

CuO₂ ELECTRON-PAIR VISUALIZATION STUDIES



European
Research
Council



Fondúireacht Eolaíochta Éireann
Science Foundation Ireland



MÁX PLANCK GRADUATE CENTER
FOR QUANTUM MATERIALS

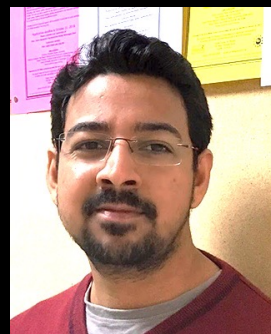




Xiaolong Liu
Notre Dame



Shuqiu Wang
Oxford



P. Choubey
IIT Delhi



Weijiong Chen
Oxford



S. Uchida
Tokyo



H. Eisaki
Tsukuba



P.D. Johnson
Oxford



Kazuhiro Fujita
BNL



P.J. Hirschfeld
Florida



M. H. Hamidian
Cornell



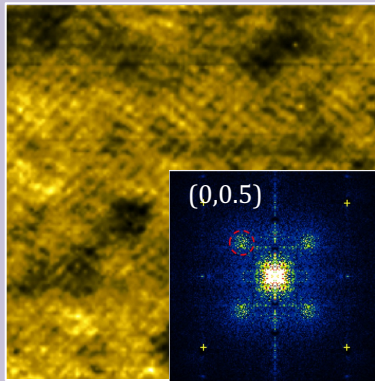
S.D. Edkins
Cornell



A.P. Mackenzie
Dresden

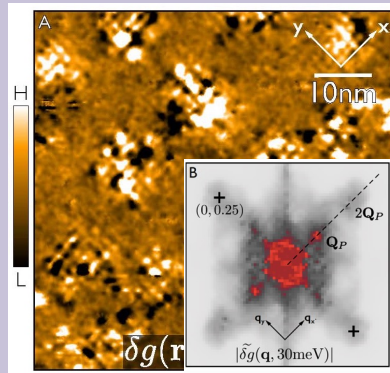
CuO₂ ELECTRON-PAIR VISUALIZATION STUDIES

PAIR DENSITY



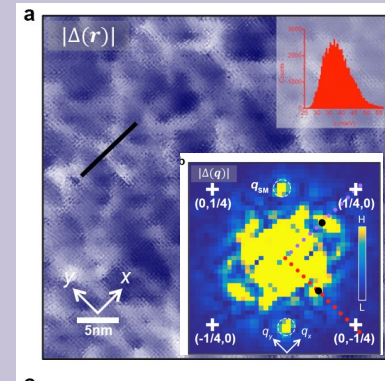
Nature 532, 343 (2016)

VORTEX HALO



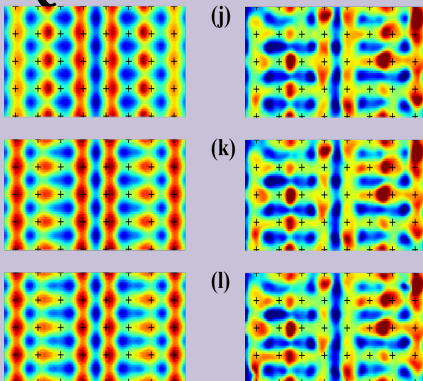
Science 364, 976 (2019)

ENERGY GAP



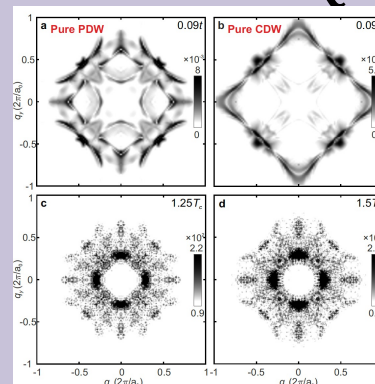
Nature 580, 6570 (2020)

QUASIPARTICLES



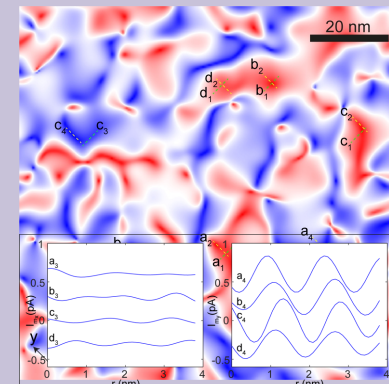
PNAS 117, 14850 (2020)

PSEUDOGAP QPI



Nat.Comm 12, 6087 (2021)

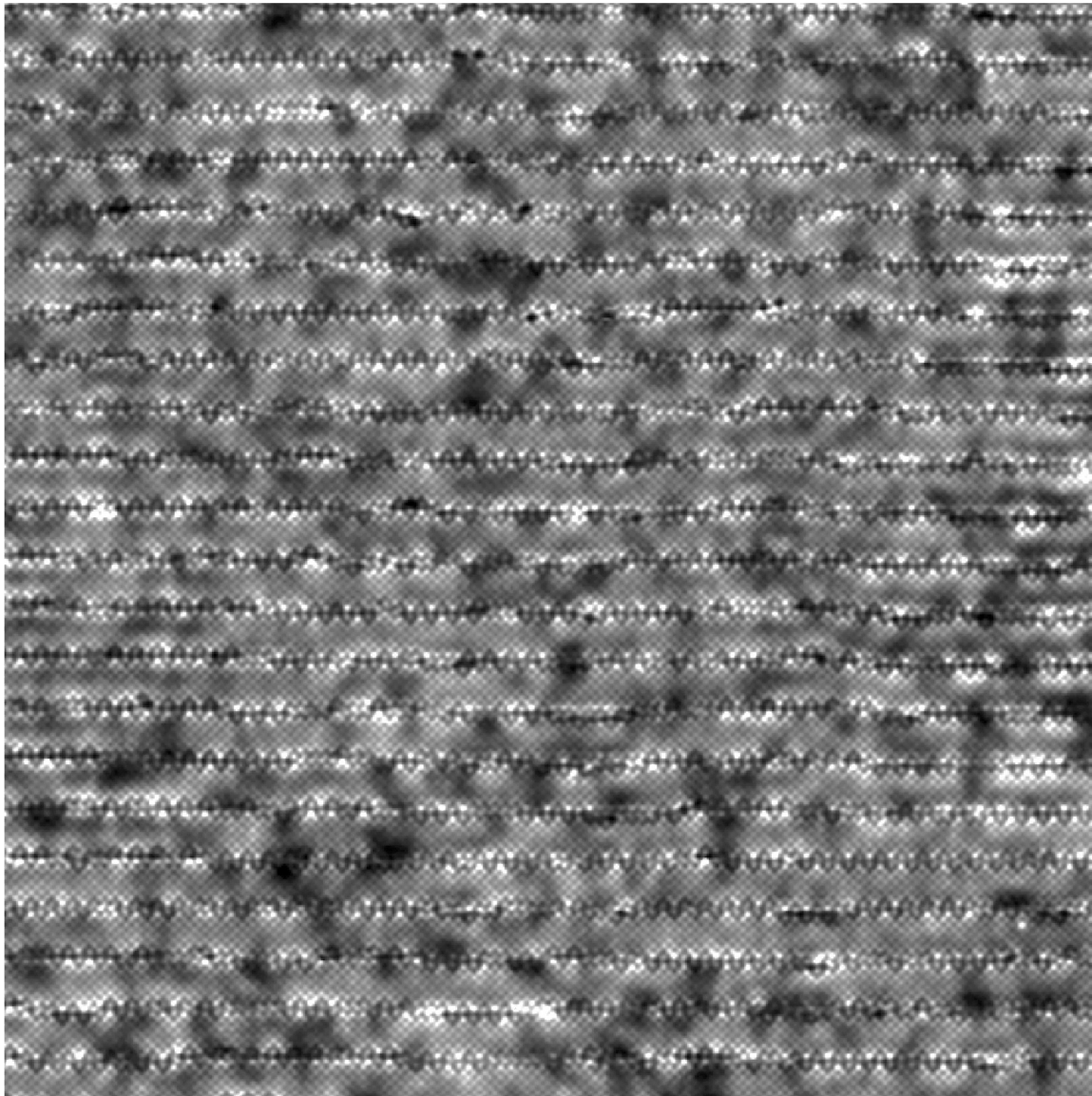
NEMATIC



Weijiong Chen *et al* (2022)

TOPOGRAPHIC IMAGING STM

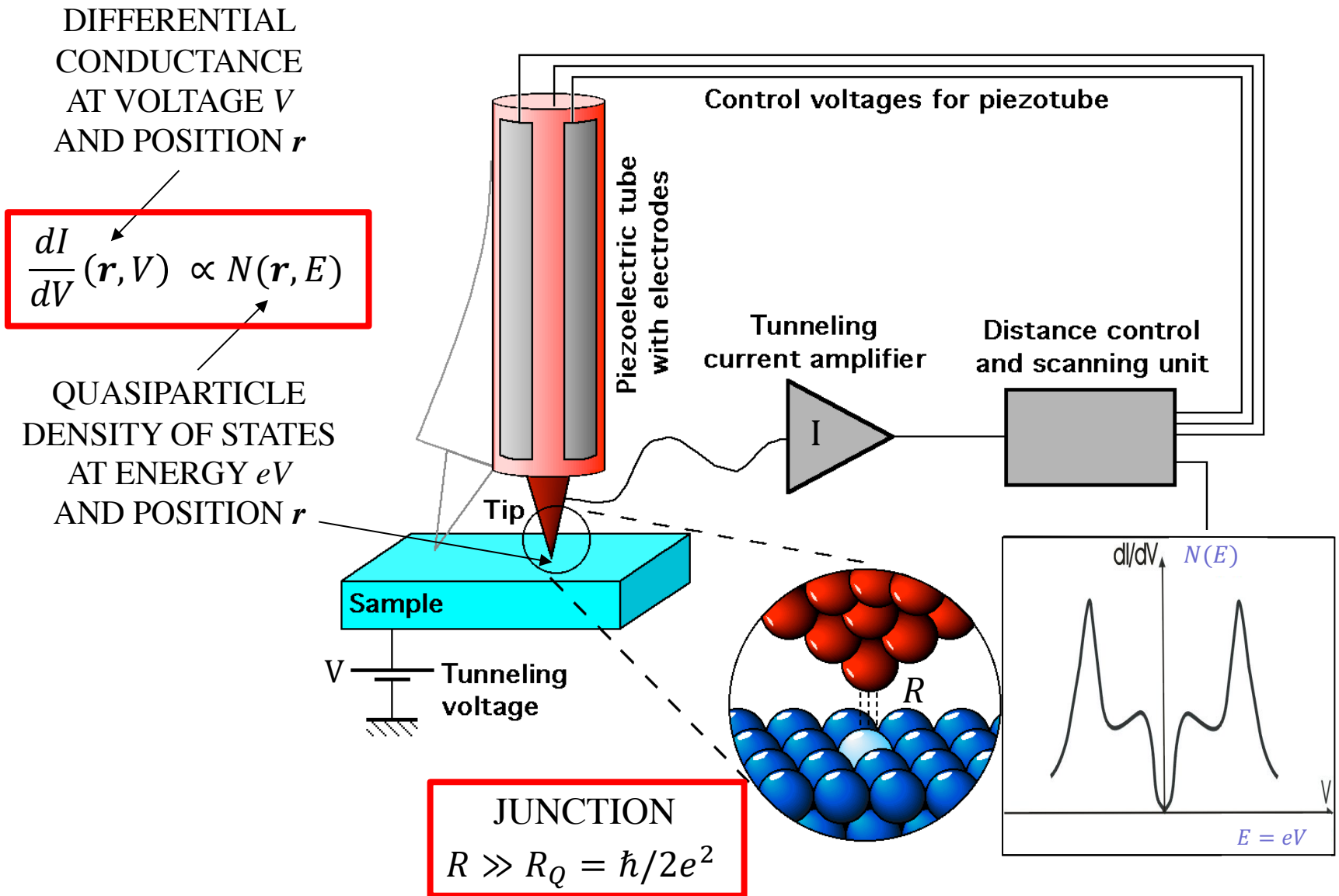
$$\ln \int_0^{V_0} N(r, \epsilon) d\epsilon$$



575 Å

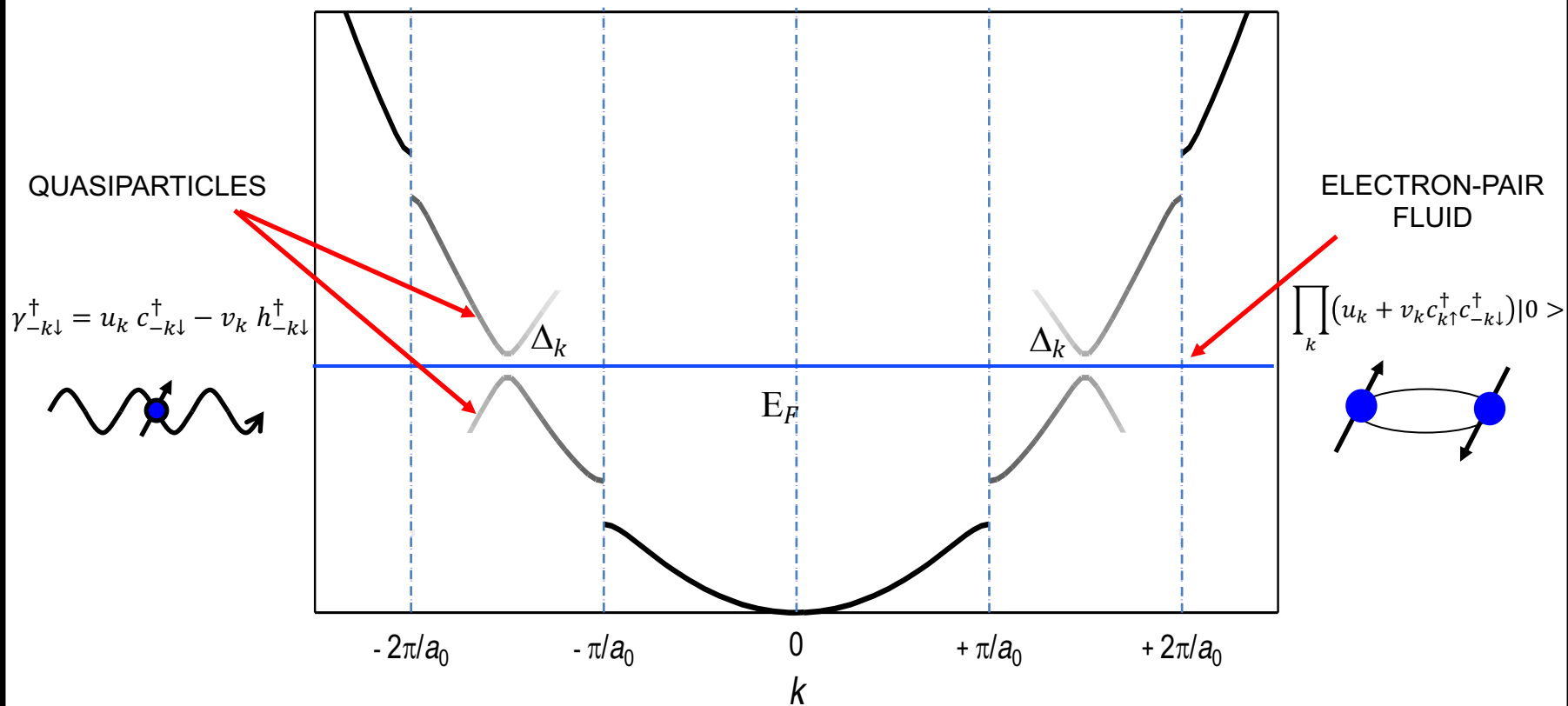
$T(r, V_0)$

SPECTROSCOPIC IMAGING STM



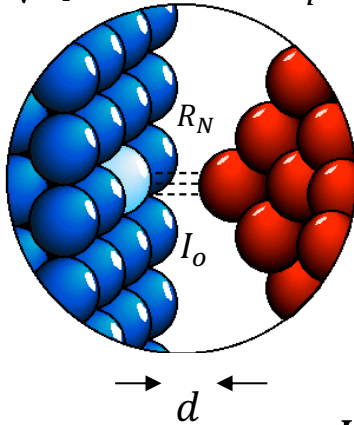
TUNNELING TO SUPERCONDUCTORS

$$\Delta_k = \langle c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger \rangle \quad \Psi = \prod_k (u_k + v_k c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger) |0\rangle$$



JOSEPHSON EFFECT : SUPERCONDUCTIVE STM TIP

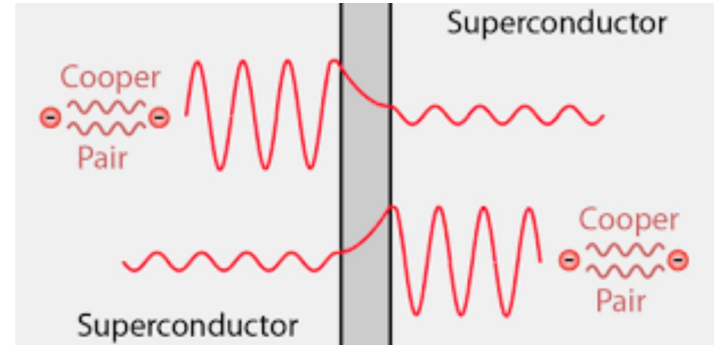
$$\psi_{\text{sample}} = \sqrt{n_P} e^{i\theta_s} \quad \psi_{\text{tip}} = \sqrt{n_t} e^{i\theta_t}$$



$$J = J_0 \sin(\theta_t - \theta_s)$$

$$J_0 = \frac{e\hbar}{m} \left\{ \frac{2}{\lambda} e^{-2d/\lambda} \right\} \sqrt{n_P} \sqrt{n_t}$$

$$\psi_{\text{sample}} = \sqrt{n_P} e^{i\theta_s} \quad \psi_{\text{tip}} = \sqrt{n_t} e^{i\theta_t}$$



ELECTRON-PAIR
CURRENT

$$I_0 R_N \propto \sqrt{n_P} \sqrt{n_t} \Rightarrow I_0^2(r) R_N^2(r) \propto n_P(r) \quad \text{VISUALIZE ELECTRON-PAIR DENSITY}$$

AMBEGAOKAR-
BARATOFF

$$I_0 R_N \approx \frac{\pi\Delta}{2e} \approx 1.5 \text{ mV}$$

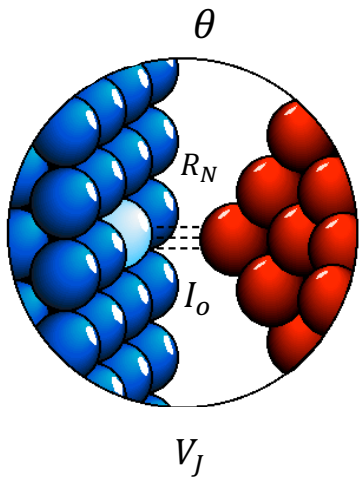
$$(T \rightarrow 0; \sqrt{n_P} = \sqrt{n_t})$$

STM TUNNEL
JUNCTION

$$I_0 \approx 1.5 \text{ pA} \quad @ \quad R_N = 1 \text{ G}\Omega \quad \Rightarrow \quad E_J = \frac{\hbar I_0}{2e} \approx 5 \text{ neV} \rightarrow 30 \mu\text{K}$$

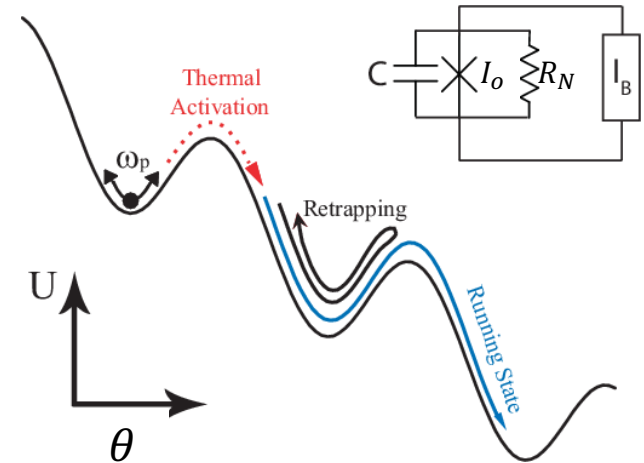
IMPOSSIBLE ?

SCANNED JOSEPHSON TUNNELING MICROSCOPY (SJTM)



$$\frac{\hbar C}{2e} \ddot{\theta} + \frac{\hbar}{2eR_N} \dot{\theta} + I_0 \sin\theta = I_B$$

$$I = I_0 \int_{-\infty}^{\infty} d\theta \sin\theta W(\theta_t, \theta_o)$$



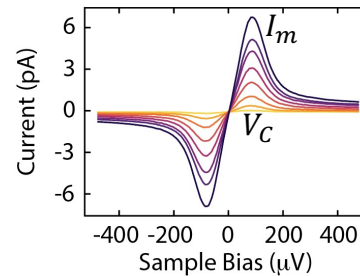
Ivanchenko & Zilberman, *JETP* **28**, 1272 (1969)

$kT \gg E_J$ and $C \rightarrow 0$

$$U(\theta) = E_J(1 - \cos\theta - I_B\theta/I_0)$$

ELECTRON-PAIR
CURRENT

$$I(V_J) = \frac{1}{2} I_0^2 Z \frac{V_J}{V_J^2 + V_C^2}$$

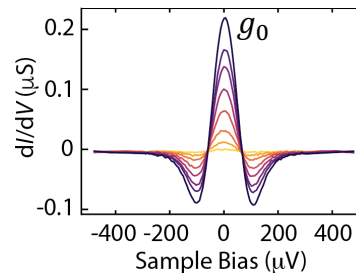


ELECTRON-PAIR
CURRENT MAX.

$$I_m = \frac{\hbar}{8ek_B T^*} I_0^2$$

ELECTRON-PAIR
V=0 CONDUCTANCE

$$g_0 \equiv \left. \frac{dI}{dV_J} \right|_{V=0} = \frac{\hbar I_m}{ek_B T^* Z}$$



**VISUALIZE
ELECTRON-PAIR
DENSITY**

$$n_p(r) \propto I_m(r) R_N^2(r)$$

$$n_p(r) \propto g_0(r) R_N^2(r)$$

SJTM EXAMPLE : SC+CDW @ NbSe₂

SUPERCONDUCTOR

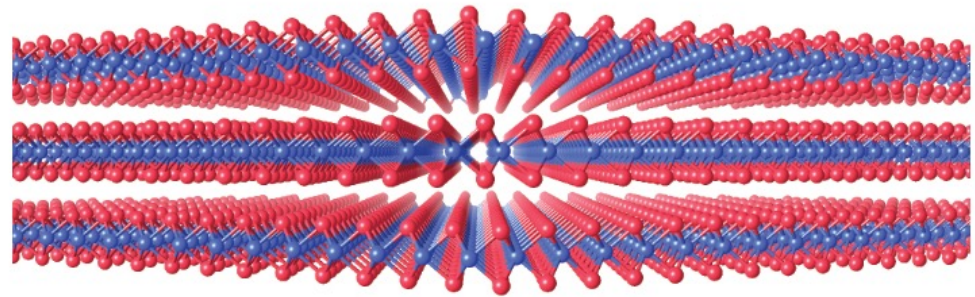
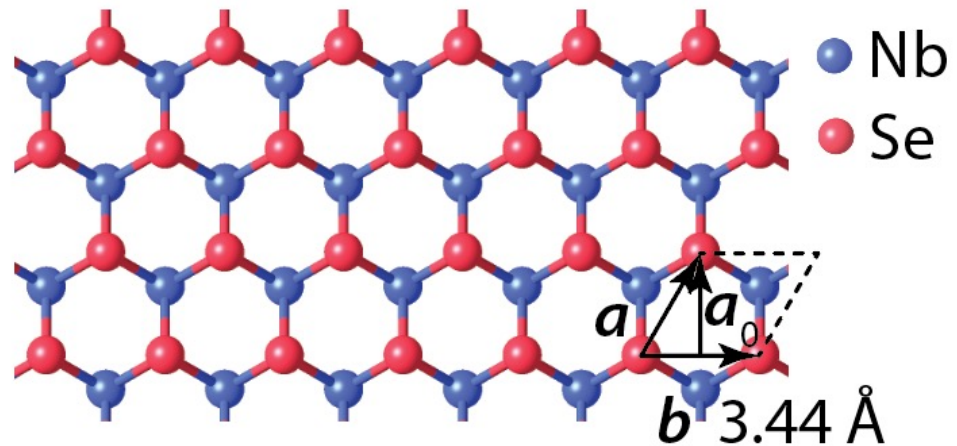


$$T_C \approx 7.2 \text{ K}$$

$$T_{CDW} \approx 33 \text{ K}$$

$$\Delta_0 \approx 1.2 \text{ meV}$$

CHARGE DENSITY WAVE



SJTM EXAMPLE : SC+CDW @ NbSe₂

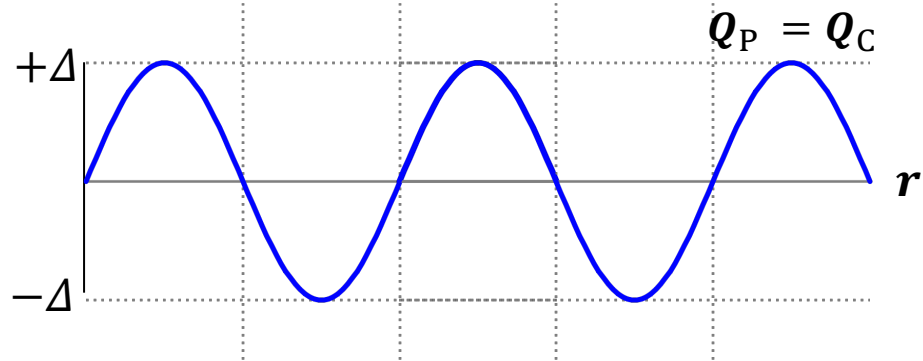
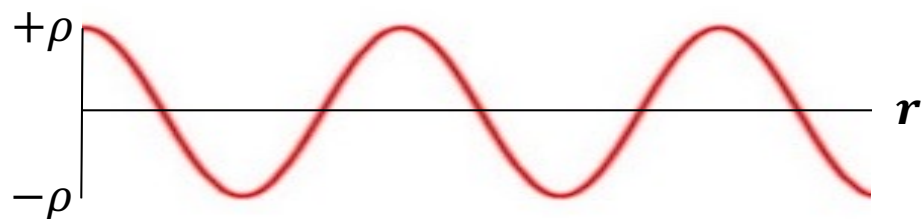
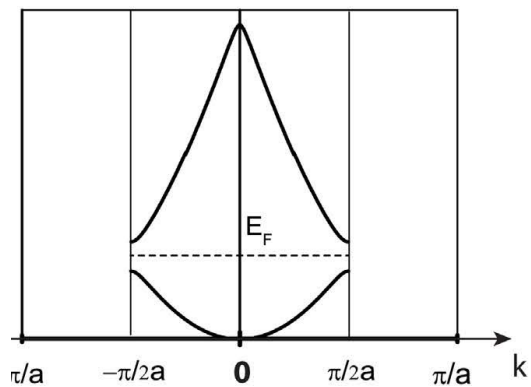
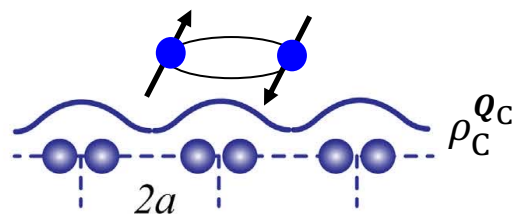
SUPERCONDUCTOR

$$\Delta_S = \Delta_0 e^{i\phi}$$

CHARGE DENSITY WAVE

$$\rho_C^{Q_C}(\mathbf{r}) = \rho e^{iQ_C \cdot \mathbf{r}} + \rho^* e^{-iQ_C \cdot \mathbf{r}}$$

$$\mathcal{F} = \mathcal{F}_S + \mathcal{F}_C + \mathcal{F}_P - \lambda \rho_C^Q \Delta_S^* \Delta_P^{-Q}$$

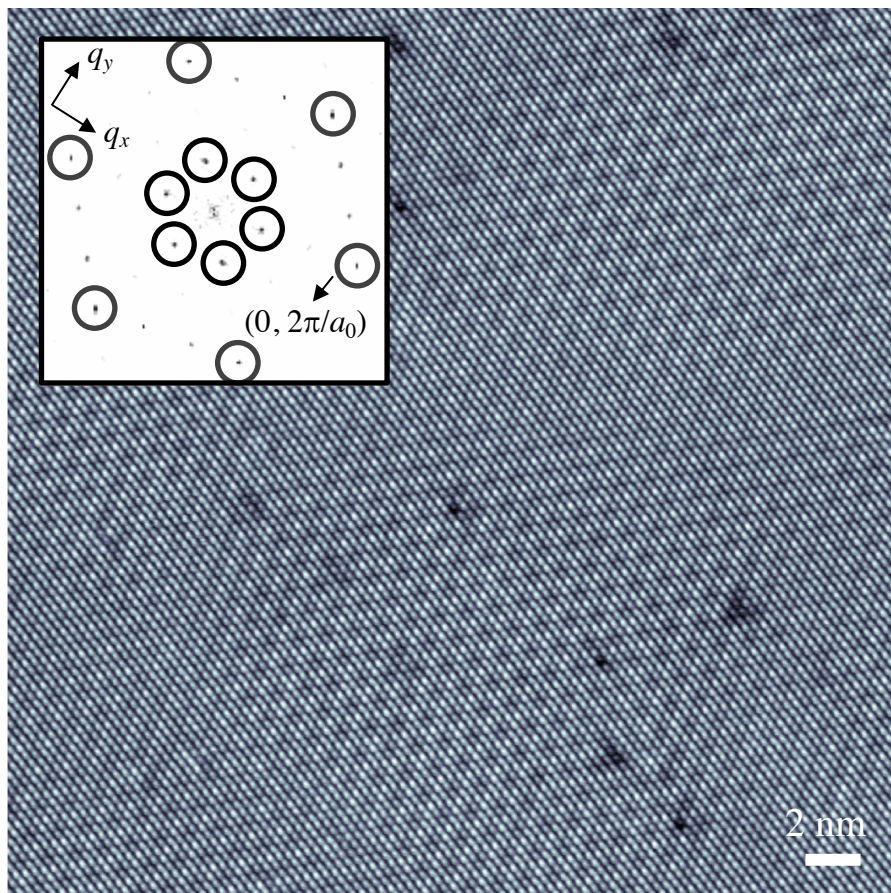


$$\Delta_P^{Q_P}(\mathbf{r}) = \left[\Delta e^{iQ_P \cdot \mathbf{r}} + \Delta^* e^{-iQ_P \cdot \mathbf{r}} \right] e^{i\phi}$$

SJTM : NbSe₂

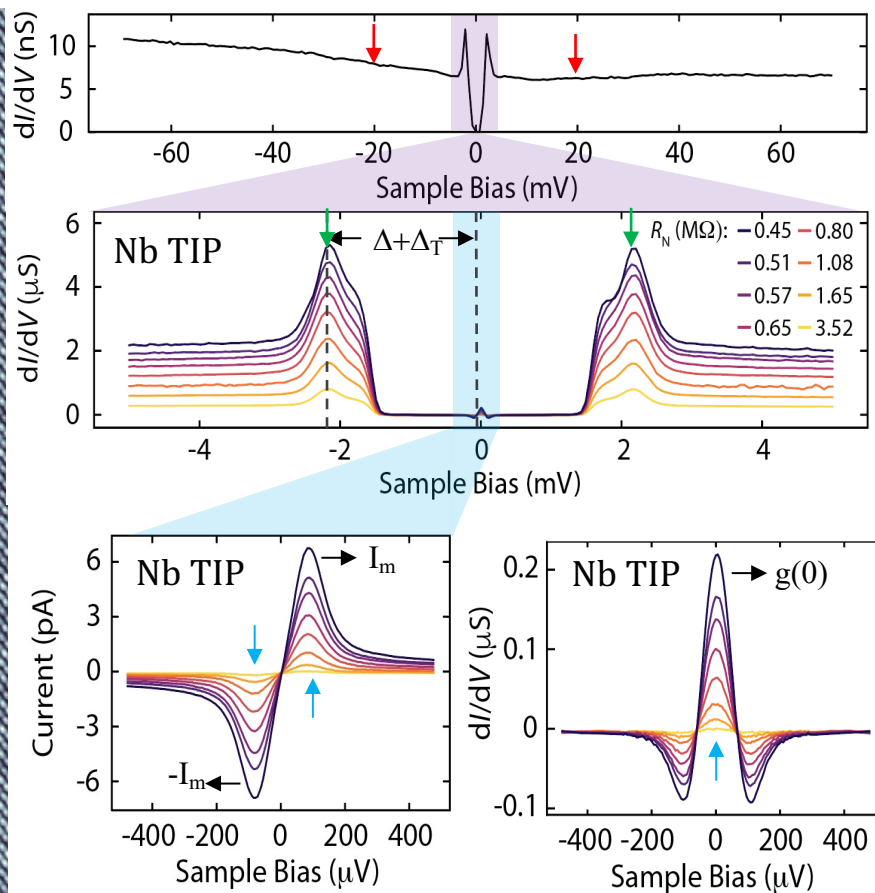
CHARGE DENSITY WAVE + SUPERCONDUCTOR

CONDUCTANCE IMAGE DYNAMIC RANGE >10⁵

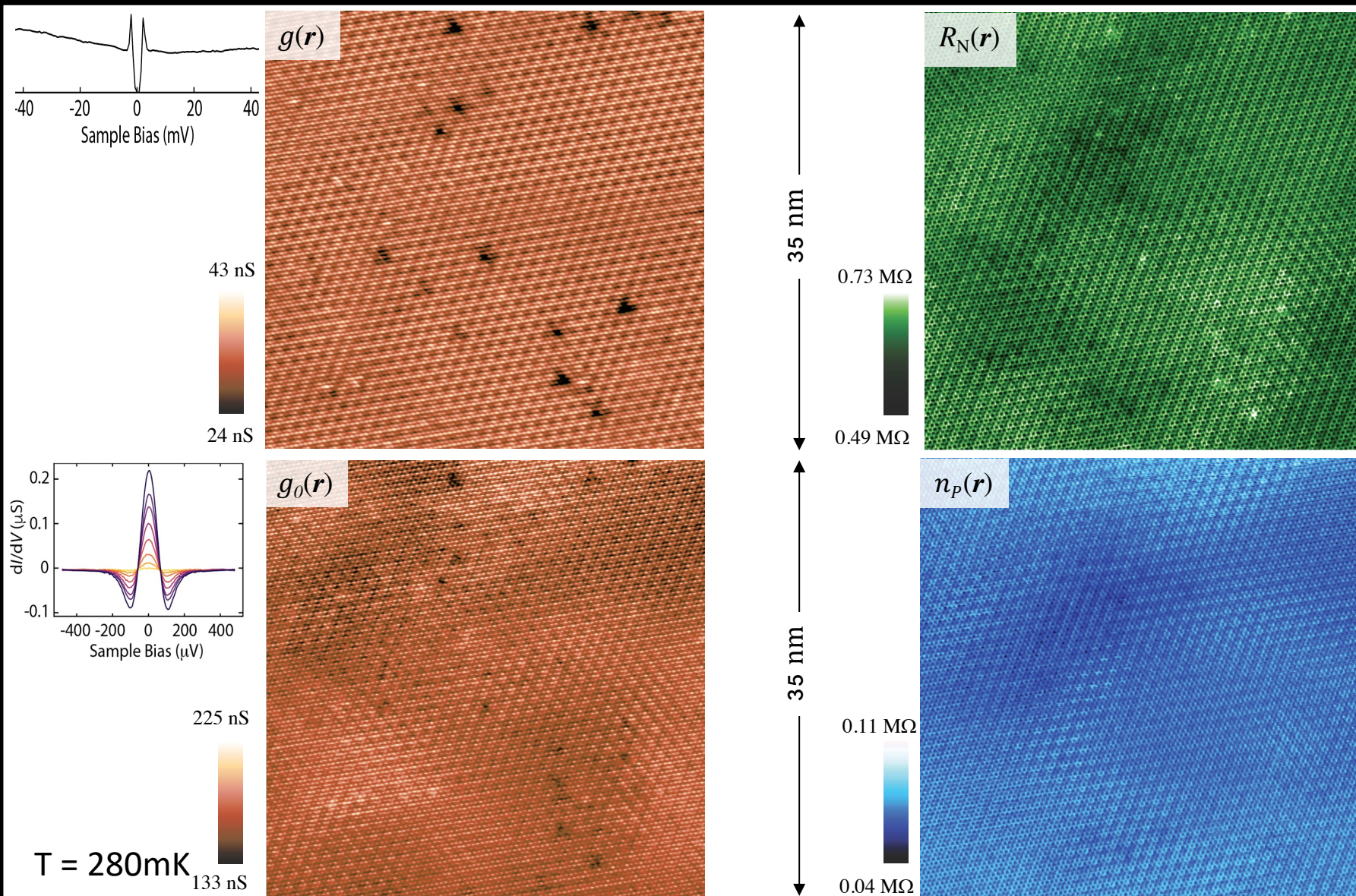


$T(r, -20 \text{ mV})$

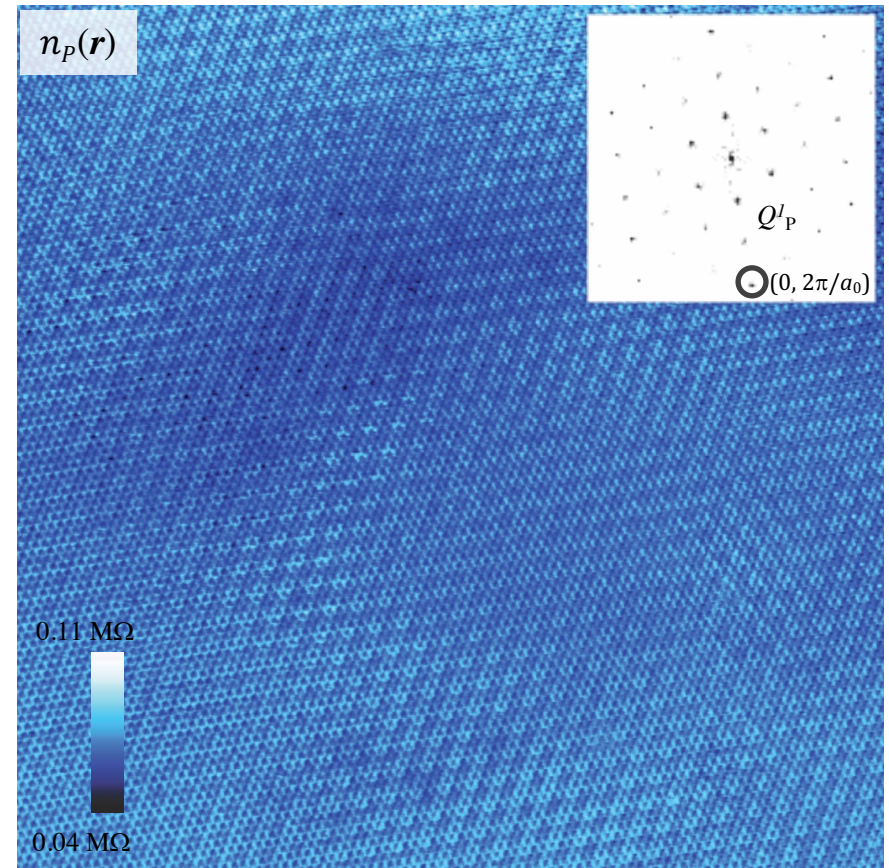
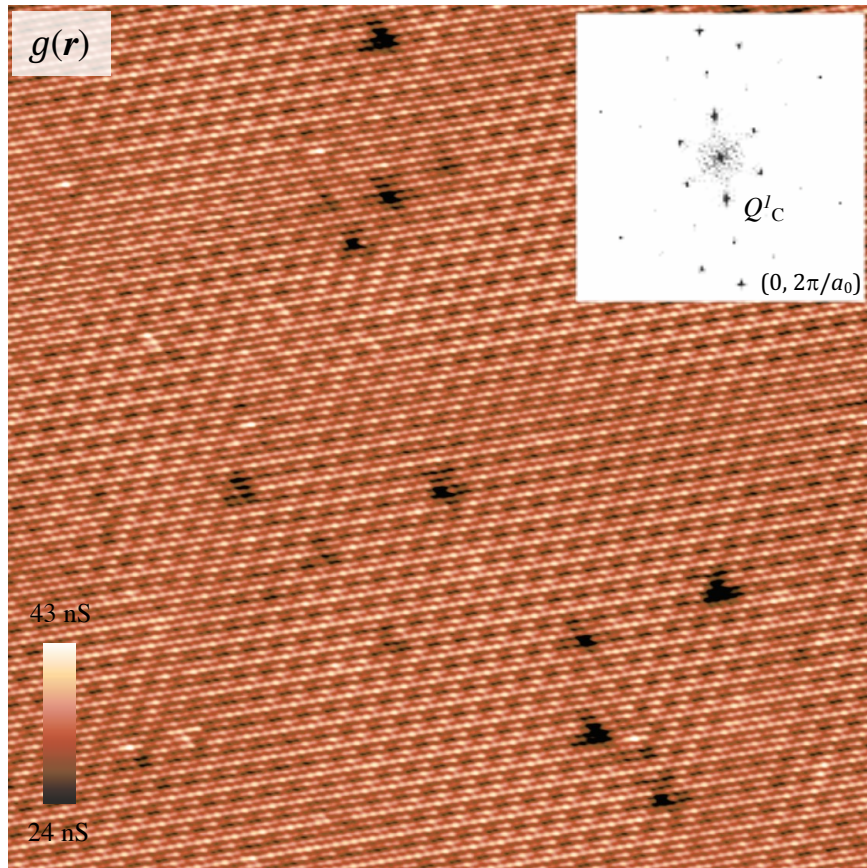
$T = 280 \text{ mK}$



VISUALIZE ELECTRON-PAIR DENSITY $n_p(\mathbf{r})$



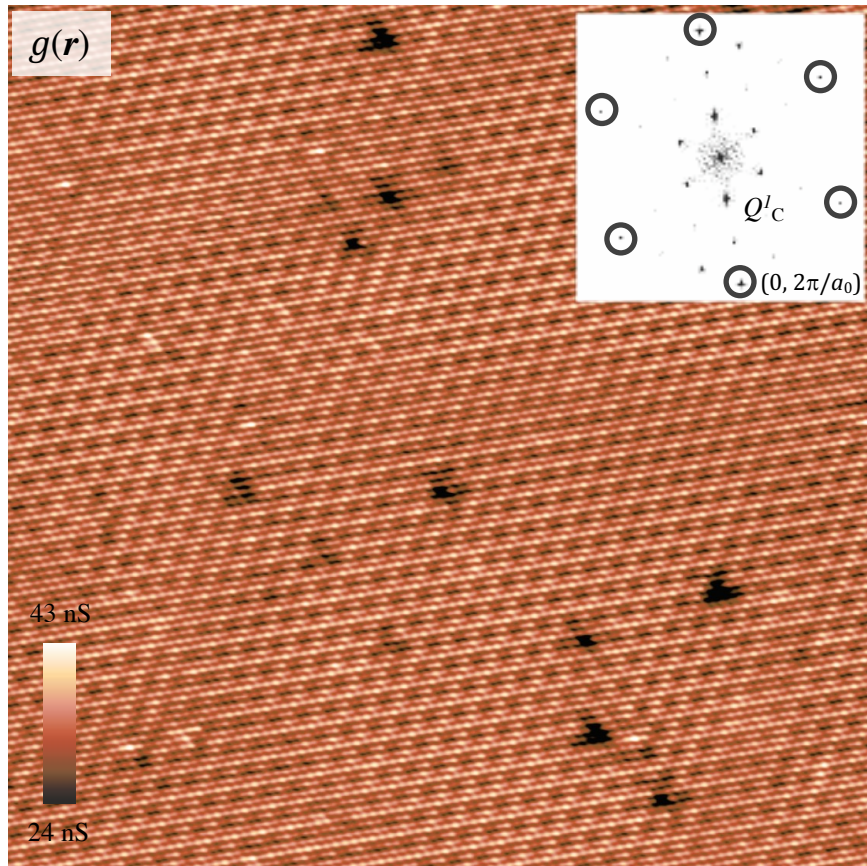
CHARGE DENSITY & PAIR DENSITY MODULATIONS



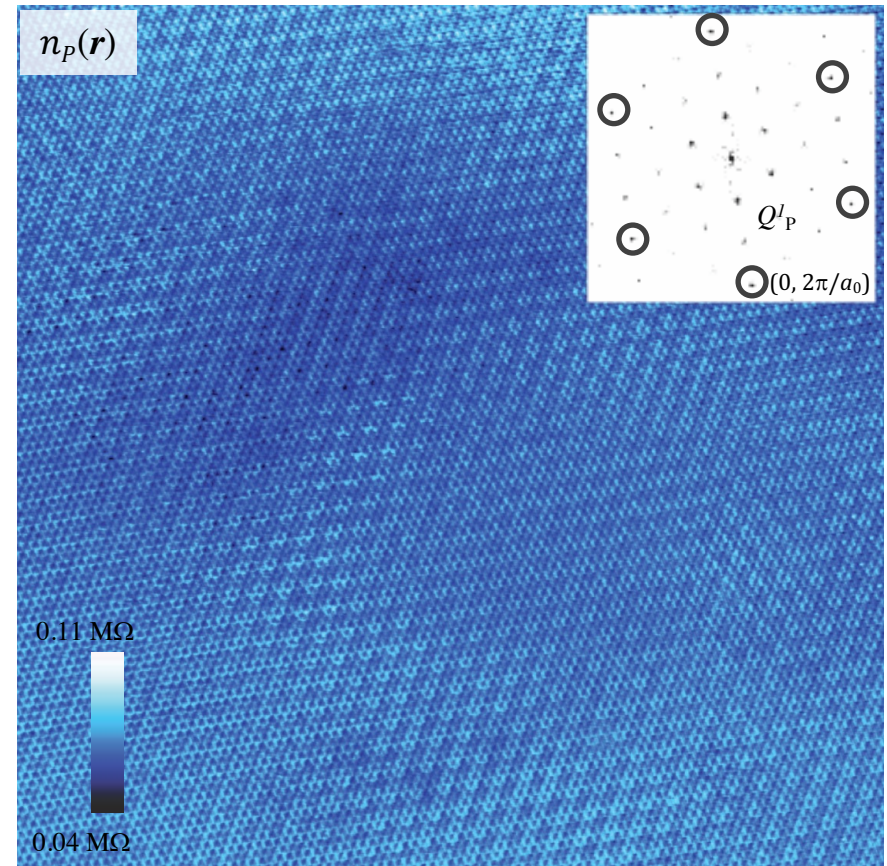
T = 280mK

CHARGE DENSITY & PAIR DENSITY MODULATIONS

CRYSTAL LATTICE



CRYSTAL LATTICE

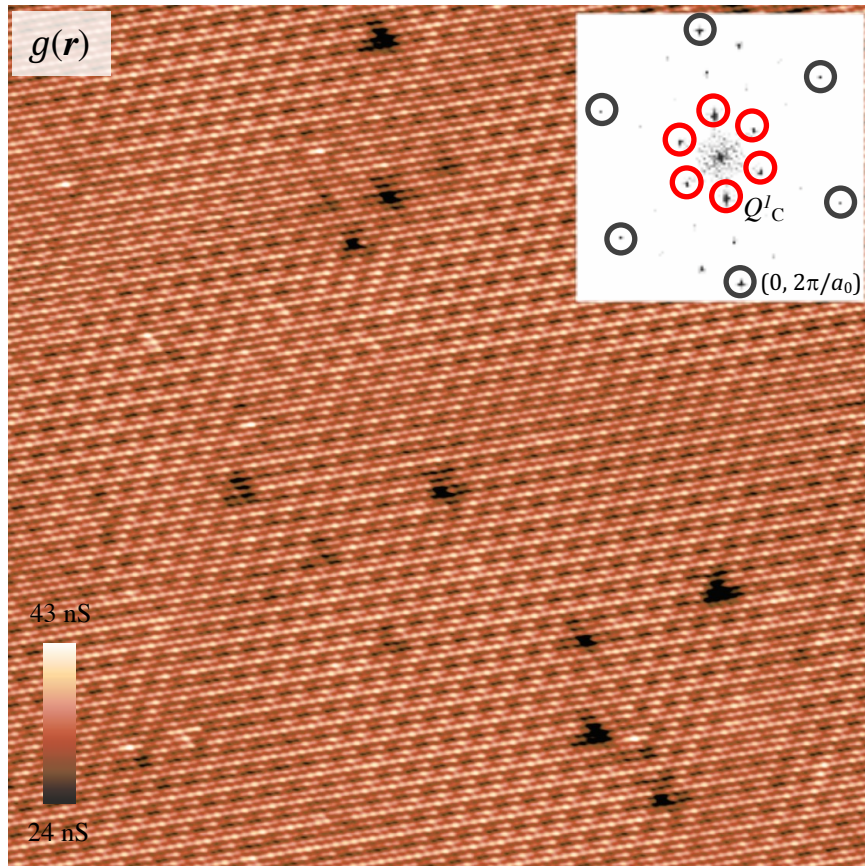


T = 280mK

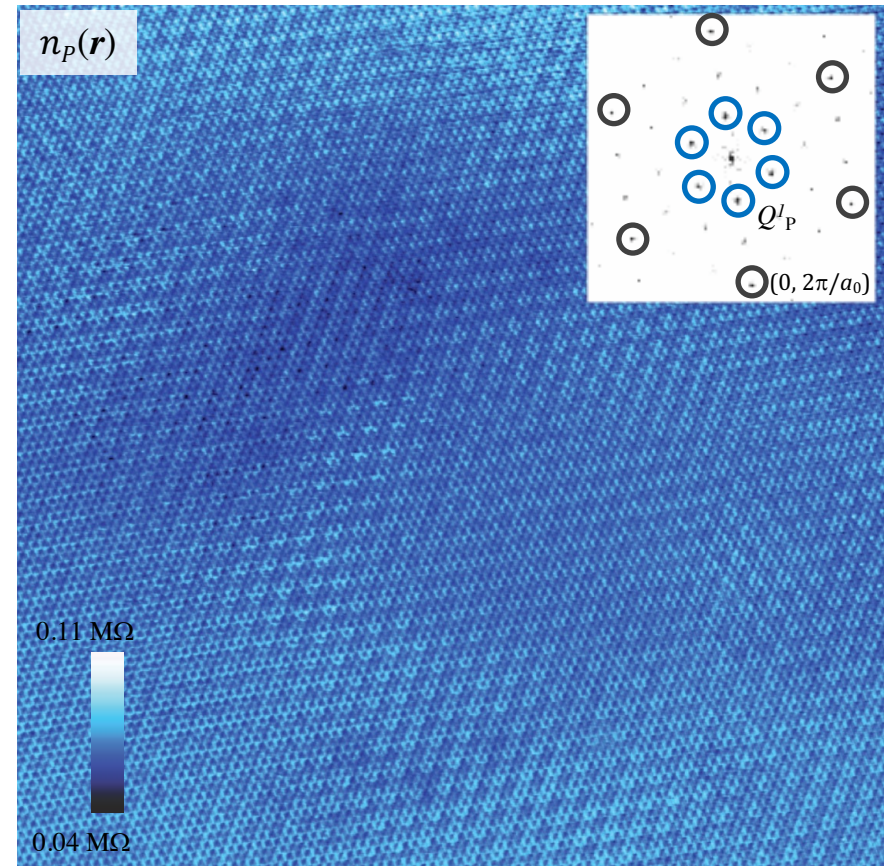
Science 372, 1447 (2021)

CHARGE DENSITY & PAIR DENSITY MODULATIONS

CHARGE DENSITY WAVE + CRYSTAL LATTICE



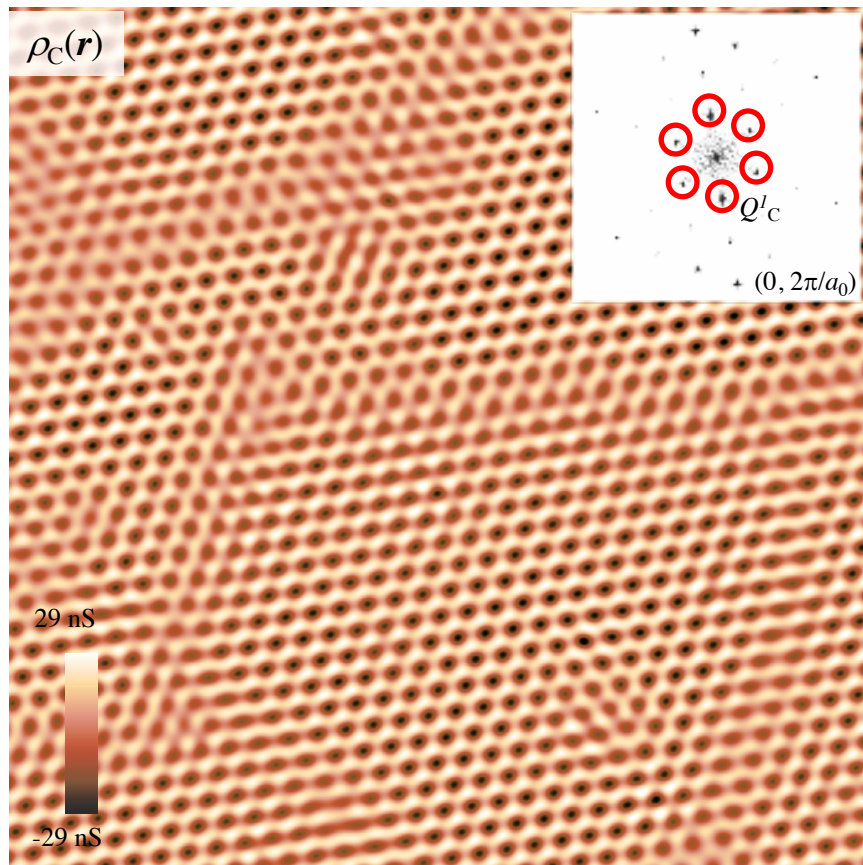
PAIR DENSITY WAVE + CRYSTAL LATTICE



T = 280mK

SIMULTANEOUS VISUALIZATION CDW AND PDW

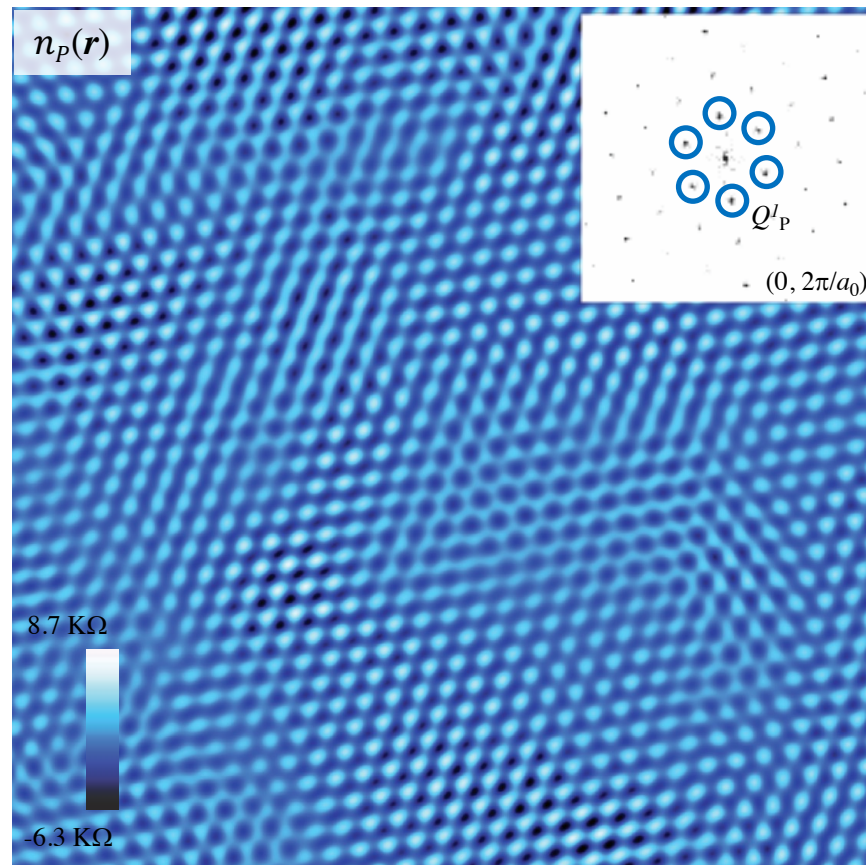
CHARGE DENSITY WAVE



$$\rho_C^{Q^c}(\mathbf{r}) = \rho e^{iQ^c \cdot \mathbf{r}} + \rho^* e^{-iQ^c \cdot \mathbf{r}}$$

T = 280mK

PAIR DENSITY WAVE



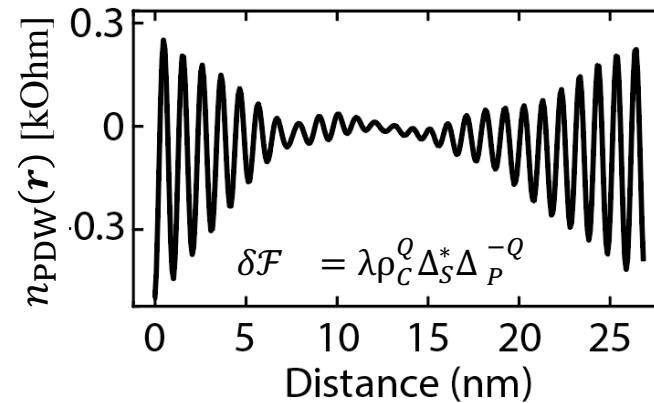
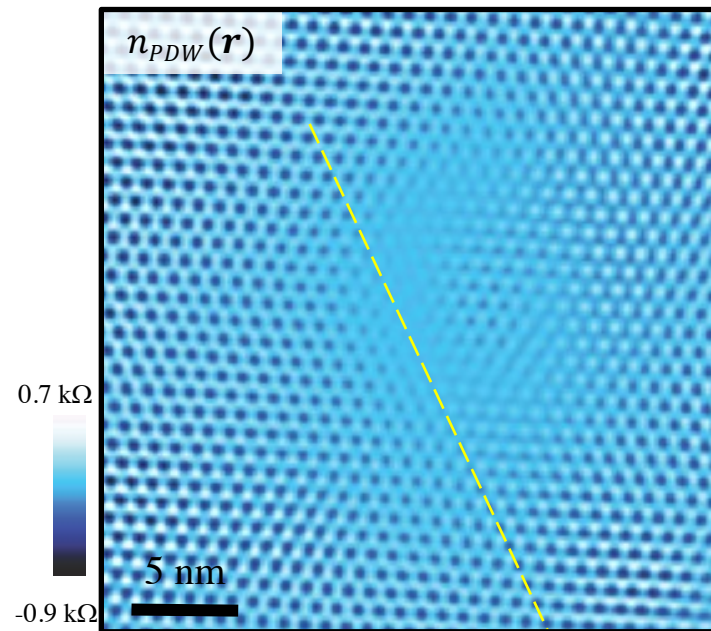
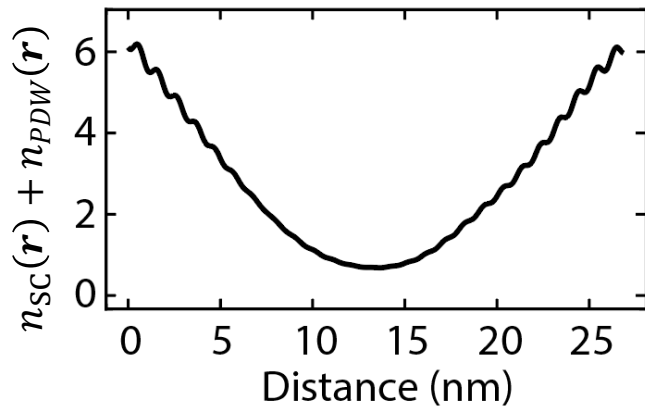
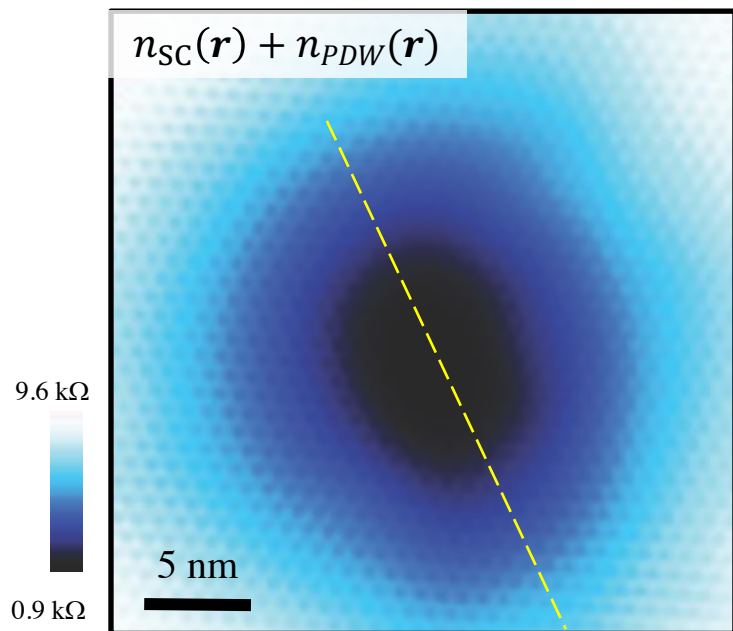
$$\Delta_P^{Q^p}(\mathbf{r}) = \left[\Delta e^{iQ^p \cdot \mathbf{r}} + \Delta^* e^{-iQ^p \cdot \mathbf{r}} \right] e^{i\phi}$$

Science 372, 1447 (2021)

VORTEX ELECTRON-PAIR DENSITY $n_p(\mathbf{r})$

$$n_p(\mathbf{r}) \propto g_0(\mathbf{r})R_N^2(\mathbf{r})$$

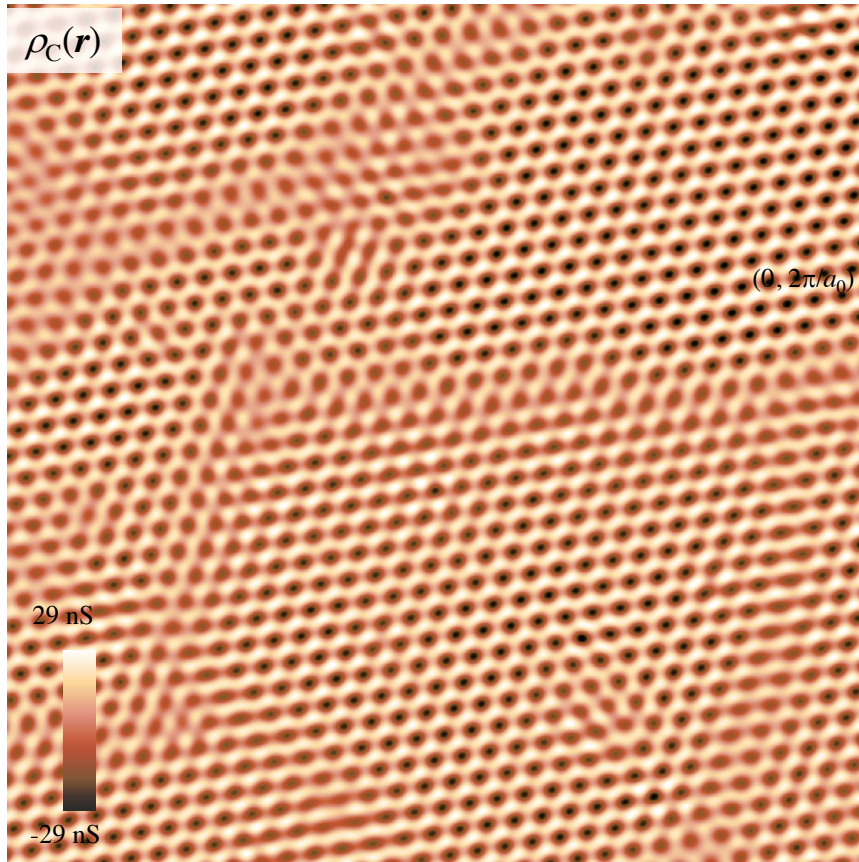
PAIR DENSITY WAVE



T = 280mK

CDW:PDW DISTINCT ATOMIC-SCALE STRUCTURES

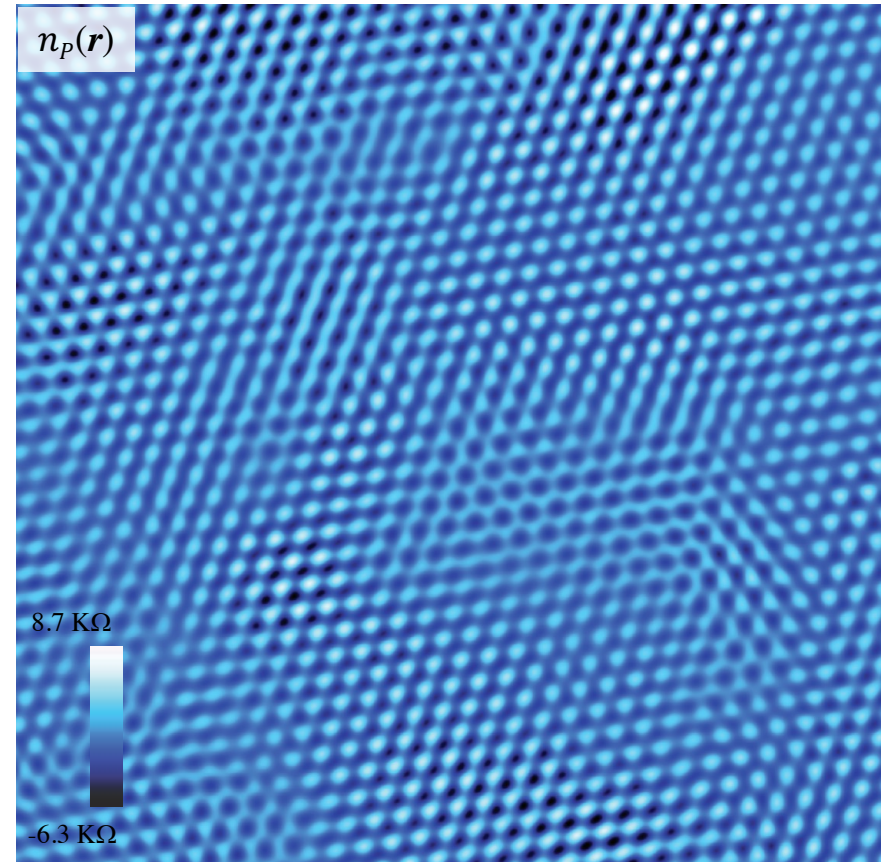
CHARGE DENSITY WAVE



$$\rho_C^{Q_C}(\mathbf{r}) = \rho e^{i\mathbf{Q}_C \cdot \mathbf{r}} + \rho^* e^{-i\mathbf{Q}_C \cdot \mathbf{r}}$$

T = 280mK

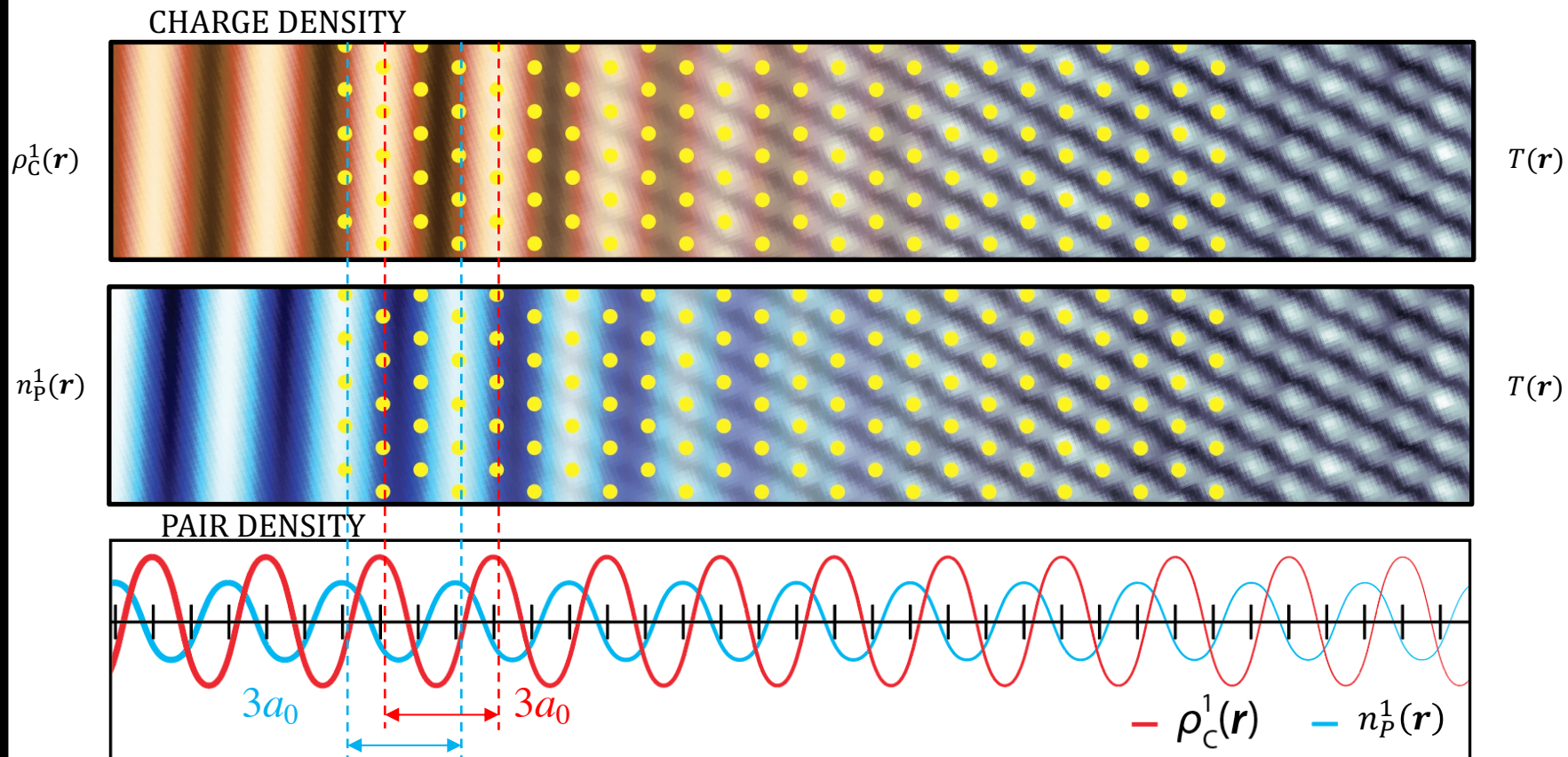
PAIR DENSITY WAVE



$$\Delta_P^{Q_P}(\mathbf{r}) = \left[\Delta e^{i\mathbf{Q}_P \cdot \mathbf{r}} + \Delta^* e^{-i\mathbf{Q}_P \cdot \mathbf{r}} \right] e^{i\phi}$$

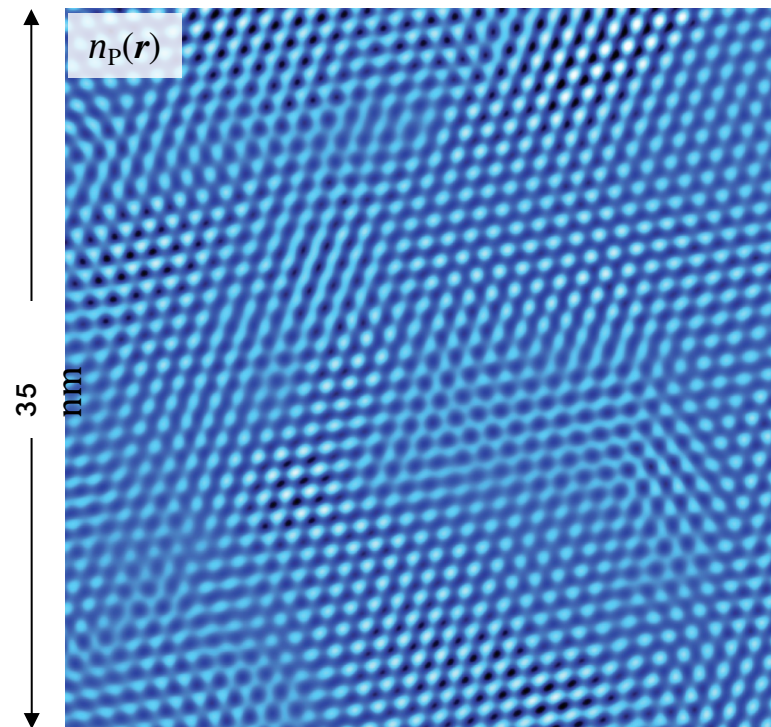
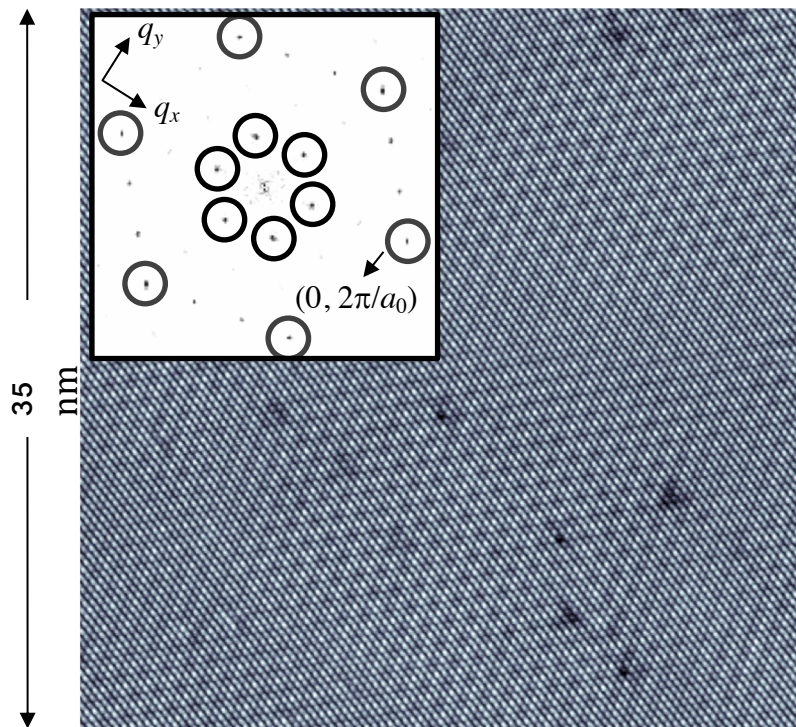
Science 372, 1447 (2021)

CDW:PDW PHASE SHIFT = $2\pi/3$ UNIVERSALLY

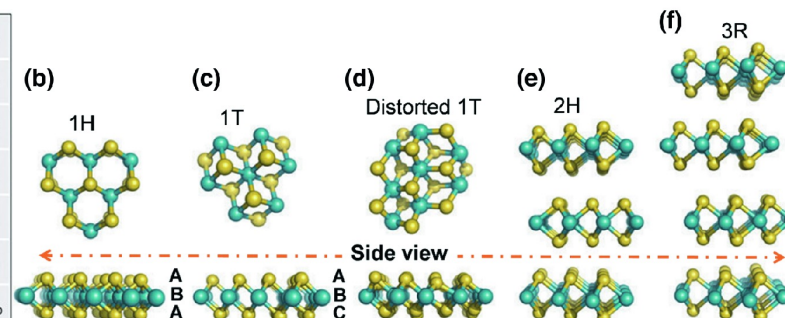


T = 280mK

SJTM: ELECTRON-PAIR VISUALIZATION

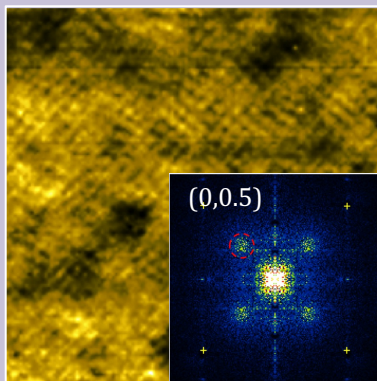


H	MX ₂ M = Transition metal X = Chalcogen																He
Li	Be											B	C	N	O	F	Ne
Na	Mg	3	4	5	6	7	8	9	10	11	12	Al	Si	P	S	Cl	Ar
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
Cs	Ba	La-Lu	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
Fr	Ra	Ac-Lr	Rf	Db	Sg	Bh	Hs	Mt	Ds	Rg	Cn	Uut	Fl	Uup	Lv	Uus	Uuo



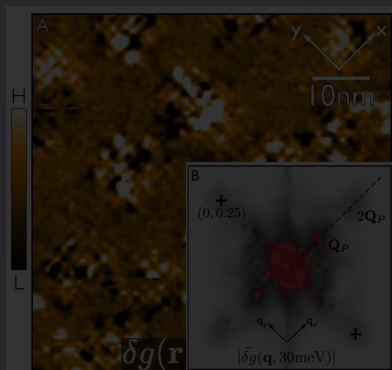
CuO₂ ELECTRON-PAIR VISUALIZATION STUDIES

PAIR DENSITY



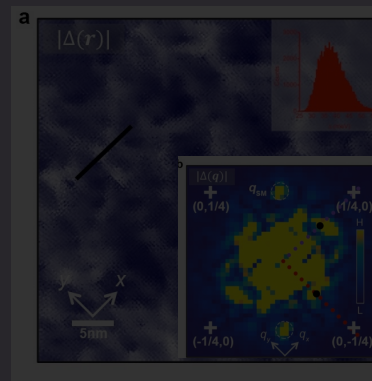
Nature 532, 343 (2016)

VORTEX HALO



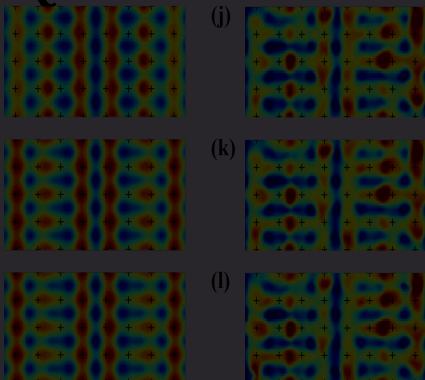
Science 364, 976 (2019)

ENERGY GAP



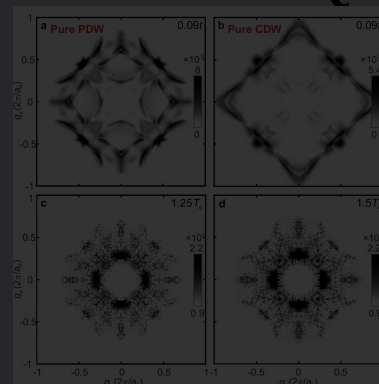
Nature 580, 6570 (2020)

QUASIPARTICLES



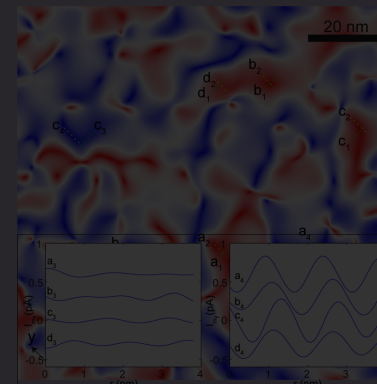
PNAS 117, 14850 (2020)

PSEUDOGAP QPI

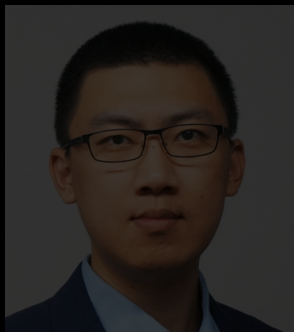


Nat.Comm 12, 6087 (2021)

NEMATIC



Weijiong Chen *et al* (2022)



Xiaolong Liu
Notre Dame



Shuqiu Wang
Oxford



P. Choubey
IIT Delhi



Weijiong Chen
Oxford



S. Uchida
Tokyo



H. Eisaki
Tsukuba



P.D. Johnson
Oxford



Kazuhiro Fujita
BNL



P.J. Hirschfeld
Florida



M. H. Hamidian
Cornell



S.D. Edkins
Cornell

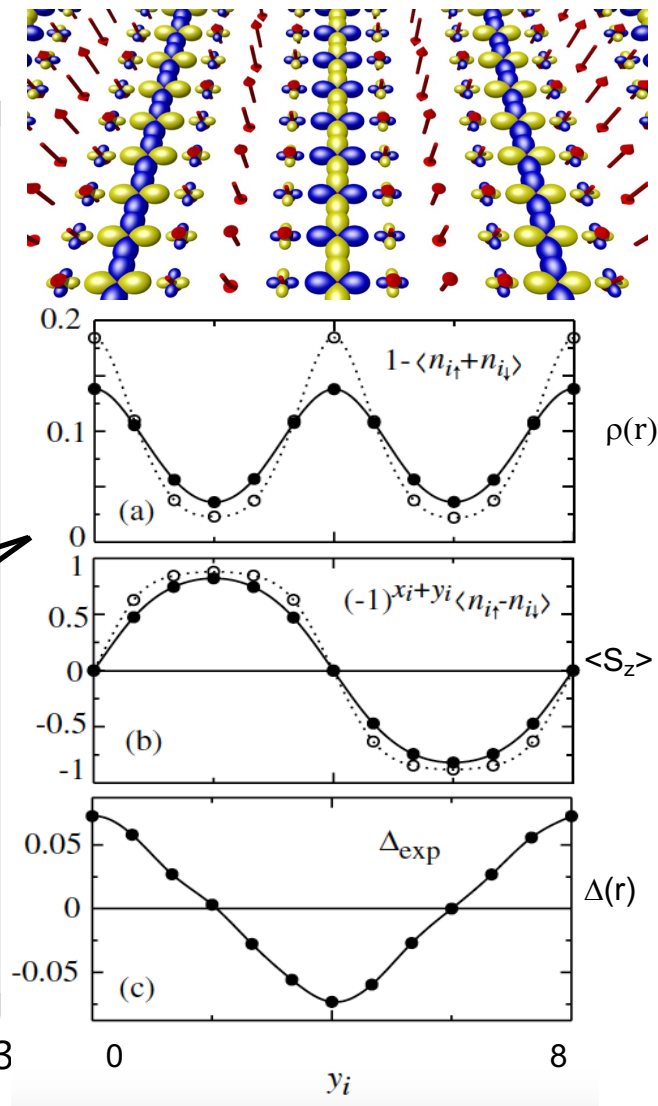
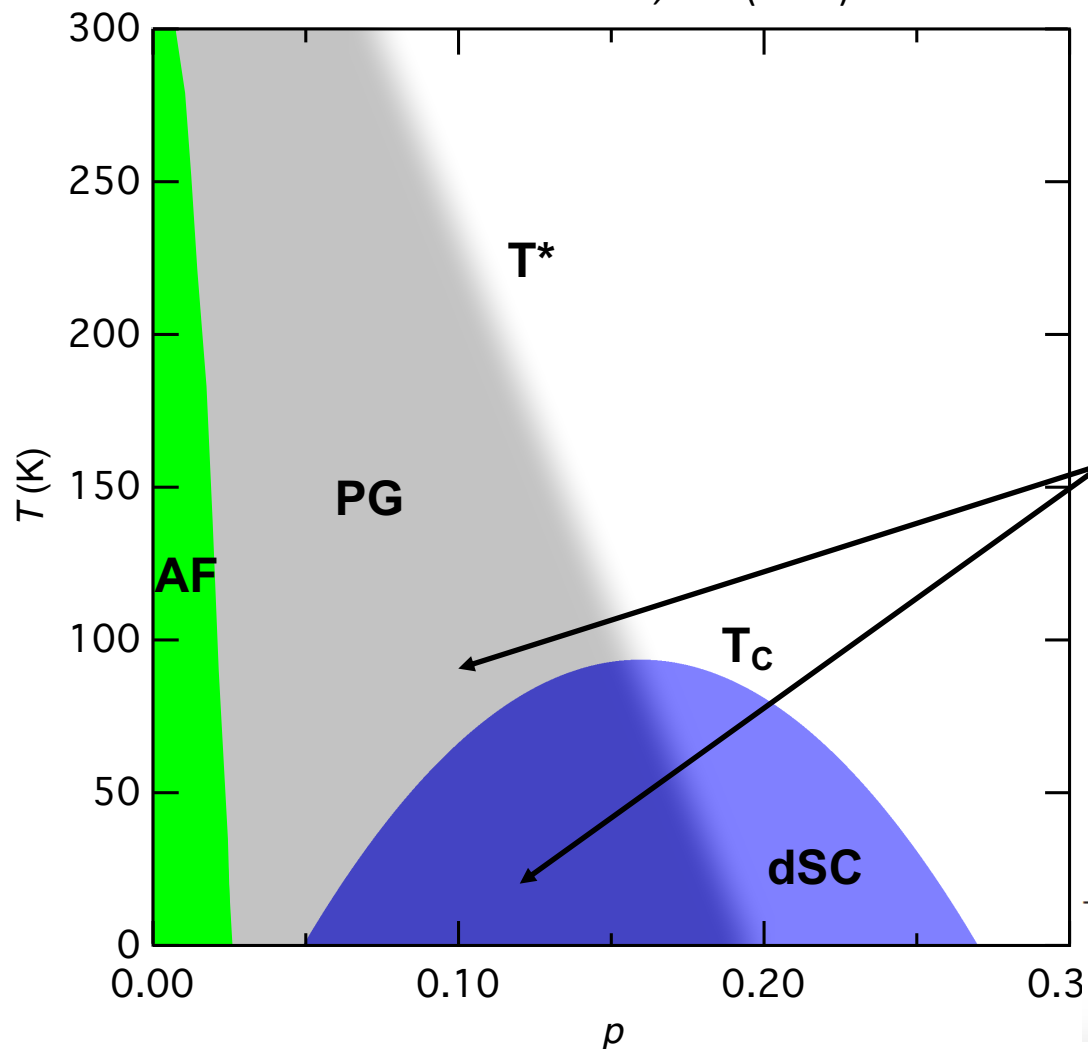


A.P. Mackenzie
Dresden

DOES STRONG-COUPLING PDW STATE EXIST IN CUPRATES?

P. A. Lee *PRX* **4**, 31017 (2014)

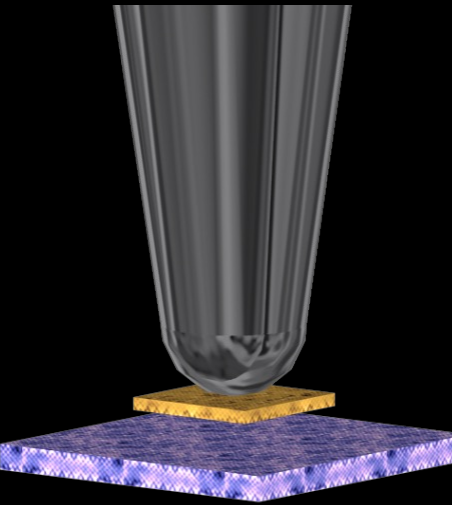
E. Fradkin *RMP* **87**, 457 (2015)



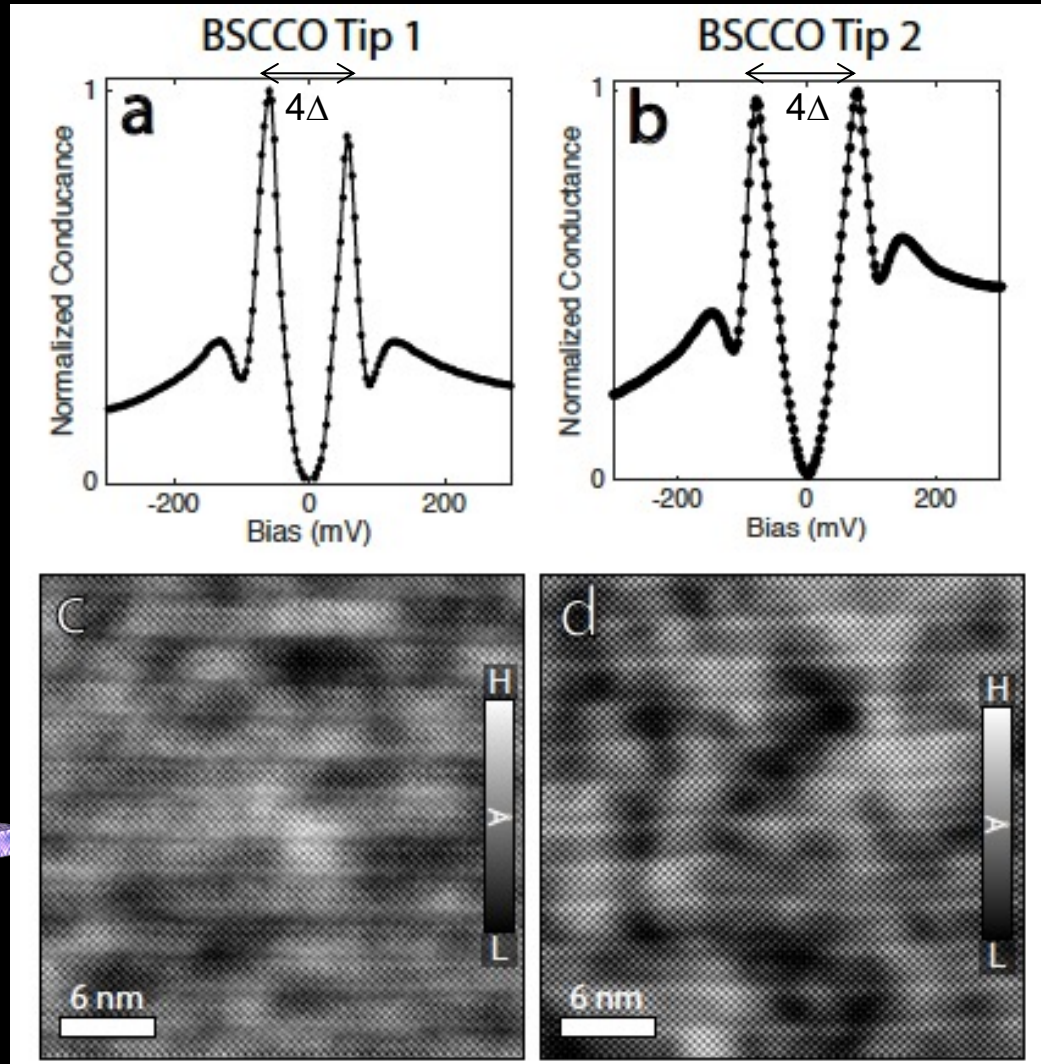
SJTM : d-WAVE HTS TIP

$$E_J = \frac{\hbar I_0}{2e} \propto \frac{\Delta}{R_N} \approx kT$$

d-WAVE HTS TIP



SIS



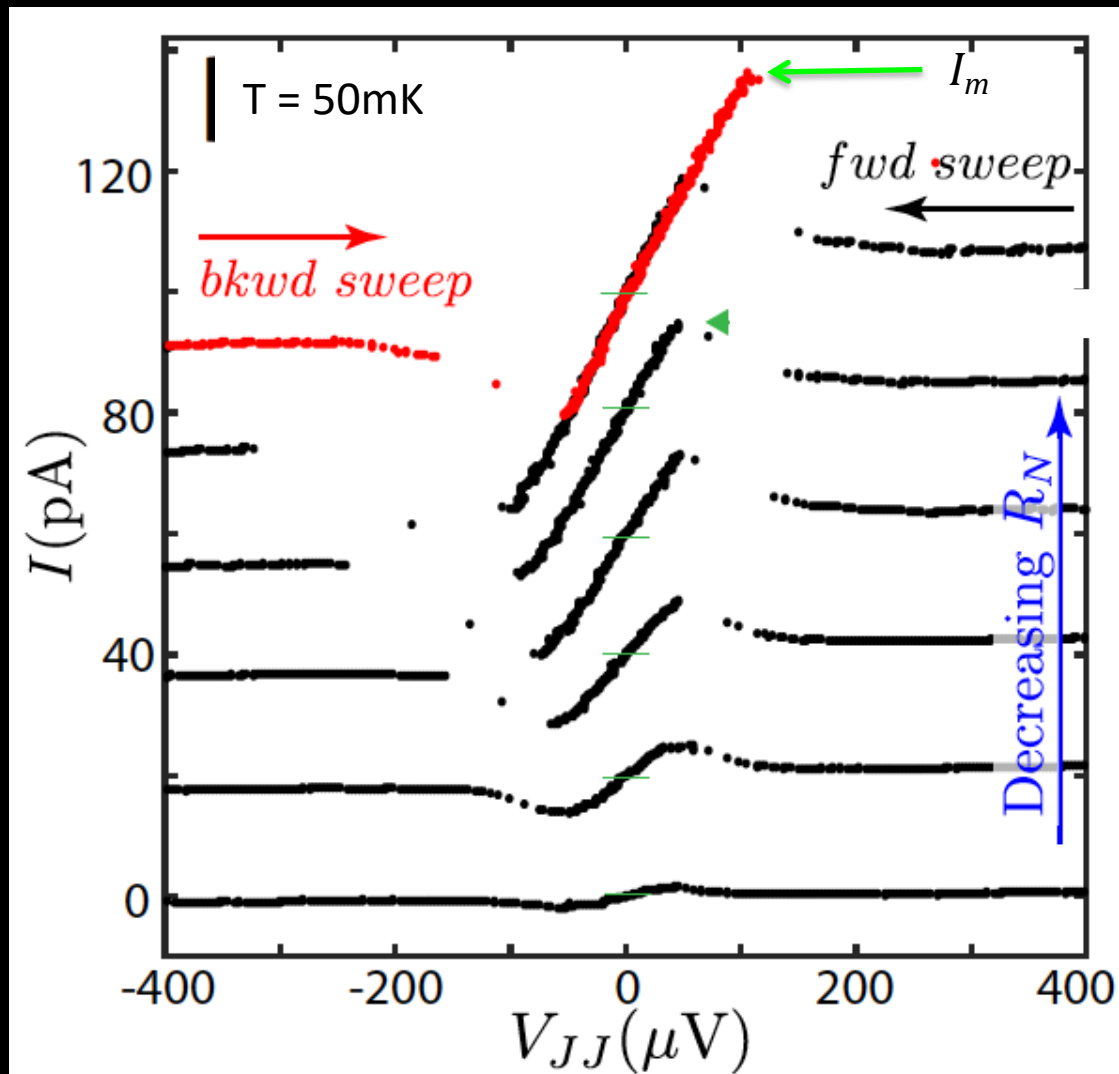
50 mK DR STM



Nature 532, 343 (2016)

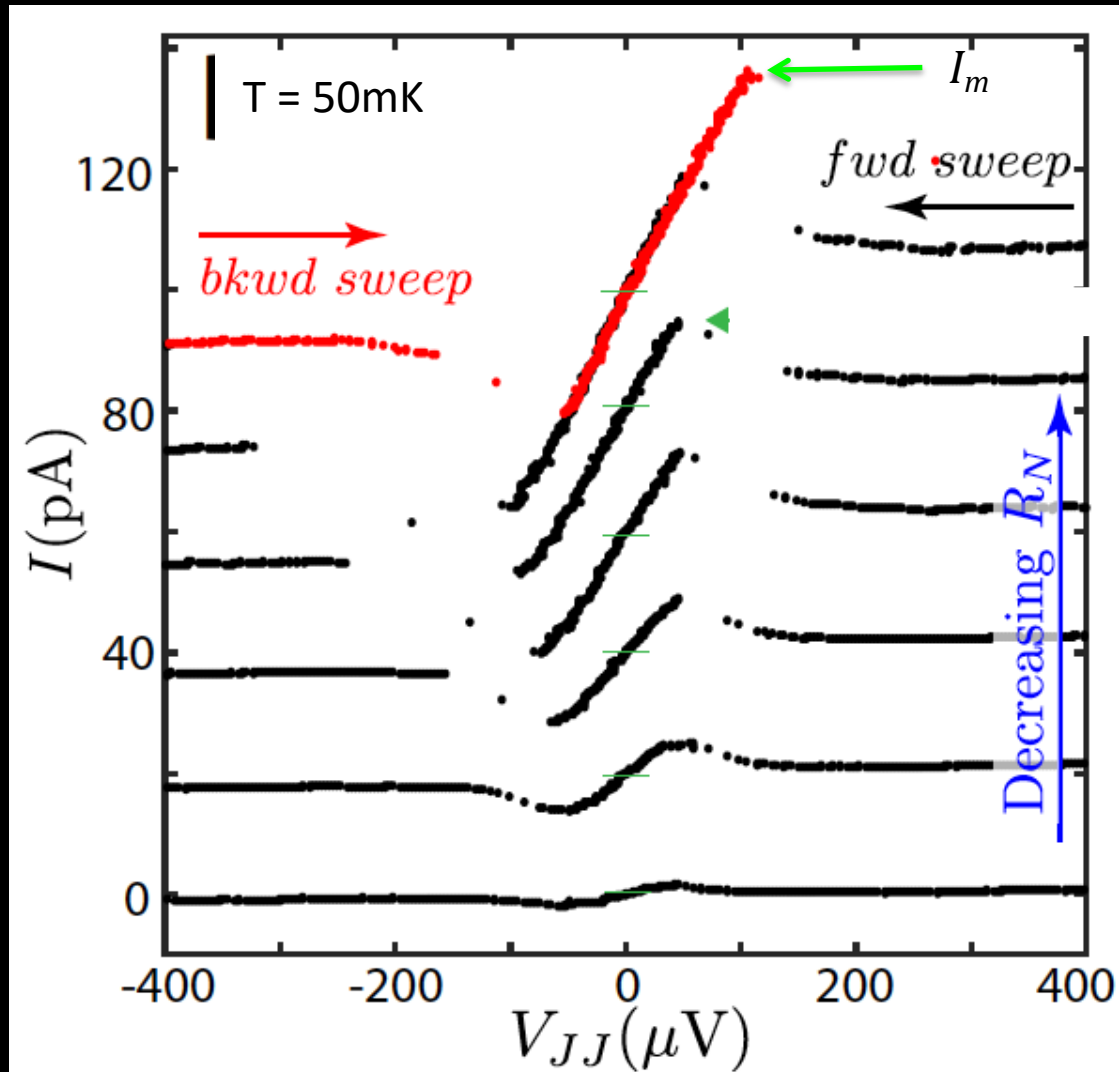
SJTM : ELECTRON-PAIR CURRENT

$$I_m = \frac{\hbar}{8ek_B T^*} I_0^2$$



Nature 532, 343 (2016)

$I_m(r)$ IMAGING OF $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{x+8}$

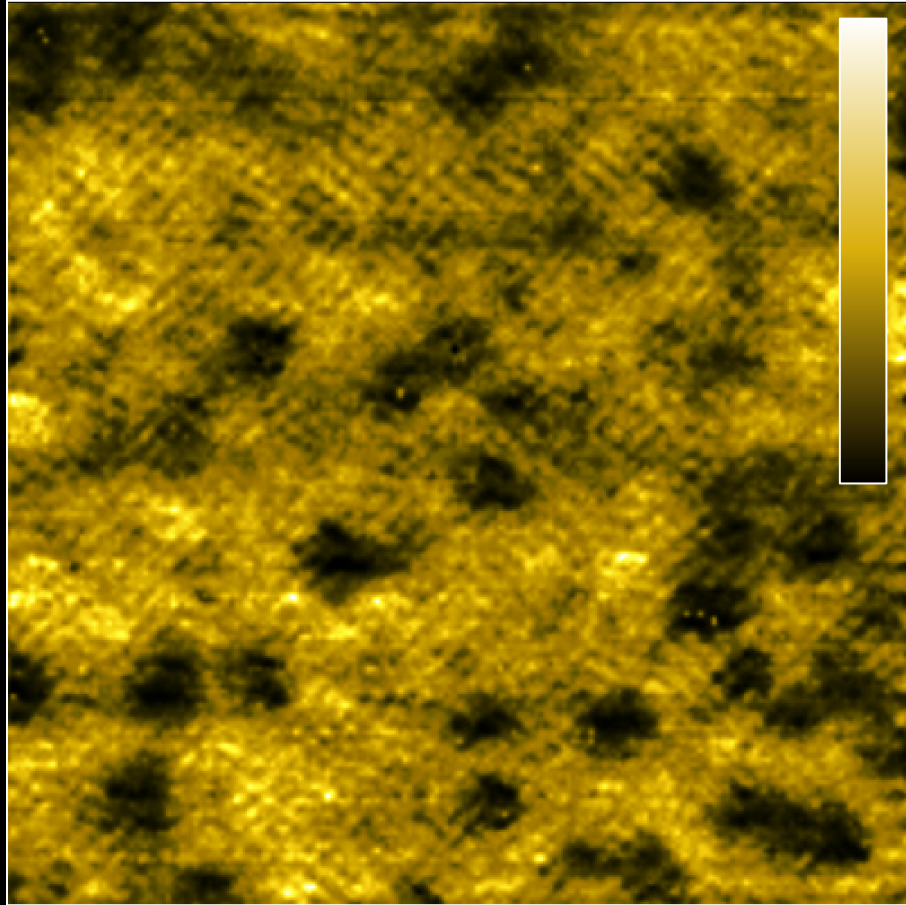


$$n_P(r) \propto I_m(r) R_N^2(r)$$

$I_m(\mathbf{r})$ IMAGING OF $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{x+8}$

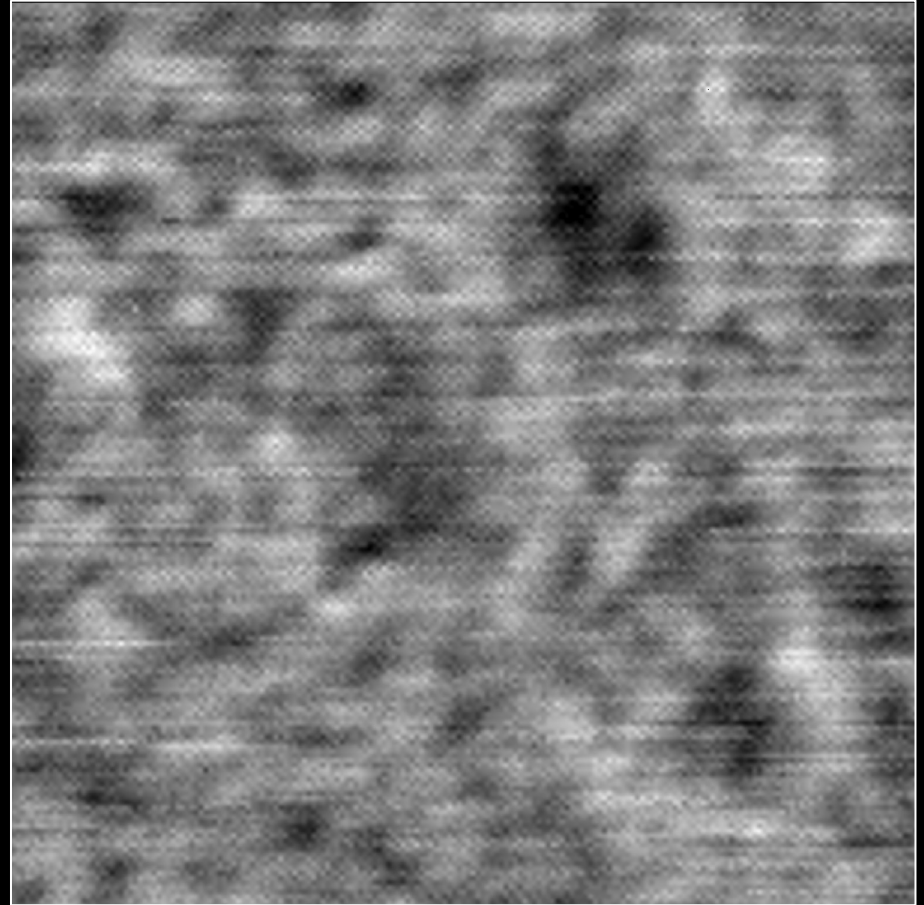
T=50mK

$I_m(\mathbf{r})$



Topography

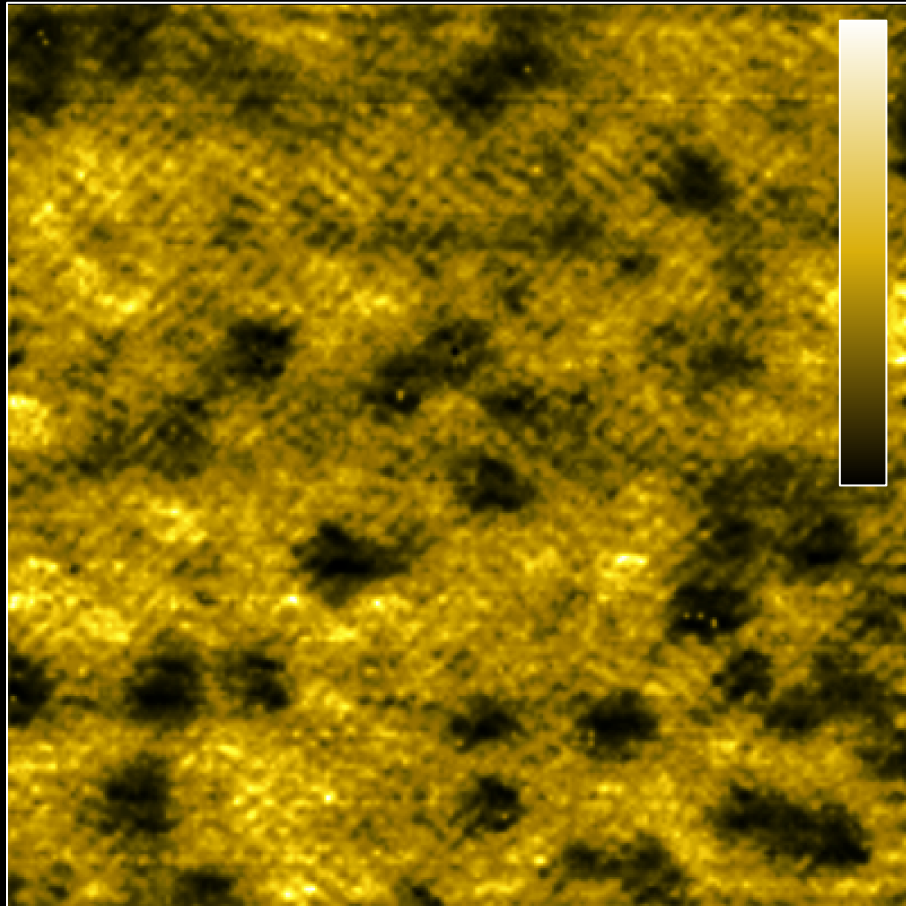
76x76nm



VALIDATE ELECTRON-PAIR DENSITY IMAGING ?

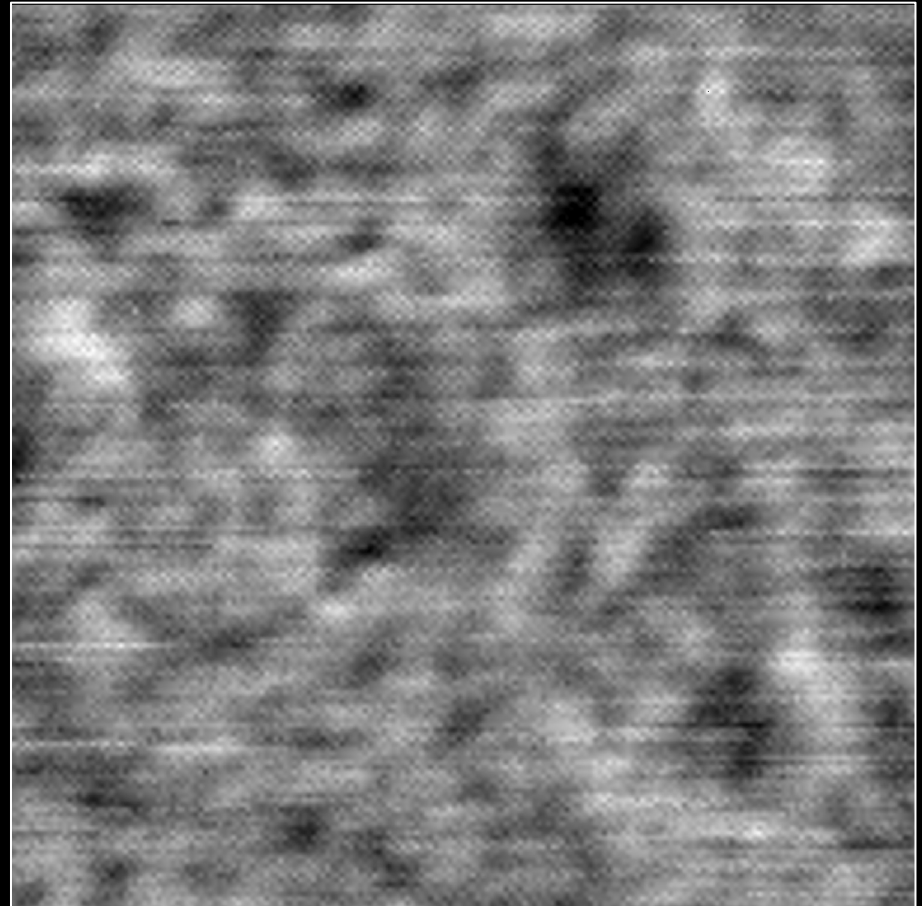
T=50mK

$I_m(\mathbf{r})$



Topography

76x76nm



VALIDATE ELECTRON-PAIR DENSITY IMAGING ?

VOLUME 77, NUMBER 27

PHYSICAL REVIEW LETTERS

30 DECEMBER 1996

Muon Spin Relaxation Studies of Zn-Substitution Effects in High- T_c Cuprate Superconductors

B. Nachumi, A. Keren, K. Kojima, M. Larkin, G. M. Luke, J. Merrin, O. Tchernyshöv, and Y. J. Uemura

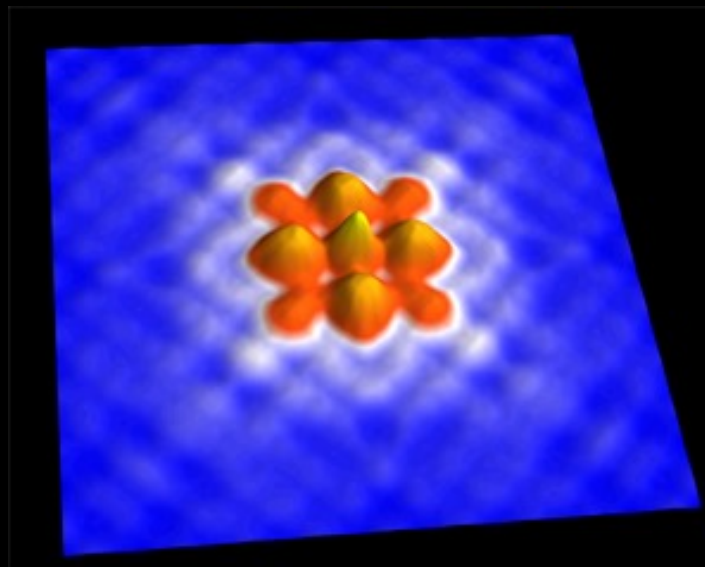
Physics Department, Columbia University, New York, New York 10027

N. Ichikawa, M. Goto, and S. Uchida

Department of Superconductivity, University of Tokyo, Bunkyo-ku, Tokyo 113, Japan

(Received 12 September 1996)

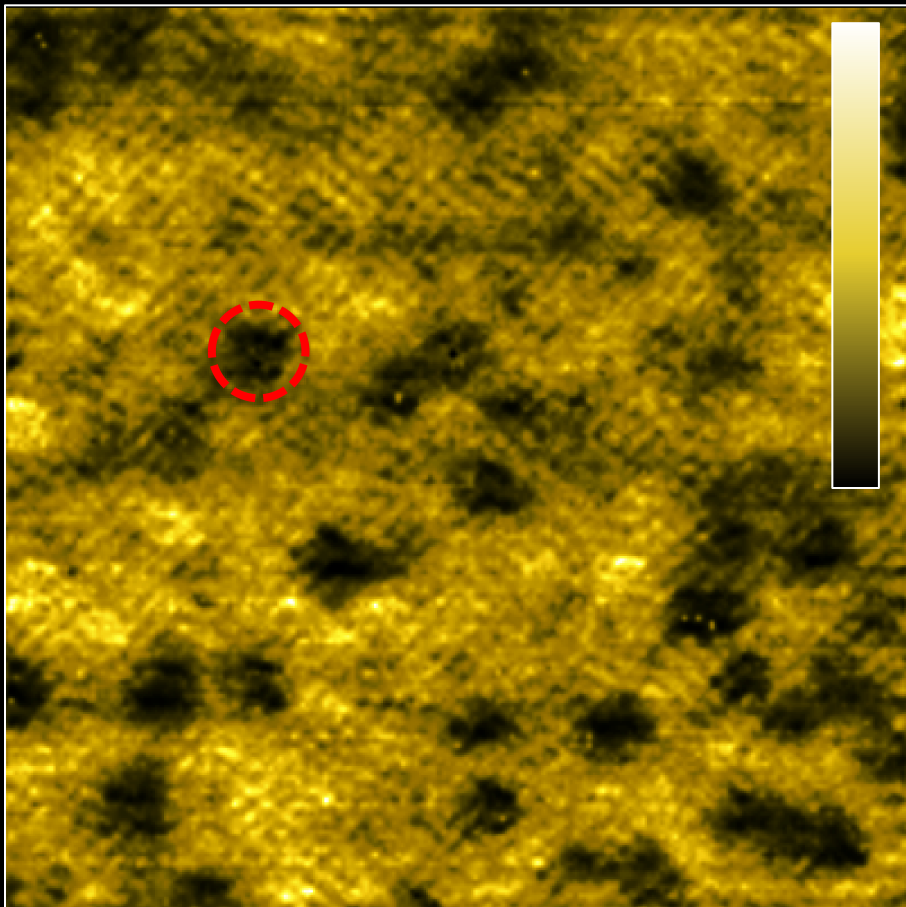
We have performed transverse-field muon spin relaxation measurements of the Zn-substituted cuprate high- T_c superconductors: $\text{La}_{2-x}\text{Sr}_x(\text{Cu}_{1-y}\text{Zn}_y)\text{O}_4$ and $\text{YBa}_2(\text{Cu}_{1-y}\text{Zn}_y)_3\text{O}_{6.63}$. The superconducting carrier density/effective mass n_s/m^* ratio at $T \rightarrow 0$ decreases with increasing Zn concentration, in a manner consistent with our “swiss cheese” model in which charge carriers within an area $\pi\xi_{ab}^2$ around each Zn are excluded from the superfluid. We discuss this result in the context of Bose condensation, pair localization, and pair breaking. [S0031-9007(96)02011-X]



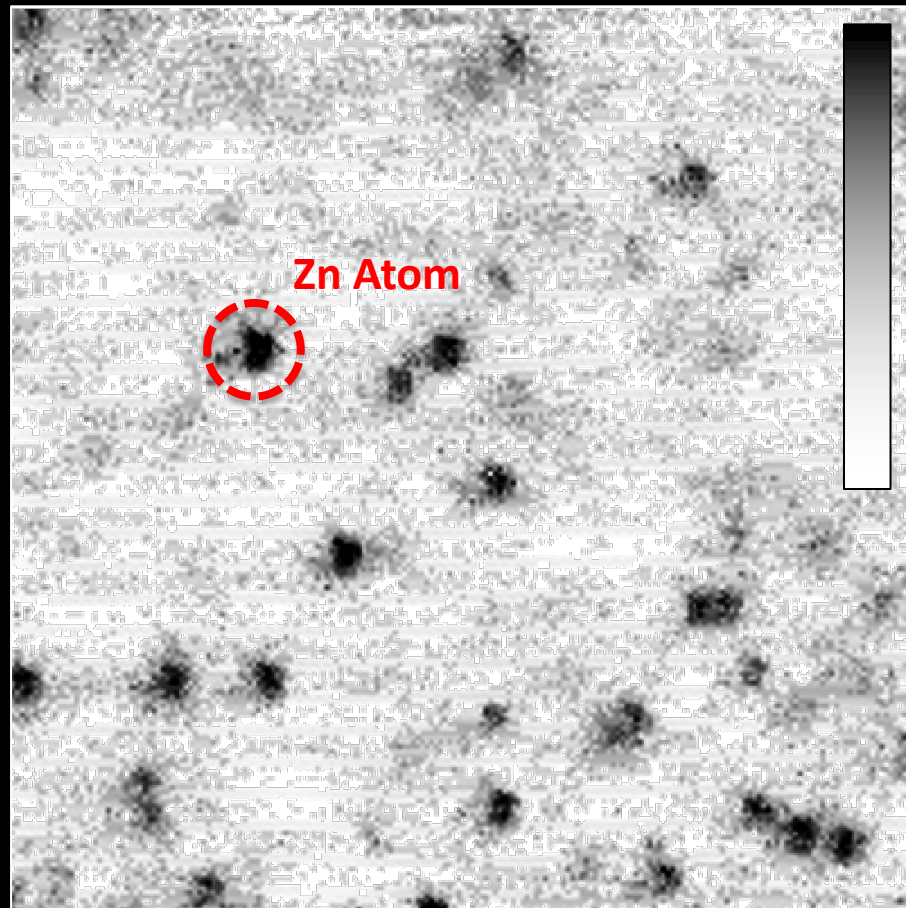
VALIDATE ELECTRON-PAIR DENSITY IMAGING

T=50mK

$I_m(r)$



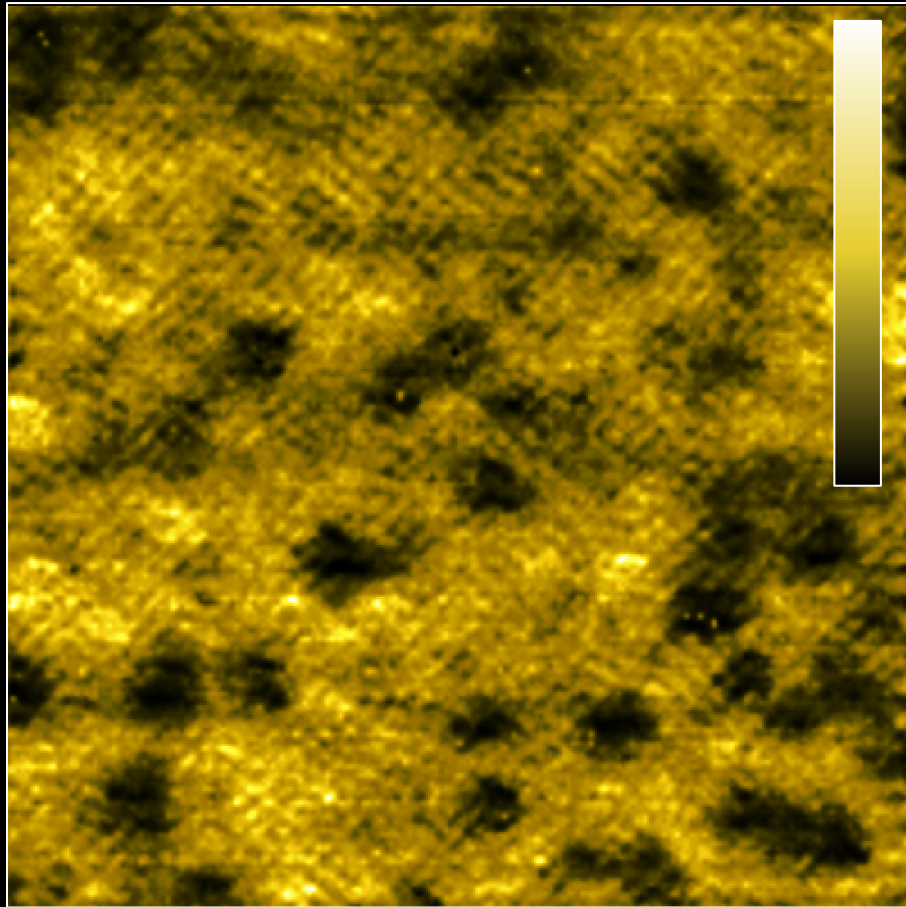
$dI/dV(r, 20mV)$



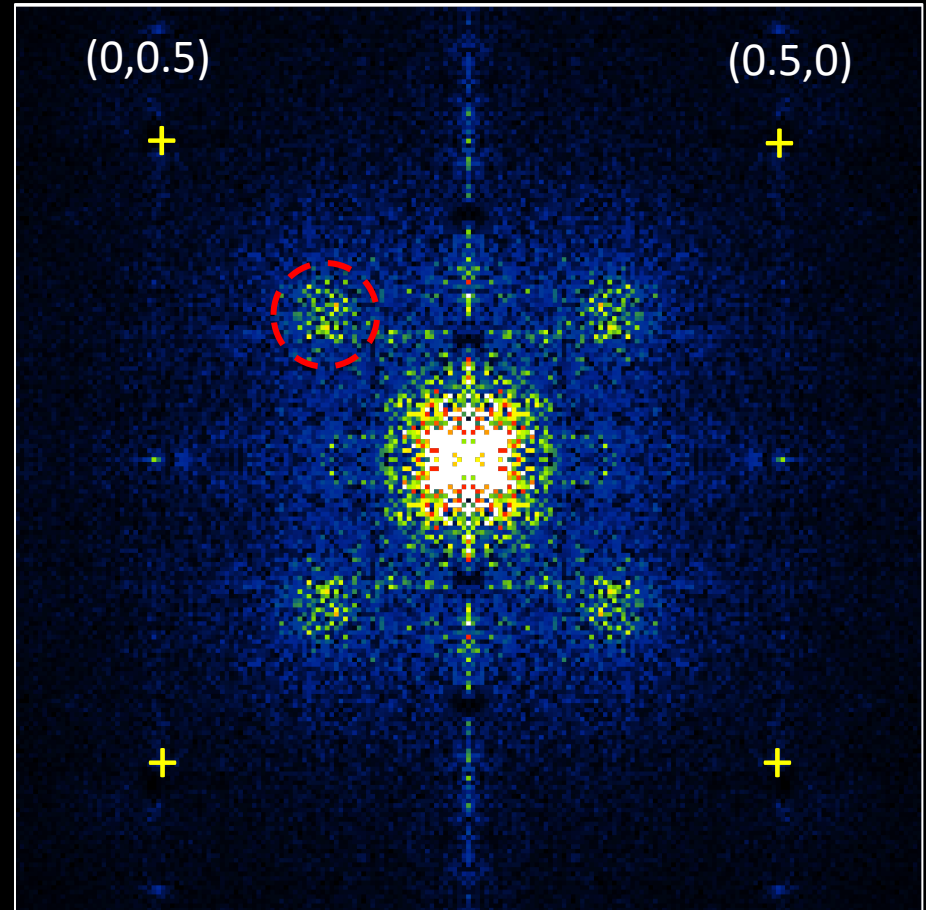
ELECTRON-PAIR DENSITY WAVE

T=50mK

$I_m(\mathbf{r})$



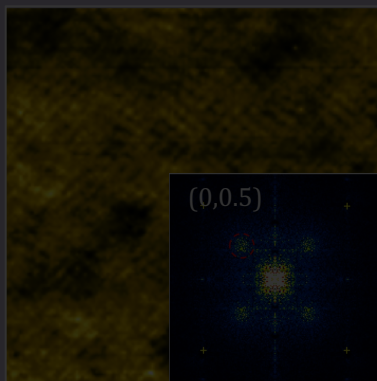
$I_m(\mathbf{q})$



$$Q = (0, 1/4)2\pi/a_0 ; (1/4, 0)2\pi/a_0$$

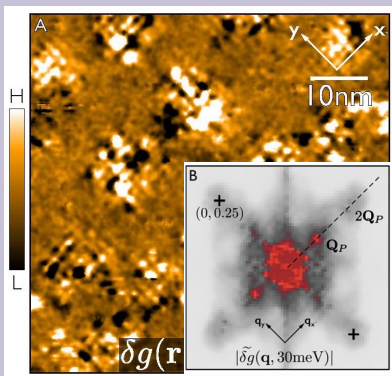
CuO₂ ELECTRON-PAIR VISUALIZATION STUDIES

PAIR DENSITY



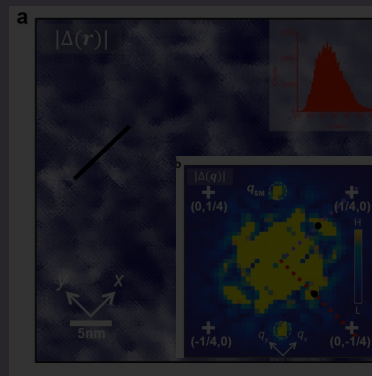
Nature 532, 343 (2016)

VORTEX HALO



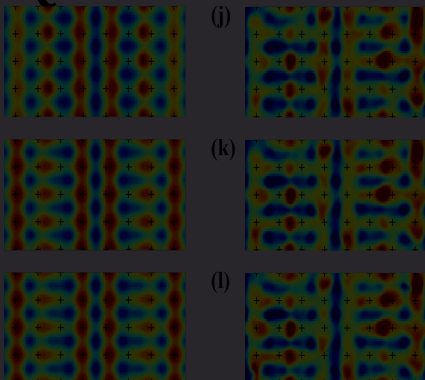
Science 364, 976 (2019)

ENERGY GAP



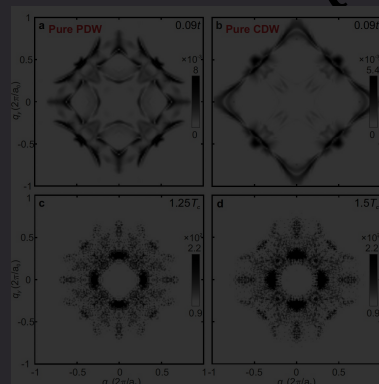
Nature 580, 6570 (2020)

QUASIPARTICLES



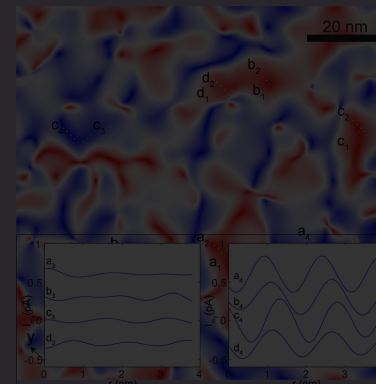
PNAS 117, 14850 (2020)

PSEUDOGAP QPI

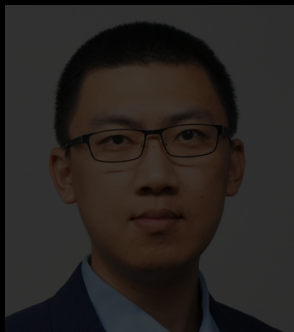


Nat.Comm 12, 6087 (2021)

NEMATIC



Weijiong Chen *et al* (2022)



Xiaolong Liu
Notre Dame



Shuqiu Wang
Oxford



P. Choubey
IIT Delhi



Weijiong Chen
Oxford



S. Uchida
Tokyo



H. Eisaki
Tsukuba



P.D. Johnson
Oxford



Kazuhiro Fujita
BNL



P.J. Hirschfeld
Florida



M. H. Hamidian
Cornell



S.D. Edkins
Cornell



A.P. Mackenzie
Dresden

MAGNETIC FIELD CONTROL OF CUPRATE DENSITY WAVE

J.E. Hoffman *et al.* *Science* 295, 466 (2002)

T. Wu *et al.*, *Nature* 477, 191 (2011)

J. Chang *et al.*, *Nat. Phys.* 8, 871 (2012)

D. LaBoeuf *et al.* *Nat. Phys.* 9, 79 (2013)

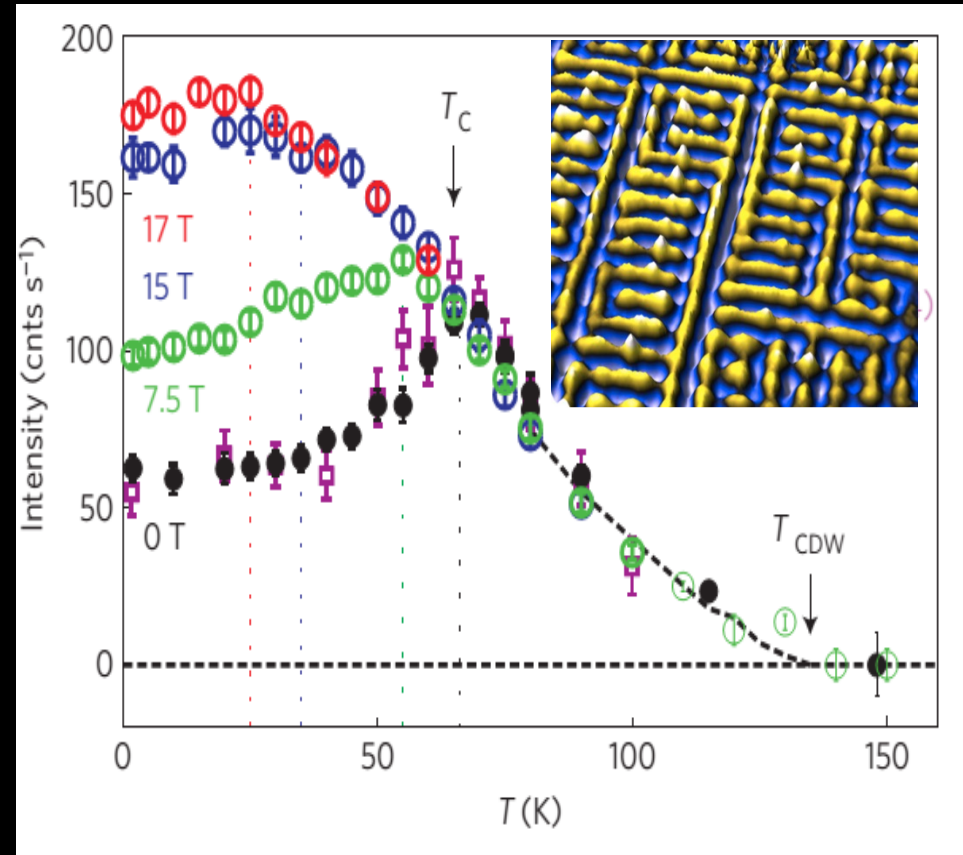
S. Blanco Canosa *et al.* *PRL* 110, 187001 (2013)

T. Wu *et al.* *Nat Comm.* 6, 1 (2015)

S. Gerber *et al.* *Science* 350, 949 (2015)

J. Chang *et al.* *Nature Comm.* 7, 11494 (2016)

H. Jang *et al.* *PNAS* 113 14645–14650 (2016)



Chang *et al.* *Nat Phys* 8, 871 (2012)

$$\text{CDW } \langle c_{k\uparrow}^\dagger, c_{k+Q_C\downarrow} \rangle \quad \text{or} \quad \text{PDW } \langle c_{k\uparrow}^\dagger, c_{-k+Q_P\downarrow} \rangle \quad ?$$

PDW @ VORTEX HALO $\Rightarrow Q_P$ & $2Q_P$ CHARGE DENSITY MODULATIONS

PHYSICAL REVIEW B 91, 104512 (2015)

Checkerboard order in vortex cores from pair-density-wave superconductivity

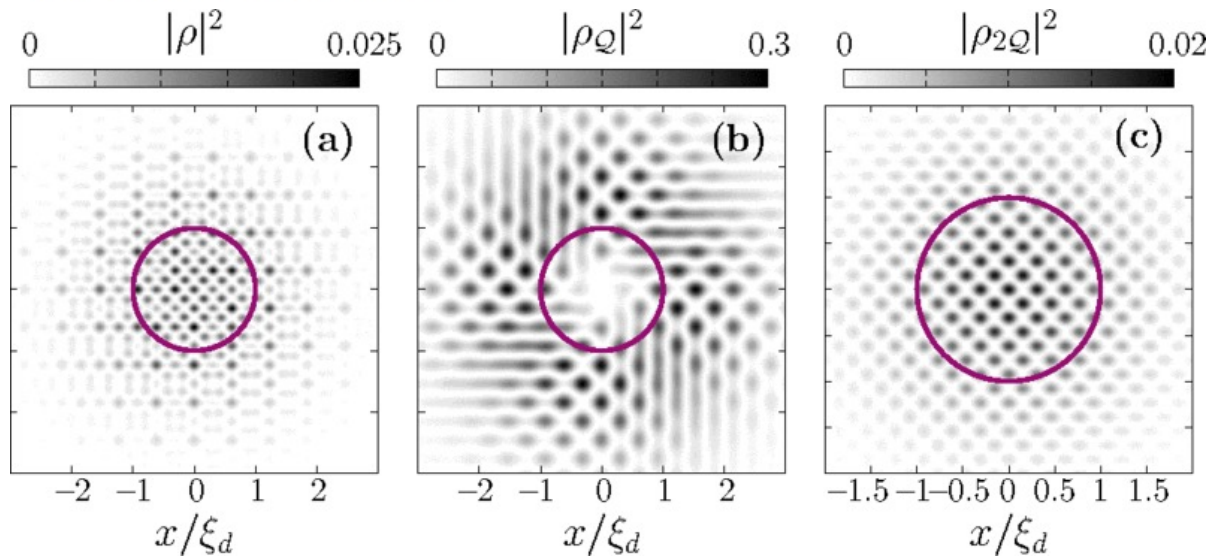
Daniel F. Agterberg¹ and Julien Garaud^{2,*}

¹*Department of Physics, University of Wisconsin-Milwaukee, Milwaukee, Wisconsin 53211, USA*

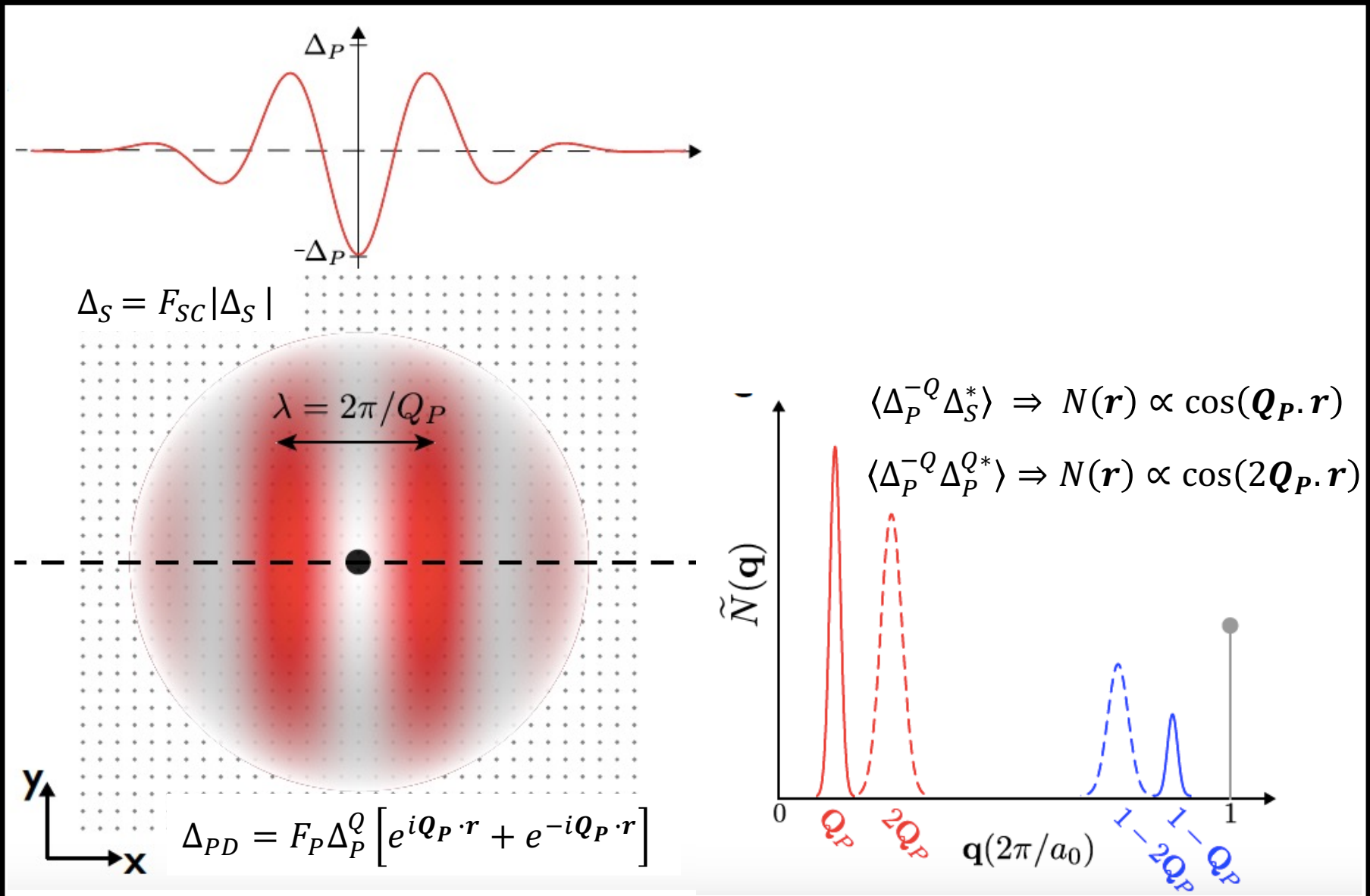
²*Department of Theoretical Physics, KTH-Royal Institute of Technology, Stockholm, SE-10691 Sweden*

(Received 16 December 2014; revised manuscript received 26 February 2015; published 16 March 2015)

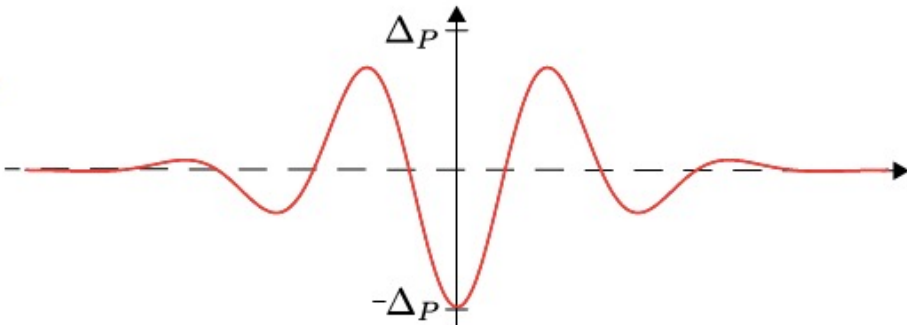
We consider competing pair-density-wave (PDW) and d -wave superconducting states in a magnetic field. We show that PDW order appears in the cores of d -wave vortices, driving checkerboard charge-density-wave (CDW) order in the vortex cores, which is consistent with experimental observations. Furthermore, we find an additional CDW order that appears on a ring outside the vortex cores. This CDW order varies with a period that is twice that of the checkerboard CDW and it only appears where both PDW and d -wave order coexist. The observation of this additional CDW order would provide strong evidence for PDW order in the pseudogap phase of the cuprates. We further argue that the CDW seen by nuclear magnetic resonance at high fields is due to a PDW state that emerges when a magnetic field is applied.



PDW @ VORTEX HALO $\Rightarrow Q_P$ & $2Q_P$ CHARGE DENSITY MODULATIONS



PDW IN VORTEX HALO \Rightarrow EXPECTED PHENOMENOLOGY



$$\Delta_S = F_{SC} |\Delta_S|$$

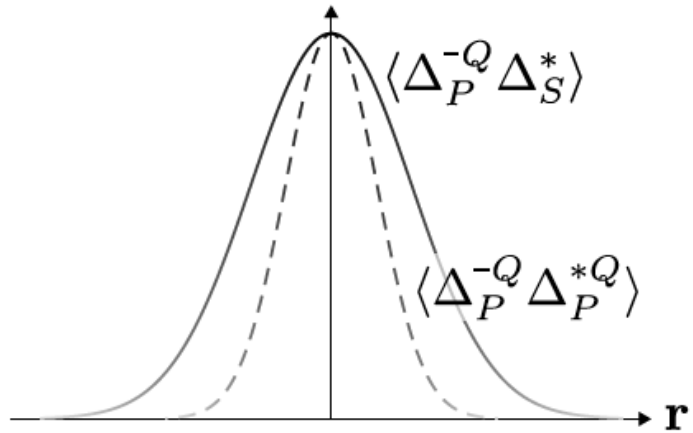
$$\Delta_P^Q = F_P |\Delta_P| \left[e^{iQ_P \cdot r} + e^{-iQ_P \cdot r} \right]$$

PDW Q_P WITHIN VORTEX HALO

Q_P & $2Q_P$ CHARGE DENSITY MODULATIONS

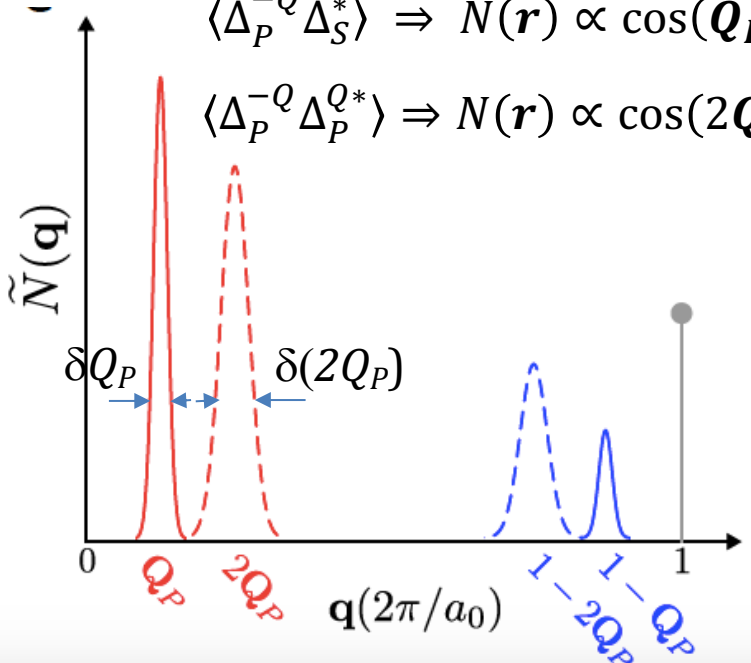
d -SYMMETRY PDW \Rightarrow s -SYMMETRY MODS

MODULATION LINEWIDTHS: $\delta(2Q_P) = 2(\delta Q_P)$

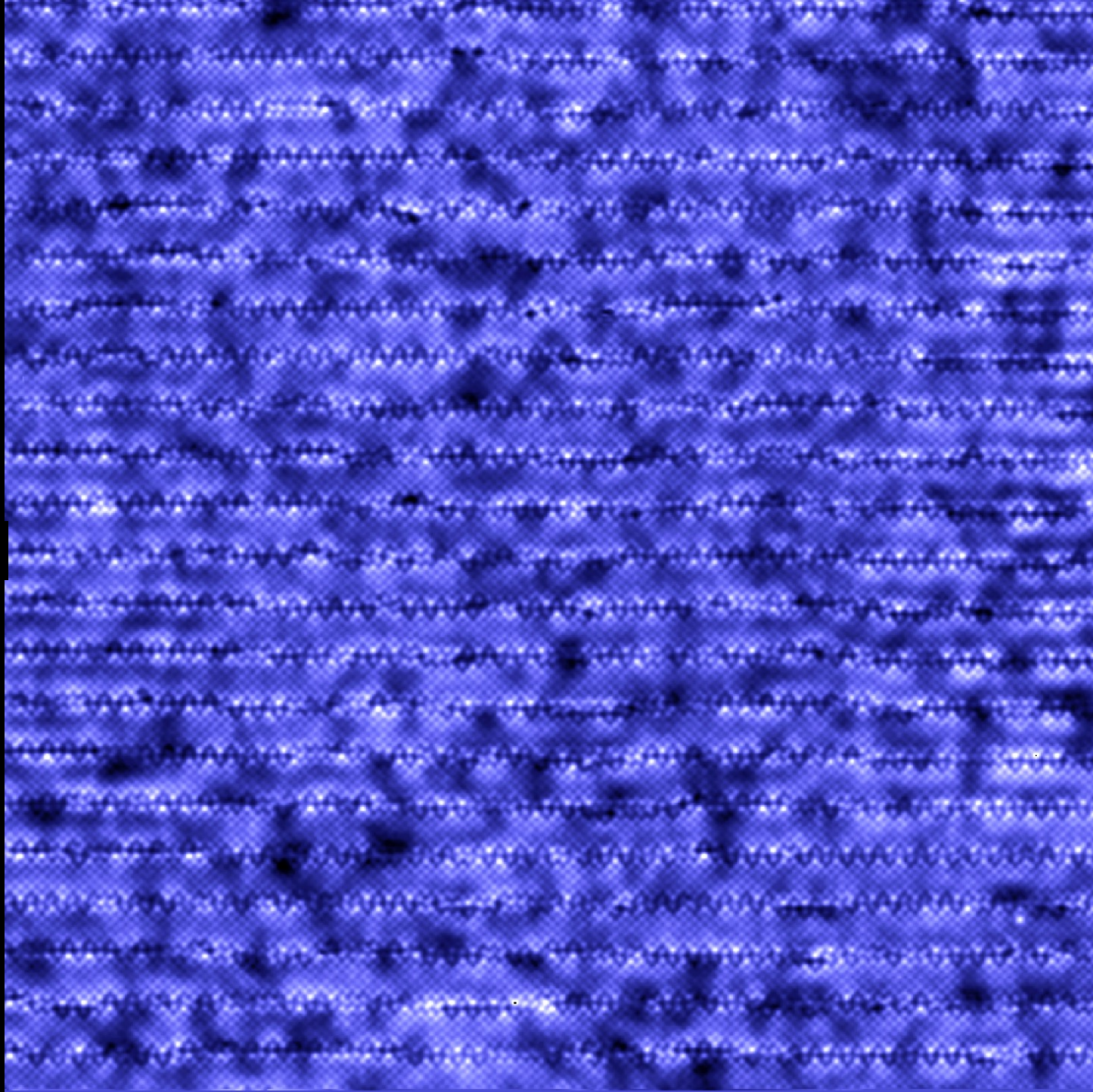


$$\langle \Delta_P^{-Q} \Delta_S^* \rangle \Rightarrow N(\mathbf{r}) \propto \cos(Q_P \cdot \mathbf{r})$$

$$\langle \Delta_P^{-Q} \Delta_P^{*Q} \rangle \Rightarrow N(\mathbf{r}) \propto \cos(2Q_P \cdot \mathbf{r})$$



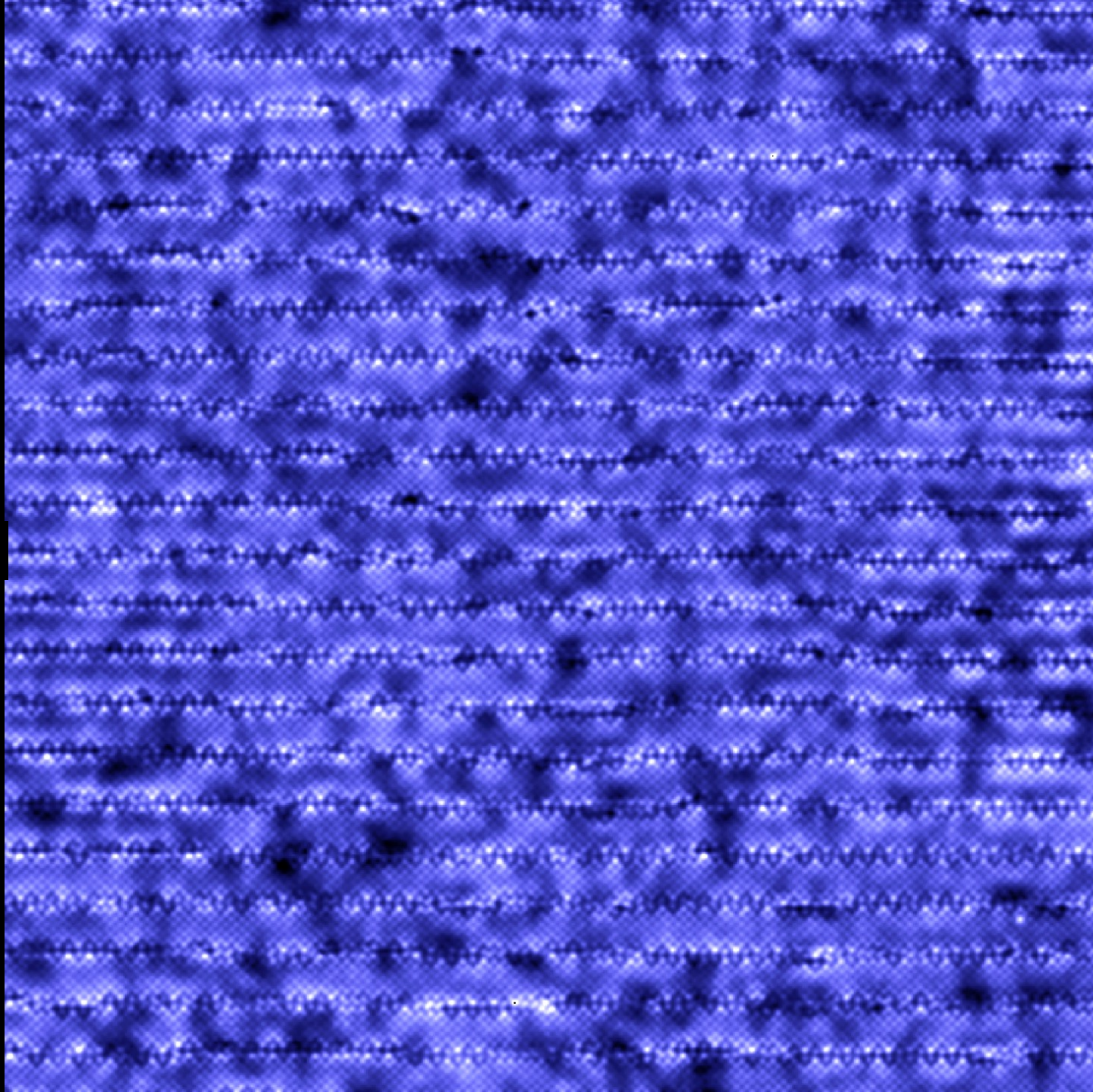
$T(r, B=0T)$



580 Å

$\sim 1K$

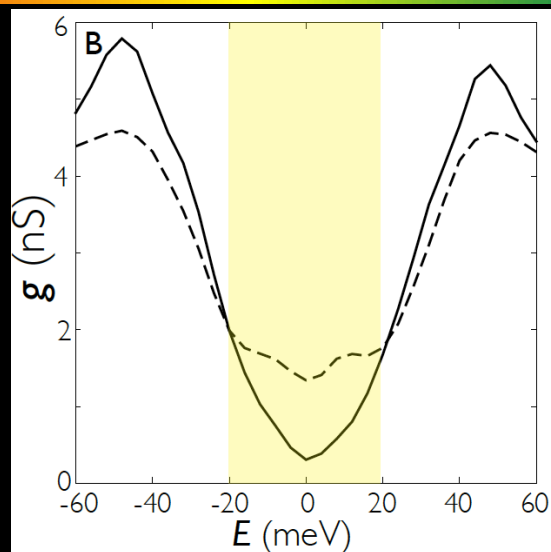
$T(r, B=8.5T)$



580 Å

~1K

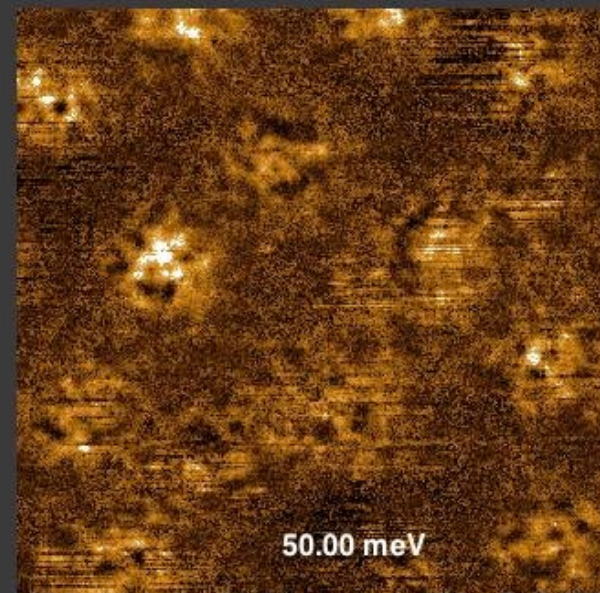
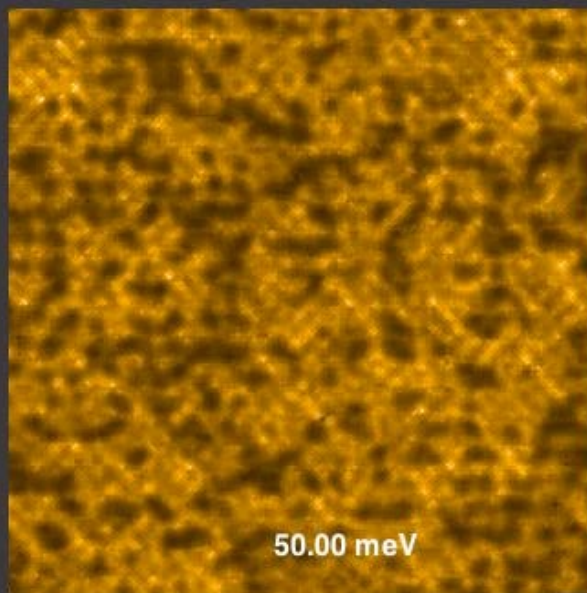
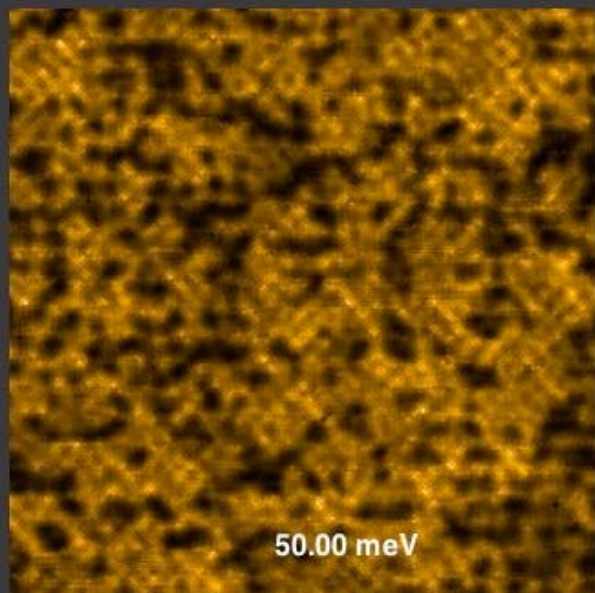
FIELD-INDUCED CHANGES TO ELECTRONIC STRUCTURE @ HALO



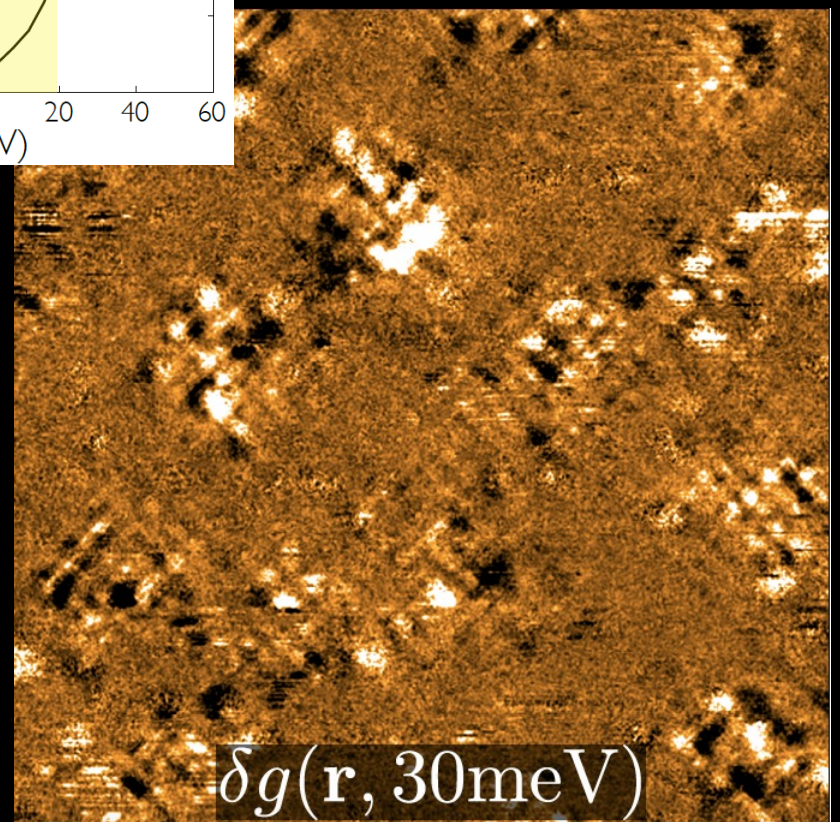
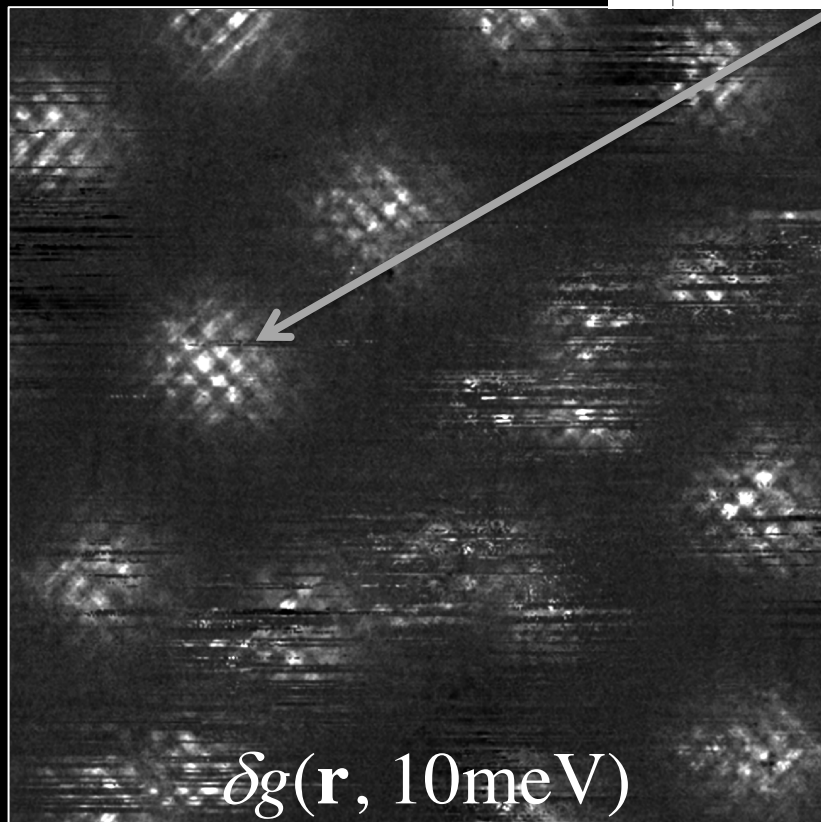
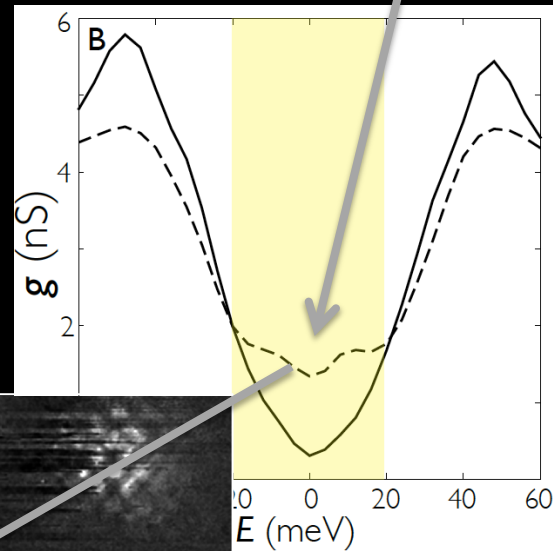
$g(B=8.5T)$

$g(B=0T)$

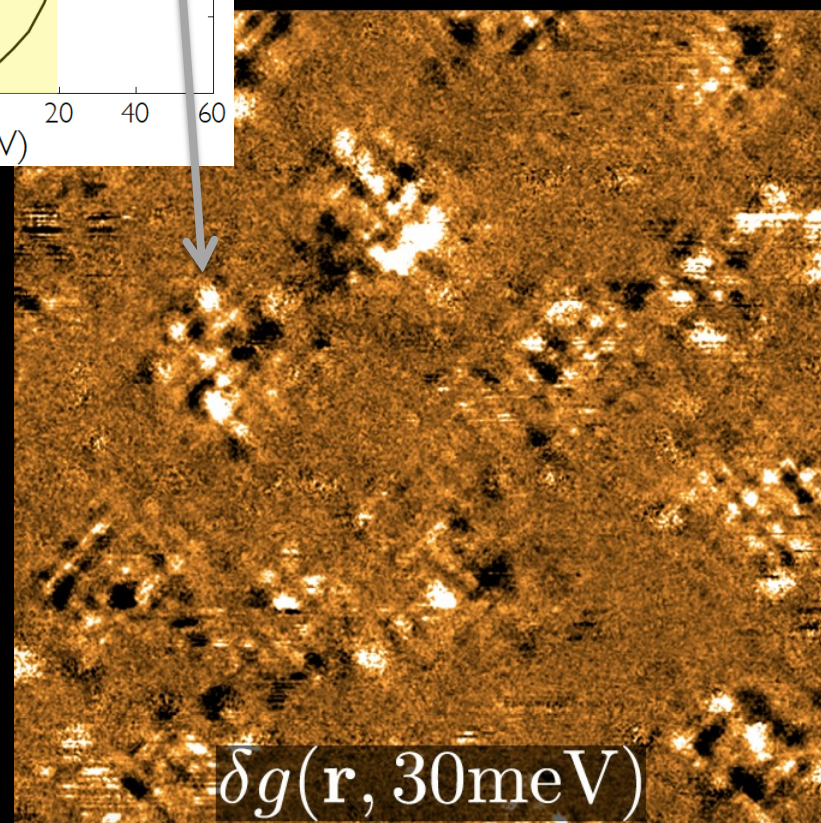
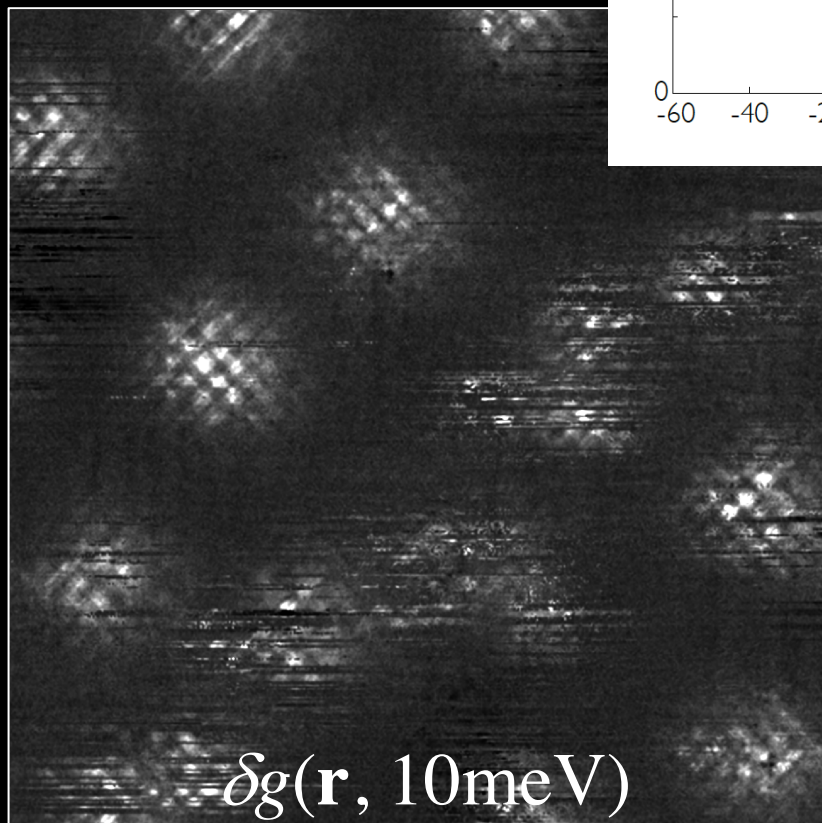
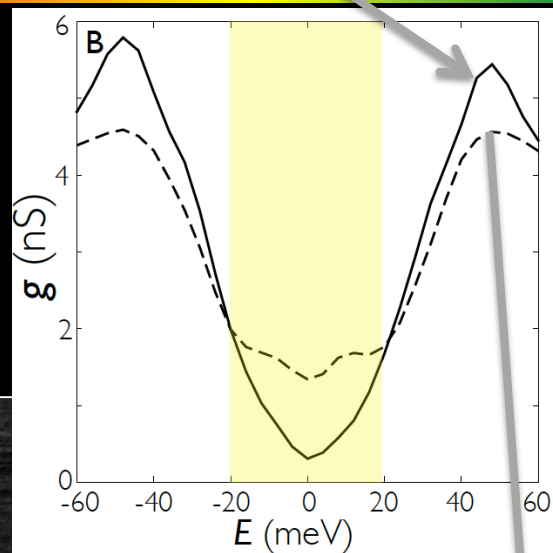
δg



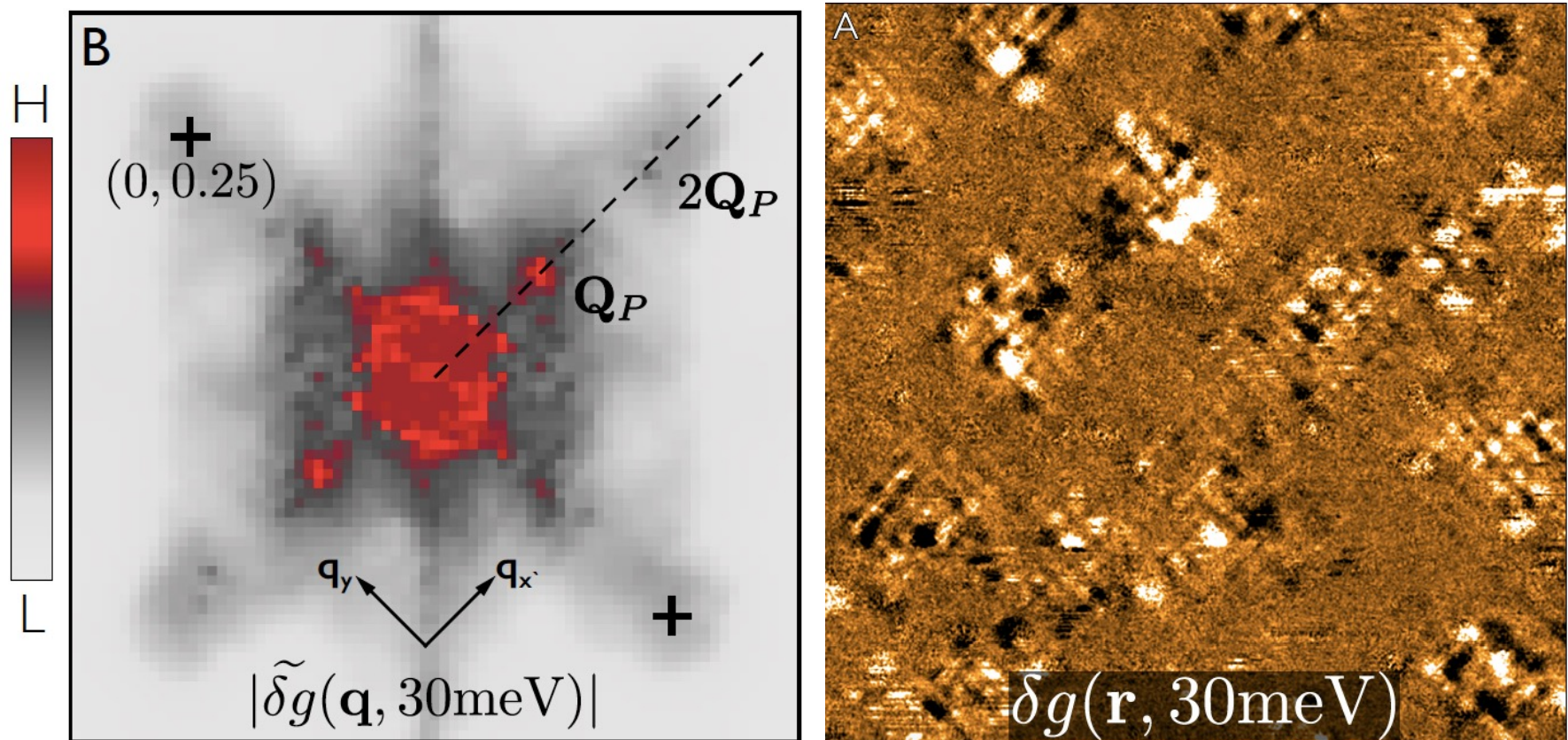
NODAL BOGOLIUBOV QUASIPARTICLES



GAP EDGE STATES

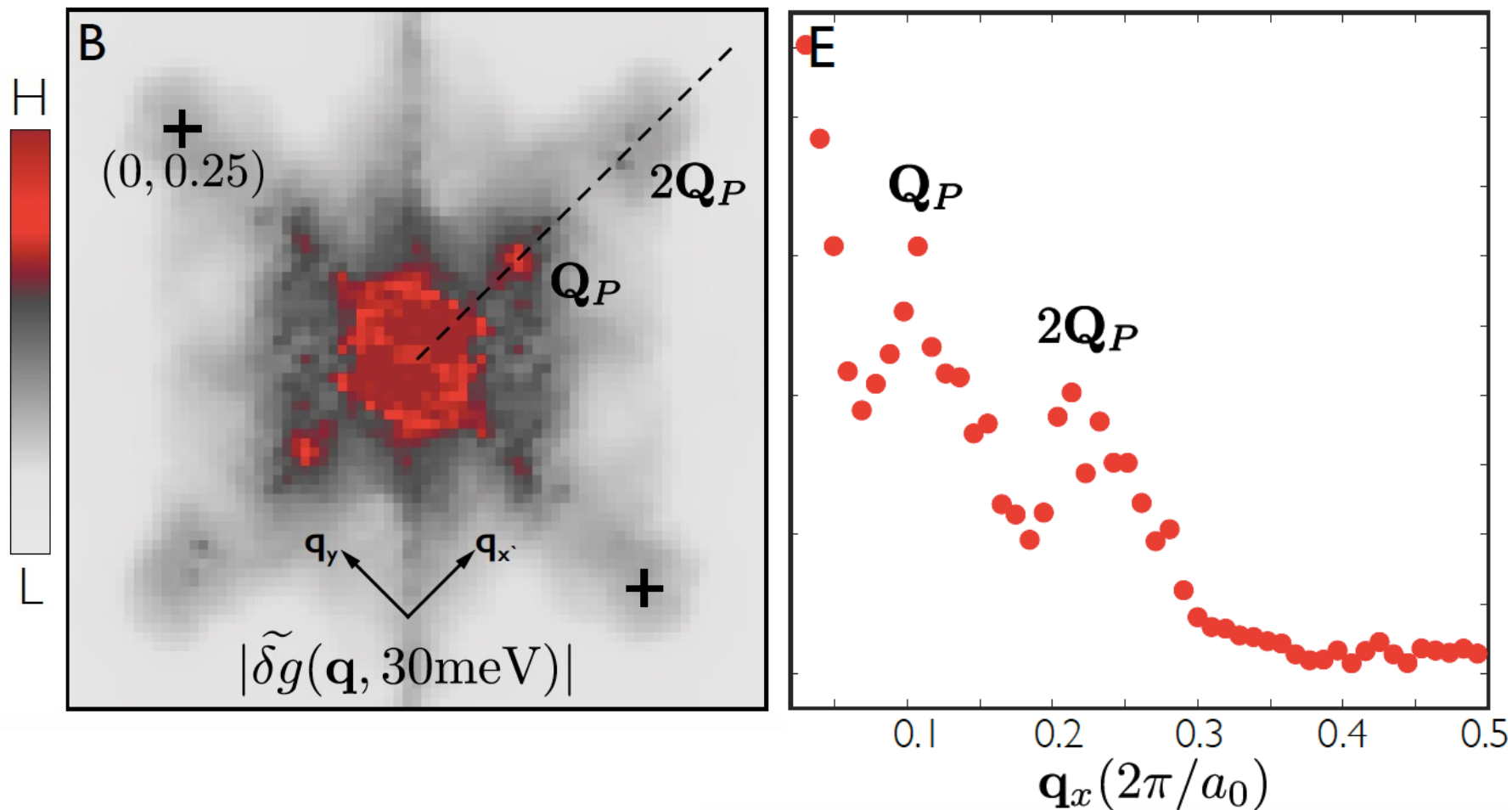


FIELD-INDUCED $N(\mathbf{r}, E \sim \Delta)$ MODULATIONS



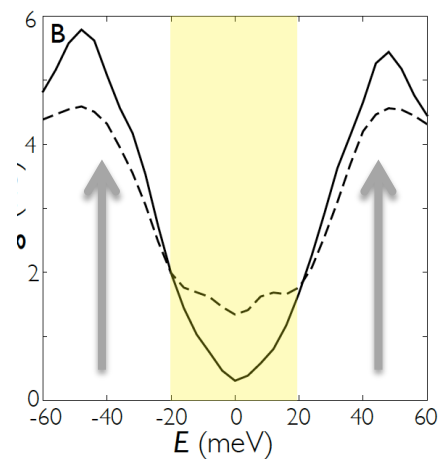
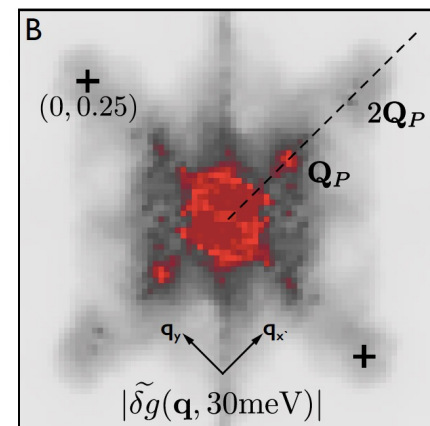
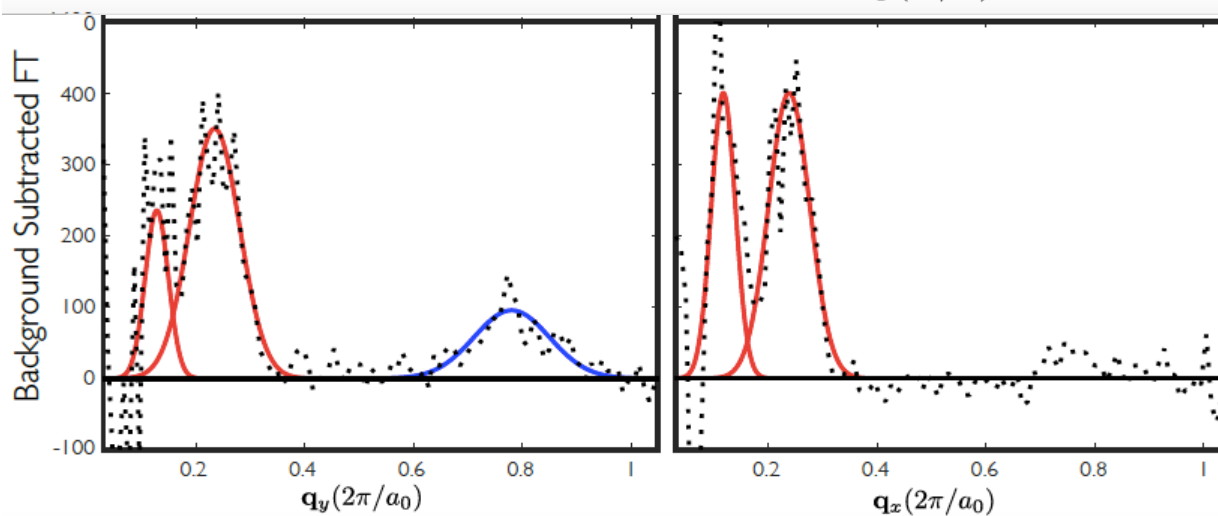
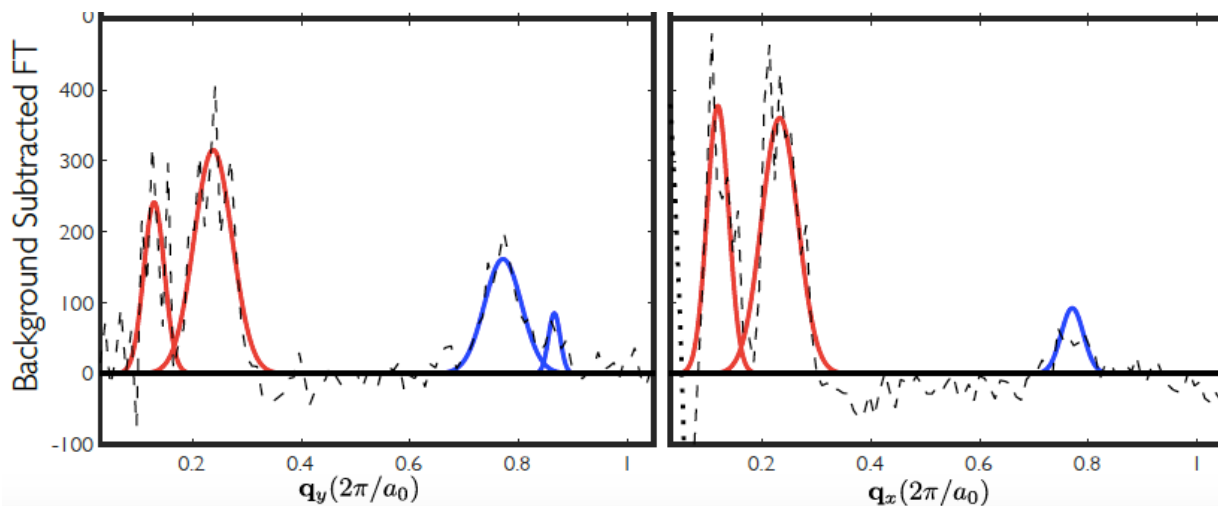
Science 364, 976 (2019)

FIELD-INDUCED $N(\mathbf{r}, E \sim \Delta)$ MODULATIONS : PEAKS at Q_P and $2Q_P$

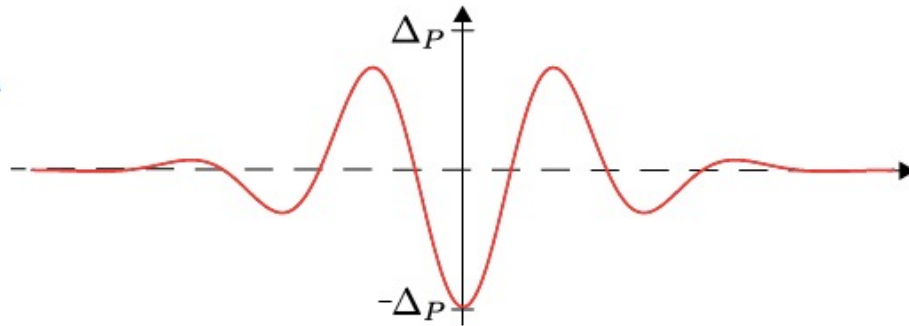


Science 364, 976 (2019)

FIELD-INDUCED $N(r, E \sim \Delta)$ MODS Q_P AND $2Q_P \rightleftharpoons \delta(2Q_P) \approx 2\delta(Q_P)$



VORTEX HALO \Rightarrow OBSERVED PHENOMENOLOGY



$$\Delta_S = F_{SC} |\Delta_P|$$

$$\Delta_{PD} = F_P \Delta_P^Q \left[e^{iQ_P \cdot r} + e^{-iQ_P \cdot r} \right]$$

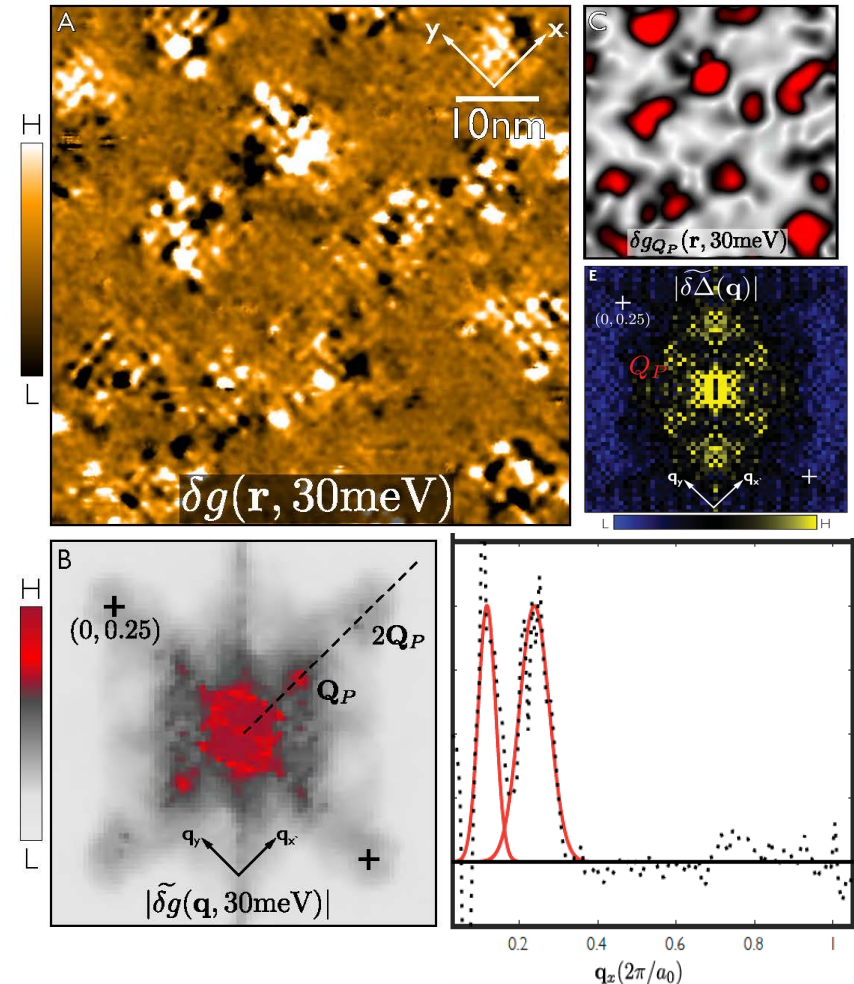
PDW Q_P WITHIN VORTEX HALO

Q_P & $2Q_P$ CHARGE DENSITY MODULATIONS

d -SYMMETRY PDW \Rightarrow s -SYMMETRY MODS

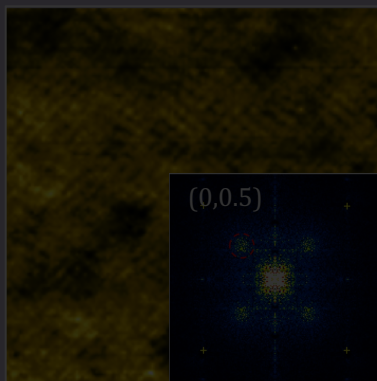
MODULATION LINEWIDTHS: $\delta(2Q_P) = 2(\delta Q_P)$

8a0 PERIODIC ENERGY GAP MODULATIONS



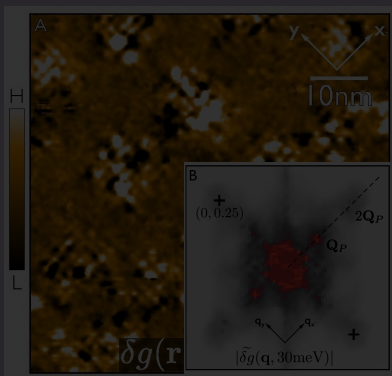
CuO₂ ELECTRON-PAIR VISUALIZATION STUDIES

PAIR DENSITY



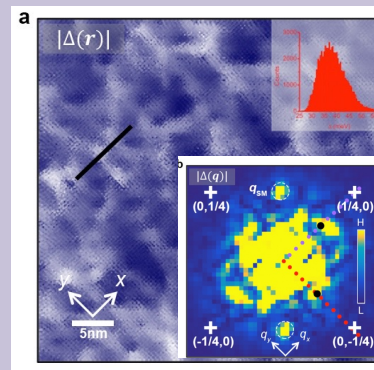
Nature 532, 343 (2016)

VORTEX HALO



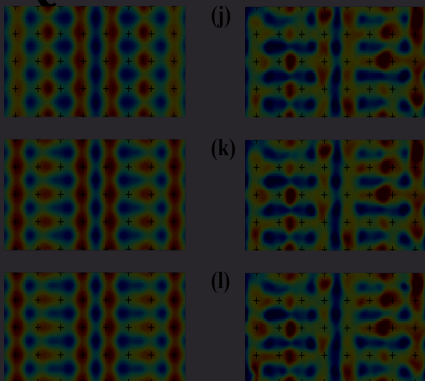
Science 364, 976 (2019)

ENERGY GAP



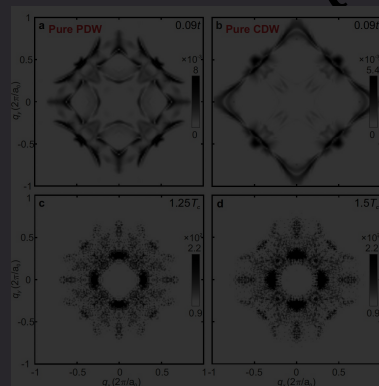
Nature 580, 6570 (2020)

QUASIPARTICLES



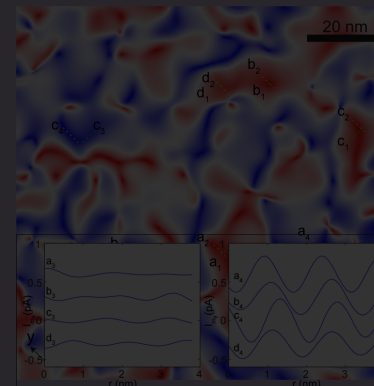
PNAS 117, 14850 (2020)

PSEUDOGAP QPI

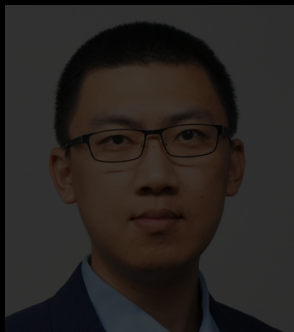


Nat.Comm 12, 6087 (2021)

NEMATIC



Weijiong Chen *et al* (2022)



Xiaolong Liu
Notre Dame



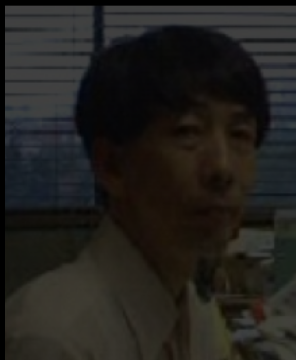
Shuqiu Wang
Oxford



P. Choubey
IIT Delhi



Weijiong Chen
Oxford



S. Uchida
Tokyo



H. Eisaki
Tsukuba



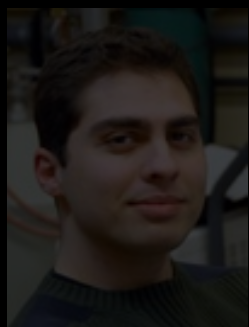
P.D. Johnson
Oxford



Kazuhiro Fujita
BNL



P.J. Hirschfeld
Florida



M. H. Hamidian
Cornell

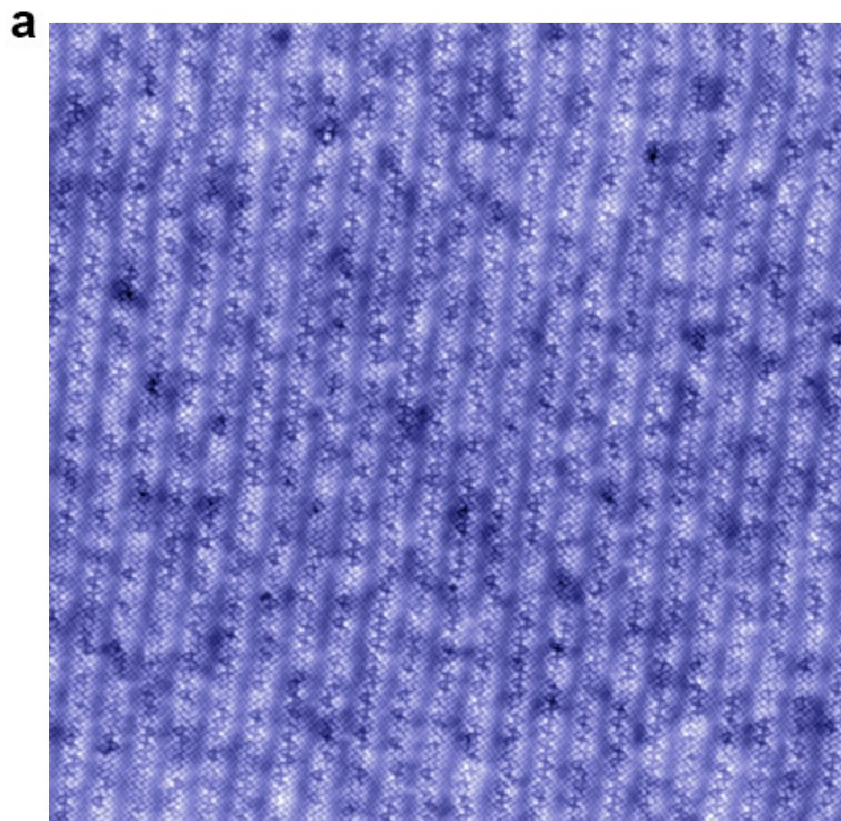
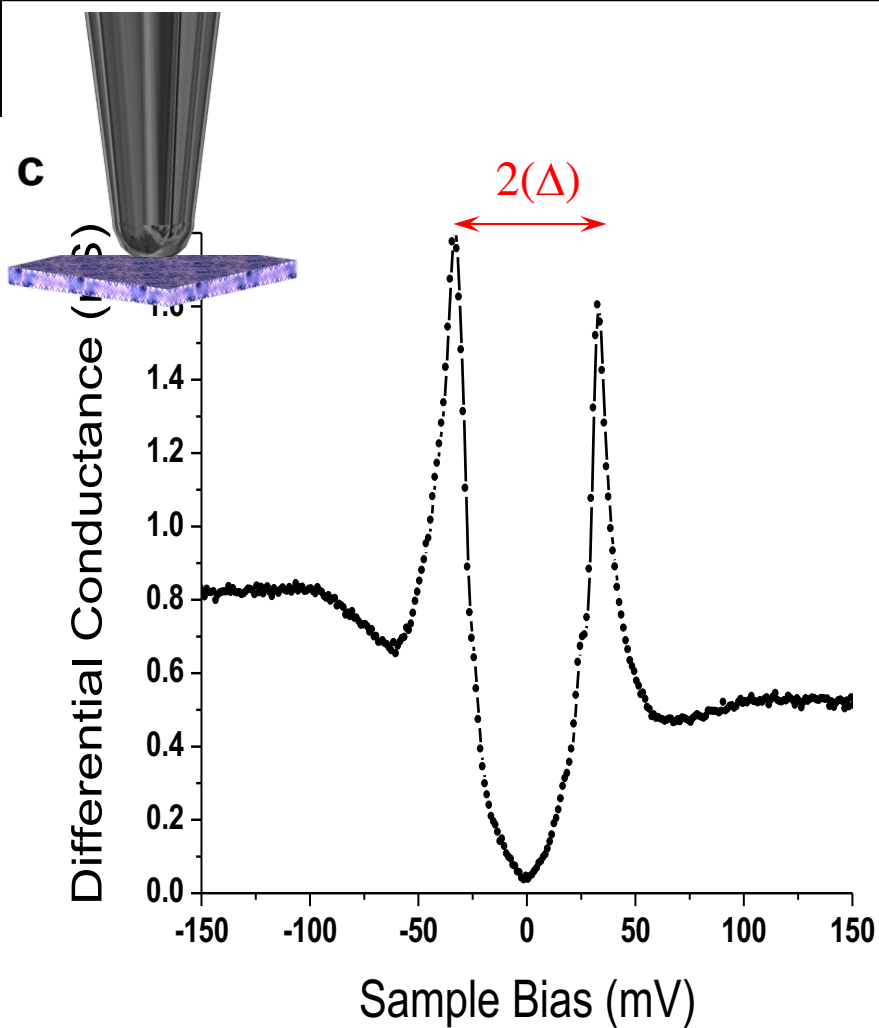


S.D. Edkins
Cornell

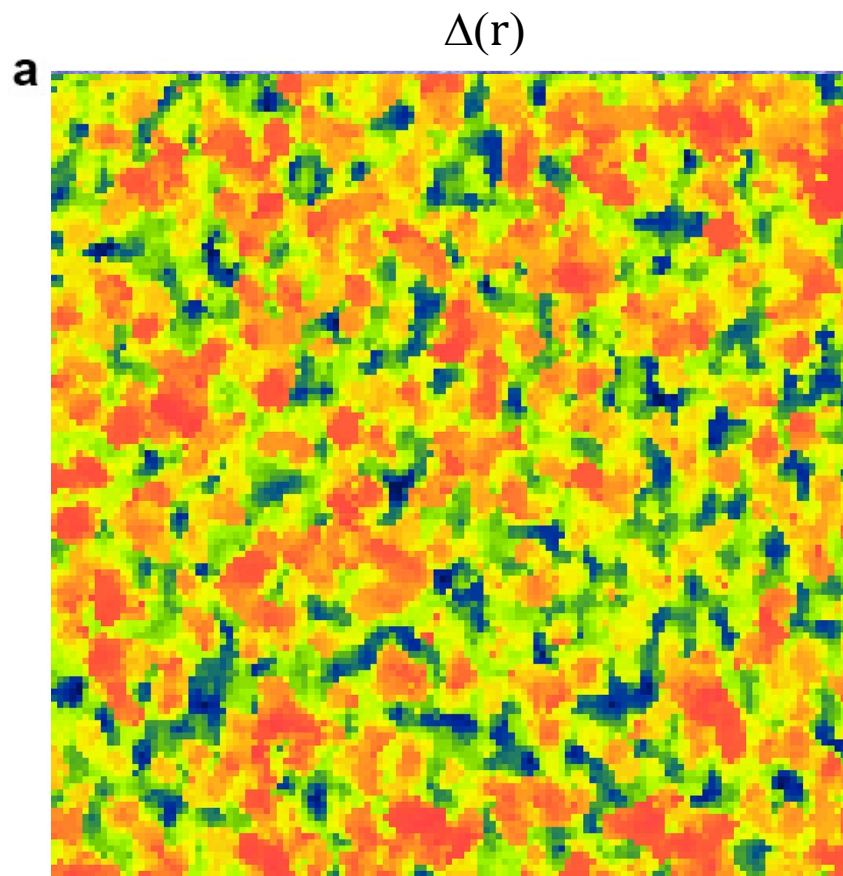
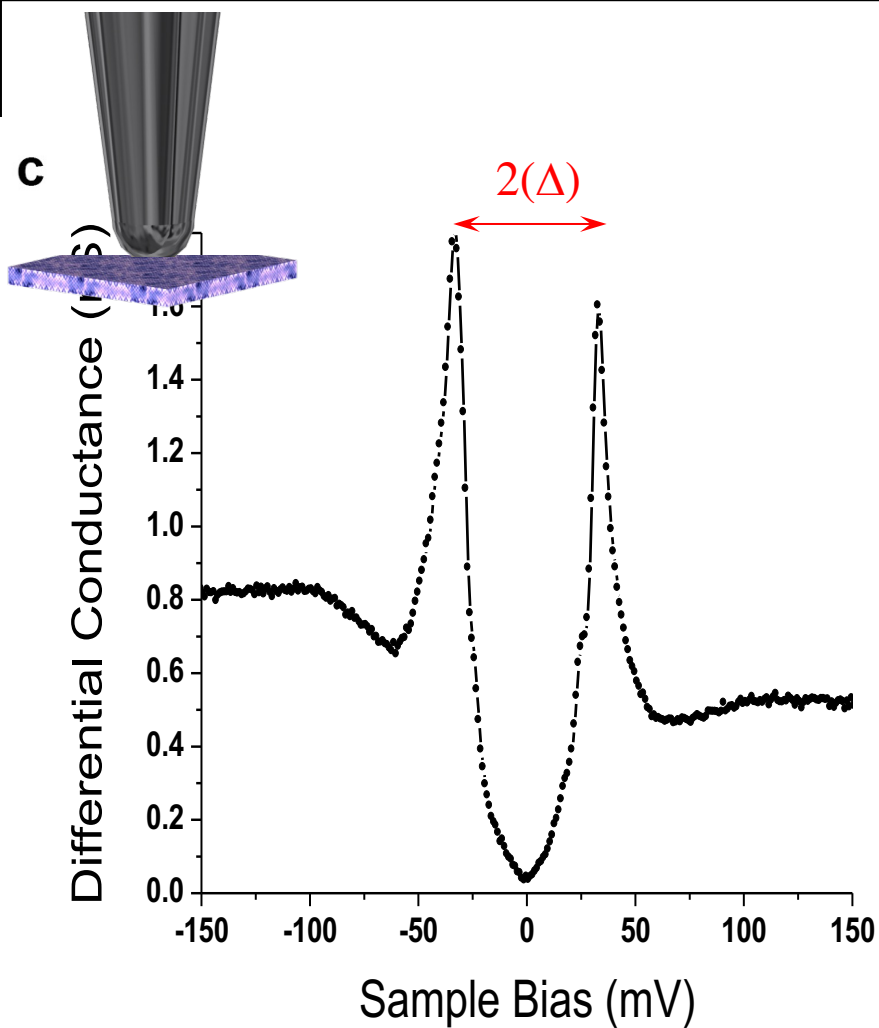


A.P. Mackenzie
Dresden

NORMAL TIP: NIS SINGLE-ELECTRON TUNNELING

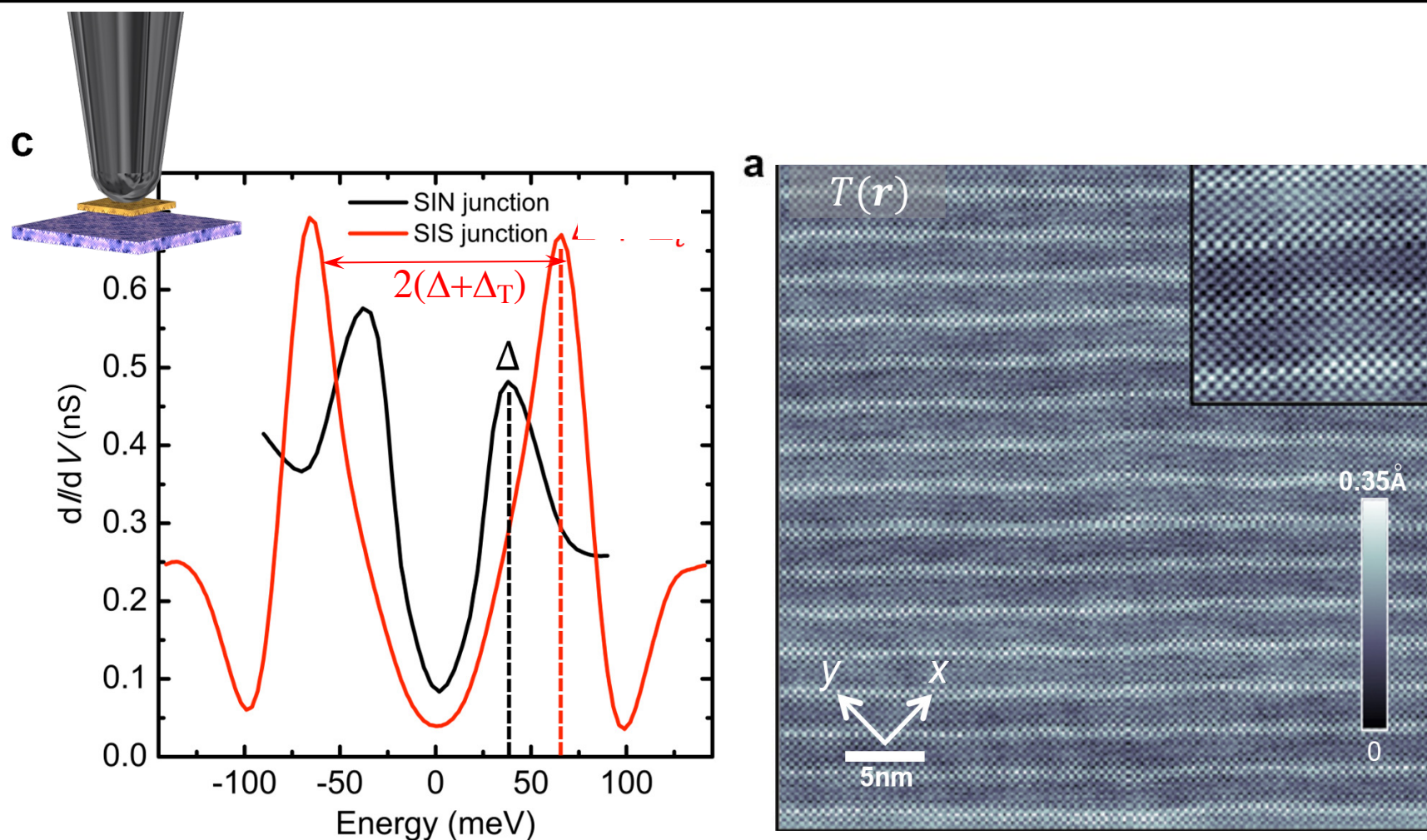


NORMAL TIP: NIS SINGLE-ELECTRON TUNNELING

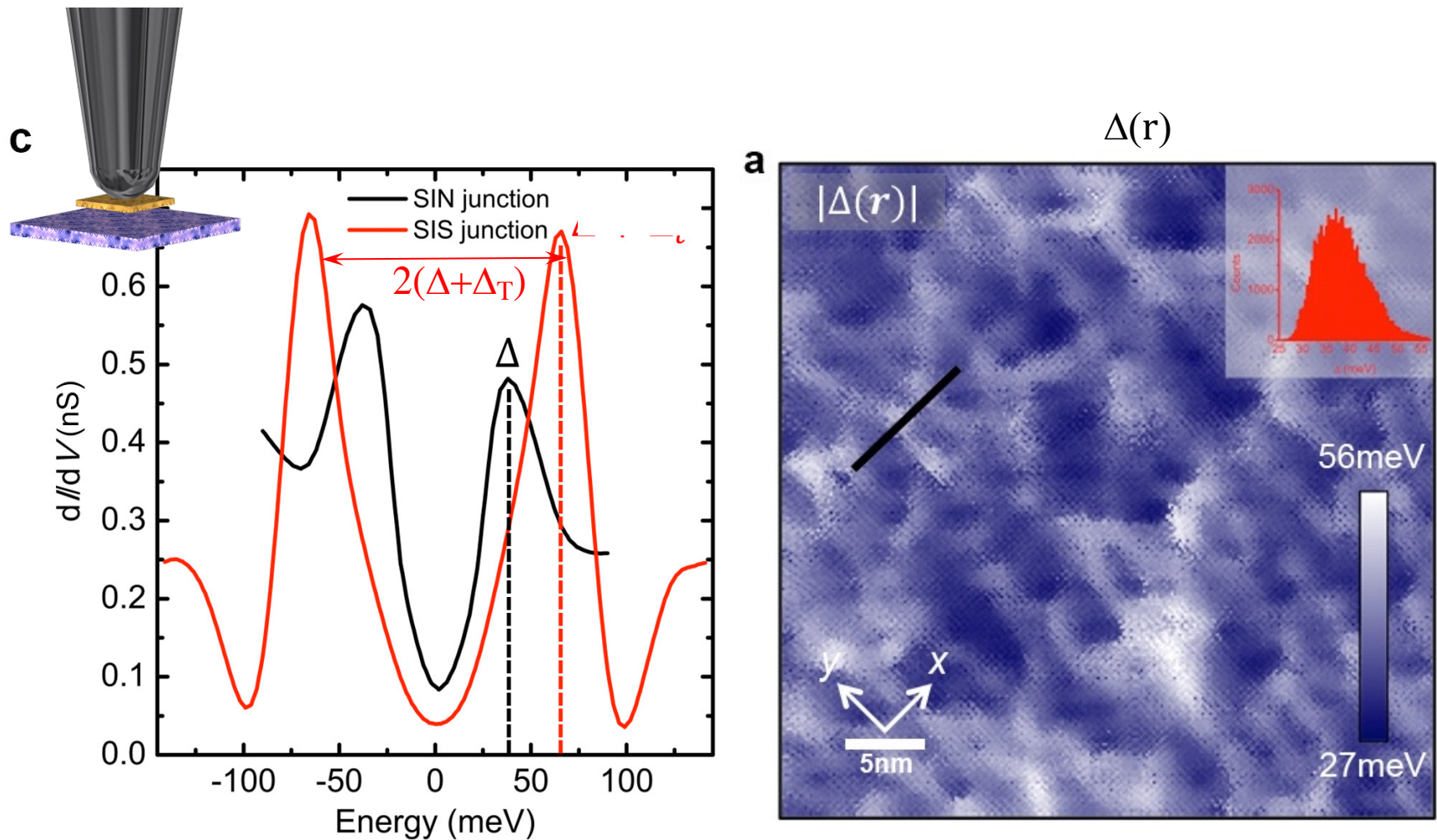


NO ENERGY GAP MODULATIONS OBSERVED

SUPERCOND TIP: SIS SINGLE-ELECTRON TUNNELING

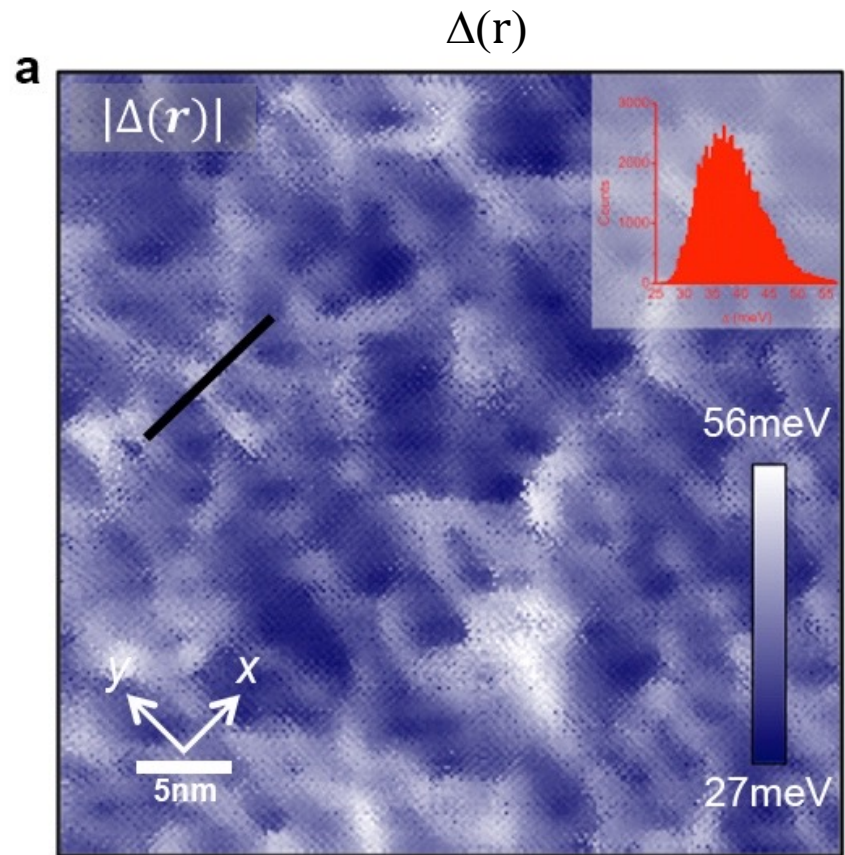
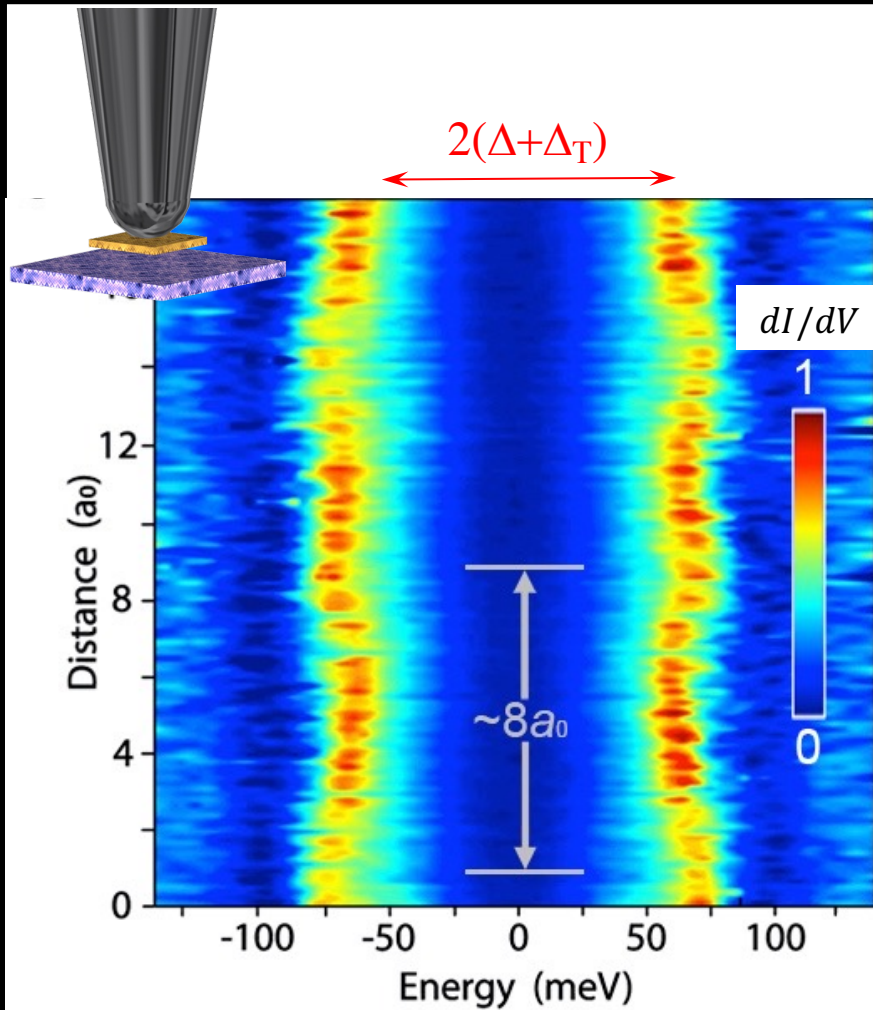


SUPERCOND TIP: SIS SINGLE-ELECTRON TUNNELING



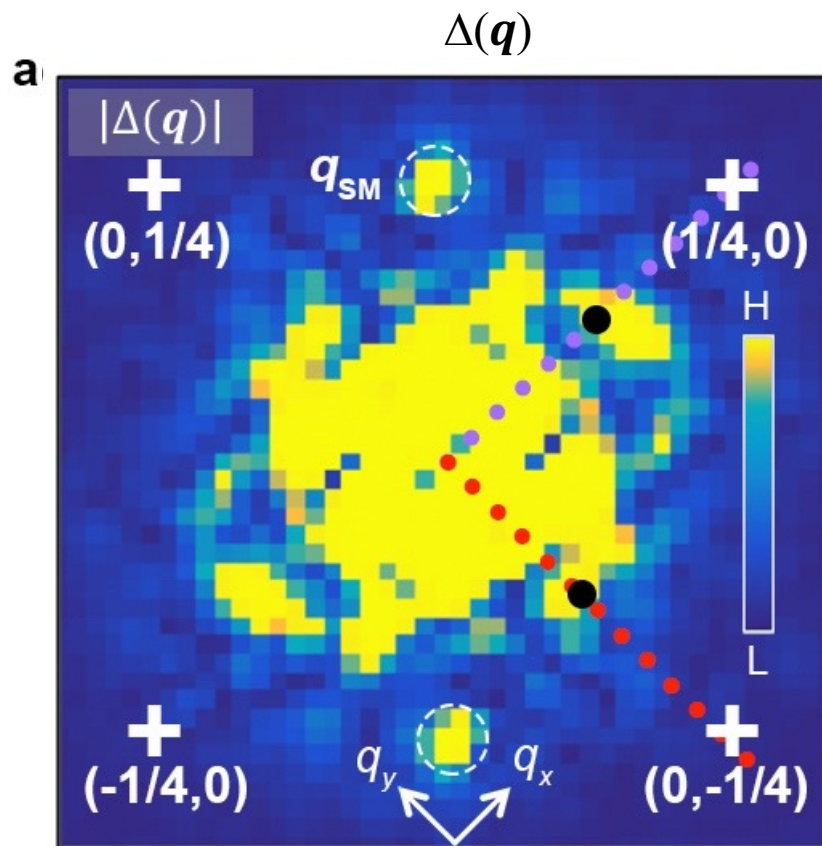
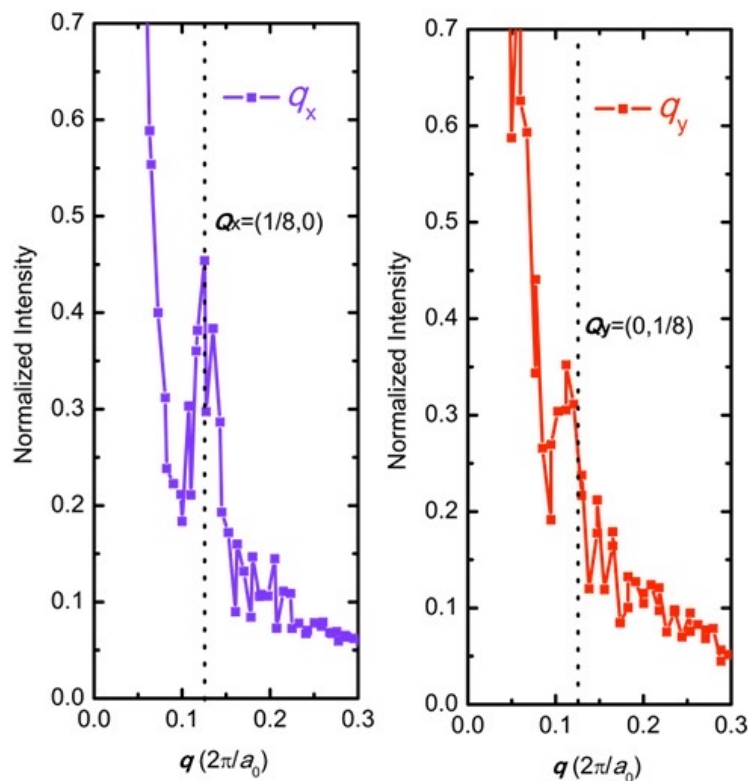
Nature 580, 6570 (2020)

ELECTRON-PAIR ENERGY GAP MODULATION



ELECTRON-PAIR ENERGY GAP MODULATION

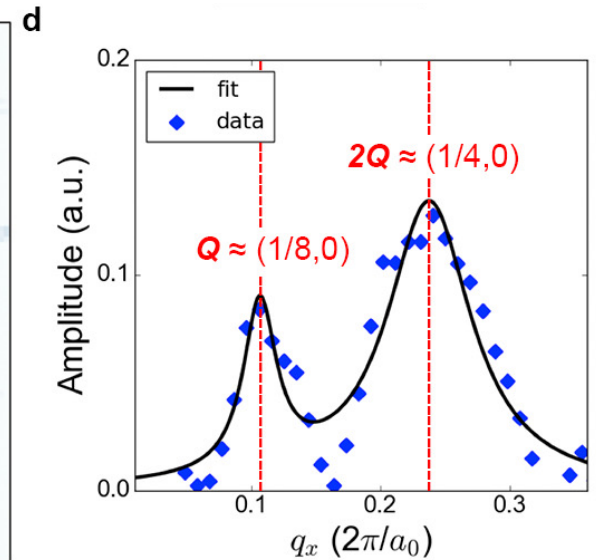
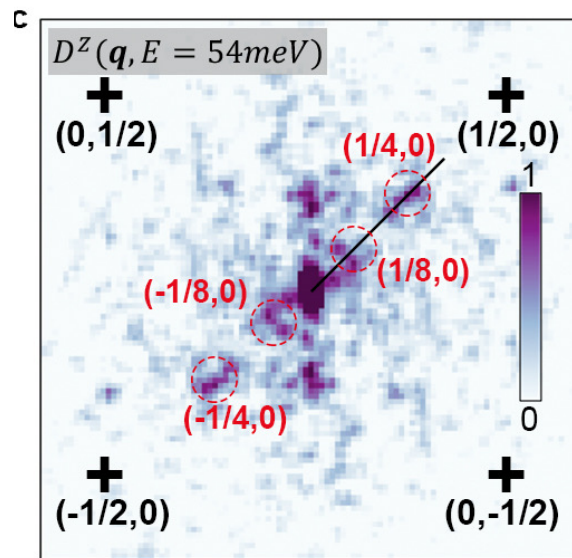
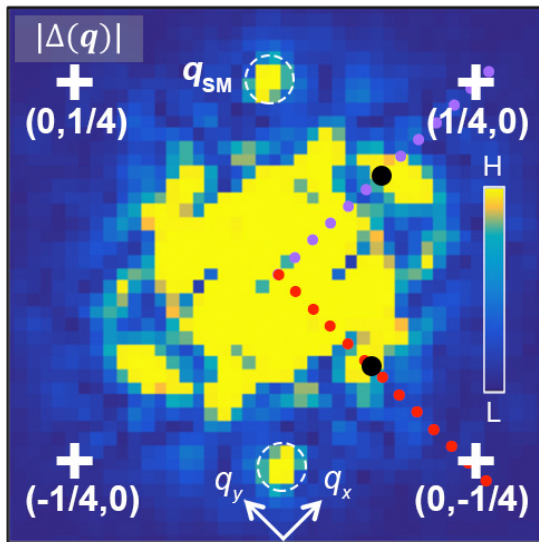
$$Q = (0, 1/8)2\pi/a_0; (1/8, 0)2\pi/a_0$$



CDW APPEARS at Q_P and $2Q_P$

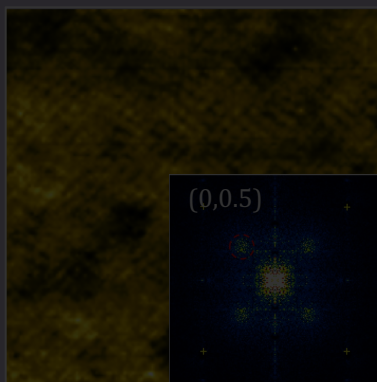
$$\langle \Delta_P^{-Q} \Delta_S^* \rangle \Rightarrow N(\mathbf{r}) \propto \cos(\mathbf{Q}_P \cdot \mathbf{r})$$

$$\langle \Delta_P^{-Q} \Delta_P^{Q*} \rangle \Rightarrow N(\mathbf{r}) \propto \cos(2\mathbf{Q}_P \cdot \mathbf{r})$$



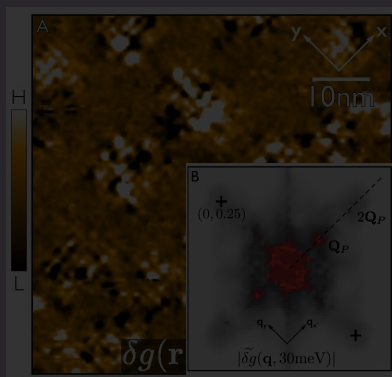
CuO₂ ELECTRON-PAIR VISUALIZATION STUDIES

PAIR DENSITY



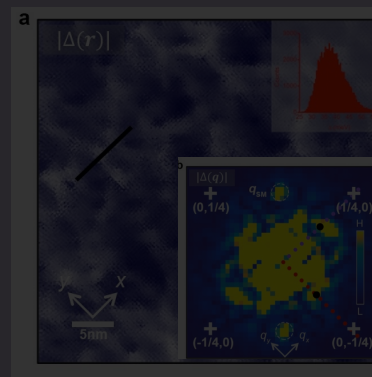
Nature 532, 343 (2016)

VORTEX HALO



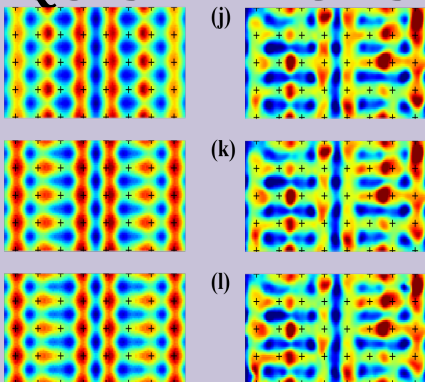
Science 364, 976 (2019)

ENERGY GAP



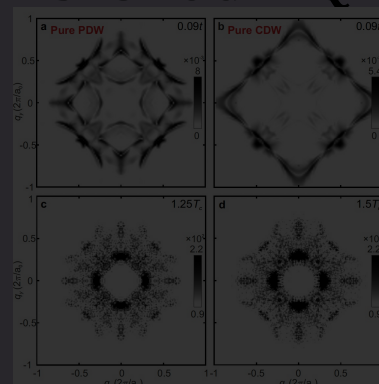
Nature 580, 6570 (2020)

QUASIPARTICLES



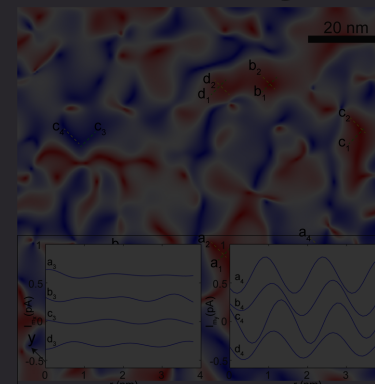
PNAS 117, 14850 (2020)

PSEUDOGAP QPI

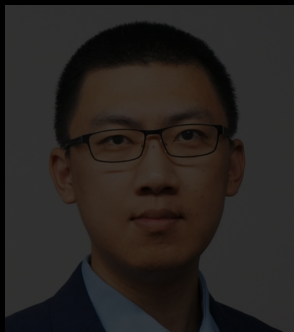


Nat.Comm 12, 6087 (2021)

NEMATIC



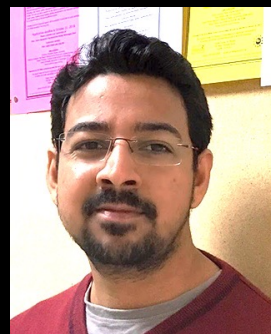
Weijiong Chen *et al* (2022)



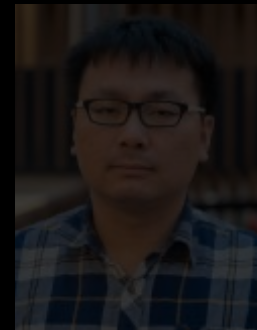
Xiaolong Liu
Notre Dame



Shuqiu Wang
Oxford



P. Choubey
IIT Delhi



Weijiong Chen
Oxford



S. Uchida
Tokyo



H. Eisaki
Tsukuba



P.D. Johnson
Oxford



Kazuhiro Fujita
BNL



P.J. Hirschfeld
Florida



M. H. Hamidian
Cornell



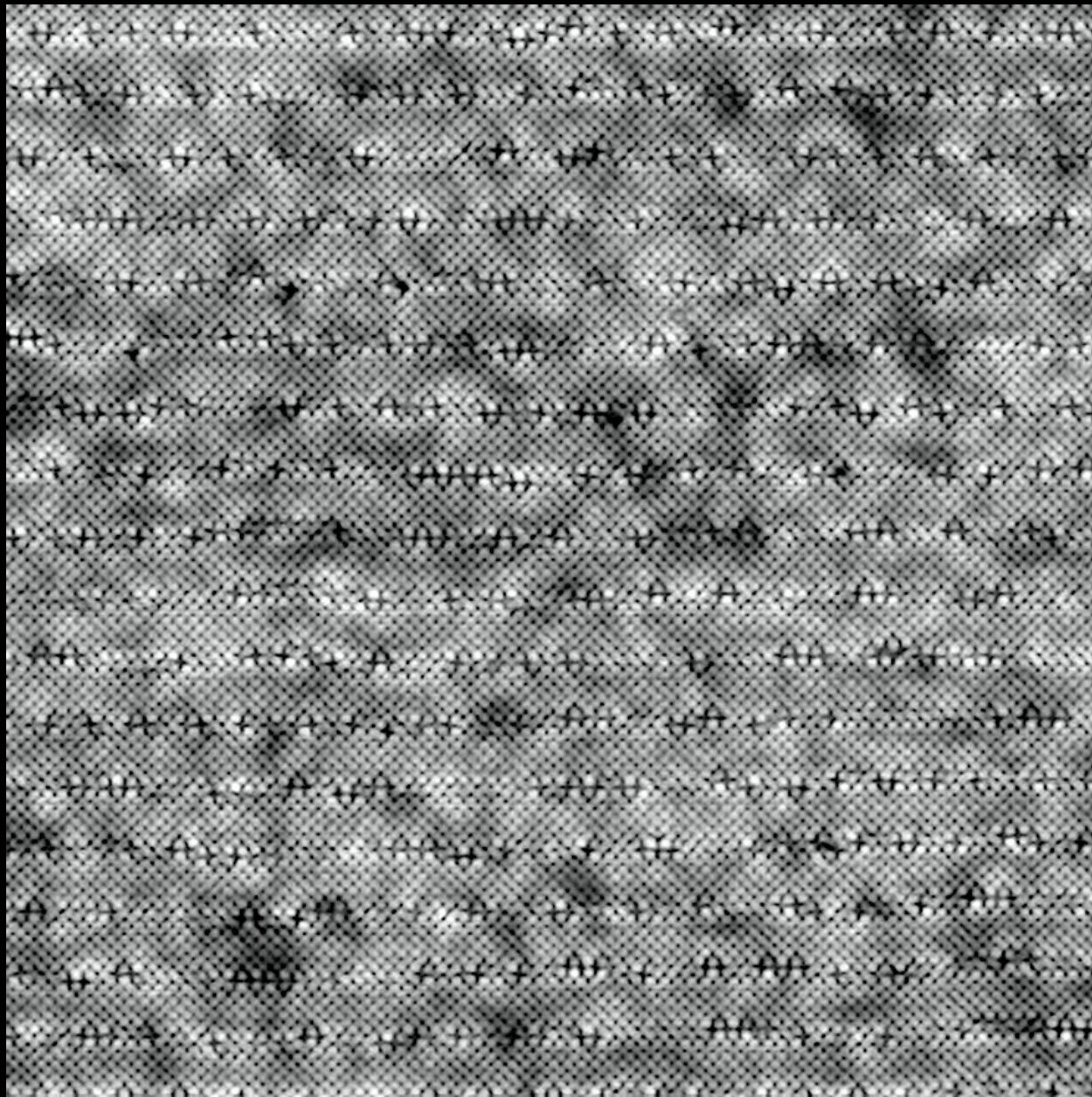
S.D. Edkins
Cornell



A.P. Mackenzie
Dresden

ACTUAL ELECTRONIC STRUCTURE of $p \sim 10\%$ CuO_2

$p \sim 10\%$

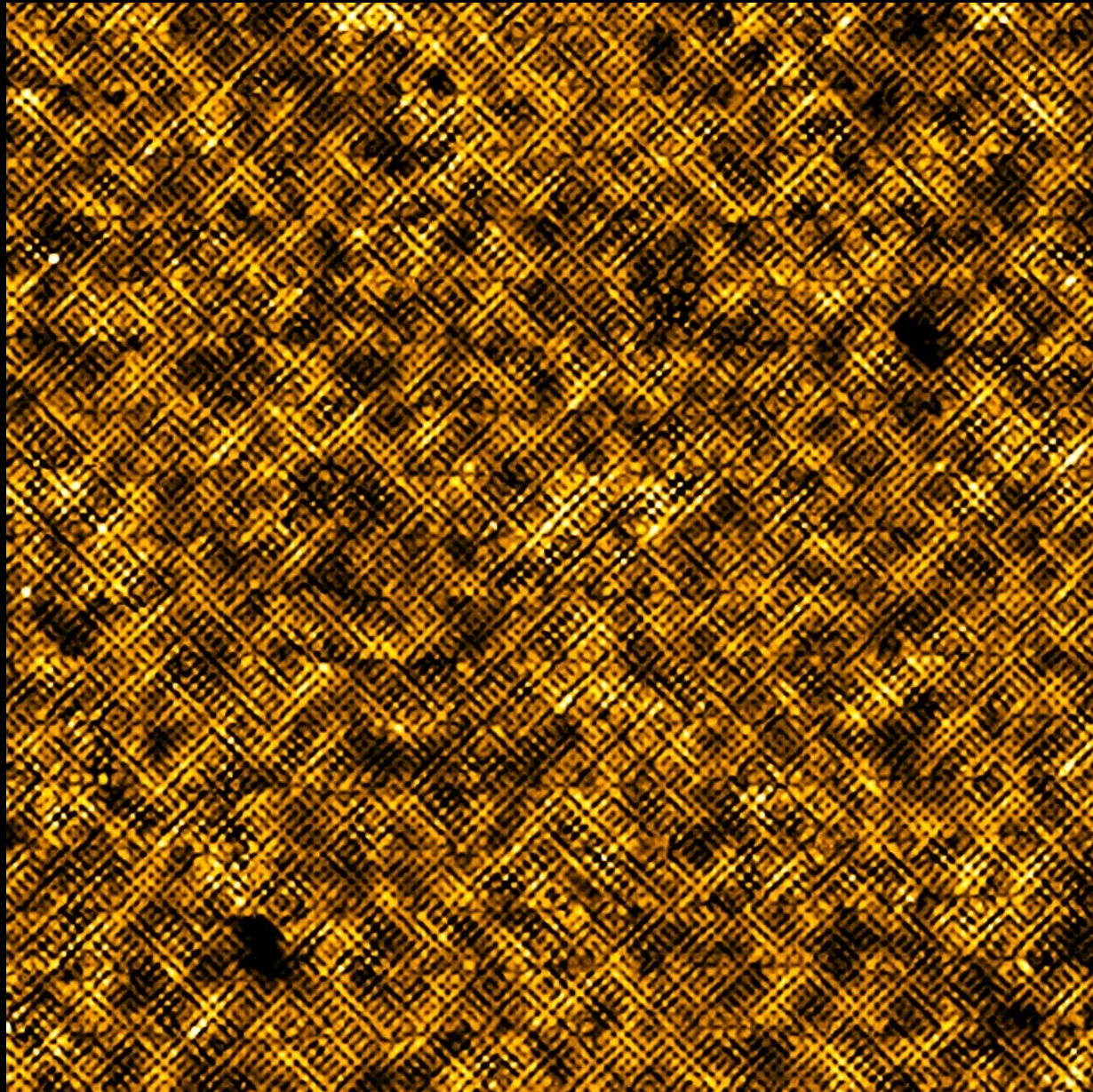


45 nm

ACTUAL ELECTRONIC STRUCTURE of $p \sim 10\%$ CuO_2

$$\frac{dI}{dV}(\mathbf{r}, \Delta_1)$$

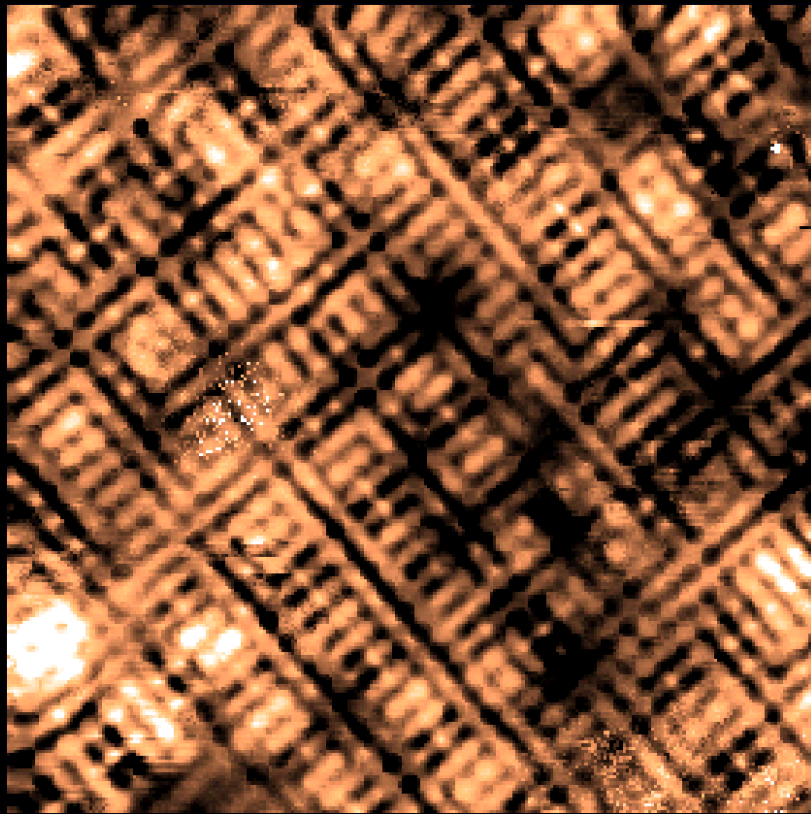
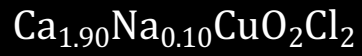
$p \sim 10\%$



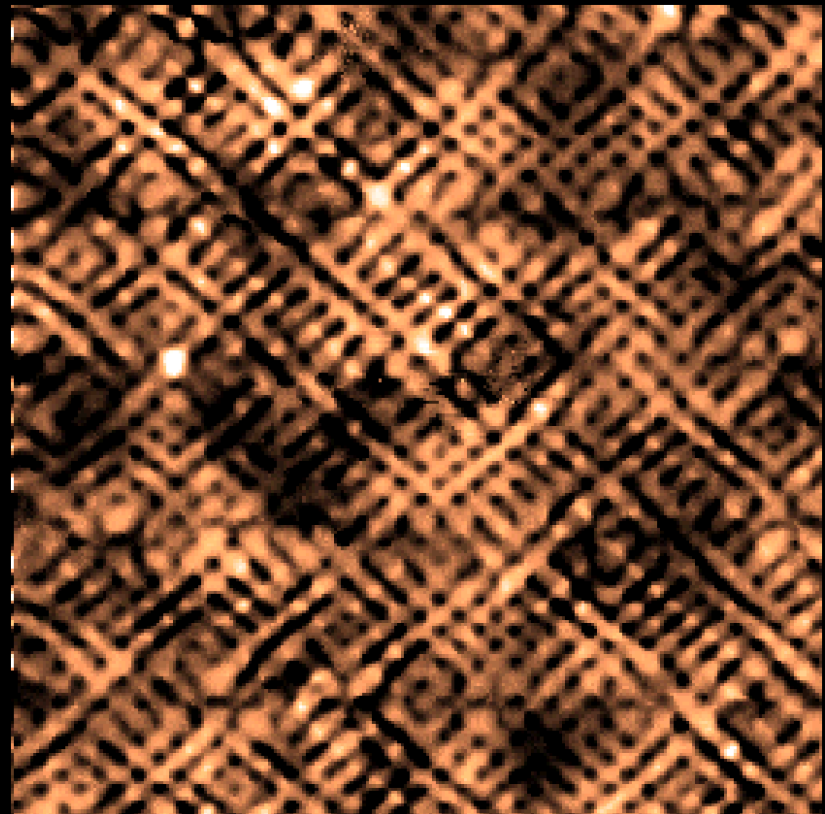
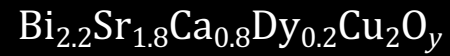
45 nm

“An Intrinsic
Bond-Centered
Electronic Glass...”
Science **315**,
138 (2007)

BOND-CENTERED d -SYMMETRY FF UNIDIRECTIONAL 'MOTIF'



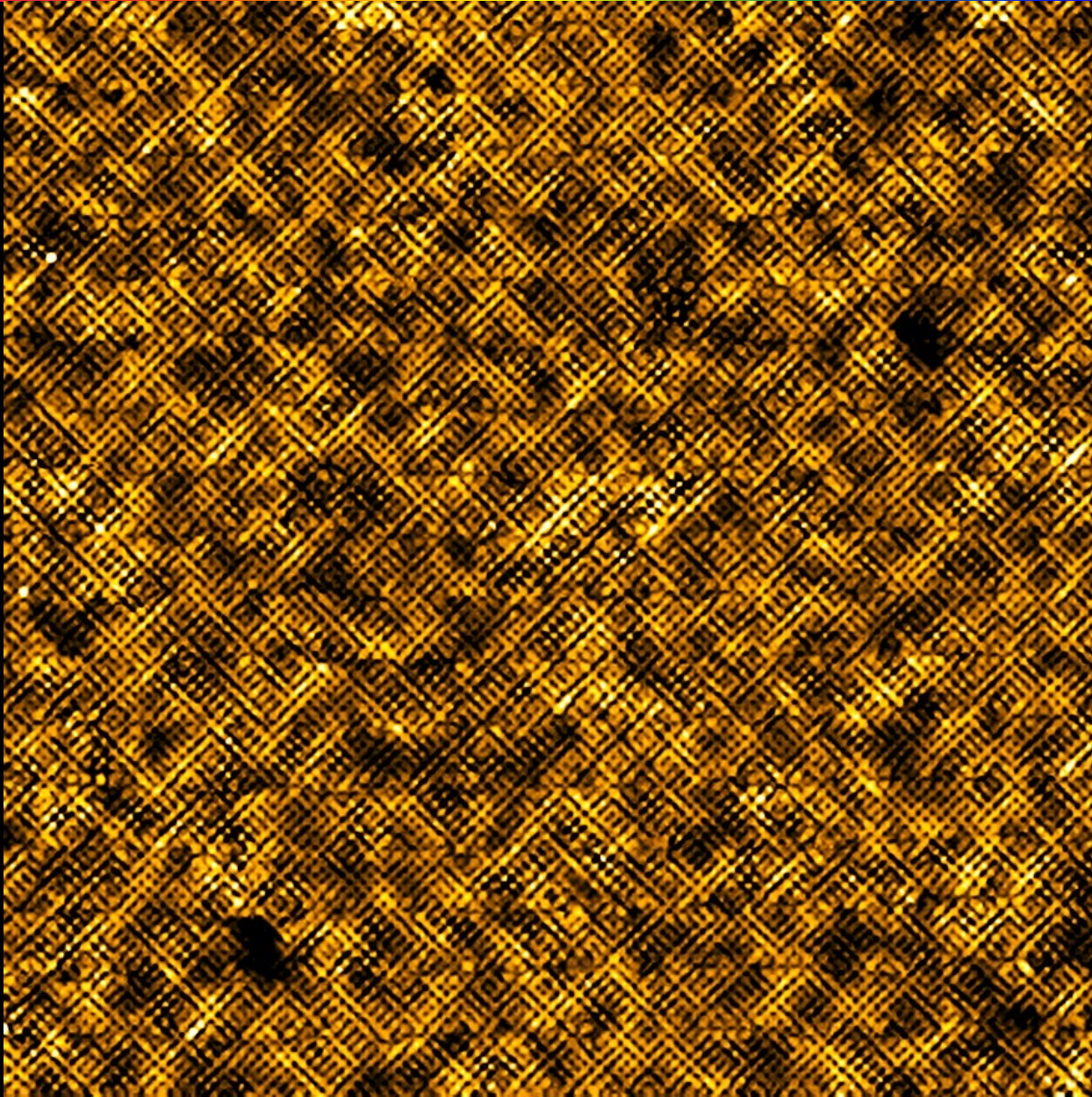
$p \sim 10\%$



$p \sim 10\%$

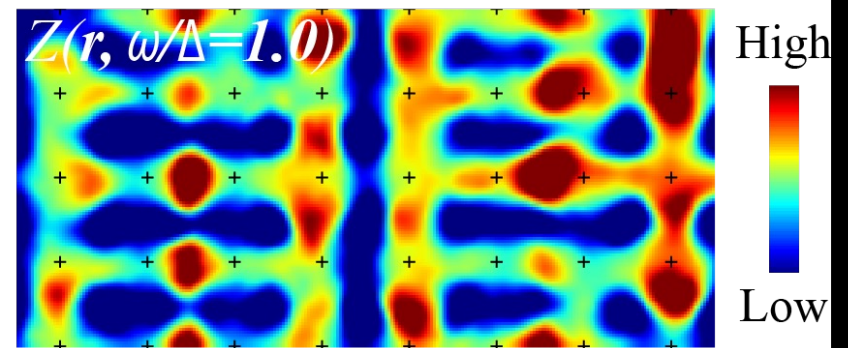
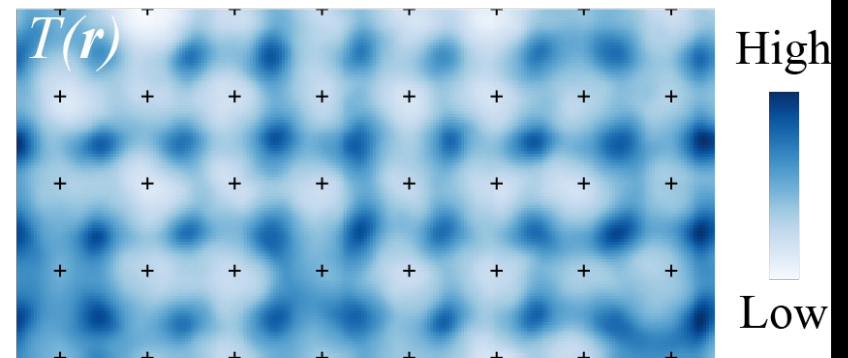
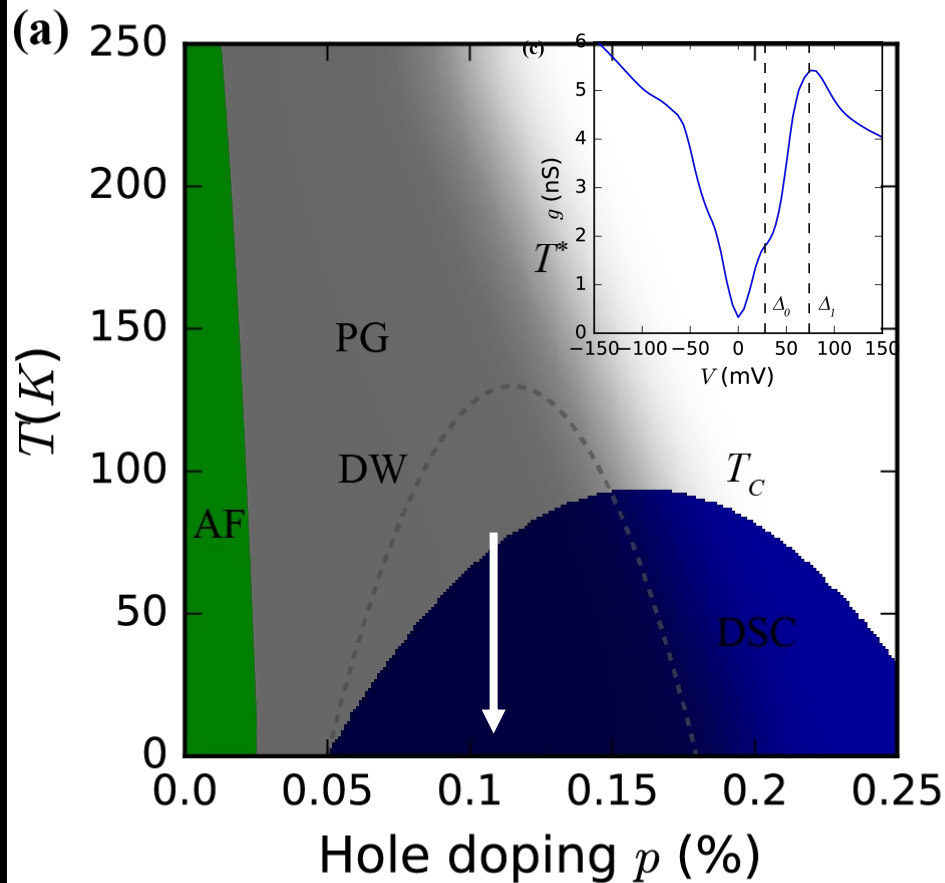
MACHINE LEARNING IDENTIFIES THIS 'MOTIF' UNIVERSALLY

Yi Zhang...
EAK. *et al*
***Nature* 570,**
484 (2019)



45 nm

QUASIPARTICLE $N(\mathbf{r}, E)$



$$g(\mathbf{r}, V) \equiv \frac{dI}{dV}(\mathbf{r}, eV) \quad Z(\mathbf{r}, E) \equiv \frac{g(\mathbf{r}, V)}{g(\mathbf{r}, -V)}$$

RENORMALIZED MEAN-FIELD t - J : $\lambda=8a_0$ PDW + DSC

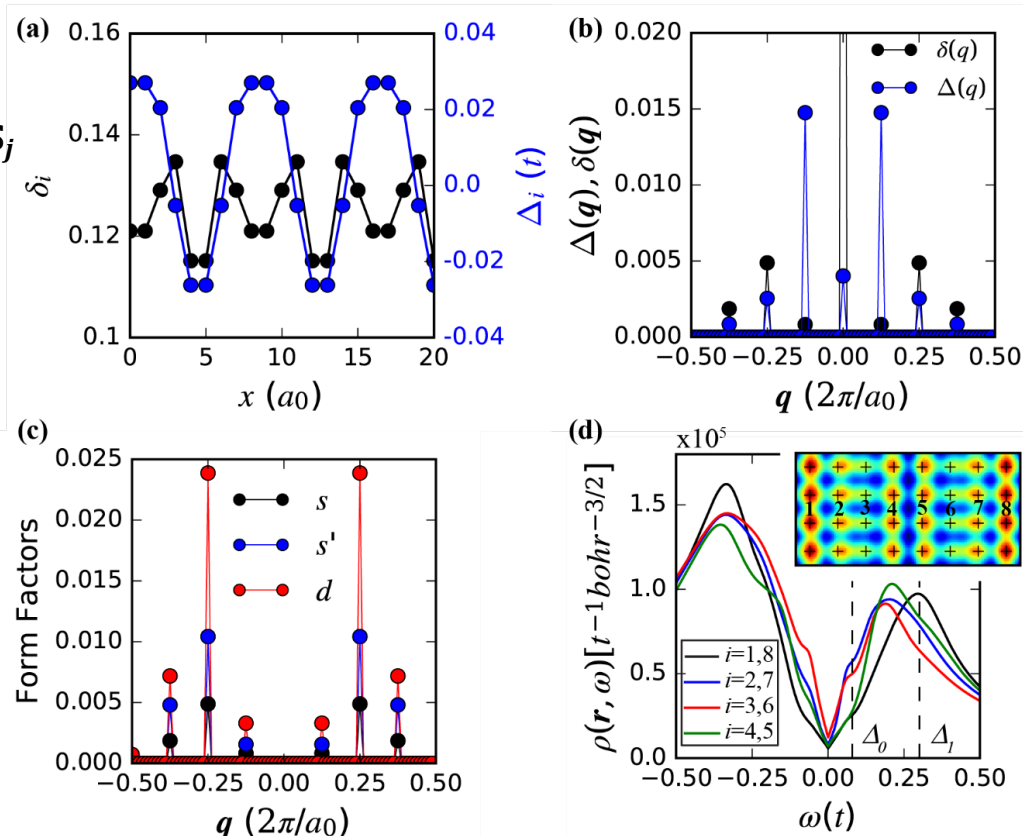
CuO₂ electronic structure & t - J Hamiltonian

$$H = - \sum_{\langle i,j \rangle, \sigma} P_G t_{ij} (c_{i\sigma}^\dagger c_{j\sigma} + h.c.) P_G + J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

- Replace PG by spin/charge renormalization factors – variable by atomic site..
- Decouple into a diagonalizable mean-field approximation using: on-site hole density δ_i , bond-order field $\chi_{ij\sigma}$, and electron-pair potential $\Delta_{ij\sigma}$.
- Initialize with a set of order parameter fields modulating at $\mathbf{Q}_P = (\pm 1/8, 0)2\pi/a_0$,

Predict:

- Net charge Cu site $\delta_i = 1 - \langle \Psi_0 | \sum_{\sigma} n_{i\sigma} | \Psi_0 \rangle$
- Bond-order field $\chi_{ij\sigma} = \langle \Psi_0 | c_{i\sigma}^\dagger c_{j\sigma} | \Psi_0 \rangle$
- Electron-pair field $\Delta_{ij\sigma} = \sigma \langle \Psi_0 | c_{i\sigma} c_{j\bar{\sigma}} | \Psi_0 \rangle$
- Quasiparticle density of states N_i at every site



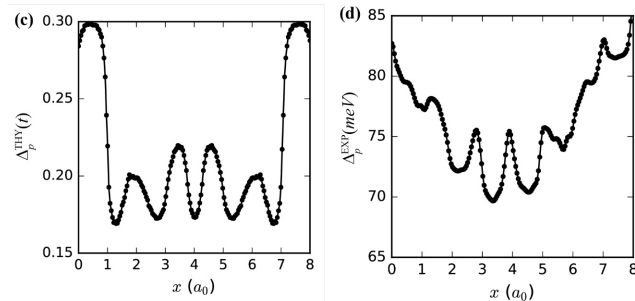
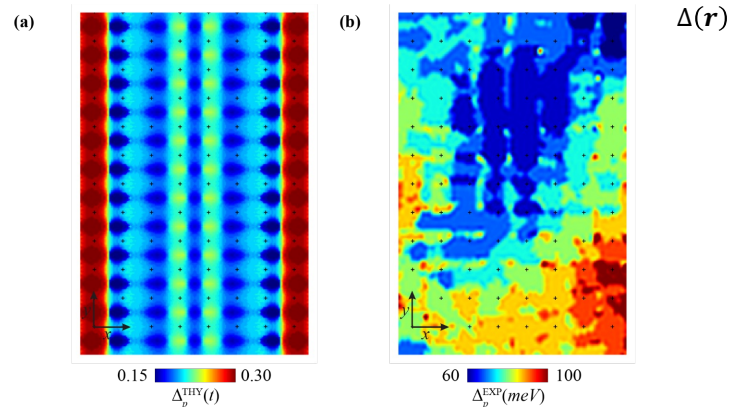
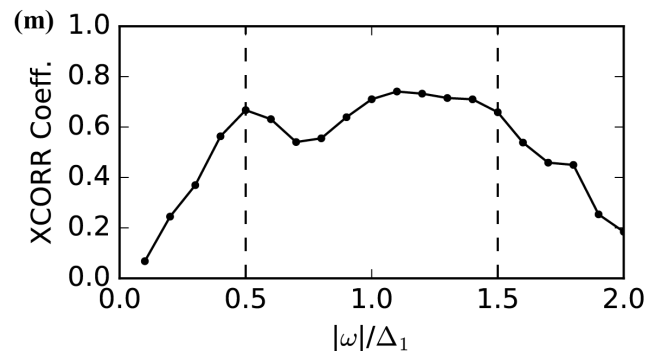
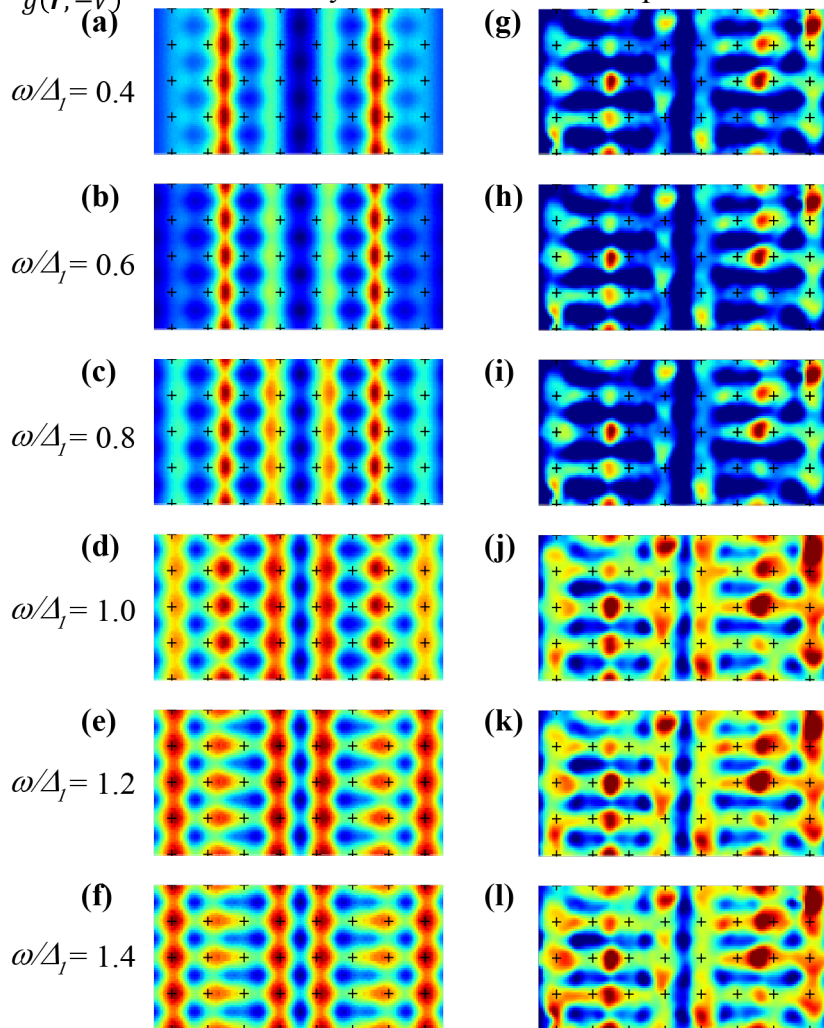
COEXISTING ORDER PARAMETERS = INTERTWINED

$\lambda=8a_0$ PDW + DSC : $N(\mathbf{r},E)$ & $\Delta(\mathbf{r})$

$$Z(\mathbf{r}, E) \equiv \frac{g(\mathbf{r}, V)}{g(\mathbf{r}, -V)}$$

Theory

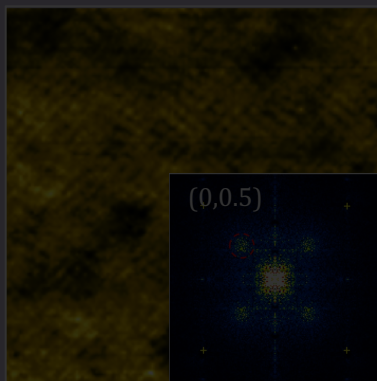
Experiment



Low High

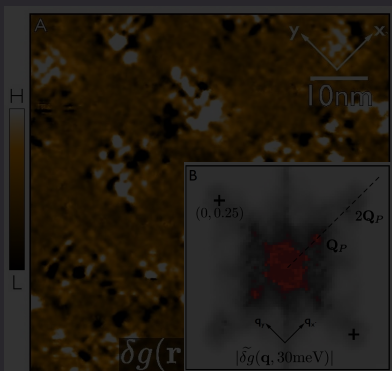
CuO₂ ELECTRON-PAIR VISUALIZATION STUDIES

PAIR DENSITY



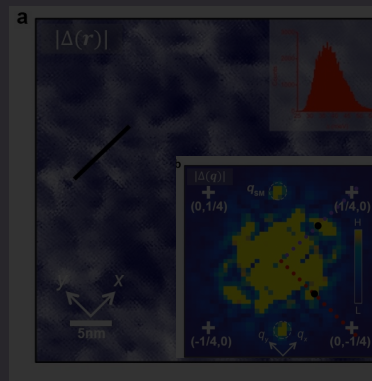
Nature 532, 343 (2016)

VORTEX HALO



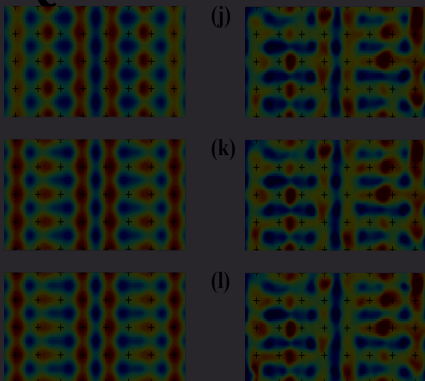
Science 364, 976 (2019)

ENERGY GAP



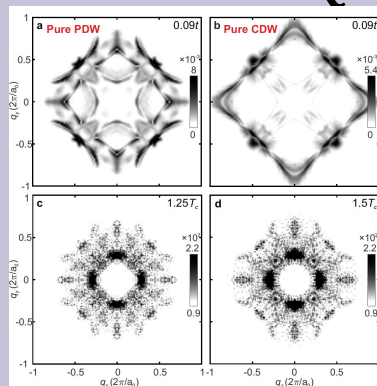
Nature 580, 6570 (2020)

QUASIPARTICLES



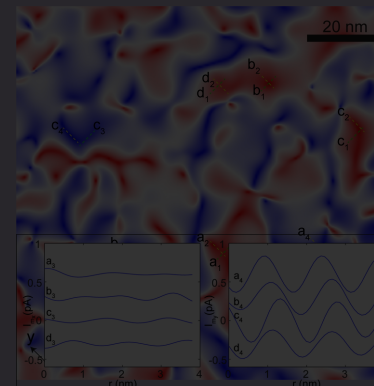
PNAS 117, 14850 (2020)

PSEUDOGAP QPI

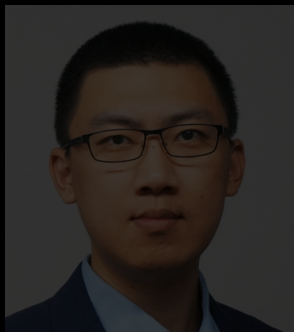


Nat.Comm 12, 6087 (2021)

NEMATIC



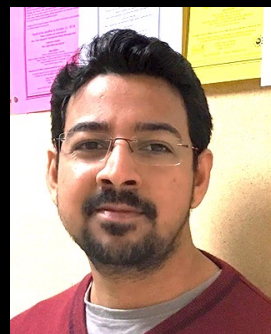
Weijiong Chen *et al* (2022)



Xiaolong Liu
Notre Dame



Shuqiu Wang
Oxford



P. Choubey
IIT Delhi



Weijiong Chen
Oxford



S. Uchida
Tokyo



H. Eisaki
Tsukuba



P.D. Johnson
Oxford



Kazuhiro Fujita
BNL



P.J. Hirschfeld
Florida



M. H. Hamidian
Cornell

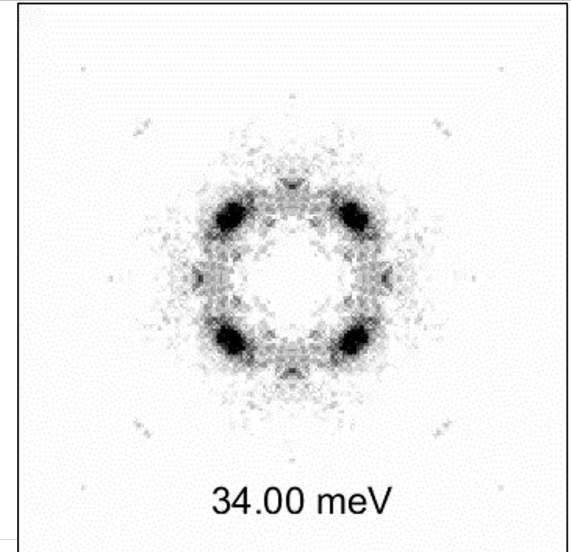
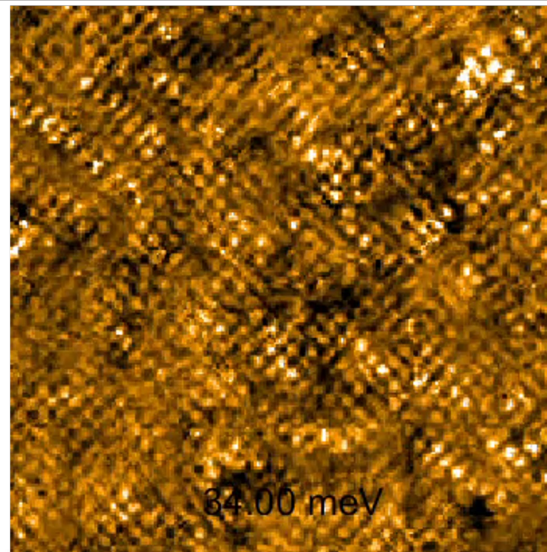
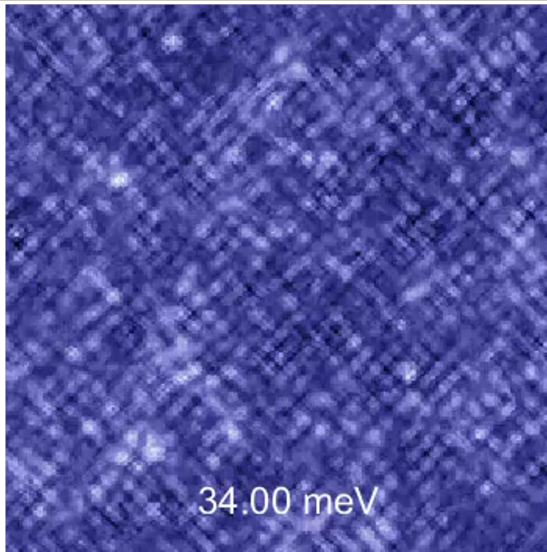


S.D. Edkins
Cornell



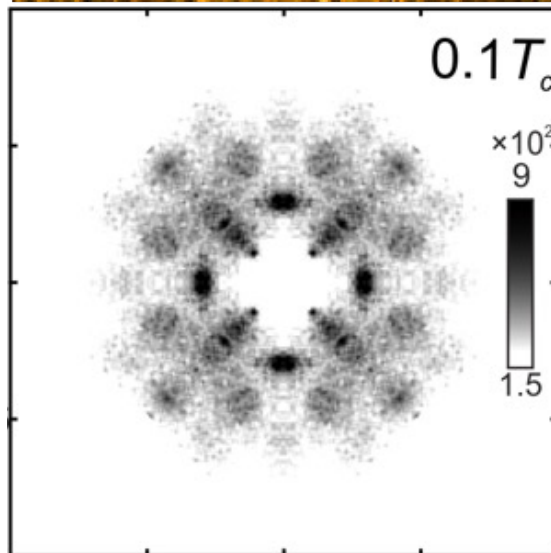
A.P. Mackenzie
Dresden

QUASIPARTICLE INTERFERENCE $\Lambda(\mathbf{q}, \Delta_0)$



$$g(\mathbf{r}, V) \equiv \frac{dI}{dV}(\mathbf{r}, E = eV)$$

$$\Lambda(\mathbf{q}, \Delta_0) \equiv \sum_{E \cong 0}^{\Delta_0} Z(\mathbf{q}, E)$$



$$Z(\mathbf{q}, E) = \mathcal{F}Z(\mathbf{r}, E).$$

- $\Lambda(\mathbf{q}, \Delta_0)$ provides an efficient characteristic “fingerprint” of any ordered state that controls scattering wavevectors.

RENORMALIZED MEAN-FIELD t - J : $\lambda=8a_0$ PDW + DSC

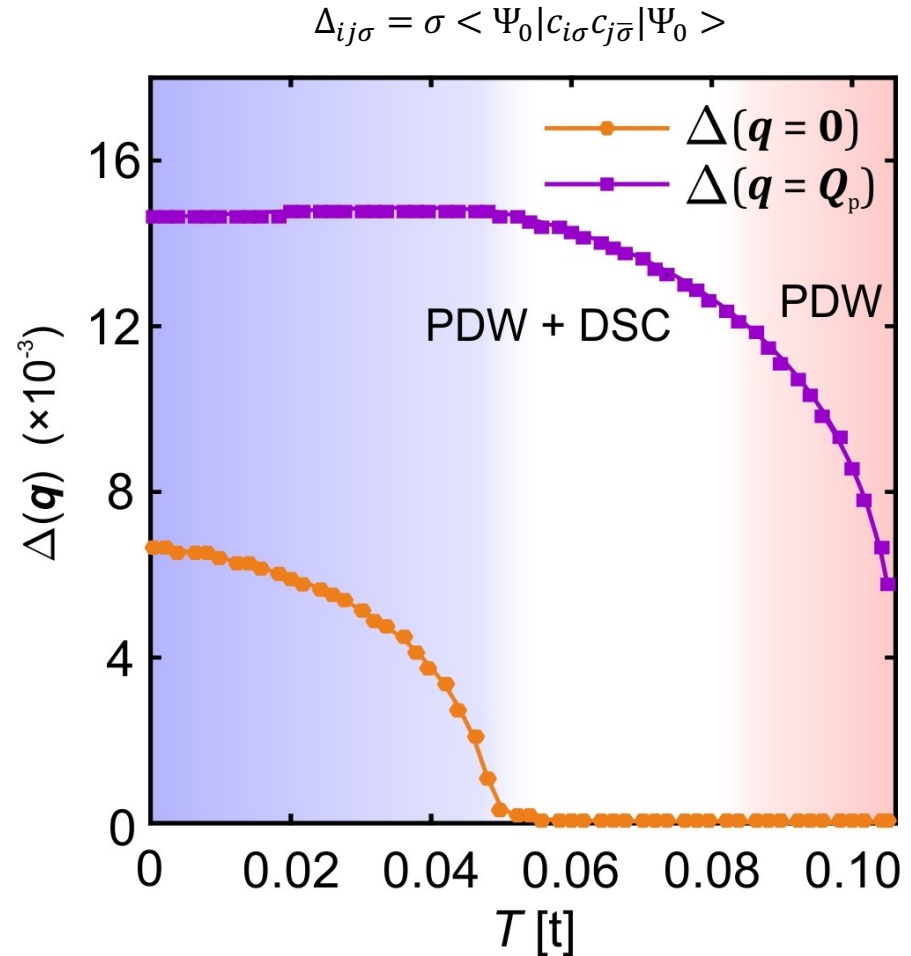
CuO₂ electronic structure & t - J Hamiltonian

$$H = - \sum_{\langle i,j \rangle, \sigma} P_G t_{ij} (c_{i\sigma}^\dagger c_{j\sigma} + h.c.) P_G + J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

- Replace PG by spin/charge renormalization factors - variable by atomic site..
- Decouple into a diagonalizable mean-field approximation using: on-site hole density δ_i , bond-order field $\chi_{ij\sigma}$, and electron-pair potential $\Delta_{ij\sigma}$.
- Initialize with a set of order parameter fields modulating at $\mathbf{Q}_p = (\pm 1/8, 0)2\pi/a_0$,

Predict:

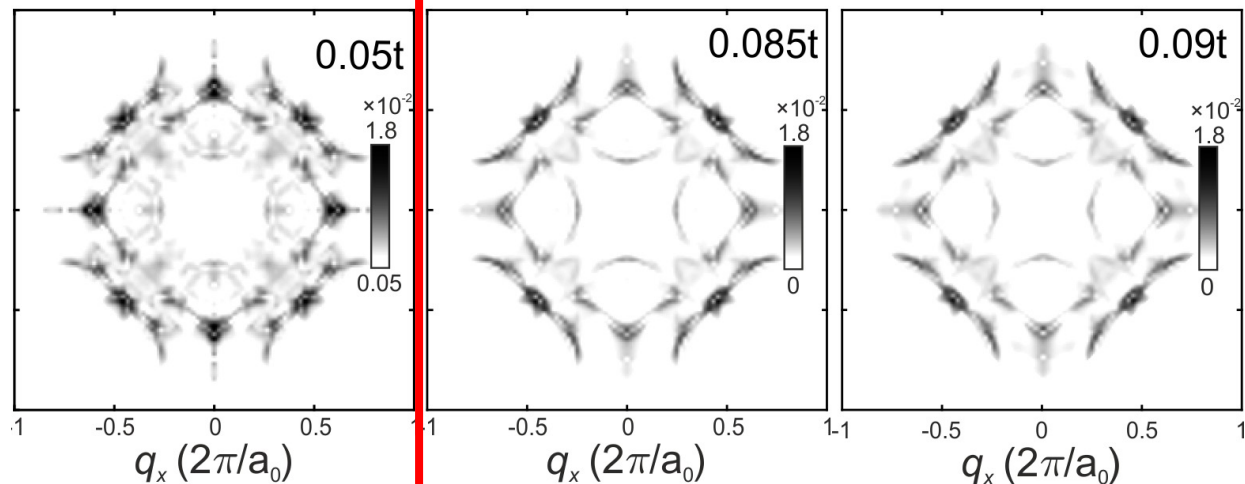
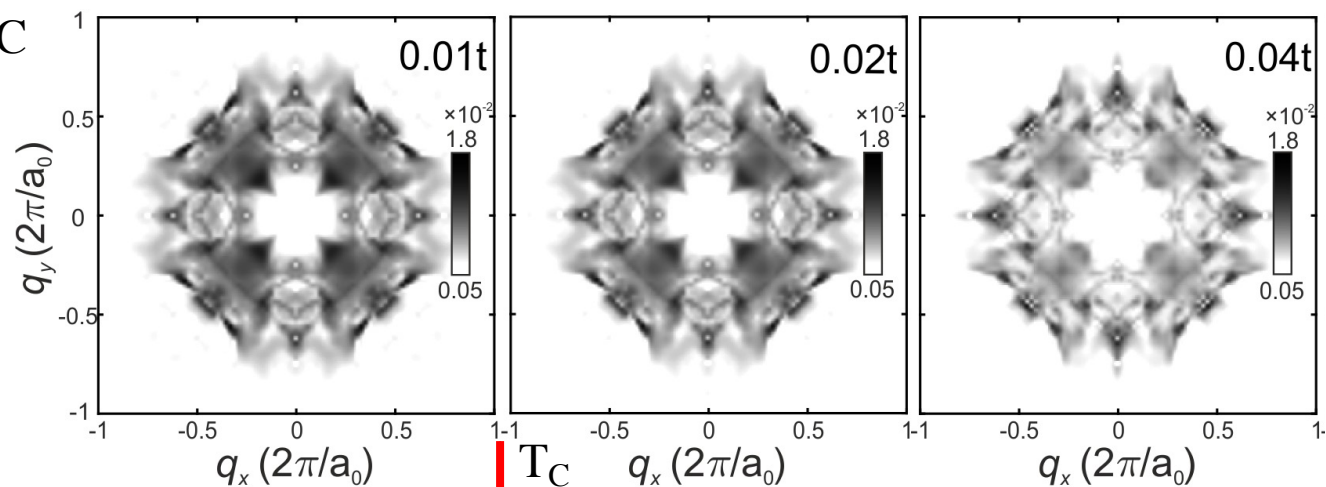
- Net charge Cu site $\delta_i = 1 - \langle \Psi_0 | \sum_{\sigma} n_{i\sigma} | \Psi_0 \rangle$
- Bond-order field $\chi_{ij\sigma} = \langle \Psi_0 | c_{i\sigma}^\dagger c_{j\sigma} | \Psi_0 \rangle$
- Electron-pair field $\Delta_{ij\sigma} = \sigma \langle \Psi_0 | c_{i\sigma} c_{j\bar{\sigma}} | \Psi_0 \rangle$
- Quasiparticle density of states N_i at every site



$\lambda=8a_0$ PDW + DSC : PREDICT QPI SIGNATURE $\Lambda(\mathbf{q}, \Delta_0)$

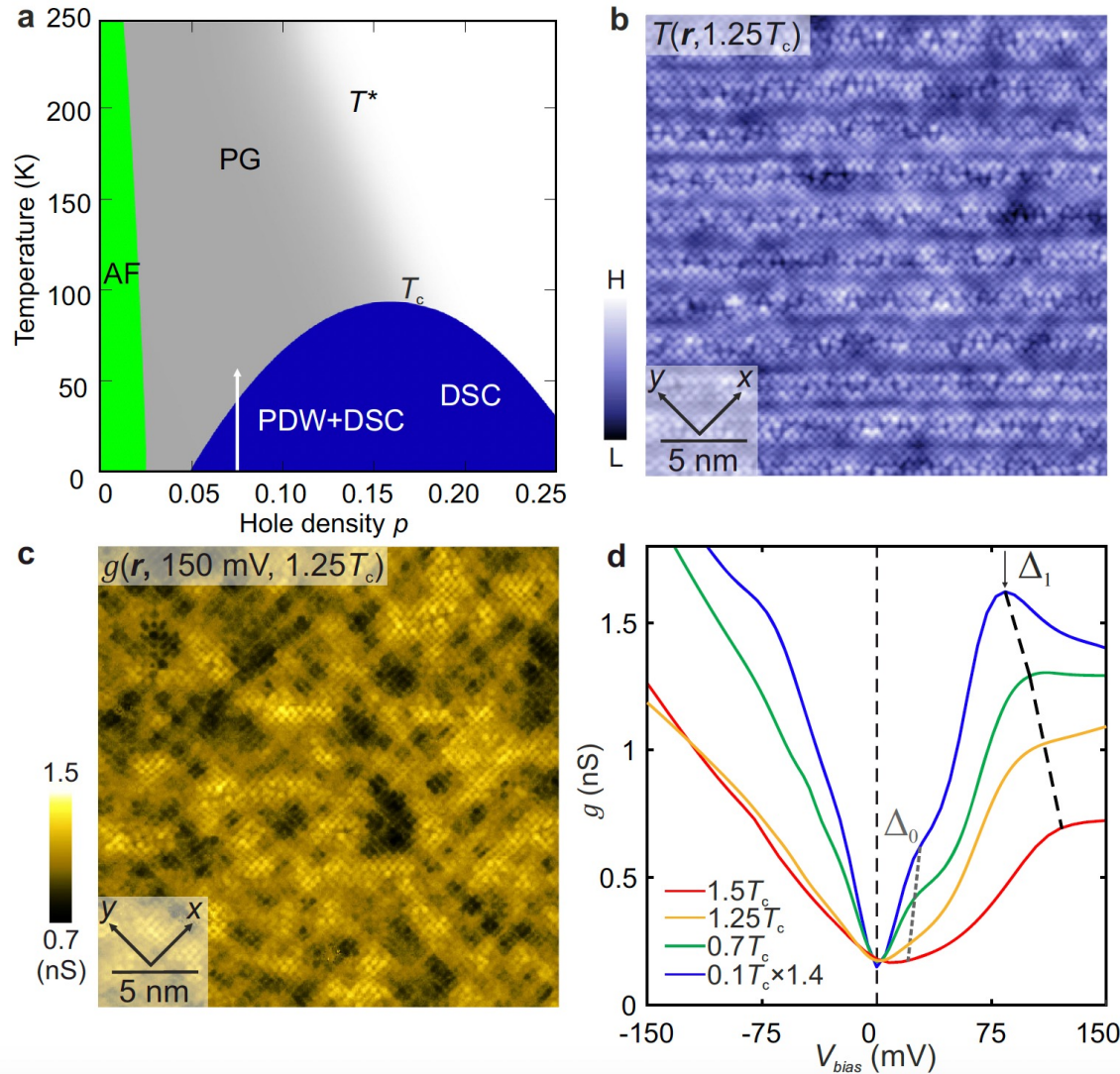
$$N(\mathbf{q}, E) \propto \text{Im}G(k, E)TG(k - \mathbf{q}, E) \Rightarrow \Lambda(\mathbf{q}, \Delta_0) \equiv \sum_{E \cong 0}^{\Delta_0} N(\mathbf{q}, E) / N(\mathbf{q}, -E)$$

PDW+DSC

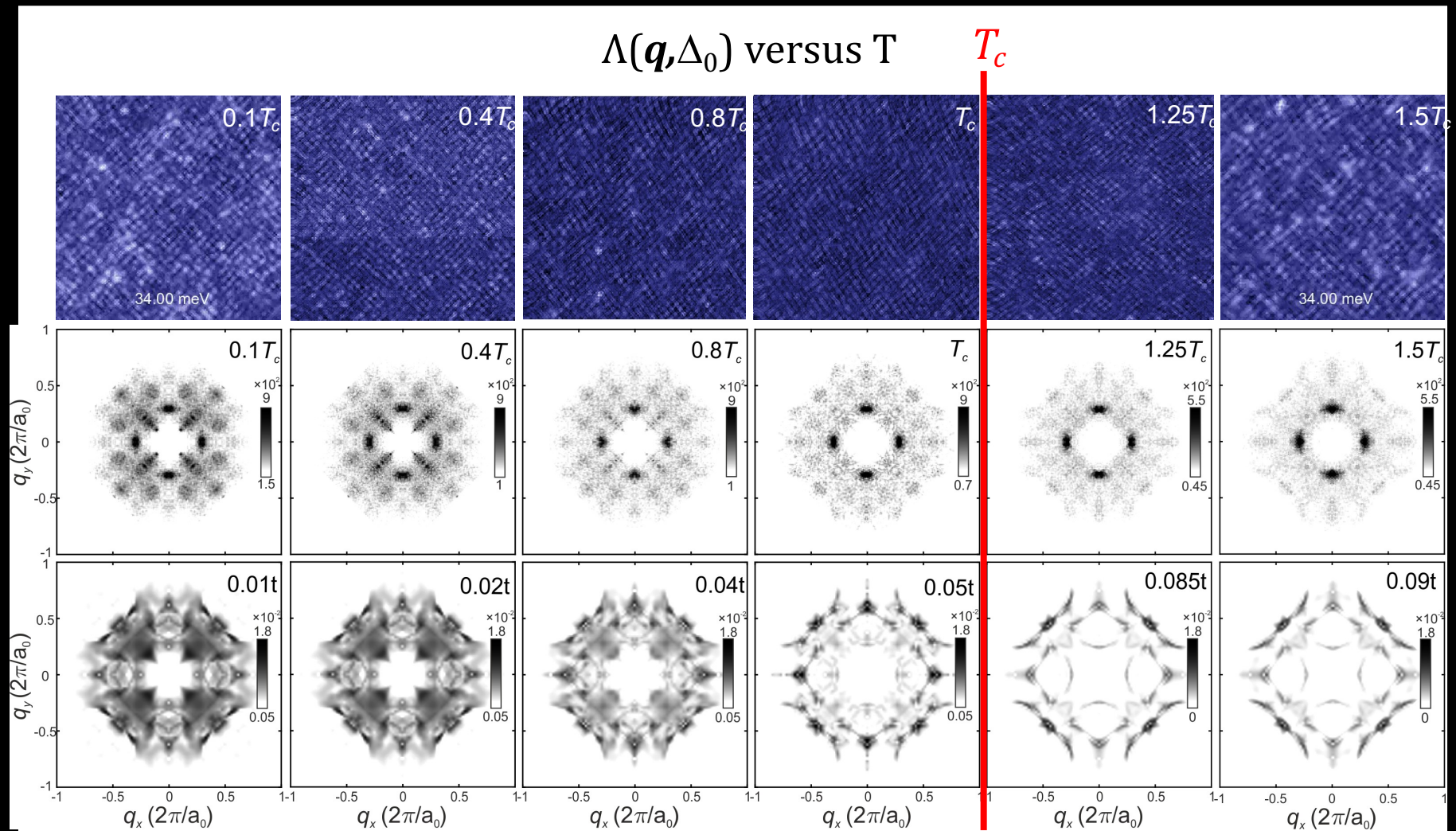


PDW

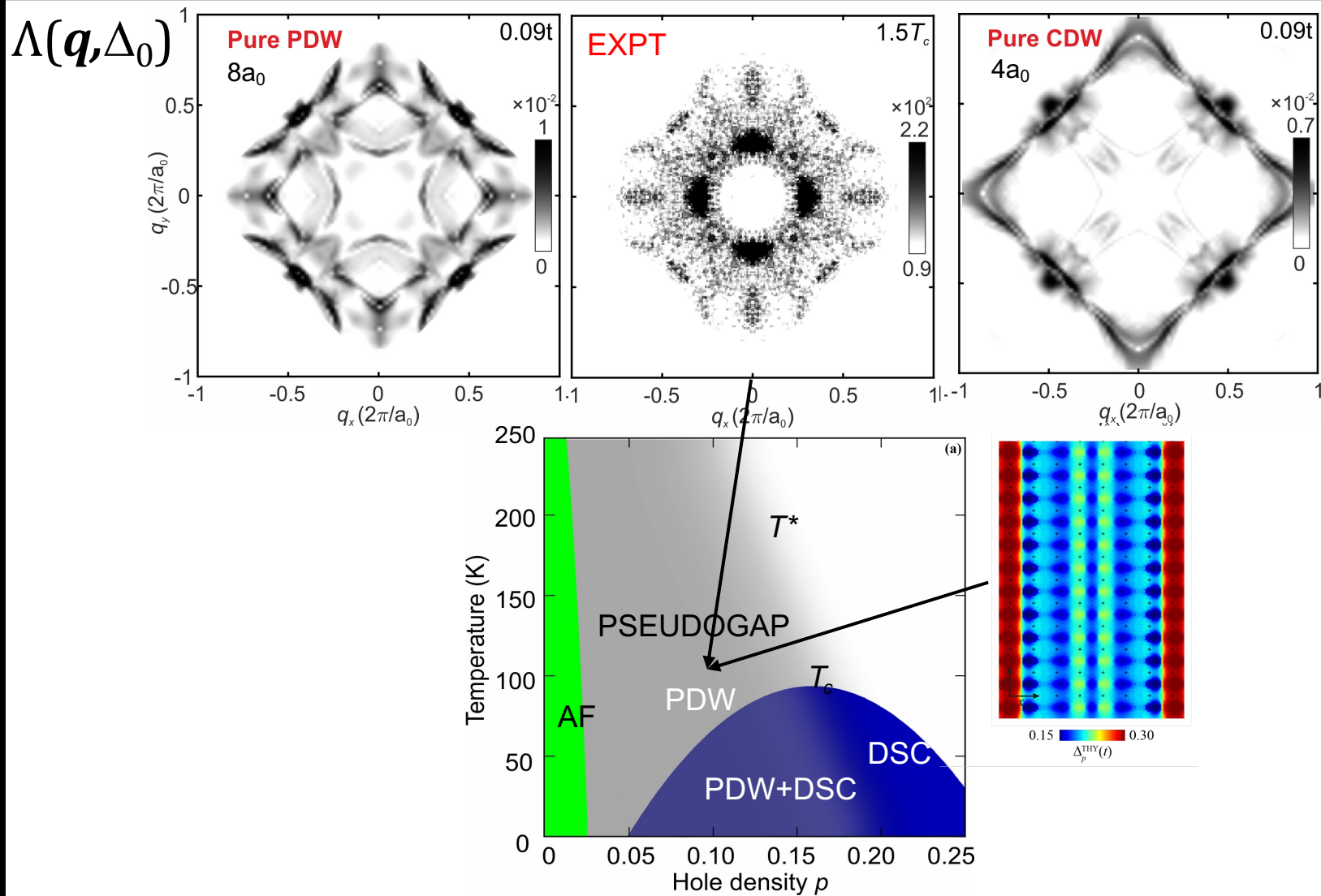
MEASURE QPI SIGNATURE $\Lambda(\mathbf{q}, \Delta_0)$



$\lambda=8a_0$ PDW + DSC : $\Lambda(\mathbf{q}, \Delta_0)$



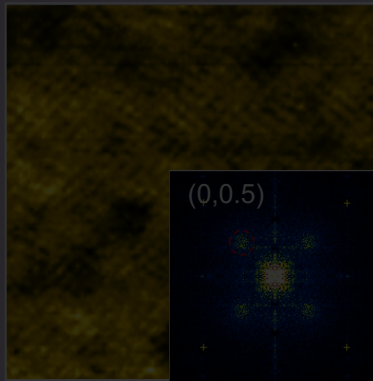
PSEUDOGAP ($T > T_c$): $\Lambda(\mathbf{q}, \Delta_0)$ OF $\lambda=8a_0$ PDW



Nat.Comm 12, 6087 (2021)

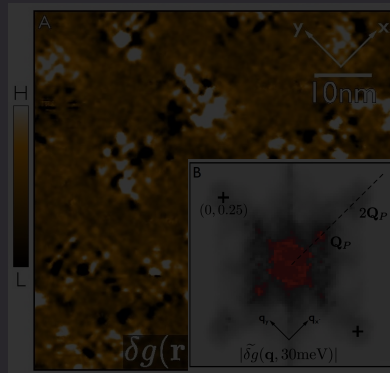
CuO₂ ELECTRON-PAIR VISUALIZATION STUDIES

PAIR DENSITY



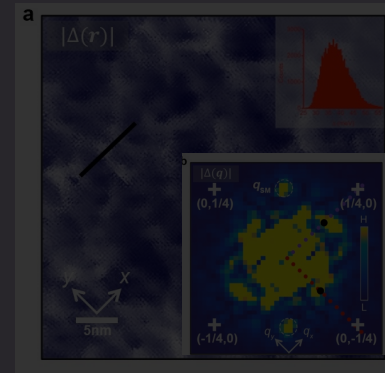
Nature 532, 343 (2016)

VORTEX HALO



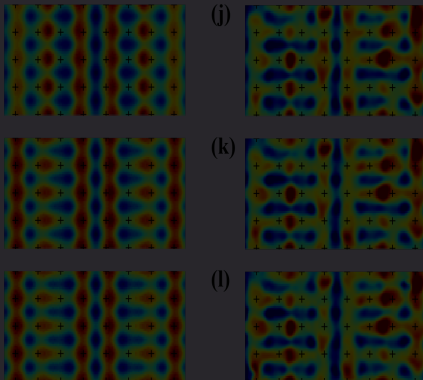
Science 364, 976 (2019)

ENERGY GAP



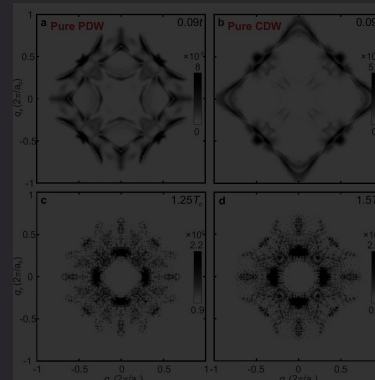
Nature 580, 6570 (2020)

QUASIPARTICLES



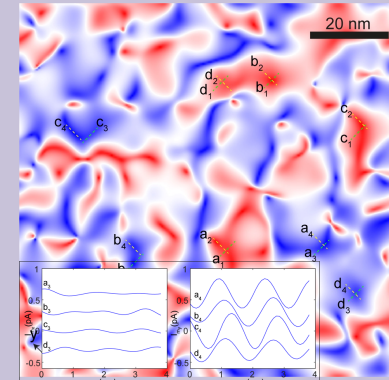
PNAS 117, 14850 (2020)

PUDOGAP QPI

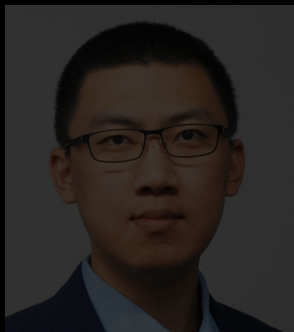


Nat.Comm 12, 6087 (2021)

NEMATIC



Weijiong Chen et al (2022)



Xiaolong Liu
Notre Dame



Shuqiu Wang
Oxford



P. Choubey
IIT Delhi



Weijiong Chen
Oxford



S. Uchida
Tokyo



H. Eisaki
Tsukuba



P.D. Johnson
Oxford



Kazuhiro Fujita
BNL



P.J. Hirschfeld
Florida



M. H. Hamidian
Cornell

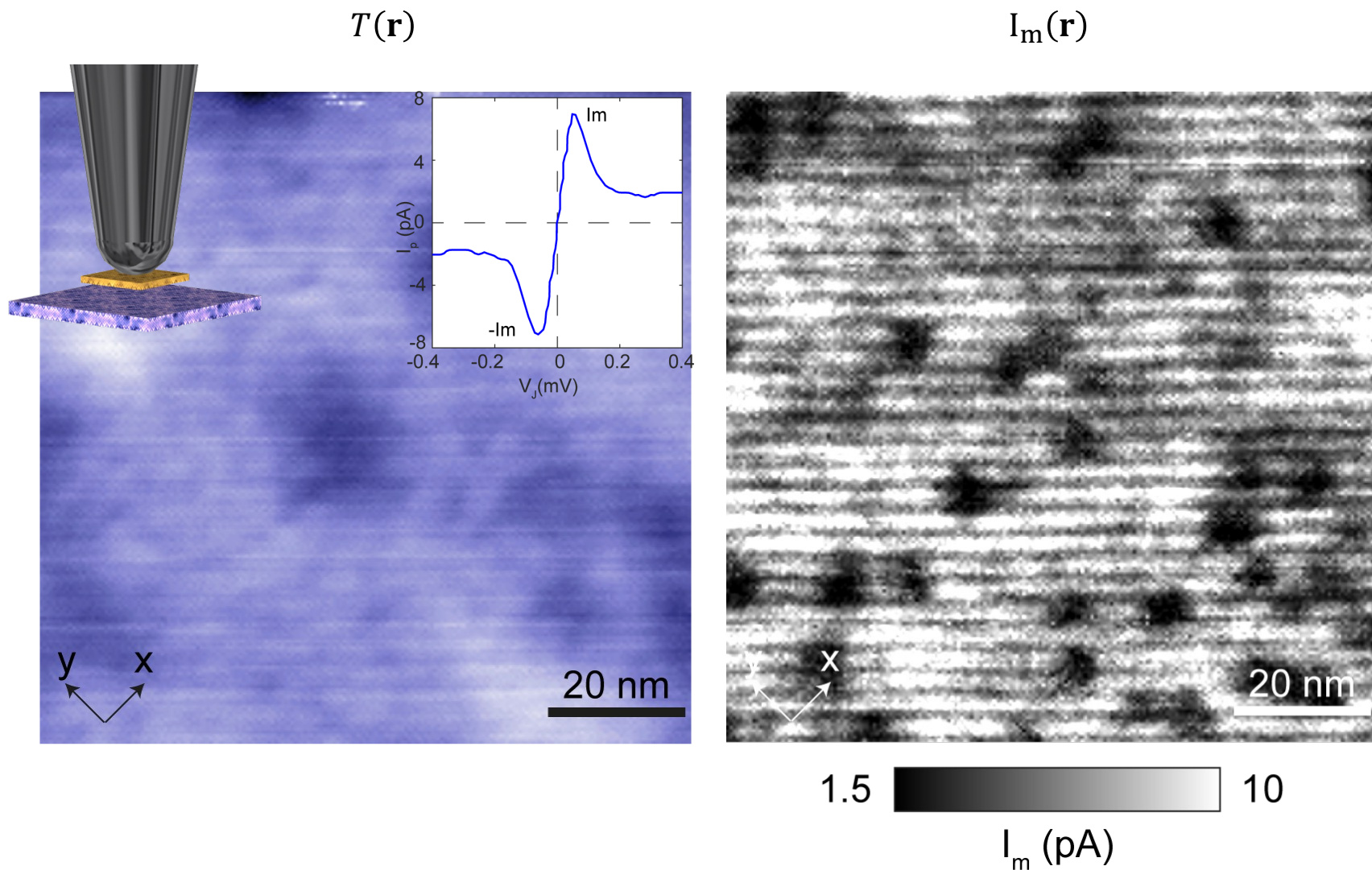


S.D. Edkins
Cornell



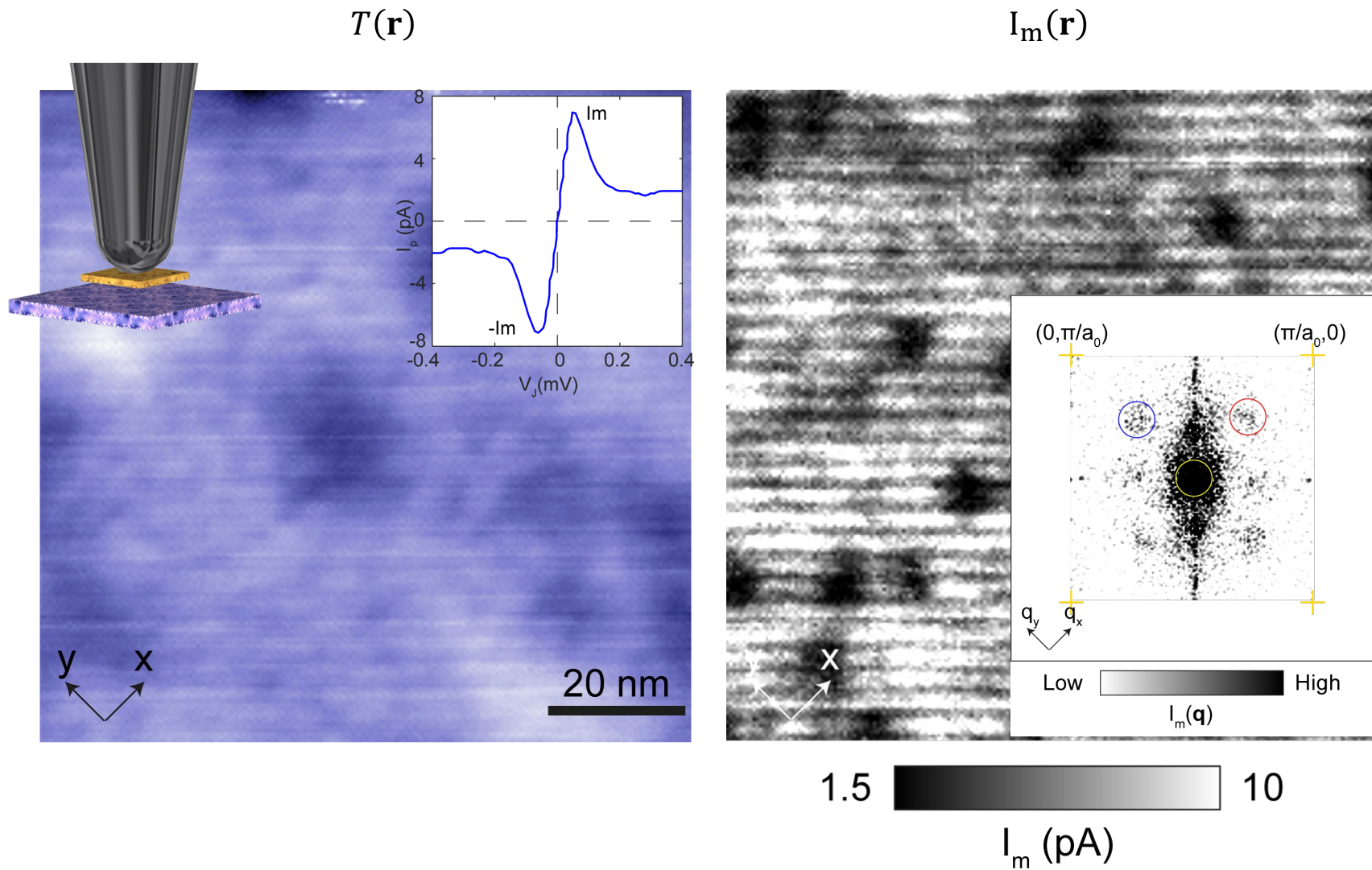
A.P. Mackenzie
Dresden

SJTM: VISUALIZATION OF ELECTRON-PAIR DENSITY $n_p(\mathbf{r})$



NO DETECTABLE CHARGE MODULATIONS

SJTM: VISUALIZATION OF ELECTRON-PAIR DENSITY $n_p(\mathbf{r})$

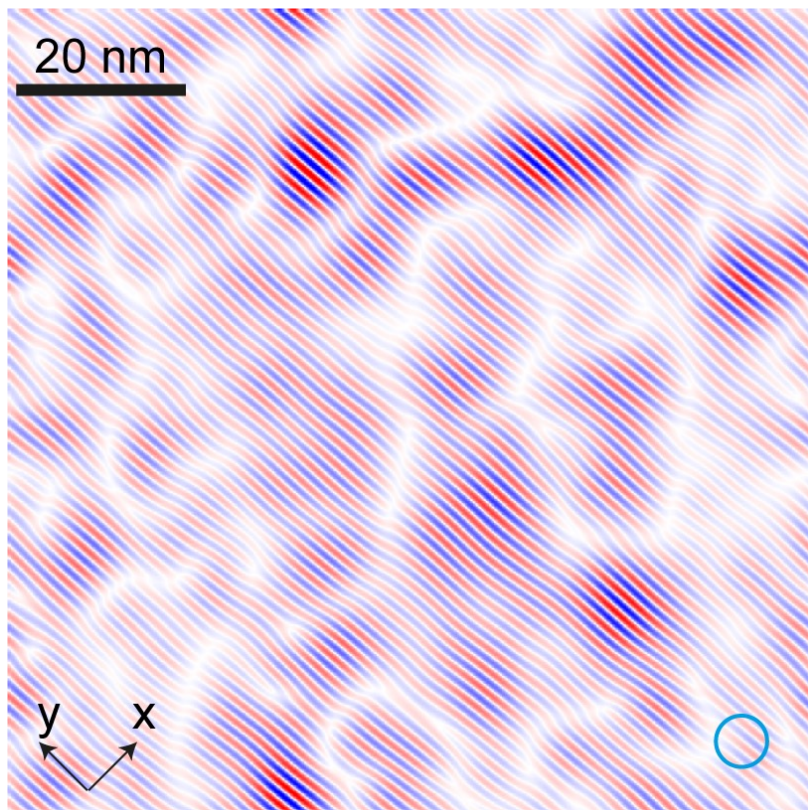


Weijiong Chen *et al* (2022)

ELECTRON-PAIR DENSITY $n_p(\mathbf{r})$ MODULATIONS

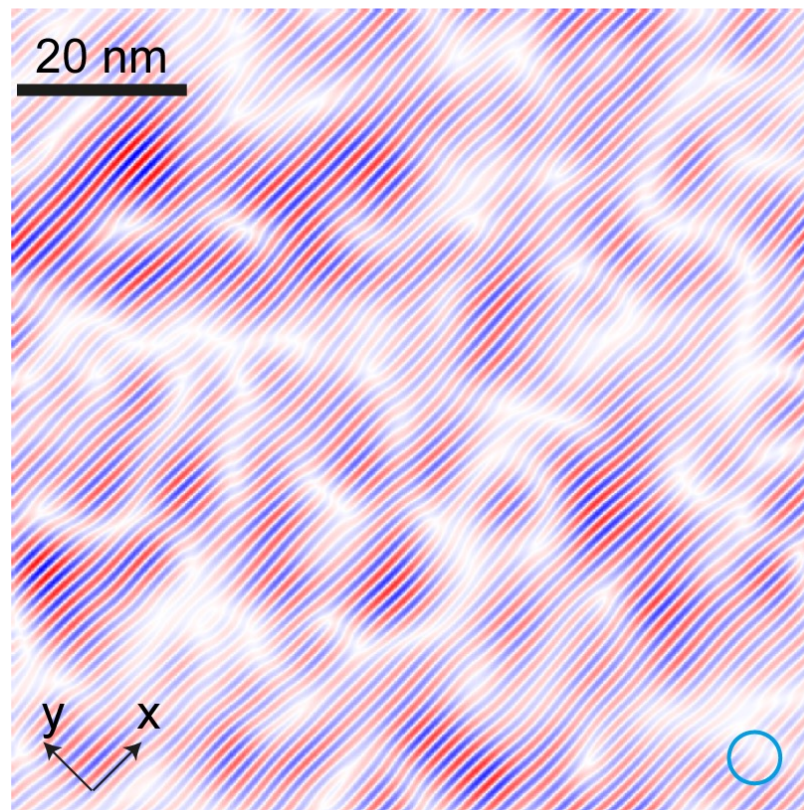
$$A_x(\mathbf{r})\cos(Q_x x + \theta_x)$$

$$A_y(\mathbf{r})\cos(Q_y y + \theta_y)$$



-0.4 0.4

$I_{m_x}(\mathbf{r})$ (pA)



-0.4 0.4

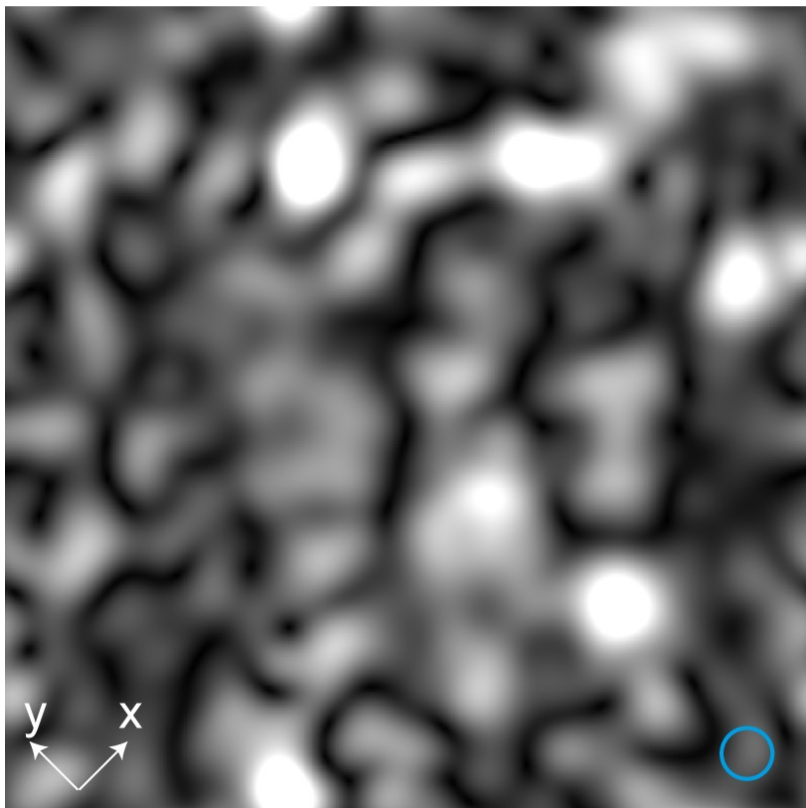
$I_{m_y}(\mathbf{r})$ (pA)

Weijiong Chen *et al* (2022)

ELECTRON-PAIR DENSITY $n_p(\mathbf{r})$ MODULATIONS

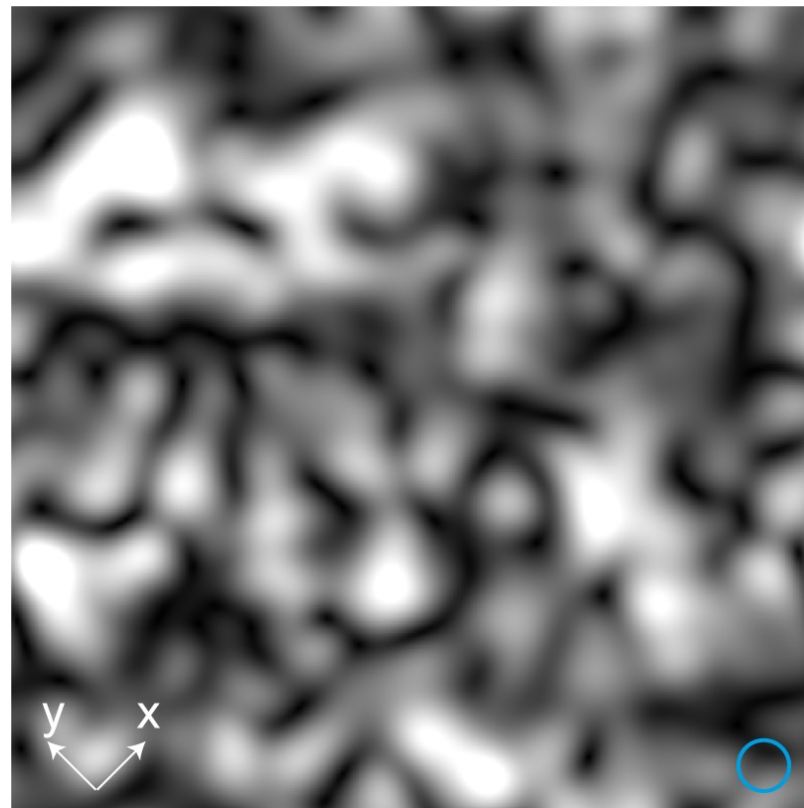
$$A_x(\mathbf{r}) \propto I_0^2 \propto |\Delta_x|^2$$

$$A_y(\mathbf{r}) \propto I_0^2 \propto |\Delta_y|^2$$



0  0.3

$A_x(\mathbf{r})$ (pA)



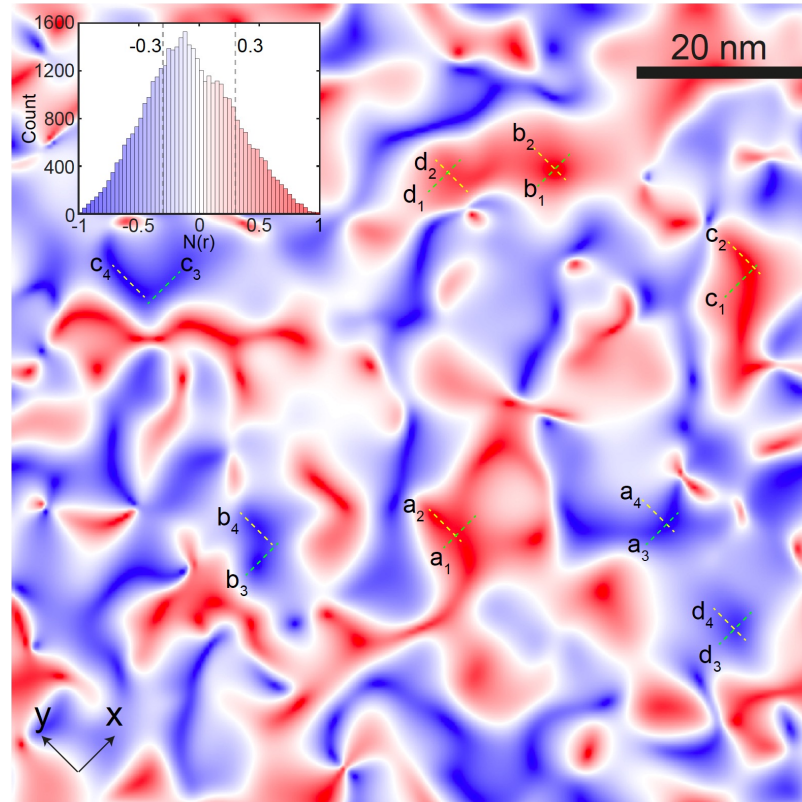
0  0.3

$A_y(\mathbf{r})$ (pA)

Weijiong Chen *et al* (2022)

NEMATIC ORDER-PARAMETER FOR $n_p(\mathbf{r})$ MODULATIONS

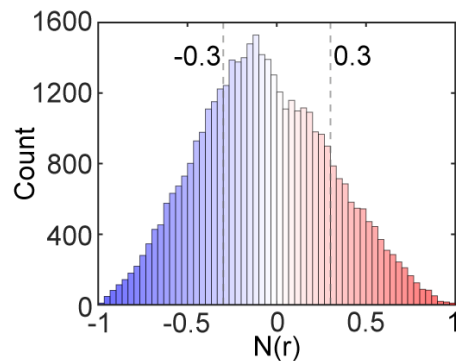
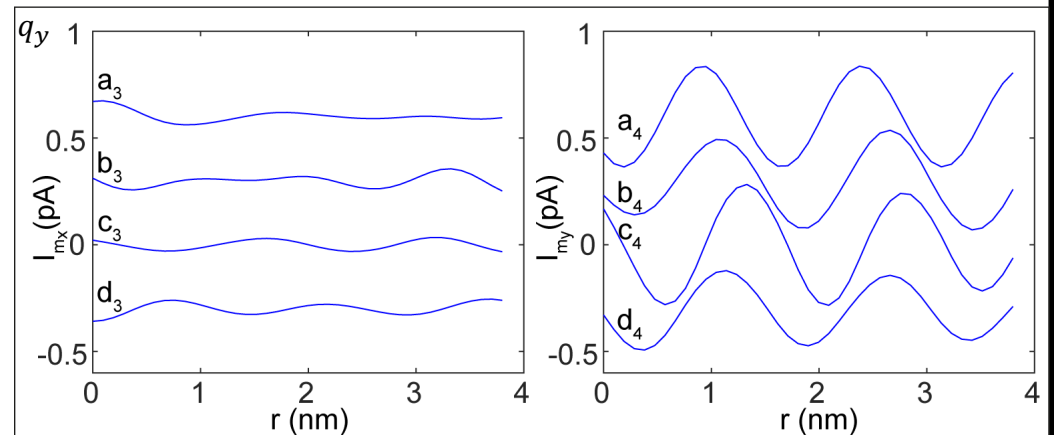
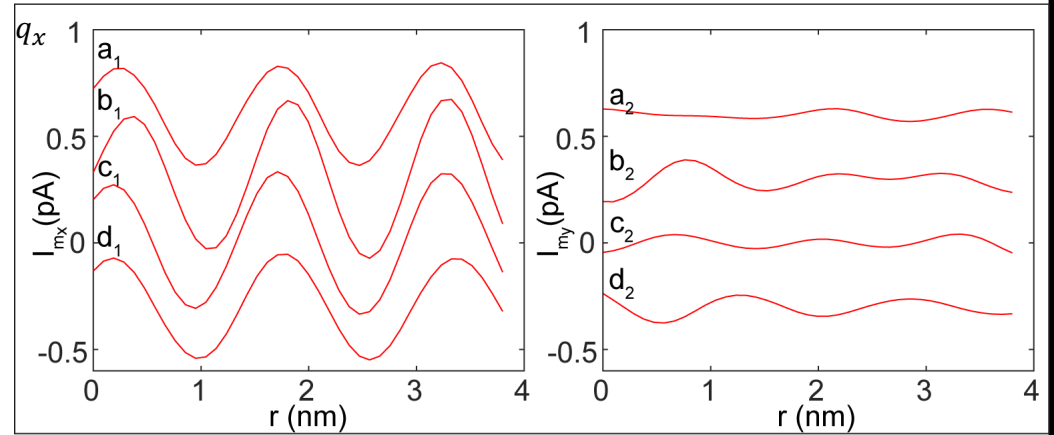
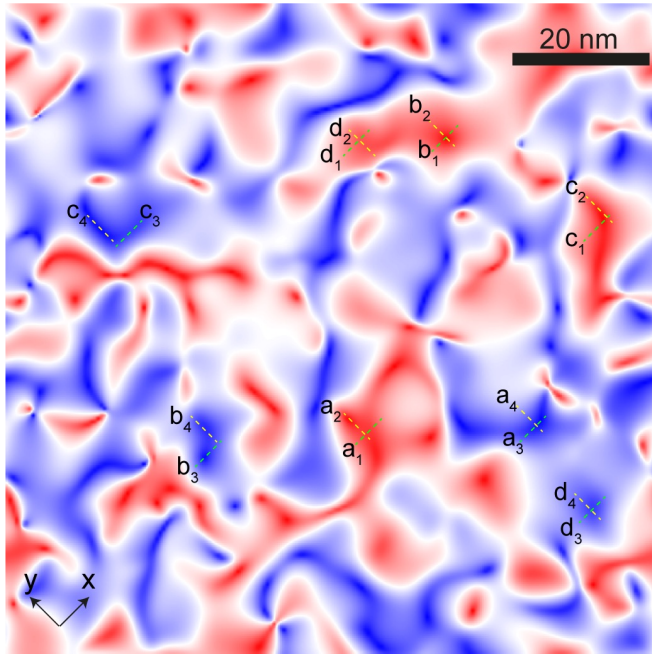
$$\mathcal{N}(\mathbf{r}) = \{A_x(\mathbf{r}) - A_y(\mathbf{r})\} / \{A_x(\mathbf{r}) + A_y(\mathbf{r})\}$$



Weijiong Chen *et al* (2022)

NEMATIC ORDER-PARAMETER FOR $n_p(\mathbf{r})$ MODULATIONS

$$\mathcal{N}(\mathbf{r}) = \{A_x(\mathbf{r}) - A_y(\mathbf{r})\} / \{A_x(\mathbf{r}) + A_y(\mathbf{r})\}$$






Weijiong Chen *et al* (2022)

Zn IMPURITY-ATOM SITES : NEMATIC DOMAIN WALLS

PHYSICAL REVIEW B **103**, L020502 (2021)

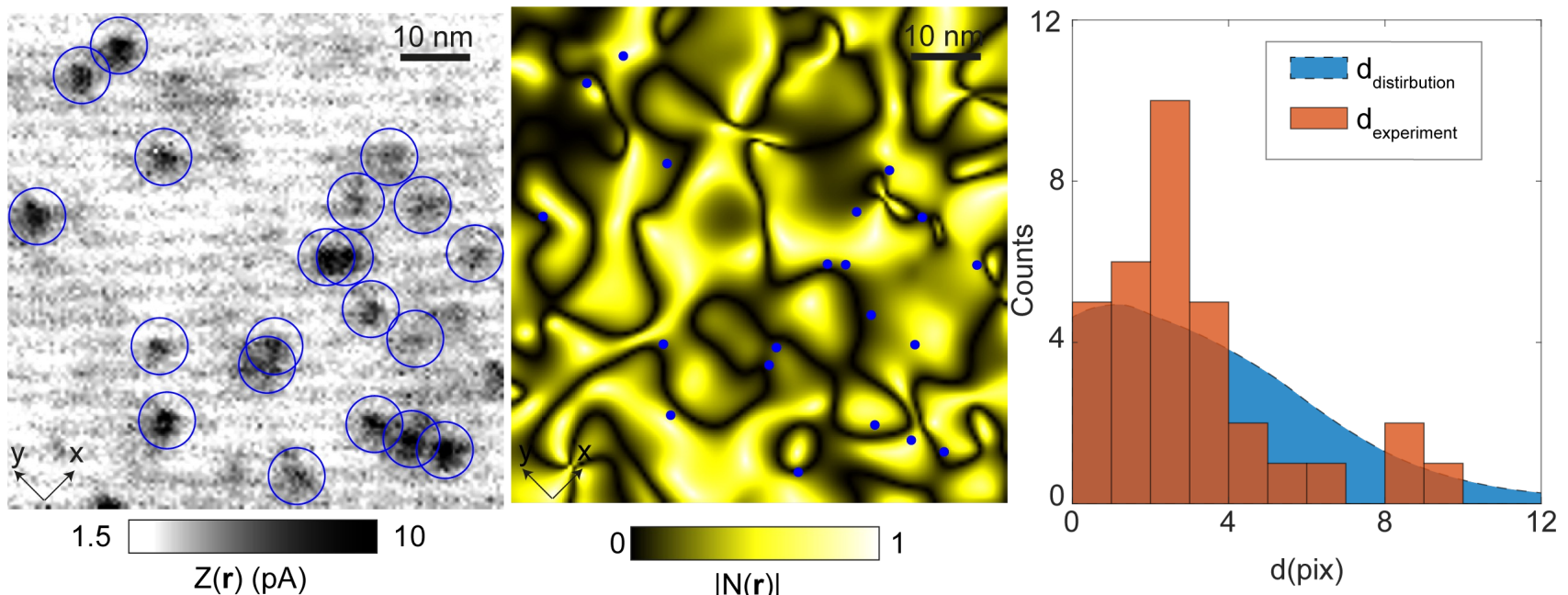
Letter

Experimental evidence that zinc impurities pin pair-density-wave order in $\text{La}_{2-x}\text{Ba}_x\text{CuO}_4$

P. M. Lozano ^{1,2}, G. D. Gu,¹ J. M. Tranquada ^{1,*} and Qiang Li ^{1,2,*}

¹Condensed Matter Physics & Materials Science Division, Brookhaven National Laboratory, Upton, New York 11973-5000, USA

²Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York 11794-3800, USA

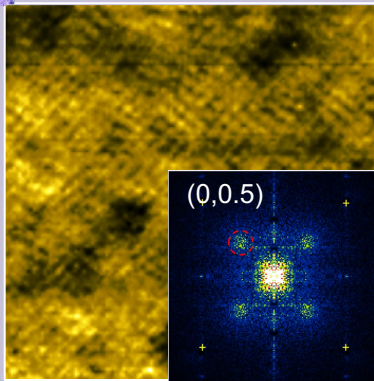


Weijiong Chen *et al* (2022)

CuO₂ ELECTRON-PAIR VISUALIZATION STUDIES

PAIR DENSITY

J
E

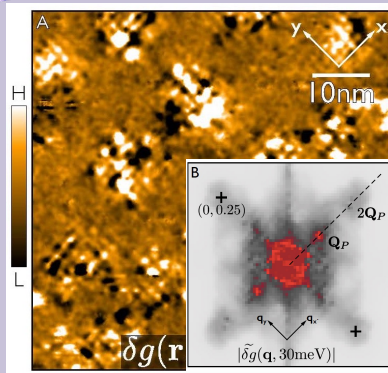


(2e)

Nature 532, 343 (2016)

VORTEX HALO

N
I
S

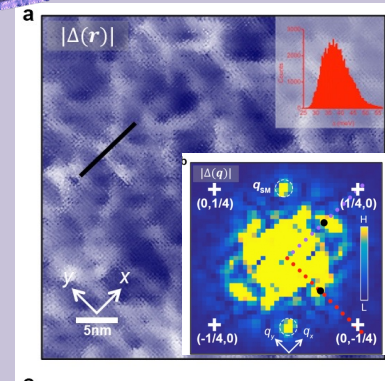


(1e)

Science 364, 976 (2019)

ENERGY GAP

S
I
S

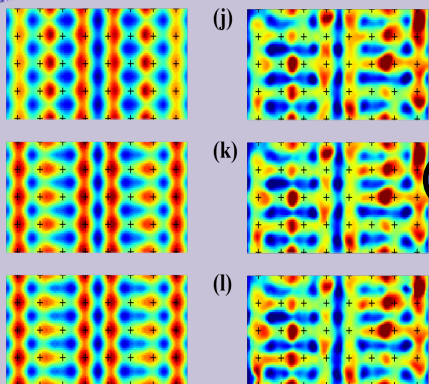


(1e)

Nature 580, 6570 (2020)

QUASIPARTICLES

N
I
S

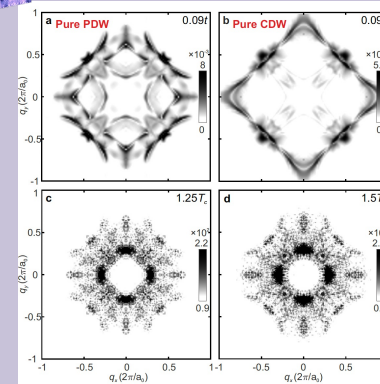


(1e)

PNAS 117, 14850 (2020)

PSEUDOGAP QPI

N
I
S

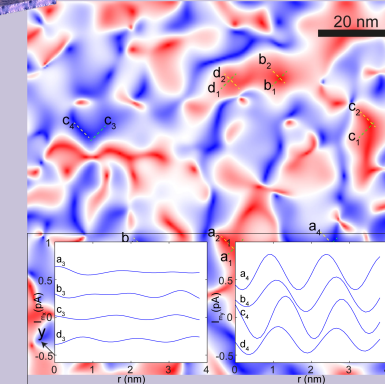


(1e)

Nat.Comm 12, 6087 (2021)

NEMATIC

J
E



(2e)

Weijiong Chen *et al* (2022)

CuO₂ ELECTRON-PAIR VISUALIZATION STUDIES

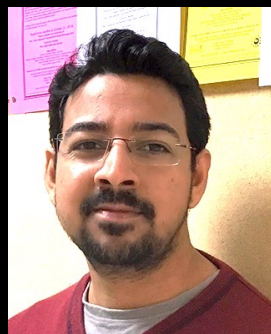
		MEASURED	PAIR MODS	CHARGE MODS	MODEL
PAIRS	PAIR DENSITY	$n_P(\mathbf{r})$	$Q_{Im}\left(\frac{2\pi}{a}\right) = 1/4$	-	IZ
VORTEX	QP DENSITY	$N(\mathbf{r}, \sim\Delta_1)$	$Q_\Delta\left(\frac{2\pi}{a}\right) = 1/8$	$Q_C\left(\frac{2\pi}{a}\right) = 1/8$ $Q_C\left(\frac{2\pi}{a}\right) = 1/4$	GLW(8a ₀)
GAP	(PSEUDO)GAP	$\Delta(\mathbf{r})$	$Q_\Delta\left(\frac{2\pi}{a}\right) = 1/8$	$Q_C\left(\frac{2\pi}{a}\right) = 1/8$ $Q_C\left(\frac{2\pi}{a}\right) = 1/4$	GLW(8a ₀)
QP DOS	QP DENSITY	$N(\mathbf{r}, \sim\Delta_1)$	$Q_P\left(\frac{2\pi}{a}\right) = 1/8$	$Q_C\left(\frac{2\pi}{a}\right) = 1/8$ $Q_C\left(\frac{2\pi}{a}\right) = 1/4$	RMFtj(8a ₀)
PG QPI	INTERFERENC E	$\Lambda(\mathbf{q}, \sim\Delta_0)$	$Q_P\left(\frac{2\pi}{a}\right) = 1/8$		RMFtj(8a ₀)
PAIRS	PAIR DENSITY	$n_P(\mathbf{r})$	$Q_{Im}\left(\frac{2\pi}{a}\right) = 1/4$	NONE	IZ



Xiaolong Liu
Notre Dame



Shuqiu Wang
Oxford



P. Choubey
IIT Delhi



Weijiong Chen
Oxford

T
H
A
N
K
S
!



S. Uchida
Tokyo



H. Eisaki
Tsukuba



P.D. Johnson
Oxford



Kazuhiro Fujita
BNL



P.J. Hirschfeld
Florida



M. H. Hamidian
Cornell



S.D. Edkins
Cornell



A.P. Mackenzie
Dresden

'FUJITA CONJECTURE'

DSC part

$$\Delta_T^S = |\Delta_0| e^{i\theta_T}$$

$$\Delta_S^S = |\Delta_0| e^{i\theta_S}$$

Josephson Current

$$I_J^S = \frac{K}{i\hbar} (\Delta_T \Delta_S^* - \Delta_T^* \Delta_S) = \frac{2K}{\hbar} |\Delta_0|^2 [\sin(\theta_T - \theta_S)]$$

PDW part

$$\Delta_T^D = |\Delta_0| \alpha e^{i(Q(x-\delta))} e^{i\theta_T}$$

$$\Delta_S^D = |\Delta_0| \alpha e^{i(Qx)} e^{i\theta_S}$$

Josephson Current

$$I_J^D = \frac{K}{i\hbar} (\Delta_T \Delta_S^* - \Delta_T^* \Delta_S)$$

$$= \frac{2K}{\hbar} |\Delta_0|^2 \alpha^2 [\sin(-Q\delta + \theta_T - \theta_S)]$$

Normal Josephson effect

$$I_J = I_c \sin(\theta_T - \theta_S)$$

$$I_m R_N^2 \propto I_c^2$$

Assume $\theta_T - \theta_S = \frac{\pi}{2}$

$$I_m R_N^2 \propto I_J^{S^2} + I_J^{D^2} = \left(\frac{2K}{\hbar}\right)^2 |\Delta_0|^4 (1 + \alpha^2 \sin(Q\delta)^2)$$