Current status of exploiting theoretical modelling of gravity waves in data analysis

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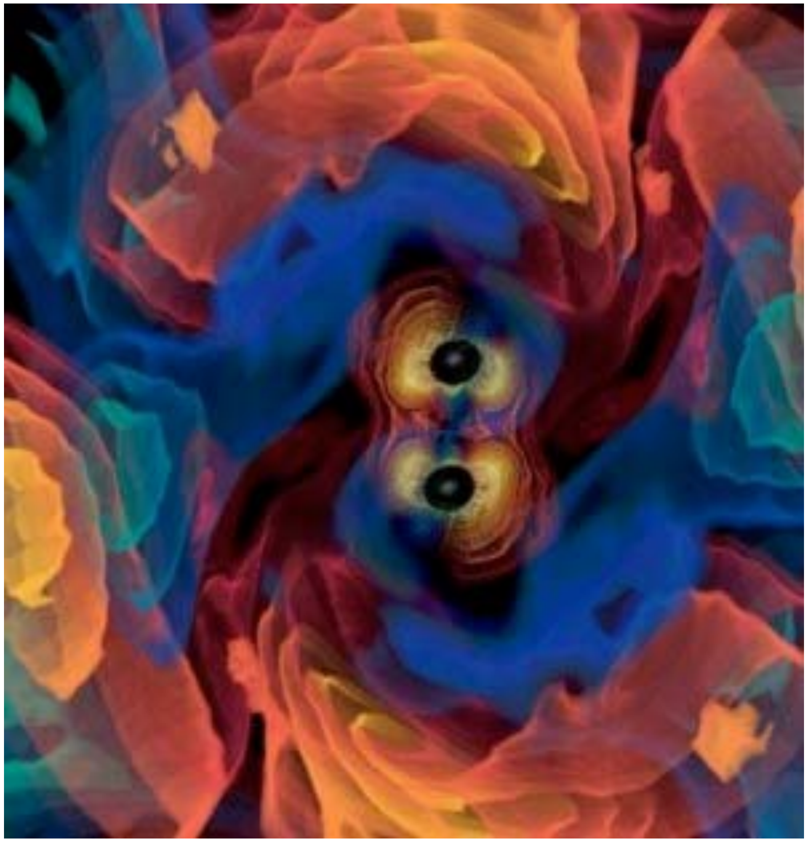
Cardiff University and

LIGO Scientific Collaboration

Outline

- Detectors
- Source Waveforms
- Search Methods
- Input from Numerical Relativity

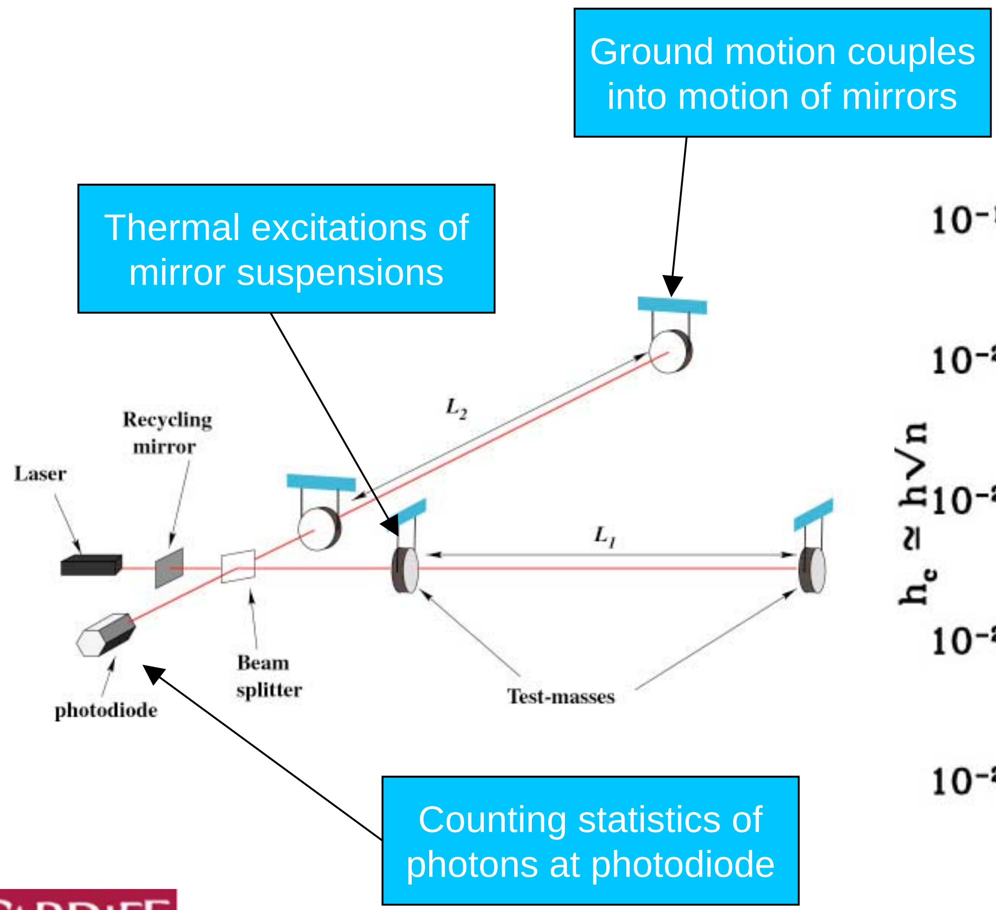


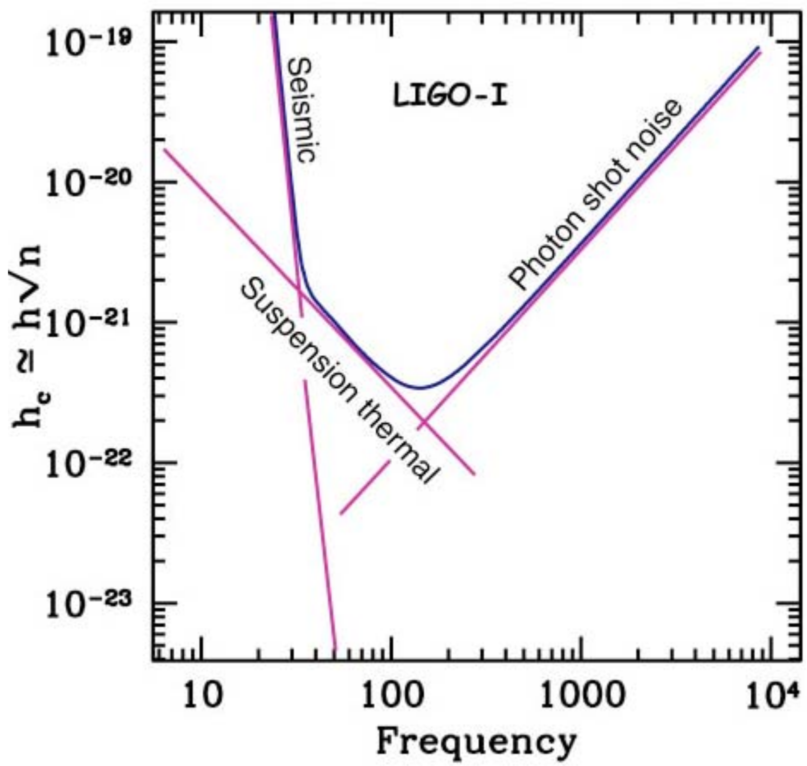
The LIGO Detectors

Livingston, LA
4km detector “L1”

Hanford, WA
4km detector “H1”
2km detector “H2”

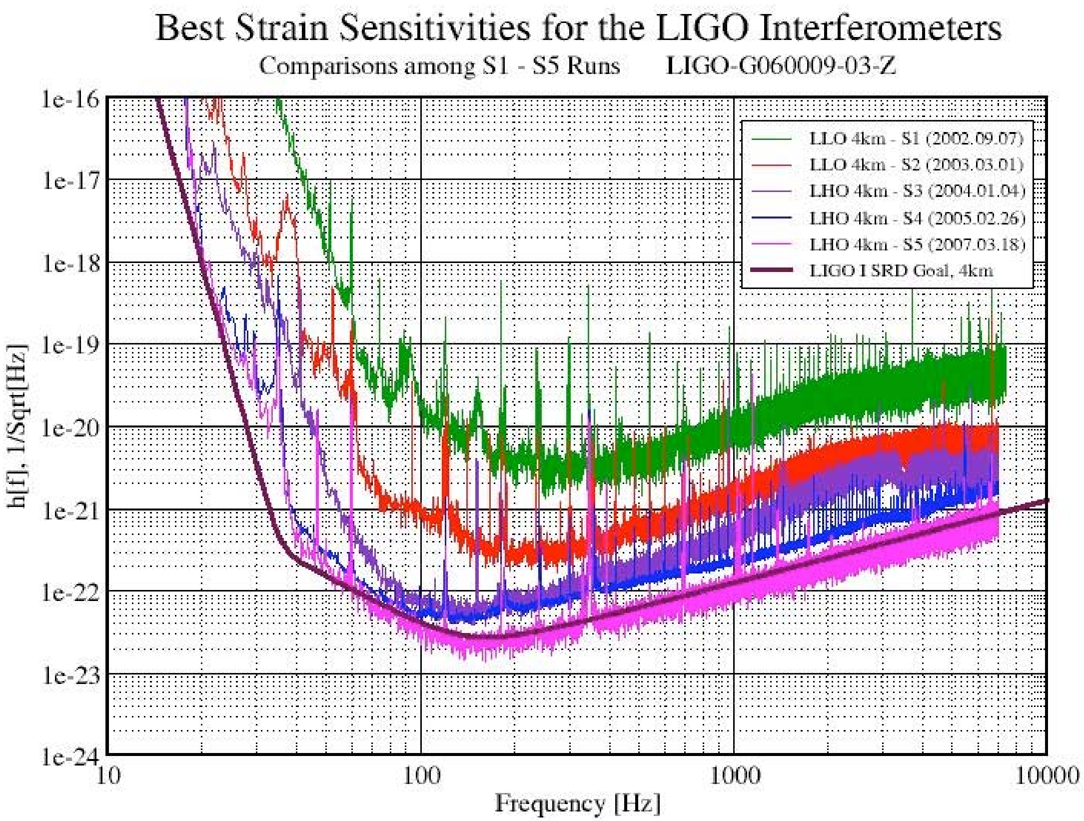
Noise Sources





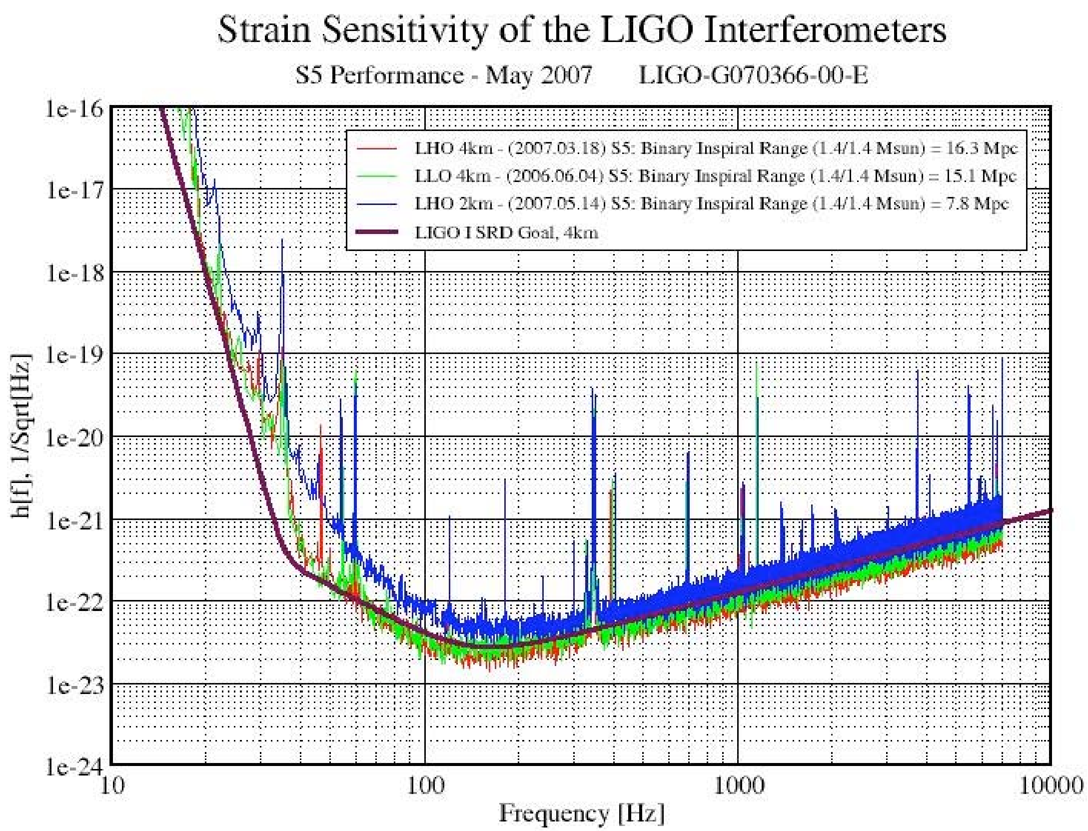
LIGO Sensitivities

- The initial LIGO detectors have completed five science runs, S1 - S5.

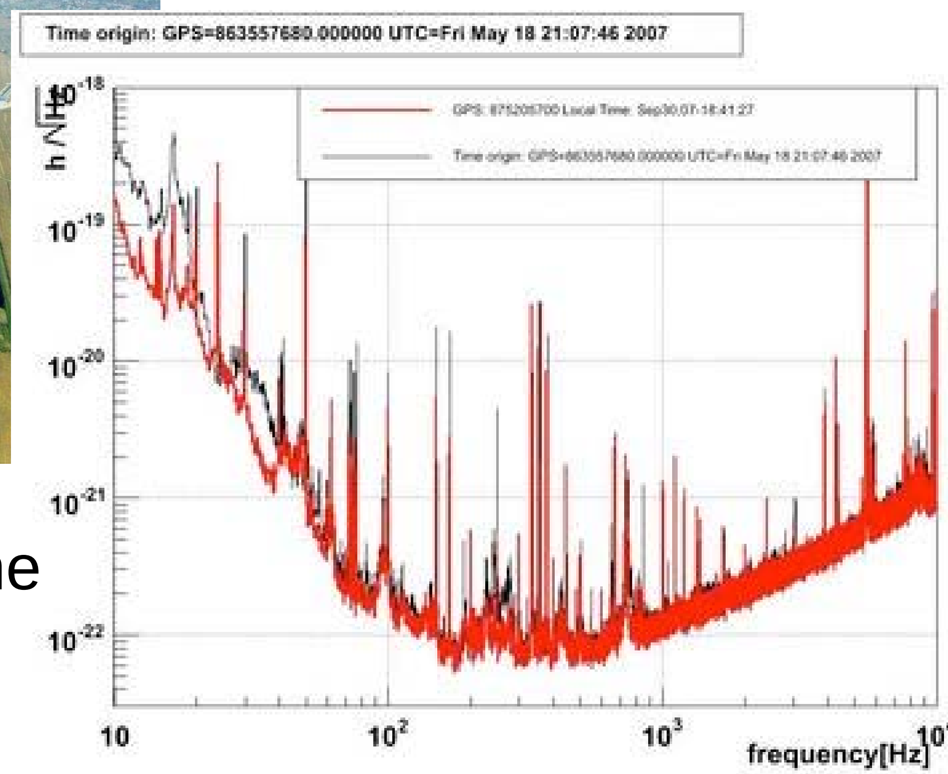


LIGO Sensitivities

- The initial LIGO detectors have completed five science runs, S1 - S5.
- S5 ended recently, having taken 1 year of coincident data at design sensitivity



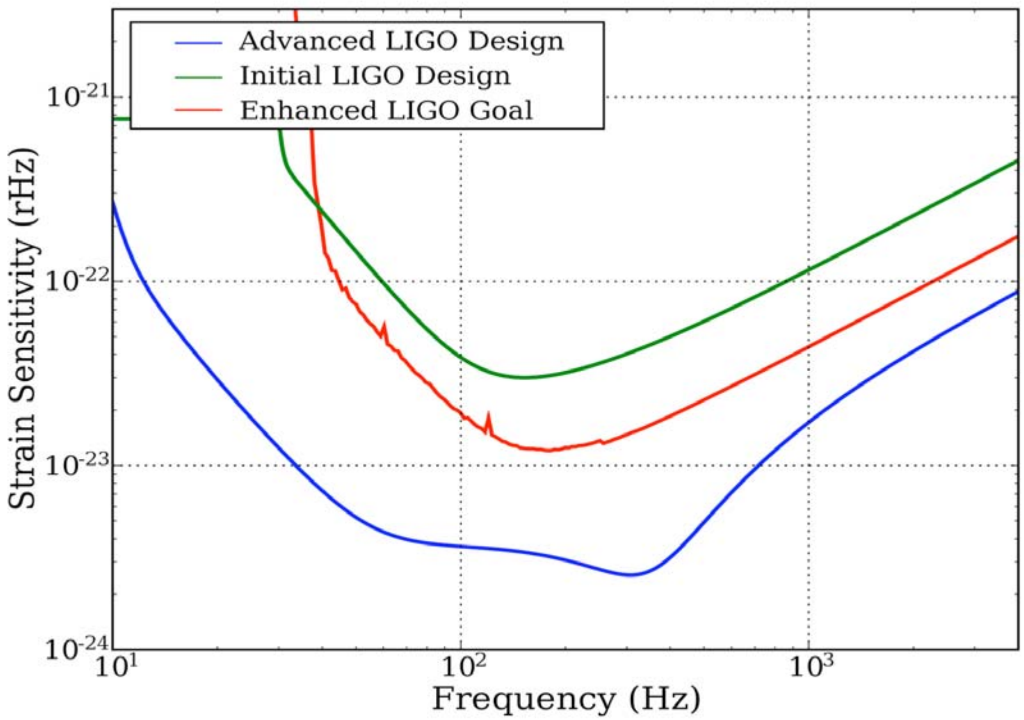
Virgo



The Virgo detector joined the last 6 months of the S5 run

Enhancements and Upgrades

- Work has begun on enhanced LIGO
  - Approx factor of 2 improvement in sensitivity
- Advanced LIGO construction will begin this year
  - Order of magnitude more sensitive than initial detectors



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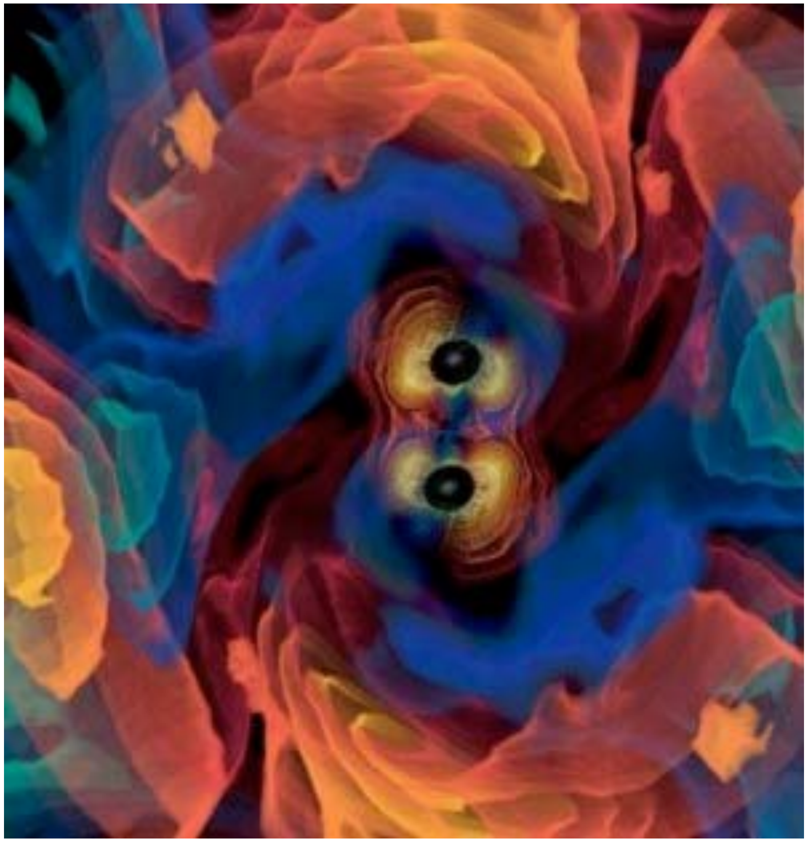


Coalescing Binaries

- Binary systems emit gravitational waves and slowly inspiral together.

- This have already been (indirectly) observed.

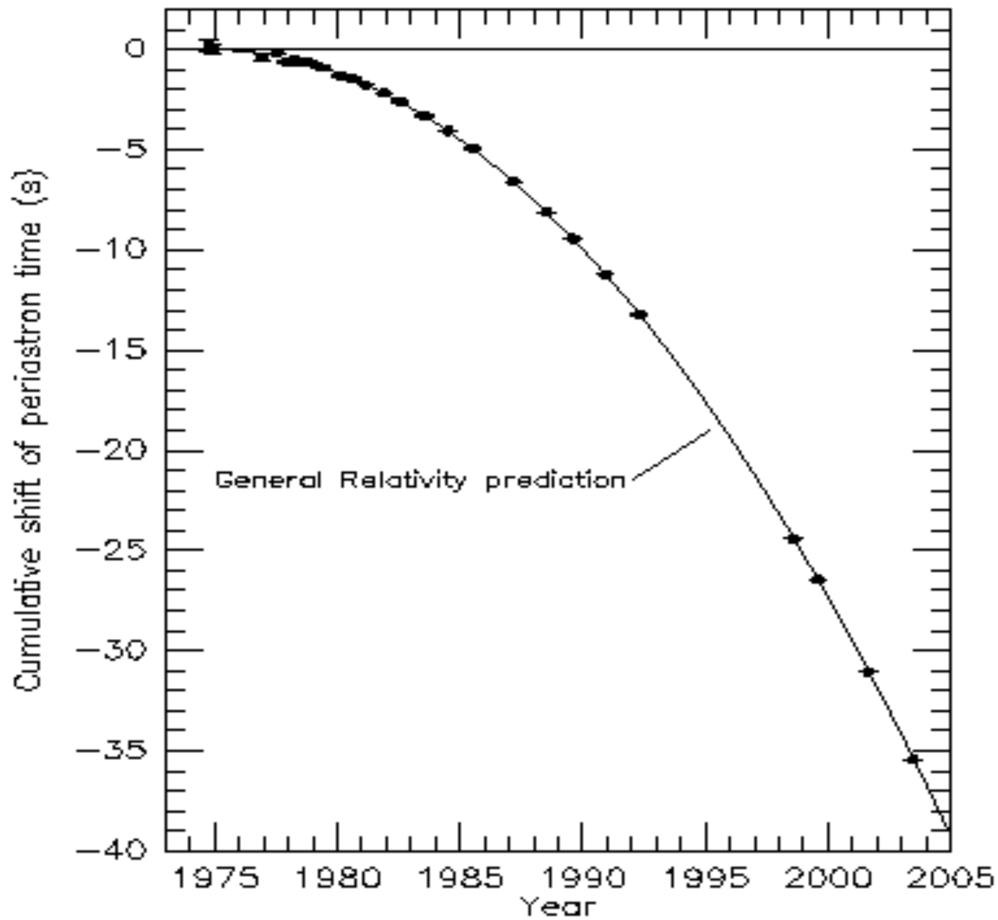
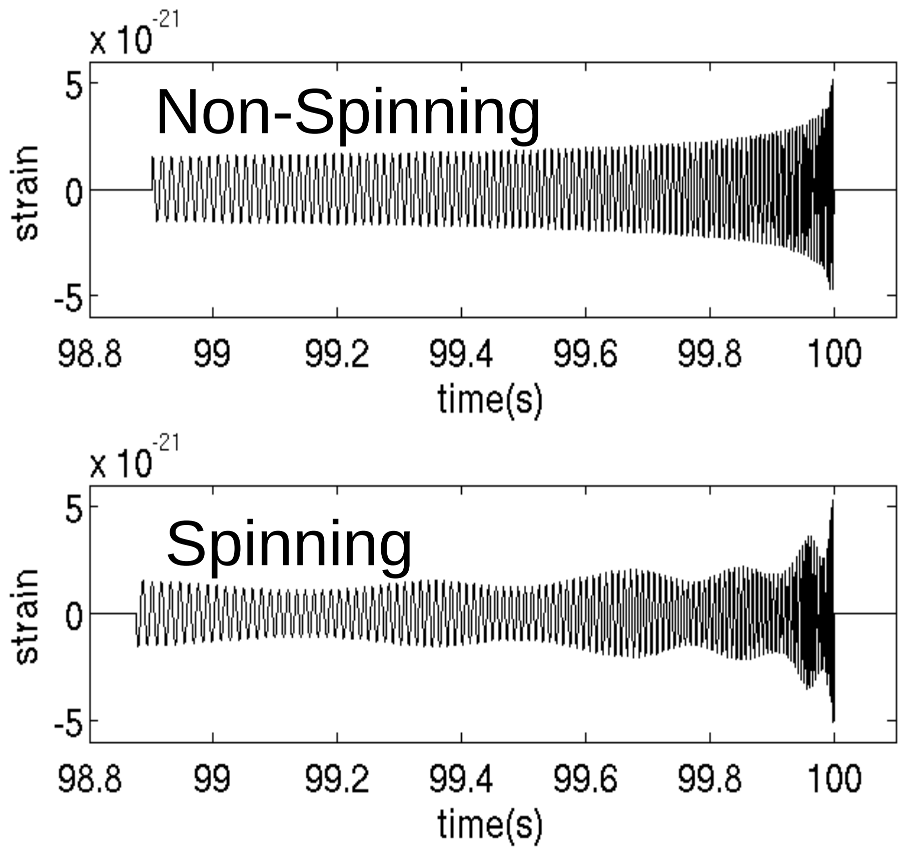


Figure from Weisberg+Taylor (2004).

The Inspiral Waveform

- Gravitational waves are emitted as the binary inspirals.
- The waveform depends upon masses, spins, binary orientation
- Waveform can be well modelled up to the last few orbits, using the post-Newtonian formalism.
- pN fails around ISCO:

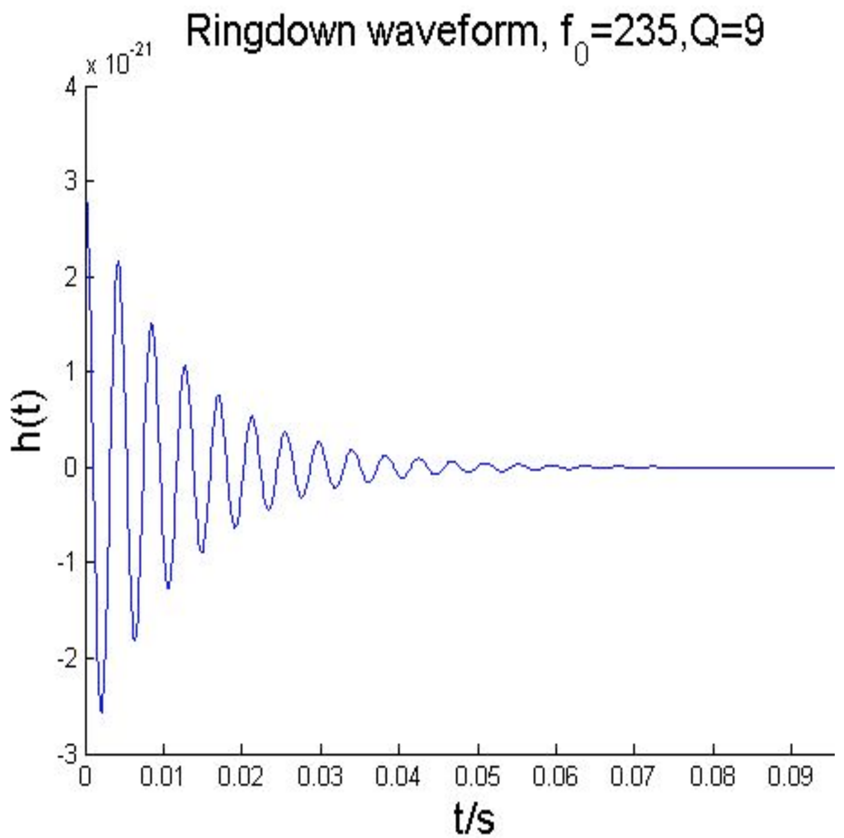
$$F_{\rm ISCO} = 220Hz \times \left(\frac{20M_\odot}{M_{\rm total}}\right)$$



The Ringdown Waveform

- Following merger, the final "ringdown" can be modelled using black hole perturbation theory

- The waveform depends upon final mass and spin
  – Currently these cannot be derived from inspiral parameters



Astrophysical Rates

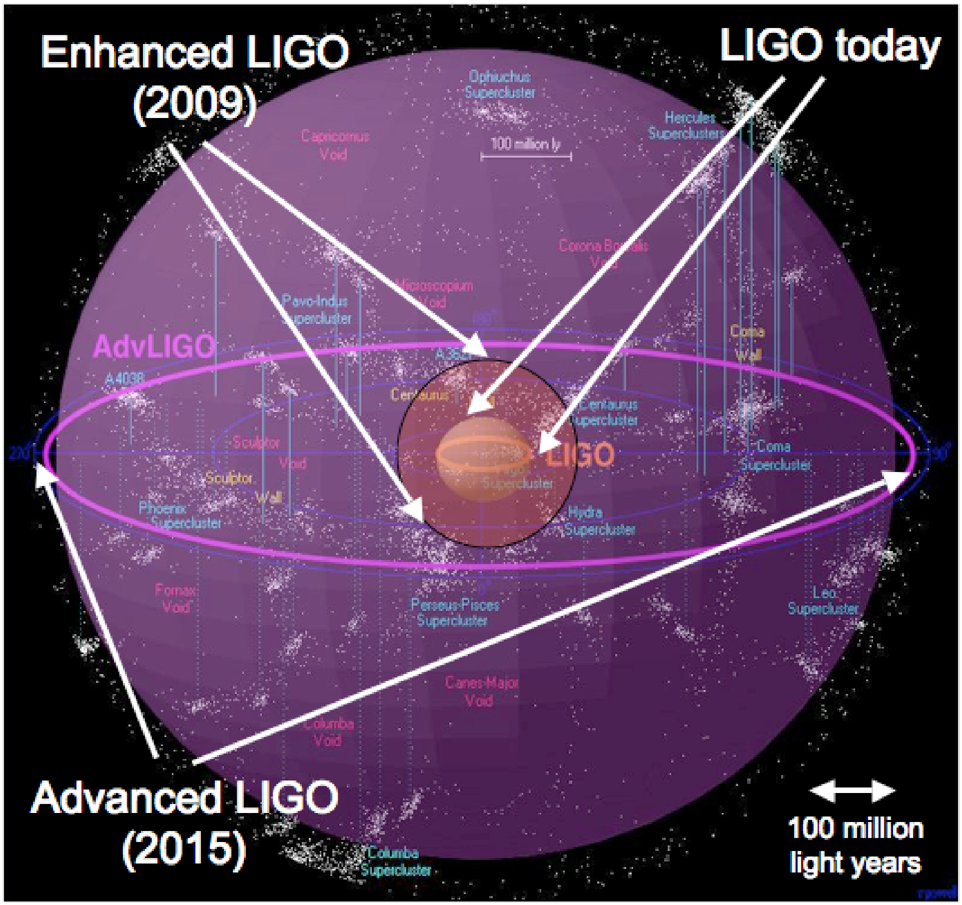
- Assume that the rate of coalescences is proportional to blue light luminosity
  - Follows star formation rate, supernova rate
  - Expected number of coalescences for a given search

$$N \approx 7.4 \times 10^{-3} \left(\frac{\mathcal{R}}{L_{10}^{-1}\ \mathrm{Myr}^{-1}} \right) \left(\frac{D_{\mathrm{horizon}}}{100\ \mathrm{Mpc}} \right)^3 \left(\frac{T}{\mathrm{yr}} \right)$$

  - For binary neutron stars: $\mathcal{R} = 10 - 170\,\mathrm{Myr}^{-1} L_{10}^{-1}$
  - For binary black holes: $\mathcal{R} = 0.1 - 15\,\mathrm{Myr}^{-1} L_{10}^{-1}$

Kopparapu et al., arXiv:0706.1283

LIGO range



- Expected rates
  - 1 event per few years for Enhanced LIGO
  - Several to many events per year for Advanced LIGO

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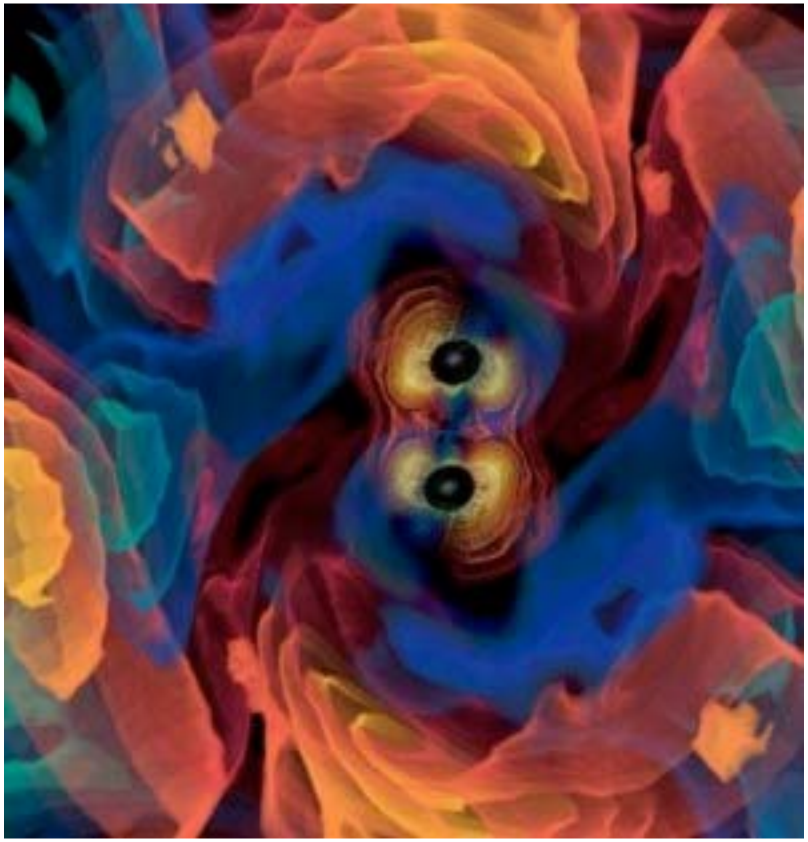
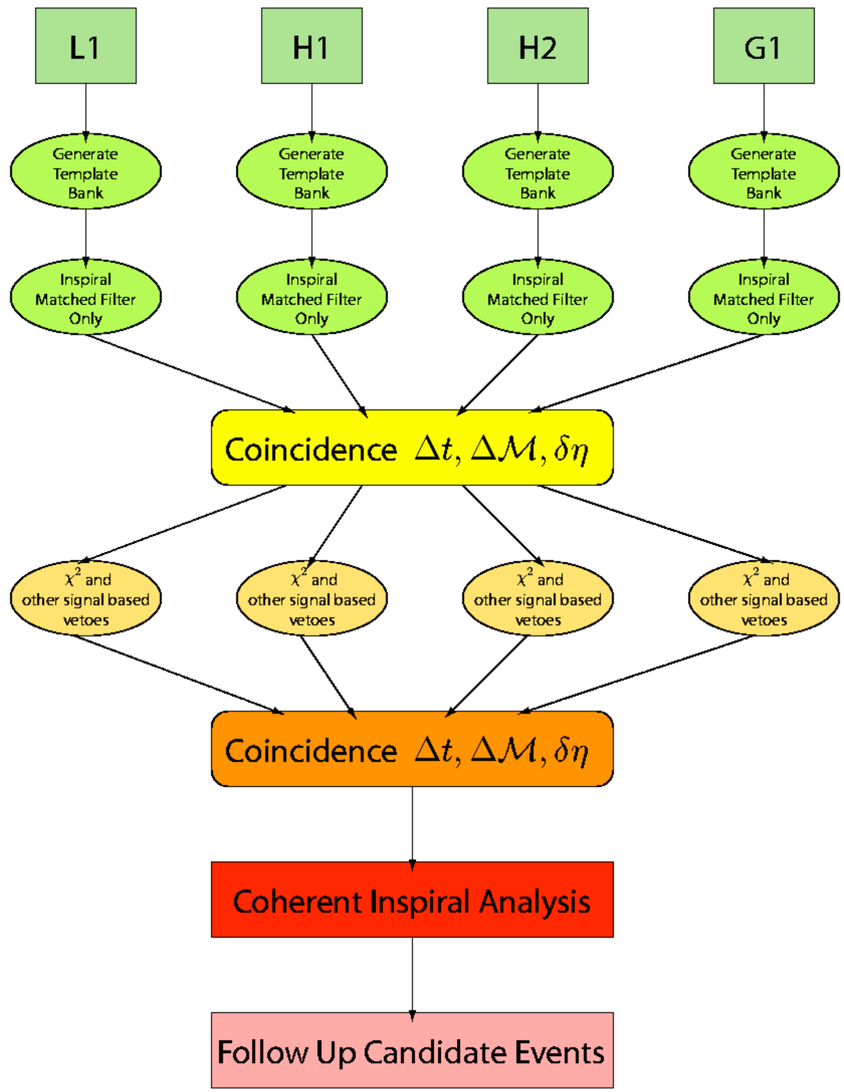


The Inspiral Search Pipeline

- Generate template bank
- Perform matched filter
- Require coincidence between detectors
- Signal consistency tests
- Interpretation of results



Search Details

- The inspiral waveform is described by masses, spins, distance, sky location, orientation.
- For non-spinning waveforms, the orbital plane does not precess, and the waveform can be written as:

$$h(t) = \left(\frac{1\mathrm{Mpc}}{D_{\mathrm{eff}}}\right) A(t - t_o) \cos(\phi(t) - \phi_o)$$

- Where D_{eff} is the effective distance ($\geq$ distance)
- D_{eff}, ϕ_o for a given detector depend upon distance, sky location, orientation relative to detector

Matched Filtering

- Define the 2 phases of the waveform

$$h_c(t) = A(t - t_o)\cos(\phi(t))$$

$$h_s(t) = A(t - t_o)\sin(\phi(t))$$

- And a normalization factor: $\sigma^2 = \int_{f_{\rm low}}^{f_{\rm high}} df\, \frac{|\tilde{h}_c(f)|^2}{S_h(f)}$
- $S_h(f)$ is the noise power spectral density

Matched Filtering

- The signal to noise ratio (SNR) ρ is given by

$$\rho^2(t) = \rho_c^2(t) + \rho_s^2(t)$$

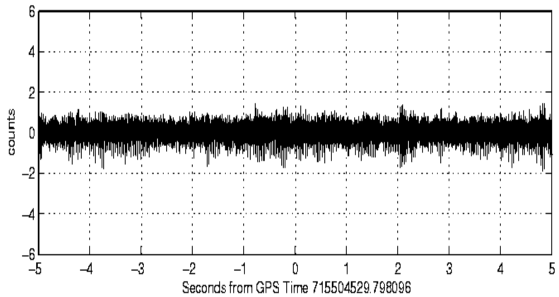
$$\rho_{c,s}(t) = \frac{1}{\sigma} \int_{f_{\rm low}}^{f_{\rm high}} df\, e^{2\pi i f t}\, \frac{\tilde{s}(f) \tilde{h}^{\star}_{c,s}(f)}{S_h(f)}$$

- From the measured SNR, we can calculate the observed effective distance and phase:

$$D_{\rm eff} = \sigma/\rho$$

$$\phi_o = \tan^{-1}(\rho_s/\rho_c)$$

In Pictures



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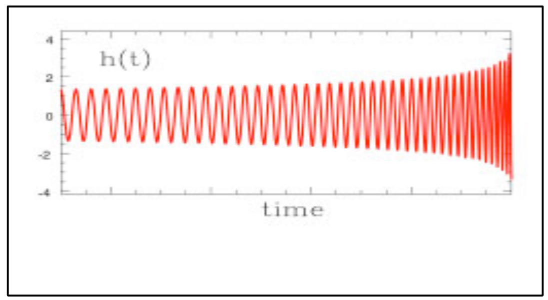

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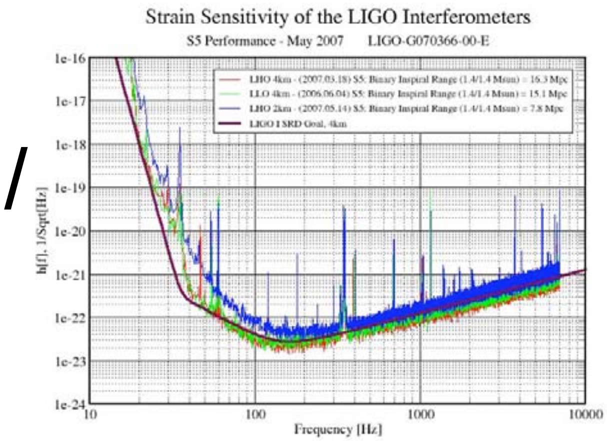

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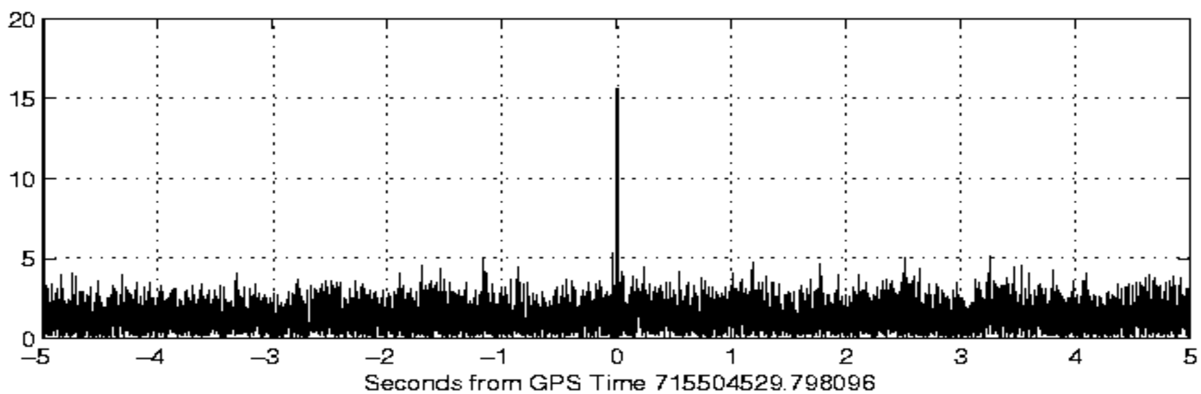
SNR



Coalescence Time

Searching the mass space

- We search over the mass space by employing many template waveforms with different masses.

- Place templates so that for any waveform h in the parameter space, the match is not less than some mininum value

$$\mathrm{Match} = \mathrm{Max}_{t_o,\phi_o,i} \frac{< h|h_i >}{|h||h_i|}$$

  – Where the inner product is

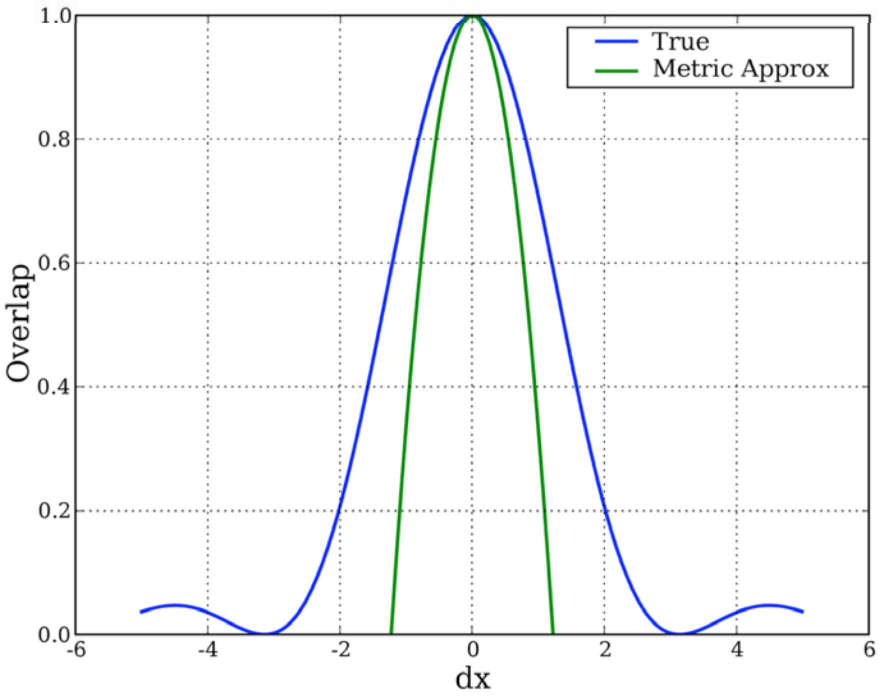
$$< a|b >= \int_{f_{\mathrm{low}}}^{f_{\mathrm{high}}} df \, \frac{\tilde{a}(f)\tilde{b}^{\star}(f)}{S_h(f)}$$

Metric on the parameter space

- Placing templates is simplified by calculating a metric on the mass space

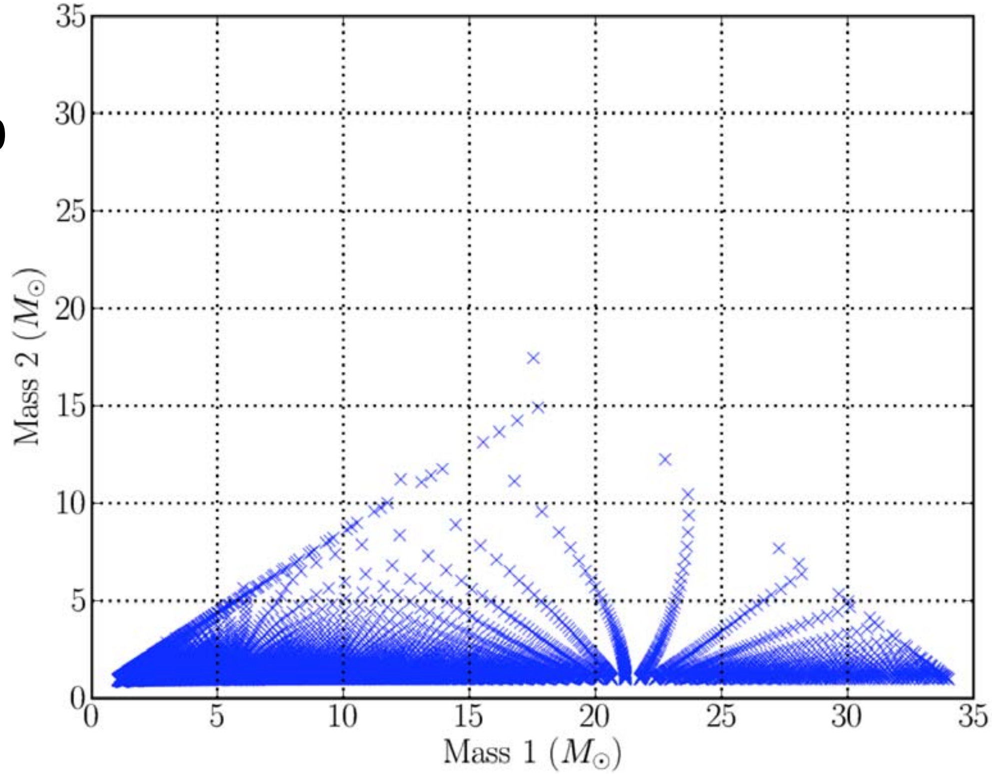
$$\frac{< h(\mathbf{x})|h(\mathbf{x}+\mathbf{dx}) >}{|h(\mathbf{x})||h(\mathbf{x}+\mathbf{dx})|} = 1 - g_{ab}(\mathbf{x})dx^a dx^b$$

- Allows for efficient placing of templates



The Template Bank

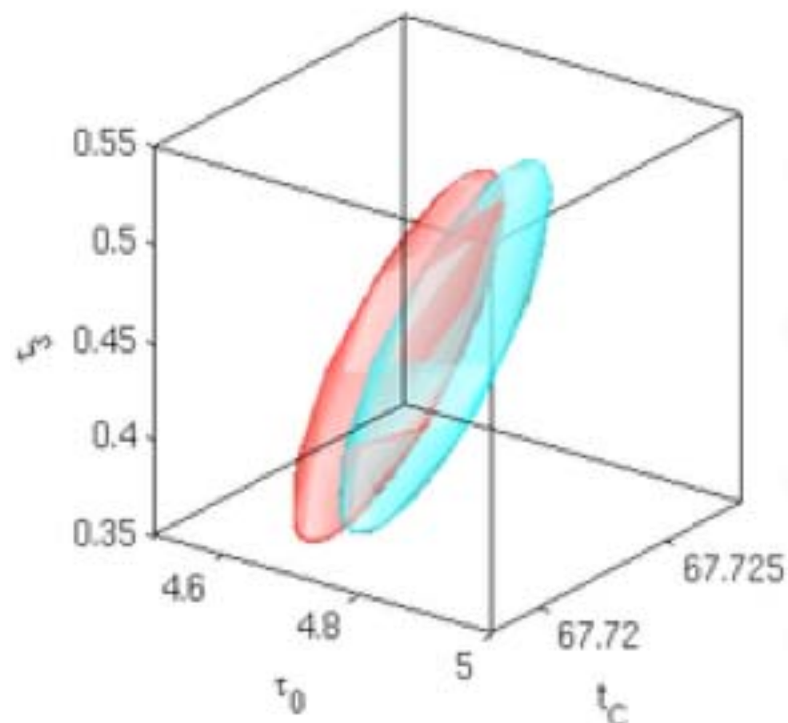
- Typically require a minimal match of 97%
- Equivalent to allowing a loss of
  - 3% in range
  - 10% in rate



Example template bank from S5

Coincidence between detectors

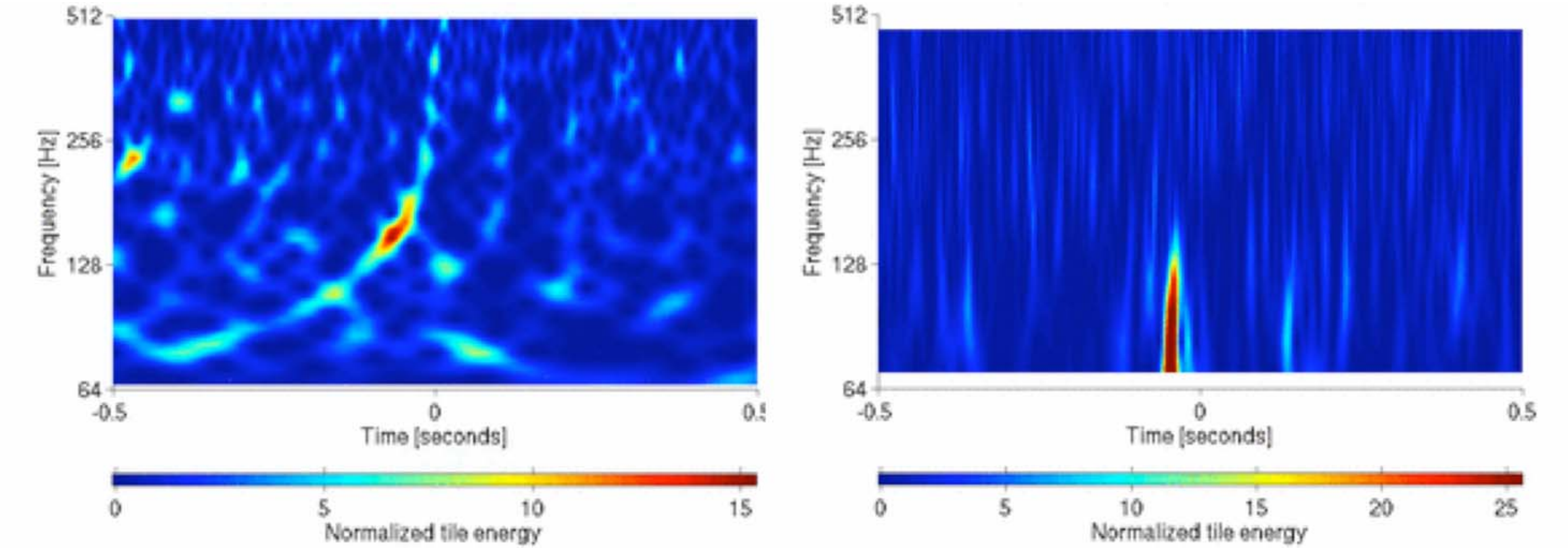
- Require that an event is seen in at least 2 detectors with similar time and masses.
  - Reduces false alarms due to environmental noise
  - Naturally account for correlations between parameters by using metric to determine coincidence windows



Life isn't Gaussian

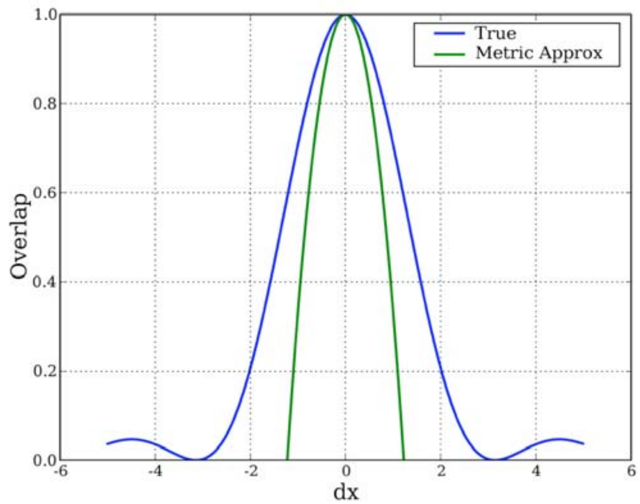
- Time-frequency Q-scans showing excess power

Inspiral Hardware Injection | Non-stationary time

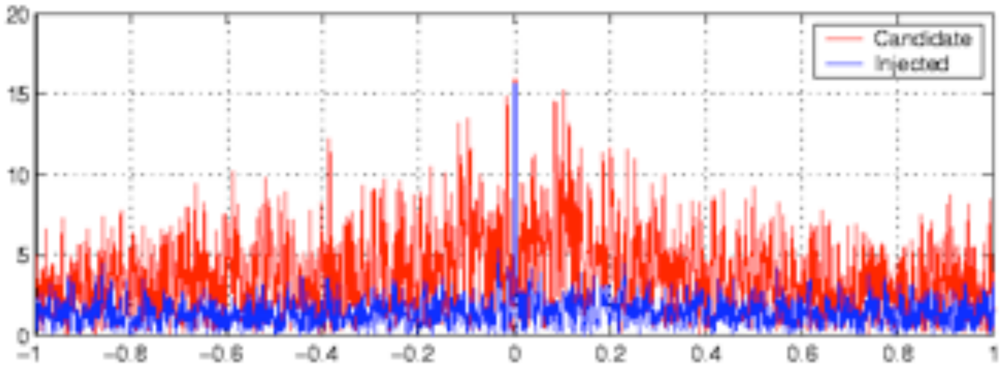


Signal Consistency Tests

- We know how the SNR will vary over parameter space for a true signal.



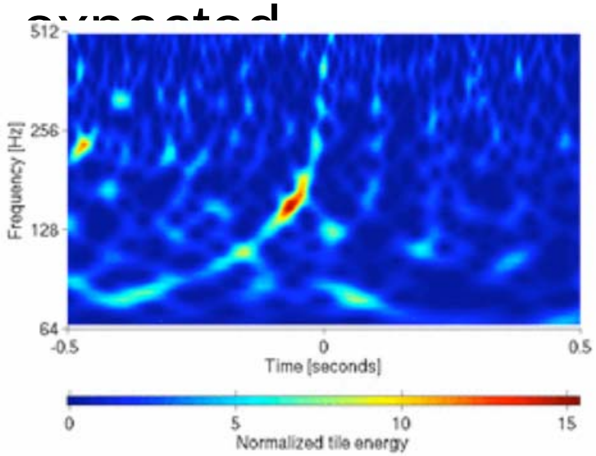
- Check if it does
  - Example: loudest surviving event in S1

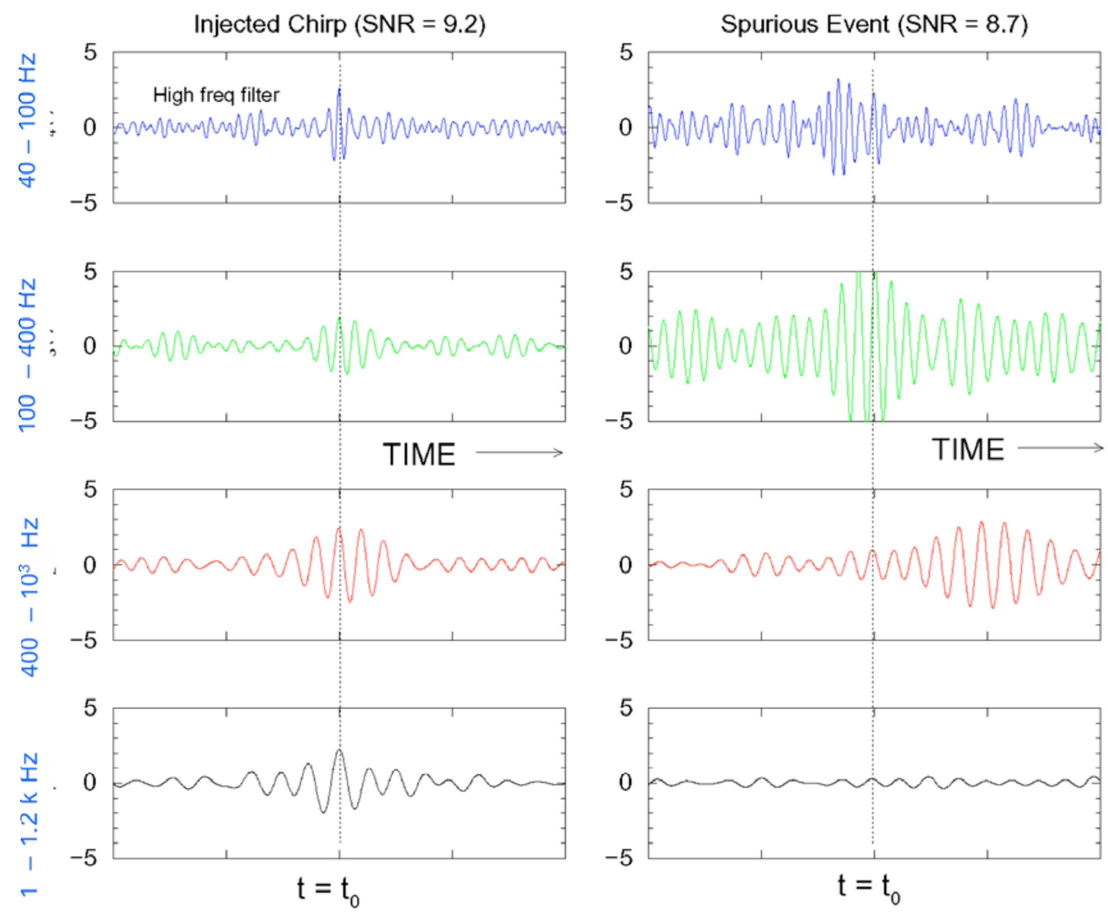


The χ^2 Test

$$\chi^2 = p \sum_{i=1}^{p} (\rho_{c,i} - \rho_c/p)^2 + (\rho_{s,i} - \rho_s/p)^2$$

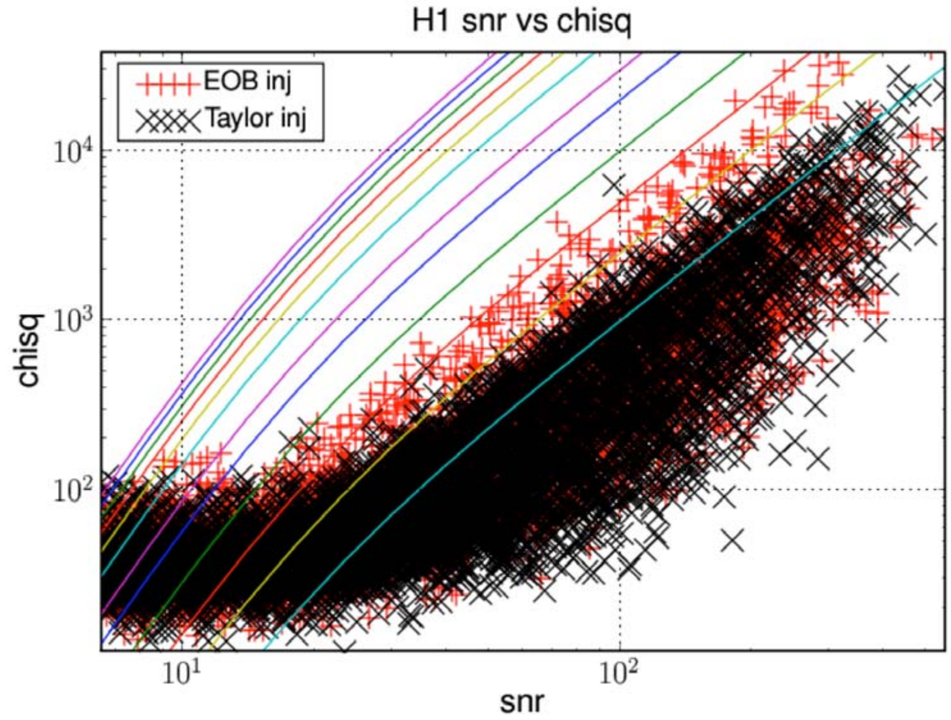
- Check the power in the signal is distributed as expected





Uncertainties in the waveform

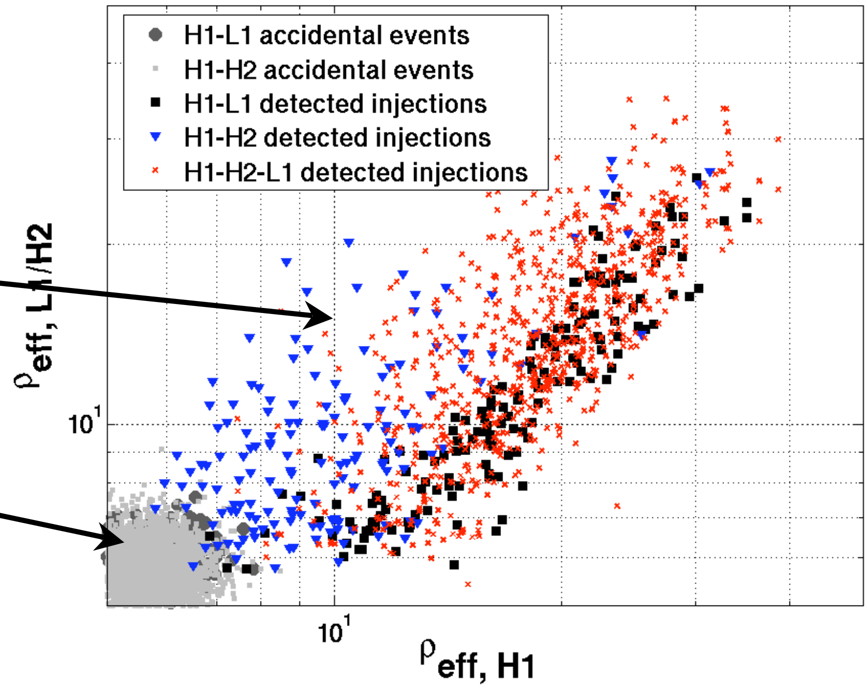
- Signal consistency tests are sensitive to inaccuracies in the waveforms used, e.g.
  - Using non-spinning waveforms
  - Not including merger/ringdown
  - Calibration uncertainty



Example: Filtering EOB waveforms with Taylor PN templates gives increased χ^2

Search Results

- To interpret results, require:
  - Sensitivity estimate from “injecting” signals into the data
  - Background estimate from time shifts
- Results from:
  “Search for gravitational waves from binary inspirals in S3 and S4 LIGO data.”
  Abbott et al. arXiv:0704.3368

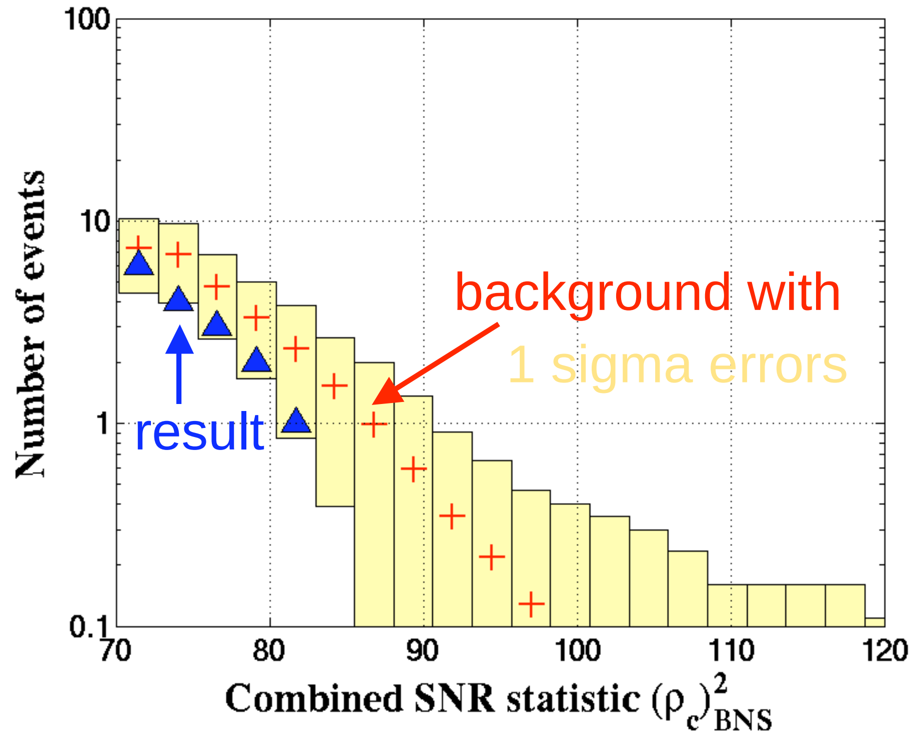


ρ_{eff} : effective SNR, calculated from ρ, χ^2

Search Results

- We wind up with a list of candidate events
  - Want to see whether any are significant
  - Plot cumulative histogram of number of events vs SNR
  - Compare to background from time shifts

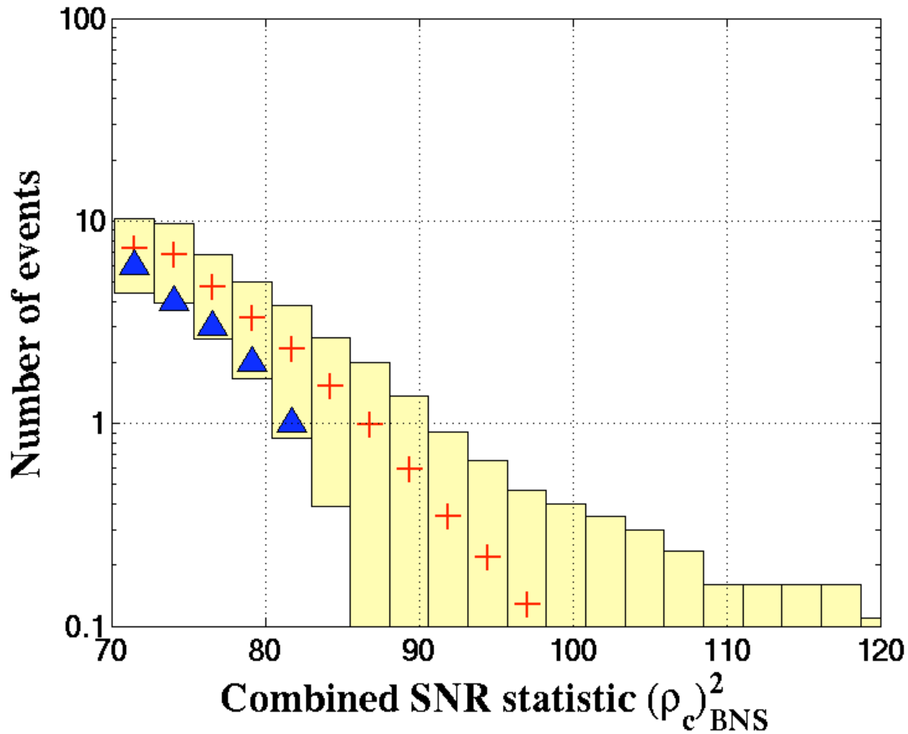
S4 BNS result



When do things get interesting?

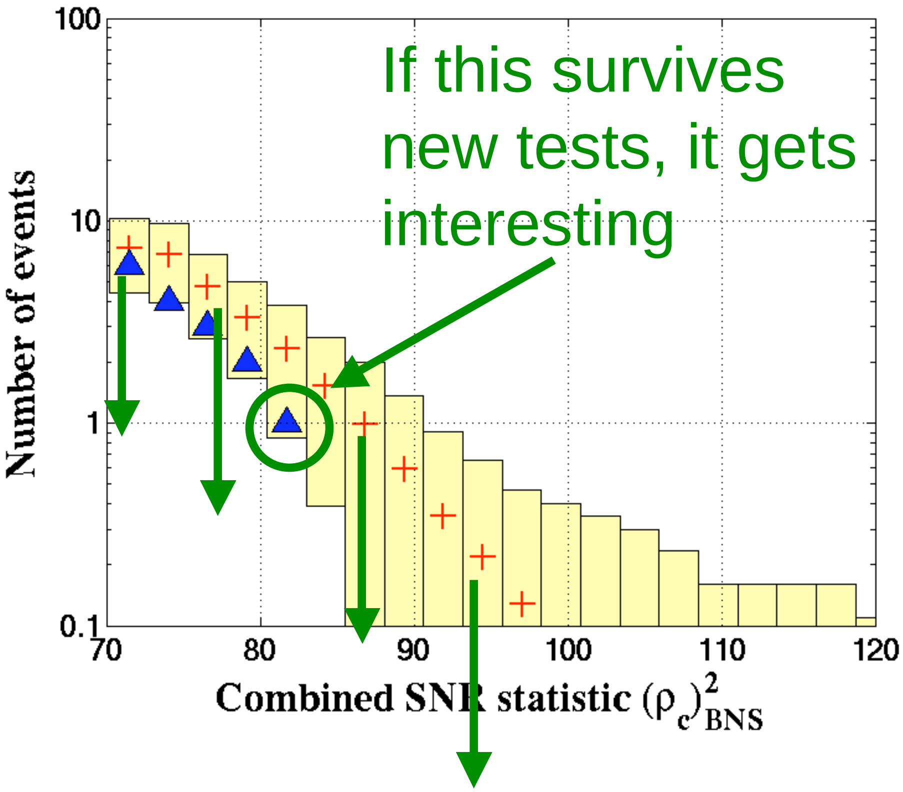
- When foreground stands out above background
- Example from S4 BNS search
  - Loudest few events are known to be due to instrumental or environmental origin, but we can dream …

S4 BNS result



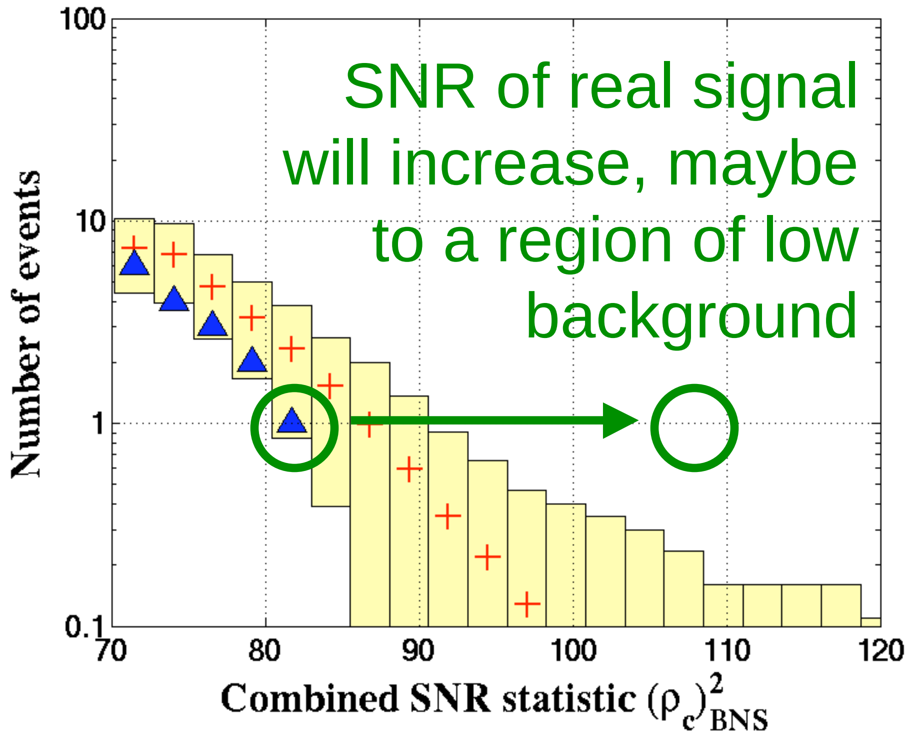
When do things get interesting?

- When foreground stands out above background
- Improved signal consistency or better "ranking statistic" will lower the background



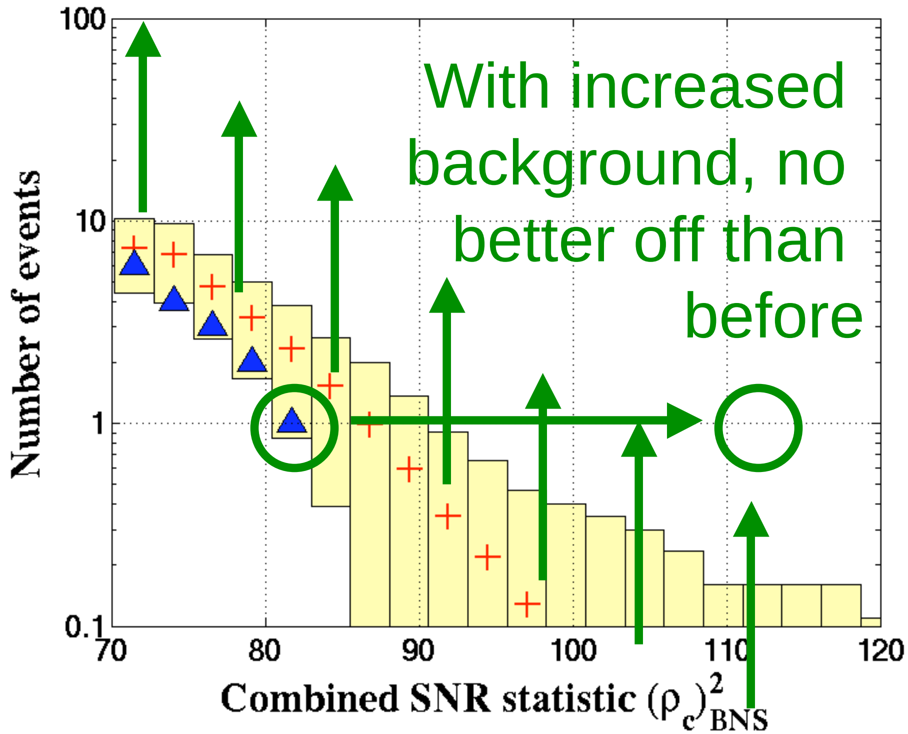
When do things get interesting?

- When foreground stands out above background
- Improved knowledge of waveforms will increase SNR of signals



When do things get interesting?

- When foreground stands out above background
- Improved knowledge of waveforms will increase SNR of signals
- However, more complicated template families run risk of increasing background as well!



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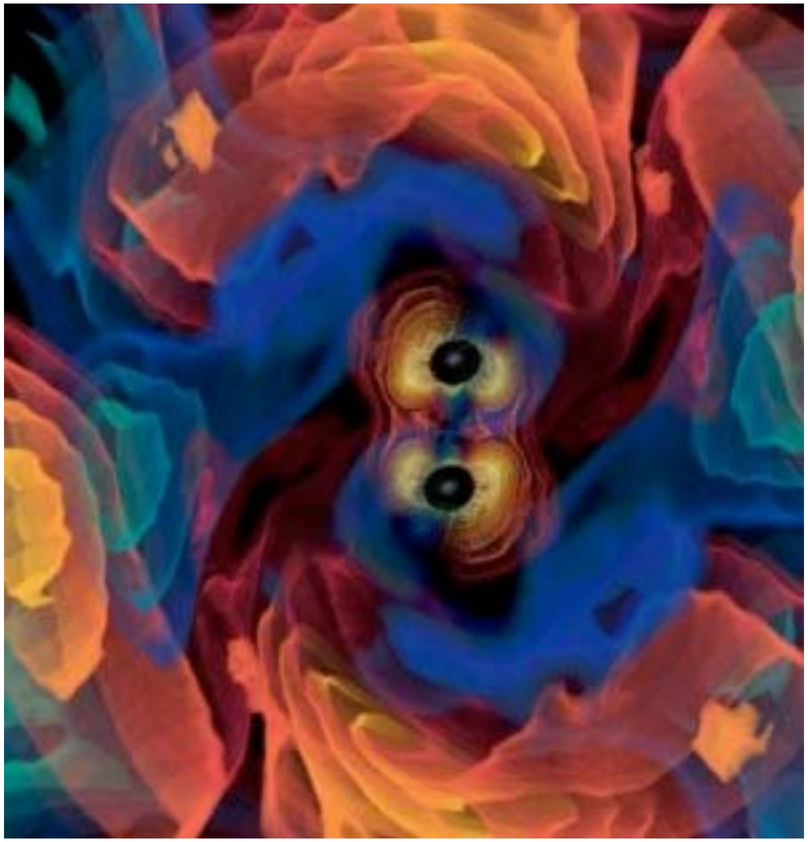
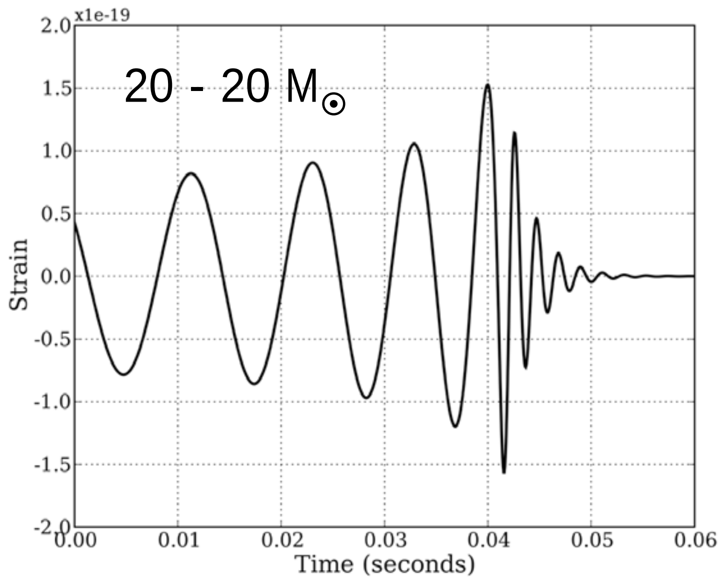


Testing our Current Analyses

- We need to test the current analyses with full inspiral-merger-ringdown waveforms.
  - Have already done blind hardware injections in S5 with "made up" waveforms.
  - Would like to use the NR waveforms, I'm sure they're more accurate.



Numerical Relativity Injections

- We can perform simulations (in software) using NR waveforms, if provided in format described in
  "Data formats for numerical relativity waves", arXiv:0709.0093
  - Scale waveform for physical masses, distance, location, orientation

- Will run inspiral and ringdown matched filter searches, and burst "excess power" searches on same set of simulations.

- Work out which search, or combination of searches, is most sensitive.

Final Thoughts

- We are sure to learn a lot when we do NR injections, but maybe not what we expect
  - It may turn out that we don't win by match filtering for the full waveform.
  - It may be that we don't win (in some mass ranges) by matched filtering at all.

- I've focused primarily on detection, have barely touched parameter estimation.