NUCLEAR PHYSICS AROUND THE UNITARITY LIMIT

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Outline

- What is essential?
- Pionless EFT
- Unitarity: atoms
- Unitarity: nuclei
- Conclusion

with

B. Bazak & M. Eliyahu

S. König,
H.W. Grießhammer
& H.-W. Hammer

What is essential in nuclear physics?

1935-50s: the pion era

- "too many divergences"
- "too many interactions"

1960s-80s: the everything era

fairwell field theory
data fitting trumps consistency

Weinberg '90'91'92
Rho '90
Ordóñez + v.K. '92
...

1990s on: the low-energy QCD era

- ✓ renormalization
- ✓ power counting

welcome effective field theory
consistency before fitting

but

naïve dimensional analysis too naïve
for non-perturbative renormalization
⚠ renormalization issues
⚠ lack of clear systematics

Kaplan, Savage + Wise '96
Cohen '96
Cohen + Phillips '97
...

Pionless EFT

Most general dynamics among nucleons with QCD symmetries

Expansion in momentum/(pion mass)

emphasis today

- A theory of (real) light nuclei

Bedaque + v.K. '97
v.K. '97'98
Kaplan, Savage + Wise '98
Bedaque, Hammer + v.K. '98
Chen, Rupak + Savage '99
...

$A \leq 6$ nuclei described up to 30% in LO,
$A \leq 3$ much better to N^2LO

bound states and low-energy reactions, including symmetry violation

- A theory of nuclei at larger quark masses ("lattice nuclei")

Barnea *et al.* '13
Beane *et al.* '15
Kirscher *et al.* '15
...

only EFT for pion masses
in most current LQCD calculations

extrapolation of LQCD to heavier nuclei and to reactions

Pionless EFT

Most general dynamics among nucleons with QCD symmetries

Expansion in momentum/(pion mass)

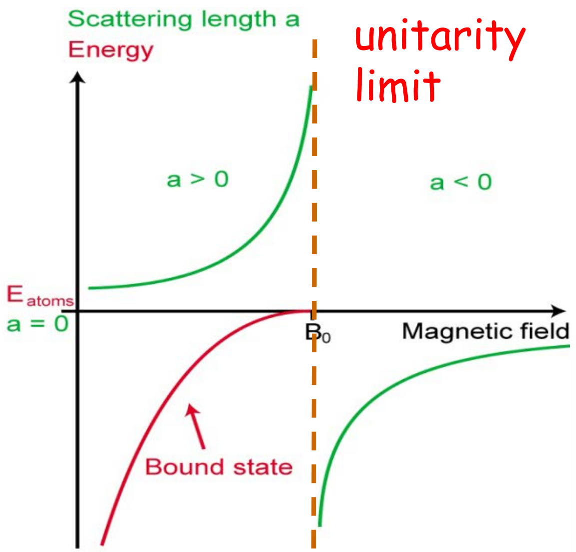
König, Grießhammer,
Hammer + v.K. '15'16
König '16

around two-nucleon unitarity

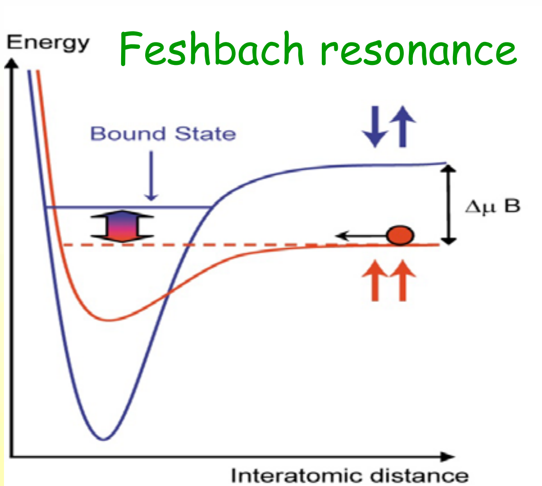
☑ One essential (three-body) interaction/parameter

☑ Everything else in perturbation theory

(no, not a cow joke!)

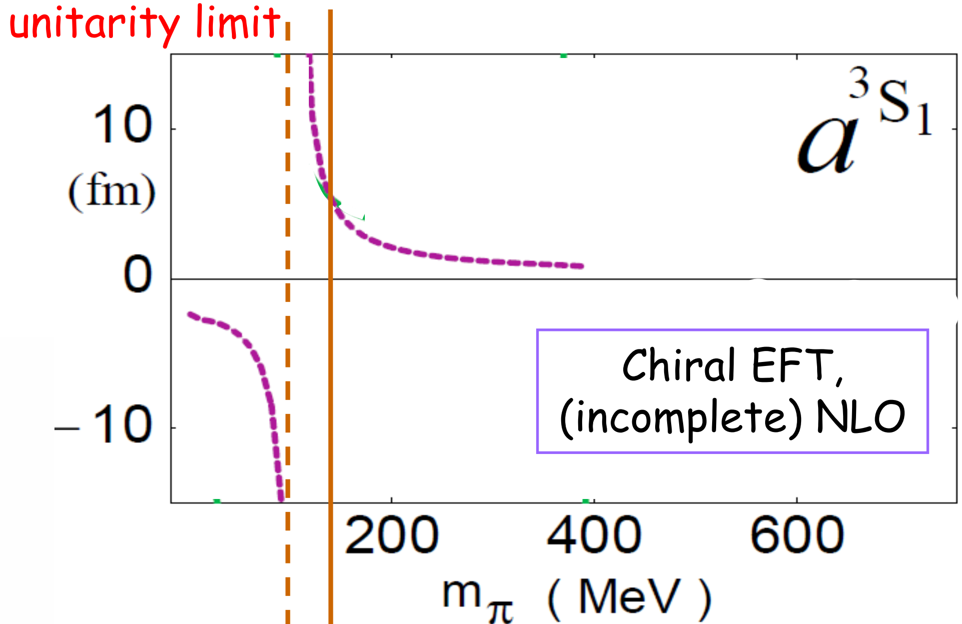


MIT webpage

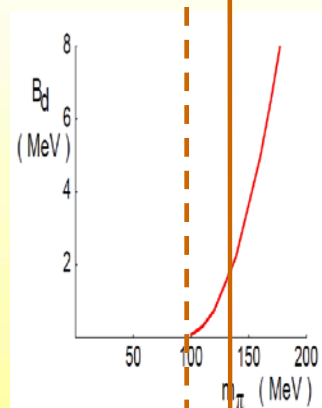


$$a_2 \gg l_{\mathrm{vdW}} \sim r_2$$

or "accidentally", *e.g.* ^{4}He atoms



$m_\pi^*\left(M_{QCD}\right)$ $\quad m_\pi \simeq 140\ \mathrm{MeV}$



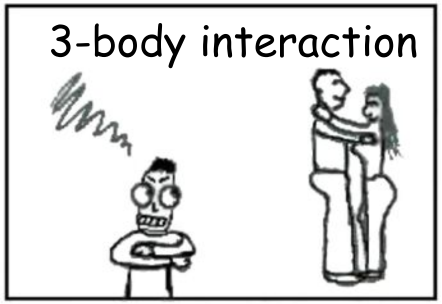
Beane, Bedaque,
Savage + v.K. '02
Beane + Savage '03'04
Epelbaum, Glöckle
+ Meißner '03
...

$$a_2 \gg 1/m_\pi \sim r_2$$

No scale at unitarity?

No!
Scale invariance anomalously broken
from countinous to discrete
in few-body systems:
in general, clusters with
non-zero binding energy exist

Efimov. ’71
...



Wikipedia*

One-parameter
three-body force:
scale emerges
Λ_*

Bedaque, Hammer + v.K. ’99’00
...

... and everything else in
perturbation theory

König, Grießhammer,
Hammer + v.K. ’15’16
König ‘16

Effective Field Theory

$$\begin{cases} T(Q\sim M_{lo} \ll M_{hi}) = \mathcal{N}(M_{lo}) \displaystyle\sum_{\nu=\nu_{\min}}^{\infty} \left[\frac{Q}{M_{hi}}\right]^{\nu} \sum_i \underbrace{\tilde{c}_i^{(\nu)}\left(\frac{\Lambda}{M_{lo}},\frac{\Lambda}{M_{hi}}\right)}_{\text{"low-energy constants"}} \underbrace{F_i^{(\nu)}\left(\frac{Q}{M_{lo}},\frac{Q}{\Lambda}\right)}_{\text{non-analytic functions, from loops}} \\ \dfrac{\partial T}{\partial \Lambda} = 0 \end{cases}$$

light scales

hard scales

norm

counting index

arbitrary UV regulator

"power counting"

Truncate … … consistently with RG invariance

$$T = T^{(\mathcal{V})}\left[1+\mathcal{O}\left(\frac{Q}{M},\frac{Q}{\Lambda}\right)\right] \Rightarrow \frac{\Lambda}{T^{(\mathcal{V})}}\frac{\partial T^{(\mathcal{V})}}{\partial \Lambda} = \mathcal{O}\left(\frac{Q}{\Lambda}\right) \ll 1$$

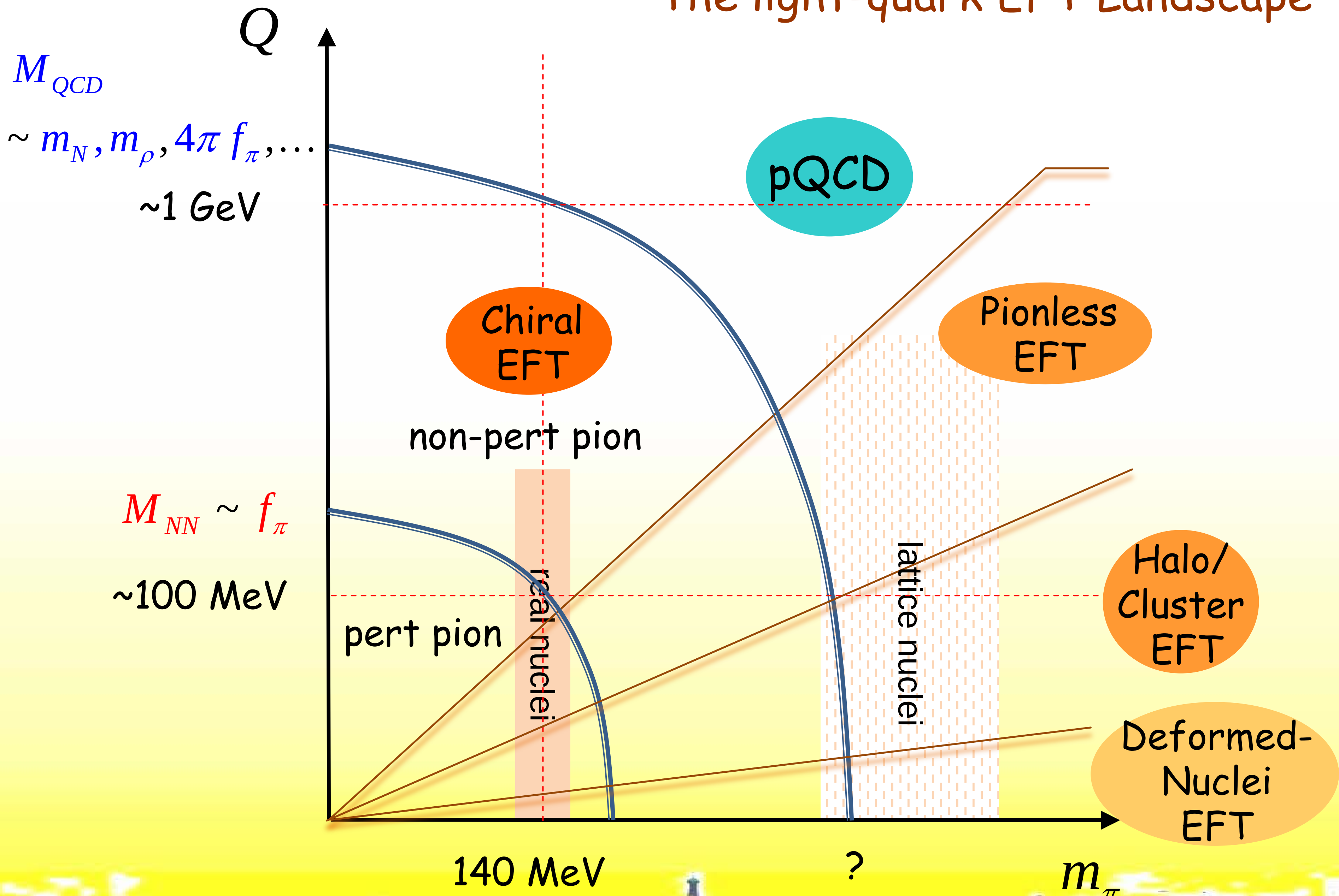
controlled

model independent

<u>If</u> so $\begin{cases} \text{to minimize cutoff errors, } \Lambda \gtrsim M \\ \text{for realistic error estimate, } \Lambda \in [M,\infty) \end{cases}$

(OTHERWISE, NOT ERROR ESTIMATE)

The light-quark EFT Landscape



Pionless EFT

$Q \ll M_{hi} \sim m_\pi$

- d.o.f.: nucleons
- symmetries: ~~Lorentz~~, ~~P~~, ~~T~~, ~~B~~, U(1)$_{em}$

$$\mathcal{L}_{EFT} = N^+ \left(i\mathrm{D}_0 + \frac{\mathrm{D}^2}{2m_N} \right) N + \sum_{I=0,1} C_{0I}\, N^+ N^+ P_I\, N\, N$$

projector on isospin *I*

$$+ D_0\, N^+ N^+ N^+\, N\, N\, N$$

$$+ \Delta C_{0I_3=1}\, N^+ N^+ P_{I_3=1} N\, N + \sum_{I=0,1} C_{2I} \left(N^+ N^+ P_I\, \mathrm{D}^2 N\, N + \ldots \right)$$

$$+ \ldots \qquad \mathrm{D}_\mu = \partial_\mu + ie \frac{1+\tau_3}{2} A_\mu$$

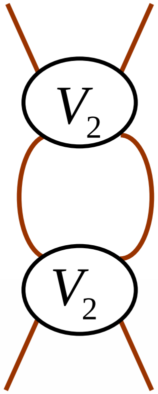
Universality: first orders apply also to neutral atoms

$m_\pi \to 1/l_{\mathrm{vdW}}$ *where* $V(r) = -\dfrac{l_{\mathrm{vdW}}^4}{2mr^6} + \ldots$

Bedaque, Hammer + v.K. '99'00
Bedaque, Braaten + Hammer '01
...

$A=2$

v.K. '97'98
Kaplan, Savage + Wise '98
...



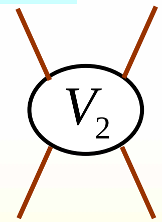
$$\sim \frac{m}{4\pi} C_{2n} C_{2n'} k^{2(n+n')} \left\{ \Lambda \sum_{i=0}^{n+n'} \theta_i \left(\frac{k^2}{\Lambda^2} \right)^{i-(n+n')} + ik + \frac{k^2}{\Lambda} \mathcal{R}_{n+n'} \left(\frac{k^2}{\Lambda^2} \right) \right\}$$

analytic, absorbed in same and/or lower order

non- analytic in $E = k^2/m$

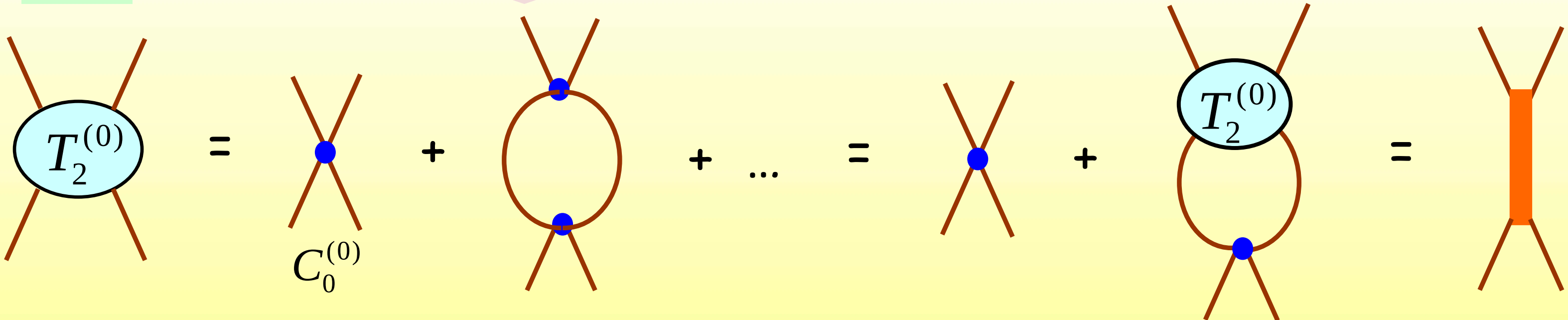
analytic, absorbed in **next** order

$$\sim \frac{mQ}{4\pi} V_2 \times$$



Want: single, shallow (real or virtual) bound state

LO

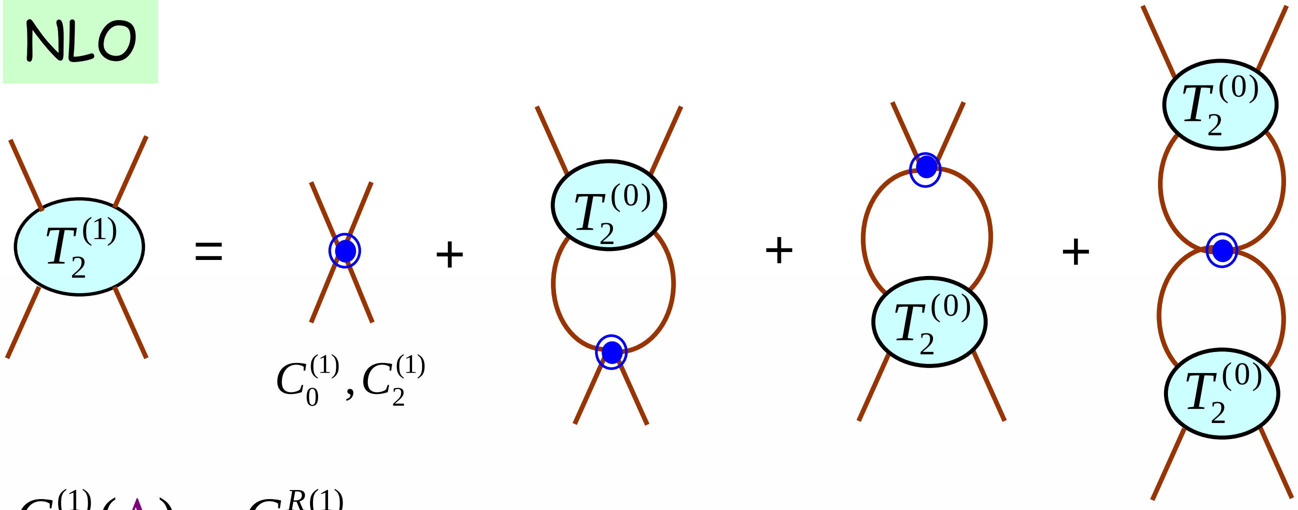


series in $\frac{mQ}{4\pi} C_0^{R(0)}$

$$T_2^{(0)}(k) = \frac{4\pi}{m} \left(\frac{4\pi}{m C_0^{R(0)}} + ik \right)^{-1} \left(1 + \mathcal{O}\left(\frac{Q}{\Lambda} \right) \right)$$

$$C_0^{(0)}(\Lambda) = \frac{4\pi}{m\theta_0 \Lambda} \left[1 + \frac{4\pi}{m\theta_0 \Lambda C_0^{R(0)}} \right]^{-1}$$

NLO



$$
\begin{cases}
\dfrac{C_0^{(1)}(\Lambda)}{C_0^{(0)}(\Lambda)} = \dfrac{C_0^{R(1)}}{C_0^{R(0)}} - \dfrac{m}{\pi^2}\theta_3\Lambda^3 C_2^{(1)}(\Lambda) \\[2ex]
\dfrac{C_2^{(1)}(\Lambda)}{C_0^{(0)2}(\Lambda)} = \dfrac{C_2^{R(1)}}{C_0^{R(0)2}} + \dfrac{m}{4\pi^2\Lambda}\mathcal{R}(0)
\end{cases}
$$

expansion in

$$\frac{Q}{M_{hi}}$$

etc.

equivalent to
- effective range expansion — Bethe '49
- pseudopotential — Fermi '37
- boundary condition at origin — Bethe + Peierls '35

N.B. Perturbative treatment of subLOs not (in general) optional

1) Except for regular interactions, iteration can destroy RG invariance

e.g. iterating $C_2 \Rightarrow r_2 < 0$ Wigner bound Cohen *et al.* '96'97

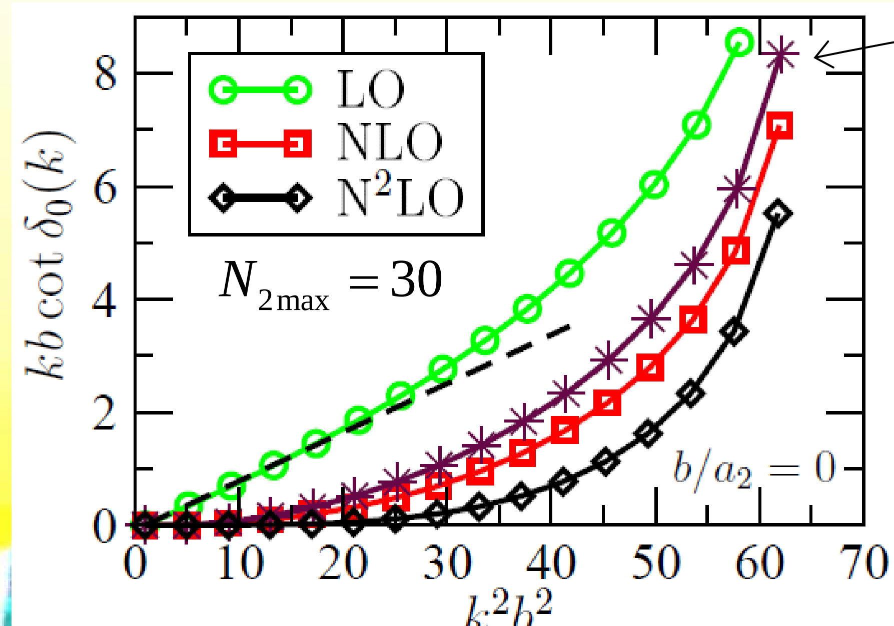
RG invariance

2) Even at fixed cutoff, iteration can give worse results

e.g.

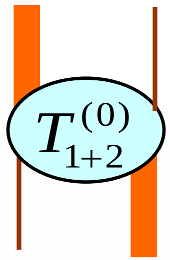
two spin-1/2 fermions at unitarity, in a harmonic oscillator of length *b* and $N_{2\max}$ shells

Rotureau, Stetcu, Barrett + v.K. '10



N²LO Hamiltonian fully diagonalized: worse than NLO!

$A = 3$



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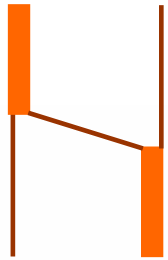

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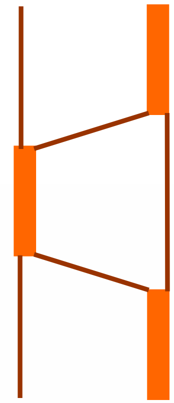
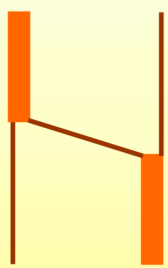


+ ...

$$\sim \frac{4\pi/m}{Q^2/m} \sim \frac{4\pi}{Q^2} \qquad \sim \frac{Q^3}{4\pi}\left(\frac{4\pi/m}{Q^2/m}\right)^2 \frac{1}{Q} \sim \frac{4\pi}{Q^2}$$

=



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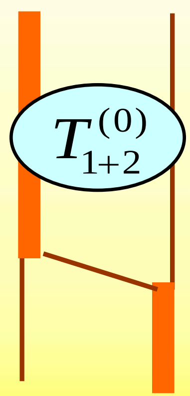
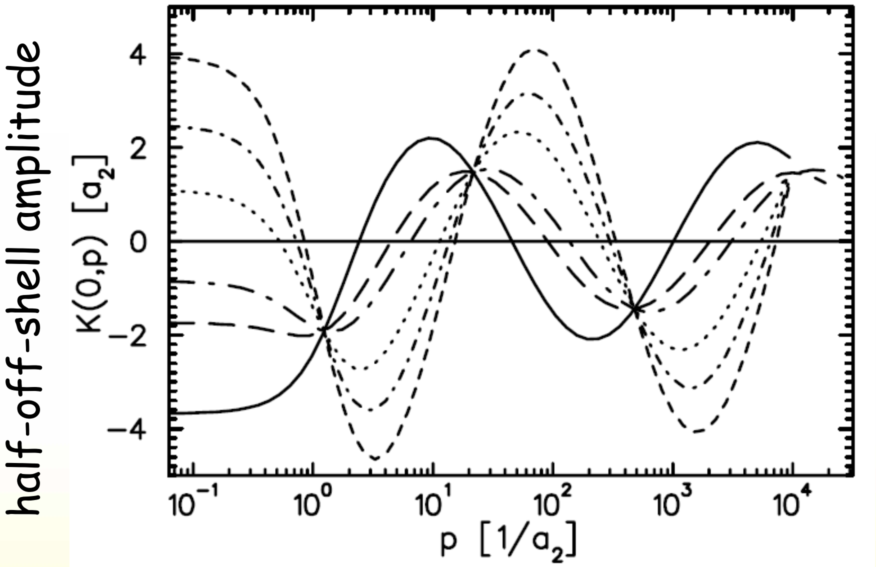


Bedaque, Hammer + v.K. '99'00

...

bosons
fermions with more than two states



no RG invariance

$$B_3 \sim \frac{\Lambda^2}{m}$$

Thomas collapse

Thomas '35

...

$$T_{2+1}^{(0)}(\Lambda \gg p \gg Q; D_0 = 0) \approx A \cos\left(s_0 \ln \frac{p}{\Lambda} + \delta \right)$$

approximate scale invariance

$$s_0 = 1.00624...$$

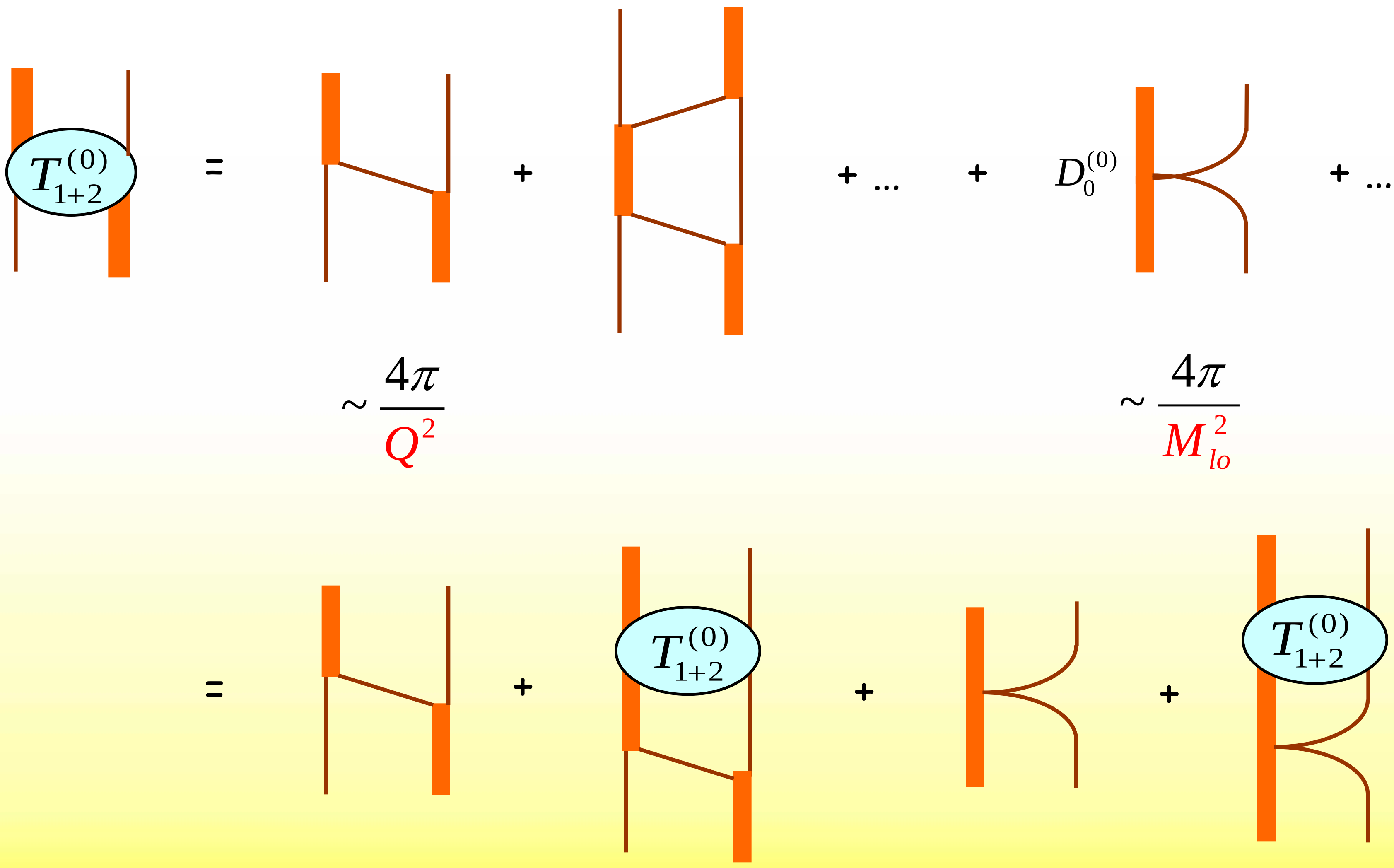
$$\Rightarrow \quad \frac{\Lambda}{T_{2+1}^{(0)}} \frac{\partial T_{2+1}^{(0)}}{\partial \Lambda}(p \sim Q; D_0 = 0) \sim 1$$

requires

$$D_0^{R(0)} = \mathcal{O}\left(\frac{(4\pi)^2}{m M_{lo}^4} \right)$$

LO

not *just* the effective-range expansion!



$T^{(0)}_{1+2}$
=
+
+ ...
+
$D_0^{(0)}$
+ ...
$\sim \frac{4\pi}{Q^2}$
$\sim \frac{4\pi}{M_{lo}^2}$
=
+
$T^{(0)}_{1+2}$
+
+
$T^{(0)}_{1+2}$

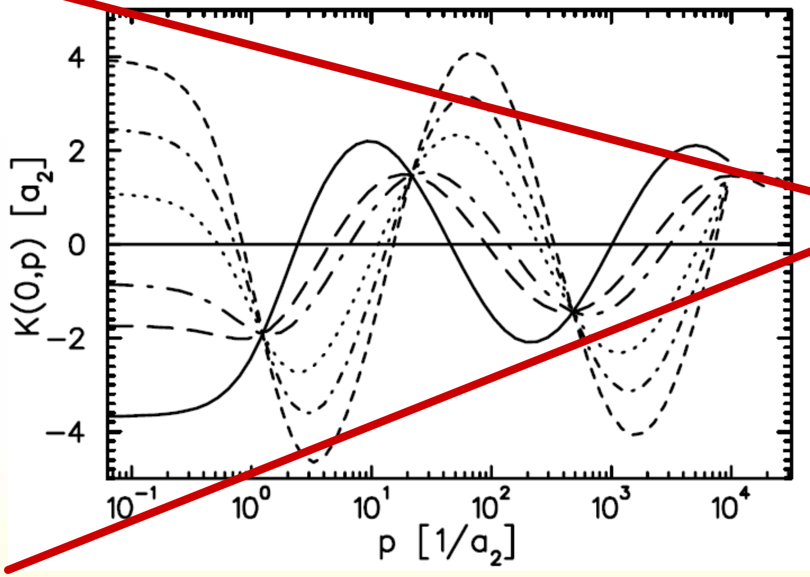
Bedaque, Hammer + v.K. '99'00

...

bosons
fermions with more than two states

no RG invariance

half-off-shell amplitude



$B_3 \sim \frac{\Lambda^2}{m}$

Thomas collapse

Thomas '35

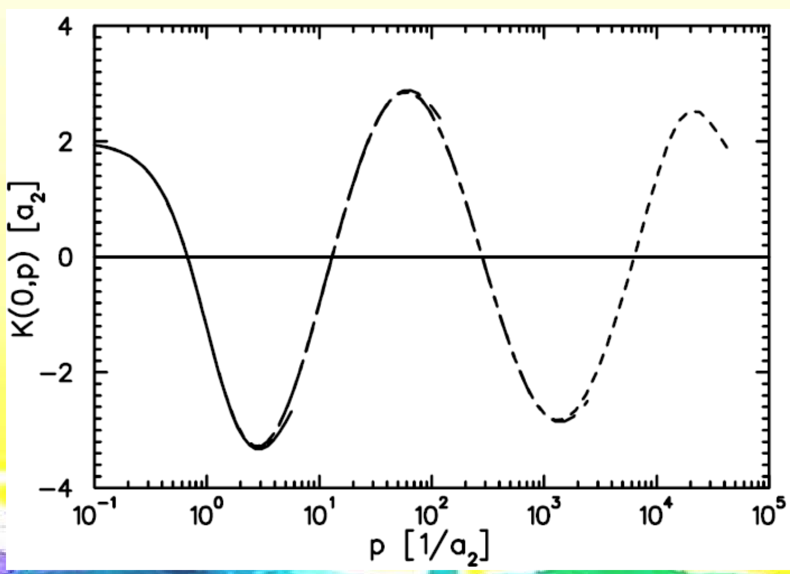
...

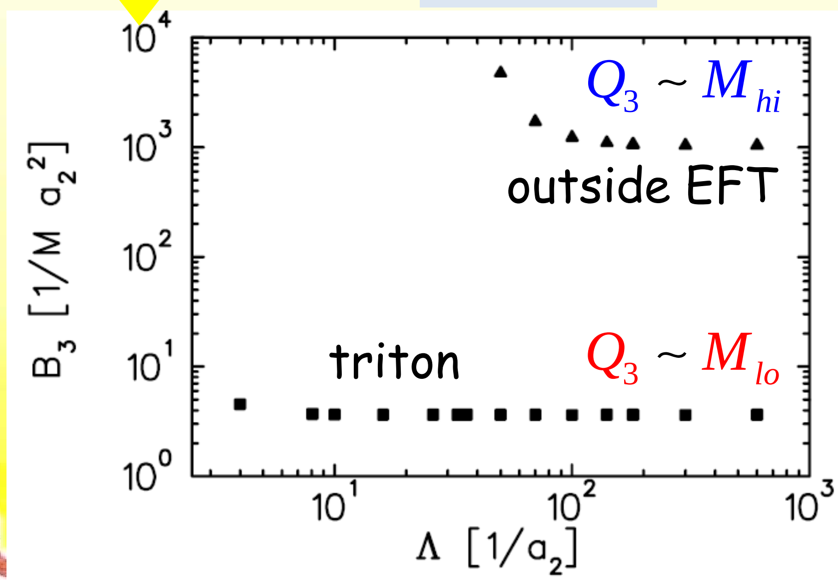
RG invariance

Efimov states

Efimov '71

...

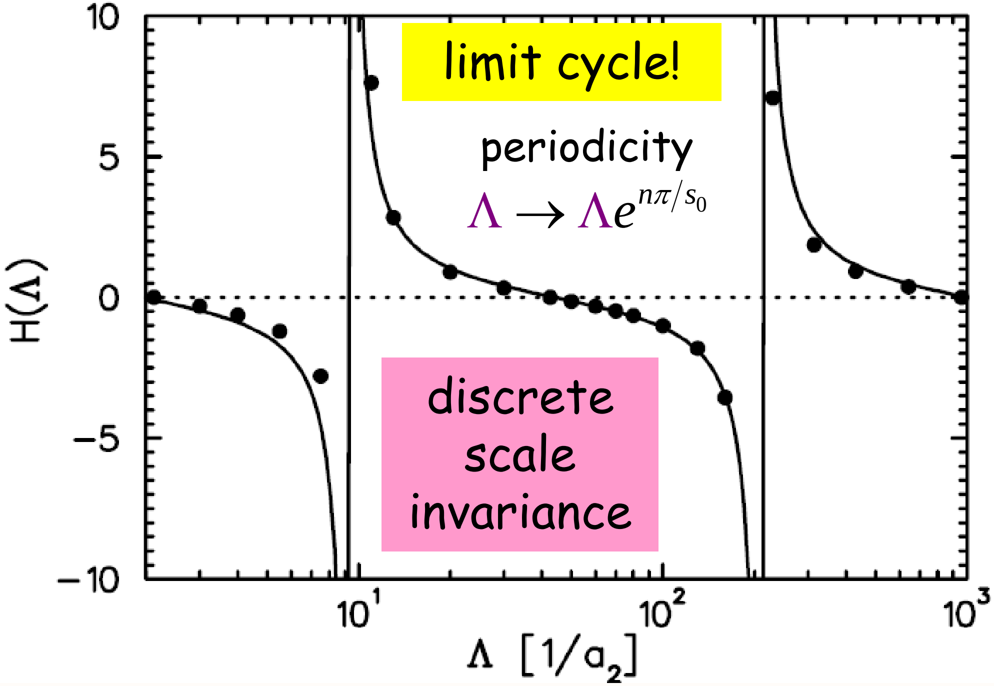




$$H(\Lambda) \equiv \frac{\Lambda^2 D_0^{(0)}(\Lambda)}{m C_0^{(0)2}(\Lambda)}$$

$$\simeq \frac{\sin\left(s_0 \ln(\Lambda/\Lambda_*) + \arctan s_0^{-1}\right)}{\sin\left(s_0 \ln(\Lambda/\Lambda_*) - \arctan s_0^{-1}\right)}$$

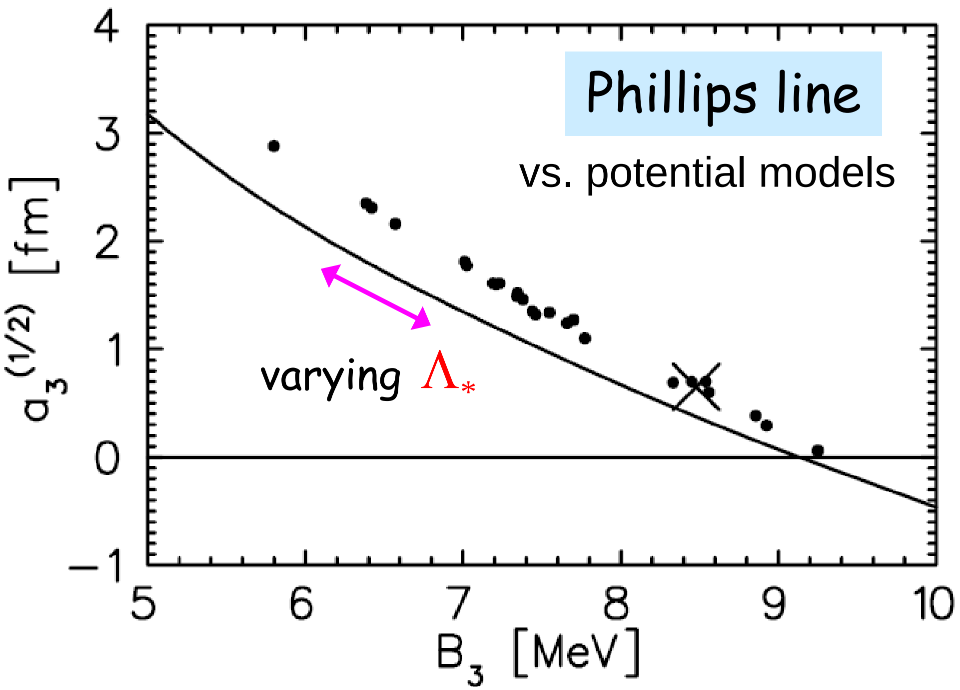
dimensionful parameter
(dimensional transmutation)



limit cycle!

periodicity
$\Lambda \to \Lambda e^{n\pi/s_0}$

discrete
scale
invariance



Phillips line

vs. potential models

varying Λ_*

$SU(4)_W$ symmetric

$$\frac{D_2^{(2)} Q^2}{D_0^{(0)}} \sim \frac{Q^2}{M_{hi}^2}$$

Bedaque, Hammer
+ v.K. '99
Ji + Phillips '13
Vanasse '13

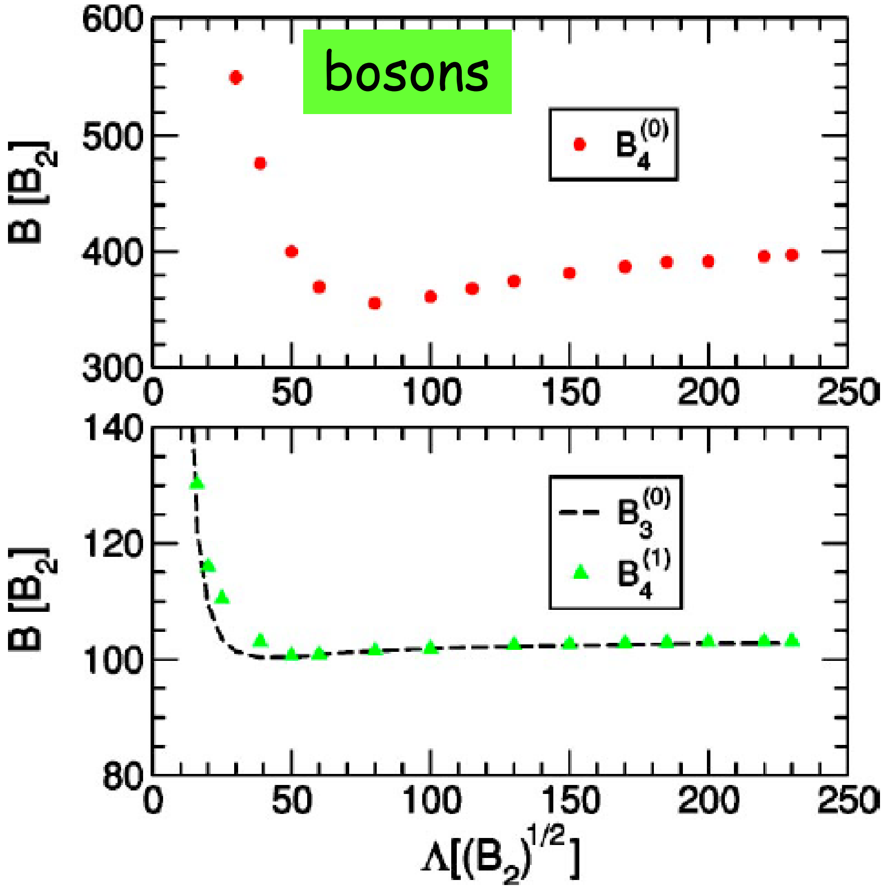
Analogous for
higher derivatives?

At what order a more-body force?

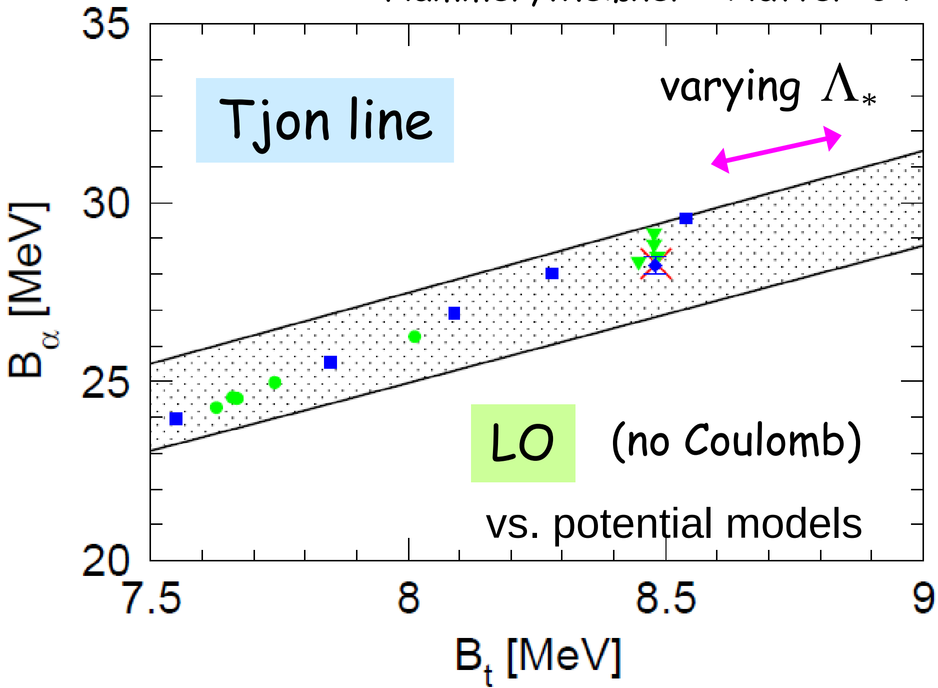
Not at LO

Hammer, Meißner + Platter '04'05
Hammer + Platter '07
Stetcu, Barrett + v.K. '07
Kirscher '11
Bazak, Eliyahu + v.K. '16

Efimov descendants



Hammer, Meißner + Platter '04



$a_{2,I=1,I_3=0}, a_{2,I=0}$

Hammer + Platter '07

$a_{2,I=1,I_3=0}, B_d$

$A = 4$

Standard Power Counting

$$M_{lo} \sim \frac{1}{|a_{2,I=0}|} \sim \frac{1}{|a_{2,I=1,I_3}|} \sim \alpha m_N$$
$$\sim Q_A \overset{?}{\approx} \sqrt{2mB_A/A}$$

(particle binding momentum)

$$M_{hi} \sim \frac{1}{|r_{2,I=0}|} \sim \frac{1}{|r_{2,I=1,I_3}|} \sim \ldots$$
$$\sim m_\pi$$

- *d*, *d**, *t*, … treated equally
- Coulomb LO and thus non-perturbative
- quark mass splitting effects?

Chen, Rupak + Savage '99
Bedaque, Hammer + v.K. '99
…

Kong + Ravndal '00
…
Kirscher *et al.* '09
Ando + Birse '10
…

Kirscher + Phillips '11
König + Hammer '14

How well does it work?

Example: ^{4}He atoms at LO with
- Gaussian regulator
- correlated Gaussian basis
- stochastic variational method

Bazak, Eliyahu + v.K. '16

potentials

Aziz + Slaman '91 — Przybytek *et al.* '10

	(in mK)	LM2M2	PCKLJS	experiment	
$C_0^{(0)}$	B_2	1.3094	1.6154	$1.3^{+0.25}_{-0.19}$, $1.76(15)$	Grisenti *et al.* '00 (+ Cencek *et al.* '12), Zeller *et al.* '16
$D_0^{(0)}$	B_3^*	2.2779	2.6502		
	$B_3^* - B_2$	0.9685	1.0348	0.98(2)	Kunitski *et al.* '15
	B_3	126.50	131.84		
	B_4^*	127.42	132.70		
	B_4	559.22	573.90		

$\underbrace{\text{LM2M2 \quad PCKLJS}}_{\text{Hiyama + Kamimura '12}}$

predictions

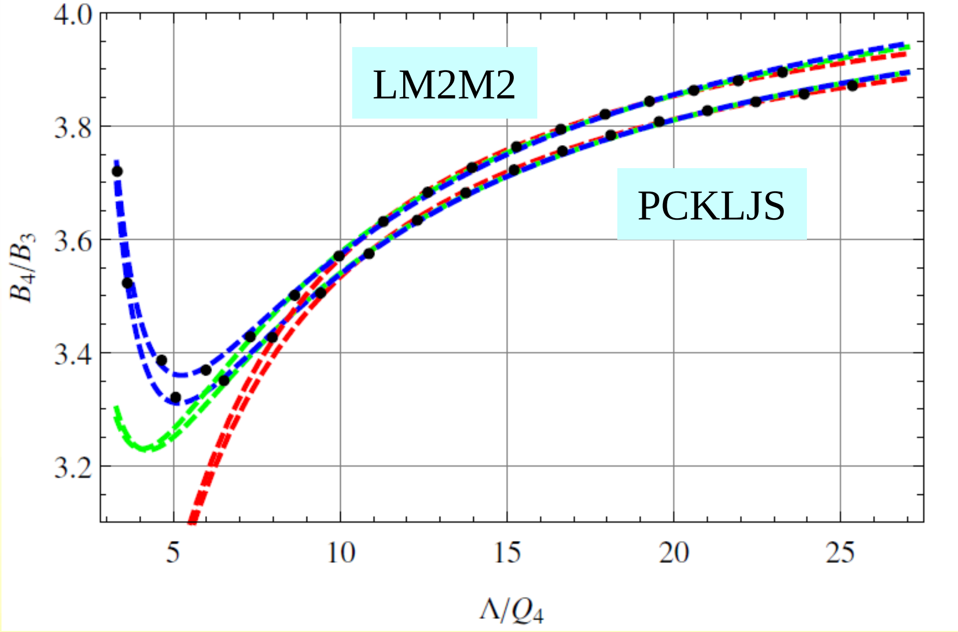
fit
- experimental data → make predictions
- potential model results → test EFT

Bazak, Eliyahu
+ v.K. '16

$$B_N(\Lambda) = B_N(\infty)\left[1 + \alpha_N \frac{Q_N}{\Lambda} + \beta_N \left(\frac{Q_N}{\Lambda}\right)^2 + \gamma_N \left(\frac{Q_N}{\Lambda}\right)^3 + \ldots\right]$$

$$Q_N \equiv \sqrt{2mB_N(\infty)/N}$$

$A = 4$



	$B_4(\infty)/B_3$	α	β	γ
LM2M2	4.128	-1.34	—	—
	4.240	-2.06	4.93	—
	4.237	-2.02	4.06	4.23
PCKLJS	4.090	-1.36	—	—
	4.165	-1.90	4.02	—
	4.157	-1.80	2.32	7.99

similar for other values of A

input	$B_3(\infty)/B_3^*$
$B_2 = 1.3094$ mK	57.15(4)
$a_2 = 100.23$ Å	65.30(3)
direct [37]	55.53
$B_2 = 1.6154$ mK	51.50(3)
$a_2 = 90.42$ Å	59.81(2)
direct [37]	49.75

Works surprisingly well!

Bazak, Eliyahu + v.K. '16

Ref.	[37]	[32]	[31]	[19]	this work
B_4/B_3	4.35	4.44(1)	4.49(2)	4.500	4.20(6)
B_5/B_3	—	10.33(1)	10.519(8)	10.495	9.5(2)
B_6/B_3	—	18.41(2)	18.50(2)	18.504	16.3(5)

[37] Hiyama + Kamimura '12 [32] Blume + Greene '00 [31] Lewerenz '97 [19] Gattobigio *et al.* '11

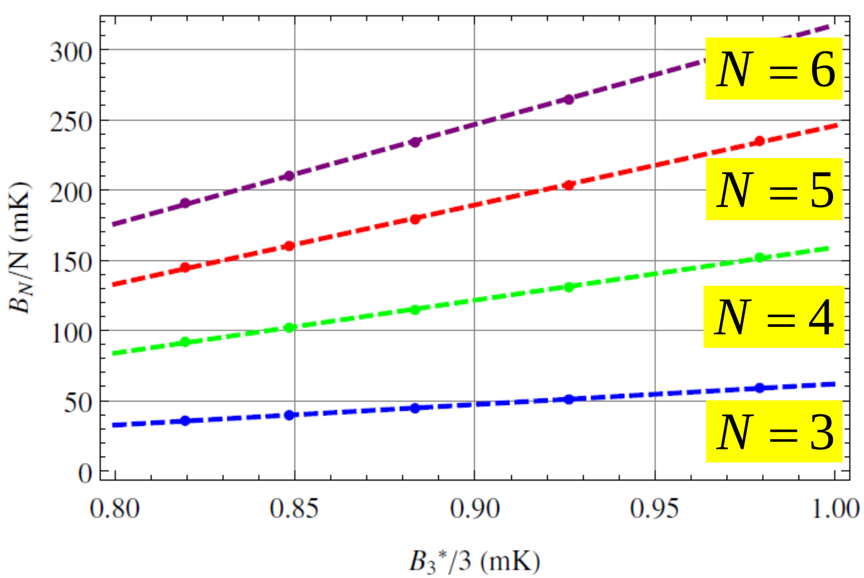
$$\frac{B_N}{N} = c_N\left(B_2/B_3^*\right)\frac{B_3^*}{3} = c_N(0)\frac{B_3^*}{3} + \ldots$$

one scale

$$\frac{B_N}{B_3} \simeq \frac{N\,c_N(0)}{3c_3(0)} \approx (N-2)^2$$

cf. Nicholson '12
Gattobigio + Kievski '14

Generalized Tjon lines



N	$Q_N\, l_{\mathrm{vdW}}$
2	0.05
3^*	0.06
3	0.4
4	0.8
5	1.3
6	1.7

very near unitarity

overestimate of characteristic expansion parameter?

König, Grießhammer, Hammer + v.K. '16

Nuclei

$$\left.\begin{array}{l}\dfrac{B_\alpha}{B_h} \simeq 3.7 \\[2ex] \dfrac{B_{\alpha*}}{B_h} \simeq 1.05\end{array}\right\} \quad \text{vs.} \quad \left\{\begin{array}{l}\dfrac{B_4}{B_3} \simeq 4.6 \\[2ex] \dfrac{B_{4*}}{B_3} \simeq 1.002\end{array}\right.$$

Deltuva '10

bosons at unitarity

+ successes of $SU(4)_W$ for larger nuclei?

ground states

A	Q_A/m_π
2	0.3
3	0.5
4	0.8
5	0.7
6	0.7
…	…
∞	0.9

$\left\|a_{2,I=1,I_3=0}m_\pi\right\|^{-1}$	0.06
$\alpha m_N/m_\pi$	0.05
$\left\|a_{2,I=1,I_3=+1}m_\pi\right\|^{-1}-\left\|a_{2,I=1,I_3=0}m_\pi\right\|^{-1}$	0.12
$\left\|a_{2,I=1,I_3=-1}m_\pi\right\|^{-1}-\left\|a_{2,I=1,I_3=0}m_\pi\right\|^{-1}$	0.02

$$\left\{\begin{array}{l} \left|a_{2,I=1,I_3=0}\right|^{-1} \equiv \aleph_0 \ll M_{lo} \\ \left|a_{2,I=1,I_3=+1}\right|^{-1}-\left|a_{2,I=1,I_3=0}\right|^{-1} \sim \alpha m_N \ll M_{lo} \\ \left|a_{2,I=1,I_3=-1}\right|^{-1}-\left|a_{2,I=1,I_3=0}\right|^{-1} \sim m_d - m_u \ll \aleph_0 \end{array}\right.$$

$$Q_{A\geq 3} \sim M_{lo}$$

$$Q_2 \equiv \aleph_1 \ll M_{lo}$$

$^3S_1$ unitarity

$^1S_0$ unitarity

König, Grießhammer, Hammer + v.K. '16
König '16

König, Grießhammer, Hammer + v.K. '15
König '16

$$m_d - m_u, \alpha m_N, \aleph_0, \aleph_1 \ll Q \sim M_{lo} \ll M_{hi}$$

five expansions
$$\left\{ \begin{array}{l} \dfrac{Q, M_{lo}}{M_{hi}} \quad \text{(standard)} \\[2ex] \dfrac{m_d - m_u, \alpha m_N, \aleph_0, \aleph_1}{Q, M_{lo}} \end{array} \right.$$

for simplicity
(can be improved later)
$$\left\{ \begin{array}{l} \alpha m_N \sim \aleph_0 \sim \aleph_1 \sim \dfrac{M_{lo}^2}{M_{hi}} \\[2ex] m_d - m_u \sim \dfrac{M_{lo}^3}{M_{hi}^2} \end{array} \right.$$

(not inconsistent with $m_n - m_p = \mathcal{O}\left(\dfrac{\alpha m_N}{4\pi}, m_d - m_u \right)$)

Consequences

1) Treat $A \geq 3$ ground states as usual

but two-nucleon S waves are expanded around unitarity

LO

$$\left.\begin{cases} C_{0,I=1}^{R(0)} = 0 \\ C_{0,I=0}^{R(0)} = 0 \\ D_0^{R(0)} = \mathcal{O}\left(\dfrac{(4\pi)^2}{m_N M_{lo}^4}\right) \end{cases}\right\} \quad T_{2,I}^{(0)}(k) = \frac{4\pi}{m_N}(ik)^{-1}\left(1+\mathcal{O}\left(\frac{Q}{\Lambda}\right)\right)$$

exact $SU(4)_W$ symmetry
discrete scale invariance

$$\Rightarrow \left.\begin{matrix} a_{2,I=1,I_3}^{-1} \neq 0 \\ a_{2,I=0}^{-1} \neq 0 \end{matrix}\right\} \text{ at NLO}$$

$$C_{0,I=1}^{R(1)} = \frac{4\pi}{m_N} a_{2,I=1,I_3=0}$$

$$C_{0,I=0}^{R(1)} = \frac{4\pi}{m_N} a_{2,I=0}$$

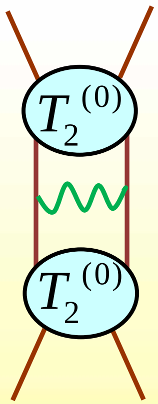
Consequences

1) Treat $A \geq 3$ ground states as usual but two-nucleon *S* waves are expanded around unitarity

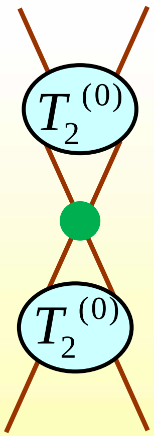
cf. Hammer, Meißner + Platter '05
Stetcu, Barrett + v.K. '07

2) Coulomb is perturbative for bound states, and needs to be resummed only near scattering thresholds

also Kirscher + Gazit '15



RG invariance



$$\propto 4\pi\alpha\left(\ln\frac{\Lambda}{\alpha m_N} + \mathcal{O}(1)\right) P_{I_3=1} \qquad \propto \Delta C_{0I_3=1}\left(\Lambda + Q\right)^2 P_{I_3=1}$$

$$\Delta C_{0I_3=1}(\Lambda) \sim 4\pi\alpha\,\frac{\ln\left(\alpha m_N/\Lambda\right)}{\Lambda^2}$$

$$\Delta C_{0I_3=1} \sim \frac{4\pi}{m_N M_{lo}}\,\frac{\alpha m_N}{M_{lo}}$$

$$\Rightarrow \quad a_{2,I=1,I_3=+1} \neq a_{2,I=1,I_3=0}$$

at NLO

+ no three-body LEC needed at NLO

Consequences

1) Treat $A \geq 3$ ground states as usual but two-nucleon *S* waves are expanded around unitarity
2) Coulomb is perturbative for bound states, and needs to be resummed only near scattering thresholds
3) smaller quark mass splitting effects ⇒ $a_{2,I=1,I_3=-1} \neq a_{2,I=1,I_3=0}$ at N²LO

can predict ³He up to NLO

in contrast to standard power counting where

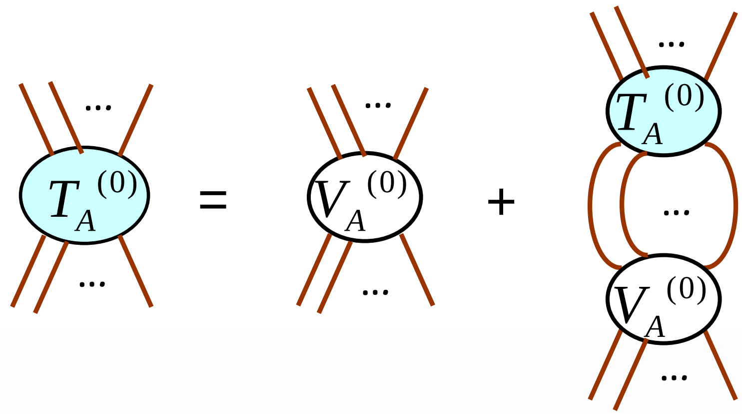
Coulomb and the associated two-body LEC are LO
a new three-body LEC appears at NLO

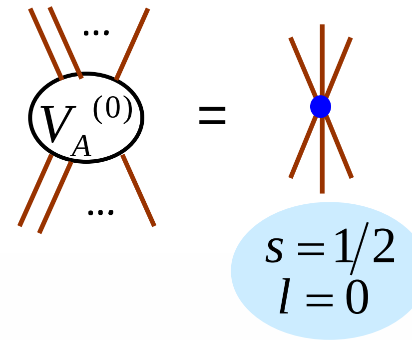
Vanasse *et al.* '14

smaller number of LECs at each order, more predictive power

LO

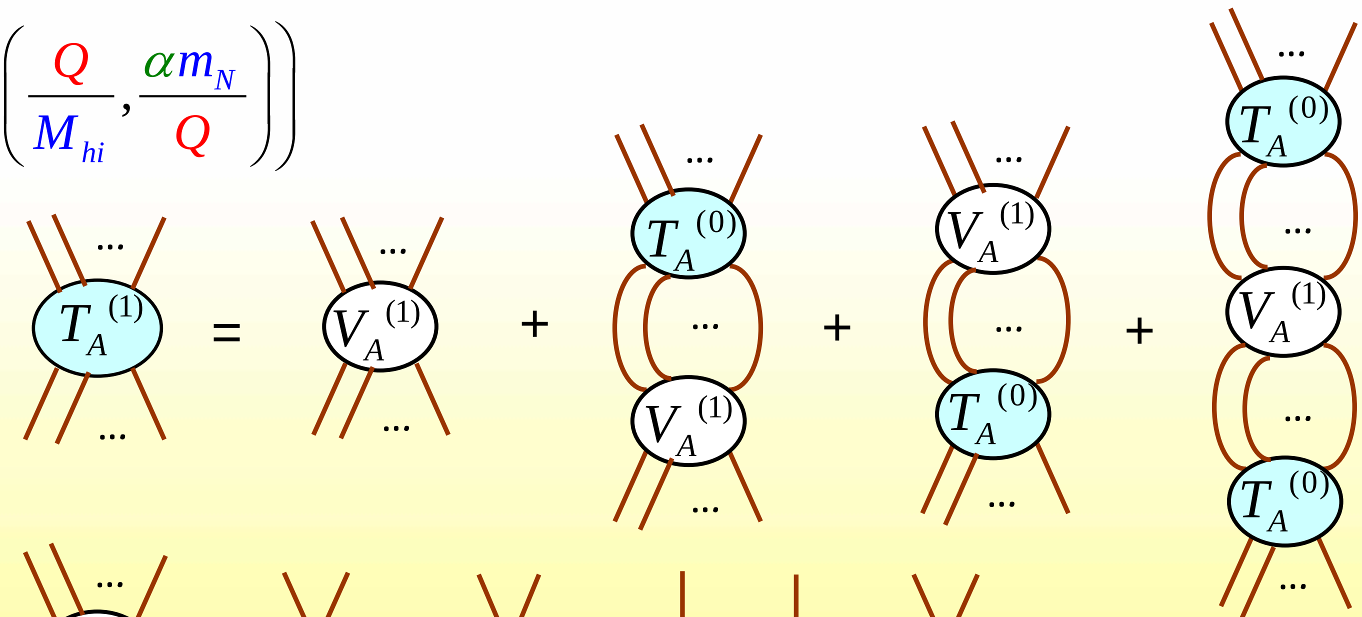
$$\mathcal{O}\left(\frac{4\pi}{m_N M_{lo}}\right)$$

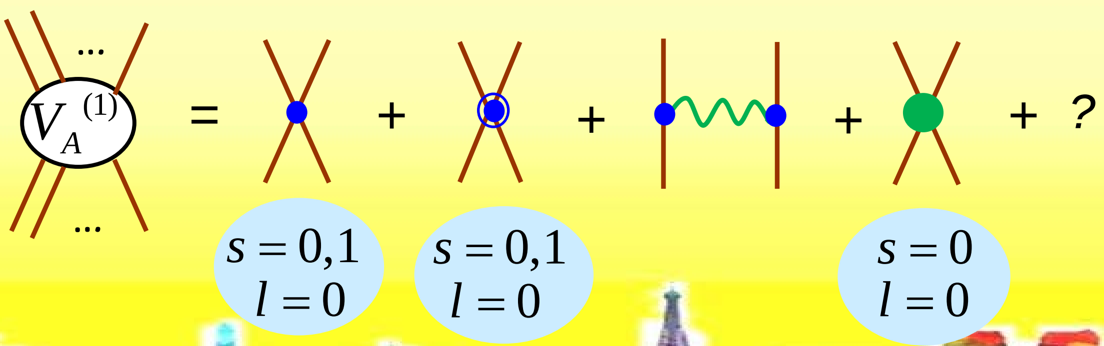




NLO

$$\mathcal{O}\left(\frac{4\pi}{m_N M_{lo}} \times \left(\frac{Q}{M_{hi}}, \frac{\alpha m_N}{Q}\right)\right)$$



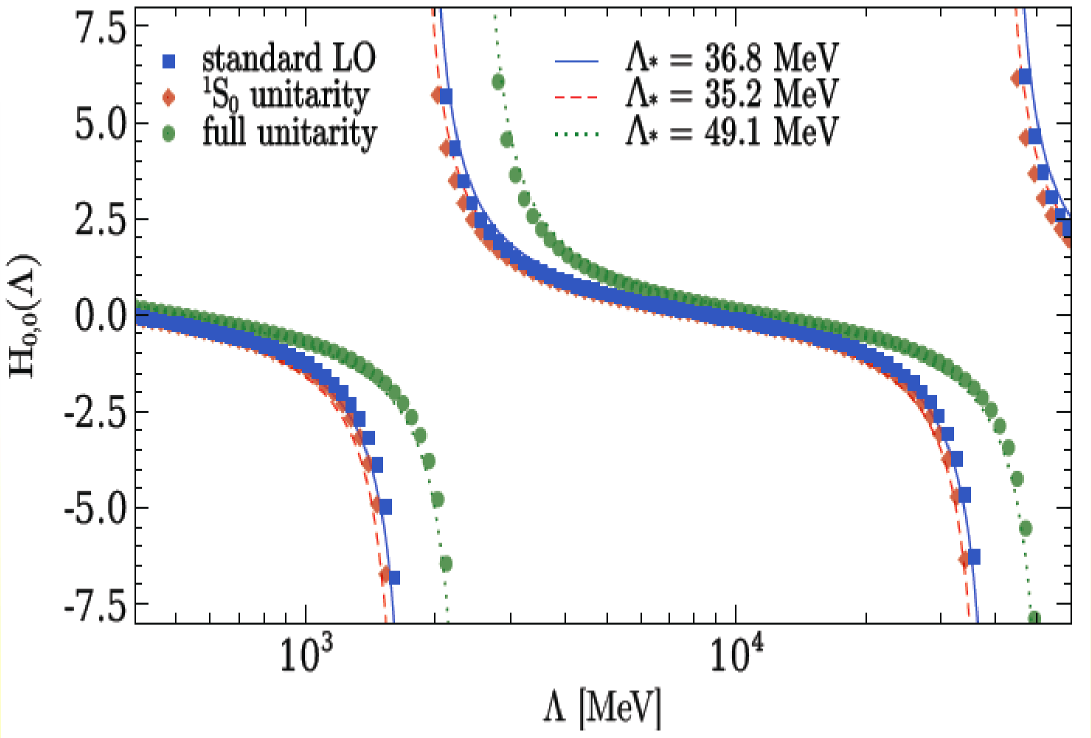


etc.

LO

König, Grießhammer, Hammer + v.K.'15'16

$$B_t = -8.48 \text{ fm (exp)} \Rightarrow D_0$$



$$H(\Lambda) \equiv \frac{\Lambda^2 D_0^{(0)}(\Lambda)}{m_N C_0^{(0)2}(\Lambda)} \simeq \frac{\sin\left(s_0 \ln(\Lambda/\Lambda_*) + \arctan s_0^{-1}\right)}{\sin\left(s_0 \ln(\Lambda/\Lambda_*) - \arctan s_0^{-1}\right)}$$

NLO

$A=2$

${}^1S_0$ unitarity

König, Grießhammer, Hammer + v.K. '15

$$T_{2,I=1,I_3\neq 1}(k)=\frac{4\pi}{m_N}\frac{1}{ik}\left\{1+\frac{1}{ik}\left[-a^{-1}_{2,I=1,I_3=0}+\frac{r_{2,I=1,I_3=0}}{2}k^2\right]+\ldots\right\}$$

$a_{2,I=1,I_3=0} = -23.714$ fm $\Rightarrow C_{0\,I=1}$

$r_{2,I=1,I_3=0} = 2.73$ fm $\Rightarrow C_{2\,I=1}$

$$T_{2,I=1,I_3=+1}(k)=T_C(k)+\frac{\Gamma\left(1+i\dfrac{\alpha m_N}{2k}\right)}{\Gamma\left(1-i\dfrac{\alpha m_N}{2k}\right)}t_{sC}(k)$$

$a_{2,I=1,I_3=+1} = -7.8063$ fm $\Rightarrow \Delta C_{0\,I_3=1}$

$$t_{sC}(k)=\frac{4\pi}{m_N}\frac{1}{ik}\left\{1+\frac{1}{ik}\left[\alpha m_N\left(C_E+\ln\frac{\alpha m_N}{2k}\right)-a^{-1}_{2,I=1,I_3=+1}+\frac{r_{2,I=1,I_3=0}}{2}k^2\right]+\ldots\right\}$$

predict
$$\begin{cases} a_{2,I=1,I_3=-1}(\mathrm{NLO}) = a_{2,I=1,I_3=0} \\ r_{2,I=1,I_3=-1}(\mathrm{NLO}) = r_{2,I=1,I_3=0} \\ r_{2,I=1,I_3=+1}(\mathrm{NLO}) = r_{2,I=1,I_3=0} \end{cases}$$
vs.
$$\begin{cases} a_{2,I=1,I_3=-1} = -18.7 \text{ fm (? exp)} \\ r_{2,I=1,I_3=-1} = \text{? (exp)} \\ r_{2,I=1,I_3=+1} = 2.79 \text{ fm (exp)} \end{cases}$$

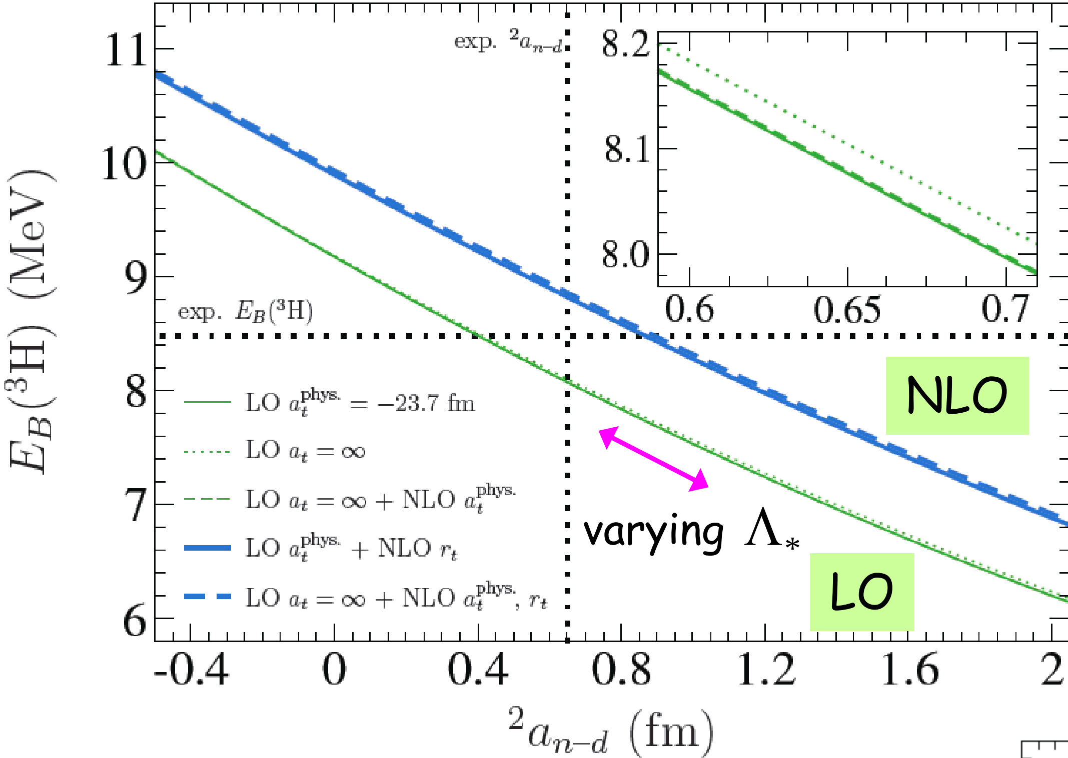
N²LO

two-photon exchange

$r_{2,I=1,I_3=+1} - r_{2,I=1,I_3=0} \simeq 0.06$ fm $\Rightarrow \Delta C_{2\,I_3=1}$

König '16

König, Grießhammer, Hammer + v.K. '15



$A = 3$

$^1S_0$ unitarity

$S_{1/2}$

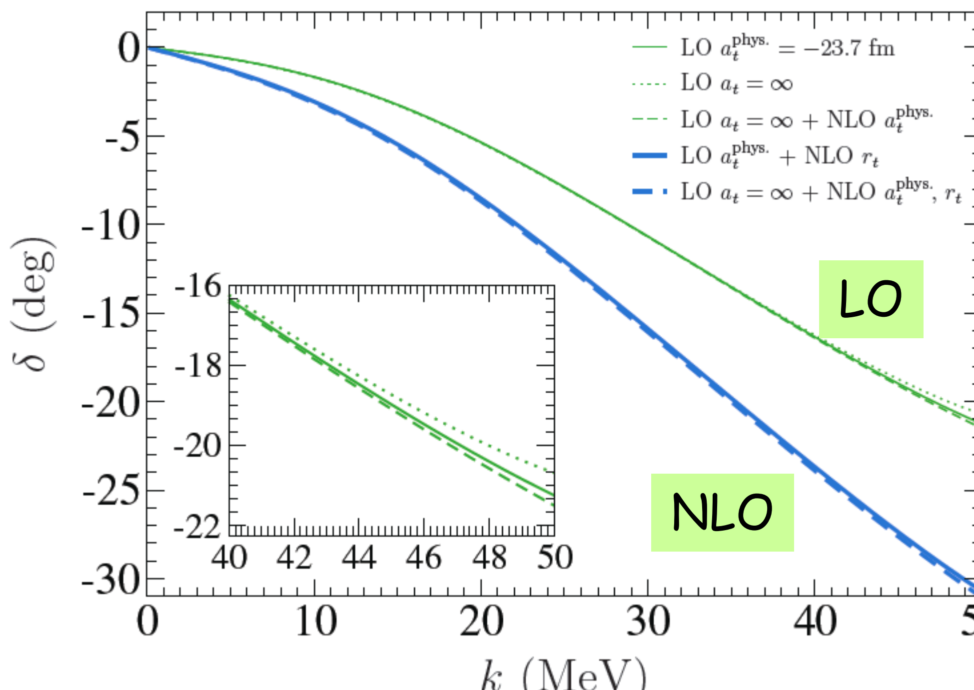
nd scattering

Phillips line

even better at

N²LO

König '16



⇨ range effects more important than unitarity corrections

König, Grießhammer, Hammer + v.K. '16

$A=2$

full unitarity

NLO

$$T_{2,I=0}(k)=\frac{4\pi}{m_N}\frac{1}{ik}\left\{1-\frac{\gamma_d}{ik}\left(1-\frac{\rho_d}{2}\frac{k^2+\gamma_d^2}{\gamma_d}\right)+\ldots\right\}$$

$$\gamma_d = 45.7\,\mathrm{MeV} \Rightarrow C_{0\,I=0}$$

$$\rho_d = 1.765\ \mathrm{fm} \Rightarrow C_{2\,I=0}$$

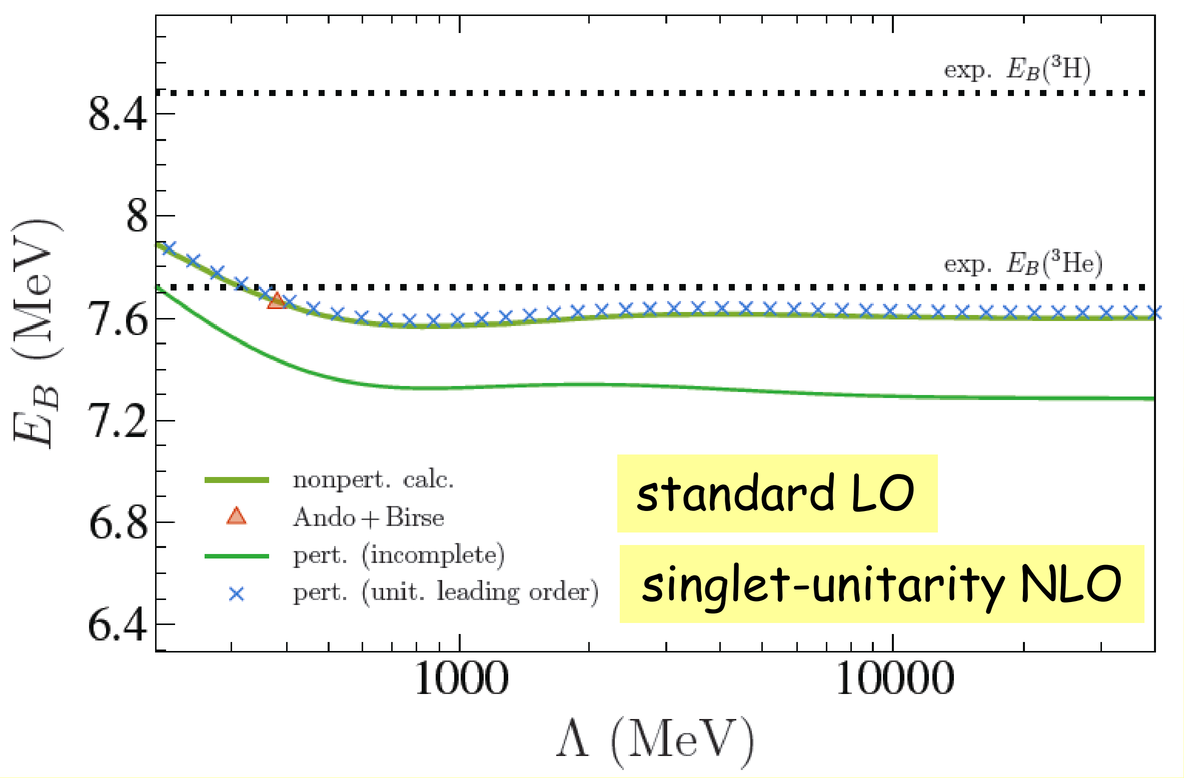
$$k_2^{(1)} = i\gamma_d \quad \Rightarrow \quad B_d^{(2)} = \frac{\gamma_d^2}{m_N}$$

N²LO

$A = 3$

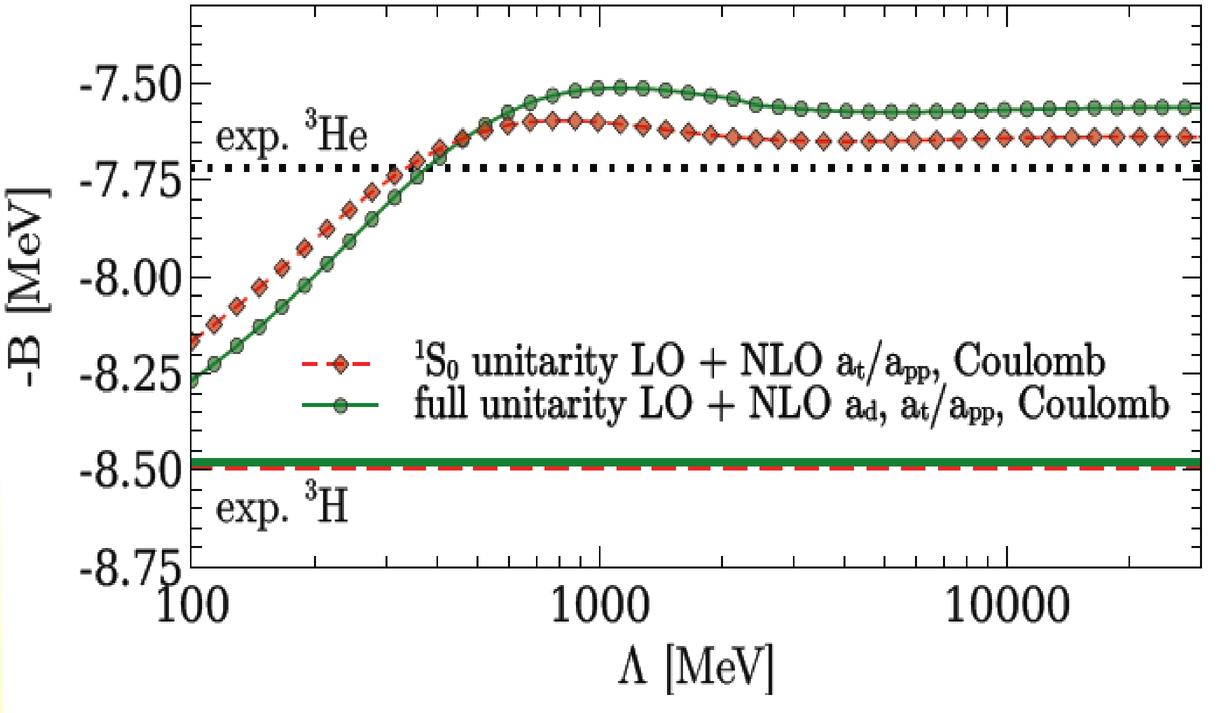
König, Grießhammer, Hammer + v.K. '15

$^{1}S_0$ unitarity



$A = 3$

König, Grießhammer,
Hammer + v.K. '16



$$(B_t - B_h)(\mathrm{NLO}) \simeq (0.92 \pm 0.18)\ \mathrm{MeV}$$

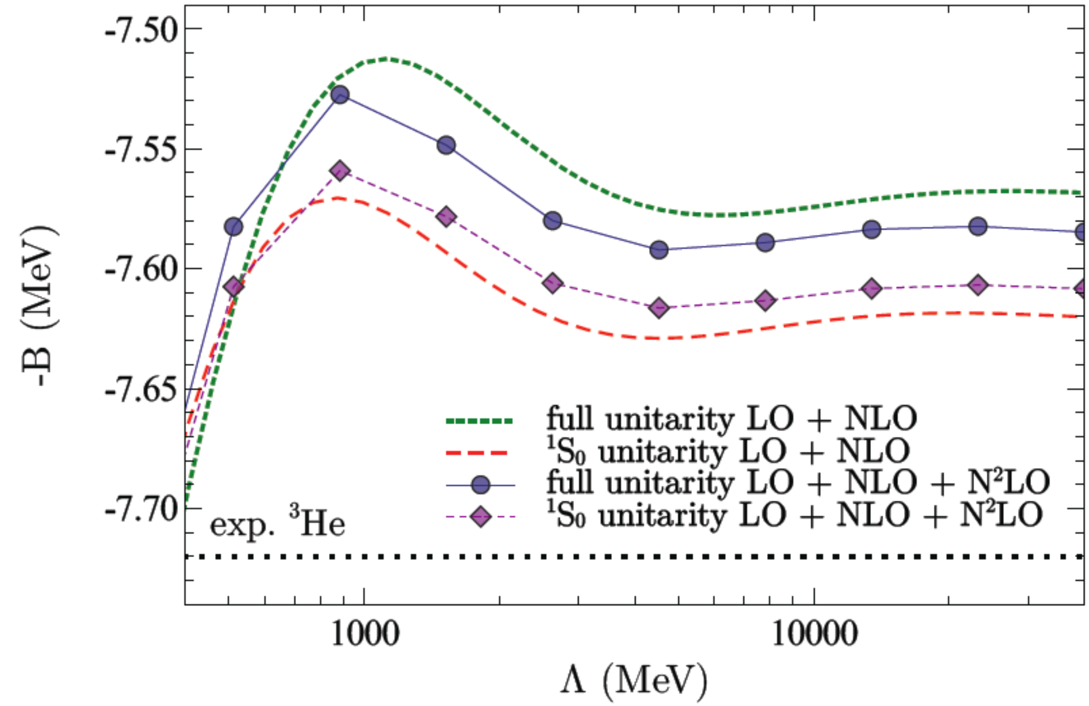
vs.

$$B_t - B_h \simeq 0.764\ \mathrm{MeV}\ (\mathrm{exp})$$

König'16

$A = 3$

N^2LO



without ranges

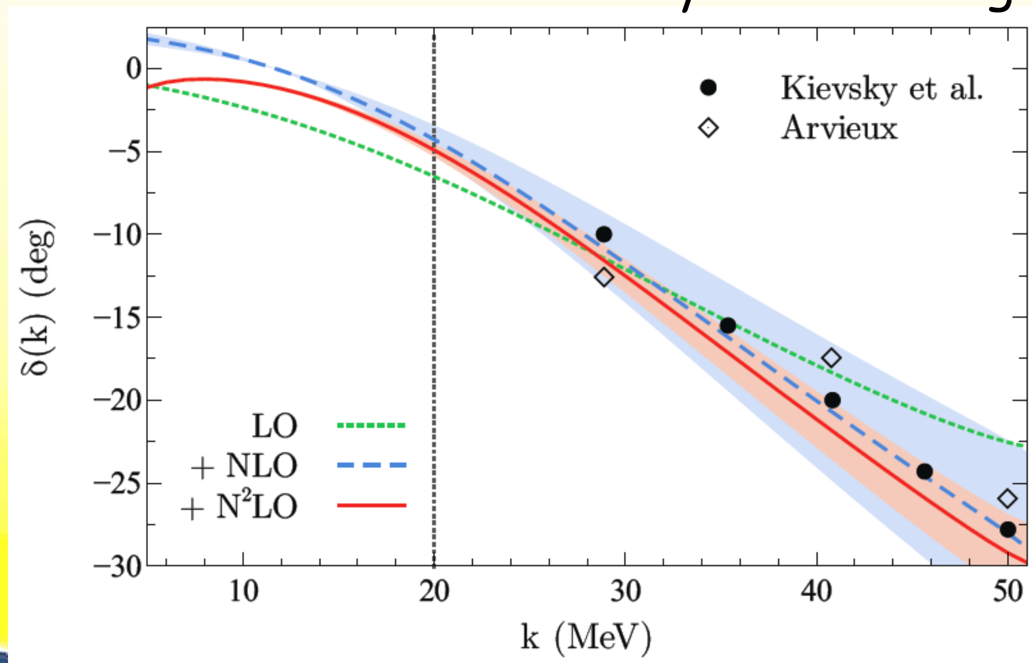
$S_{1/2}$

pd scattering

$^1S_0$ unitarity

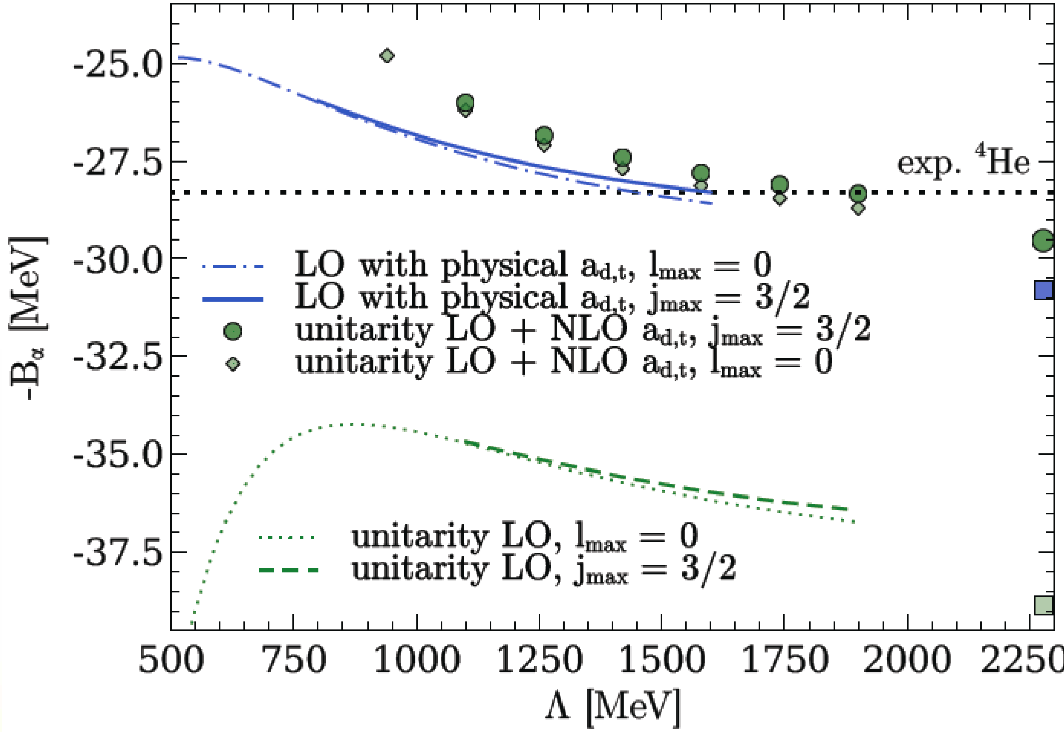
with ranges → cutoff dependence
→ isospin-breaking 3-nucleon force
fitted to

$$B_t - B_h \simeq 0.764 \text{ MeV (exp)}$$



König, Grießhammer,
Hammer + v.K. '16

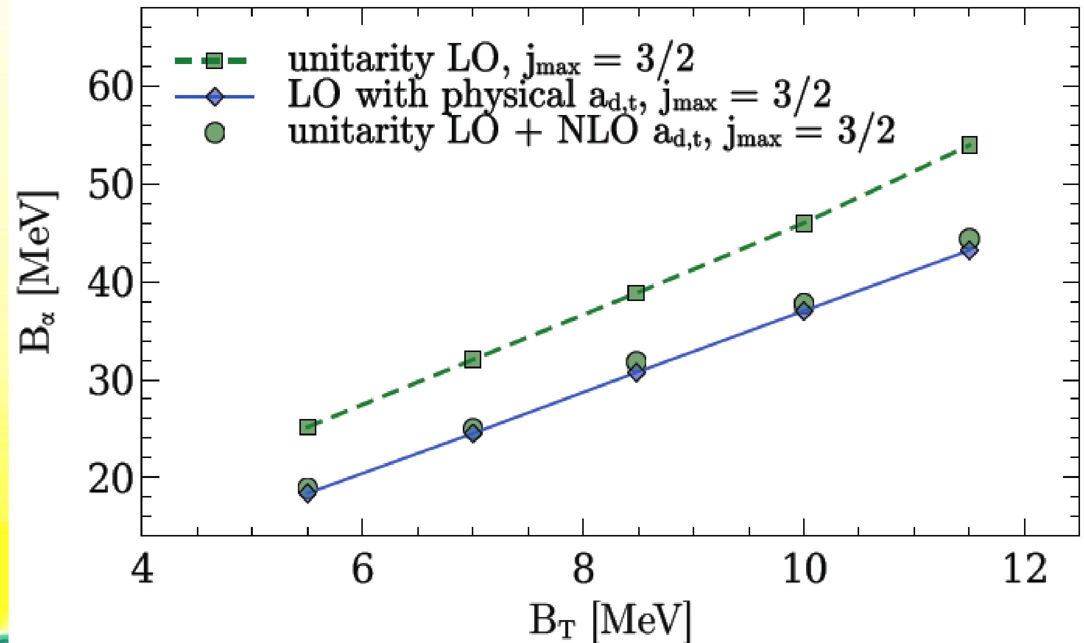
$A = 4$



extrapolated values

$$B_4(\Lambda) = B_4(\infty)\left[1 + \beta_4\left(\frac{Q_N}{\Lambda}\right)^2 + \ldots\right]$$

Tjon line



full unitarity

(no Coulomb,
no ranges)

Conclusion

- Few-body systems near unitarity can be described model-independently by Pionless EFT

- For ^4He atoms and light nuclei,

  energies given by essentially one parameter
  details obtained in perturbation theory

- How far can we go this way?