## New Features of Baryon Magnetic Moments Uncovered from Lattice QCD



B C Tiburzi 16 September 2016

My work funded by





Done in collaboration with Nuclear Physics Lattice QCD =



Strong interactions in unphysical environments, e.g.  $m_u = m_d = m_s$ 



#### **Magnetic Moments of Light Nuclei**

Home Physics General Physics February 2, 2015

PHYS

#### Pinpointing the magnetic moments of nuclear matter

February 2, 2015 by Kathy Kincade



Artist's impression of a triton, the atomic nucleus of a tritium atom. The image show a red neutron with quarks inside; the arrows indicate the alignments of the spins. Credit: William Detmold, MIT

A team of nuclear physicists has made a key discovery in its quest to shed light on the structure and behavior of subatomic particles. Beane, Chang, Cohen, Detmold, Lin, Orginos, Parreño, Savage, and Tiburzi (*NPLQCD*), Phys.Rev.Lett.**113** (2014)

**First Computation**:

$$m_u = m_d = (m_s)_{\text{phys}}$$

$$m_\pi\sim 800\,{
m MeV}$$

#### First "Nuclear Reaction" from QCD

 $\mu_1$ 

 $\overline{L}_1$ 

0.0

0.0

0.2

Dominant M1 transition @ low energy



Two-body contribution isolated & compares favorably with EFT(T) phenomenology

$$n + p \rightarrow d + \gamma \qquad \gamma^* + d \rightarrow n + p$$
Magnetically Coupled Channels
$$|\Delta I| = |\Delta J| = 1 \qquad I_3 = j_z = 0$$

$$\mathbf{C}(t; \mathbf{B}) = \begin{pmatrix} C_{3S_1, 3S_1}(t; \mathbf{B}) & C_{3S_1, 1S_0}(t; \mathbf{B}) \\ C_{1S_0, 3S_1}(t; \mathbf{B}) & C_{1S_0, 1S_0}(t; \mathbf{B}) \end{pmatrix}$$

Beane, Chang, Detmold, Orginos, Parreño, Savage, and Tiburzi (*NPLQCD*), Phys.Rev.Lett.**115** (2015)

 $m_{\pi}^2$  [GeV<sup>2</sup>]

0.6

0.8

1.0

0.4

# 

## Lattice QCD in Magnetic Fields

Lattice QCD spectroscopy

 $G(t) = \langle \mathcal{O}(t)\mathcal{O}^{\dagger}(0) \rangle = Ze^{-Mt} + \cdots$ 



Add classical magnetic field to QCD

 $G_B(t) = \langle \mathcal{O}(t)\mathcal{O}^{\dagger}(0)\rangle_B = Z(B)e^{-E(B)t} + \cdots$ 

$$E_s(B) = M + \frac{|QeB|}{M} \left(n + \frac{1}{2}\right) - 2\mu sB + \cdots$$

• Zeeman effect  $\Delta E = E_{+\frac{1}{2}}(B) - E_{-\frac{1}{2}}(B) = -2\mu B + \cdots$ 



 $U_{\mu}(x) = e^{igG_{\mu}(x)} \in SU(3)$  $U_{\mu}^{\text{e.m.}}(x) = e^{iqA_{\mu}(x)} \in U(1)$ 

#### Magnetic Field on a Periodic Lattice

Seek uniform B-field  $U_{\mu}(x) = e^{-iqx_2B\delta_{\mu 1}}$ 



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#### Magnetic Field on a Periodic Lattice

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0,						1
	qB(1-N)	qB(1-N)	qB(1-N)	qB(1-N)	qB(1-N)	qB(1-N)
N-1	qB	qB	qB	qB	qB	qB
	qB	qB	qB	qB	qB	qB
	qB	qB ,	qB	qB	qB	qB
$x_2$	qB	qB	qB	qB	qB	qB
0	qB	qB	qB	qB	qB	qB
J	0	$x_1$			N-1 0	

 $U_1(x)U_2(x+\hat{i})U_2^{\dagger}(x+\hat{i}+\hat{j})U_1^{\dagger}(x+\hat{j}) = e^{iqF_{12}} = e^{iqB}$ 

 $U_{\mu}(x) = e^{igG_{\mu}(x)} \in SU(3)$  $U_{\mu}^{\text{e.m.}}(x) = e^{iqA_{\mu}(x)} \in U(1)$ 

#### Magnetic Field on a Periodic Lattice

Seek uniform B-field  $U_{\mu}(x) = e^{-iqx_2B\delta_{\mu 1}} e^{+iqx_1BN\delta_{\mu 2}\delta_{x_2,N-1}}$ 

't Hooft Flux quantization



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Parreño, Savage, Tiburzi, Wilhelm, Chang, Detmold, Orginos (NPLQCD), arXiv/1609.03985



**Compute Zeeman splitting** using Lattice QCD + Uniform Magnetic fields



**Units!** 

$$\mu_p = 2.560(09)(52) \text{ [LatM]}$$
$$\text{[LatM]} = \frac{e \, a}{2}$$

 $a=0.145(2)\,{\rm fm}$ 

 $\mu_p = 1.770(06)(36)(19)$  [NM]  $[\texttt{NM}] = \frac{e}{2M_N}$ 

**Compute Zeeman splitting** using Lattice QCD + Uniform Magnetic fields

 $m_u = m_d = m_s$   $m_\pi \sim 800 \, \mathrm{MeV}$ Proton 0.10 I I  $n_{\Phi} = -6$  $E_{\downarrow}$ 0.05 0.00  $E_{\uparrow}$ -0.05  $n_{\Phi} = 3$ -0.10 -0.15 -0.20  $n_{\Phi} = 12$ ł t 5 10 15 0 0.20 0.15 0.10 0.05 0.00 <sup>上</sup> 0.00 0.02 0.04 0.06 0.08

 $|a^2 e B|$ 

$$\mu_p = 1.770(06)(36)(19) \text{[NM]}$$
$$\text{[NM]} = \frac{e}{2M_N}$$

Ruler Mass Rule (Walker-Loud, LHPC)

 $M_N(m_{\pi}) = 800 \; {\rm MeV} + m_{\pi} \sim 1,600 \, {\rm MeV}$ 



**Compute Zeeman splitting** using Lattice QCD + Uniform Magnetic fields

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$$\mu_p = 1.770(06)(36)(19) \text{[NM]}$$
$$\text{[NM]} = \frac{e}{2M_N}$$

Natural nucleon magnetons

$$\label{eq:nnm} \begin{split} [\texttt{nNM}] &= \frac{e}{2M_N(m_\pi)} \\ u_p &= 3.087(10)(62) \; \texttt{[nNM]} \end{split}$$

Dirac part is short-distance & guaranteed to  $\mathcal{O}(a^2 \Lambda_{\rm QCD}^2)$   $\delta \mu_p = 2.087(10)(62) \text{ [nNM]}$  $\delta \mu_p^{\rm exp} = 1.7929... \text{ [NM]}$ 



**Compute Zeeman splitting** using Lattice QCD + Uniform Magnetic fields



Natural baryon magnetons

$$[\texttt{nBM}] = \frac{e}{2M_B(m_\pi)}$$

Anomalous magnetic moments

 $\delta\mu_B \text{ [nBM]} = \mu_B \text{ [nBM]} - Q_B$ 

U-spin

$$\begin{pmatrix} d \\ s \end{pmatrix} \xrightarrow{SU(2)} U \begin{pmatrix} d \\ s \end{pmatrix}$$



Compute Zeeman splitting using Lattice QCD + Uniform Magnetic fields





**Compute Zeeman splitting** using Lattice QCD + Uniform Magnetic fields



[Actually more complicated, our sea quarks are neutral]



#### **Coleman-Glashow Relations**

$$\mathcal{H} = -\frac{e\,\vec{\sigma}\cdot B}{2M_B} \Big[ \mu_D \langle \overline{B}\{Q,B\} \rangle + \mu_F \langle \overline{B}[Q,B] \rangle \Big] \qquad \qquad m_u = m_d = m_s$$





#### **Coleman-Glashow Magnetic Moments**





## **Coleman-Glashow Magnetic Moments**

Estimate **SU(3)** moments in **SU(3)** chiral limit? 2.1  $\langle \overline{B}\{Q,B\}\rangle\langle m_q\rangle \qquad \langle \overline{B}[Q,B]\rangle\langle m_q\rangle$ 2.0 Quark-mass dependence subsumed! Meißner, Steininger (1997) <sup>L</sup> 1.9 Durand, Ha (1998) Puglia, Ramsey-Musolf (2000) [nBM] 1.8 Requires chiral limit octet baryon mass! 1.7 Dürr et al., BMWc (2012) 1.6  $\mu_D(m_\pi = 0 \, \text{MeV}) = 3.8(1.1)$  $\mu_F(m_{\pi} = 0 \, \text{MeV}) = 2.5(0.6)$ Need SU(3) lattice calculation near chiral limit...





#### **Coleman-Glashow Magic Moments?**

$$\mu_p = \frac{1}{3}\mu_D + \mu_F$$

$$\mu_n = -\frac{2}{3}\mu_D$$
[nBM]

Whole number **CG** moments imply counting?

$$\mu_p = \frac{4}{3}\mu_u - \frac{1}{3}\mu_d = 1 \text{ [cQM]}$$
$$\mu_n = -\frac{1}{3}\mu_u + \frac{4}{3}\mu_d = -\frac{2}{3} \text{ [cQM]}$$

$$\mathbf{NRQM} \quad \frac{e}{2M_Q} = [cQM]$$

 $\mu_D = [\texttt{cQM}] / [\texttt{nBM}] = M_B / M_Q$ 





#### Naïve Quark Model

#### Isovector moments in **NRQM** give light constituent quark mass



 $[cQM] / [nNM] = M_N / M_Q$  $[cQM] / [nBM] = M_B / M_Q$ 



#### Naïve Quark Model

Isovector moments in NRQM give light constituent quark mass





#### Naïve Quark Model Moment Ratio

Grand success of **NRQM** is the ratio





#### Naïve Quark Model Moment Ratios

Grand success of **NRQM** is the ratio

 $1 = R_N = -\frac{2}{3} \frac{\mu_p}{\mu_n}$ But there are other *NRQM* ratios too  $1 = R_{N\Sigma} = \frac{5}{4} \frac{\Delta \mu_{\Sigma}}{\Delta \mu_N}$   $\stackrel{0.6}{\simeq} 0.6$   $\stackrel{0.4}{\simeq} 0.6$ 



## Naïve Quark Model Moment Ratios

#### Grand success of **NRQM** is the ratio



Lattice **QCD** results generally agree better with **NRQM** than experiment

Why do some NRQM predictions work better than others?



Dashen, Jenkins, Manohar (1994)

Our calculations & nature have  $N_c = 3 \dots$ 



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Dashen, Jenkins, Manohar (1994)

Our calculations & nature have  $N_c = 3 \dots$ 

 $\mathcal{R}_{V10_1} = \frac{\Delta \mu_N}{\Delta \mu_{\Sigma}}$  $= 1.25 \quad NRQM$  $= 1 + \mathcal{O}(1/N_c)$ 



Why do some NRQM predictions work better than others?



#### Dashen, Jenkins, Manohar (1994)

Our calculations & nature have  $N_c = 3 \dots$ 

 $\mathcal{R}_{V10_1} = \frac{\Delta \mu_N}{\Delta \mu_{\Sigma}}$   $= 1.25 \quad NRQM$   $= 1 + \mathcal{O}(1/N_c)$   $\mathcal{R}_{V10_2} = (1 - 1/N_c) \frac{\Delta \mu_N}{\Delta \mu_{\Sigma}}$   $= 0.83 \quad NRQM$   $= 1 + \mathcal{O}(\Delta m_q/N_c)$   $= 1 + \mathcal{O}(1/N_c^2)$ 





#### Dashen, Jenkins, Manohar (1994)

Our calculations & nature have  $N_c = 3 \dots$ 



$$\begin{aligned} \mathcal{R}_{S/V1} &= \frac{\frac{1}{2}(\mu_p + \mu_n) + 3(1/N_c - 2/N_c^2)\Delta\mu_N}{\mu_{\Sigma^+} + \mu_{\Sigma^-} - \frac{1}{2}(\mu_{\Xi^0} + \mu_{\Xi^-})} \\ &= 0.62 \quad NRQM \\ &= 1 + \mathcal{O}(\Delta m_q) + \mathcal{O}(\Delta m_q/N_c) + \mathcal{O}(1/N_c^2) \end{aligned}$$

Dashen, Jenkins, Manohar (1994)

Our calculations & nature have  $N_c = 3 \dots$ 



 $= 1 + \mathcal{O}(\Delta m_q) + \mathcal{O}(\Delta m_q/N_c) + \mathcal{O}(1/N_c^2)$ 



Parreño, Savage, Tiburzi, Wilhelm, Chang, Detmold, Orginos (NPLQCD), arXiv/1609.03985



$$H_{I_3=0} = \Delta_{\Lambda\Sigma} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} - \frac{e \,\vec{\sigma} \cdot \vec{B}}{2M_N} \begin{pmatrix} \mu_{\Sigma^0} & \mu_{\Lambda\Sigma} \\ \mu_{\Lambda\Sigma} & \mu_{\Lambda} \end{pmatrix} + \mathcal{O}(B^2)$$

Parreño, Savage, Tiburzi, Wilhelm, Chang, Detmold, Orginos (NPLQCD), arXiv/1609.03985

## $\Sigma^0 \Lambda$ Coupled-Channels Analysis

Diagonalize matrix of correlation functions

$$\mathbb{G}^{(s)}(t,n_{\Phi}) = \begin{pmatrix} G^{(s)}_{\Sigma\Sigma}(t,n_{\Phi}) & G^{(s)}_{\Sigma\Lambda}(t,n_{\Phi}) \\ G^{(s)}_{\Lambda\Sigma}(t,n_{\Phi}) & G^{(s)}_{\Lambda\Lambda}(t,n_{\Phi}) \end{pmatrix}$$

$$\begin{split} E_{\lambda_{-}}^{(-\frac{1}{2})}(B_{z}) &= M_{B} + \mu_{n} \frac{eB_{z}}{2M_{B}} - 2\pi\beta_{n}B_{z}^{2}, \\ E_{\lambda_{+}}^{(+\frac{1}{2})}(B_{z}) &= M_{B} + \mu_{n} \frac{eB_{z}}{2M_{B}} - 2\pi\left(\beta_{n} + \frac{4}{\sqrt{3}}\beta_{\Lambda\Sigma}\right)B_{z}^{2}, \\ E_{\lambda_{-}}^{(+\frac{1}{2})}(B_{z}) &= M_{B} - \mu_{n} \frac{eB_{z}}{2M_{B}} - 2\pi\beta_{n}B_{z}^{2}, \\ E_{\lambda_{+}}^{(-\frac{1}{2})}(B_{z}) &= M_{B} - \mu_{n} \frac{eB_{z}}{2M_{B}} - 2\pi\left(\beta_{n} + \frac{4}{\sqrt{3}}\beta_{\Lambda\Sigma}\right)B_{z}^{2}, \end{split}$$

Coleman-Glashow + magnetic polarizability

 $\mu_{\lambda_{\pm}} = \mp \mu_n \sim \pm 2 \text{ [nBM]}$ 

 $\beta_n = 3.48(12)(26)(04) [10^{-4} \text{ fm}^3]$  $\beta_{\Lambda\Sigma} = -1.82(06)(12)(02) [10^{-4} \text{ fm}^3]$ 

 $m_u = m_d = m_s$ 



# **New Features = New Puzzles**





Walker-Loud LAT2013

1.5

1.4

M<sup>1.3</sup> M<sup>1.2</sup> M<sup>1.1</sup>

1.0

0.9

0.8

0.0

0.1

0.2

0.3

0.4

 $\sqrt{\overline{m}_q}$ 

- Mild pion-mass dependence [nBM]
- Nearness to SU(3) really nearness to SU(6)?
- NRQM explains large-Nc relations for Nc = 3?

\*Need to compute octet, decuplet, and their transition moments \*Need further pion masses, even light SU(3) symmetric ensembles

Why is NRQM successful @ spectrum

physical LHPC 2008

xOCD 2012

**RBC: Preliminary DSDR** 

RBC:  $a^{-1} = 1.75(3)$  GeV

RBC:  $a^{-1} = 2.31(4)$  GeV

0.7

0.6

0.5







# **Future Directions**

#### Magnetic Structure of Baryons & Nuclei

**Move beyond initial studies**: remove systematics, lower pion mass, better treat Landau levels, sea quarks, ...

#### Electric Structure of Baryons & Nuclei

**Electric polarizabilities**? **EDMs of light nuclei** from  $\theta$ -term?, BSM sources?

#### Baryons & Nuclei in other classical fields...

Gravitational?, Weak?





