Errors in Numerical Relativity

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KITP Miniprogram: Interplay between
Numerical Relativity and Data Analysis

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Goal: Enumerate various sources of "error", to raise awareness of what *might* be wrong, and to stimulate discussion.

Personal view: NR is in excellent shape for GW-detection. Error analysis uncomfortably weak for parameter extraction and LISA data-analysis.

Numerical error

Numerical error: Given initial data, how accurately can a code predict the future development of this initial data and the waveform observed at infinity?

In an ideal world, all conceivable sources of numerical error would be checked and quantified. There are many such sources:

- "Standard" Truncation error
  - Well documented by every group
- Effect of outer boundary conditions?
  – $R \approx 800M$, but $T \gtrsim 1500M$.
  – Error would be **convergent**, i.e. not visible in Δx-convergence tests.
- Effect of underresolved region at intermediate/large distances?

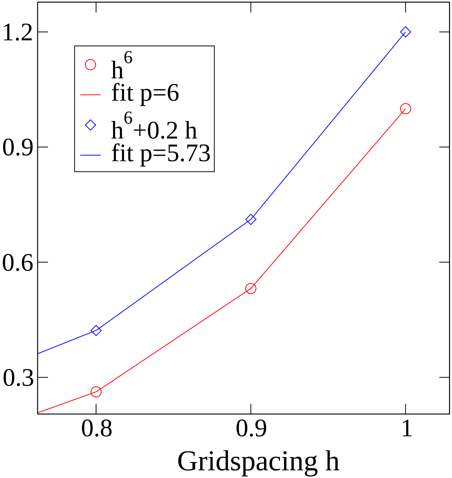
Numerical error

- Choice of tetrad, extrapolation $r \to \infty$; ambiguities at ∞ (see Luis Lehner talk)
- "t" is coordinate time, not proper time at infinity.
  – Need Lapse $N \to 1$.
  – N depends on gauge conditions; $N \neq 1$ would be **convergent**.
- Does solution remain asymptotically flat?
- Extraction surfaces usually *coordinate* spheres.
  – Change in physical radius $\Rightarrow$ time-shift of extracted waveform.
  – Change in physical shape of surface $\Rightarrow$ mixes Y^{lm}'s
  – Change in coordinates on surface $\Rightarrow$ mixes Y^{lm}'s

Numerical error

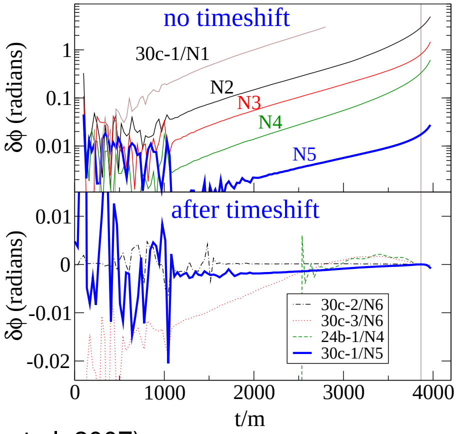
Convergence test

Convergence tests for high-order FD methods may be insensitive to low-order error terms.



"Error" is not a simple number

E.g. Phase-error depends on matching time



(Boyle et al, 2007)

"Gain" in accuracy:

$$\frac{\delta\phi_1}{\delta\phi_2} \approx \frac{\omega_{\rm match2}}{\omega_{\rm match1}}$$

Astrophysical modelling errors

How close can we get to simulating a desired astrophysical situation?
($M_1 = 4.67 M_\odot$, $M_2 = 7.43$, $e = 10^{-6}$, $\vec{S}_1/M_1^2 = ...$)

- Concerns **Initial data**
- Want desired properties *after* junk-radiation is gone
  – Should measure properties after junk-radiation is gone.
  – Probably a minor effect
- How to measure and control eccentricity for non-aligned spins?

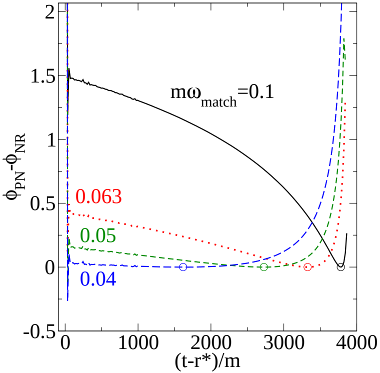
Errors introduced in PN-comparison procedure

- Match close to merger: PN unreliable
- Early match $\Rightarrow$ large time-uncertainty δt:

$$\omega(t_{\mathrm{match}}) = \omega_{\mathrm{match}}$$

$$\delta t_{\mathrm{match}} = \frac{\delta\omega}{\dot{\omega}} \sim \omega^{-8/3}\frac{\delta\omega}{\omega}$$

- Compare different physical scenarios e.g. $e_{\mathrm{NR}} = 0.001$ vs. $e_{\mathrm{PN}} = 0$



Exemplary error budget

Table: Summary of uncertainties for the Caltech/Cornell PN-NR comparison.

Effect	$\delta\phi$ (radians)	$\delta A/A$
Numerical truncation error	0.003	0.001
Finite outer boundary	0.005	0.002
Extrapolation $r \to \infty$	0.005	0.002
GW extraction at $r_{\rm areal}$=const?	0.002	10^{-4}
Drift of mass m	0.002	10^{-4}
Coordinate time = proper time?	0.002	10^{-4}
Lapse spherically symmetric?	0.01	4×10^{-4}
residual eccentricity	0.02[1]	0.004
residual spins	0.03	0.001
root-mean-square sum	**0.04**[1]	**0.005**

[1] For the case of matching at $m\omega_m = 0.04$, the phase uncertainty due to residual eccentricity increases to 0.05 radians, thus increasing the root-mean-square sum to 0.06 radians.

Overlap & Matches

Differences between different numerical resolutions