Nucleon structure from Lattice QCD

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Frontiers in Nuclear Physics
Kavli Institute for Theoretical Physics,
Santa Barbara, *Sep 1, 2016*

Outline

- Nucleon structure on a lattice
  Methods and challenges
- Nucleon form factors
  Nucleon form factors, radii, magnetic moment at nearly-physical point
  Strangeness in nucleon form factors
  Axial vector current
- Neutron-antineutron oscillation matrix elements
- Quark momentum and spin
  Quark contributions to the nucleon momentum
  Nucleon spin puzzle and quark spin and angular momentum

Hadron Correlators in Lattice QCD

Lattice Field Theory ⇔ Numerical evaluation of the Path Integral

$$\langle q_x \bar{q}_y \ldots \rangle = \int \mathcal{D}\Big(Glue\Big) \int \mathcal{D}\Big(Quarks\Big)\; e^{-S_{Glue} - \bar{q}\left(\not{D}+m\right)q} \; \left[q_x \bar{q}_y \ldots\right]$$

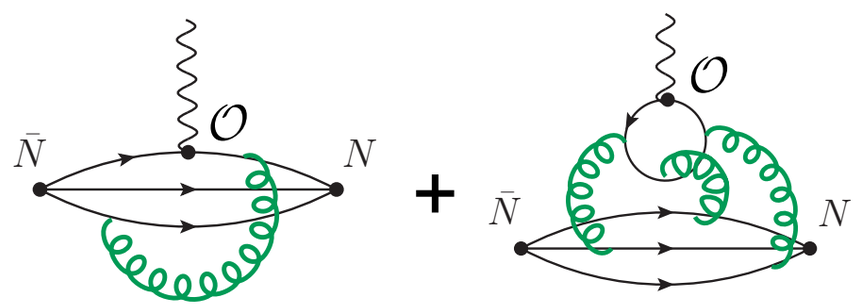
Grassmann integration

$$= \int \underbrace{\mathcal{D}\Big(Glue\Big)\; e^{-S_{Glue}}\; \mathrm{Det}\left(\not{D}+m\right)}_{\text{Hybrid Monte Carlo}} \; \left[\left(\not{D}+m\right)^{-1}_{x,y} \ldots\right]$$

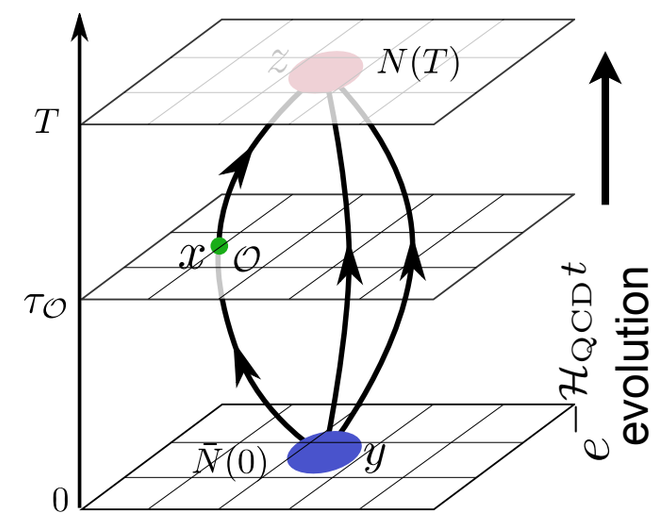
$(\not{D}+m)\cdot q = 0$
quark motion in
gluon background

Hadron Matrix Elements:

$$C^{\mathcal{O}}_{3\mathrm{pt}}(T) = \langle N(T)\mathcal{O}(\tau)\bar{N}(0)\rangle =$$



"connected" "disconnected"



Each quark line = $(\not{D}+m)^{-1}\cdot\psi$

$$\langle N(T)\mathcal{O}(\tau)N(0)\rangle = \sum_{n,m} Z_m e^{-E_n(T-\tau)} \langle n|\mathcal{O}|m\rangle e^{-E_m\tau} Z_n^*$$

$$\xrightarrow[T\to\infty]{} Z_{00} e^{-M_N T}\Big[\langle P'|\mathcal{O}|P\rangle + \mathcal{O}\big(\underbrace{e^{-\Delta E_{10}T}, e^{-\Delta E_{10}\tau}, e^{-\Delta E_{10}(T-\tau)}}_{\text{excited states}}\big)\Big]$$

Excited states contribute to correlators and may (and do) bias results

Computational Challenges in Lattice QCD

Taking limit $V \to \infty, \quad a \to 0, \quad m_\pi \to m_\pi^{\rm phys}$ is challenging

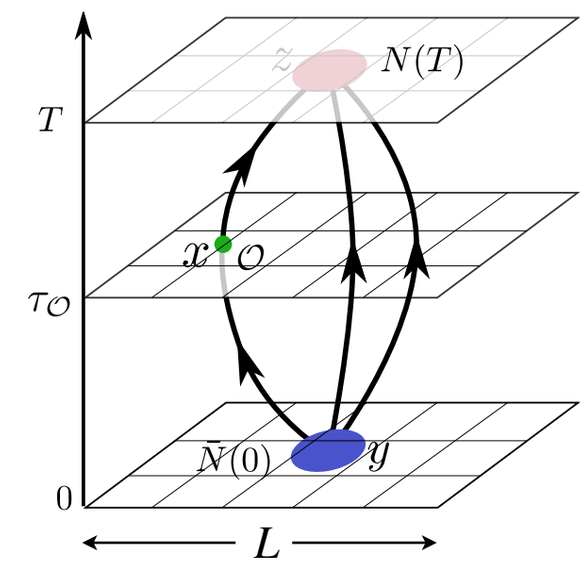
- MC noise is determined by the lightest degree of freedom

$$|(\not{D} + m_q)^{-1}_{x,y}| \sim e^{-\frac{1}{2} m_\pi |x-y|}$$

for N quarks, $\quad \text{Noise} \sim \exp\left[-\frac{N m_\pi}{2} t\right]$

for nucleons, $\quad \frac{\text{Signal}}{\text{Noise}} \sim \exp\left[-(m_N - \frac{3}{2} m_\pi) t\right]$

[Lepage (1989)]



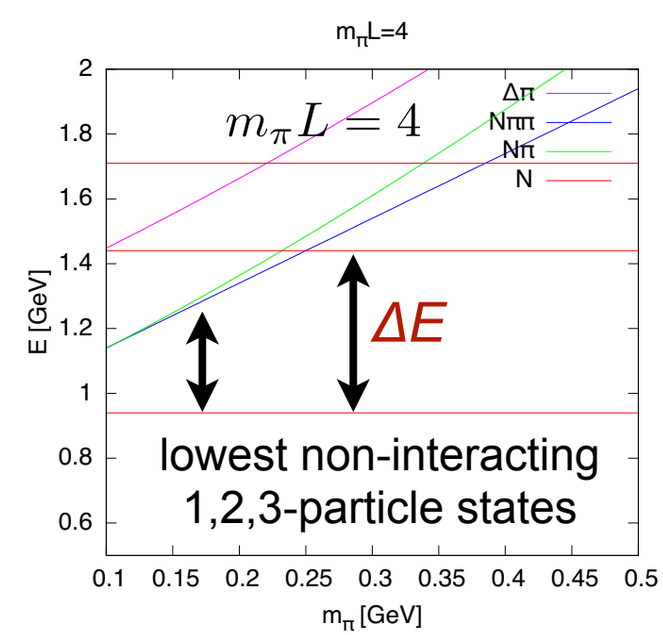
- finite volume effects

require box size $\quad L \gtrsim (4 \ldots 6) \cdot \frac{1}{m_\pi}$

- as $m_\pi \to$ physical, excited states become denser

Excited state corrections to the ground state: $\quad \sim \mathcal{O}(|Z_{10}|^2 e^{-\Delta E_{10} T})$

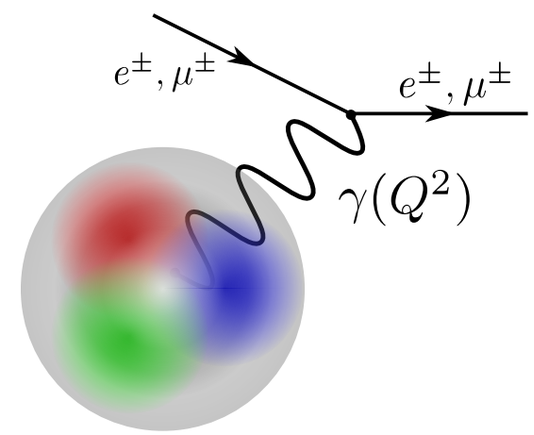
Addressing excited states requires
  - Multi-state fits
  - Variational methods

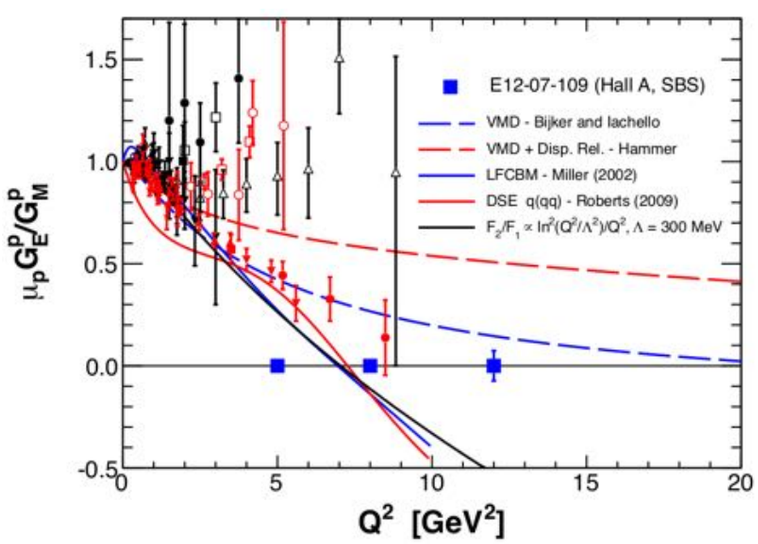


Nucleon Electromagnetic Form Factors

$$\langle P+q|\,\bar{q}\gamma^{\mu}q\,|P\rangle \;=\; \bar{U}_{P+q}\Big[\,F_1(Q^2)\,\gamma^{\mu} + F_2\,(Q^2)\,\frac{i\sigma^{\mu\nu}q_{\nu}}{2M_N}\Big]U_P$$

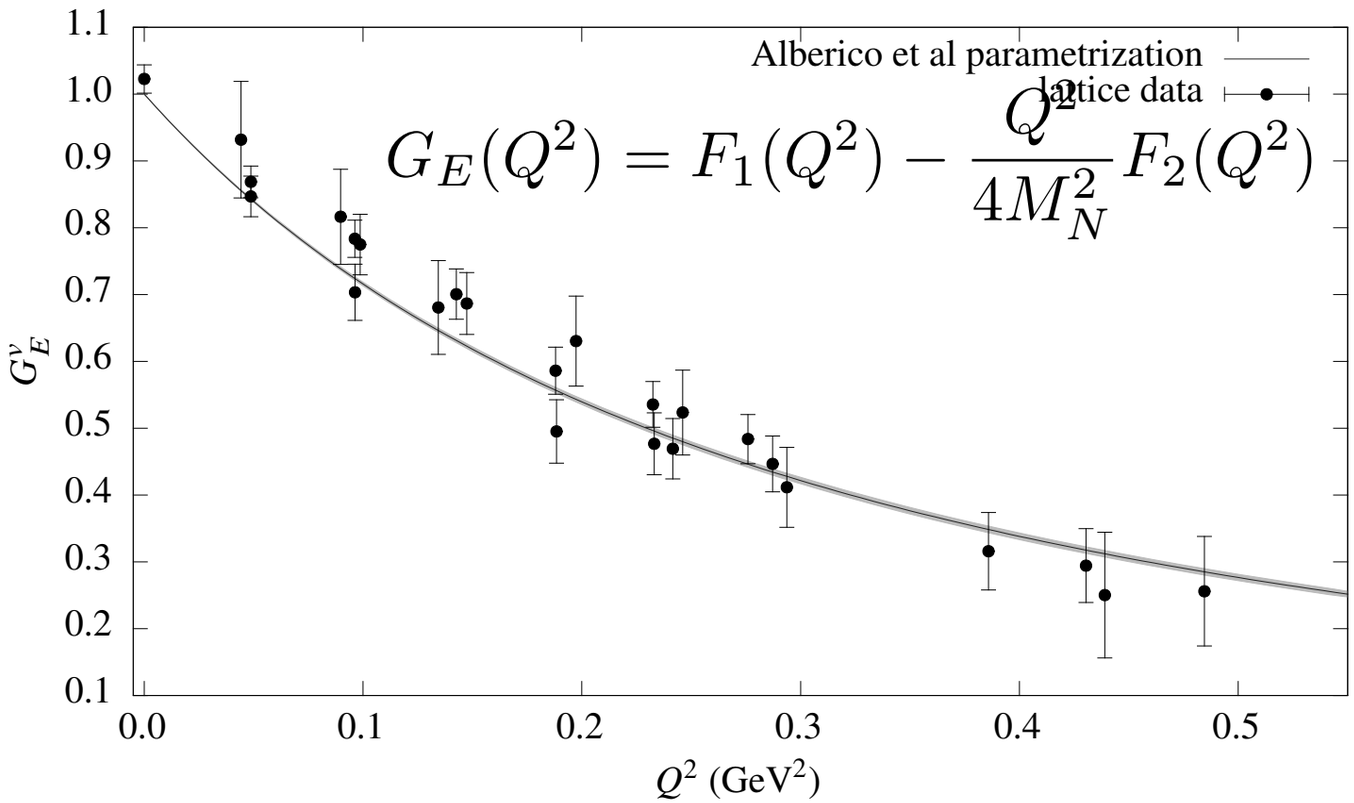
- JLab@12GeV : explore form factors at $Q^2 \gtrsim 10$ GeV^2
  - (F_1 / F_2) scaling at $Q^2 \to \infty$
  - (G_E/G_M) dependence up to Q^2=18 GeV^2
  - *u*-, *d*-flavor contributions to form factors
- Proton radius puzzle: 7σ difference
  - JLab pRAD experiment
  - MUSE@PSI : $e^{\pm}$ / $\mu^{\pm}$-scattering off the proton

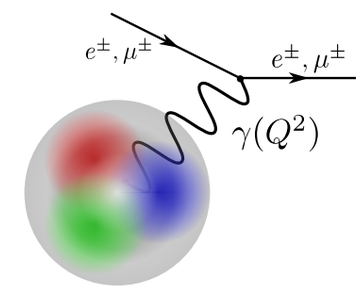




[Research Mgmt. Plan for SBS(JLab Hall A)]

Nucleon (p-n) Form Factors vs Pheno





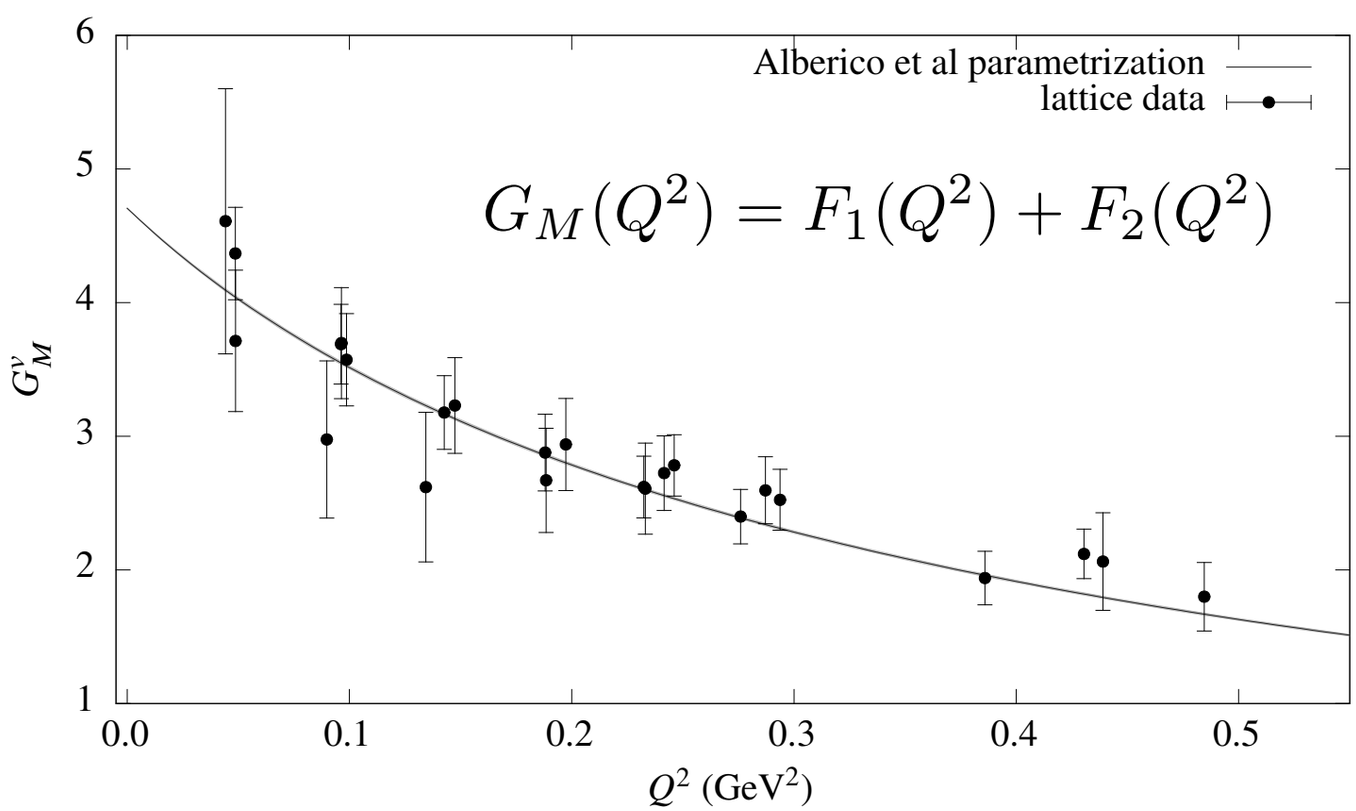
$$\langle P+q|\,\bar{q}\gamma^\mu q\,|P\rangle = \bar{U}_{P+q}\Big[F_1(Q^2)\gamma^\mu + F_2(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}\Big]U_P$$

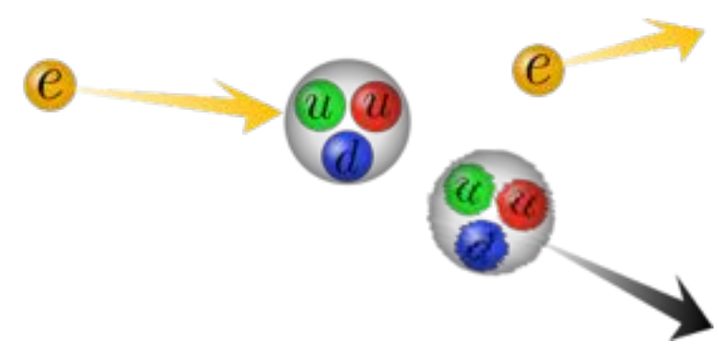
Lattice calculations with m_π=149 MeV

[J.Green, SNS, et al 1209.1687; PLB734:290]

vs phenomenology

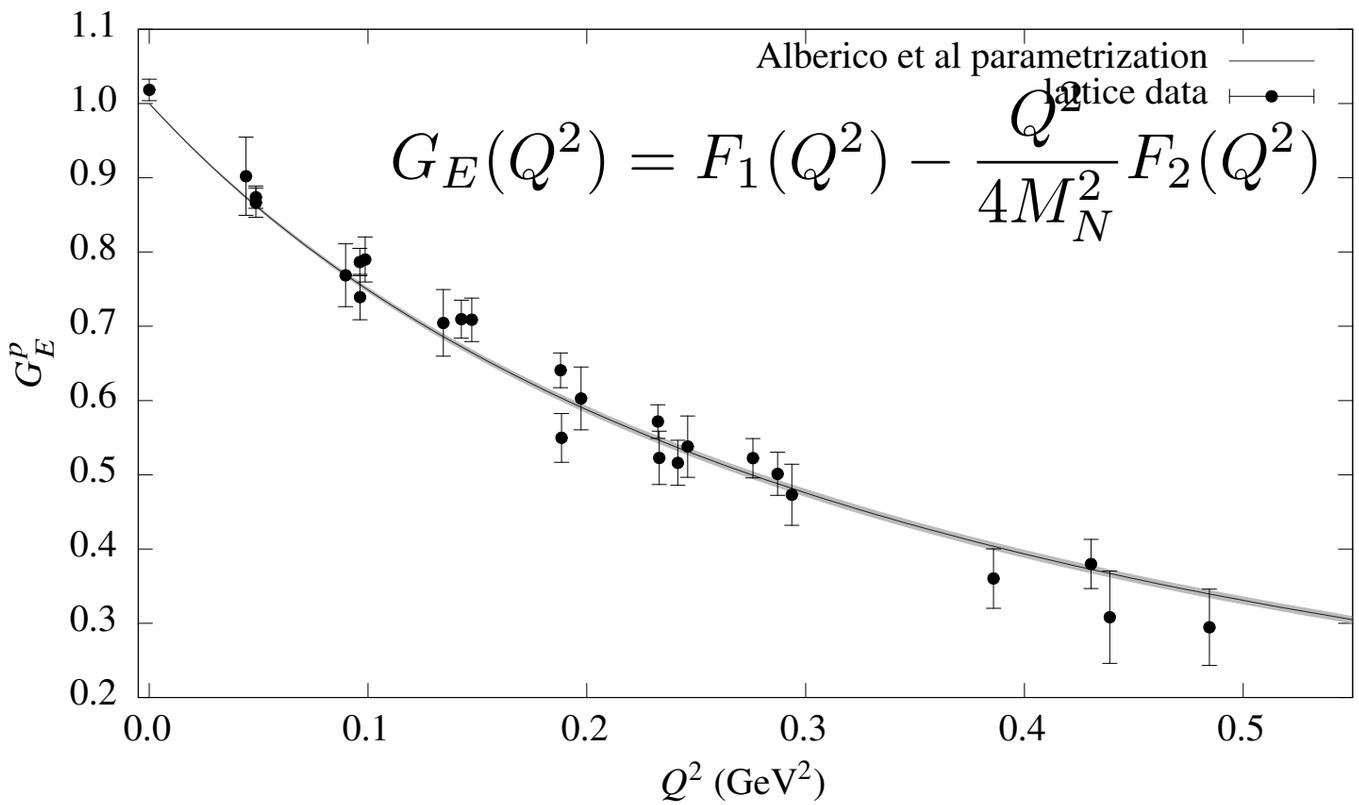
[W.M.Alberico et al, PRC79:065204(2009)]

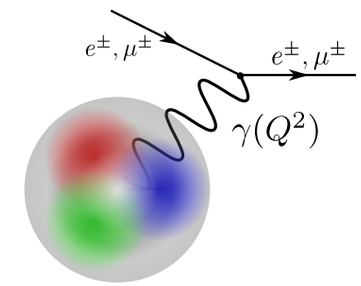




Elastic scattering with electromagnetic probes $e^\pm$, $\mu^\pm$

Proton Form Factors vs Pheno (conn. only)





$$\langle P+q|\,\bar{q}\gamma^\mu q\,|P\rangle = \bar{U}_{P+q}\Big[F_1(Q^2)\gamma^\mu + F_2(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}\Big]U_P$$

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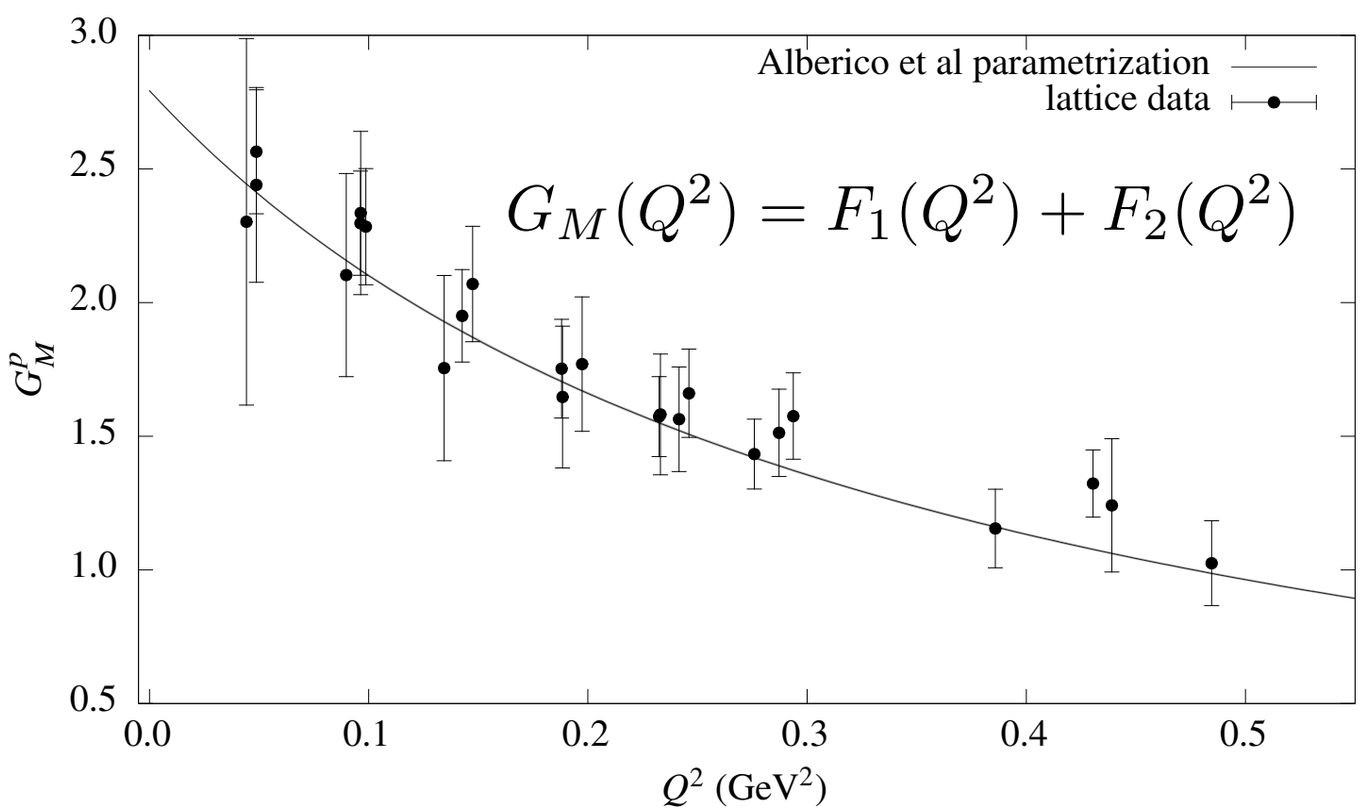
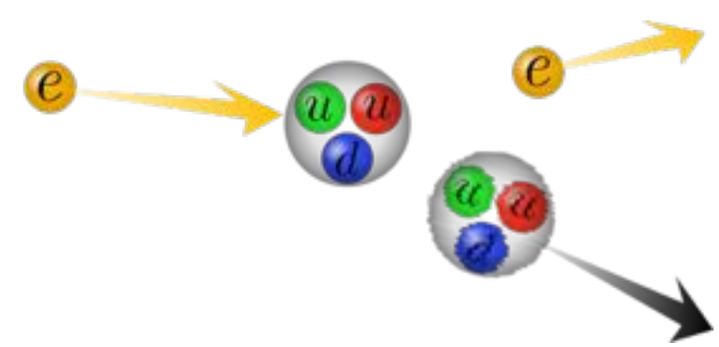




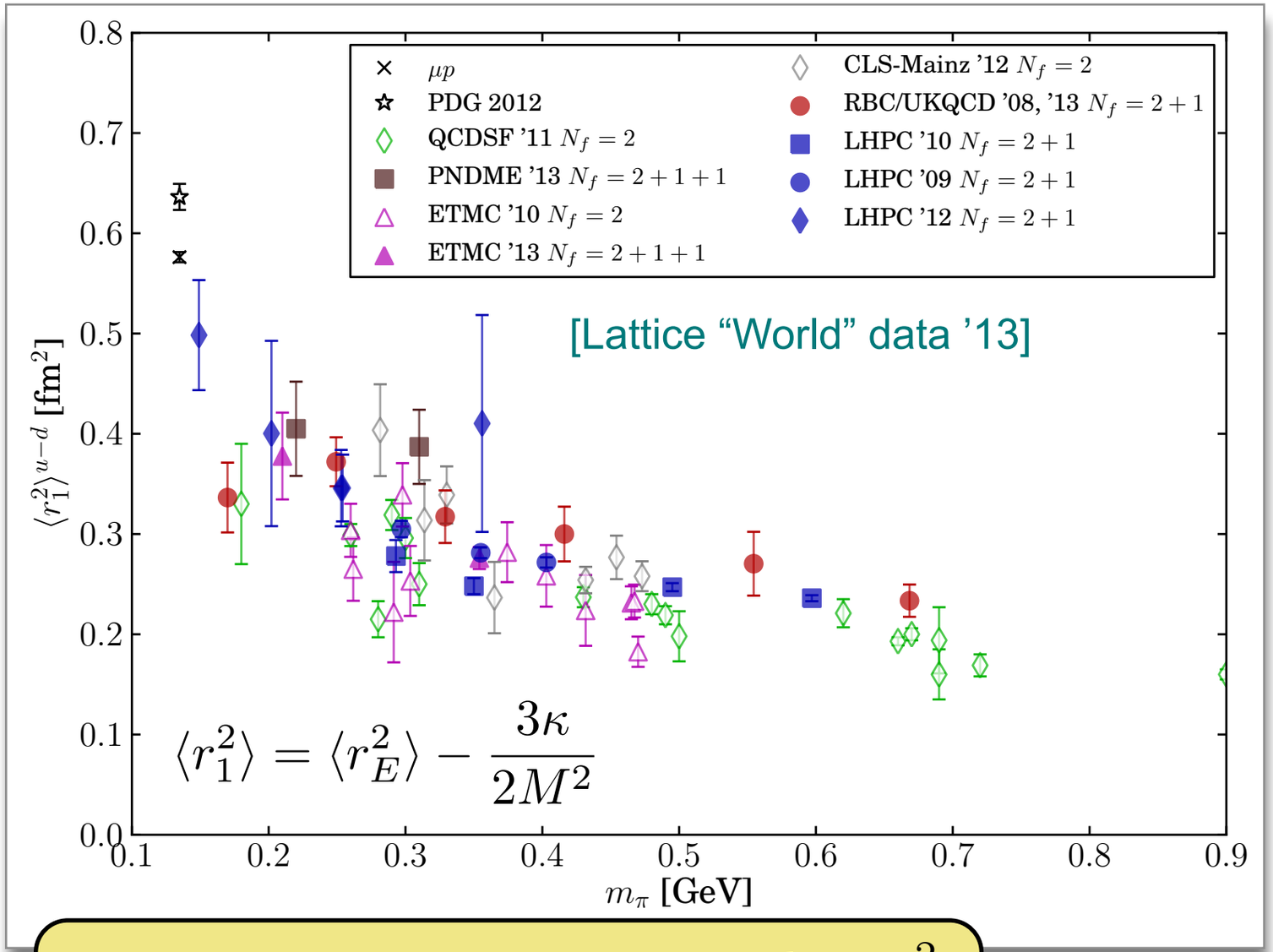
Elastic scattering with electromagnetic probes $e^\pm$, $\mu^\pm$

No disconnected part! (negligible in this case)

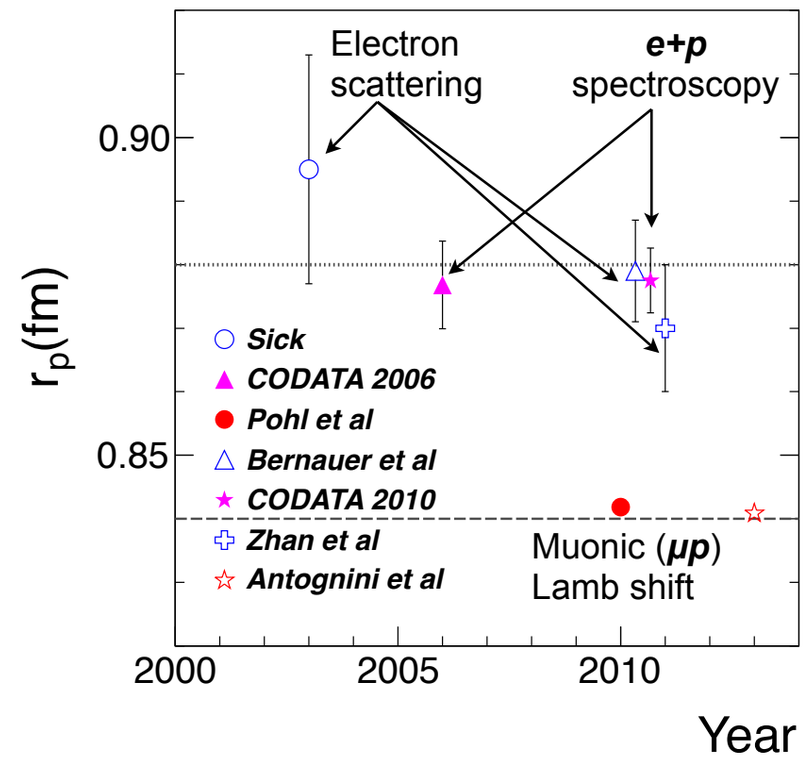
Dirac Radius vs. m_π and Proton Size Puzzle

$$F_1^{p-n}(Q^2) = F_1^{u-d}(Q^2) \approx 1 - \frac{1}{6}Q^2\langle r_1^2\rangle^{u-d} + O(Q^2)$$

$$G_{Ep}(Q^2) \approx 1 - \frac{1}{6}Q^2\langle r_E^2\rangle^{p} + O(Q^4)$$



ChPT predicts divergence $\sim \log m_\pi^2$



[MuSE white-paper, 1303.2160]

Issues with e-p experiments?

- *underestimated combined error*
- *use Q^2 fits up to 1 GeV^2*

Dirac Radius vs. m_π and Proton Size Puzzle

$$F_1^{p-n}(Q^2) = F_1^{u-d}(Q^2) \approx 1 - \frac{1}{6}Q^2\langle r_1^2\rangle^{u-d} + O(Q^2)$$

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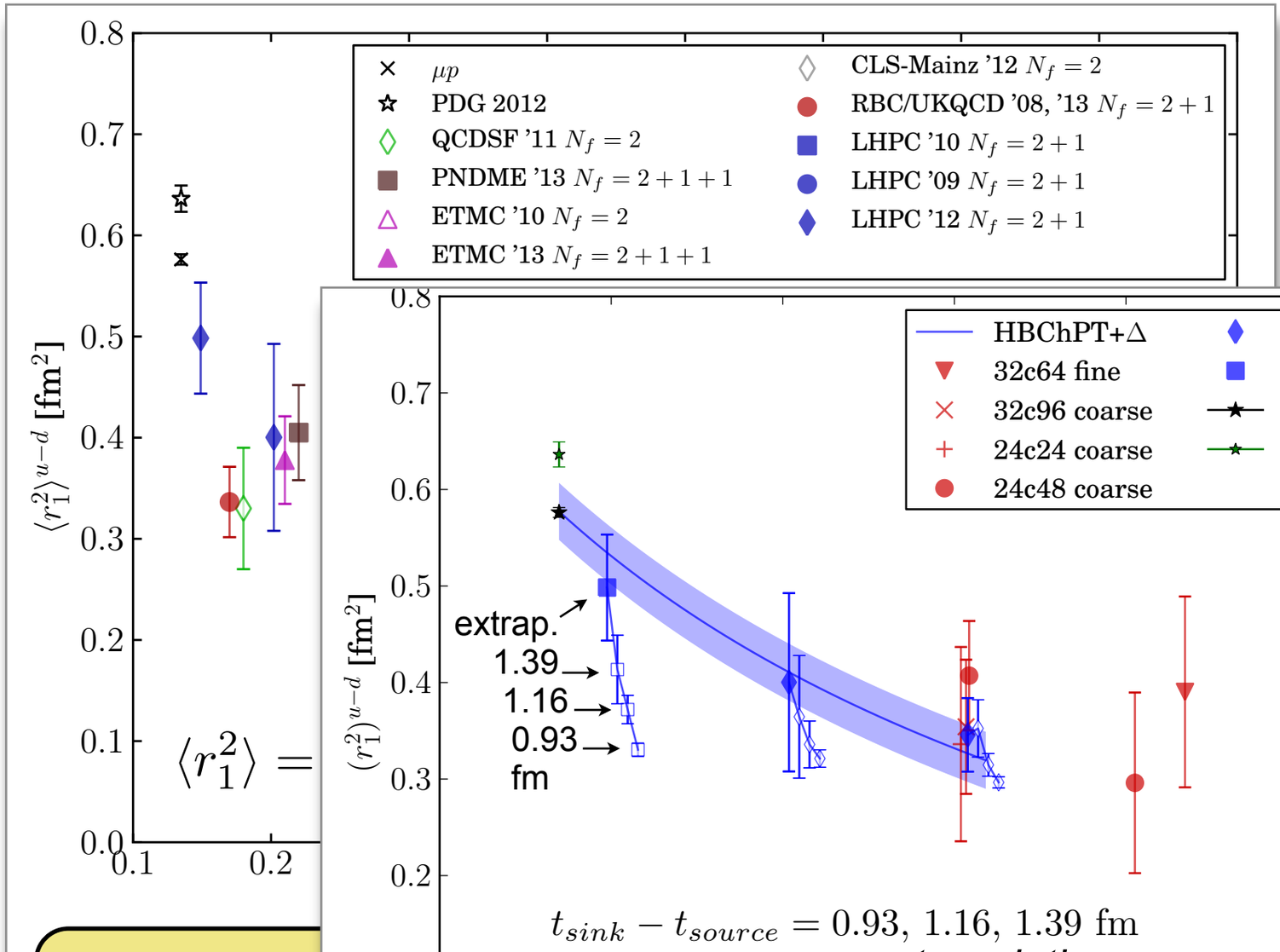
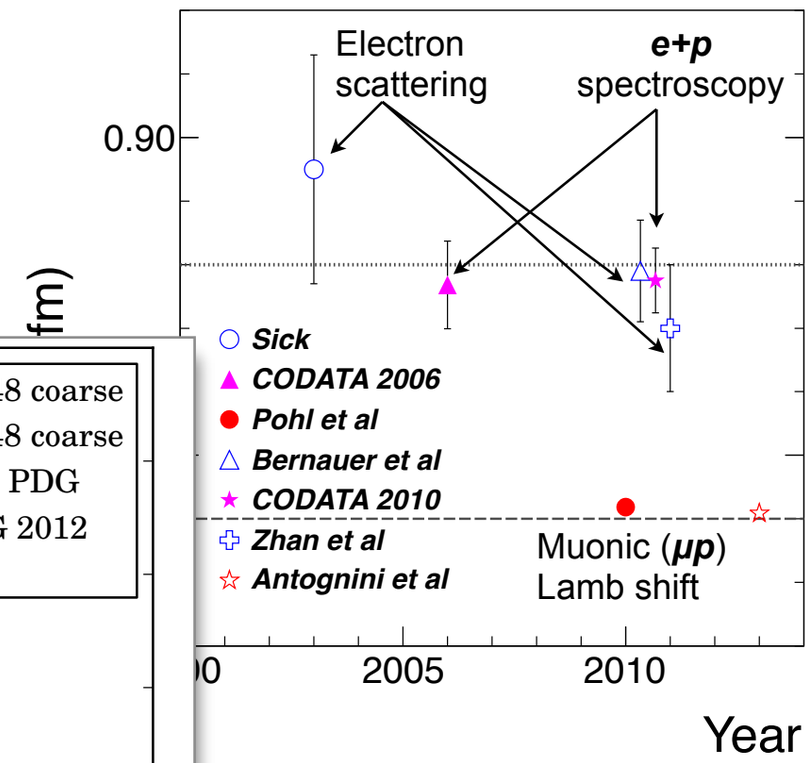




MuSE white-paper, 1303.2160]

es with e-p experiments?
nderestimated combined error
se Q^2 fits up to 1 GeV^2

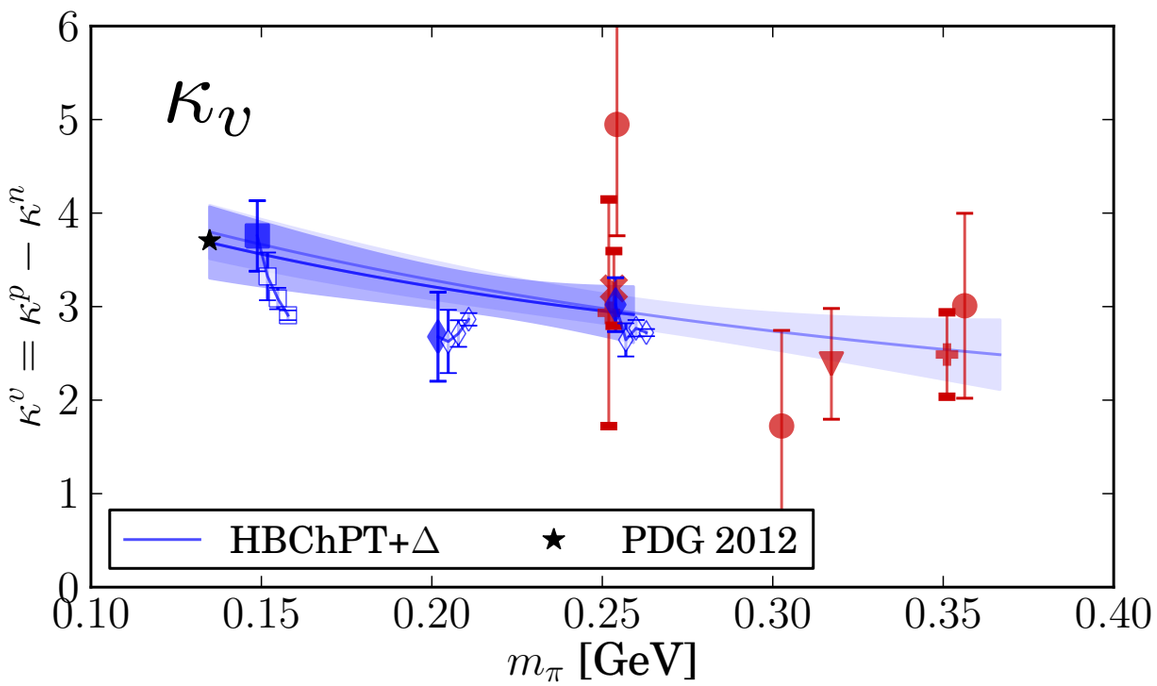
ChPT pred

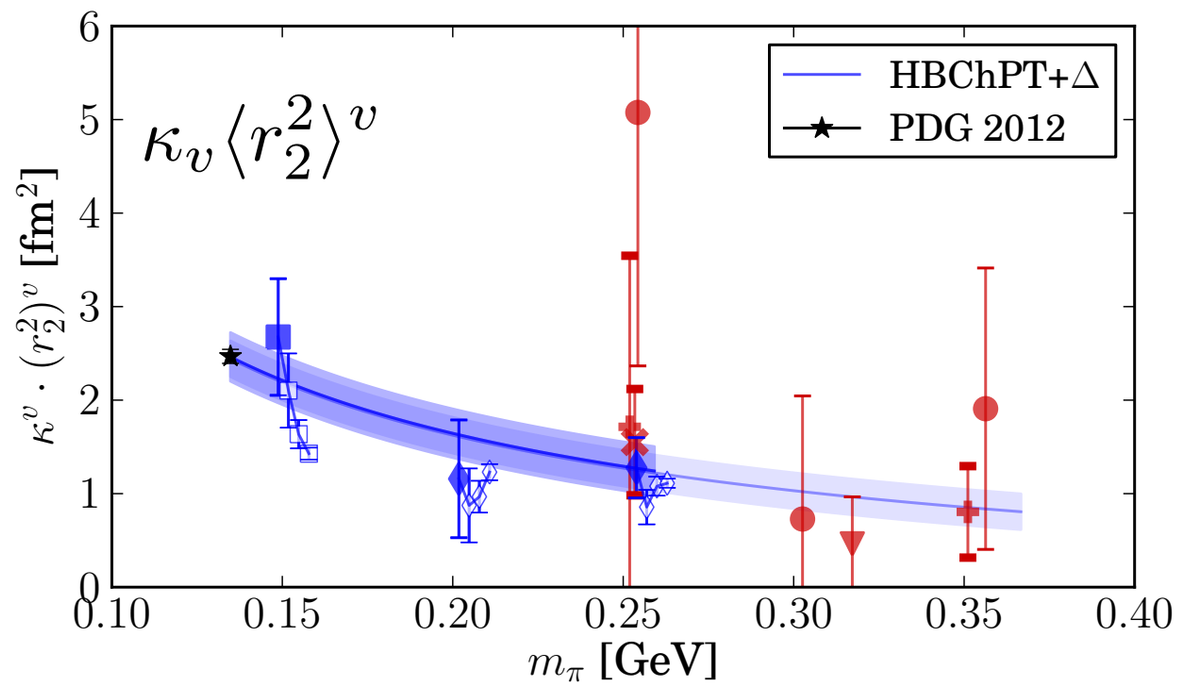
Excited states are worse for m_π<200 MeV

Isovector Magnetic Moment vs. m_π

$$\langle P+q|\,\bar{q}\gamma^\mu q\,|P\rangle = \bar{U}_{P+q}\Big[F_1(Q^2)\,\gamma^\mu + F_2(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}\Big]U_P$$

$$F_2^{u-d}(Q^2) \approx \kappa_v\Big[1-\frac{1}{6}Q^2\langle r_2^2\rangle^v + \mathcal{O}(Q^4)\Big]$$





m_π=149 MeV N_f=2+1 clover-imp.Wilson [J.R.Green, SNS et al (LHPc)]

$F_2(0)$ value is extrapolated from $Q_{min} \approx 0.05\ \mathrm{GeV}^2$ $\quad F_2(Q^2) = \dfrac{\kappa}{(1+Q^2/M^2)^2}$

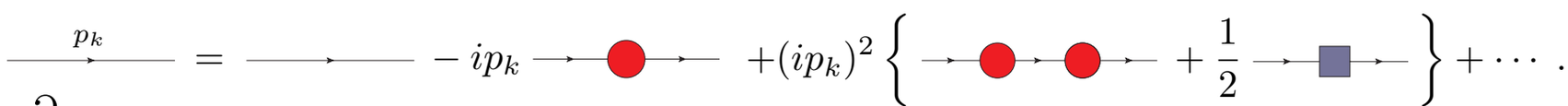
Larger L_s, smaller $Q^2_{\min}$ are desirable

OR use twisted boundary conditions

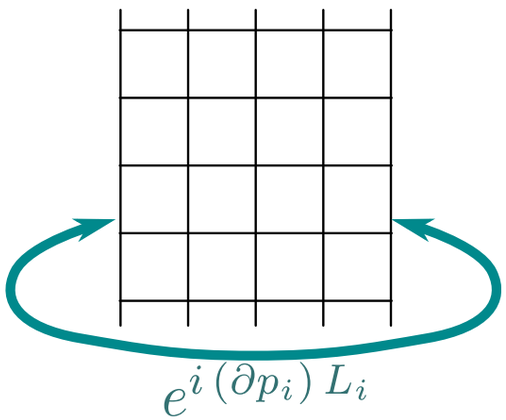
Expansion in Boundary Conditions

Derivative of correlators w.r.t. momentum = infinitesimal BC twisting

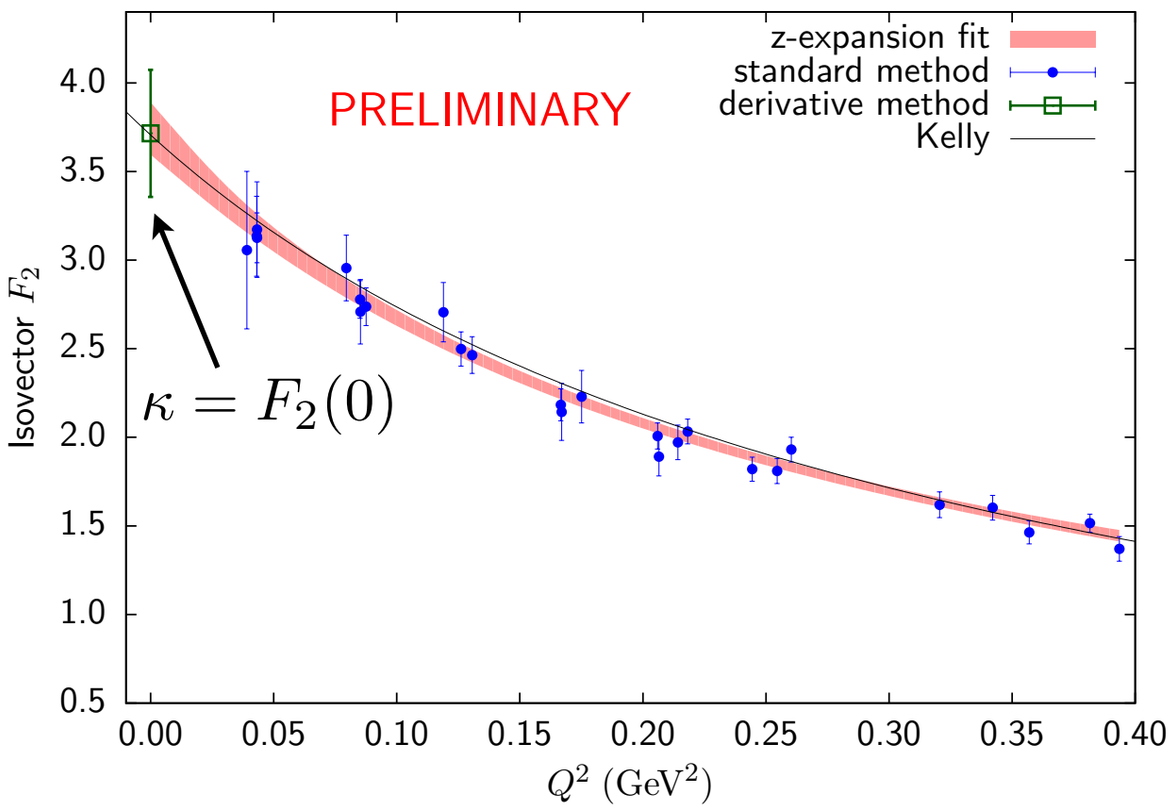
Rome method: (Phys. Lett. B 718, 589 (2012) [arXiv:1208.5914])

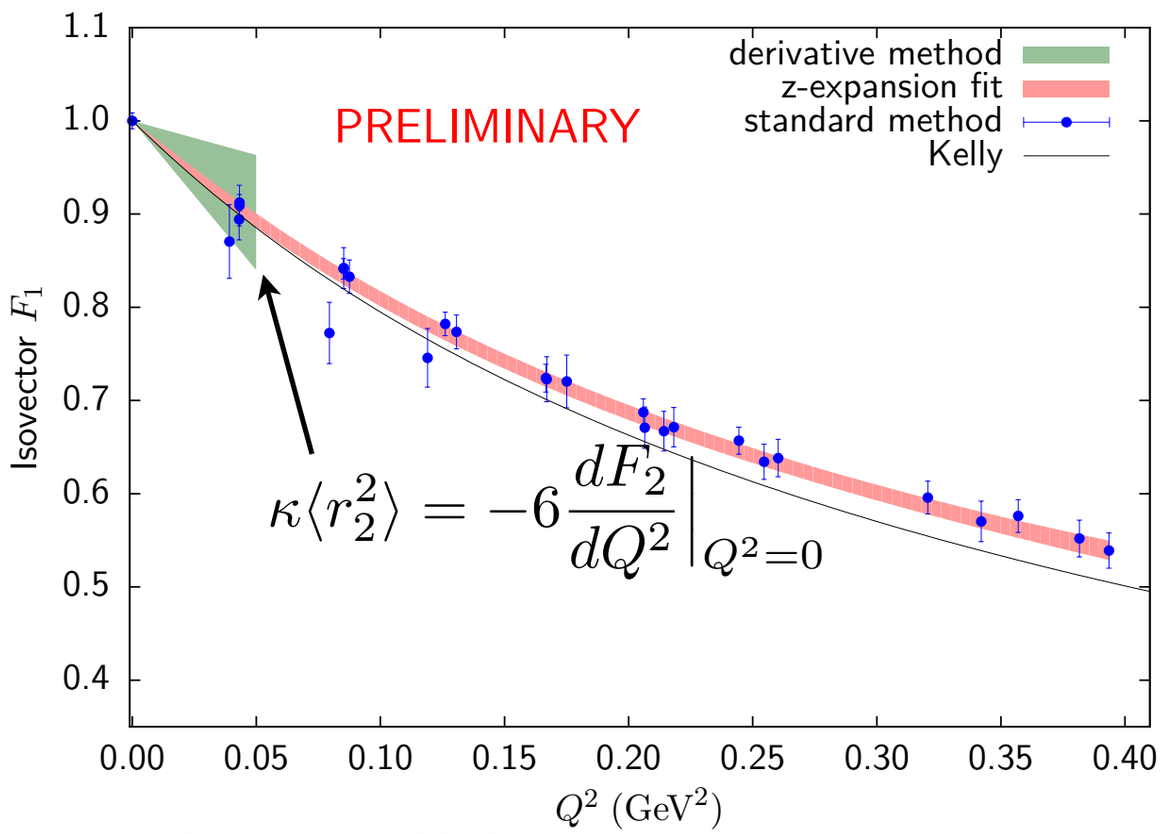


$$\frac{\partial}{\partial p_k} G(x,y;\vec{p})\Big|_{\vec{p}=0} = -i \sum_z G(x,z)\, \Gamma_V^k\, G(z,y)$$



Physical point m_π=134 MeV





[N.Hasan, J.Green, S.Meinel et at (LHPc), Lattice 2016]

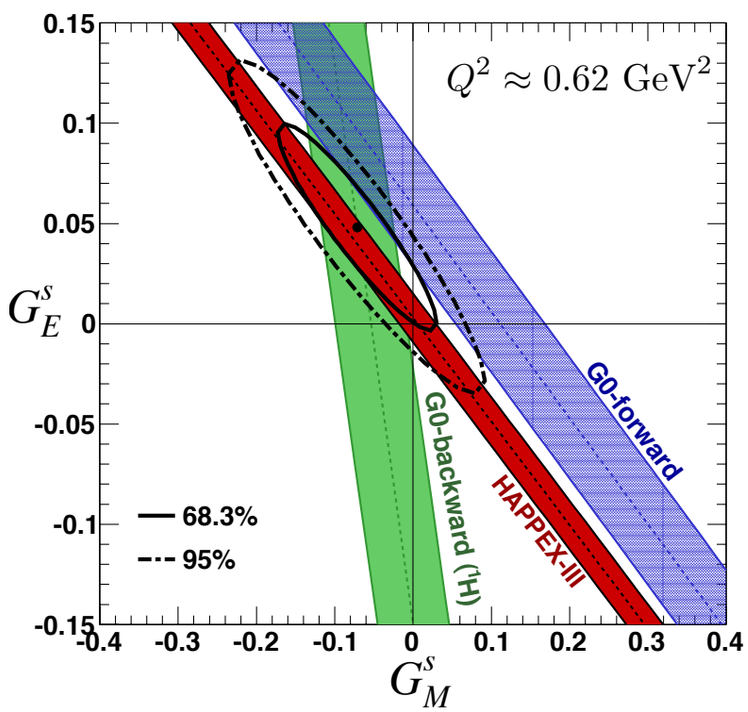
Strangeness in EM form factors

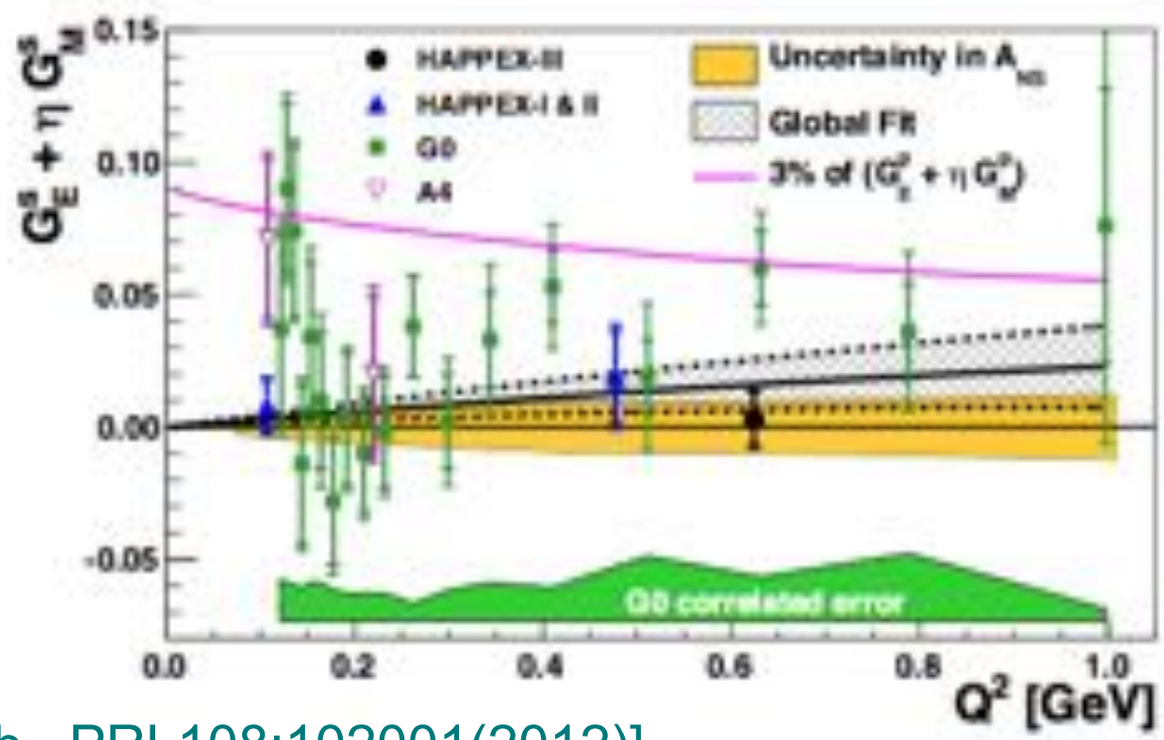
Strange quark contribution to EM: the next after light quarks

$$G_{E,M}^{p,\gamma} = \frac{2}{3}G_{E,M}^{u} - \frac{1}{3}(G_{E,M}^{d} + G_{E,M}^{s})$$
$$G_{E,M}^{n,\gamma} = \frac{2}{3}G_{E,M}^{d} - \frac{1}{3}(G_{E,M}^{u} + G_{E,M}^{s})$$
$$G_{E,M}^{p,Z} = (1 - \frac{8}{3}s_W^2)G_{E,M}^{u} + (-1 + \frac{4}{3}s_W^2)(G_{E,M}^{d} + G_{E,M}^{s})$$

$G_{E,M}^{s}$ are measured e.g. in *e–p* elastic scattering asymmetry (SAMPLE, HAPPEX, G0, A4) from

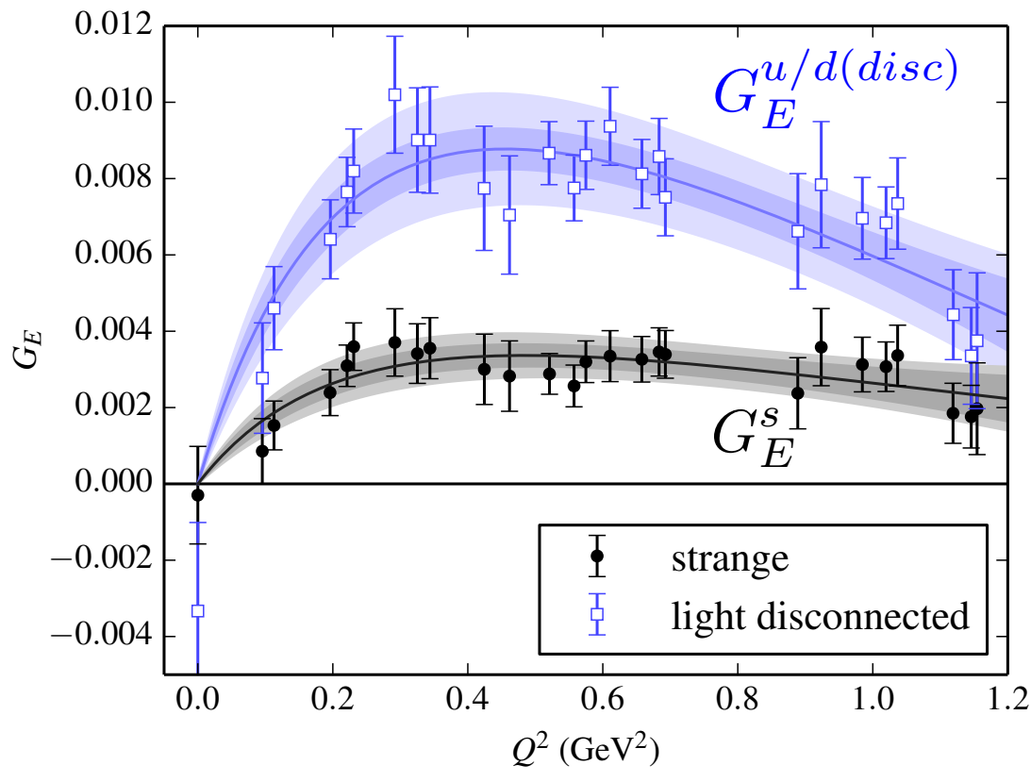
$$A_{LR} = \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L} \sim A_{non-strange} + \eta_E G_E^s + \eta_M G_M^s$$





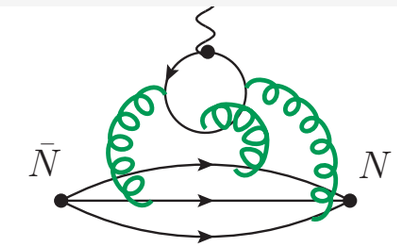
[HAPPEX collab., PRL108:102001(2012)]

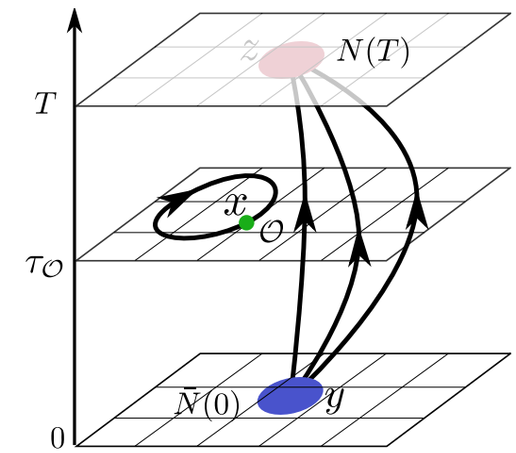
Disconnected Contractions for Nucleon FF's

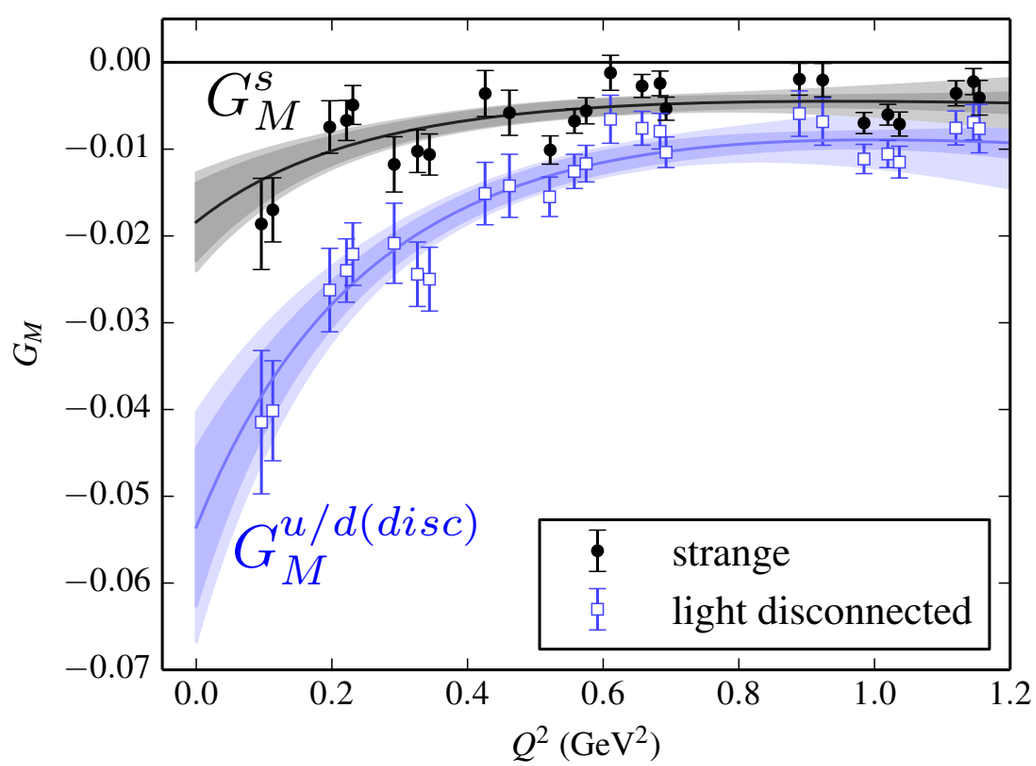


Calculation with
mπ=319 MeV
(USQCD/JLab lattices)

$$|G_{E,M}^{s,u/d(disc)}| \lesssim 1\% |G_{E,M}|$$







Strange contributions to EM radii
and magnetic moment of the proton

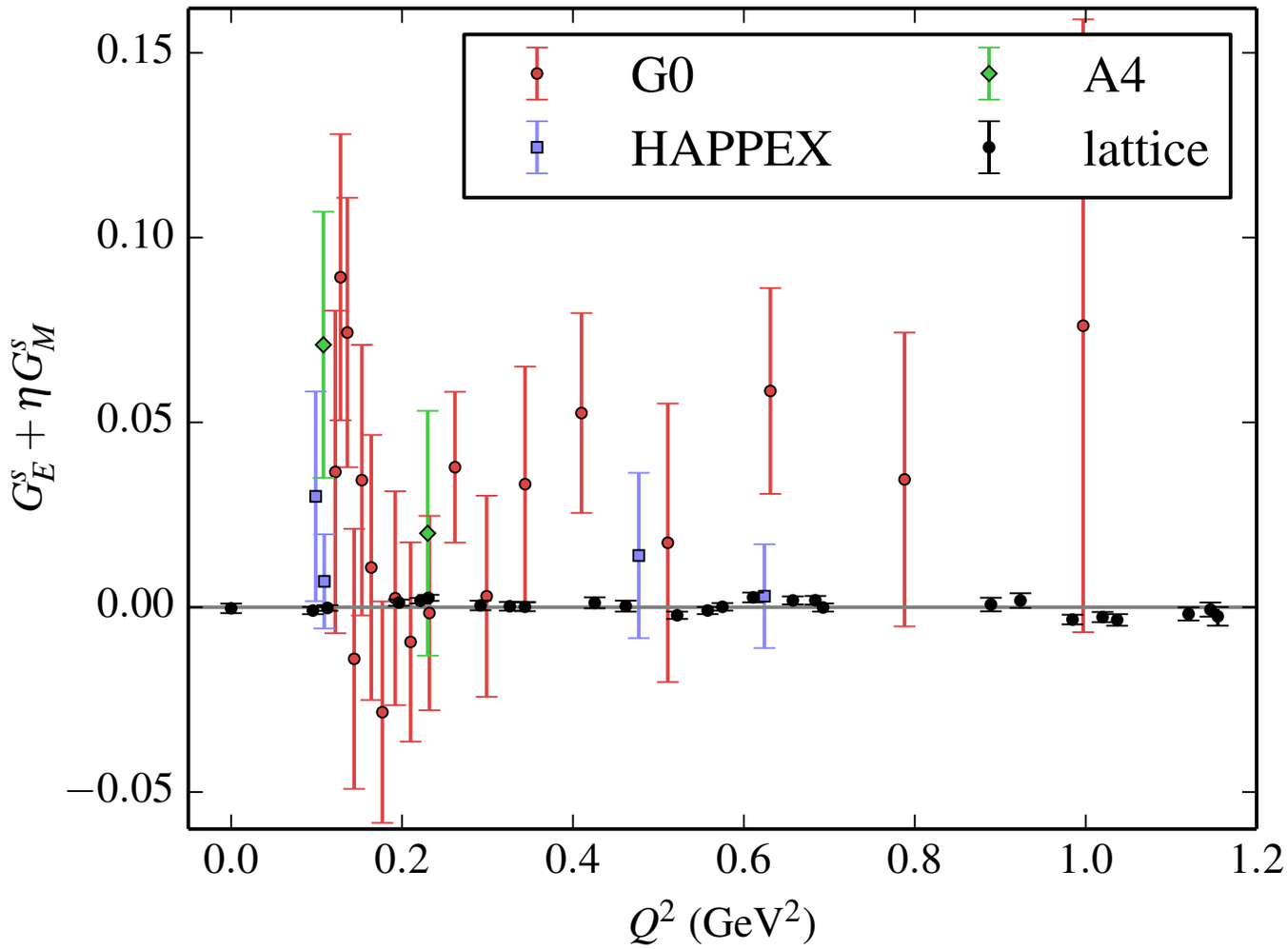
$$(r_E^2)^2 = -0.00535(89)(56)(113)(20)\ \mathrm{fm}^2$$

$$(r_M^2)^2 = -0.0147(61)(28)(34)(5)\ \mathrm{fm}^2$$

$$\mu^s = -0.0184(45)(12)(32)(1)\ \mu_N^{\mathrm{lat}}$$

[J. Green, S. Meinel, et al (LHPc)
PRD92:031501(2014)]

Strange Form Factors : PVES vs. Lattice



HAPPEX, G0, A4 data
[PRL108:102001(2012)]
vs.
Lattice QCD (m_π = 317 MeV)
[J. Green, S. Meinel, et al (LHPc) PRD92:031501(2014)]

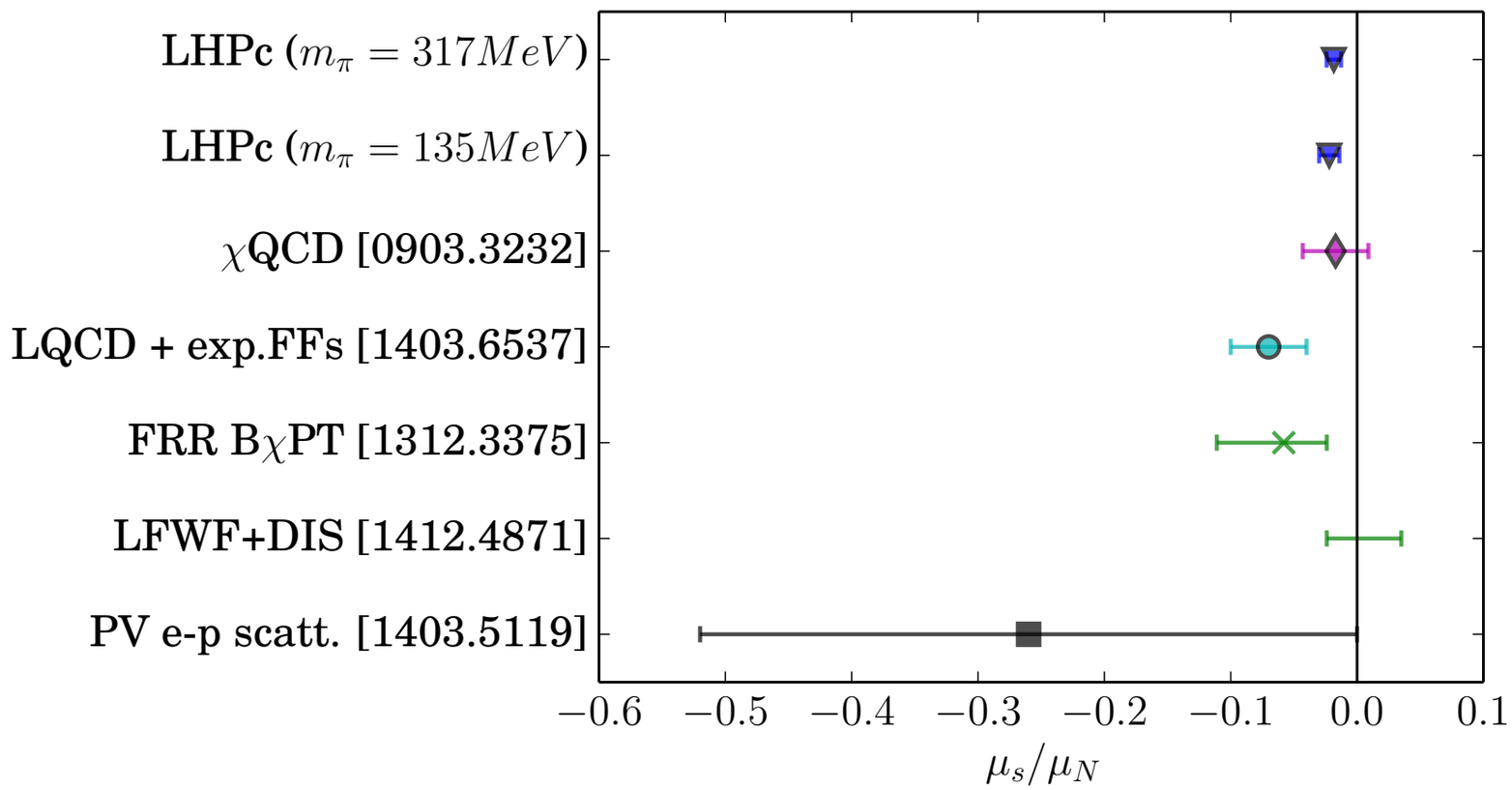
$(G_E^s + \eta G_M^s) \sim$ Strange part in the elastic *e–p* scattering asymmetry (forward angles)

$$\eta = \frac{\tau G_M^p}{\epsilon G_E^p} \simeq \frac{Q^2}{GeV^2}$$

Magnetic moment from strange quarks

Data for strange & light quarks: use PQChPT-inspired linear extrapolation in $(m_{loop})^2 \sim (m_{light} + m_{disconn})$

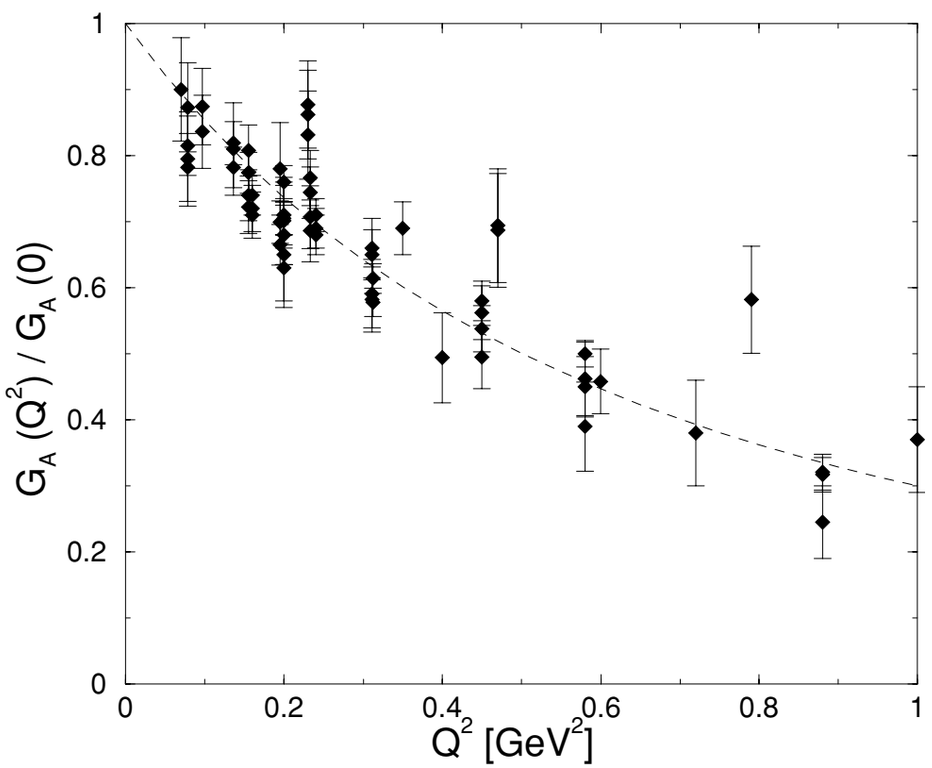
[J. Green, S. Meinel, et al (LHPc) PRD92:031501(2014)]



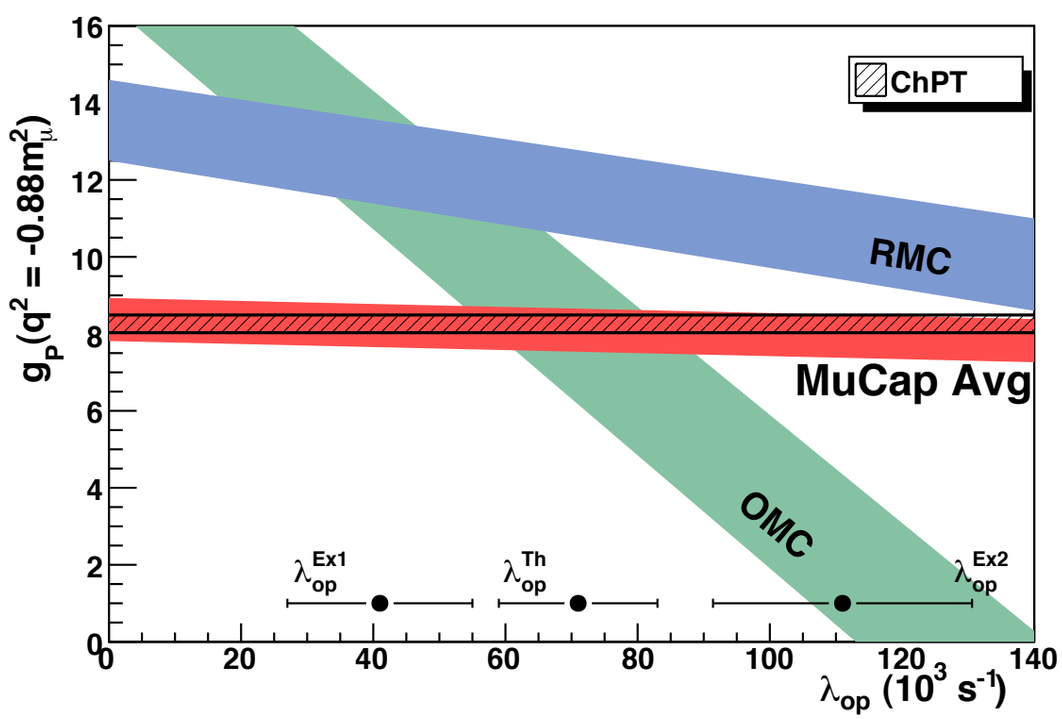
Nucleon Axial Charge and Form Factors

$$\langle P+q|\,\bar{q}\gamma^\mu\gamma^5 q\,|P\rangle = \bar{U}_{P+q}\Big[G_A(Q^2)\,\gamma^\mu\gamma^5 + G_P(Q^2)\frac{\gamma^5 q^\mu}{2M_N}\Big]U_P$$

- Axial form factor $G_A(Q^2)$
  - Interaction with neutrinos: MiniBooNE
- Induced pseudoscalar form factor $G_P(Q^2)$
  - Charged pion electroproduction
  - Muon capture (MuCAP): $g_P \sim G_P(Q^2 = 0.88\ m_\mu^2)$
- Strange axial form factor $G_A^s(Q^2)$: studied at MiniBooNE



[V.Bernard et al, JPhysG28:R1-35(2001)]



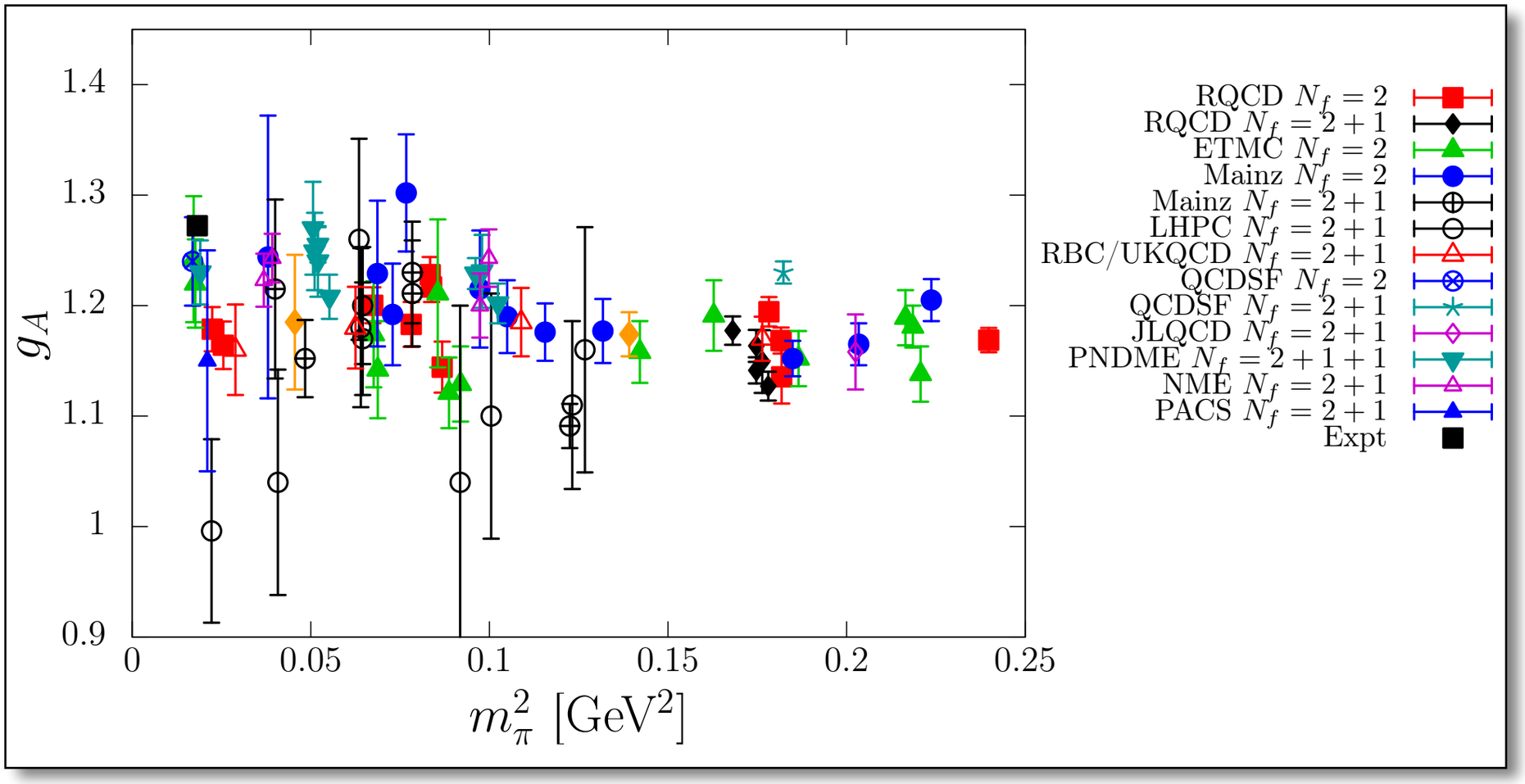
[Andreev et al (muCap), PRL110:012504(2012)]

Axial Charge

Neutron β-decay, forward limit of axial form factor

$G_A(Q^2) \longrightarrow G_A(0) = g_A$

$$\langle p|\bar{u}\gamma^\mu\gamma^5|n\rangle = g_A \bar{u}_p \gamma^\mu \gamma^5 u_n$$



Lattice data summary

[S.Collins, LATTICE 2016]

Axial Charge

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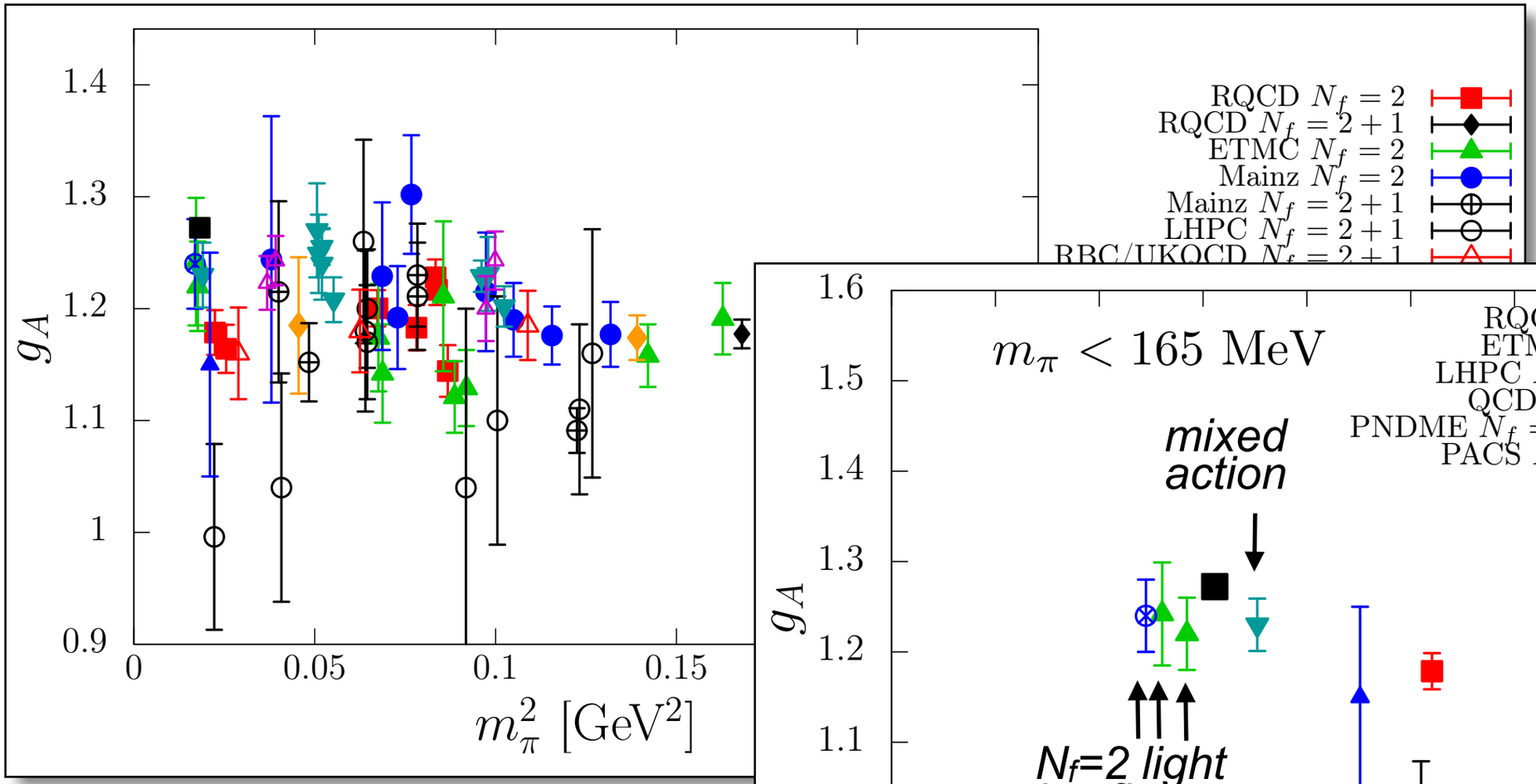
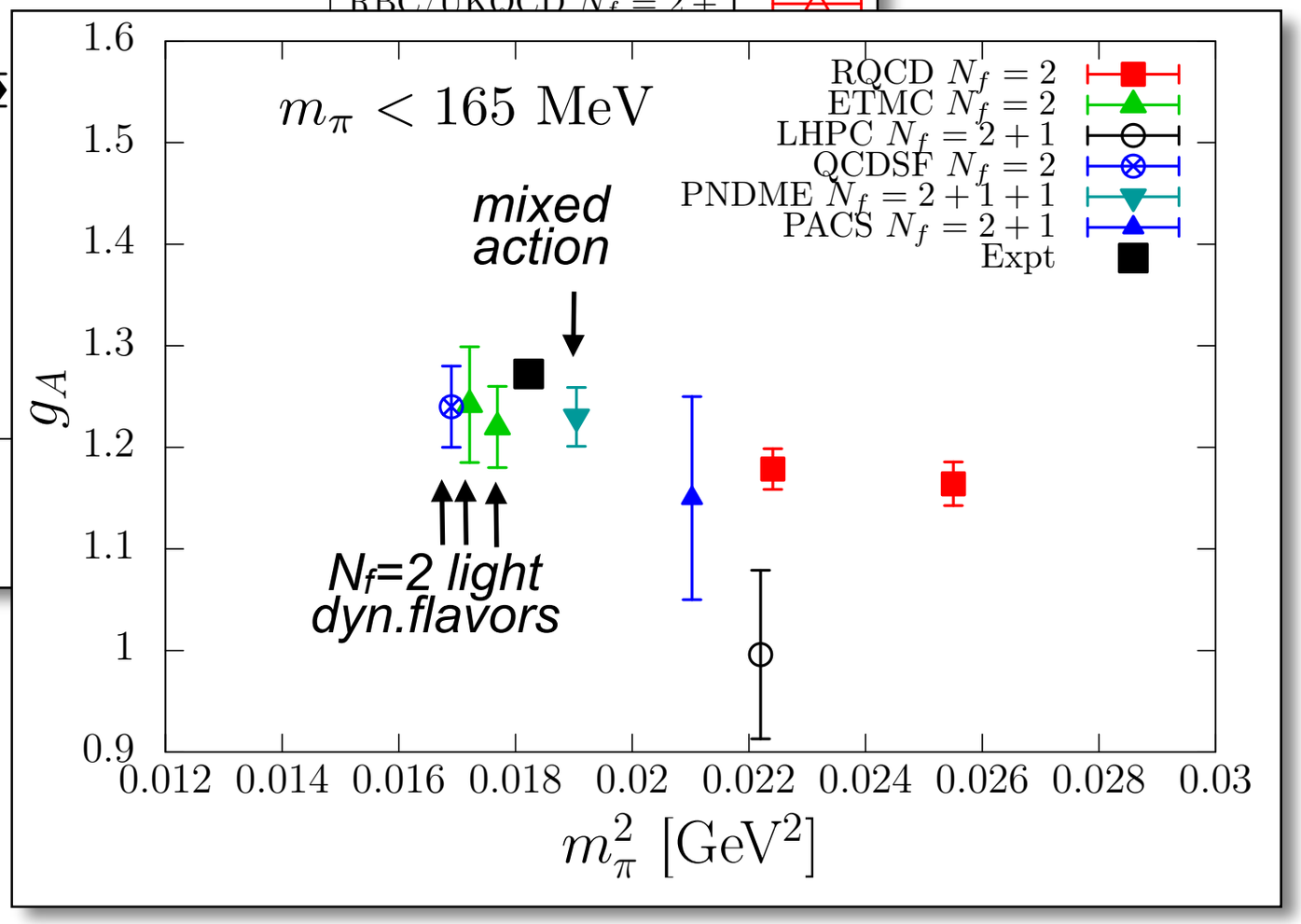
$$\langle p|\bar{u}\gamma^\mu\gamma^5|n\rangle = g_A \bar{u}_p \gamma^\mu \gamma^5 u_n$$

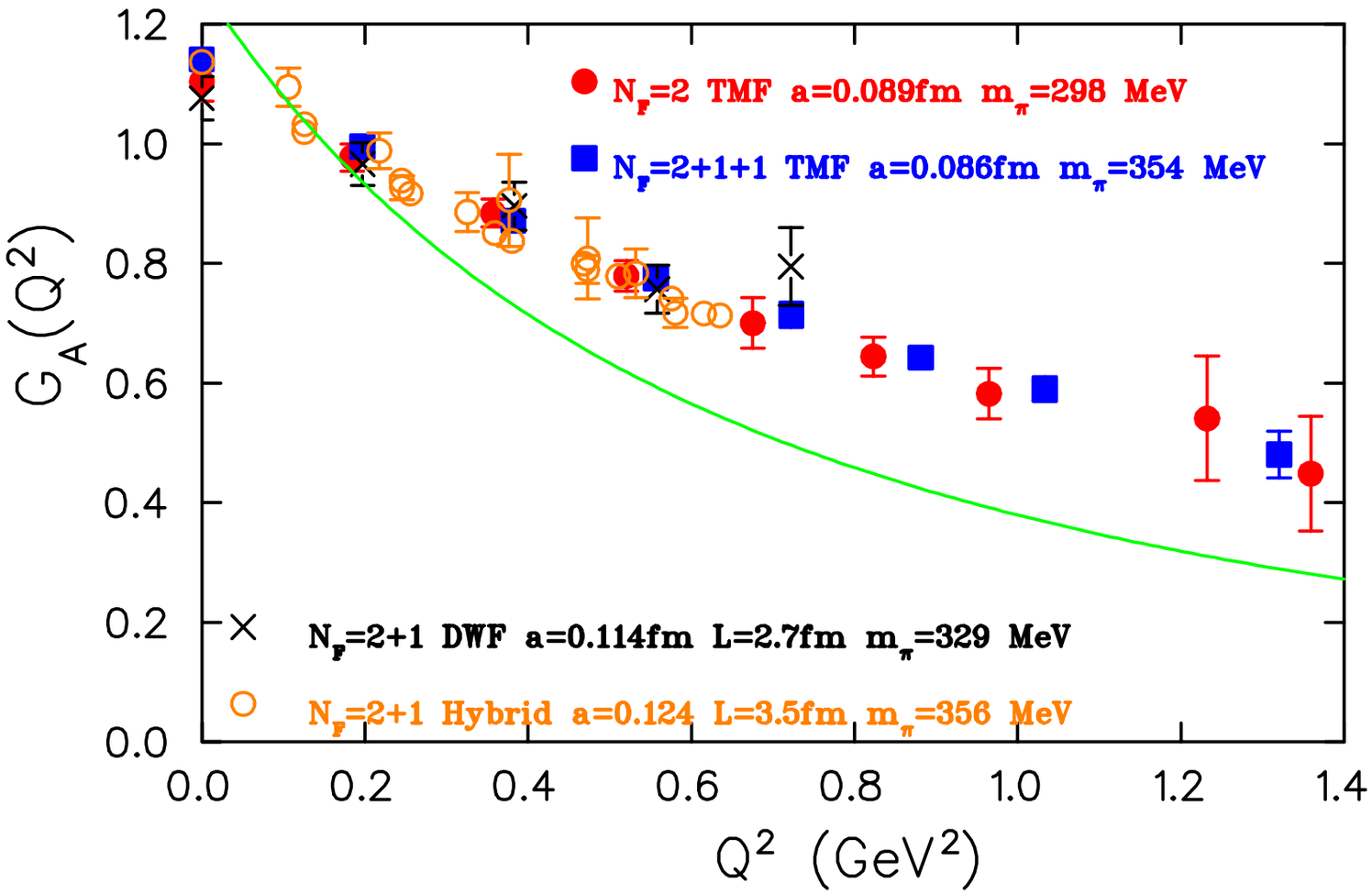




Lattice data summary
[S.Collins, LATTICE 2016]

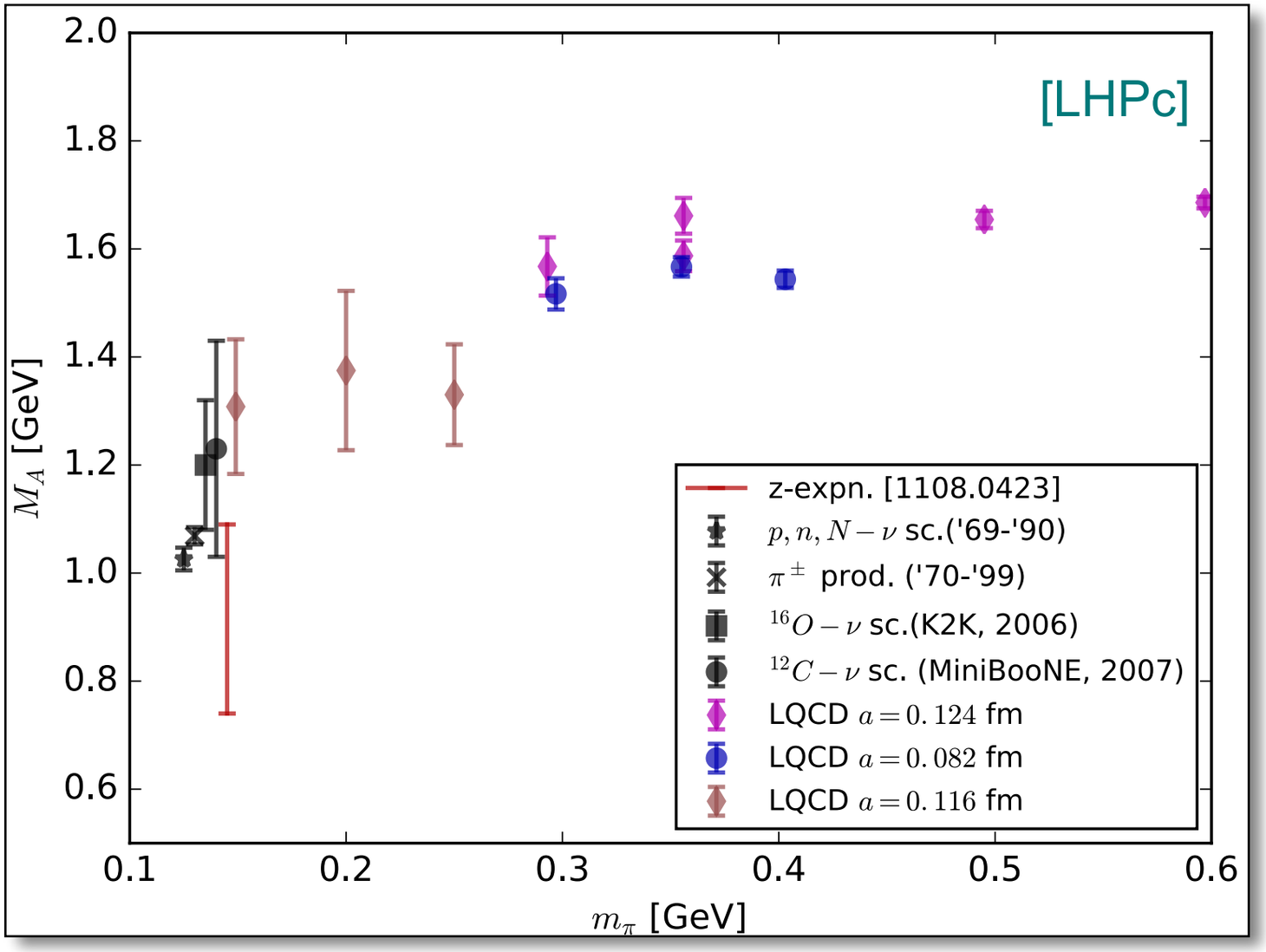
Nucleon Axial Form Factor $G_A(Q^2)$

$$\langle P+q|\,\bar{q}\gamma^\mu\gamma^5 q\,|P\rangle \;=\; \bar{U}_{P+q}\Big[\,G_A(Q^2)\,\gamma^\mu\gamma^5 \;+\; G_P(Q^2)\frac{\gamma^5 q^\mu}{2M_N}\Big]U_P$$



[C.Alexandrou (ETMC), 1303.5979]

Nucleon Axial Radius



- ν-scattering off p,n,nuclei
- $\pi^\pm$ electroproduction
- ν-scattering off ^{16}O, ^{12}C

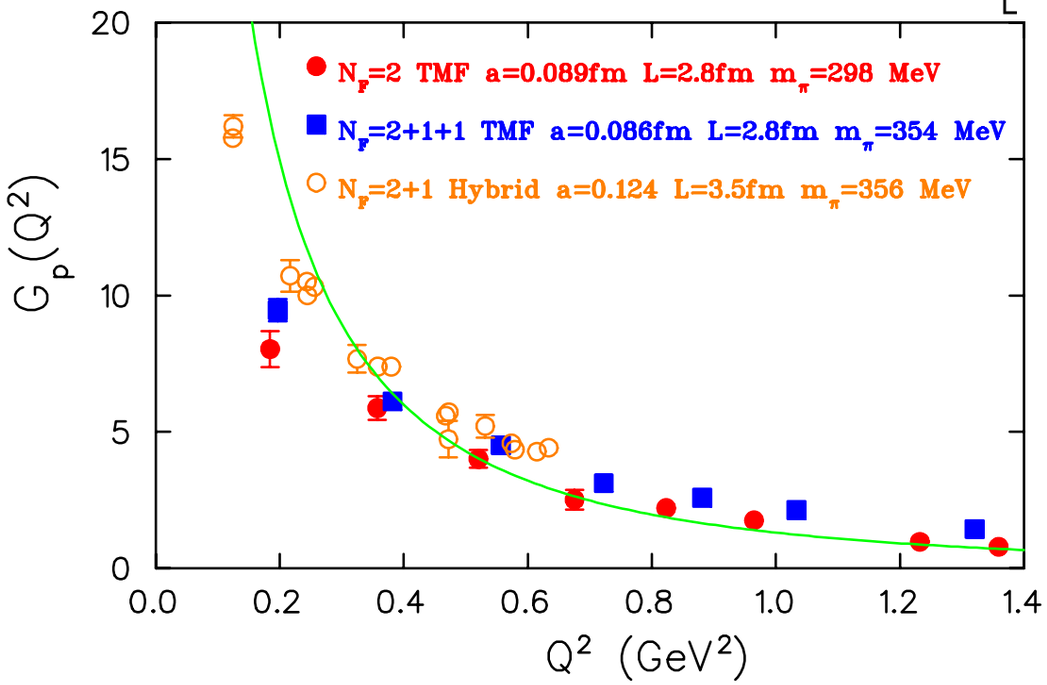
$$G_A(Q^2) \simeq \frac{g_A}{(1+Q^2/M_A^2)^2}$$

- 5% discrepancy between averages of ν-scattering and $\pi^\pm$ production [V.Bernard et al, JPhysG28:R1-35(2001)]
- Reliance on dipole fits leads to underestimated errors [B.Bhattacharya, R.Hill, G.Paz, PRD]

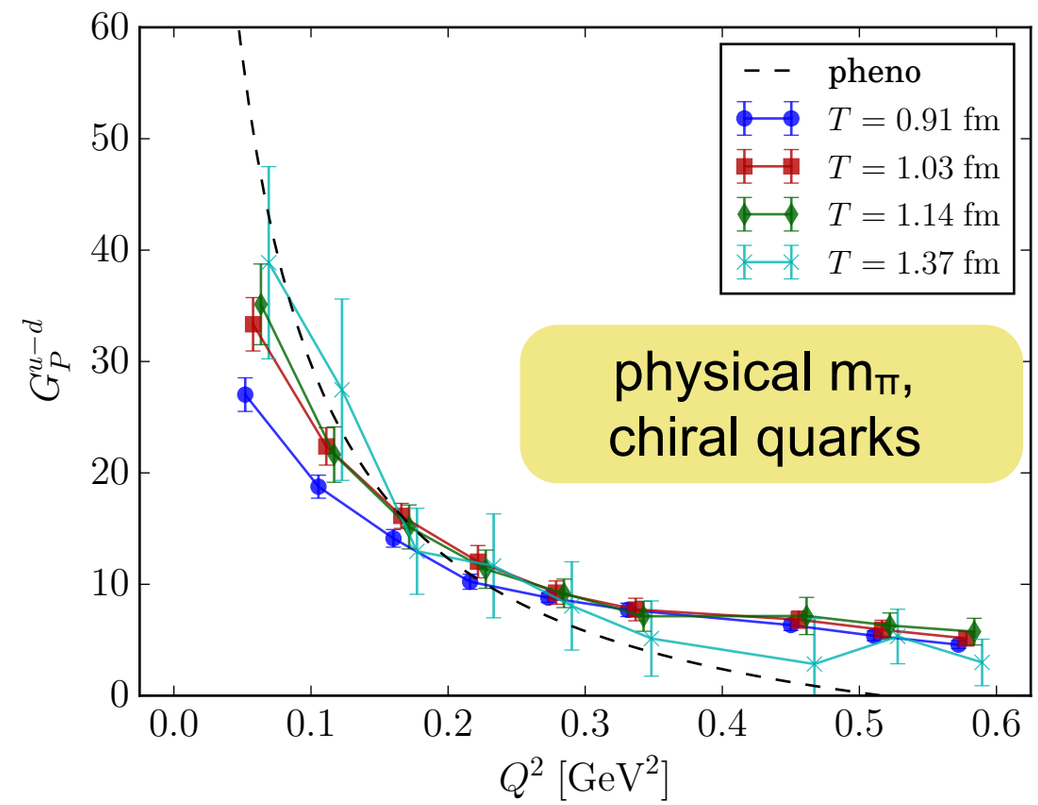
Nucleon Pseudoscalar Form Factor $G_P(Q^2)$

$$\langle P+q|\,\bar{q}\gamma^\mu\gamma^5 q\,|P\rangle = \bar{U}_{P+q}\Big[G_A(Q^2)\,\gamma^\mu\gamma^5 + G_P(Q^2)\frac{\gamma^5 q^\mu}{2M_N}\Big]U_P$$

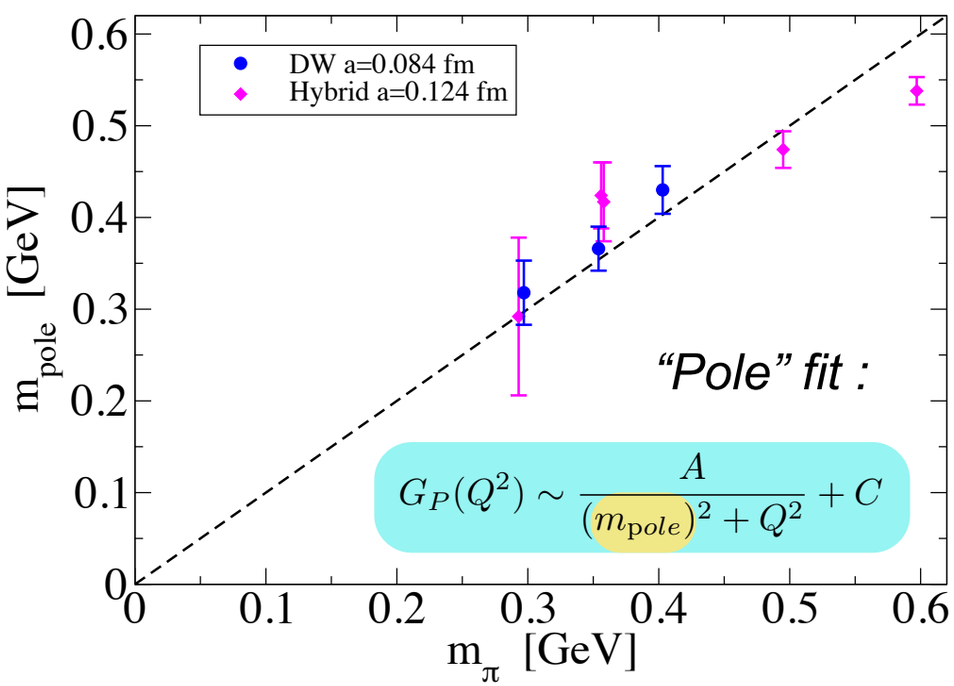
$$G_P(Q^2) \sim \frac{1}{m_\pi^2 + Q^2}$$



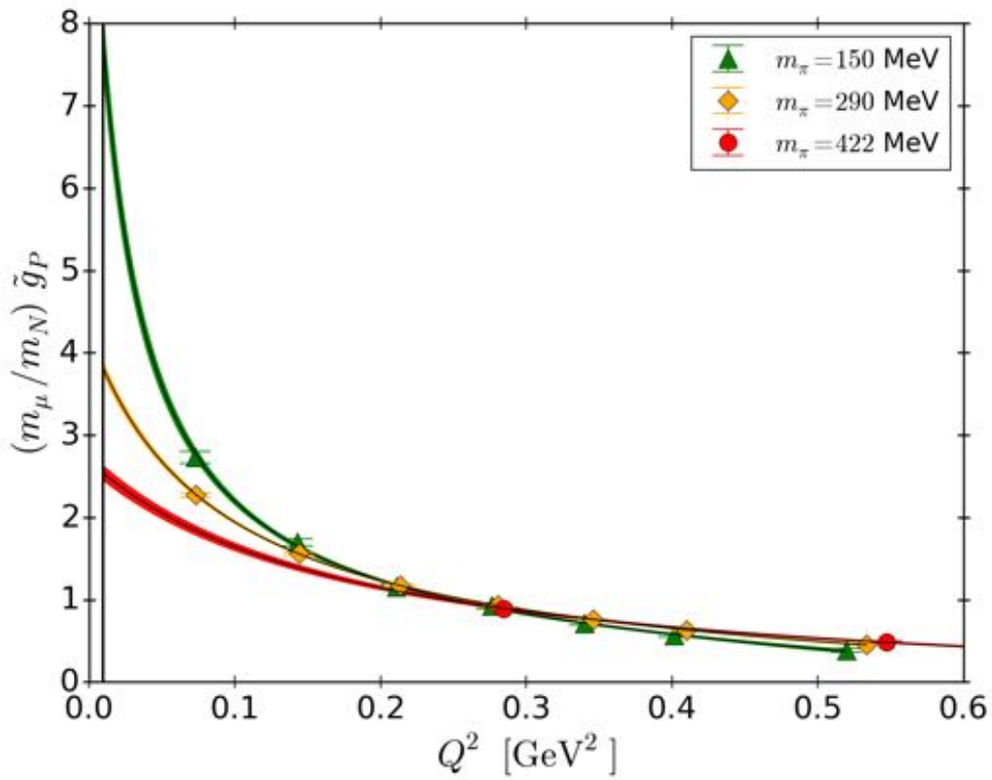
- G_P at the physical point : large excited state contributions



- Is G_P dominated by the pion pole ?



G_p Form Factor and μ-capture

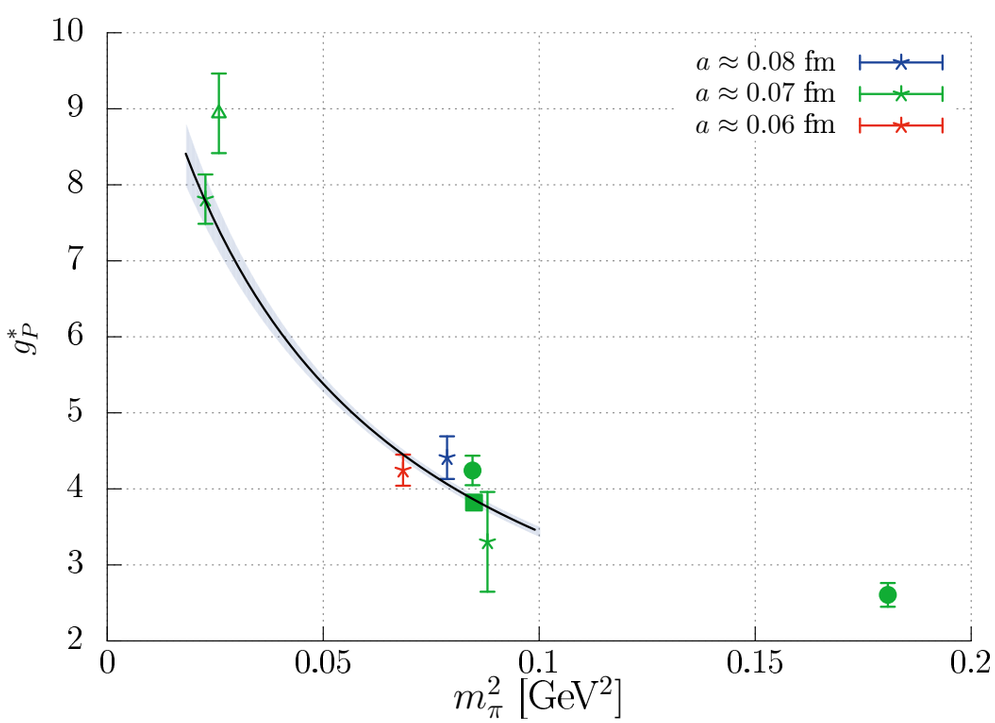


Muon-capture coupling $g_P^* = \frac{m_\mu}{m_N} g_P(0.88 m_\mu^2)$

N_f=2 calculation with Wilson-Clover fermions
[G.Bali et al (RQCD), PRD91:054501]

pion-pole extrapolation to extract g_P^*

$$\frac{m_\mu}{m_N} g_P(Q^2) = \frac{b_1}{Q^2 + m_\pi^2} + b_2 + b_3 Q^2$$



Fit & exptrapolation to phys.point

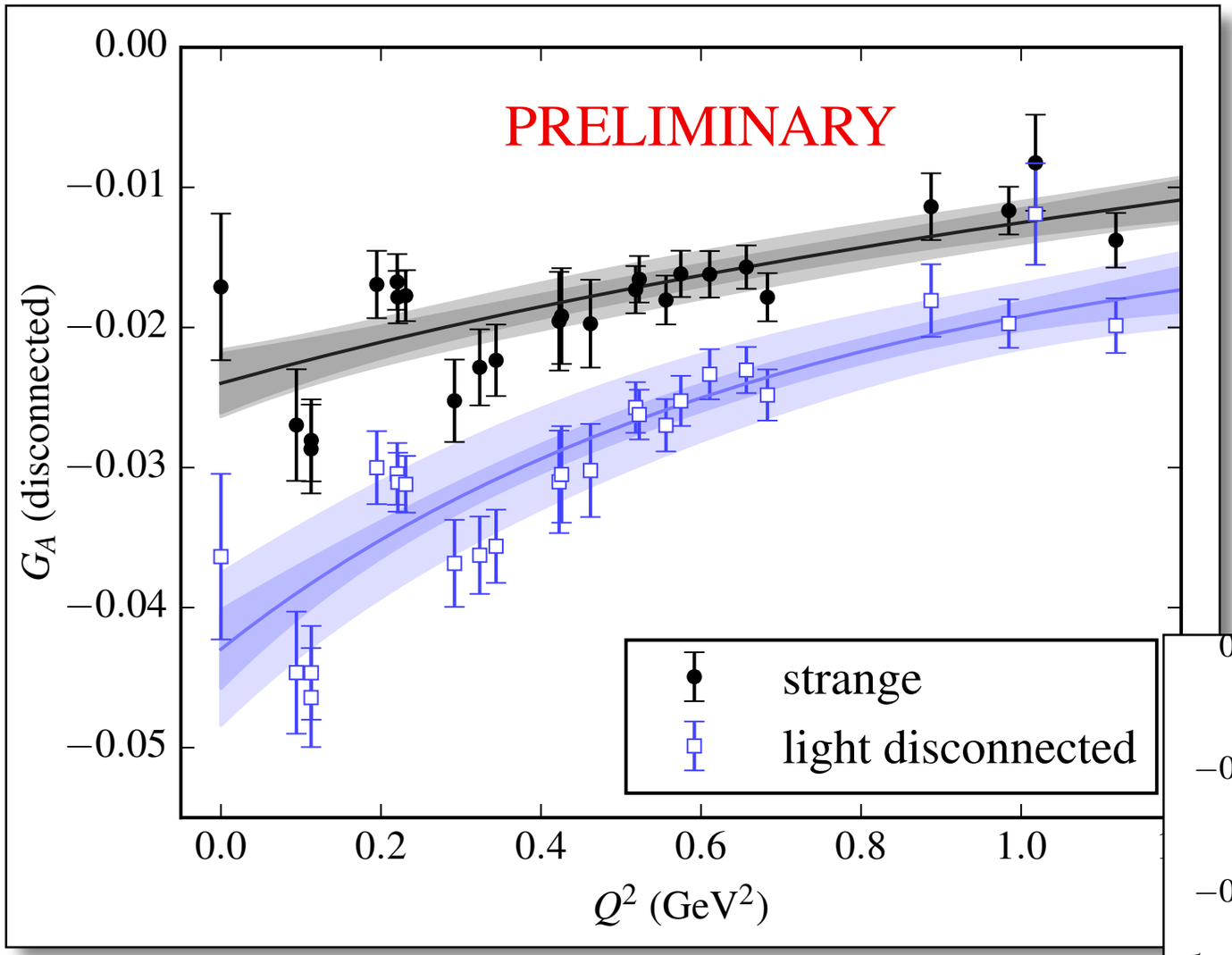
$$g_P^*(m_\pi^2) = \frac{a_1}{a_2 + m_\pi^2} \longrightarrow 8.40(40)$$

Agrees with MuCap result [PRL 110:012504]

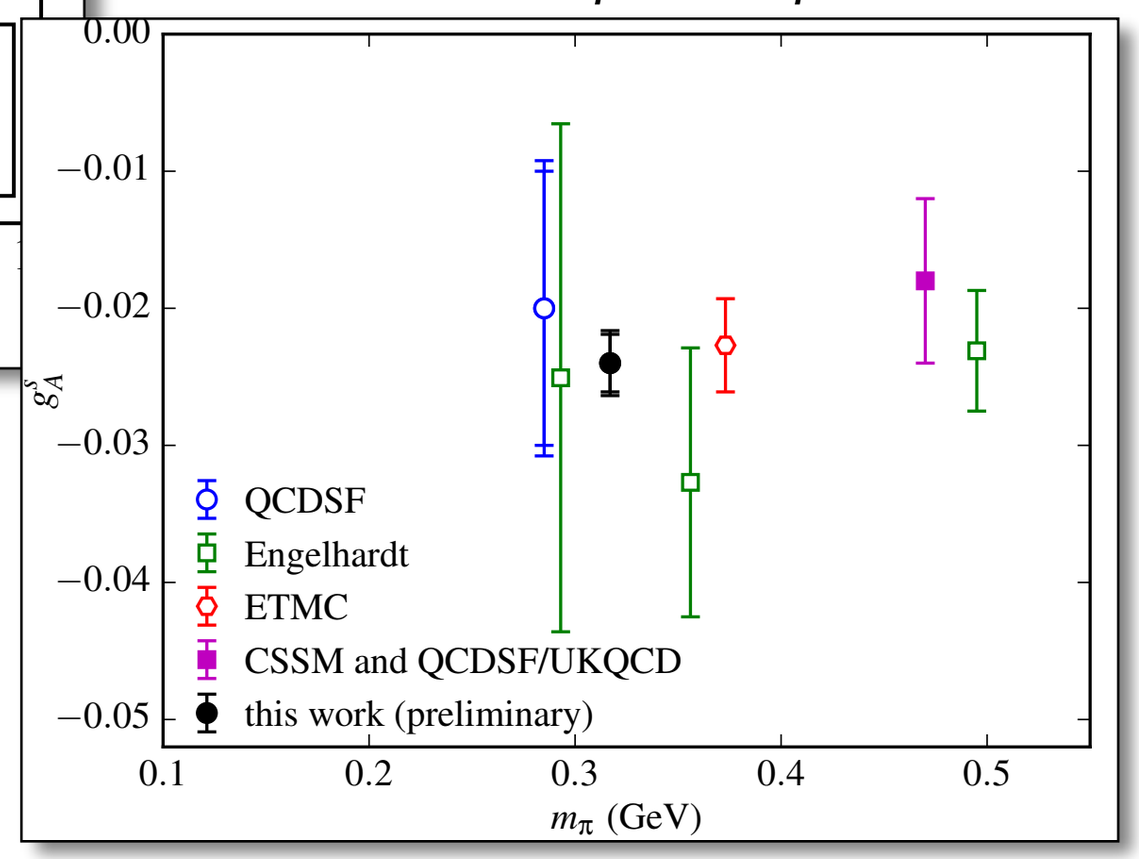
$$g_P^* = 8.06(55)$$

Strangeness in the Axial form factor

[J. Green et al (LHPc) LATTICE 2016]

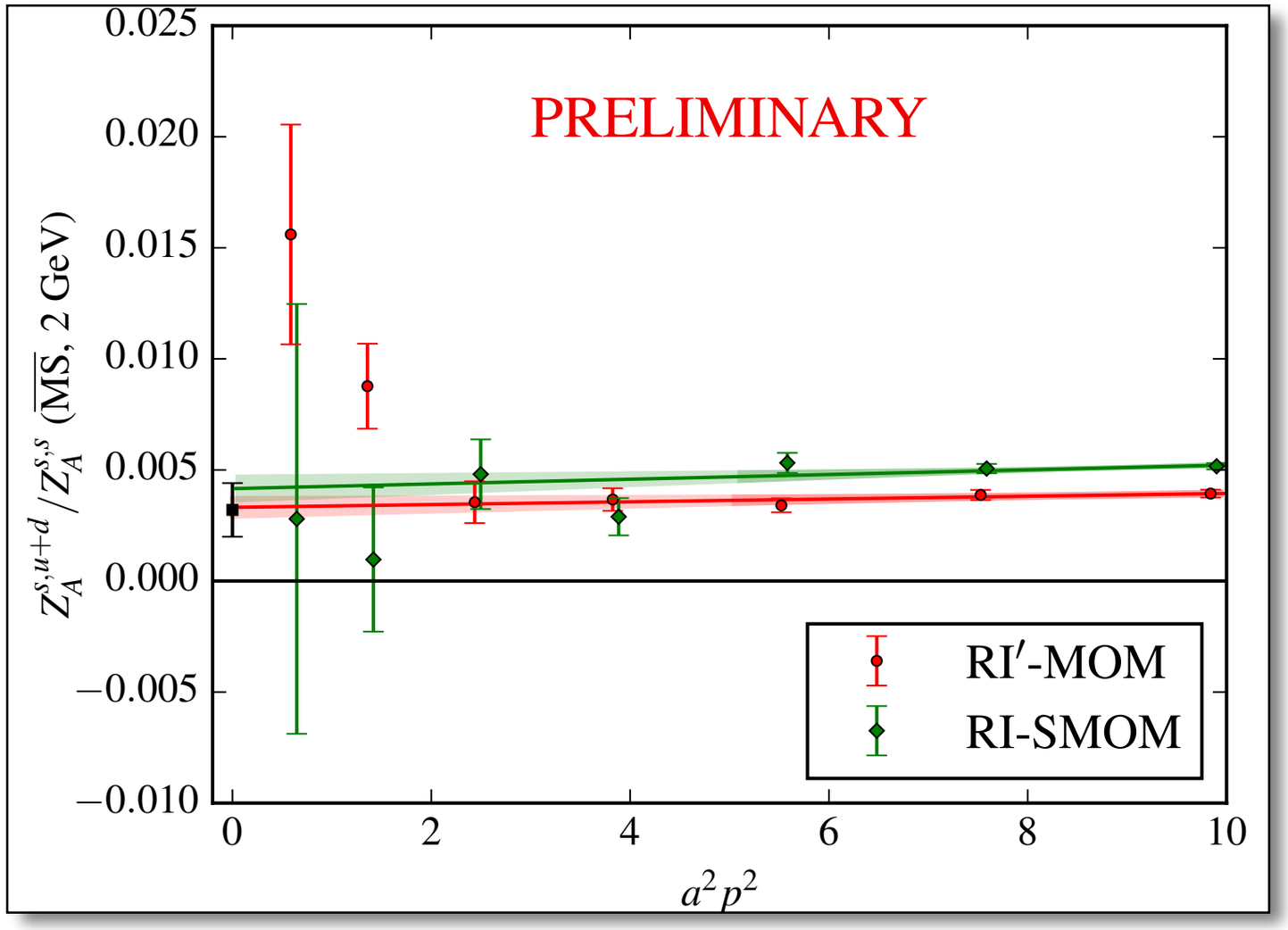


contribution of the strange quark spin to the proton spin



Light-strange Mixing in Axial Structure

[J. Green et al (LHPc) LATTICE 2016]

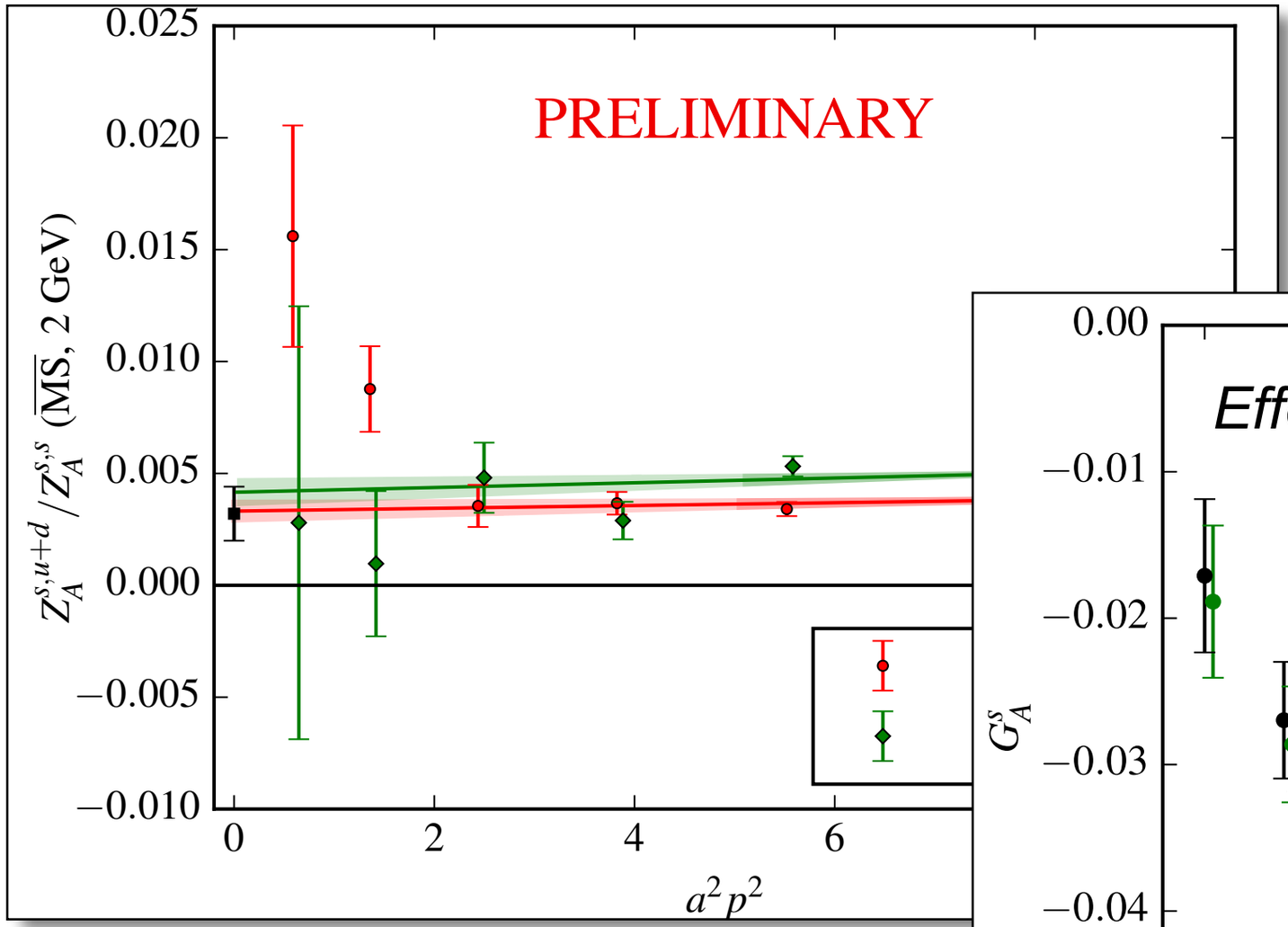


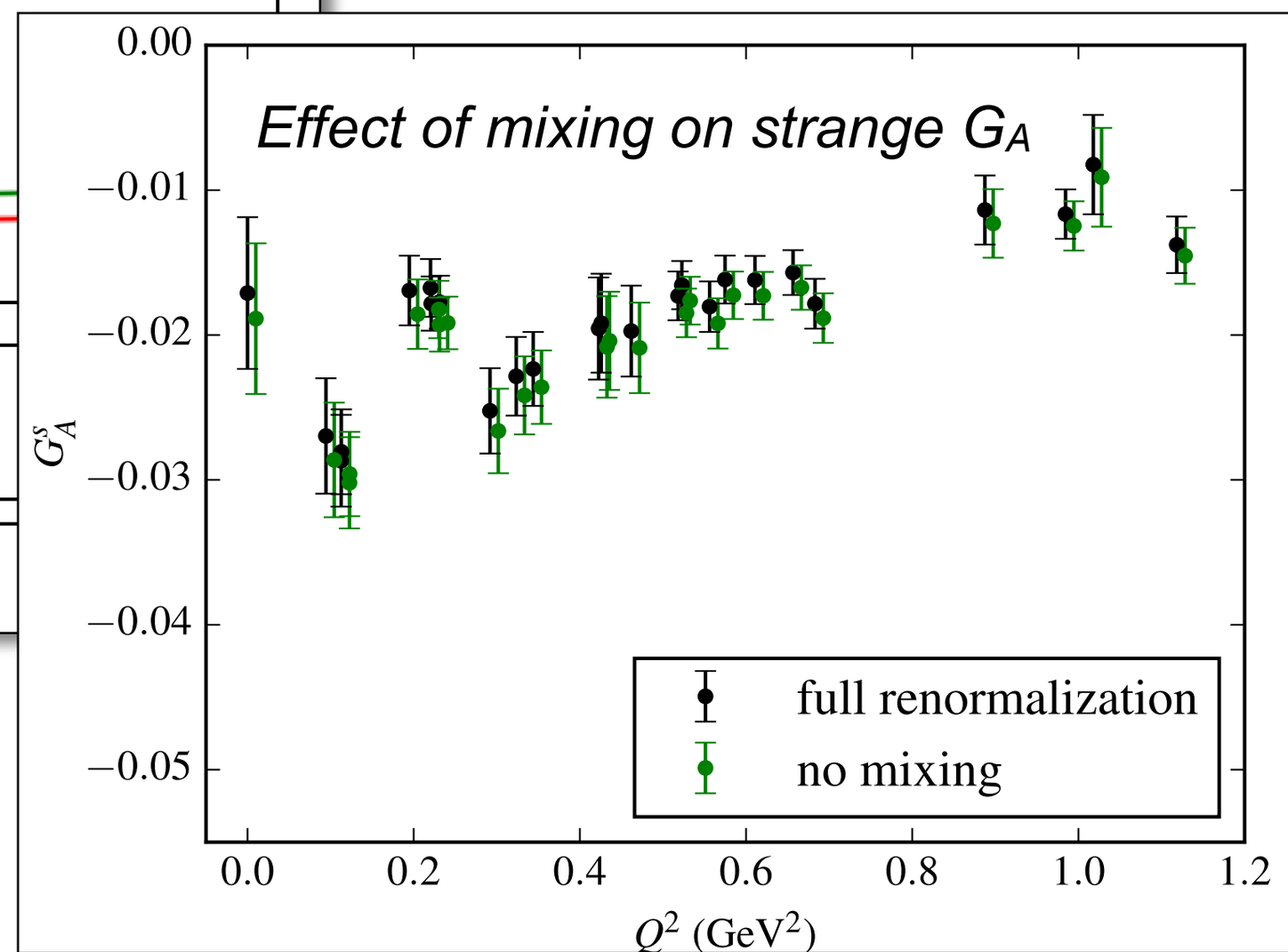
$$\begin{pmatrix} A_\mu^{R,u-d} \\ A_\mu^{R,u+d} \\ A_\mu^{R,s} \end{pmatrix} = \begin{pmatrix} Z_A^{3,3} & 0 & 0 \\ 0 & Z_A^{u+d,u+d} & Z_A^{u+d,s} \\ 0 & Z_A^{s,u+d} & Z_A^{s,s} \end{pmatrix} \begin{pmatrix} A_\mu^{u-d} \\ A_\mu^{u+d} \\ A_\mu^{s} \end{pmatrix}$$

$$= \begin{pmatrix} 0.8623(1)(71) & 0 & 0 \\ 0 & 0.8662(26)(45) & 0.0067(8)(5) \\ 0 & 0.0029(10)(5) & 0.9126(11)(98) \end{pmatrix} \begin{pmatrix} A_\mu^{u-d} \\ A_\mu^{u+d} \\ A_\mu^{s} \end{pmatrix}$$

Light-strange Mixing in Axial Structure

[J. Green et al (LHPc) LATTICE 2016]



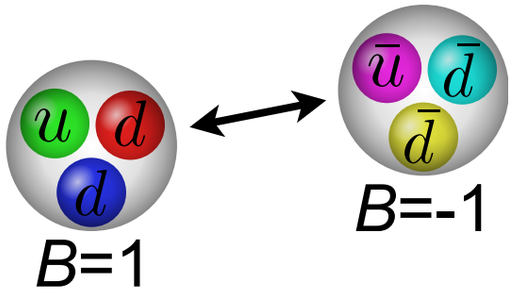


$$\begin{pmatrix} A_\mu^{R,u-d} \\ A_\mu^{R,u+d} \\ A_\mu^{R,s} \end{pmatrix} = \begin{pmatrix} Z_A^{3,3} & 0 & 0 \\ 0 & Z_A^{u+d,u+d} & Z_A^{u+d,s} \\ 0 & Z_A^{s,u+d} & Z_A^{s,s} \end{pmatrix} \begin{pmatrix} A_\mu^{u-d} \\ A_\mu^{u+d} \\ A_\mu^{s} \end{pmatrix}$$

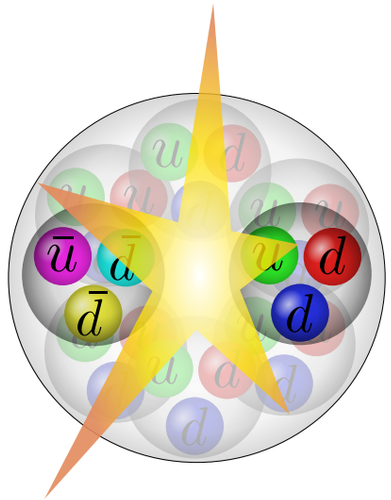
$$= \begin{pmatrix} 0.8623(1)(71) & 0 & 0 \\ 0 & 0.8662(26)(45) & 0.0067(8)(5) \\ 0 & 0.0029(10)(5) & 0.9126(11)(98) \end{pmatrix} \begin{pmatrix} A_\mu^{u-d} \\ A_\mu^{u+d} \\ A_\mu^{s} \end{pmatrix}$$

Neutron-Antineutron Oscillations

Motivation for searches :



- Baryon number must be violated for baryogenesis (Sakharov's conditions)
  - *N->Nbar transition : ΔB=2*
  - *Proton decay: ΔB=1*
  - *Which one (or both?) realized in nature?*



- Nuclear matter stability
  - *Decay of nuclei through (nn)-annihilation*

- Probing BSM physics : *Δ(B–L)=2*
  - *Connections to lepton number violation ΔL=2 ?*
  - *to neutrino mass mechanism?*
  - *unification with Majorana neutrinos ?*
  - *e.g. [R.Mohapatra, R.Marshak (1980)]*

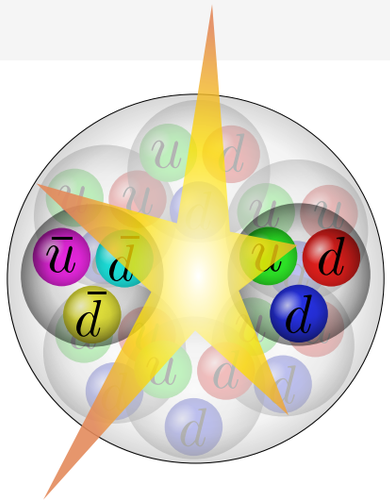
Searches for $n \to \bar{n}$ in Nuclei

Nucleus lifetime:

$$T_d = R\tau_{n\bar{n}}^2$$

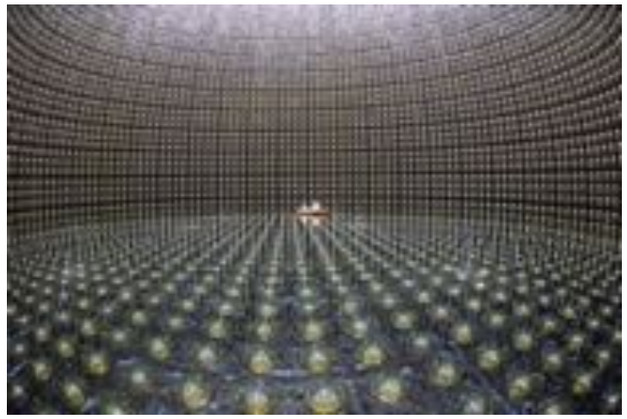
$R \sim 10^{23}\ s^{-1}$

Some nuclear model dependence:
e.g. ~ 10-15% for ^{16}O
[E.Friedman, A.Gal (2008)]



Stability of nuclei :

- ^{56}Fe [Soudan 2] $T_d(^{56}Fe) > 0.72 \cdot 10^{32}$ yr $\longrightarrow$ $\tau_{n\bar{n}} > 1.4 \cdot 10^8$ s
- ^{16}O [Super-K] $T_d(^{16}O) > 1.77 \cdot 10^{32}$ yr $\longrightarrow$ $\tau_{n\bar{n}} > 3.3 \cdot 10^8$ s
- ^{2}H [SNO] $T_d(^{2}H) > 0.54 \cdot 10^{32}$ yr $\longrightarrow$ $\tau_{n\bar{n}} > 1.96 \cdot 10^8$ s



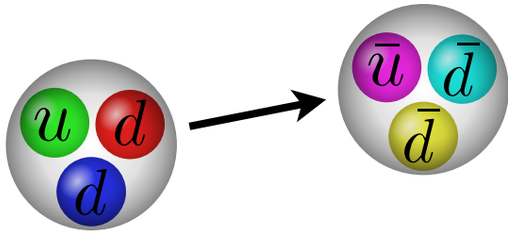
Sensitivity is limited by atmospheric neutrinos

Searches for $n \to \bar{n}$: Reactor Neutrons

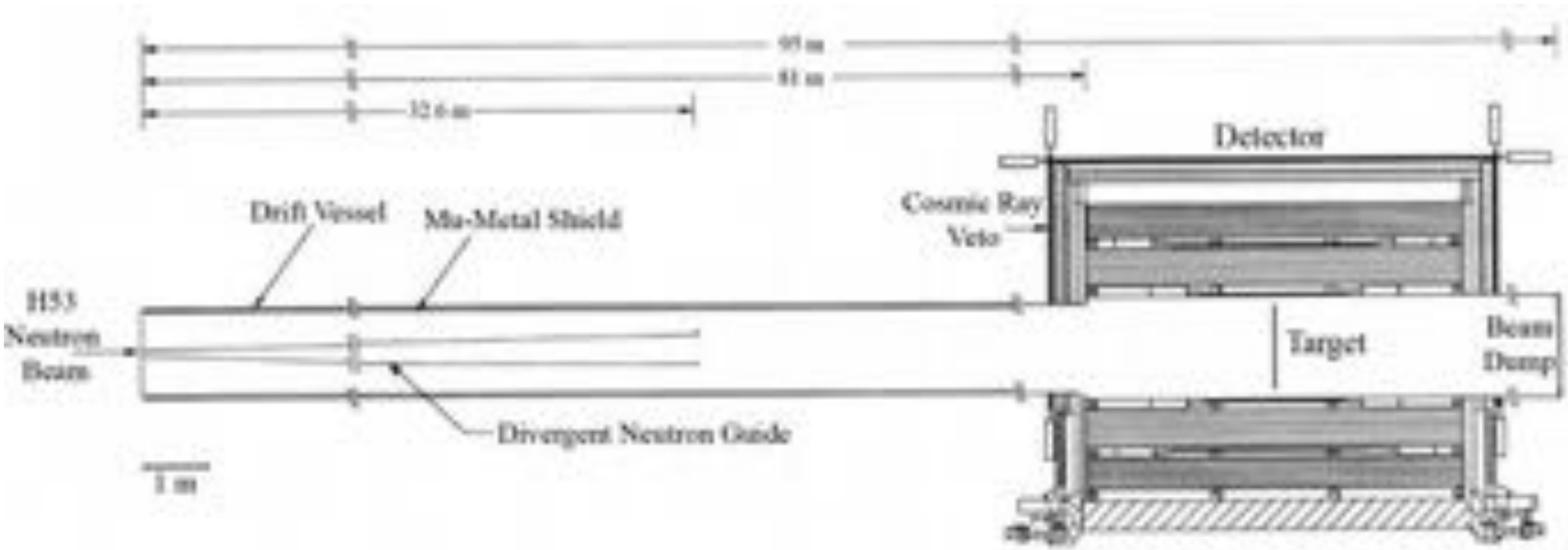
Quasi-free neutrons (*ΔEt* <<*1*) in vacuum:

$$P_{n\to\bar{n}}(t) \approx (\delta m\, t)^2 = (t/\tau_{n\bar{n}})^2$$

$$N_{\text{events}} = \text{eff} \cdot \Phi_n \cdot T \cdot \left(\frac{1}{\tau_{n\bar{n}}}\right)^2 \left(\frac{L}{v}\right)^2$$



ILL Grenoble high-flux reactor,1990 [M.Baldo-Ceolin et al, 1994)]



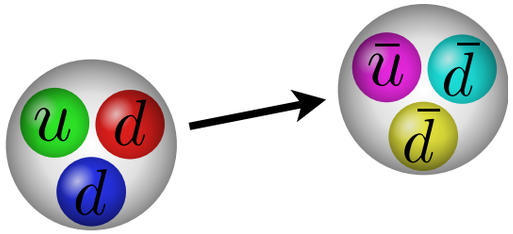
B < 10 nT
L/v ~ 0.1 s
T ~ 1 year

$$\tau_{n\bar{n}} > 0.86 \cdot 10^8 sec$$

= 2.7 years

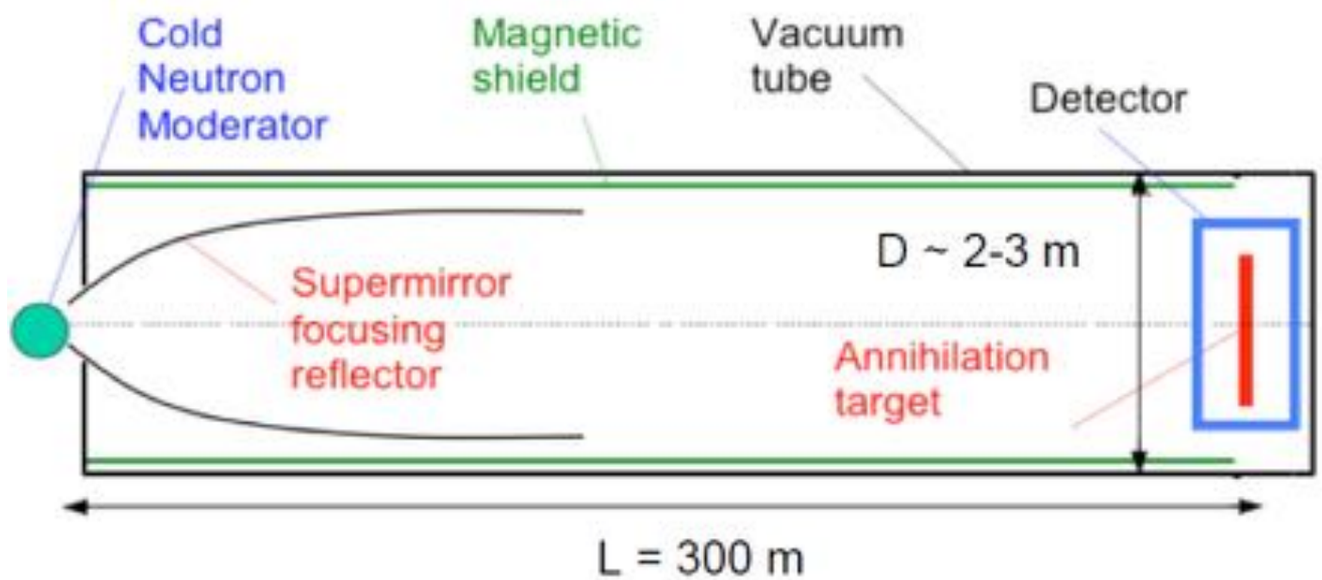
Searches for $n \to \bar{n}$: Proposed Improvements

[Phillips et al, arXiv:1410.1100]



- Free-neutron oscillation (similar to ILL):

  Maximize oscillation *Probability* ~ $N_n * (t_{free})^2$
  - Neutrons from spallation sources:
    e.g. European Spallation source: x12 neutron flux
  - Elliptic mirror for slow neutrons (reflect ~70% of $v_\perp \lesssim 40$m/s neutrons)
  - Better mag.field shielding (B < 1 nT) $\Rightarrow$ longer flight time



*Expected to increase sensitivity x**10^2-10^3** ILL, $\tau_{n\text{-}n} \gtrsim 10^9$-$10^{10}$ s*

- Other proposed experiments:
  - stored ultra-cold neutrons (4-5m/s)
  - vertical cold neutron beams

Neutron ↔ Antineutron Transitions and QCD

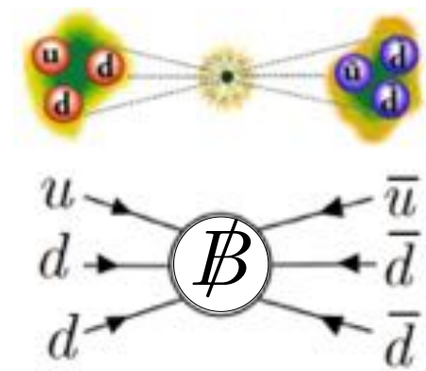
Effective $\Delta B=2$ operator: (quark field)^6

From Standard Model extensions:

interaction with a massive Majorana lepton,

unified theories, etc

[T.K.Kuo, S.T.Love, PRL45:93 (1980)]
[R.N.Mohapatra, R.E.Marshak, PRL44:1316 (1980)]



$$\mathcal{L}_{\rm eff} = \sum_i \left[c_i \mathcal{O}_i^{6q} + \text{h.c.} \right]$$

$$\delta m = -\langle \bar{n} | \int d^4x \, \mathcal{L}_{\rm eff} \, | n \rangle = -\sum_i \frac{c_i}{M_X^5} \langle \bar{n} | \mathcal{O}_i^{6q} | n \rangle$$

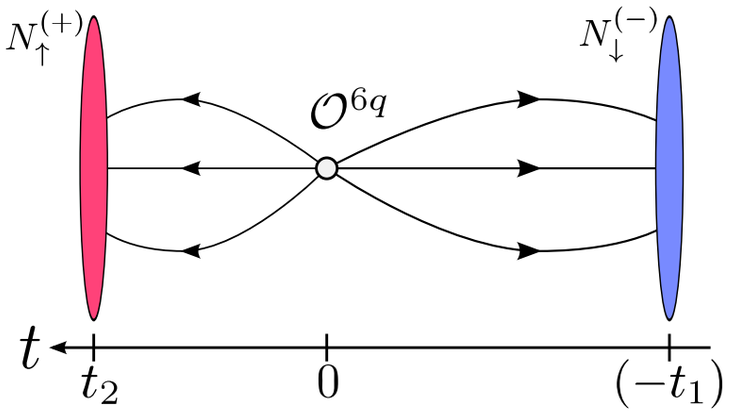
BSM scale suppression of 6-quark Dim-9 operators

nucleon sensitivity to BN-violating eff.interactions

What is the scale for new physics behind $n \leftrightarrow \bar{n}$?

- Current experimental lower bound on $\tau_{n\text{-}\bar{n}}$ requires $M_X \gtrsim 10^2$ TeV
- baryon asymmetry puts upper bound on $\tau_{n\text{-}\bar{n}}$ in models with $\Delta B=2$ mechanism (assuming SM-only CPv) e.g. [Babu et al, PRD87:115019(2013)]

Lattice Results & Comparison to Bag Model



$$\langle N_\uparrow^{(+)}(t_2)\,\mathcal{O}^{6q}(0)\,N_\downarrow^{(-)}(-t_1)\rangle \underset{t_1,\,t_2,\,t_1+t_2\to\infty}{\sim} e^{-M_n(t_2+t_1)}\langle n_\uparrow|\mathcal{O}^{6q}|\overline{n}_\uparrow\rangle$$

- On a lattice : Calculations with physical chirally symmetric quarks [SNS, M.Buchoff, J.Wasem, C.Schroeder (LATTICE 2015)]

	$\mathcal{O}^{\overline{MS}}(2\text{ GeV})$	Bag "A"	$\frac{\text{LQCD}}{\text{Bag "A"}}$	Bag "B"	$\frac{\text{LQCD}}{\text{Bag "B"}}$	
$[(RRR)_{\mathbf{3}}]$	0	0	–	0	–	
$[(RRR)_{\mathbf{1}}]$	45.4(5.6)	8.190	5.5	6.660	6.8	EW-singlet n-$\overline{\text{n}}$ tree-lev.
$[R_{\mathbf{1}}(LL)_{\mathbf{0}}]$	44.0(4.1)	7.230	6.1	6.090	7.2	
$[(RR)_{\mathbf{1}}L_{\mathbf{0}}]$	-66.6(7.7)	-9.540	7.0	-8.160	8.1	
$[(RR)_{\mathbf{2}}L_{\mathbf{1}}]^{(1)}$	-2.12(26)	1.260	-1.7	-0.666	3.2	EW non-singlet n-$\overline{\text{n}}$ at 1 loop
$[(RR)_{\mathbf{2}}L_{\mathbf{1}}]^{(2)}$	0.531(64)	-0.314	-1.7	0.167	3.2	
$[(RR)_{\mathbf{2}}L_{\mathbf{1}}]^{(3)}$	-1.06(13)	0.630	-1.7	-0.330	3.2	
		$[10^{-5}\,\text{GeV}^{-6}]$	$[10^{-5}\,\text{GeV}^{-6}]$		$[10^{-5}\,\text{GeV}^{-6}]$	

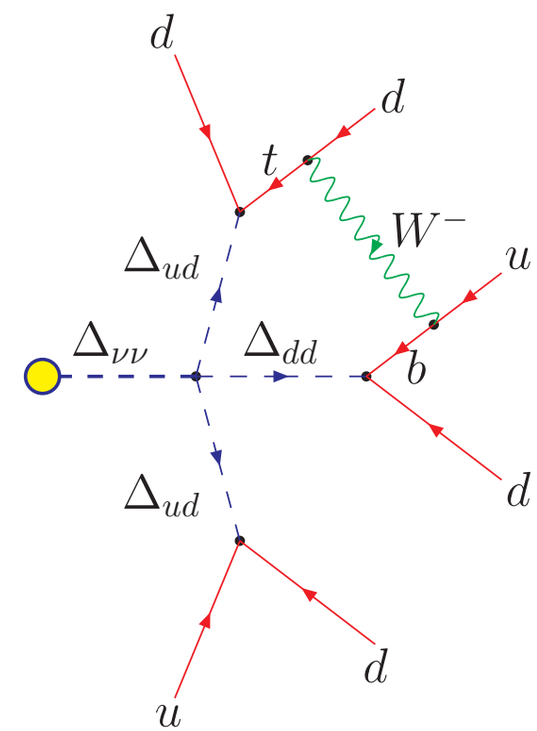
Lattice Results, In preparation

Comparison to MIT Bag model results [S.Rao, R.Shrock, PLB116:238 (1982)]

n-$\overline{\text{n}}$ oscillation is x(5-10) more sensitive to BSM physics
and (Hopefully) will motivate new n-$\overline{\text{n}}$ experiments

Constraints from Post-Sphaleron Baryogenesis

Baryogenesis below the T_{EW} in quark-lepton unified model
[K. Babu, et al, PRD87:115019 (2013)]



$n \longrightarrow \bar{n}$

Assuming SM-only CPv the prediction

$$\tau_{n-\bar{n}} \lesssim 5 \cdot 10^{10} \text{ s}$$

relying on the Bag model n-n̄ calculation for m.e.

$$\langle \bar{n} | \mathcal{O}^2_{RLR} | n \rangle \Big|_{bag} = (-0.34 \ldots + 0.17) \cdot 10^{-5} \text{ GeV}^{-6}$$

Lattice QCD calculation yields

$$\langle \bar{n} | \mathcal{O}^2_{RLR} | n \rangle \Big|_{LQCD} \approx 0.78(9) \cdot 10^{-5} \text{ GeV}^{-6}$$

and improves the upper bound for osc.time

$$\tau_{n-\bar{n}} \lesssim 2 \cdot 10^{10} \text{ s}$$

Summary

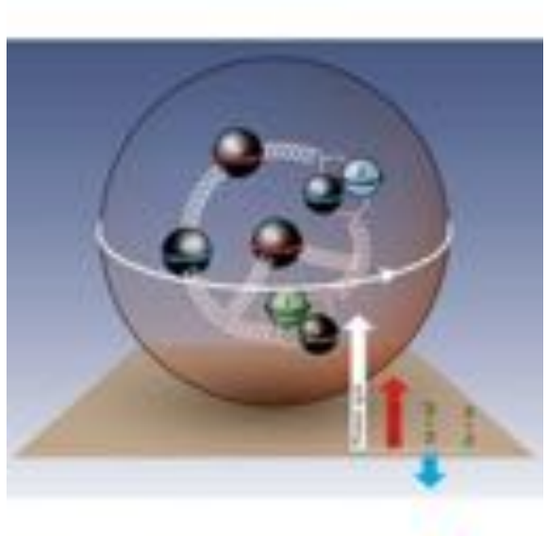
- Calculations near the physical point produce encouraging results
  Vector form factors, radii, magnetic moment
- Lattice QCD gives access to quantities hard for experiments
  E.g. strangeness contributions to the nucleon form factors
- Coupling of nucleons to BSM effective operators
  neutron-antineutron transition, proton decay, tensor charge...

Proton Spin Puzzle

EMC experiment (1989):
polarized Deep-Inelastic
μ-p Scattering :

Spin of Quarks

$$S_q = \frac{1}{2}\sum_q \left(\Delta q + \Delta\bar{q}\right) \approx \frac{1}{2}\cdot 0.3$$

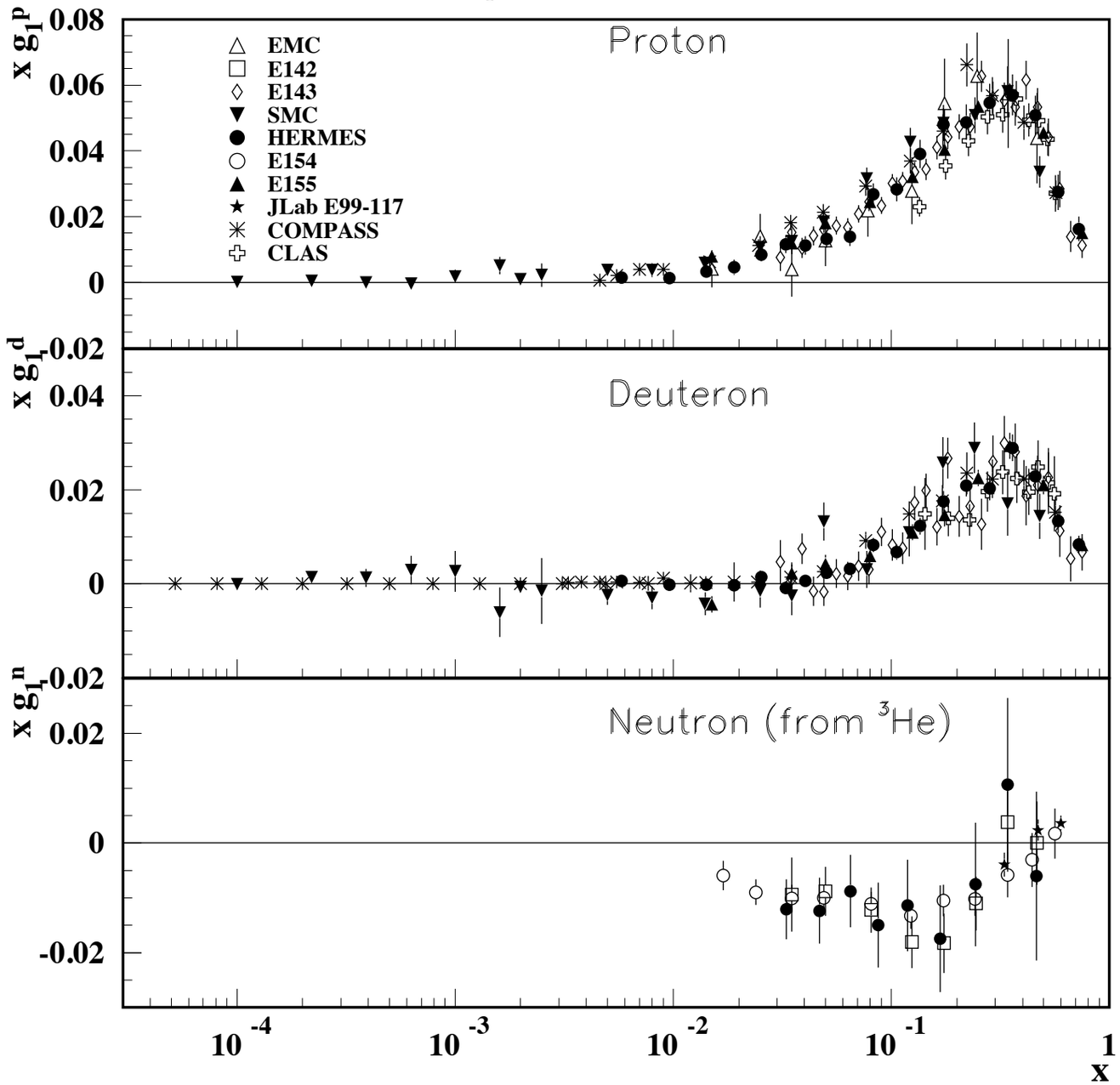


Quark spin = *33 %* of the Proton Spin

Where is the rest ?
- Quark Orbital Motion?
- Gluon Angular Momentum ?

structure functions from polarized beam & target

$$g_1(x) = \frac{1}{2}\sum_q e_q^2\left[\Delta q(x) + \Delta\bar{q}(x)\right]$$



[PDG 2014]

Proton Spin Decomposition and Sum Rule

Deeply-Virtual Compton Scattering(DVCS)

log(Q^2)-corrections, J/ψ - production at Electron-Ion Collider

polarized p-p collisions at RHIC scaling violations in g_1

$$\frac{1}{2} = J^{quarks} + J^{Glue}$$

$$(S^q + L^q)$$

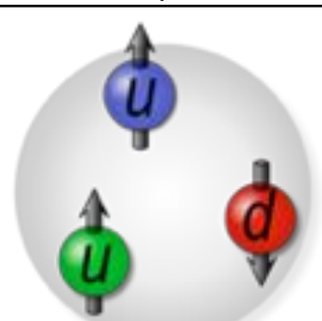
$$(S^G + L^G + L^{qG\text{-int}})$$

No local QFT definitions for S^G, L^G, L^{qG}; interpretation depends on experiment

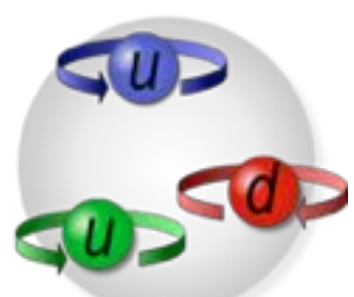
Review of recent theory developments :
[K.F.Liu and C.Lorce, contribution to EPJA "3D Structure of the Nucleon", 1508.00911]

polarized parton distributions $\Delta q(x)$ from DIS (EMC exp)

Wigner-like parton distributions (GTMDs)? Twist-3 corrections to DVCS?



$$\frac{1}{2}\bar{q}\Sigma^{0i}q$$



$$\vec{L}^q = \vec{r} \times \vec{p}$$

Proton Spin Decomposition and Sum Rules

Angular momentum

$$J^i = \frac{1}{2}\epsilon^{ijk} \int d^3x \left[x^j\, T^{0k} - x^k\, T^{0j}\right]$$

Belinfante–Rosenfeld energy-momentum tensor in QCD:

$$T^q_{\mu\nu} = \bar{q}\,\gamma_{\{\mu}\overleftrightarrow{D}_{\nu\}}\, q \qquad \text{Quarks}$$

$$T^{\rm glue}_{\mu\nu} = G^a_{\mu\lambda}G^a_{\nu\lambda} - \frac{1}{4}\delta_{\mu\nu}(G_{\mu\nu})^2 \qquad \text{Gluons}$$

Nucleon form factors of the EM tensor

$$\langle N(p+q)|\, T^{q,glue}_{\mu\nu}\, |N(p)\rangle \to \Big\{A_{20},\, B_{20},\, C_{20}\Big\}(Q^2)$$

= Mellin Moments of GPDs

$$A_{20}(Q^2) = \int dx\, x\, H(x,0,Q^2)$$

$$B_{20}(Q^2) = \int dx\, x\, E(x,0,Q^2)$$

Quark & Gluon Angular Momentum

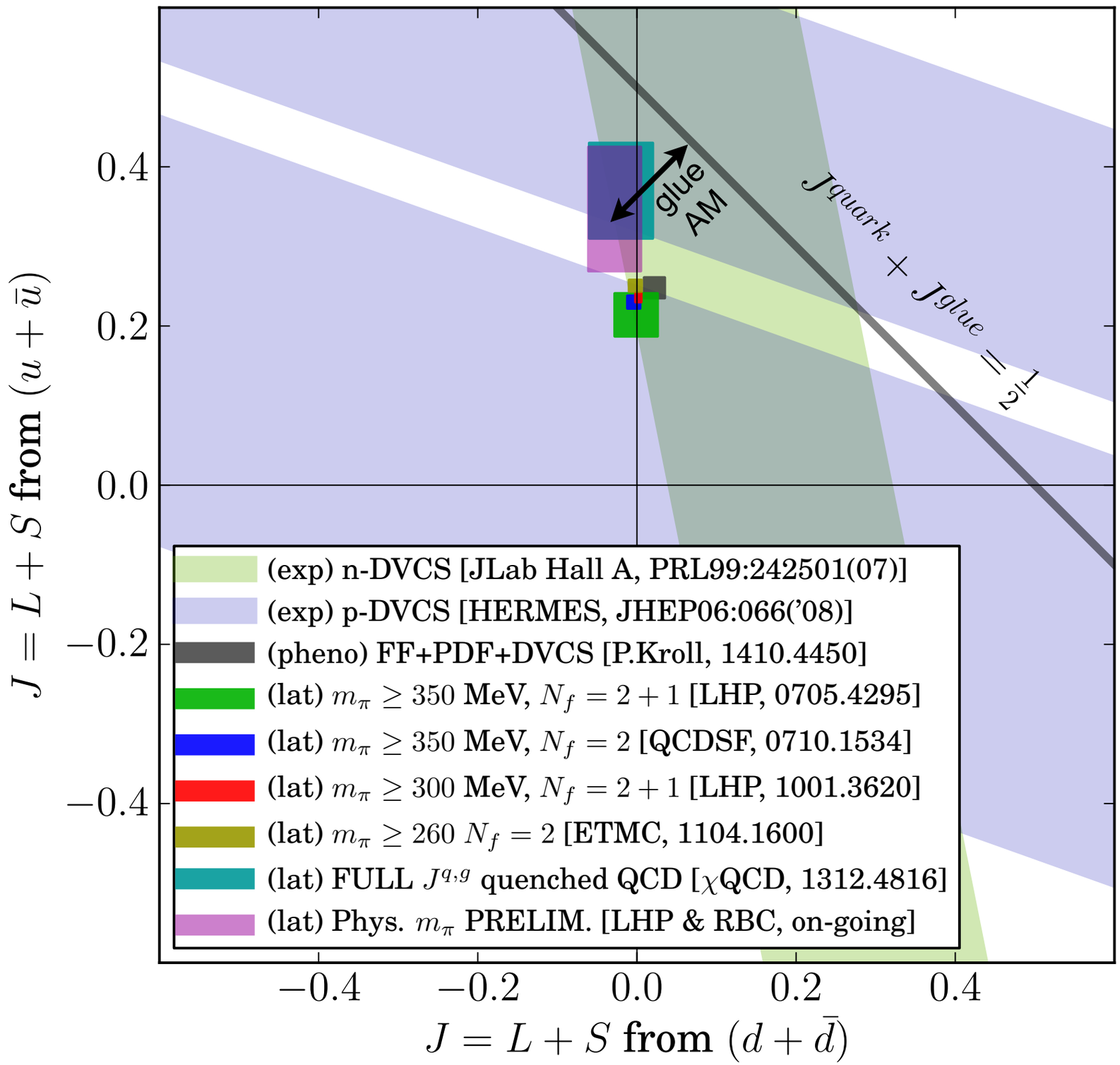
$$J_{q,glue} = \frac{1}{2}\Big[A^{q,glue}_{20}(0) + B^{q,glue}_{20}(0)\Big]$$

[X.Ji, PRL 78:610 (1997)]

Quark spin

$$\langle N(p)|\bar{q}\gamma^\mu\gamma^5 q|N(p)\rangle = (\Delta\Sigma_q)\left[\bar{u}_p\gamma^\mu\gamma^5 u_p\right]$$

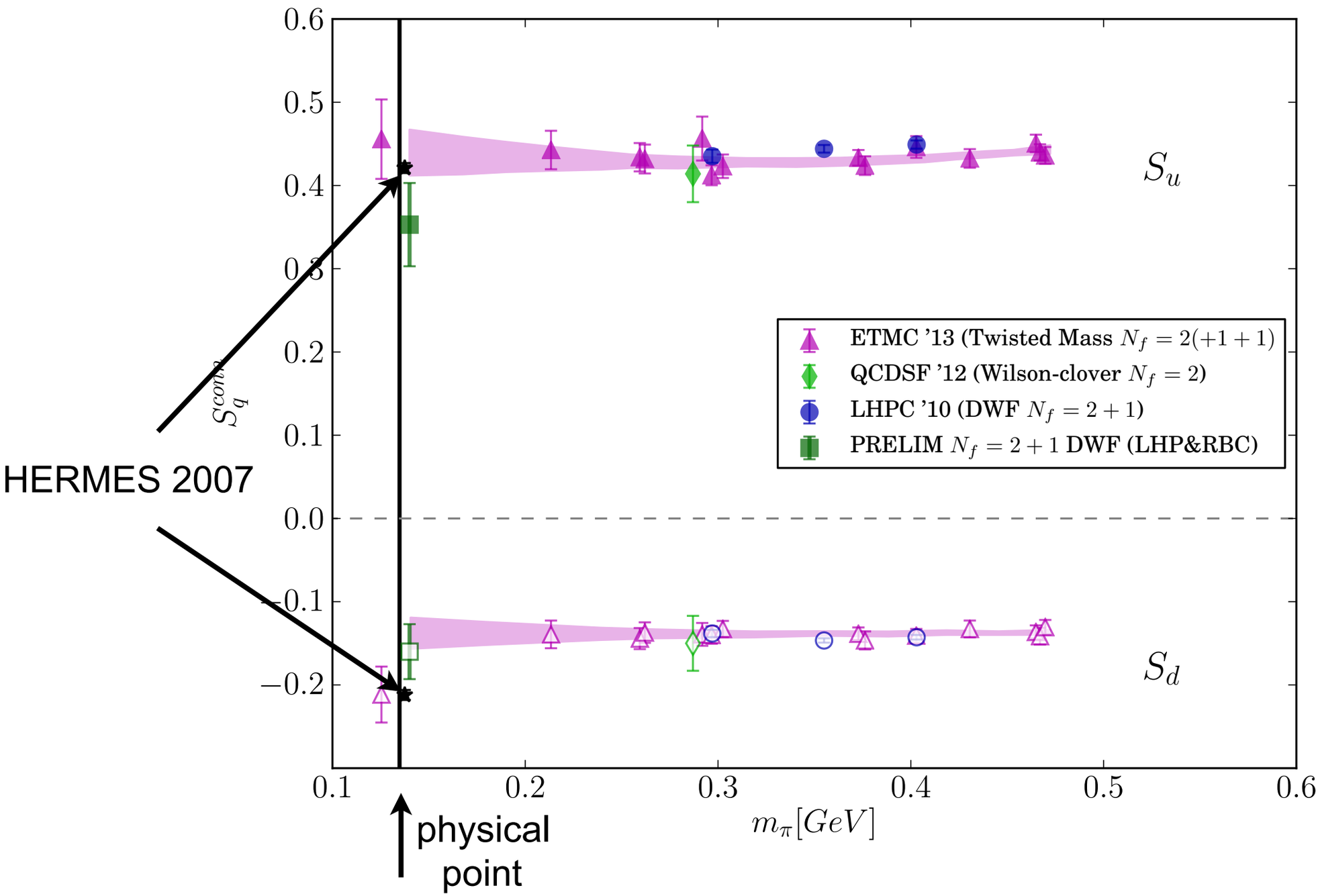
Light Quark Angular Momenta in the Proton

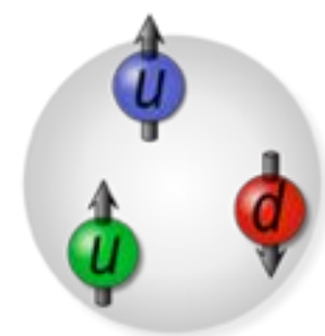


Challenges for Lattice QCD:

- "disconnected" quarks
- gluon angular momentum
- renormalization & mixing

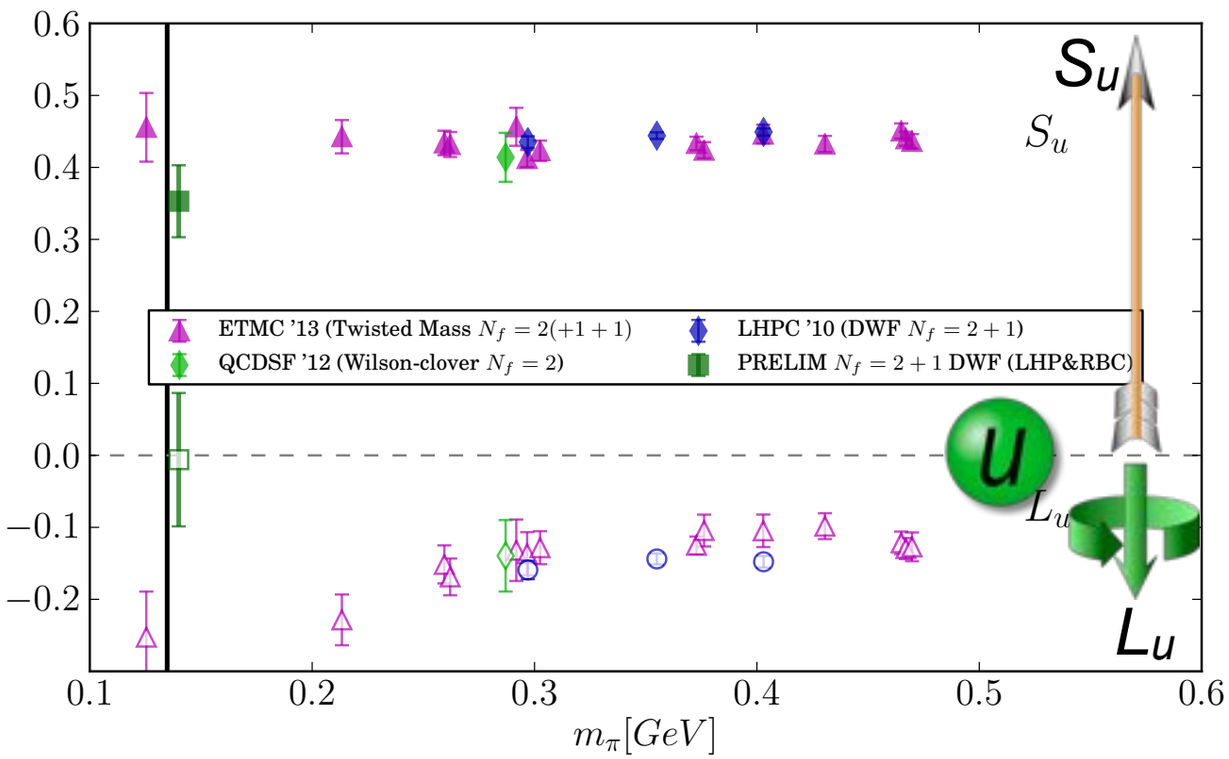
Light Quark Spin

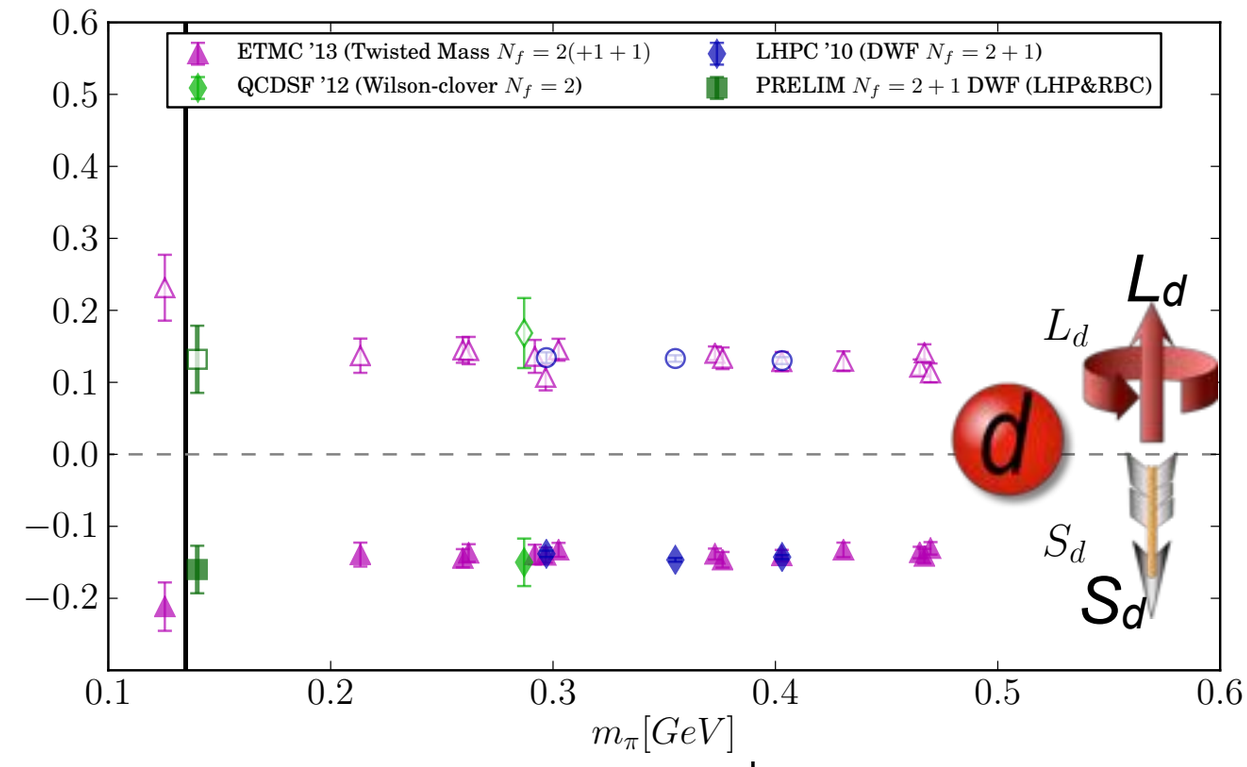


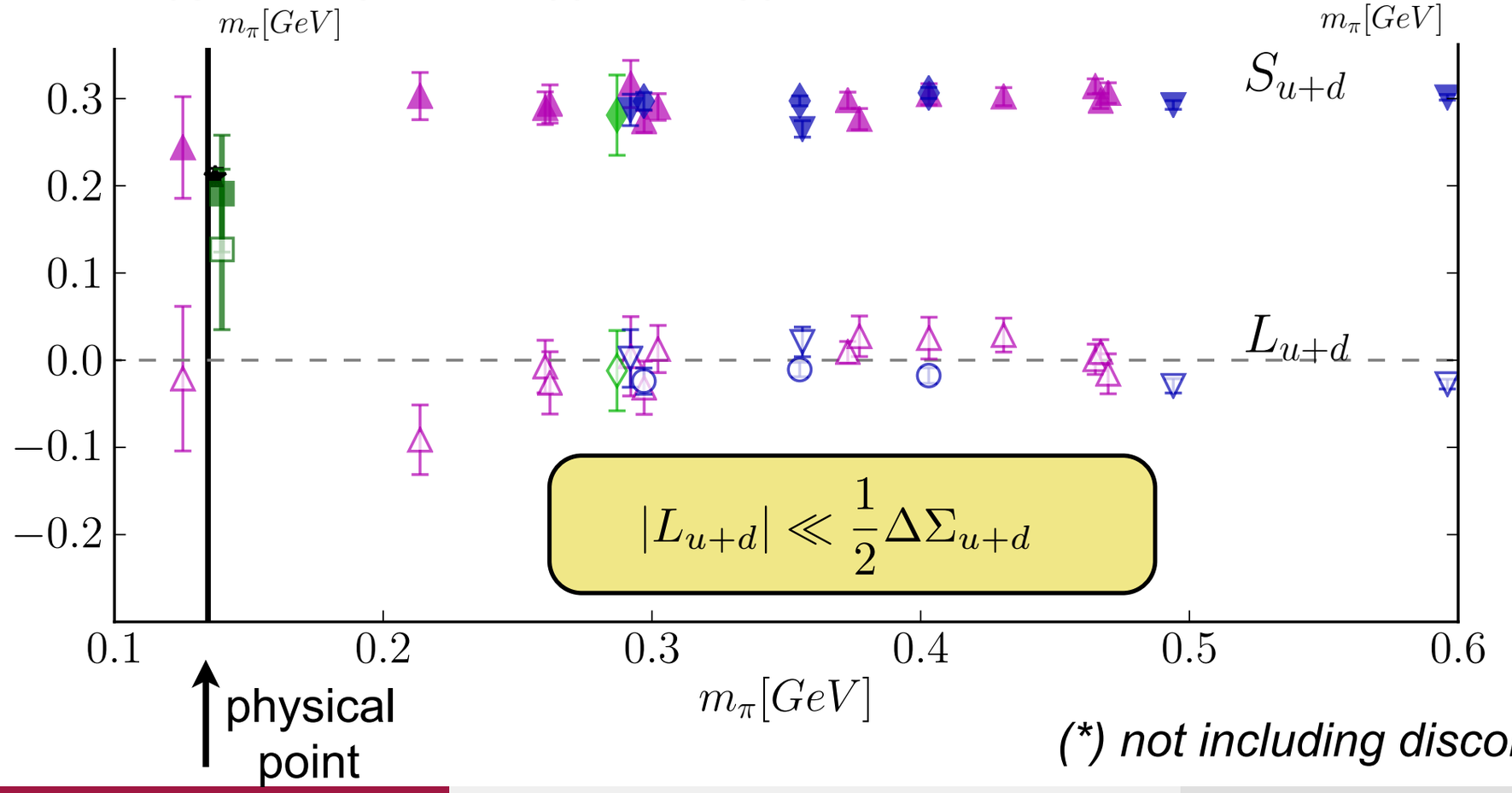


(*) not including disconnected diagrams!

Quark Orbital Angular Momentum $L_q = J_q - S_q$







(*) not including disconnected diagrams

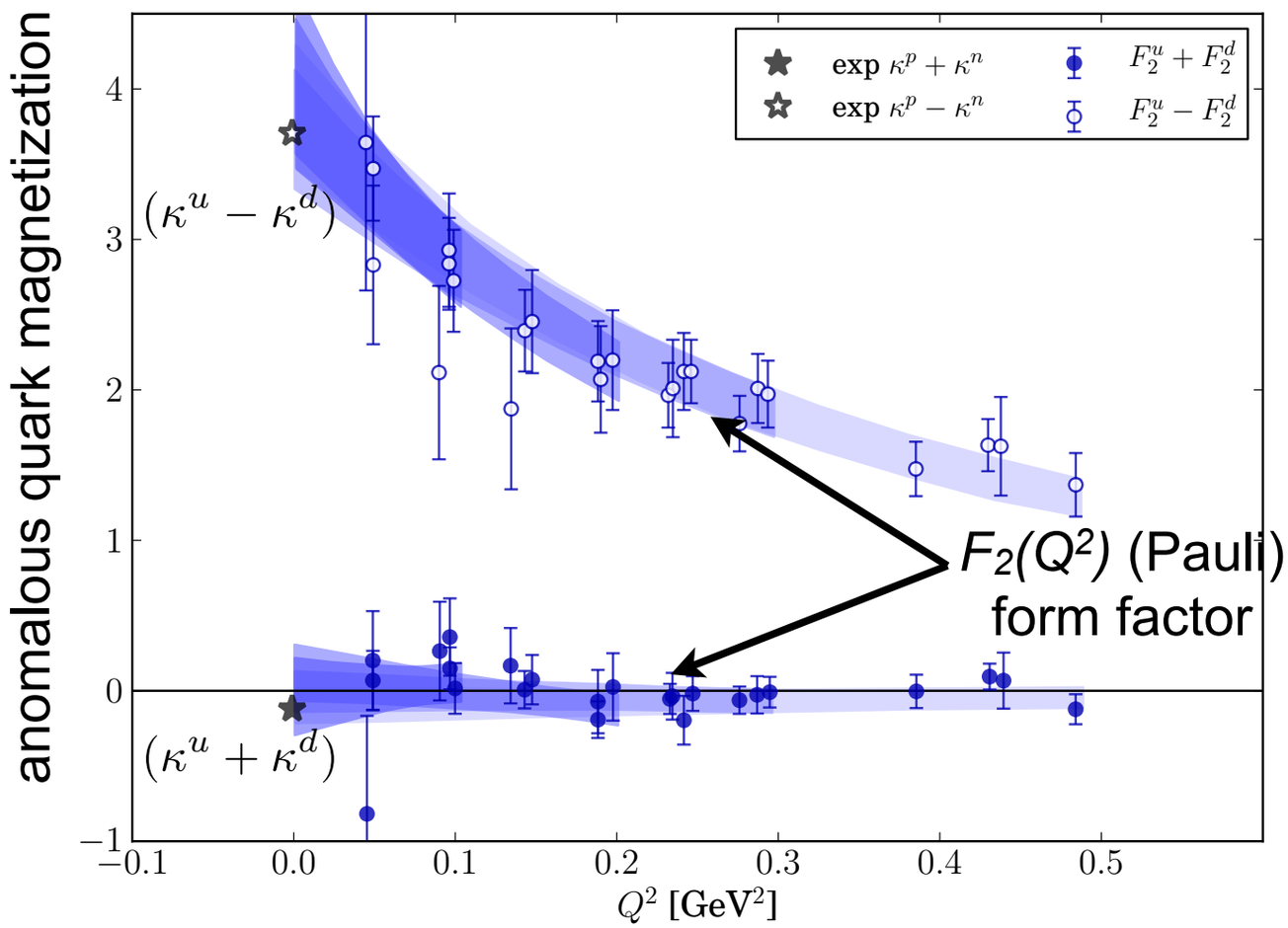
Quark OAM vs Quark Anomalous Magnetization

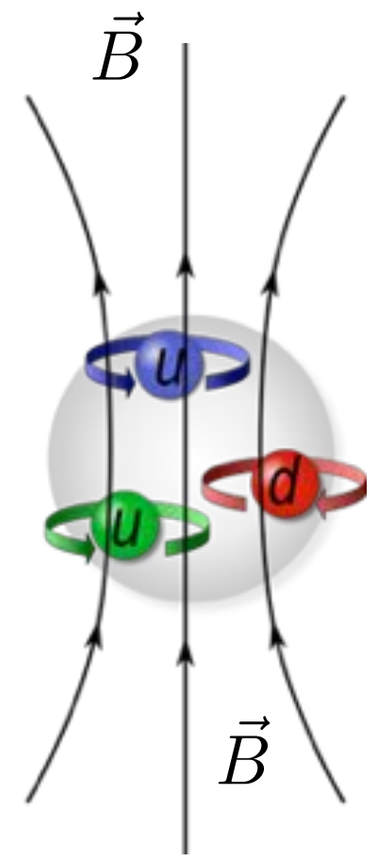
Light Cone Wave functions: Quark OAM is required for non-zero anomalous magnetization from quarks

$$|L^u + L^d| \ll |L^u|, |L^d| \quad \Longrightarrow \quad |\kappa^u + \kappa^d| \ll |\kappa^u|, |\kappa^d|$$

(same prediction for certain TMD PDFs)

[S.J.Brodsky and S.D.Drell (1980); M.Burkardt and G.Schnell (2006); X.-D.Ji, J.-P.Ma, and F.Yuan (2003)]





Lattice data near the physical point ($m_\pi = 149$ MeV):
[J.R.Green, SNS, et al (LHPc) PRD90:074507 (2014)]