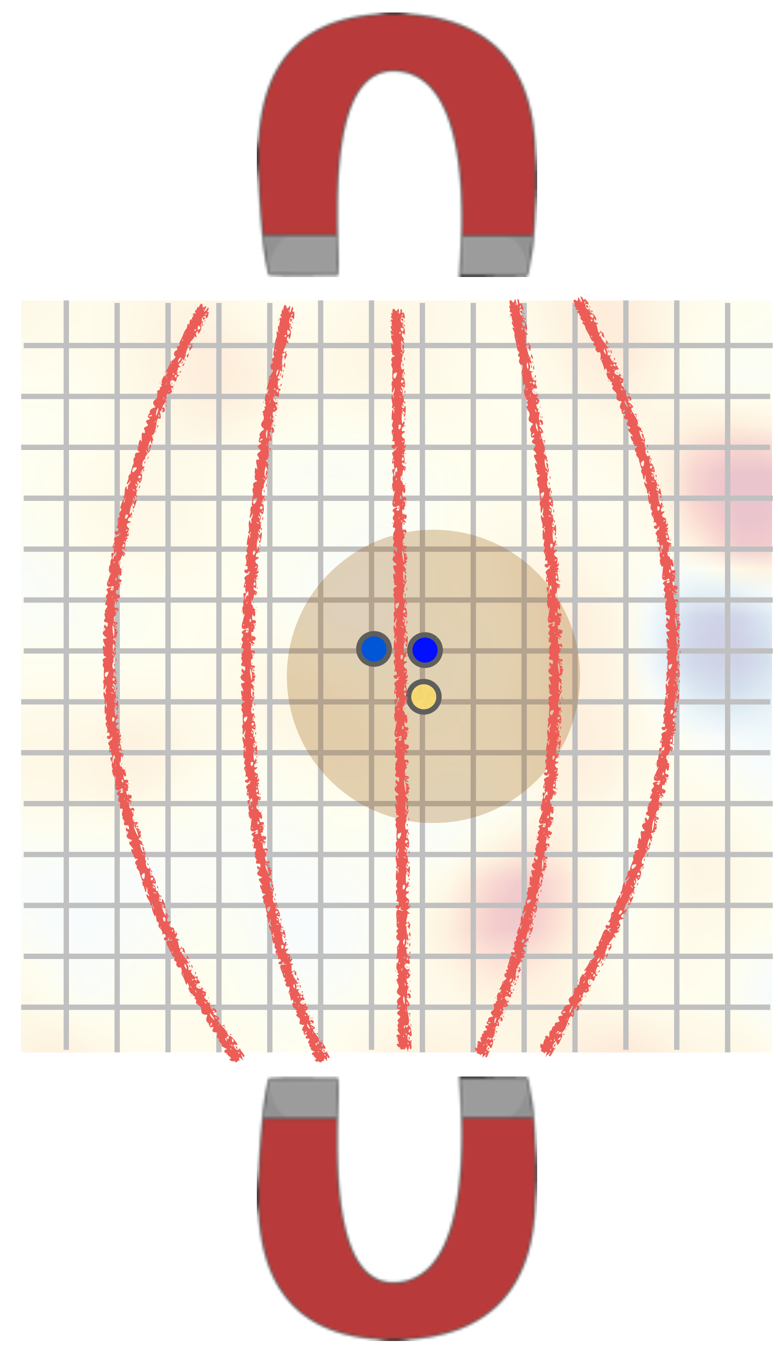
Hadron/nuclear energies are modified by presence of fixed/constant external fields

Example: fixed magnetic field

landau level — mag. mmt

$$E(\vec{B}) = \sqrt{M^2 + (2\boxed{n} + 1)|Qe\vec{B}|} - \boxed{\vec{\mu}} \cdot \vec{B}$$
$$-2\pi \boxed{\beta_{M0}} |\vec{B}|^2 - 2\pi \boxed{\beta_{M2}} \boxed{T_{ij}} B_i B_j + \ldots$$

polarisabilities — traceless, sym tensor



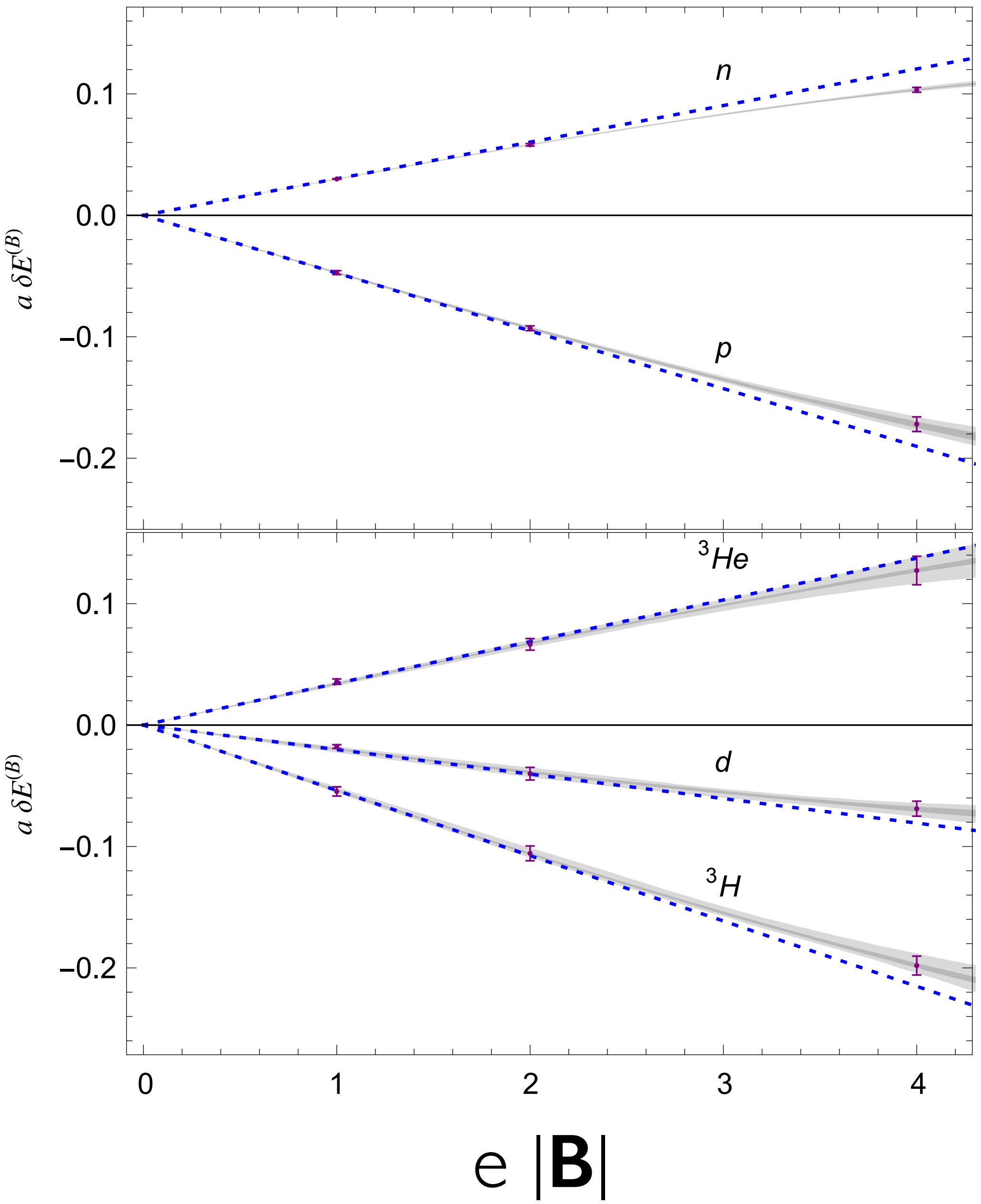
- Calculations with multiple fields
  ➡ extract coefficients of response
  e.g., magnetic moments, polarisabilities, …

- Not restricted to simple EM fields
  This work: uniform axial background field

[NPLQCD PRL **113**, 252001 (2014)]

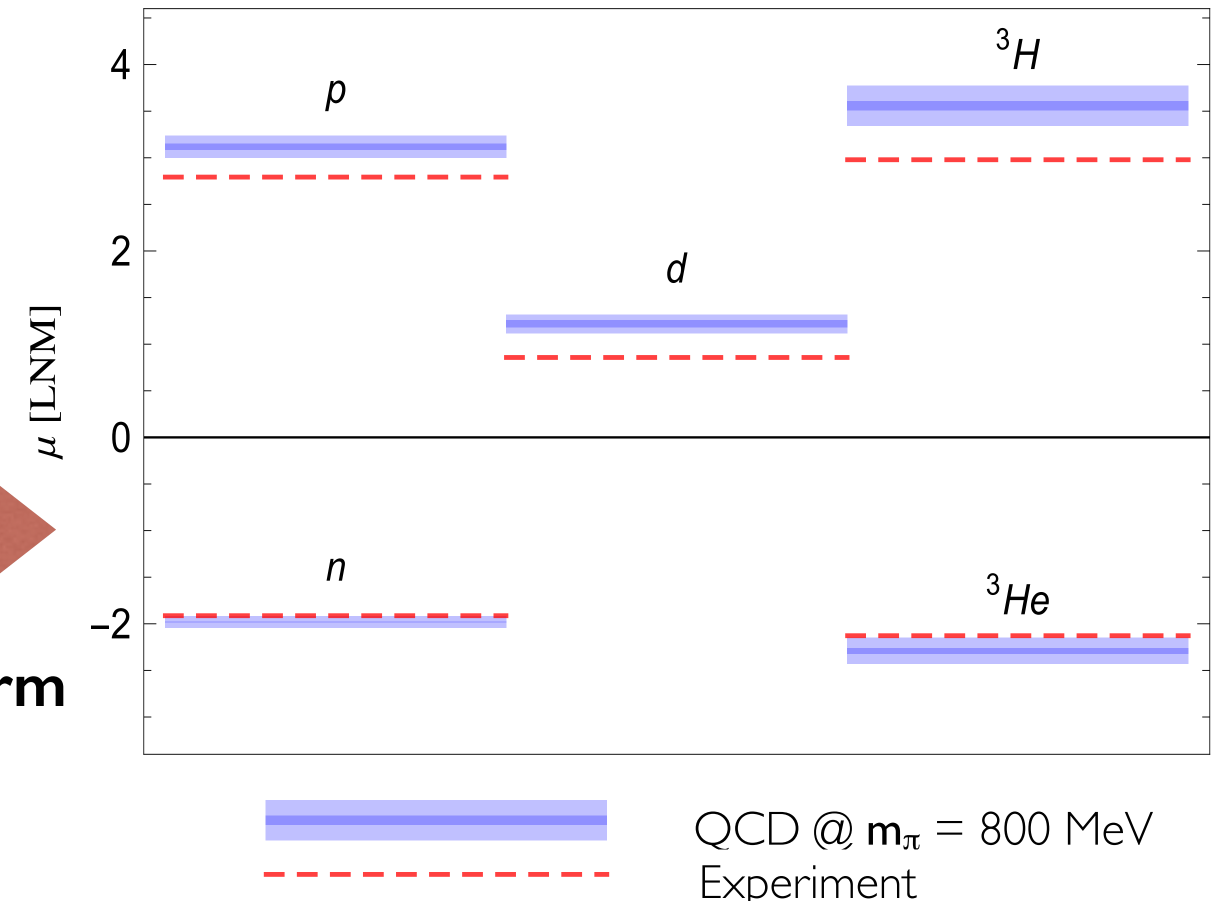
Magnetic moments

Energy shift between ground states spin-aligned/anti-aligned with B



linear term

Magnetic moments



	n	p	d	^{3}He	^{3}H
μ	-1.98(1)(2)	3.21(3)(6)	1.22(4)(9)	-2.29(3)(12)	3.56(5)(18)

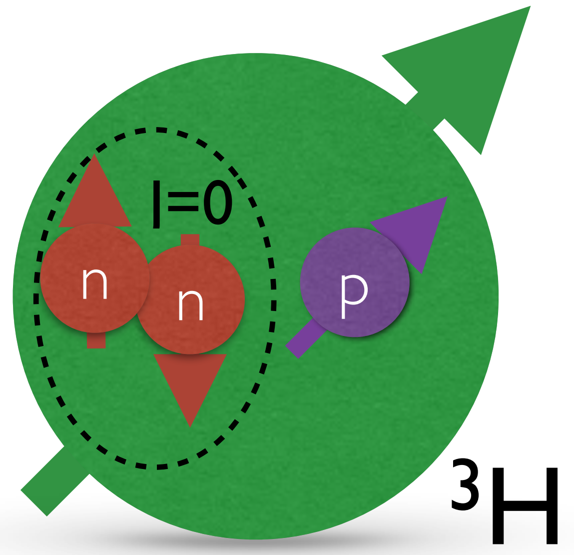
In units of appropriate nuclear magnetons (heavy M_N)

[NPLQCD PRL **113**, 252001 (2014)]

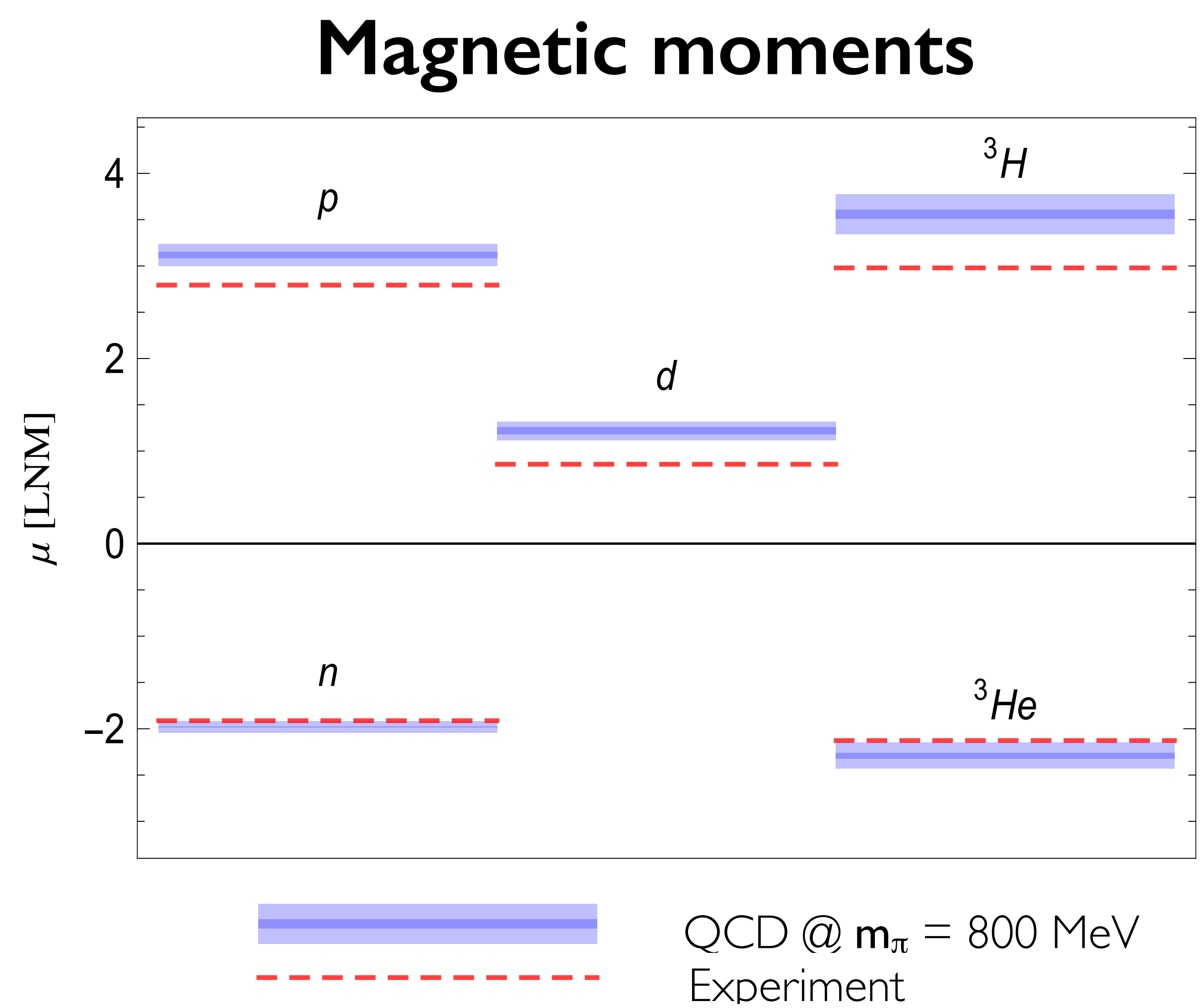
Magnetic moments

Results are surprisingly consistent with shell model expectations

$$\mu_d = \mu_p + \mu_n$$
$$\mu_{^3\mathrm{H}} = \mu_p$$
$$\mu_{^3\mathrm{He}} = \mu_n$$



Suggests that heavy-quark nuclei are shell-model like!



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In units of appropriate nuclear magnetons (heavy M_N)

[NPLQCD PRL **113**, 252001 (2014)]

Axial background field

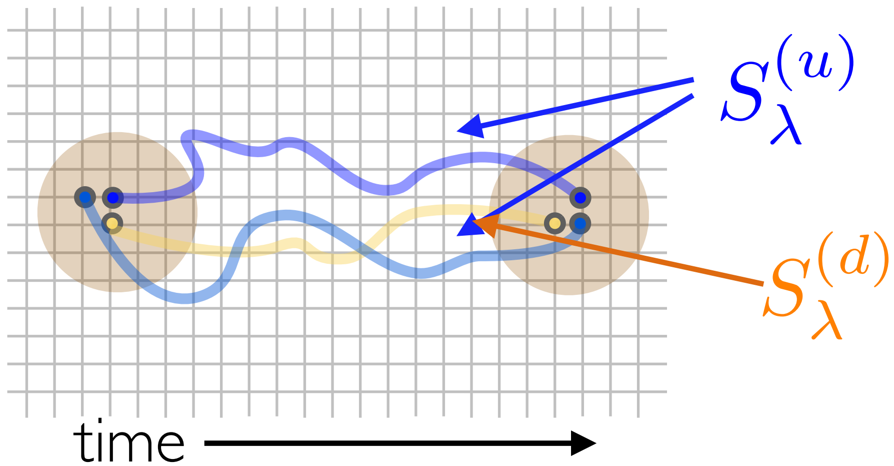
Example: fixed magnetic field ➡ moments, polarisabilities
This work: fixed axial background field ➡ axial charges, other matrix elts.

Construct correlation functions from propagators modified in axial field

compound propagator **constant**

$$S_\lambda^{(q)}(x,y) = S^{(q)}(x,y) + \lambda_q \int dz\, S^{(q)}(x,z)\,\gamma_3\gamma_5\, S^{(q)}(z,y)$$

$C_{\lambda_u;\lambda_d}(t) =$

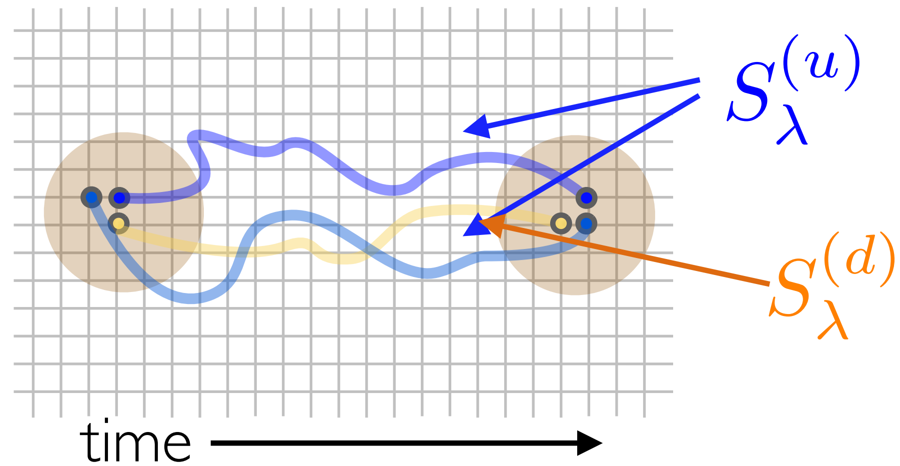


Linear response ⟷ axial matrix element

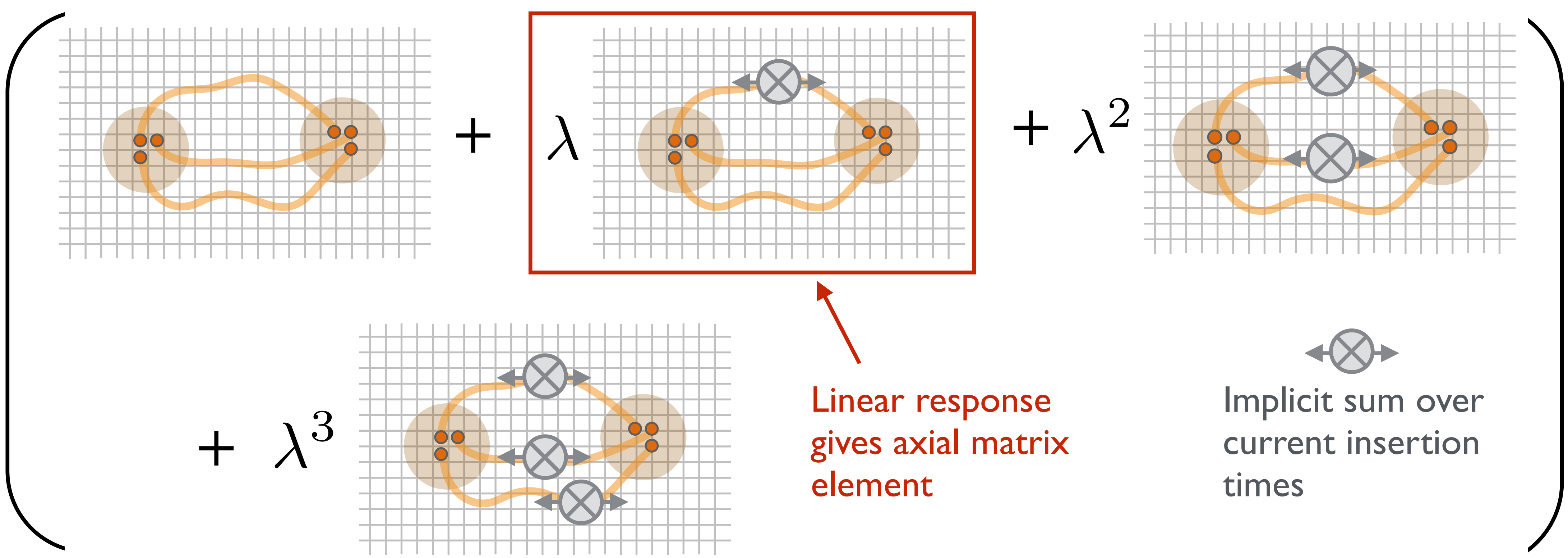
[NPLQCD Nucl. Phys. **A743**, 170 (2004)]

Axial background field

$$C_{\lambda_u;\lambda_d}(t) =$$

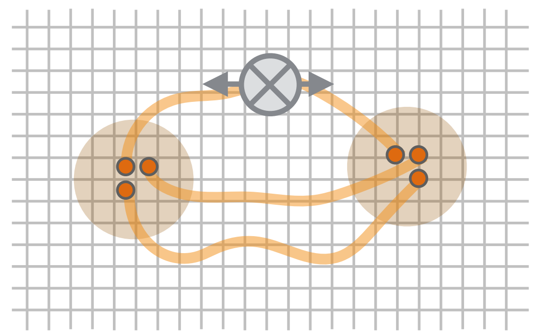


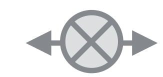
$$C_{\lambda_u;\lambda_d}(t) =$$



Axial background field

$$C_{\lambda_u;\lambda_d}(t)\Big|_{\mathcal{O}(\lambda)} =$$





Implicit sum over current insertion times

Example: determination of the proton axial charge

$$\begin{aligned} C_{\lambda_u;\lambda_d}(t)\Big|_{\mathcal{O}(\lambda)} &= \sum_{\tau=0}^{t} \langle 0|\chi^\dagger(t) J(\tau)\chi(0)|0\rangle \\ &= \ldots \\ &= Z_0 e^{-M_p t}\left[C + t\,\langle p|A_3^{(u)}(0)|p\rangle + \mathcal{O}(e^{-\delta t})\right] \end{aligned}$$

Excited states

Awful constants

Matrix element

Time difference isolates matrix element part

$$\left(C_{\lambda_u;\lambda_d}(t+1) - C_{\lambda_u;\lambda_d}(t)\right)\Big|_{\mathcal{O}(\lambda)} = Z_0 e^{-M_p t}\langle p|A_3^{(u)}(0)|p\rangle + \mathcal{O}(e^{-\delta t})$$

Proton axial charge

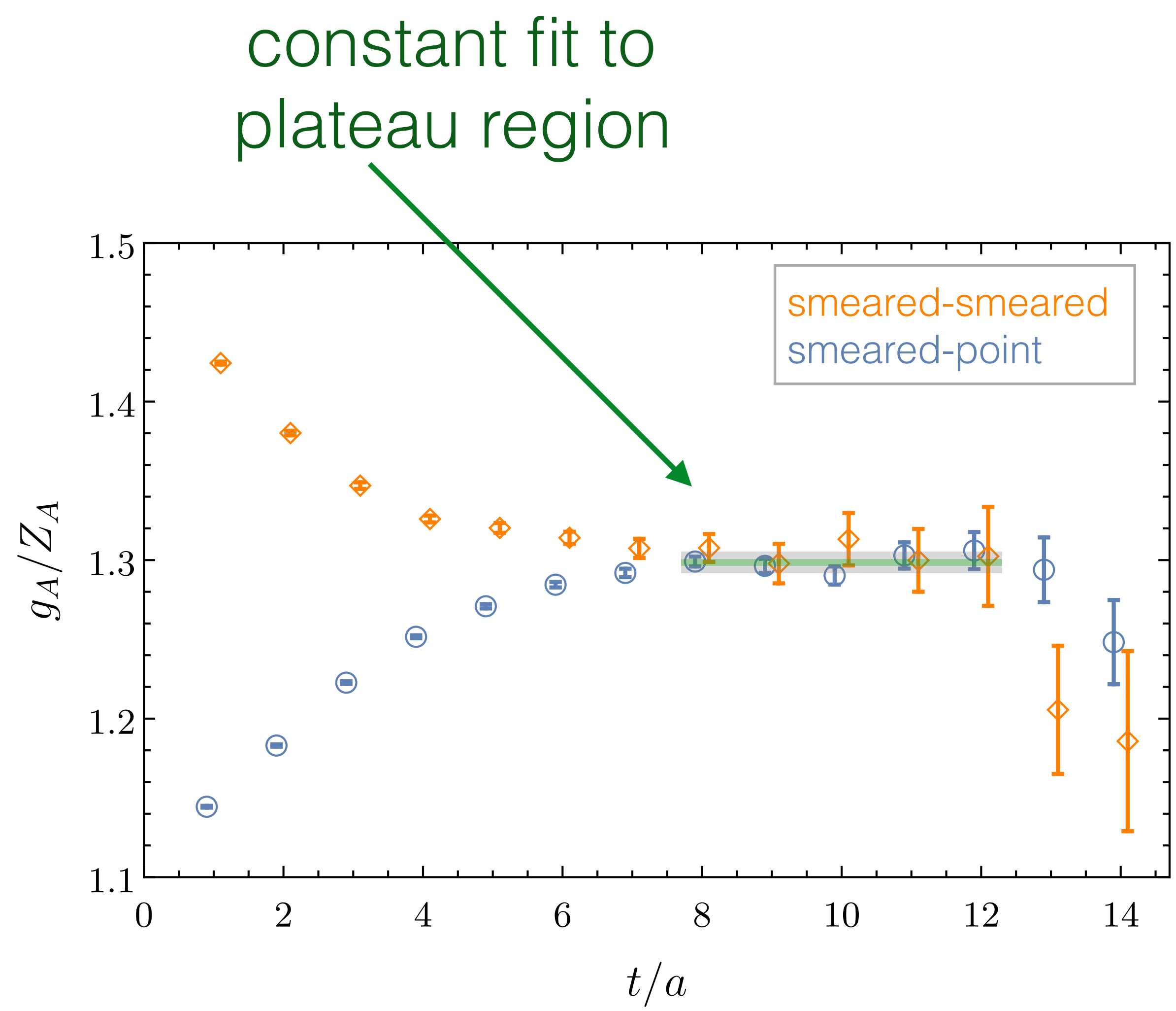
- Extract matrix element through linear response of correlators to the background field
- Form ratios to cancel leading time-dependence

$$R_p(t) = \frac{\left(C^{(p)}_{\lambda_u;\lambda_d=0}(t) - C^{(p)}_{\lambda_u=0;\lambda_d}(t)\right)\Big|_{\mathcal{O}(\lambda)}}{C^{(p)}_{\lambda_u=0;\lambda_d=0}(t)}$$

At late times:

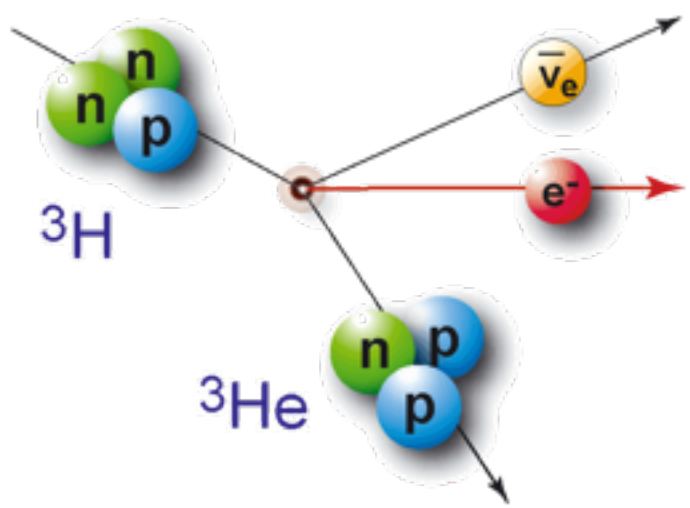
$$R_p(t+1) - R_p(t) \xrightarrow{t\to\infty} \frac{g_A}{Z_A}$$

- Matrix element revealed through "effective matrix elt. plot"



Tritium β-decay

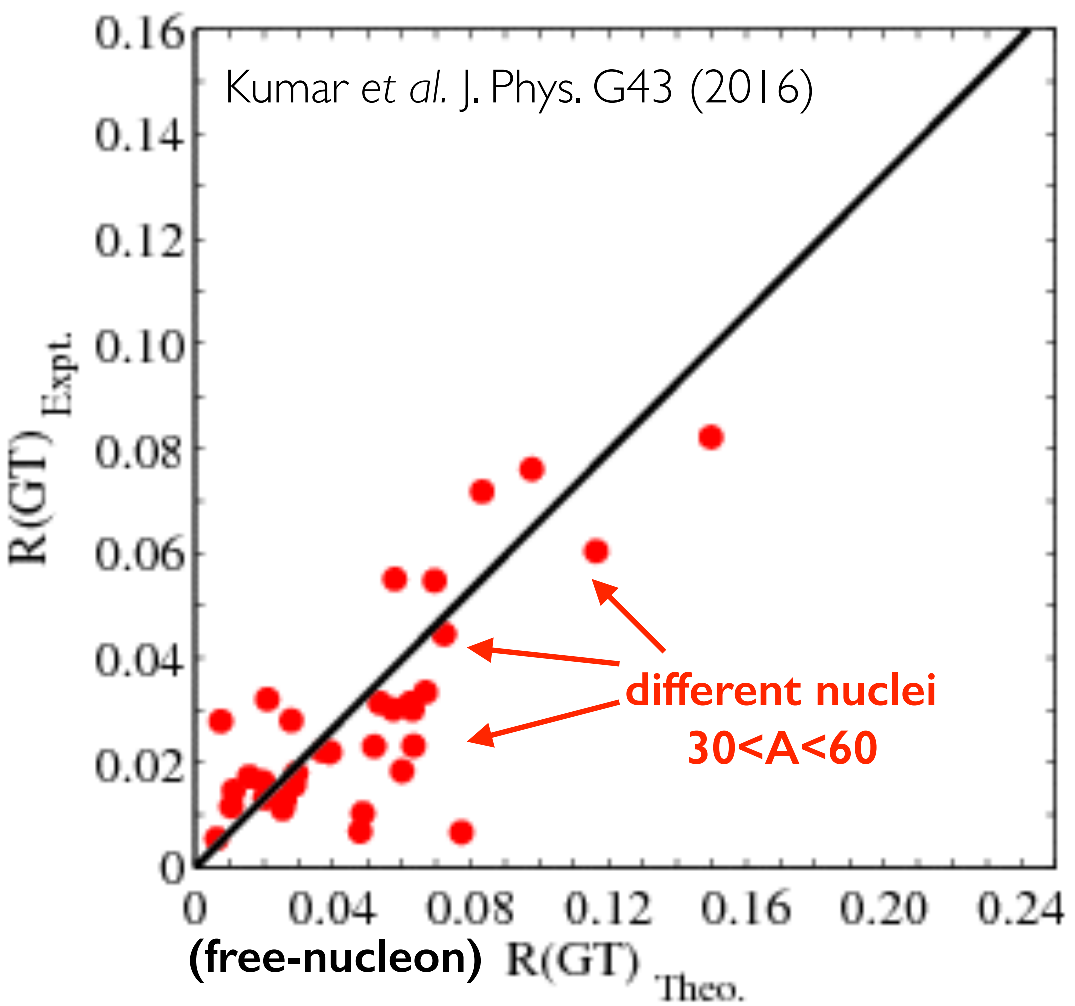
- Simplest semileptonic weak decay of a nuclear system



- Gamow-Teller (axial current) contribution to decays of nuclei not well-known from theory
- Understand multi-body contributions to $\langle \mathbf{GT} \rangle$ ➡ better predictions for decay rates of larger nuclei

We calculate

$$g_A \langle \mathbf{GT} \rangle = \langle {}^{\mathbf{3}}\mathrm{He} | \overline{\mathbf{q}} \gamma_{\mathbf{k}} \gamma_{\mathbf{5}} \tau^- \mathbf{q} | {}^{\mathbf{3}}\mathrm{H} \rangle$$



Tritium β-decay

$$\frac{(1+\delta_R)f_V}{K/G_V^2} t_{1/2} = \frac{1}{\langle \mathbf{F} \rangle^2 + f_A/f_V\, g_A^2 \langle \mathbf{GT} \rangle^2}$$

half-life

vector ME

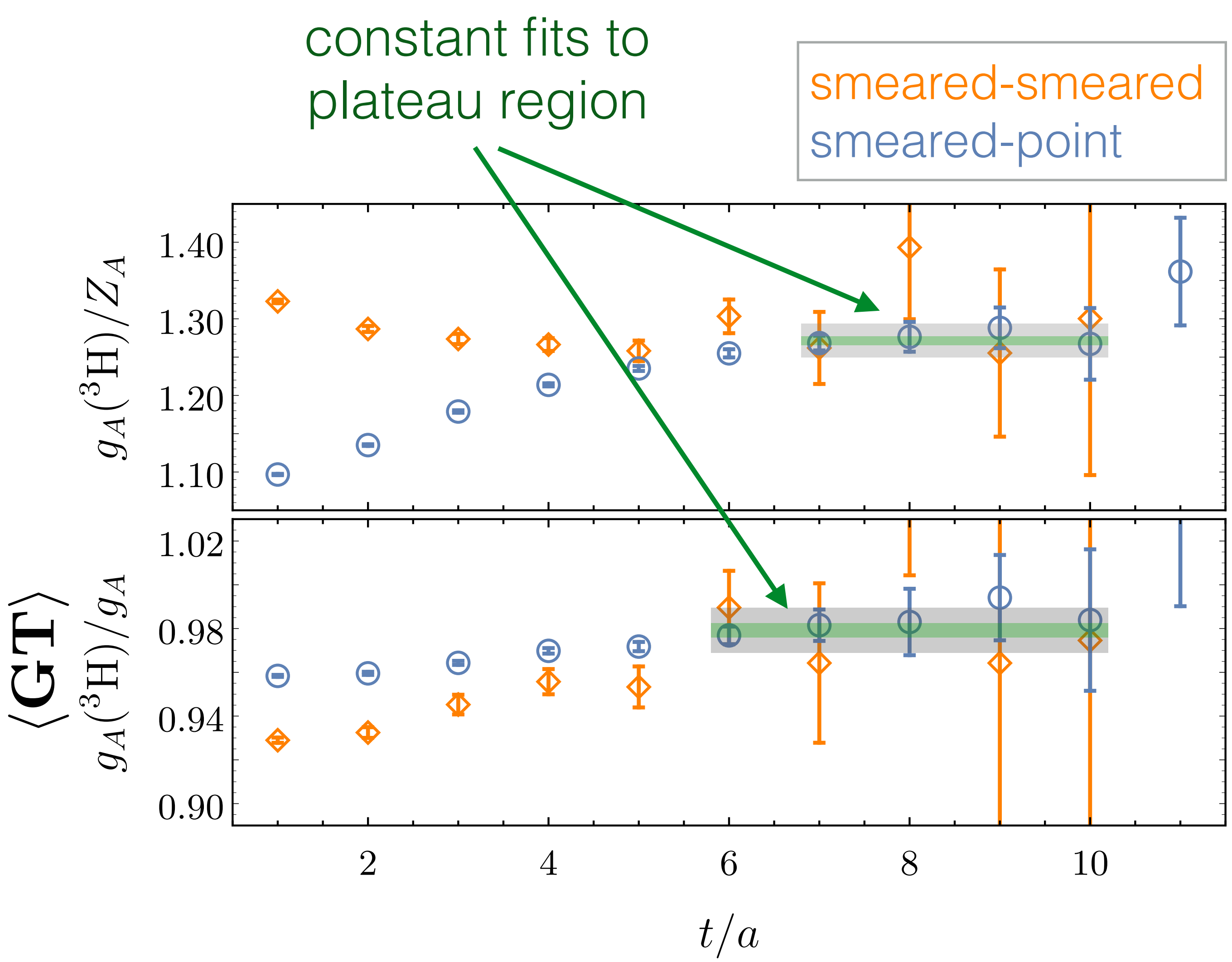
axial ME

known from theory or expt.

- Form ratios of compound correlators to cancel leading time-dependence:

$$\frac{\overline{R}_{^3\mathrm{H}}(t)}{\overline{R}_p(t)} \xrightarrow{t\to\infty} \frac{g_A(^3\mathrm{H})}{g_A} = \langle \mathbf{GT} \rangle$$

- Ground state ME revealed through "effective ME plot"



Proton-proton fusion

- Stars emit heat/light from conversion of H to He
- Sun + cooler stars: proton-proton fusion chain reaction

We calculate $\langle d;3|A_3^3|pp\rangle$
$pp \to de^+\nu$ cross-section
$\rightarrow L_{1,A},\ \ell_{1,A},\ \overline{L}_{1,A},\ldots$

- Related to:
  - Neutrino breakup reaction (SNO)
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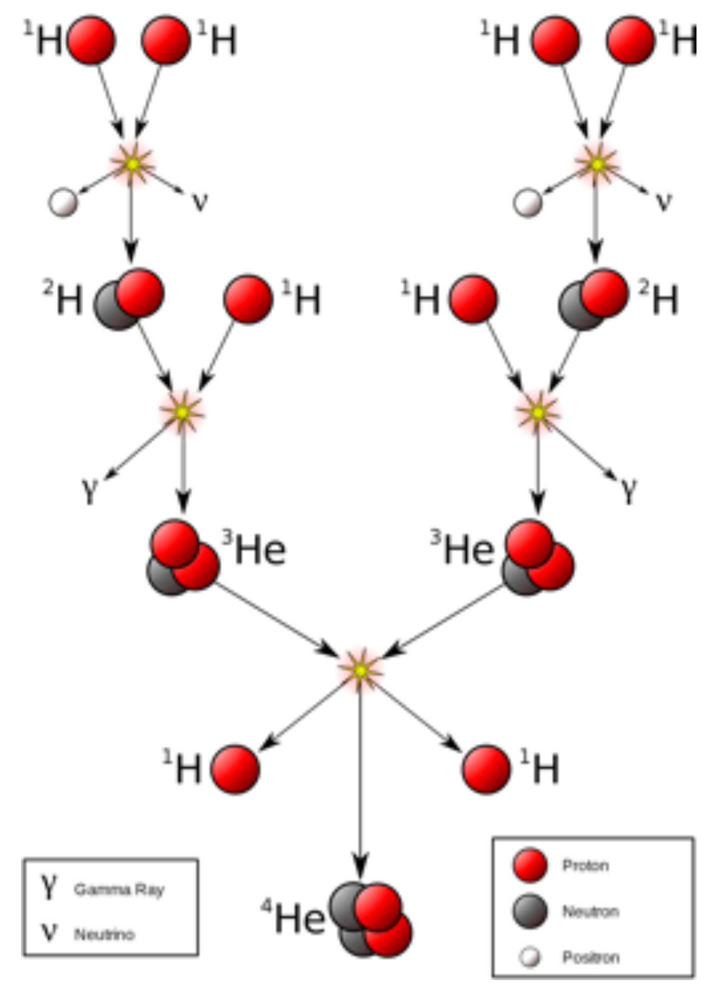


pp		
$p + p$	$\to$	$^2H + e^+ + \nu_e$
$^2H + p$	$\to$	^3He
$^3He + {}^3He$	$\to$	$^4He + 2p$

Proton-proton fusion

- Extract matrix element through linear response of correlators to the background field

matrix elt. is linear in Λ

$$C^{({}^3S_1,{}^1S_0)}_{\lambda_u;\lambda_d=0}(t) = \lambda_u \sum_{\tau=0}^{t} \sum_{\mathbf{x}} \langle 0|\chi^3_{{}^3S_1}(\mathbf{x},t) A^u_3(\tau) \chi^\dagger_{{}^1S_0}(0)|0\rangle + c_2\lambda_u^2 + c_3\lambda_u^3,$$

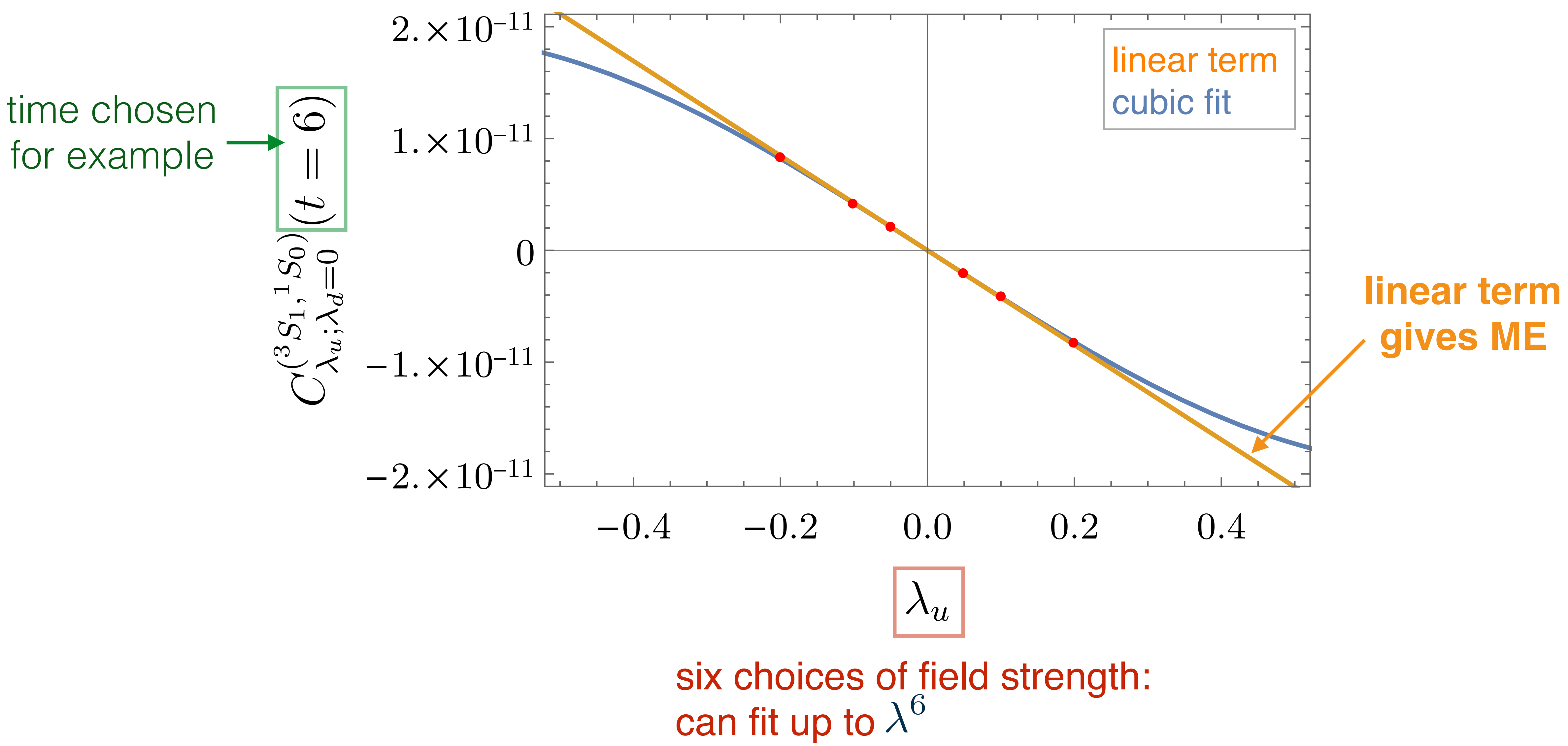
correlator formed with background field coupling to u quark

irrelevant consts.

- Calculate correlators at multiple values of λ_u, λ_d
  → extract matrix element pieces

Proton-proton fusion

- Example: correlator formed with background field coupling to u quark



Proton-proton fusion

- Form ratios of compound correlators to cancel leading time-dependence

transition pieces linear in Λ

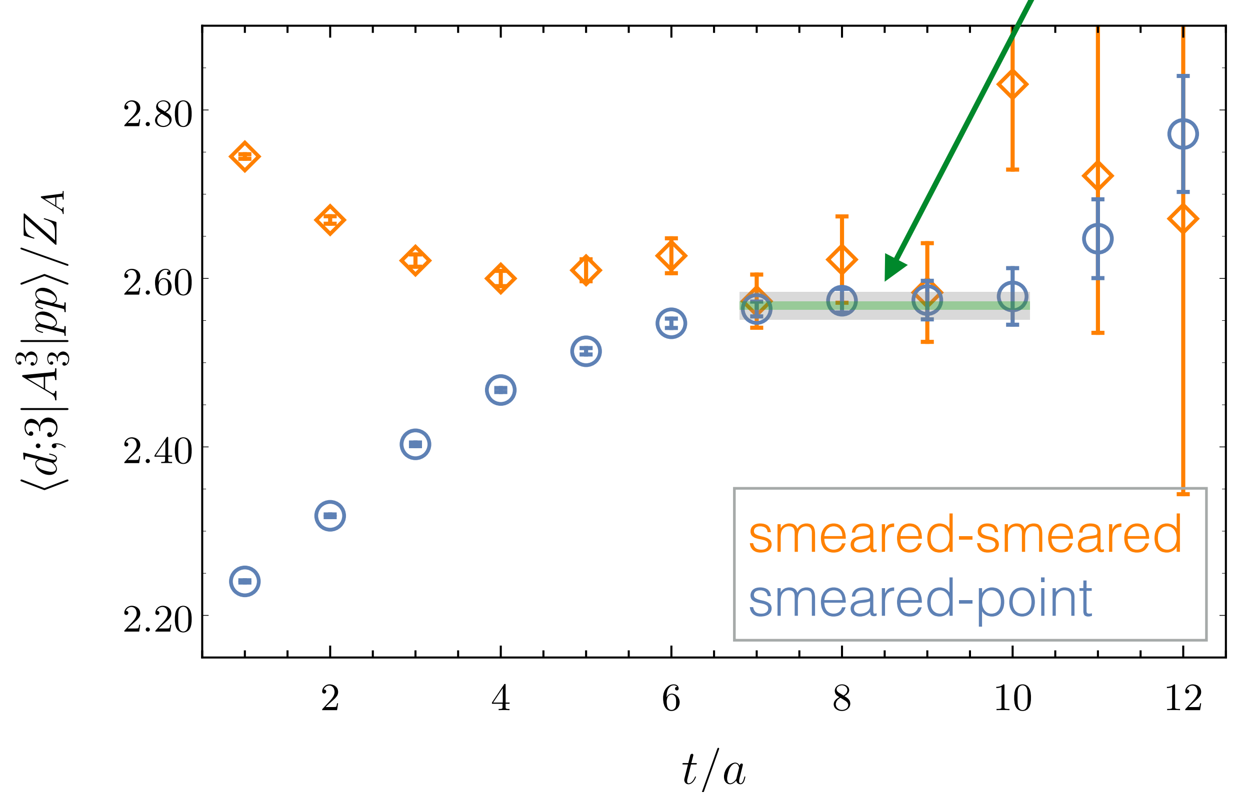
$$R_{^3S_1,^1S_0}(t) = \frac{C^{(^3S_1,^1S_0)}_{\lambda_u,\lambda_d=0}(t)\Big|_{\mathcal{O}(\lambda_u)} - C^{(^3S_1,^1S_0)}_{\lambda_u=0,\lambda_d}(t)\Big|_{\mathcal{O}(\lambda_d)}}{\sqrt{C^{(^3S_1,^3S_1)}_{\lambda_u=0,\lambda_d=0}(t)\,C^{(^1S_0,^1S_0)}_{\lambda_u=0,\lambda_d=0}(t)}}$$

diagonal pieces with no field

- Fit a constant to the 'effective matrix element plot' at late times

$$R_{^3S_1,^1S_0}(t+1) - R_{^3S_1,^1S_0}(t) \xrightarrow{t\to\infty} \frac{\langle ^3S_1; J_z=0|A_3^3|^1S_0; I_z=0\rangle}{Z_A} = \frac{\langle d;3|A_3^3|pp\rangle}{Z_A}$$

constant fit to plateau region



Proton-proton fusion

Treatment of uncertainties: MEs at $m_\pi \sim 800$MeV

- Statistical

  bootstrap/jackknife over configs.

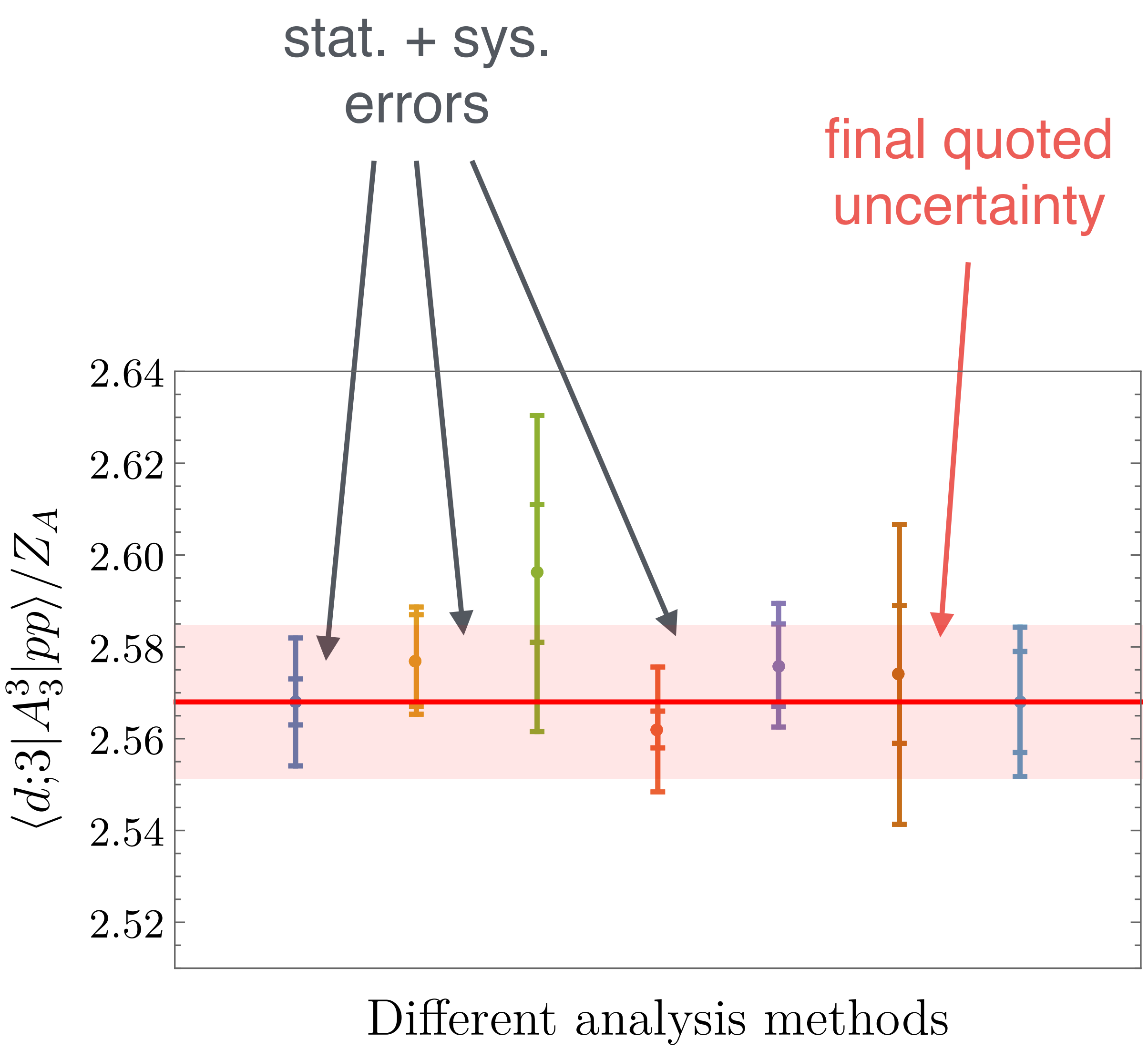
  correlated ratios of correlation functions

- Systematics in fit

  range of field strengths in fit
  t-range of plateau fit to ratio

- Systematic in analysis method

  range of analysis procedures chosen
  by different collaboration members



Proton-proton fusion

Want to relate lattice QCD ME to

- LECs of EFTs
- pp-fusion cross section

- Finite-volume quantisation condition: relate $\langle d; 3|A_3^3|pp\rangle$ to scale-indep. LECs
  - Pionless EFT: $\overline{L}_{1,A}$
  - Dibaryon formalism: $\overline{\ell}_{1,A}$
- Define a new related quantity, $L_{1,A}^{sd-2b}$, which should have mild pion-mass dependence (remove effective range terms in $\overline{L}_{1,A}$)
- Extrapolate $L_{1,A}^{sd-2b}$ to the physical point

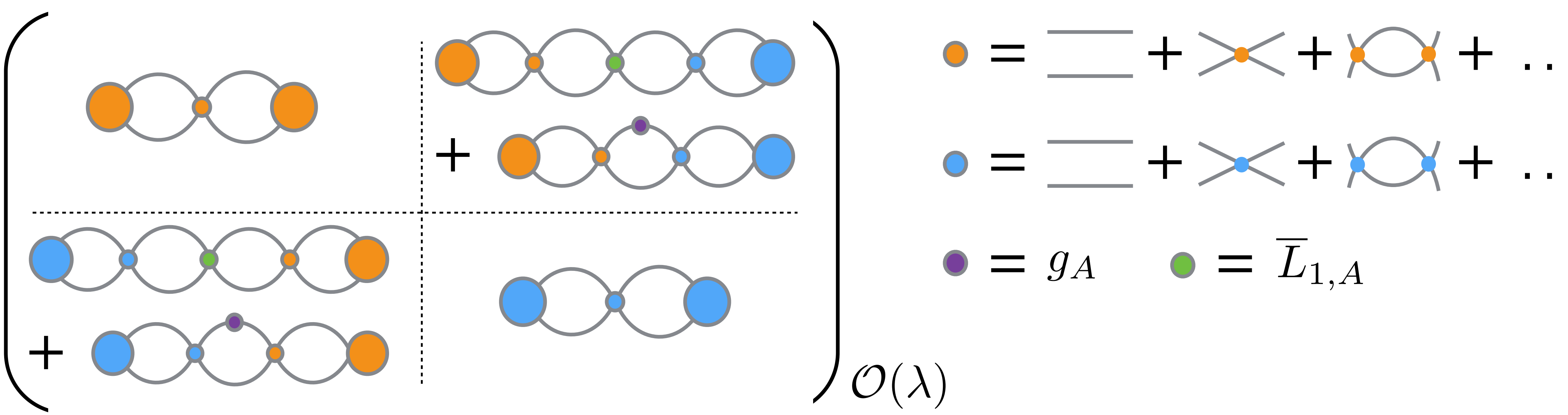
→ Prediction for $\overline{L}_{1,A}$, $\overline{\ell}_{1,A}$ at the physical point

→ Prediction for physical cross-section

Detmold and Savage, Nucl. Phys. A743, 170 (2004).

Finite-volume quantisation

- Axial field splits degeneracy of the nucleon doublet
- $^3S_1$ and $^1S_0$ channels mix
- Construct 2x2 inverse scattering amplitude matrix in background field



- Continuum integrals from bubble diagrams → discrete sums
- Det = 0 ↔ poles of scattering amplitude ↔ eigenenergies

Briceno, Davoudi ,Phys.Rev. D88 (2013) 094507

Finite-volume quantisation

- Det of inverse scattering matrix = 0 ⟷ eigenenergies are solutions of

two-body LEC

$$\left[p\cot\delta^{^3S_1} + \delta G_0^V(p;L)\right]\left[p\cot\delta^{^1S_0} + \delta G0^V(p;L)\right] = \left[W_3 g_A M \overline{L}_{1,A} - W_3 g_A G_1^V(p;L)\right]^2$$

from effective range expansion finite-volume sums weak coupling

➡ Matrix element related to scale-indep. LEC

$$|\delta E^{^3S_1-^1S_0}|/W_3 = |\langle ^3S_1\,|A_3^3|^1S_0\rangle| = Z_d^2(4g_A\gamma\overline{L}_{1,A} + 2g_A)$$

- Define combination that characterises two-nucleon contribution
  Expect mild pion-mass dependence ➡ can extrapolate

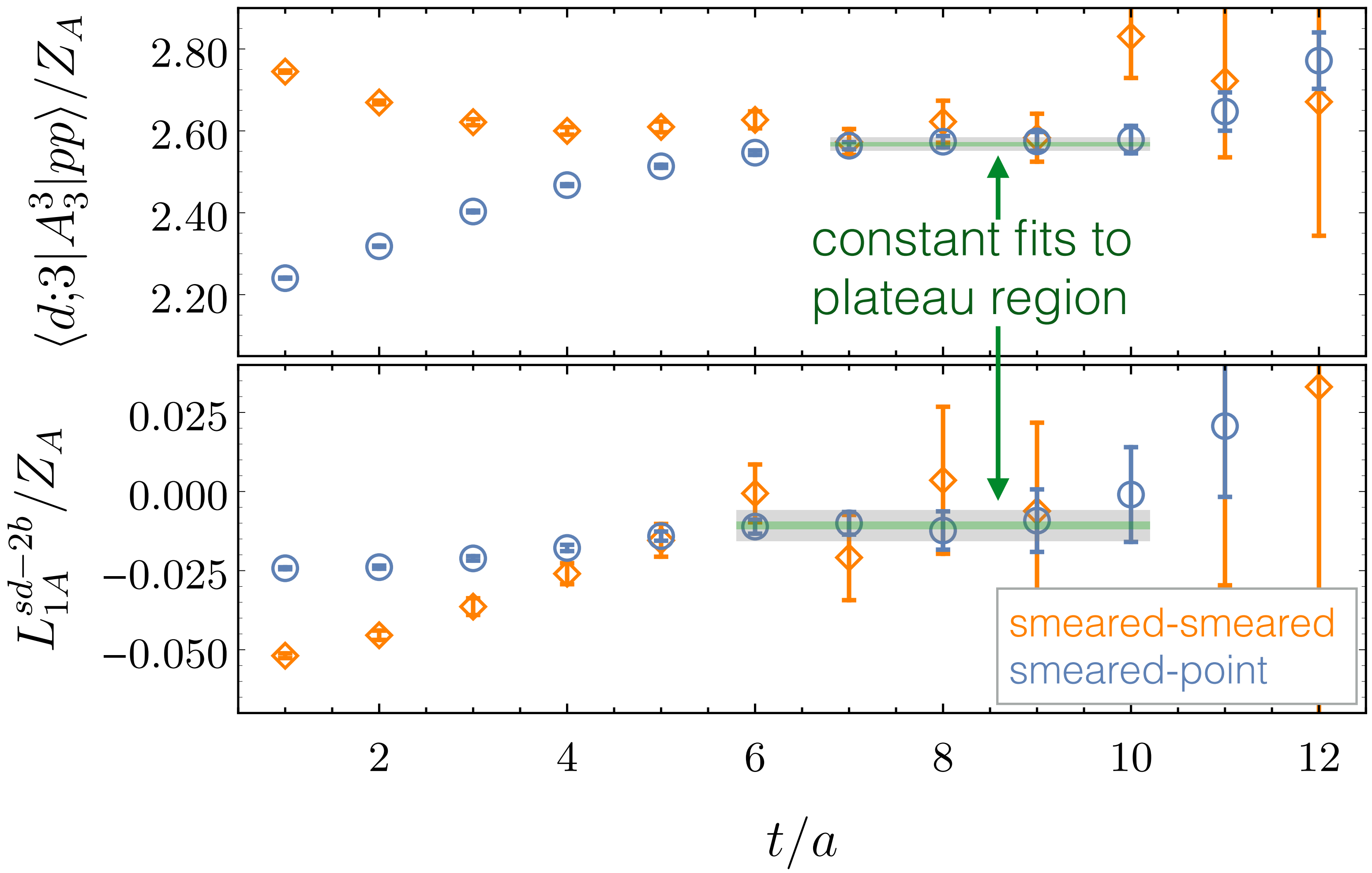
experience from $np \to d\gamma$

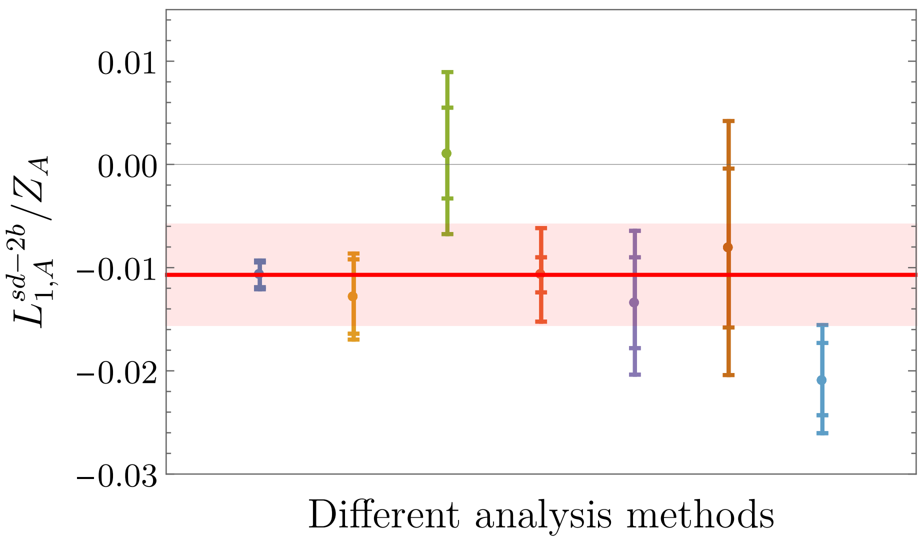
$$L_{1,A}^{sd-2b} \equiv (\langle d;3|A_3^3|pp\rangle - 2g_A)/2$$

$$Z_d = 1/\sqrt{1-\rho\gamma}$$

Briceno, Davoudi ,Phys.Rev. D88 (2013) 094507

Proton-proton fusion





$$\frac{L_{1,A}^{sd-2b}}{Z_A} = -0.0107(12)(49)$$

→ Extrapolate, predict physical cross-section

Proton-proton fusion

Low-energy cross section for $pp \to de^+\nu$ dictated by the matrix element

$$\left|\langle d; j \,|A_k^-|\, pp\rangle\right| \equiv g_A C_\eta \sqrt{\frac{32\pi}{\gamma^3}}\,\Lambda(p)\,\delta_{jk}$$

Relate $\Lambda(0)$ to extrapolated LEC using EFT

$$\Lambda(0) = \frac{1}{\sqrt{1-\gamma\rho}}\{e^\chi - \gamma a_{pp}[1 - \chi e^\chi \Gamma(0,\chi)] + \frac{1}{2}\gamma^2 a_{pp}\sqrt{r_1\rho}\} - \frac{1}{2g_A}\gamma a_{pp}\sqrt{1-\gamma\rho}\,\boxed{L_{1,A}^{sd-2b}}$$

extrapolated lattice value

C_η	Sommerfield factor
γ	Deuteron binding mtm
$r_1,\ \rho$	Effective ranges
a_{pp}	pp scattering length
$\Gamma(0,\chi)$	Incomplete gamma func.
	$\chi = \alpha M_p/\gamma$

N[2]LO $\not\pi$ EFT with effective range contributions resummed using the dibaryon approach

Butler and Chen, Phys. Lett. B520, 87 (2001)
Detmold and Savage, Nucl. Phys. A743, 170 (2004).

Proton-proton fusion

Physical cross-section dictated by

$$\Lambda(0) = 2.6585(6)(72)(25)$$

statistical

systematic

- fitting
- analysis
- uncertainties of phys. mass inputs

quark mass extrap. (50% additive)

Can also extract

$$L_{1,A} = 3.9(0.1)(1.0)(0.3)(0.9)\ \mathrm{fm}^3$$

renormalisation scale $\mu = m_\pi$

higher-order $\not\pi$ EFT corrections (power-counting)

Proton-proton fusion

Physical cross-section dictated by

$$\Lambda(0) = 2.6585(6)(72)(25)$$

$$\Lambda(0) = 2.652(2)$$ (phenomenology)

E. G. Adelberger et al., Rev. Mod. Phys. 83, 195 (2011)

Can also extract

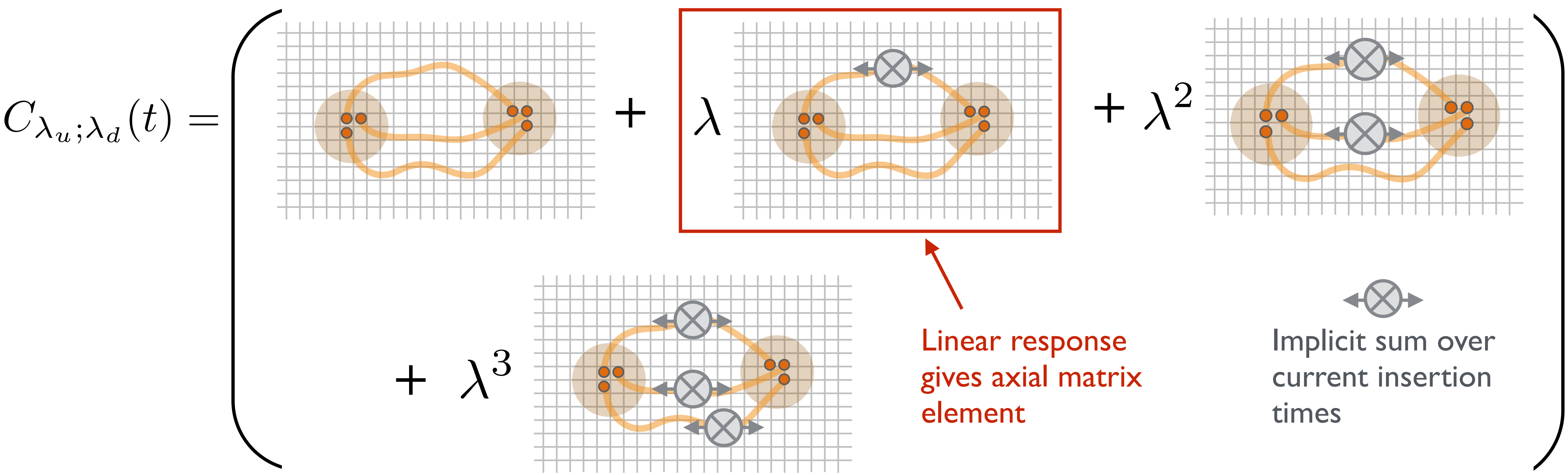
$$L_{1,A} = 3.9(0.1)(1.0)(0.3)(0.9)\ \mathrm{fm}^3$$

$$L_{1,A} = 3.6(5.5)\ \mathrm{fm}^3$$ (reactor expts.)

M. Butler, J.-W. Chen, and P. Vogel, Phys. Lett. B549

Higher-order insertions

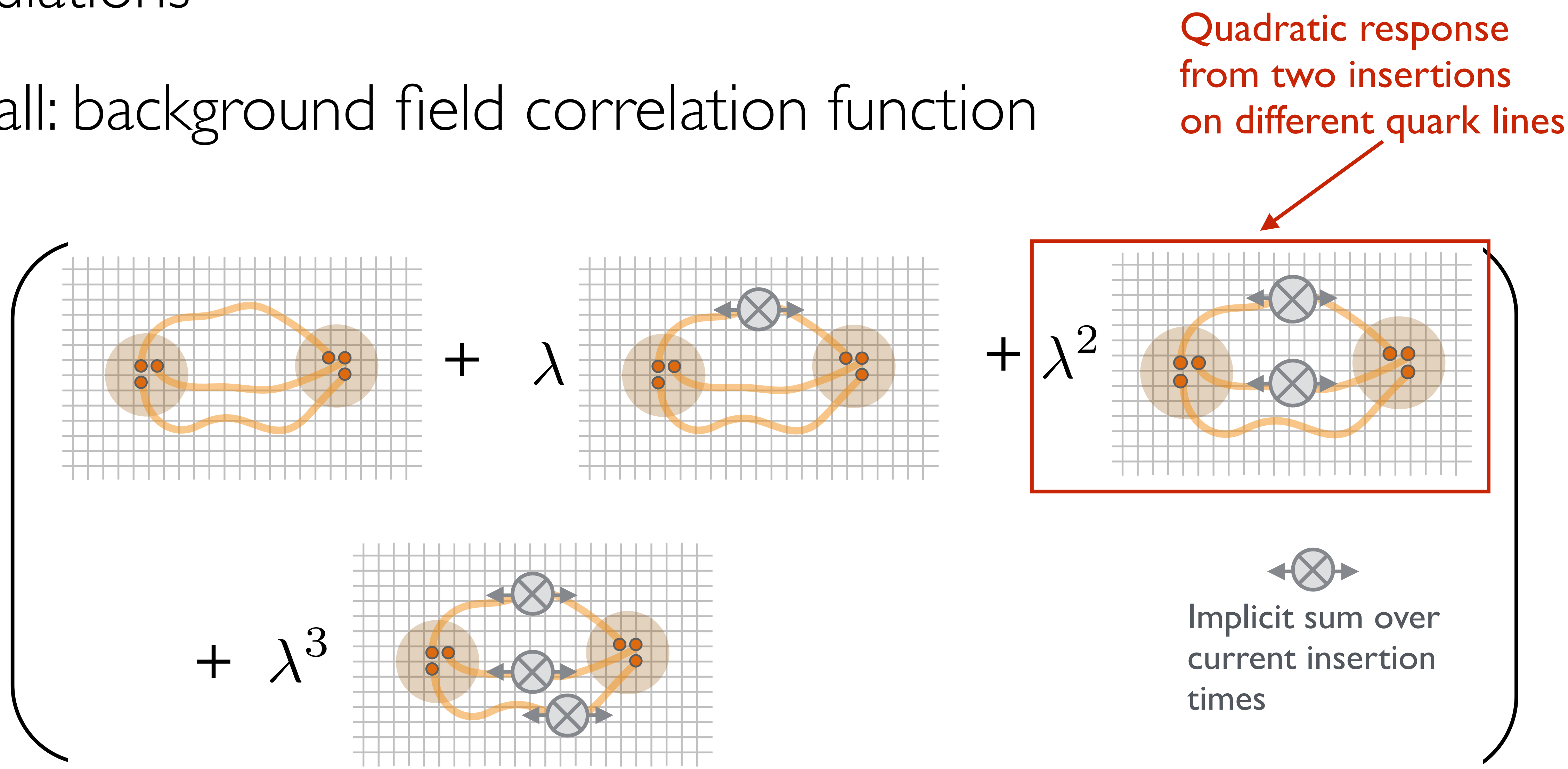
- Can access terms with more current insertions from same calculations
- Recall: background field correlation function



Higher-order insertions

- Can access terms with more current insertions from same calculations
- Recall: background field correlation function

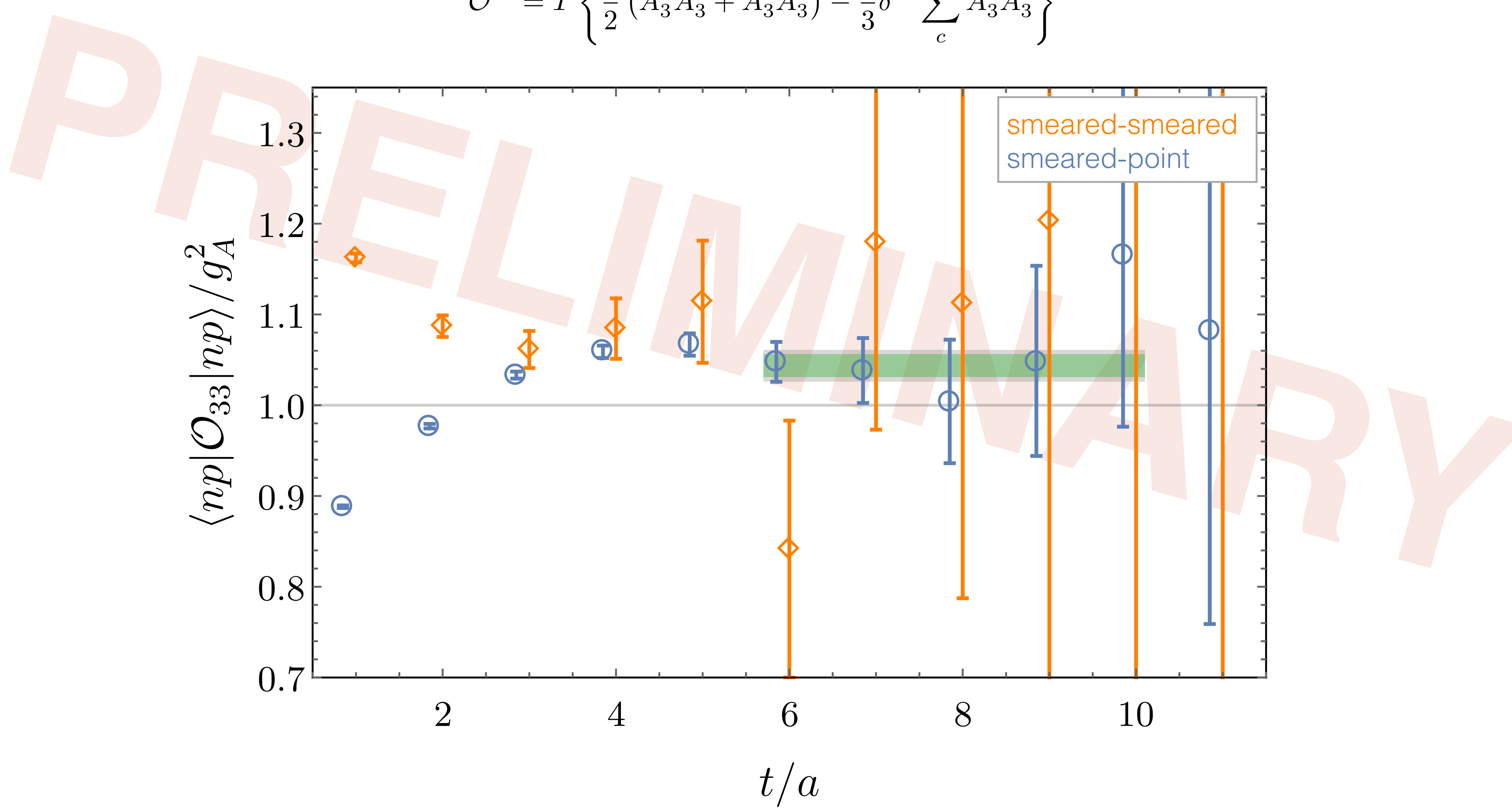
$$C_{\lambda_u;\lambda_d}(t) =$$



Axial isotensor polarisability

e.g., Dibaryon axial isotensor polarisability relevant to neutrinoful double-beta decay

$$\mathcal{O}^{ab} = T\left\{\frac{1}{2}\left(A_3^a A_3^b + A_3^b A_3^a\right) - \frac{1}{3}\delta^{ab}\sum_c A_3^c A_3^c\right\}$$



Summary

- Nuclei can be studied directly from QCD
- Current state-of-the-art: significant systematics but phenomenologically interesting at current precision
  - Spectroscopy of nuclei
  - Structure, i.e., magnetic moments, polarisabilities
  - **Electroweak interactions**
- Nuclear matrix elements important to experimental programs e.g,
  - Neutrino breakup reaction (SNO)
  - Muon capture reaction (MuSun)
  - Double-beta decay