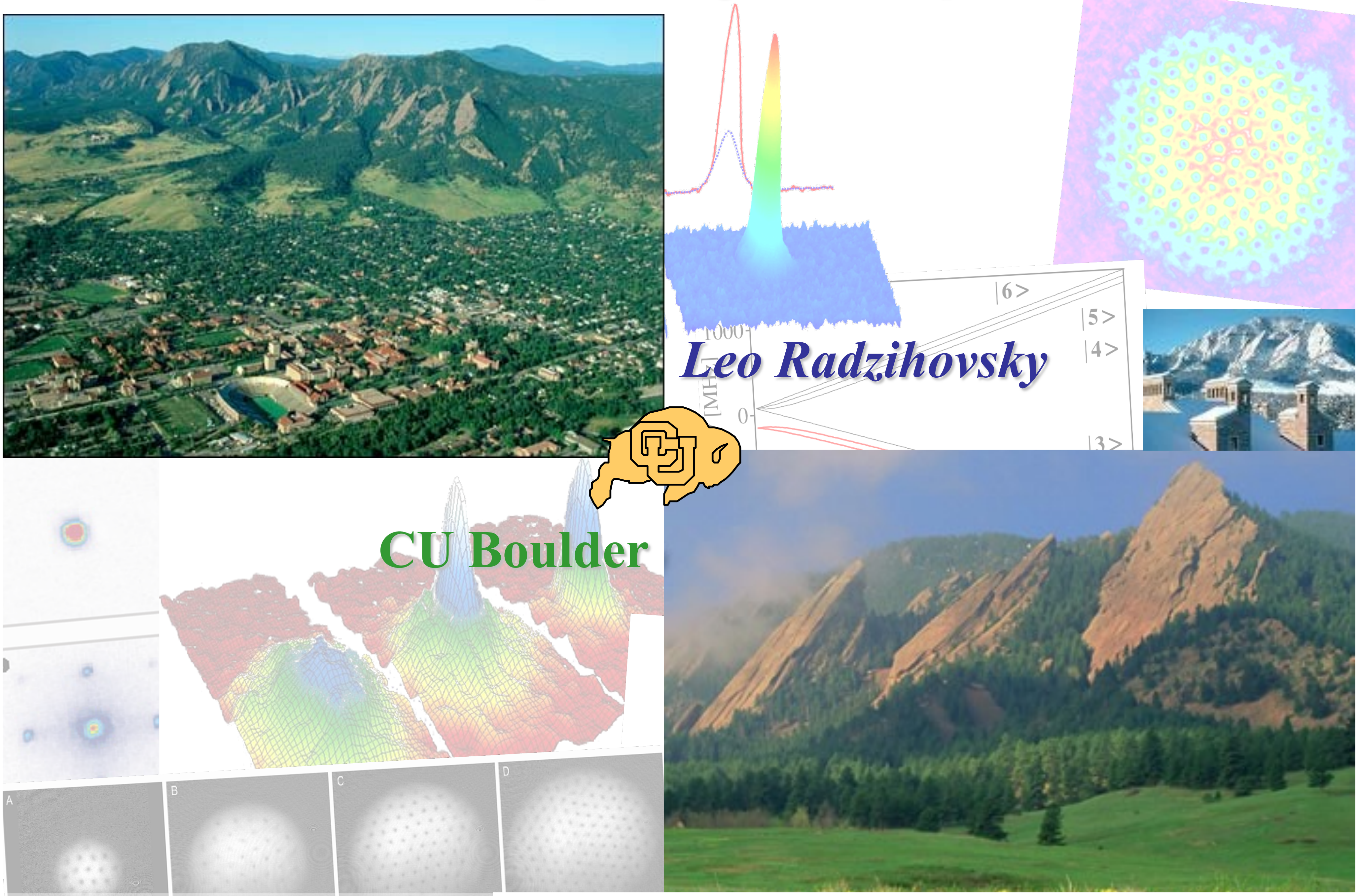
Quantum liquid-crystal superfluids



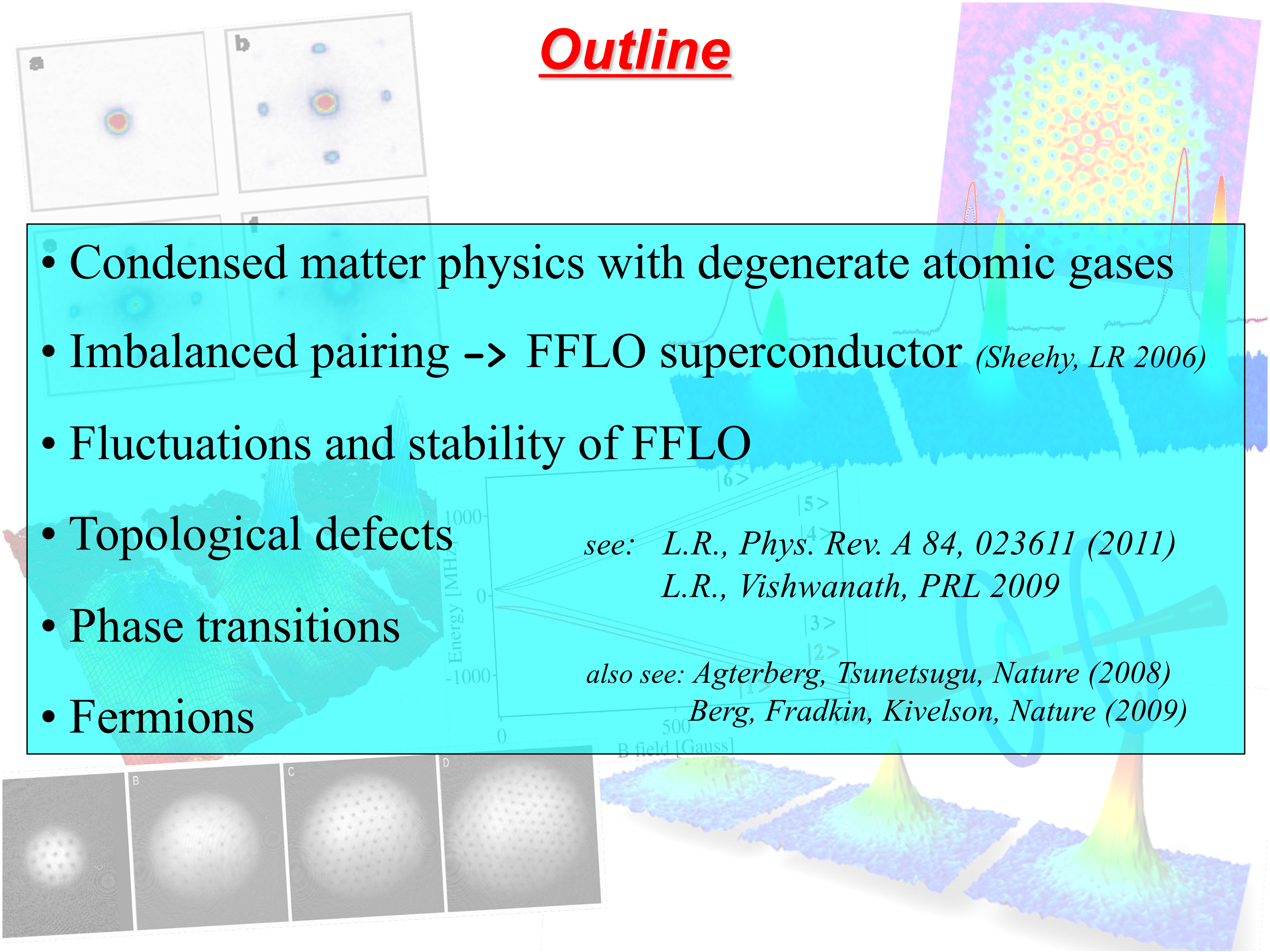
Leo Radzihovsky

CU Boulder

$: NSF, Simons Foundation

PDW Workshop, KITP, 9-20, 2022

Outline



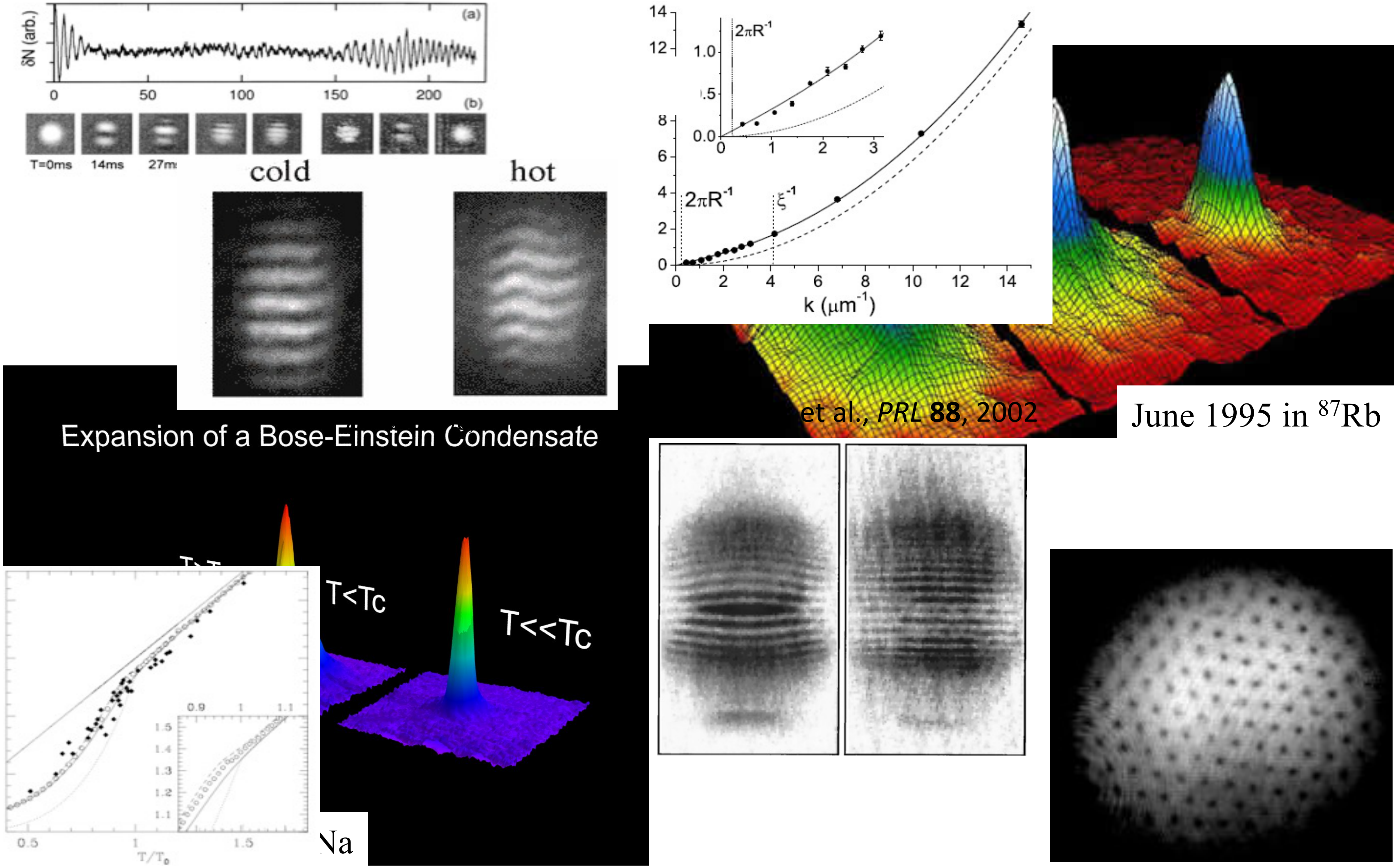
- Condensed matter physics with degenerate atomic gases
- Imbalanced pairing -> FFLO superconductor *(Sheehy, LR 2006)*
- Fluctuations and stability of FFLO
- Topological defects
- Phase transitions
- Fermions

see: L.R., Phys. Rev. A 84, 023611 (2011)
L.R., Vishwanath, PRL 2009

also see: Agterberg, Tsunetsugu, Nature (2008)
Berg, Fradkin, Kivelson, Nature (2009)

Revolution in AMO physics

- degenerate Bose and Fermi atomic gases



Expansion of a Bose-Einstein Condensate

et al., *PRL* **88**, 2002

June 1995 in ^{87}Rb

New playground for condensed matter physics

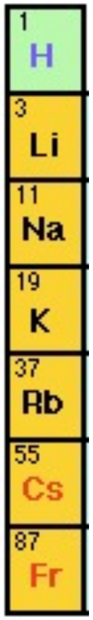
- highly coherent
- nonequilbrium
- tunable/designable
- weakly/strongly interacting

quantum many-body systems

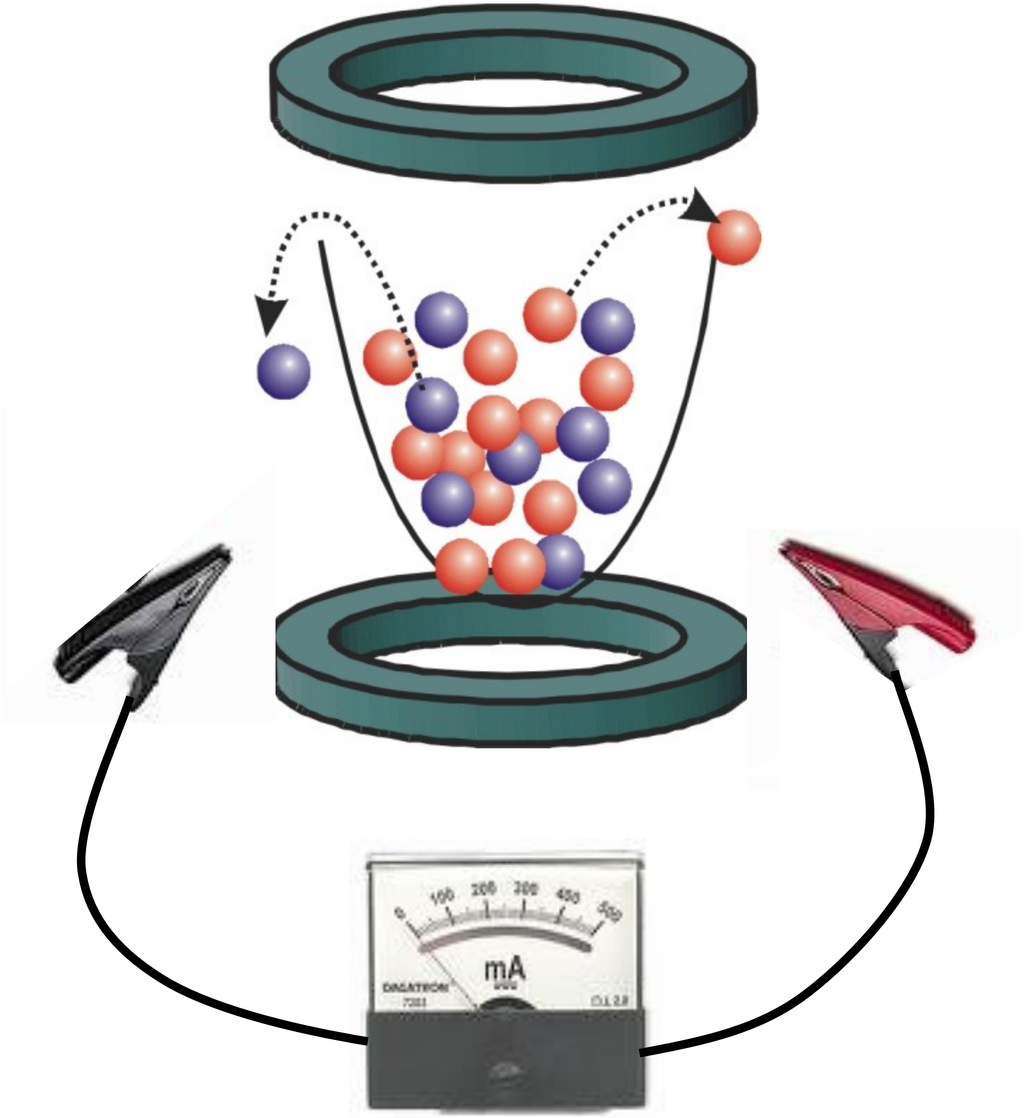
Holy Grail for 30 years, spin-polarized hydrogen *(Dan Klepner)*

now:

- degenerate gases in most AMO labs, in most alkali's
- exploring thermodynamics, excitations,…
- quantum dynamics and interactions
- huge number of probes



Condensed matter with cold atomic gases?



need strong interactions

Feshbach resonances on youtube

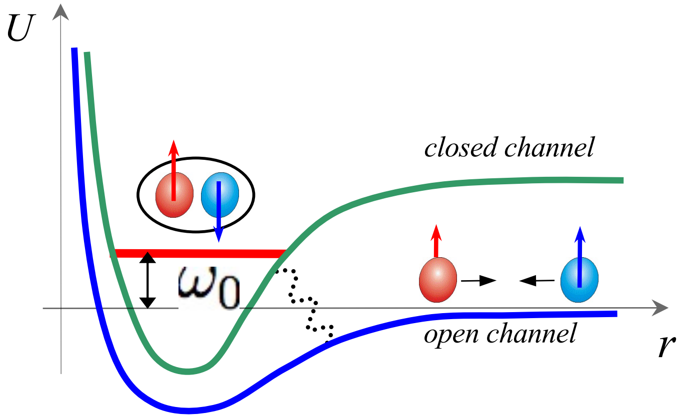
"*Quantum decoupling transition in a one-dimensional Feshbach-resonant superfluid*"
Sheehy and Radzihovsky, PRL (2005)

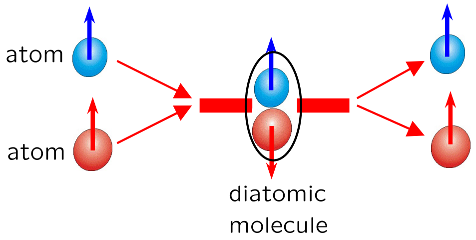
I am writing a song a day.

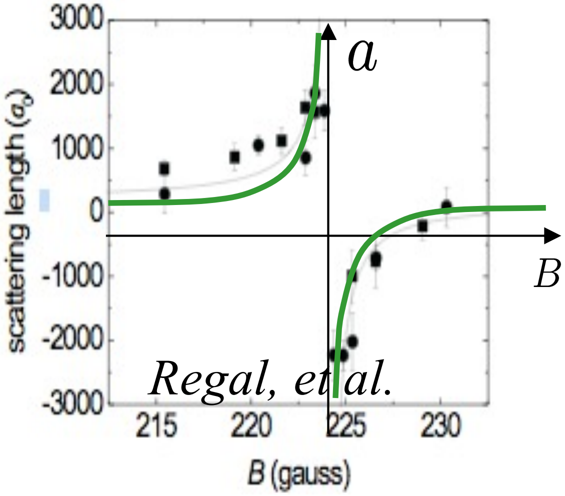
(song by Jonathan Mann, 2009)

Strong correlations via Feshbach resonance

- tunability (strength and sign) of interactions (sudden and adiabatic)







Regal, et al.

- s-wave BCS-BEC superfluidity
- p-wave superfluidity *(see e.g., Gurarie and LR, AOP 2007)*
- polarized superfluidity *(see e.g., Sheehy and LR, AOP 2007)*

…quite well understood:

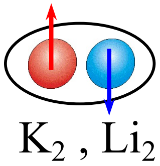
- *quantitatively for narrow (Γ/ε_F <<1) resonance*
- *qualitatively for broad (Γ/ε_F >>1) resonance*

-> mft, 1/N, ε-expansions ⟶ universality

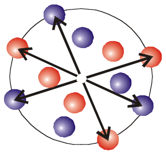
(Veillette, Sheehy, LR '07; Nikolic, Sachdev '07; Nishida, Son '06)

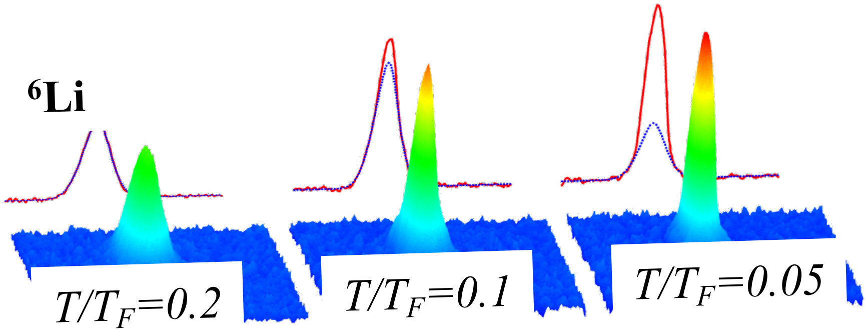
Paired fermionic superfluids via FBR

- molecular BEC *(Regal, Jin '03)*



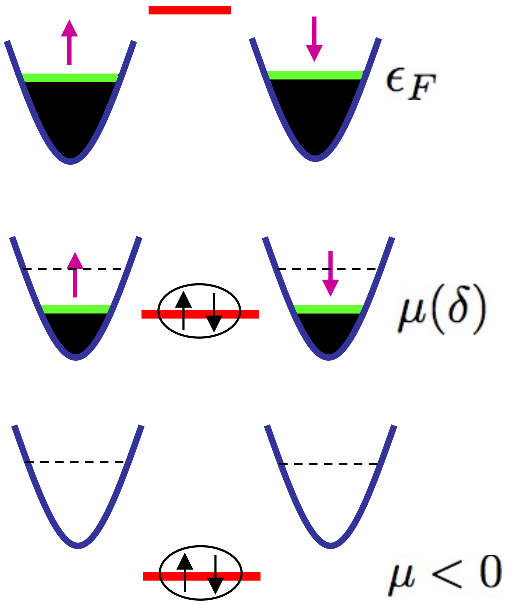
- BCS superfluid *(Regal, Jin 04 Zwierlein, Ketterle '04)*

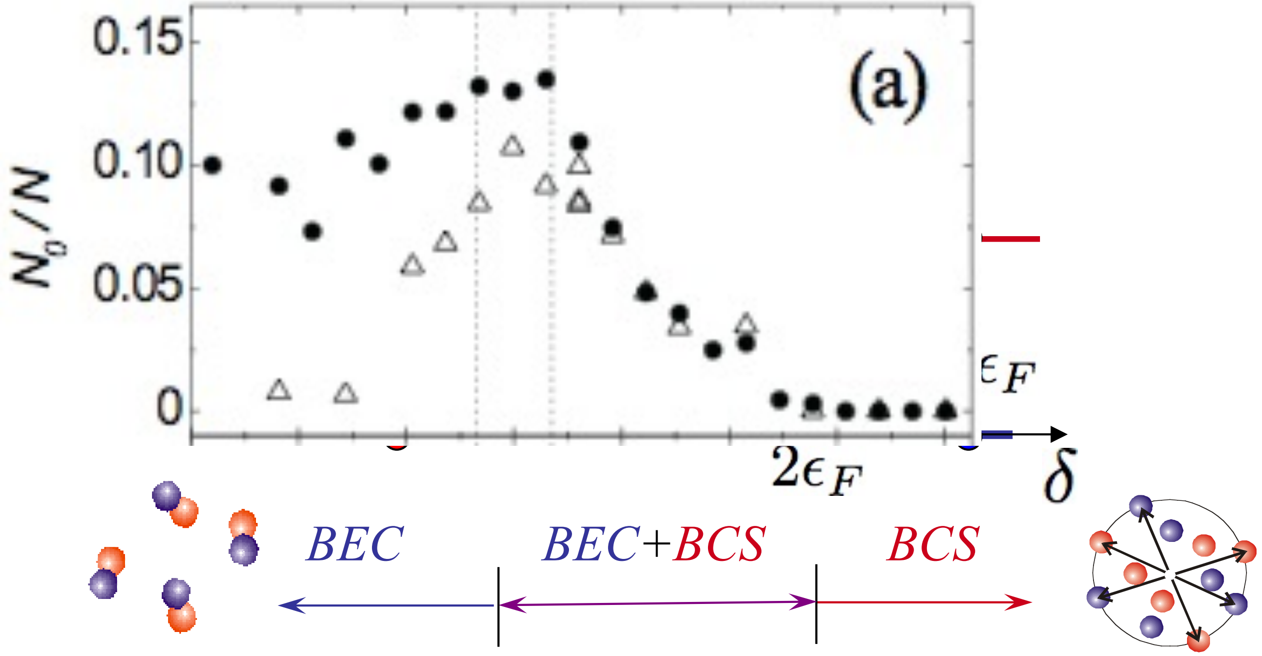




- BCS-BEC crossover:

$$\mathcal{H}_{2ch} = \psi_\sigma^\dagger\left(\frac{\hat{p}^2}{2m}-\mu\right)\psi_\sigma + \phi^\dagger\left(\frac{\hat{p}^2}{4m}-2\mu+\epsilon_0\right)\phi - g\phi\psi_\uparrow^\dagger\psi_\downarrow^\dagger$$



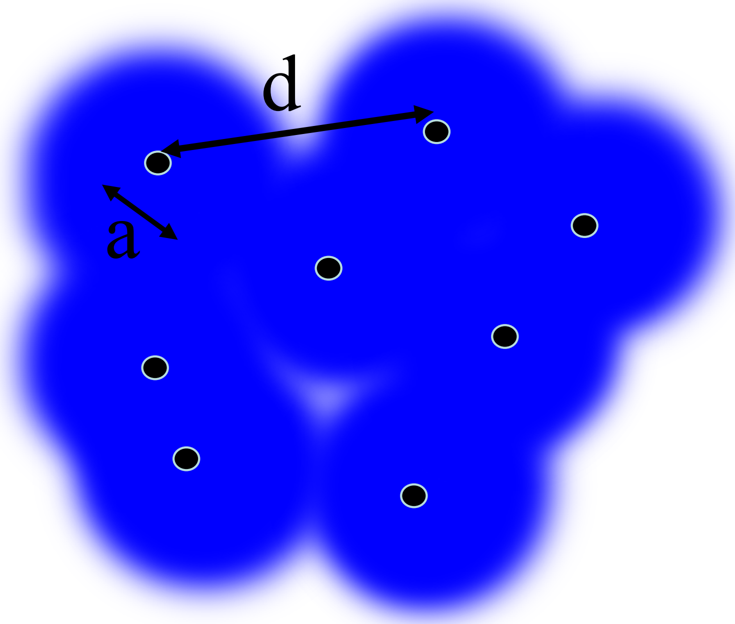


$\gamma >> 1$, $k_F a \to \infty$

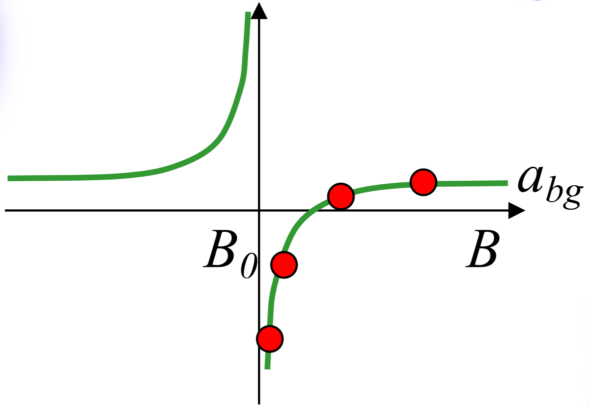
Universality at unitary point

$\mathcal{H}_{2ch}$ → $\mathcal{H}_{1ch} = \psi^\dagger_\sigma(\frac{p^2}{2m} - \mu_\sigma)\psi_\sigma + \lambda\psi^\dagger_\uparrow\psi^\dagger_\downarrow\psi_\downarrow\psi_\uparrow$

$f_k = -1/(a^{-1} + i\,k) \rightarrow i/k$, --> $d = 1/k_F$ is the only scale

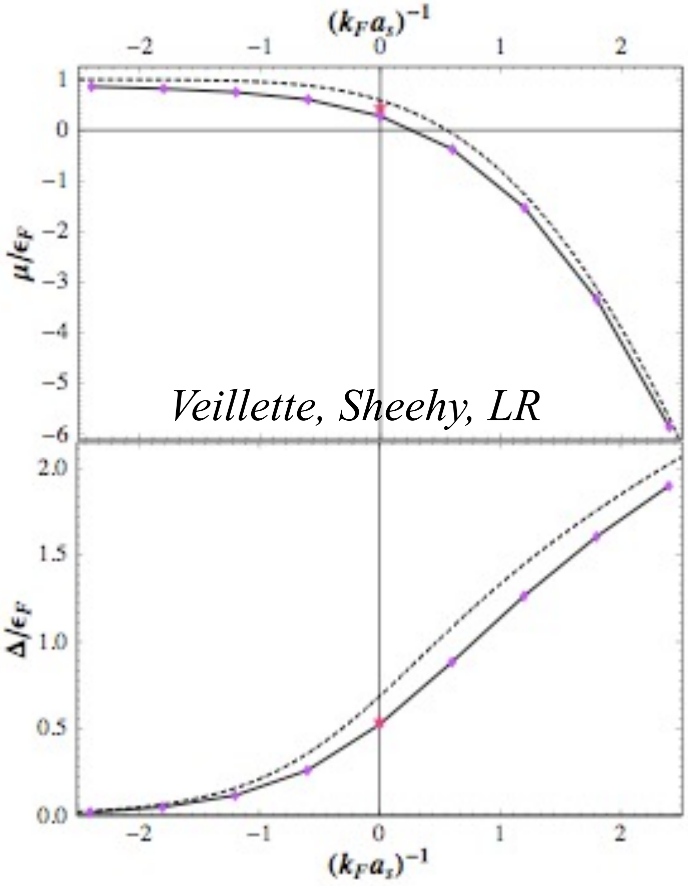


$$a = a_{bg}(1 - \frac{\Delta B}{B - B_0})$$



check in $N \to \infty$ (BCS) limit:

$$\frac{m}{2\pi\hbar^2 a} \to 0 = \int_k \left(\frac{1}{E_k} - \frac{1}{\epsilon_k}\right)$$



Veillette, Sheehy, LR

$$\boxed{f(T,n) = n\epsilon_F \hat{f}(k_B T/\epsilon_F)}$$

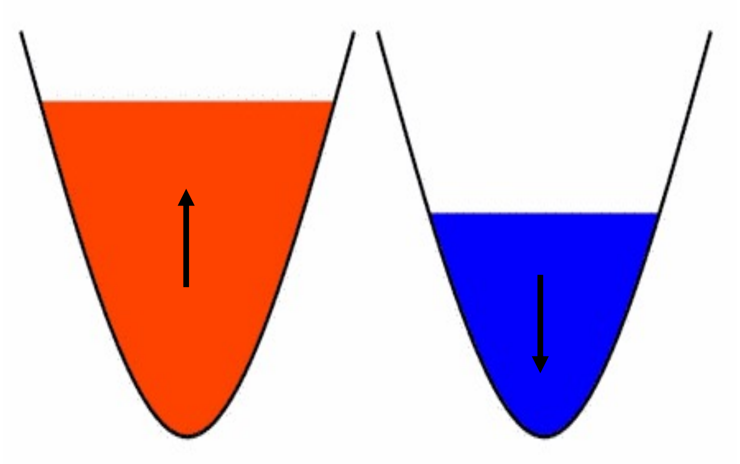
$$\begin{aligned}
\epsilon &= \xi\frac{3}{5}\epsilon_F \\
\mu &= \xi\epsilon_F \\
\Delta &= \alpha\epsilon_F \\
\Delta_{exc} &= \alpha_{exc}\epsilon_F \\
k_B T_c &= \gamma\epsilon_F \\
B &= \xi\frac{2}{3}n\epsilon_F
\end{aligned}$$

1/N theory $\xi = 0.5906 - 0.312/N + \cdots$

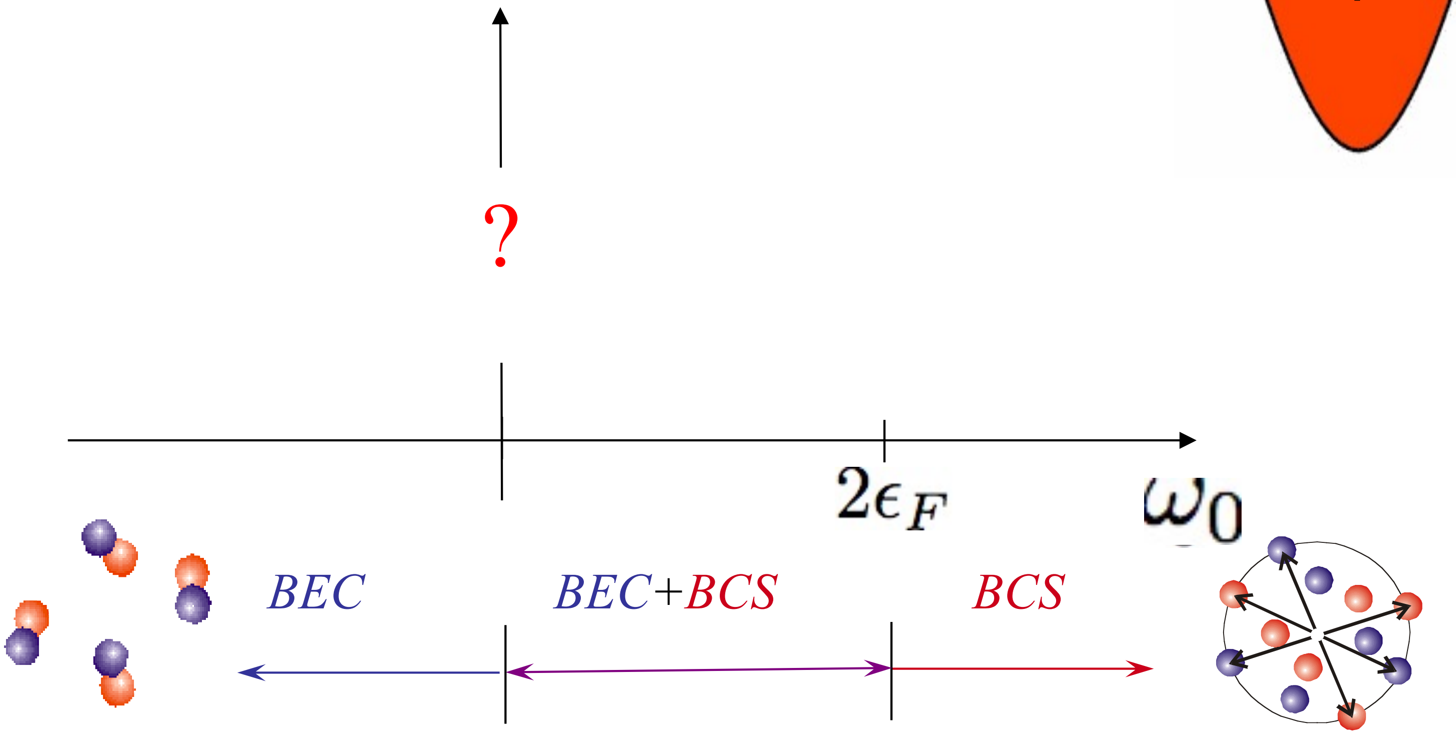
Exp with ⁴⁰K $\xi = 0.46^{+0.05}_{-0.12}$

Imbalanced ("magnetized") BEC-BCS

- motivation: *superconductivity in B field, quarks-gluon plasma,…*
- natural realization in cold atoms: $H_h = H - h(N_\uparrow - N_\downarrow)$



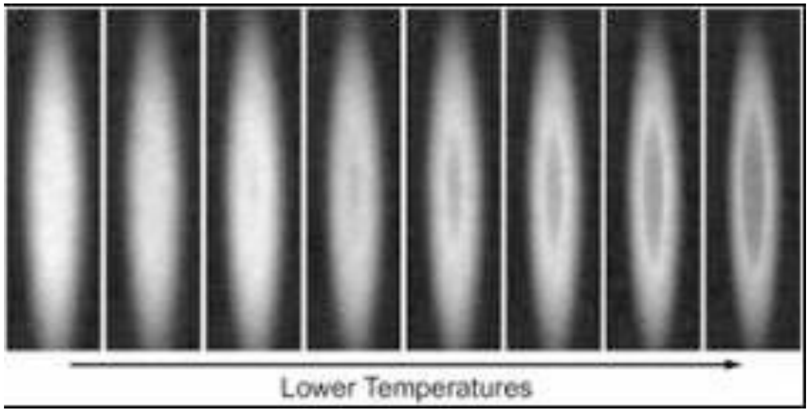
$\Delta N = N_\uparrow - N_\downarrow$

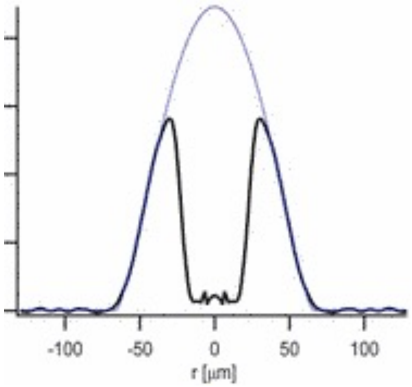


Imbalanced BEC-BCS experiments

- MIT experiments (vortices, phase separation)

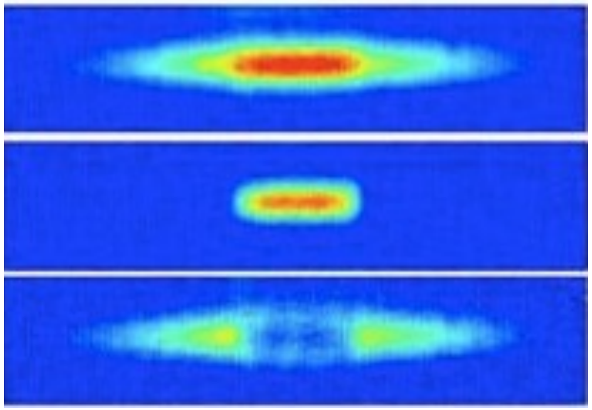
Science (2006)

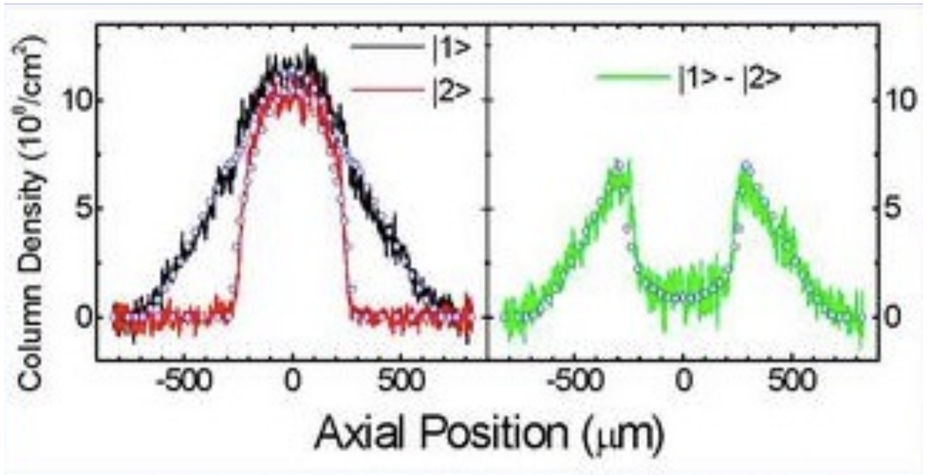




- Rice experiments (phase separation, surface tension)

Science (2006)



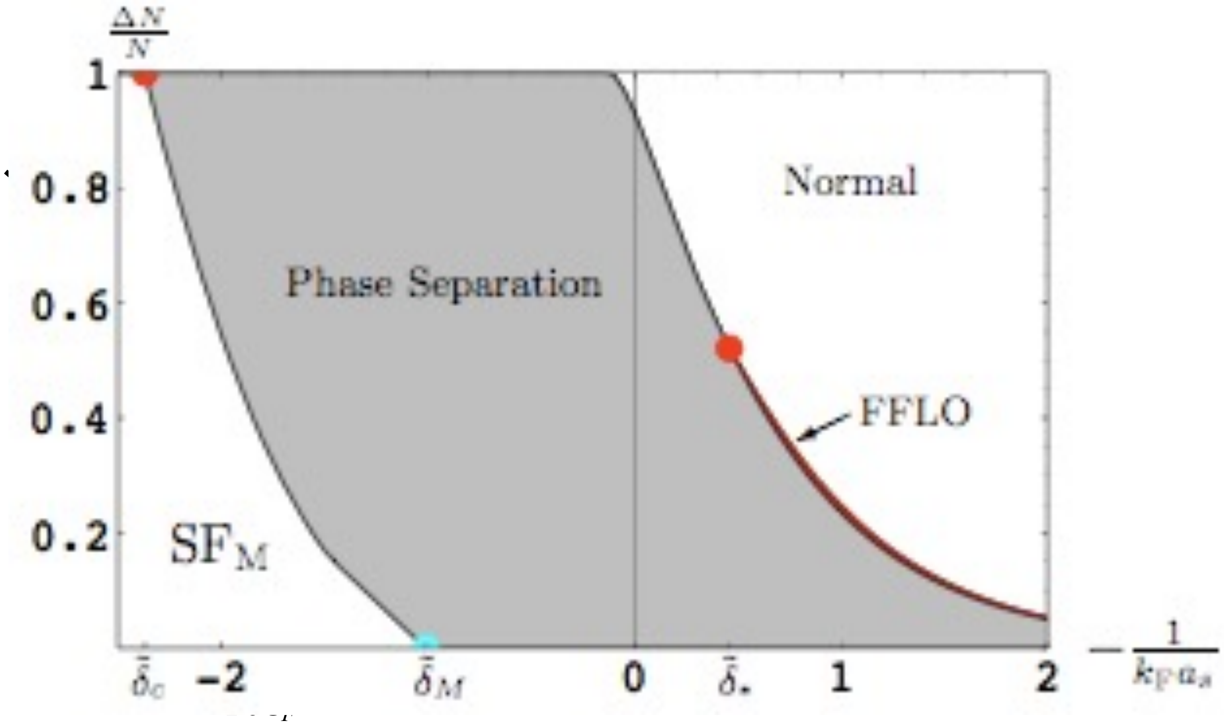


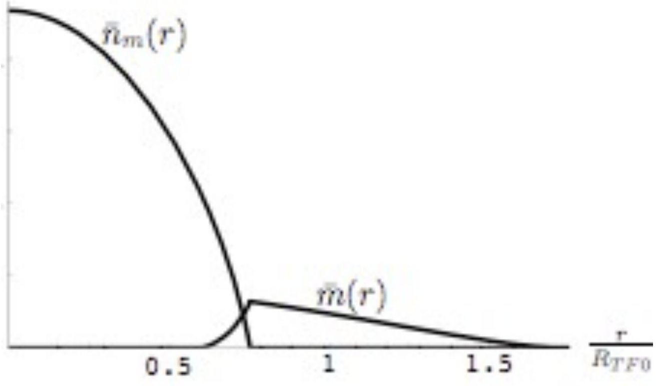
Imbalanced BEC-BCS

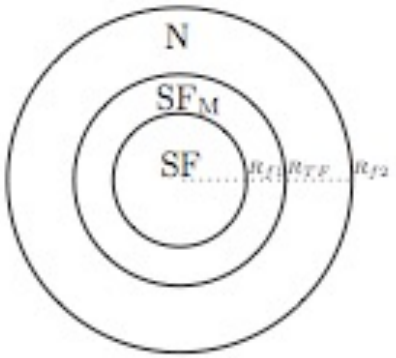
Sheehy, L.R. '05

- 1st order transitions and phase separation

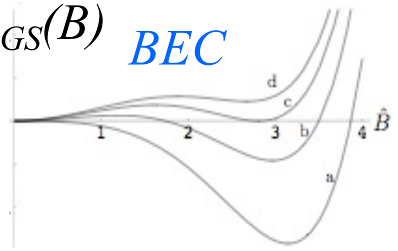
T=0







$E_{GS}(B)$ BEC

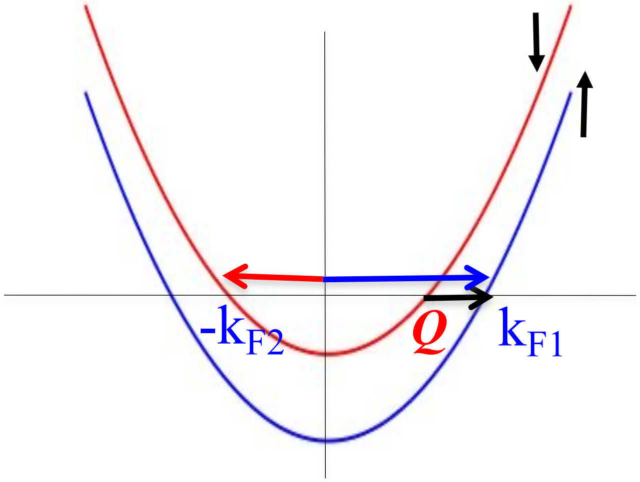


$E_{GS}(\Delta)$ BCS

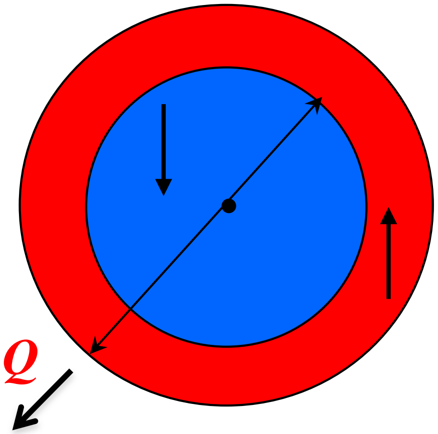
Fulde-Ferrell-Larkin-Ovchinnikov states

$$\mathcal{H}_{\text{pairing}} \sim \psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow$$

F-F, '64
L-O, '64



generic in d = 1 (bosonization)



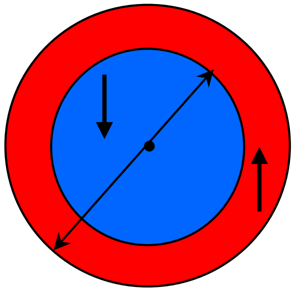
fragile in d > 1

$$\longrightarrow \quad \mathcal{H}_{\text{pairing}} \sim a^\dagger_{-k\downarrow} a^\dagger_{k+Q\uparrow}$$

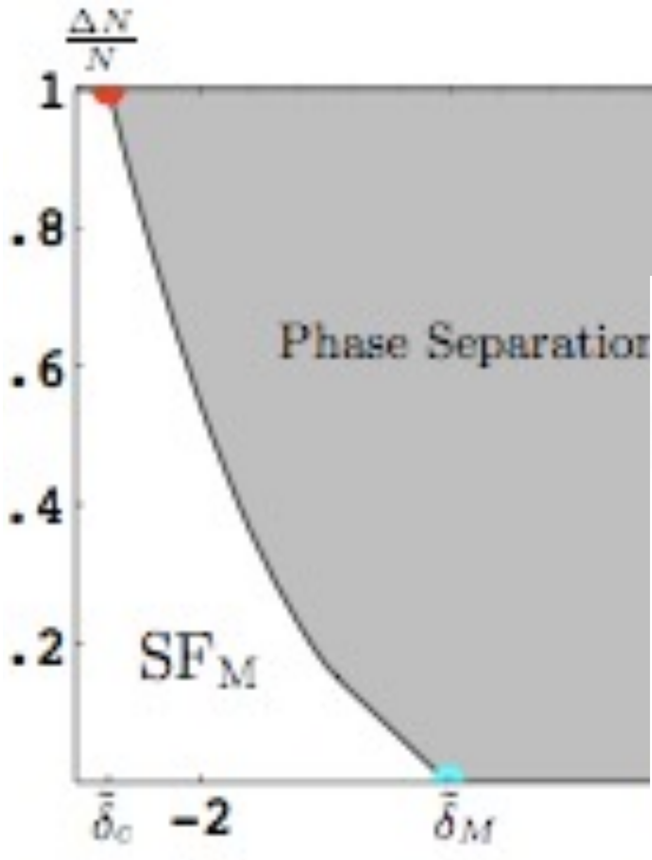
- Fermi surface mismatch: $k_{F1} > k_{F2}$ -> ***pair at $Q = k_{F1} - k_{F2}$***
- Pair-density wave: $\Delta = \sum_{\mathbf{Q}} \Delta_{\mathbf{Q}} e^{i\mathbf{Q}\cdot\mathbf{x}}$

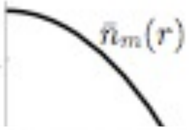
Imbalanced BEC-BCS

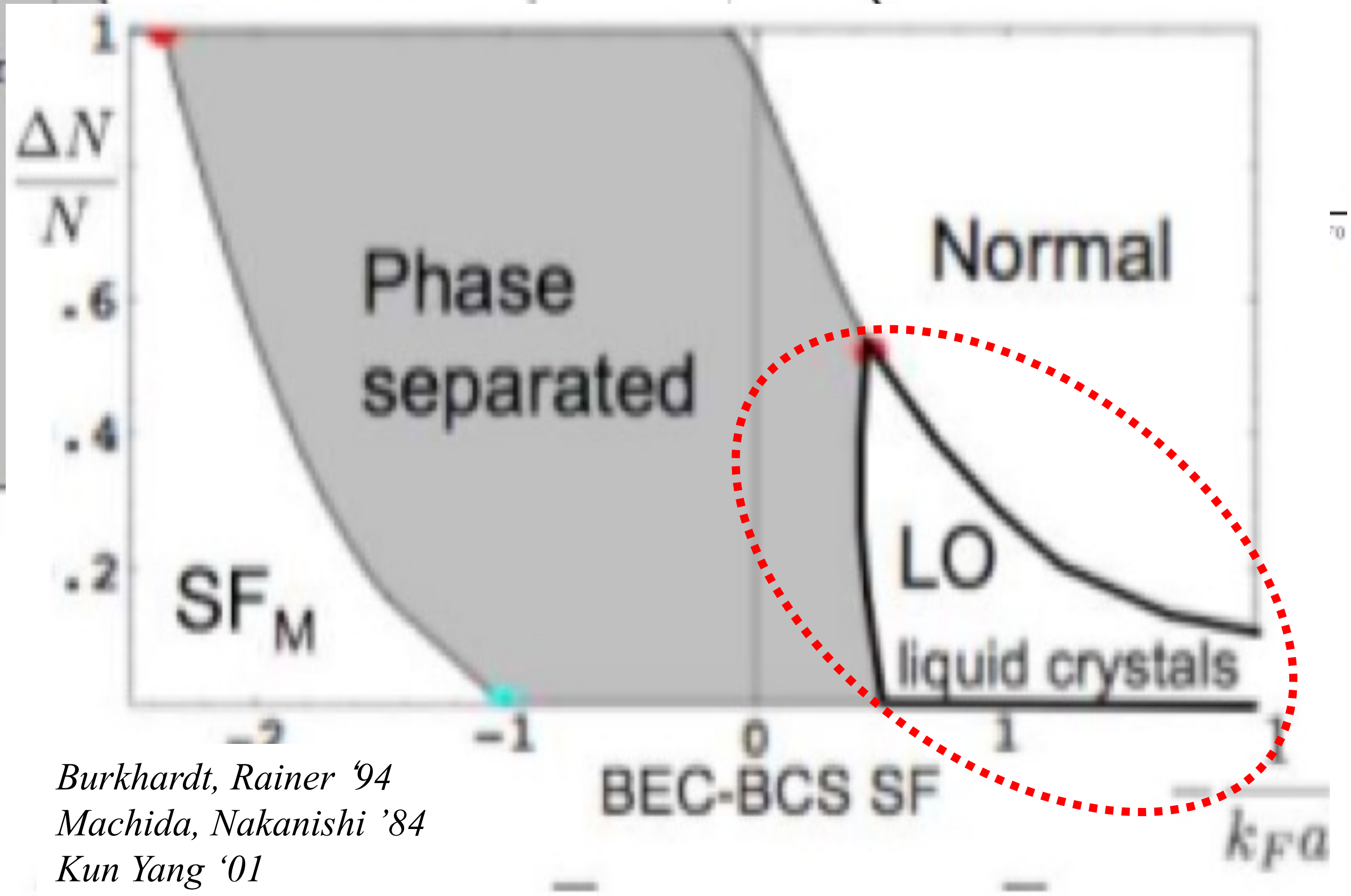
Sheehy, L.R. '05



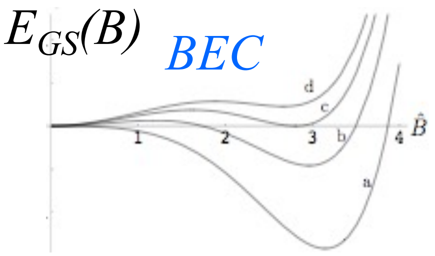
- 1st order transitions and phase separation *T=0*

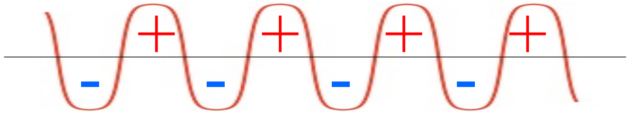




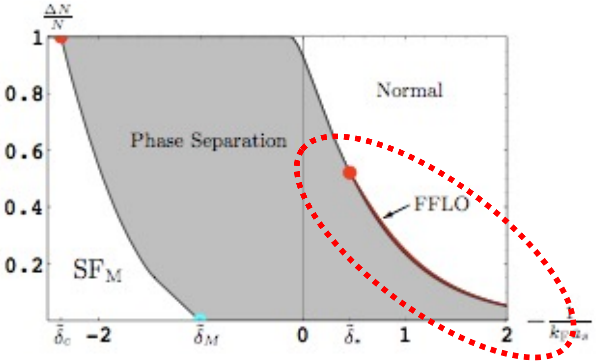


Burkhardt, Rainer '94
Machida, Nakanishi '84
Kun Yang '01



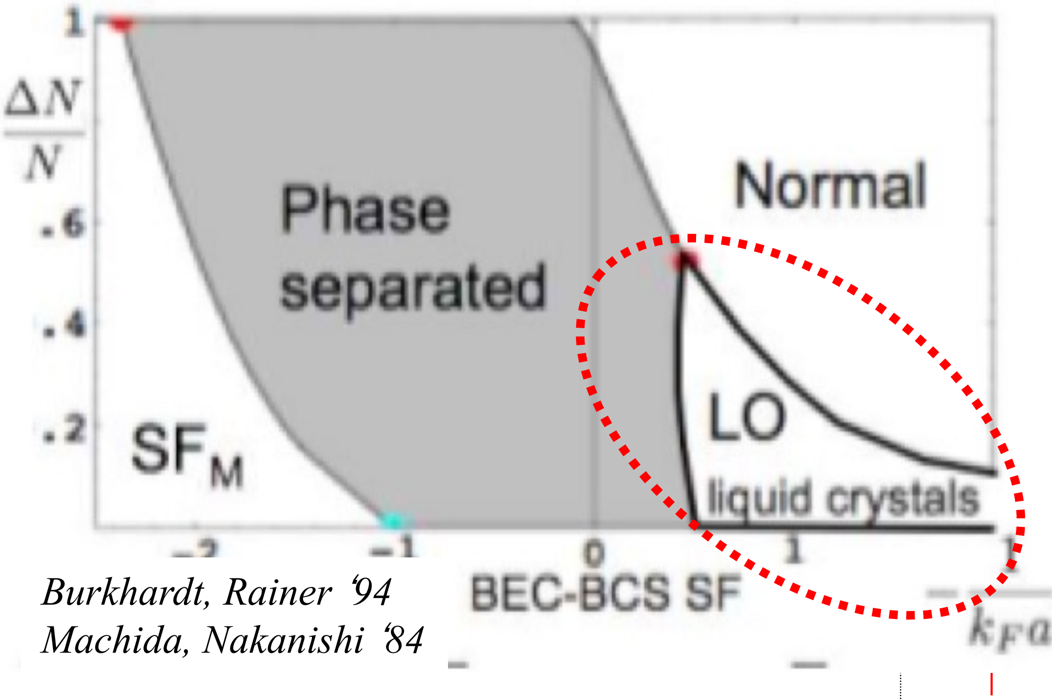


FFLO state

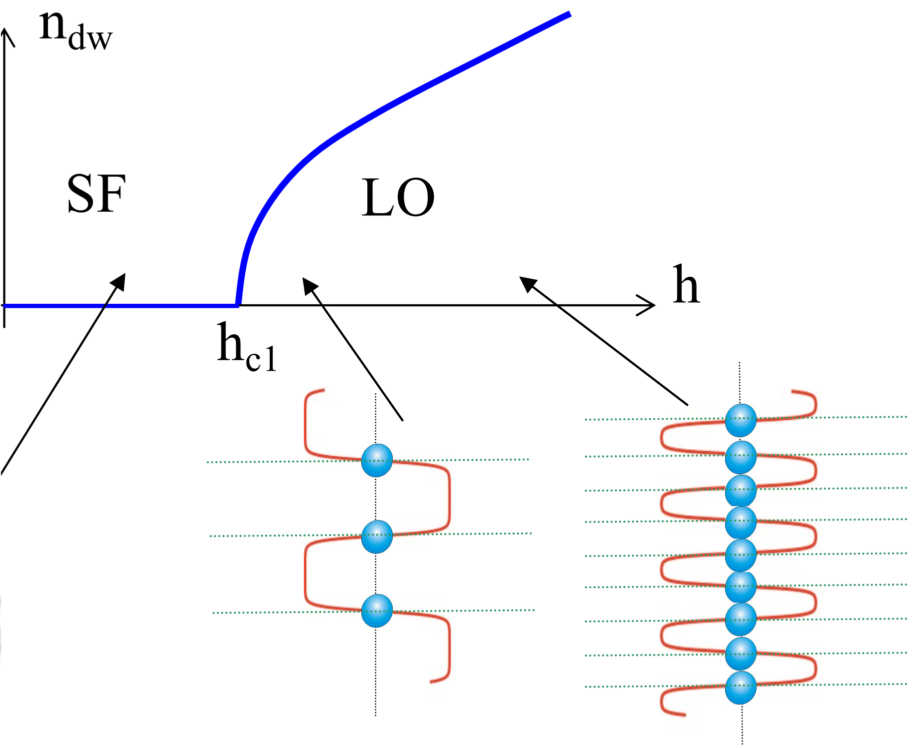


- pair "density" wave: $\Delta = \sum_{\mathbf{Q}} \Delta_{\mathbf{Q}} e^{i\mathbf{Q}\cdot\mathbf{x}}$
- motivation:
  - *stabilized in lower dimensions*
  - *negative surface tension for ± Δ domain wall*

⟹ *SF → LO: C-I transition of domain-wall proliferation*



Burkhardt, Rainer '94
Machida, Nakanishi '84



- excess fermions sit on domain walls *(cf. polyacetylene of Schrieffer, Su, Heeger)*
- microphase separation *(cf. H_{c1} transition to vortex state in type II sc's)*

Microscopics to Ginzburg-Landau

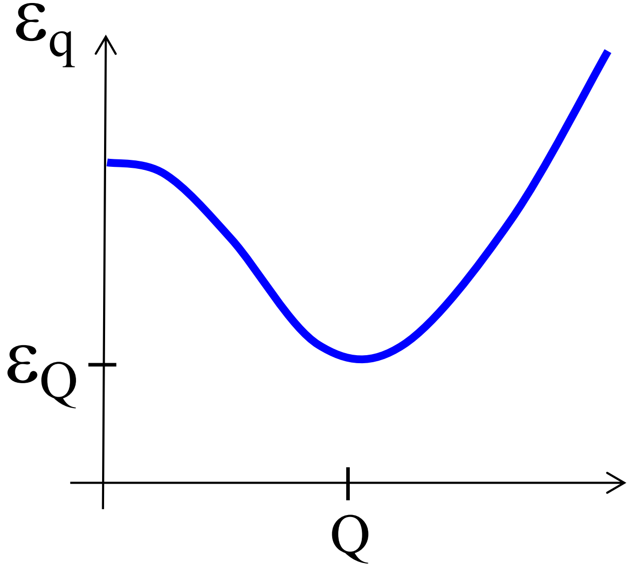
LR, AV, PRL'09
Samokhin '10

$H_{BCS}[c_\sigma, c_\sigma^\dagger] \xrightarrow{near\ h_{c2}}$ with $\Delta = V\langle c_\downarrow c_\uparrow \rangle$

$$H_{GL}[\Delta] = \sum_{\mathbf{q}} \bar{\Delta}_{\mathbf{q}} \varepsilon_q \Delta_{\mathbf{q}} + \sum_{\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3} v_{\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3,\mathbf{q}_4} \bar{\Delta}_{\mathbf{q}_1} \Delta_{\mathbf{q}_2} \bar{\Delta}_{\mathbf{q}_3} \Delta_{\mathbf{q}_4} + \ldots$$

$$\approx J\bar{\Delta}\left(-\nabla^2 - Q^2\right)^2 \Delta + \varepsilon_Q |\Delta|^2 + \frac{v_1}{2}|\Delta|^4 + \frac{v_2}{2}\,\mathbf{j}^2 + \ldots$$

$$\varepsilon_q \approx \frac{3n}{4\epsilon_F}\left[-1 + \frac{1}{2}\ln\frac{v_F^2 q^2 - 4h^2}{\Delta_{BCS}^2} + \frac{h}{v_F q}\ln\frac{v_F q + 2h}{v_F q - 2h}\right]$$



$$J \approx \frac{n}{\epsilon_F Q^4}$$

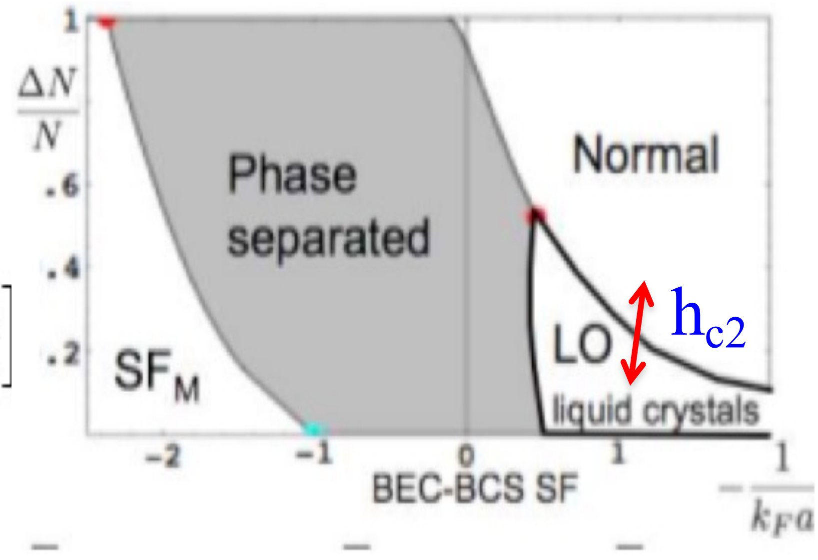
$$Q \approx \frac{\Delta_{BCS}}{\hbar v_F}$$

$$\varepsilon_Q \approx \frac{n}{\epsilon_F}\ln\left[\frac{h}{h_{c2}}\right]$$

$$h_{c2} \approx \frac{3}{4}\Delta_{BCS}$$

$$v_1 \approx \frac{n}{\epsilon_F \Delta_{BCS}^2}$$

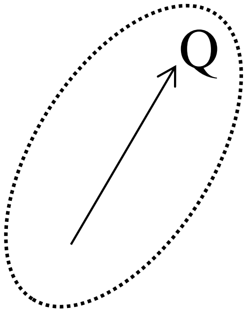
$$v_2 \approx \frac{nm^2}{\epsilon_F \Delta_{BCS}^2 Q_0^2}$$



Broken symmetries in LO/FF states

LR, Vishwanath PRL,'09
H. Shimahara, J.Phys '98

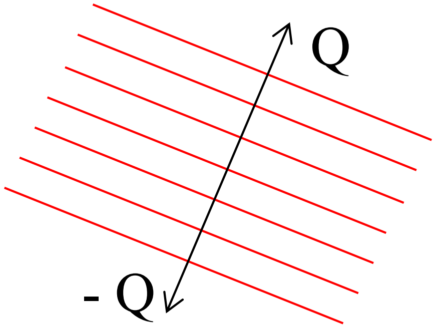
- Fulde-Ferrell: $\Delta_{FF}(\mathbf{x}) = \Delta_{\mathbf{Q}} e^{i\mathbf{Q}\cdot\mathbf{x}}$



\- broken: *time reversal, orientational, off-diagonal*

"*vector*" superfluid

- Larkin-Ovchinnikov: $\Delta_{LO}(\mathbf{x}) = \Delta_{\mathbf{Q}} \cos \mathbf{Q}\cdot\mathbf{x}$



\- broken: *orientational, translational, off-diagonal*

"*smectic*" superfluid

superfluid liquid crystals

Low-energy excitations in LO/FF states

- order parameter: $\Delta_{LO}(\mathbf{x}) = \Delta_0 e^{i\theta_+} e^{i\mathbf{Q}\cdot\mathbf{x}} + \Delta_0 e^{i\theta_-} e^{-i\mathbf{Q}\cdot\mathbf{x}}$

$$= 2\Delta_0 e^{i\theta} \cos[\mathbf{Q}\cdot\mathbf{x} - Qu]$$

- superfluid phase and phonon: $\theta = \frac{1}{2}(\theta_- + \theta_+)$ $u = \frac{1}{2Q}(\theta_- - \theta_+)$

- coupled incommensurate smectics u_+, u_- :

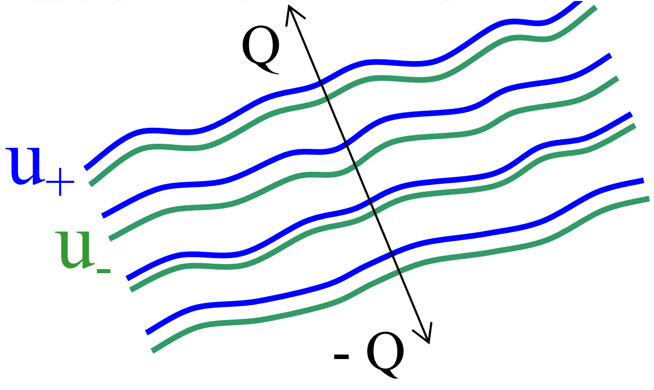
$$\mathcal{H}_{LO} \approx \underbrace{\frac{K}{2}(\nabla_\perp^2 u)^2 + \frac{B}{2}(\partial_z u)^2}_{\textit{smectic elasticity}} + \underbrace{\frac{\rho_s^i}{2}(\nabla_i \theta)^2}_{\textit{superfluid stiffness}}$$

cf: SF with SOI

$$E[u_\pm^0(\mathbf{x})] = 0 \quad \text{for} \quad u_\pm^0(\mathbf{x}) = z(\cos\phi - 1) + x\sin\phi$$

- superfluid stiffness *anisotropy*:

$$\frac{\rho_s^\perp}{\rho_s^\parallel} = \left(\frac{\Delta_Q}{\Delta_{BCS}}\right)^2 \approx \ln\left(\frac{h_{c2}}{h}\right) \ll 1$$

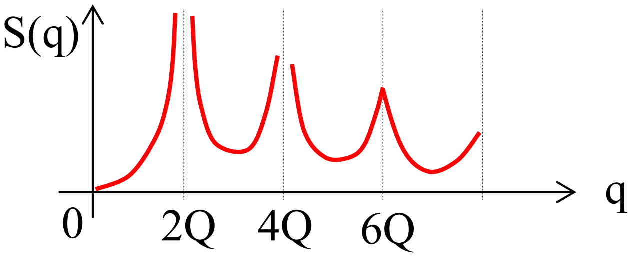


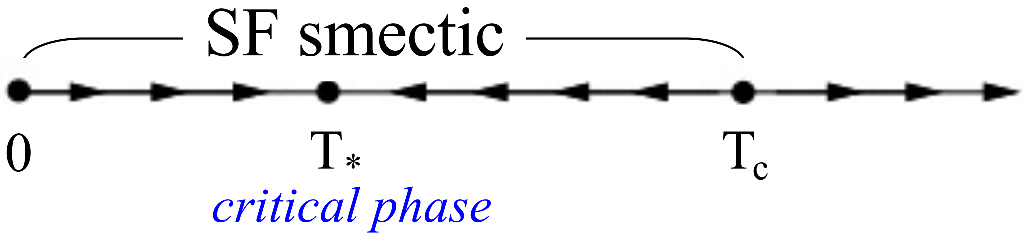
Fluctuations and stability of LO/FF states

- fluctuations at T=0: $\mathcal{L}_{LO} = \frac{\chi}{2}(\partial_\mu\theta)^2 + \frac{\rho}{2}(\partial_t u)^2 + \frac{B}{2}(\partial_z u)^2 + \frac{K}{2}(\nabla^2 u)^2$
  - $\langle\theta^2\rangle$, $\langle u^2\rangle$ ~ finite for d > 1 ⇒ *LO stable to quantum fluctuations*

- fluctuations at T≠0: $C_v \sim T^2 + cT^3$
  - $\langle\theta^2\rangle$ ~ finite for d > 2 ⇒ *SF order stable to $k_B T$ fluctuations*
  - $\langle u^2\rangle$ ~ diverges for d ≤ 3 ⇒ *positional order unstable*

➡ LO = superfluid smectic (SF_{sm}) with:
  - quasi-Bragg peaks (3d), Lorentzian (2d)
  - anomalous elasticity *(Grinstein and Pelcovits)*
  - transitions to superfluid nematic (SF_N)




SF smectic

0

T_*

critical phase

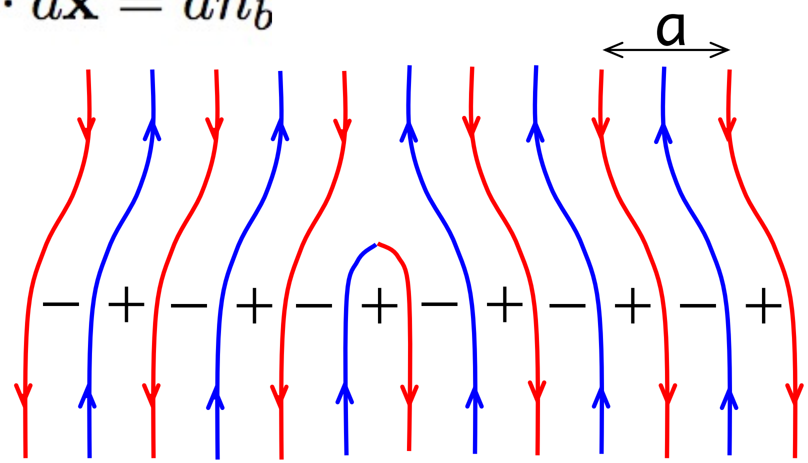
T_c

Topological defects

$$\Delta_{LO}(\mathbf{x}) = \Delta_0 e^{i\theta(\mathbf{x})}\cos[Q(z-u(\mathbf{x}))]$$

$$\frac{U(1)\times U(1)}{Z_2}$$

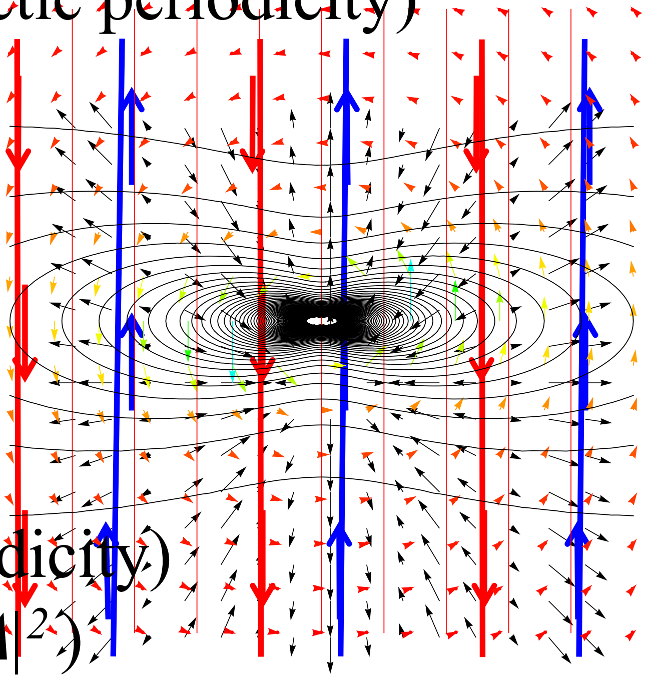
- integer dislocations in u: $\oint \nabla u \cdot d\mathbf{x} = a n_b$



$(n_v,n_b) = (0,1)$

  - destroy LO order ("charge"-*2* SF *and* full smectic periodicity)
  - retain "charge" ≥ *4* homogeneous SF (Δ^2)

- integer vortices in θ: $\oint \nabla\theta \cdot d\mathbf{x} = 2\pi n_v$



$(n_v,n_b) = (1,0)$

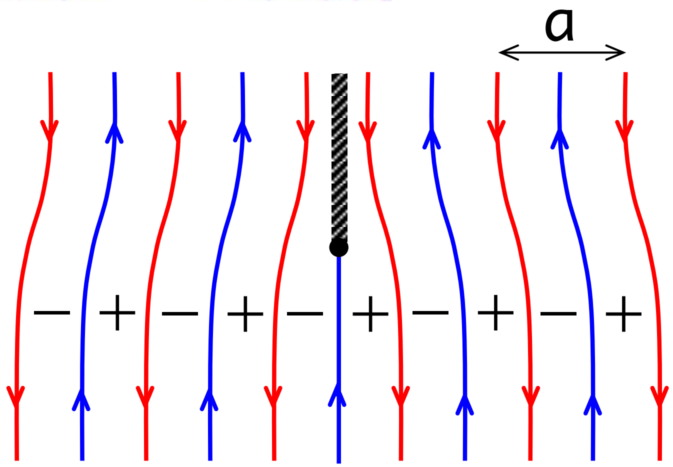
  - destroy LO order (full SF *and* *Q* smectic periodicity)
  - retain wavevector ≥ *2Q* smectic periodicity ($|\Delta|^2$)

Fractional topological defects

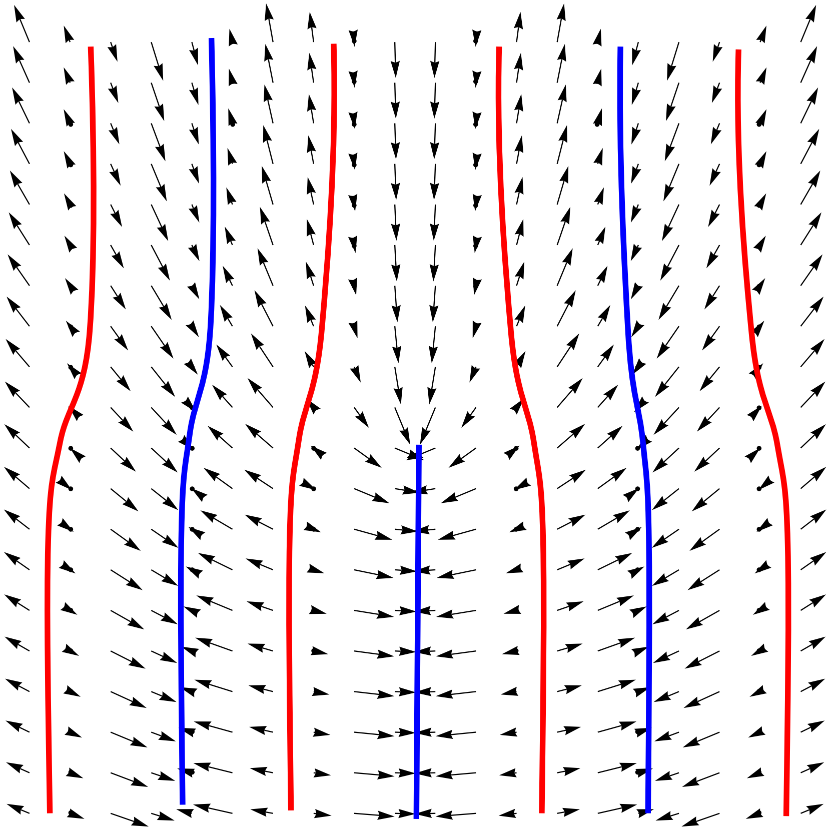
$$\frac{U(1)\times U(1)}{Z_2}$$

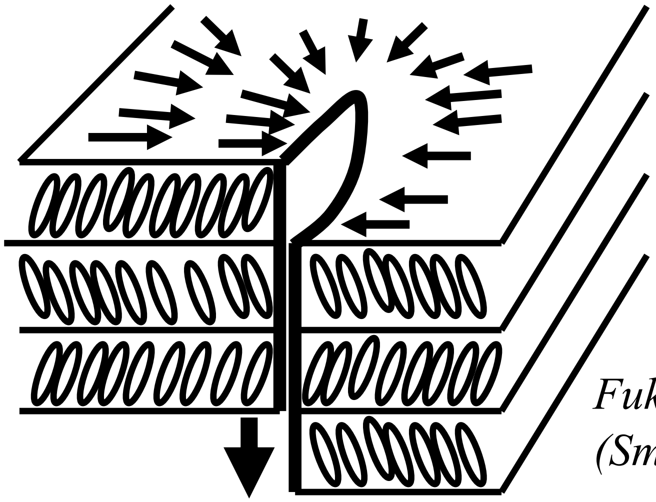
$$\Delta_{LO}(\mathbf{x}) = \Delta_0 e^{i\theta(\mathbf{x})}\cos[Q(z-u(\mathbf{x}))]$$

- π-vortex — a/2 dislocation pairs:



$(n_v,n_b) = (1/2,1/2)$ $(n_v,n_b) = (1/2,-1/2)$





dispiration in $\mathrm{SmC_A}$

Fukuda, et al., PRB 45 '92
(Smalyukh)

- destroy LO order
- restore full translational invariance and atom "conservation"

Topological defects energetics

$$\frac{U(1)\times U(1)}{Z_2}$$

$$\Delta_{LO}(\mathbf{x}) = \Delta_0 e^{i\theta(\mathbf{x})}\cos[Q(z-u(\mathbf{x}))]$$

- integer a-dislocation in u (composite): $\boxed{E_{(0,\ a)} \approx K\ L}$

- integer 2π-vortex in θ (composite): $E_{(2\pi,\ 0)} \approx \rho_s\ L \log L$

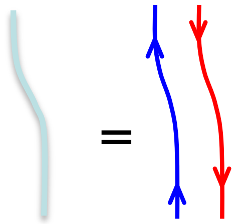
- π-vortex – a/2-dislocation (elementary): $E_{(\pm\pi,\ a)} \approx \frac{1}{4}\ \rho_s\ L \log L + \frac{1}{4}\ K\ L$

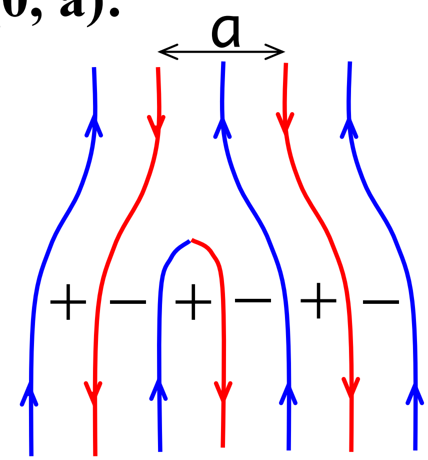
Composite defects (a-dislocation) unbind 1st -> "fractionalized" phases

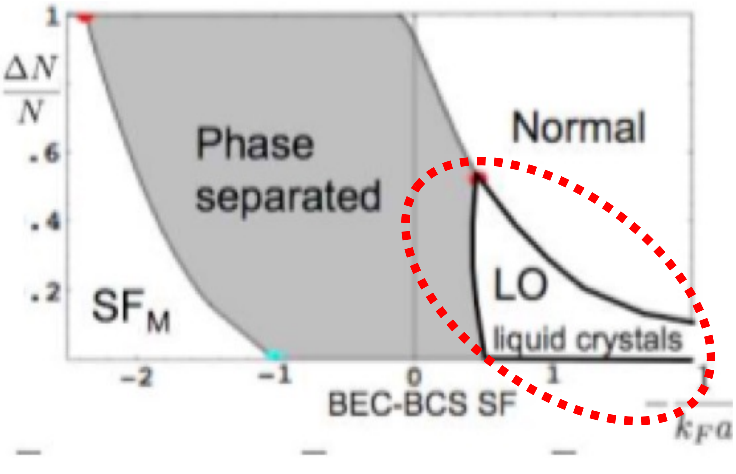
$\Delta_{LO}(\mathbf{x}) = \Delta_0 e^{i\theta(\mathbf{x})} \cos[Q(z - u(\mathbf{x}))]$

Phase transitions

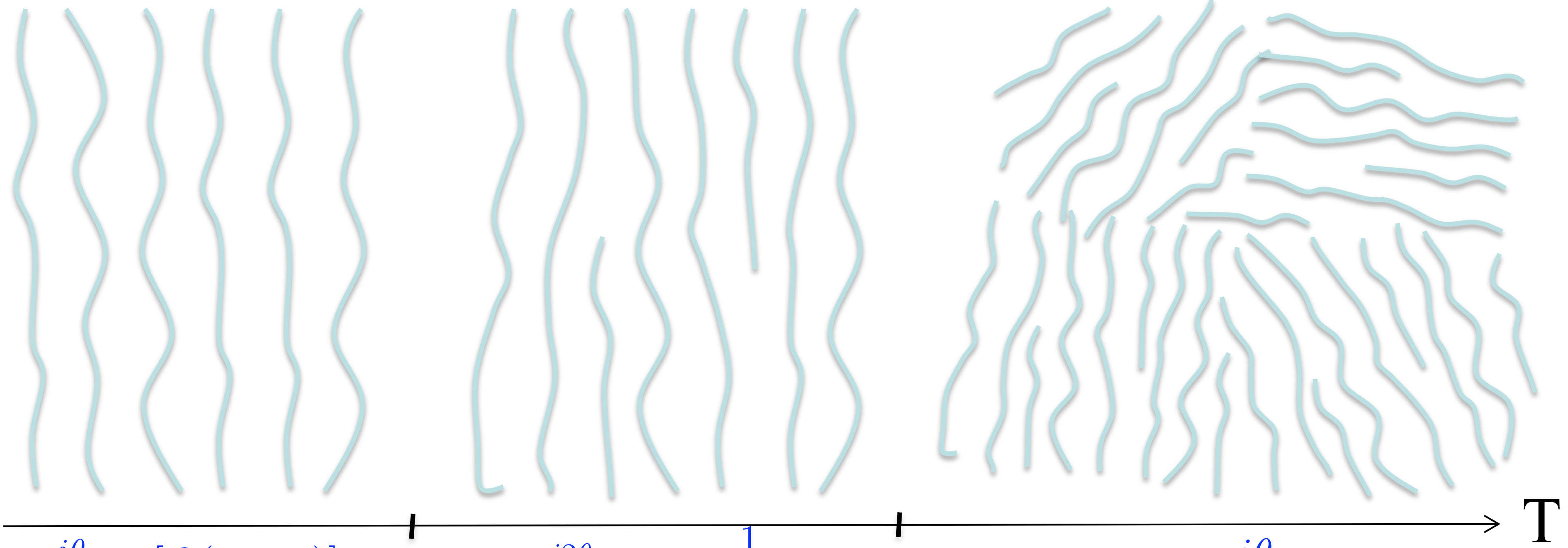
unbind defects, e.g., dislocations (0, a):







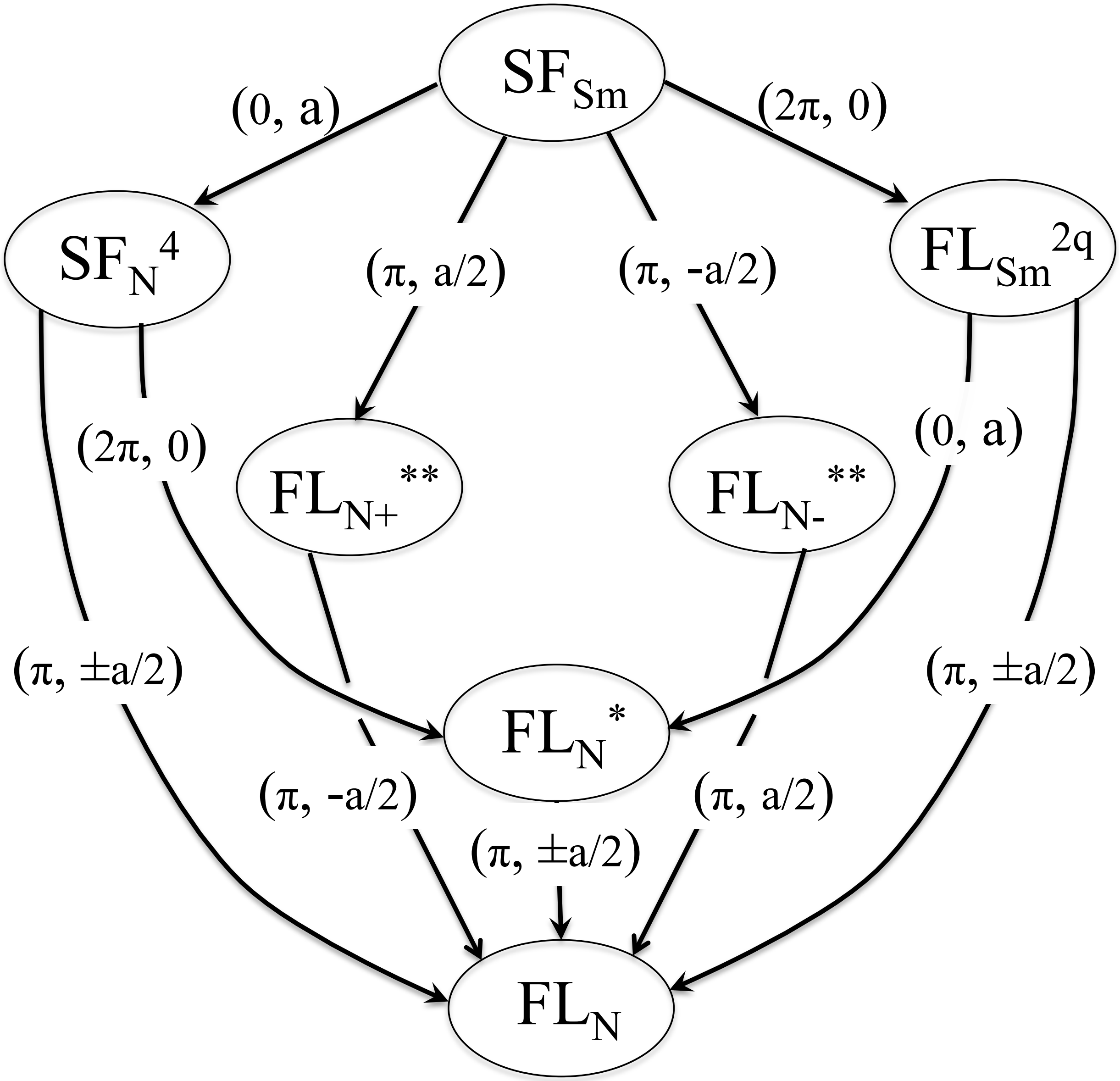
LO Smectic (SF_{Sm}) Nematic Superfluid (SF_N) Isotropic Superfluid (SF_I)



$e^{i\theta}\cos[Q(z-u)]$ T_{NSm} $e^{i2\theta}(\hat{n}_i\hat{n}_j - \frac{1}{3}\delta_{ij})$ T_{IN} $e^{i\theta}$

Proliferation of novel states

various patterns of defects unbinding:



SF_{Sm}

(0, a)

(2π, 0)

$SF_N{}^4$

(π, a/2)

(π, -a/2)

$FL_{Sm}{}^{2q}$

(2π, 0)

$FL_{N+}{}^{**}$

$FL_{N-}{}^{**}$

(0, a)

(π, ±a/2)

$FL_N{}^*$

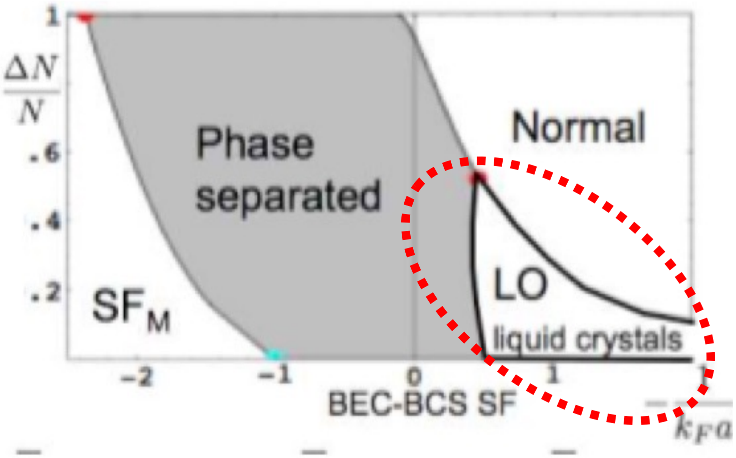
(π, ±a/2)

(π, -a/2)

(π, a/2)

(π, ±a/2)

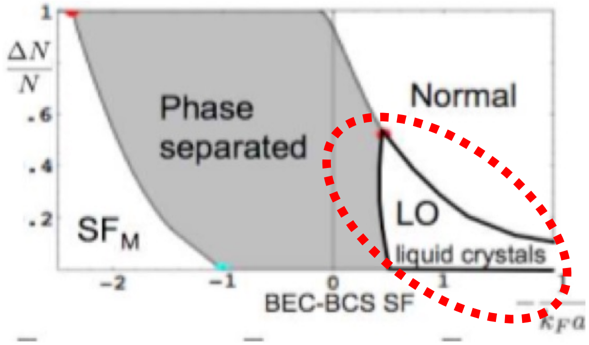
FL_N

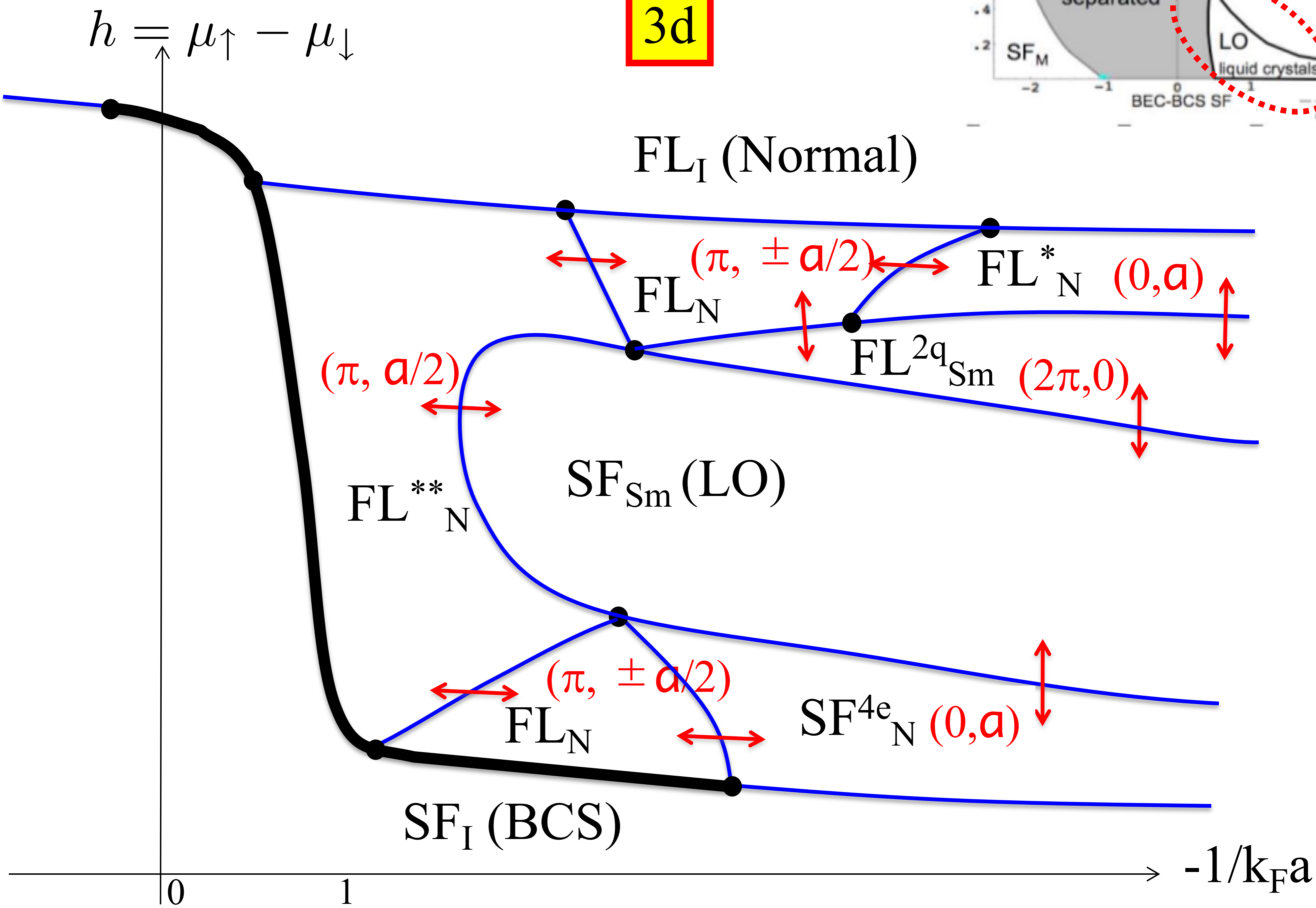


$$\Delta_{LO}(\mathbf{x}) = \Delta_0 e^{i\theta(\mathbf{x})} \cos[Q(z - u(\mathbf{x}))]$$

Phase transitions

3d



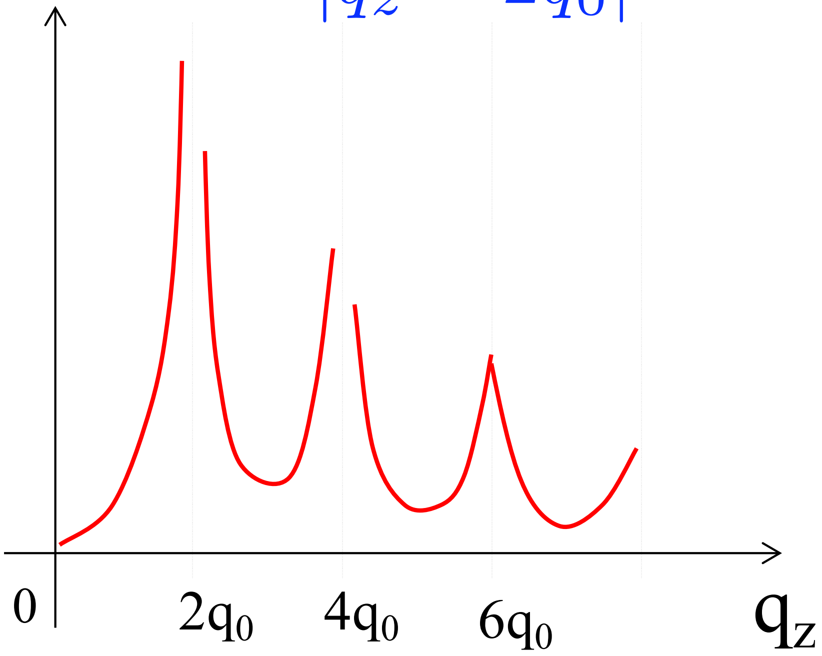


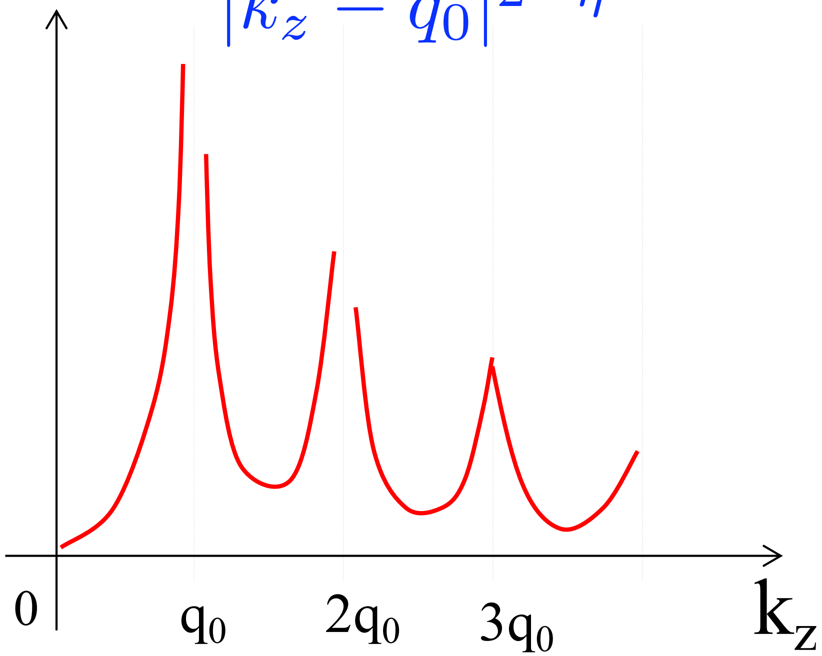
Structure function and time of flight

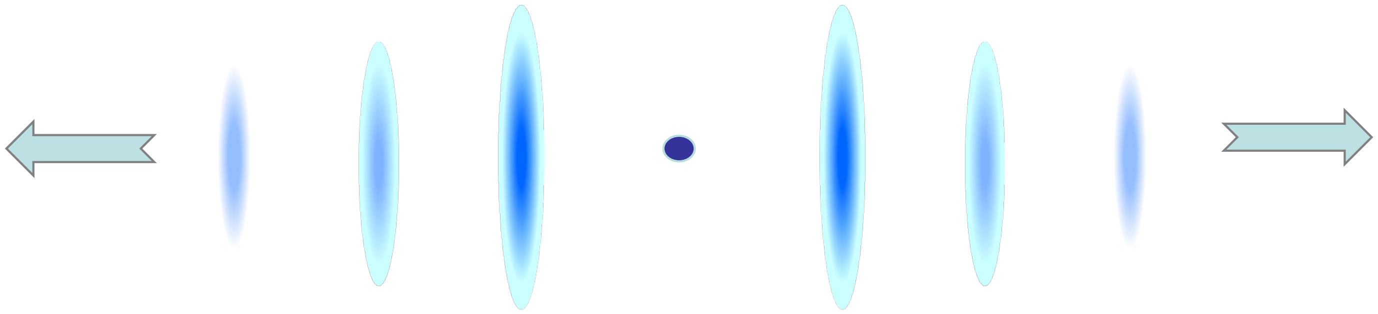
quasi-long-range order in 3d for T > 0

$$S(q_z) \sim \frac{1}{|q_z - 2q_0|^{2-4\eta}}$$

$$n_k \sim \frac{1}{|k_z - q_0|^{2-\eta}}$$





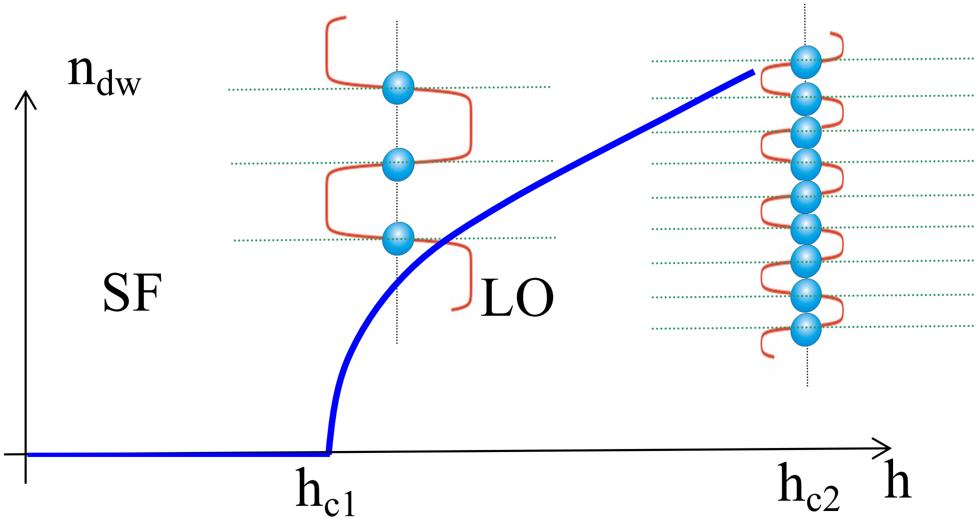


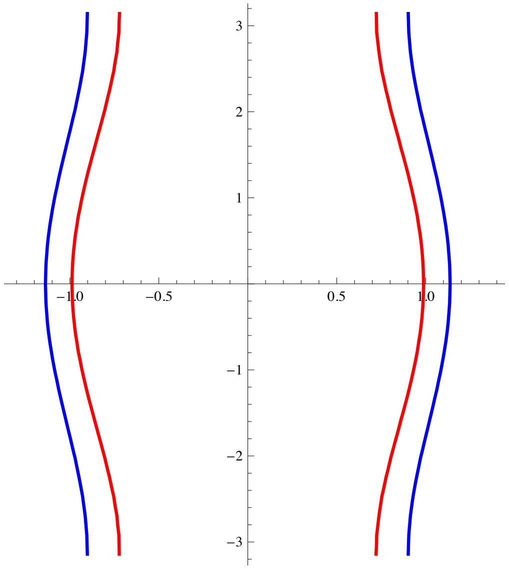
Fermionic sector of LO state

- ***ground state:***
$$\begin{aligned}|LO_{\mathbf{Q}}\rangle &= \prod_{\mathbf{k},\mathbf{Q}_i \in E_{\mathbf{k}\sigma\mathbf{Q}_i}<0} \alpha^{\dagger}_{\mathbf{k}\sigma\mathbf{Q}_i}|BCS_{\mathbf{Q}}\rangle, \\ &= \prod_{\mathbf{k},\mathbf{Q}_i\in\mathbf{k}_3} c^{\dagger}_{\mathbf{k}+\frac{\mathbf{Q}_i}{2}\downarrow} \prod_{\mathbf{k},\mathbf{Q}_i\in\mathbf{k}_2} c^{\dagger}_{-\mathbf{k}+\frac{\mathbf{Q}_i}{2}\uparrow} \prod_{\mathbf{k},\mathbf{Q}_i\in\mathbf{k}_1} (u_{\mathbf{k}} + v_{\mathbf{k}} c^{\dagger}_{\mathbf{k}+\frac{\mathbf{Q}_i}{2}\downarrow} c^{\dagger}_{-\mathbf{k}+\frac{\mathbf{Q}_i}{2}\uparrow})|0\rangle\end{aligned}$$

- ***excitation spectrum:***
  (gapped and gapless k's)
$$E_{\mathbf{k}\uparrow/\downarrow\mathbf{Q}_i} = (\varepsilon_k^2 + \Delta_Q^2)^{1/2} \mp (h + \frac{\mathbf{k}\cdot\mathbf{Q}_i}{2m})$$

- ***gapless fermionic excitations band of Andreev states:*** **superfluid with a Fermi surface**

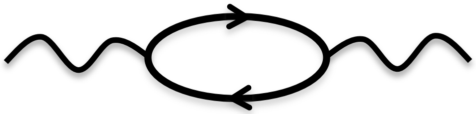




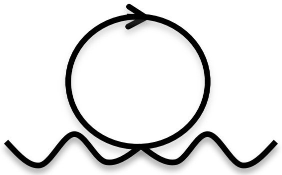
Fermion-Goldstone modes coupling in LO state

- ***supercurrent-current:*** $H_{j_s,j} \sim \nabla\theta \cdot \overline{\psi} i \nabla \psi + h.c.$

⇨ $|\omega|\sigma_{ij}(\omega,\mathbf{q})\nabla_i\theta\nabla_j\theta$



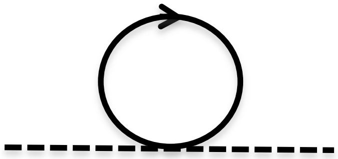
- ***supercurrent-density:*** $H_{j_s,n} \sim (\nabla\theta)^2\overline{\psi}\psi$



⇨ $n_f(\nabla\theta)^2$

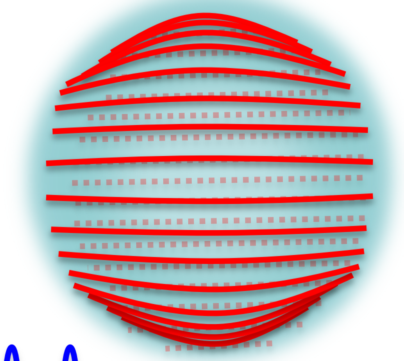
- ***atom-phonon:*** $H_{a-p} \sim \left(\partial_z u + \frac{1}{2}(\nabla u)^2\right)\overline{\psi}\psi + (\nabla u \cdot \overline{\psi} i \nabla \psi)^2 + h.c.$

⇨ $n_f\left(\partial_z u + \frac{1}{2}(\nabla u)^2\right)$



- **affects on Goldstone modes and fermions?**
  - *(weak) Landau damping, finite corrections to q_0, ρ_s, K, B,…*
  - *fermions retain their anisotropic pocket Fermi surface*

Experiments

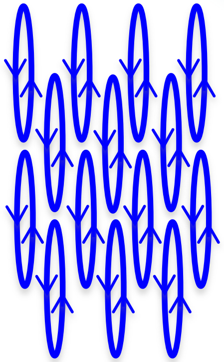


- ***trap suppression of fluctuations:***

⇨ $$\Delta_{LO} \sim R^{-\eta(T)} \sim N^{-\frac{1}{5}\eta(T)} \sim \omega_{\rm tr}^{\eta(T)} \to 0$$

- ***anisotropic, π-vortices:***

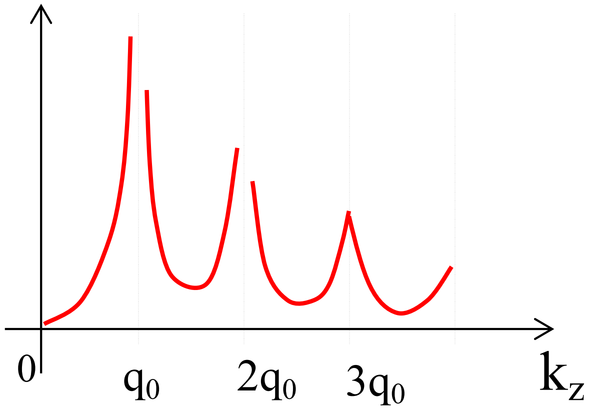
⇨ $$\mathbf{v} = \sqrt{\rho_x^s \rho_y^s}\,\frac{(-y, x)}{\rho_y^s x^2 + \rho_x^s y^2} \qquad n_v = 4\Omega_r \frac{m}{\hbar}$$



novel vortex phases?

- ***heat capacity:***

⇨ $$C_v \sim aT + bT^2$$

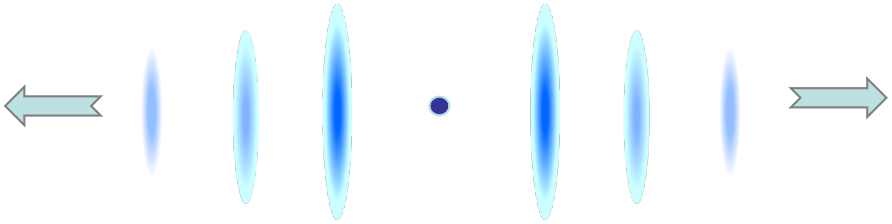


- ***momentum distribution (time of flight):***

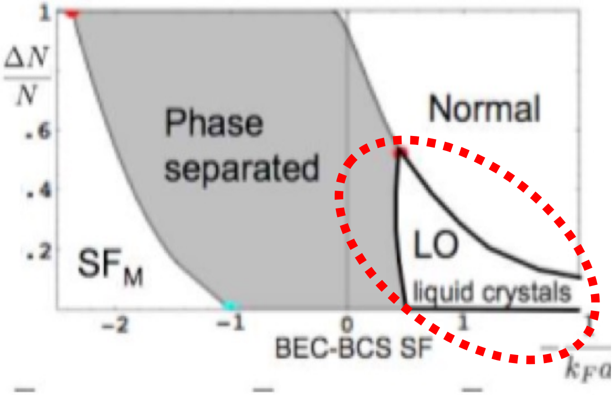
⇨ $$n_k \sim \frac{1}{|k_z - q_0|^{2-\eta}}$$

- ***structure function:***

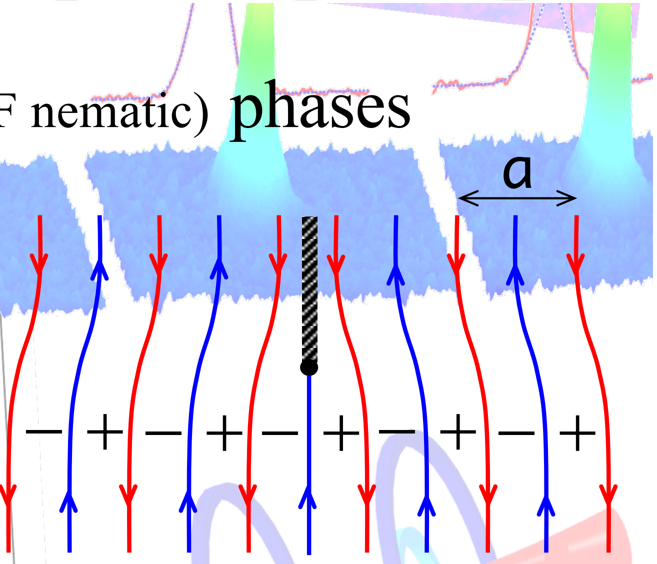
⇨ $$S(k_z) \sim \frac{1}{|k_z - 2q_0|^{2-4\eta}}$$

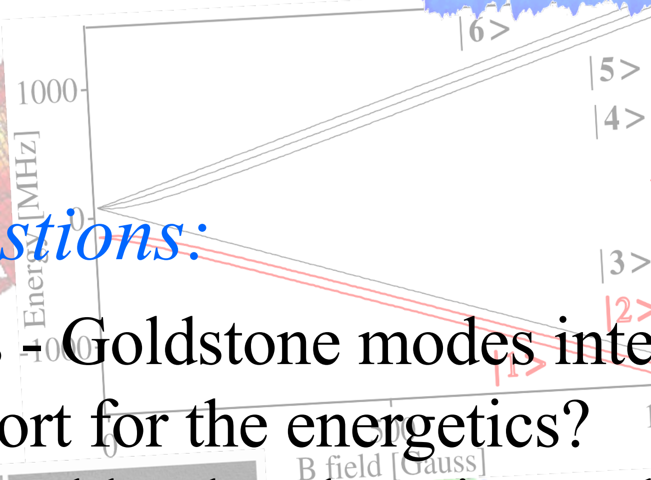


Summary and directions



- Larkin-Ovchinnikov state ⇔ superfluid smectic
- critical phase at finite T with universal properties
- half-integer vortex and dislocation defects
- transitions to N-Sm$_{2Q}$ and SF$_4$-Nm (“charge”-4 SF nematic) phases
- rich variety of descendent states and transitions





...many remaining questions:

- effects of Fermi pockets - Goldstone modes interactions?
- better microscopic support for the energetics?
- connection to experimental knobs: detuning and imbalance?
- explore further experimental consequences, detection signals?
- charge-4e SC? …