Nonequilibrium Quantum Field Theory

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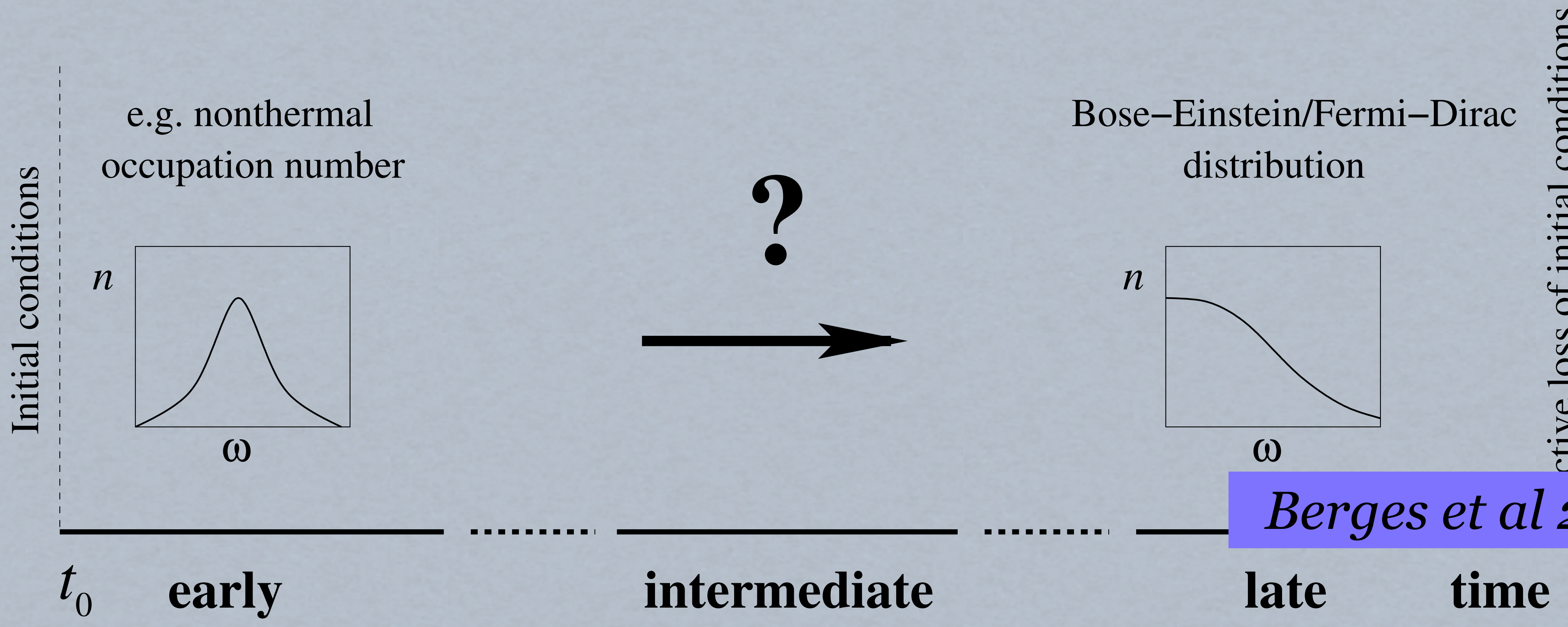
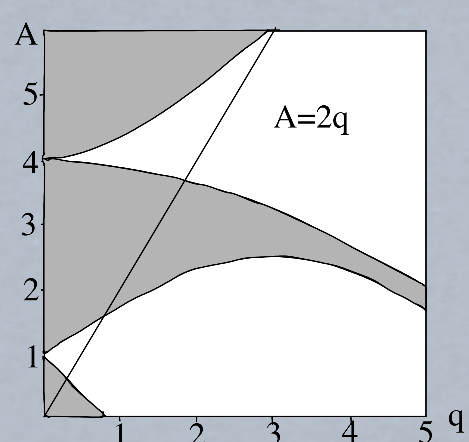
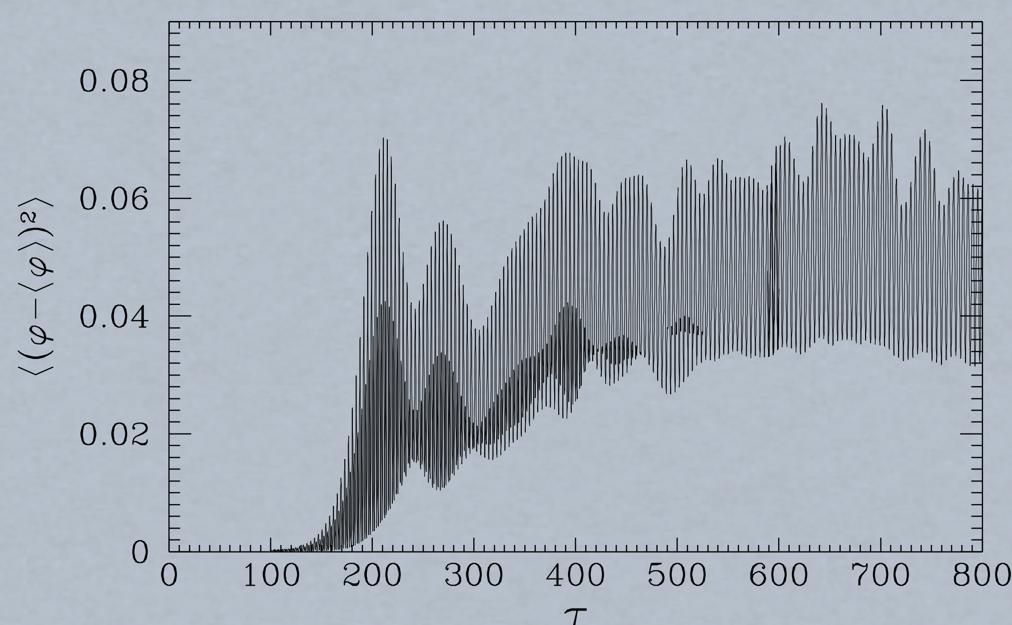
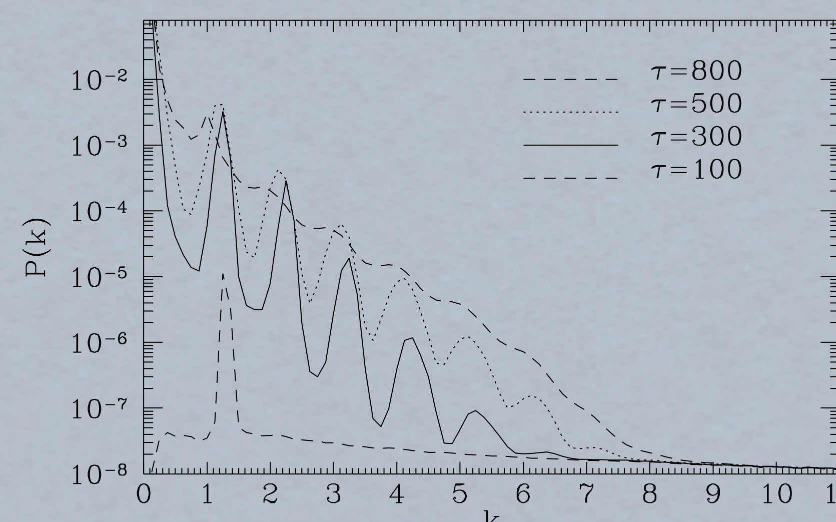
What is nonequilibrium?



Berges et al 2000-..







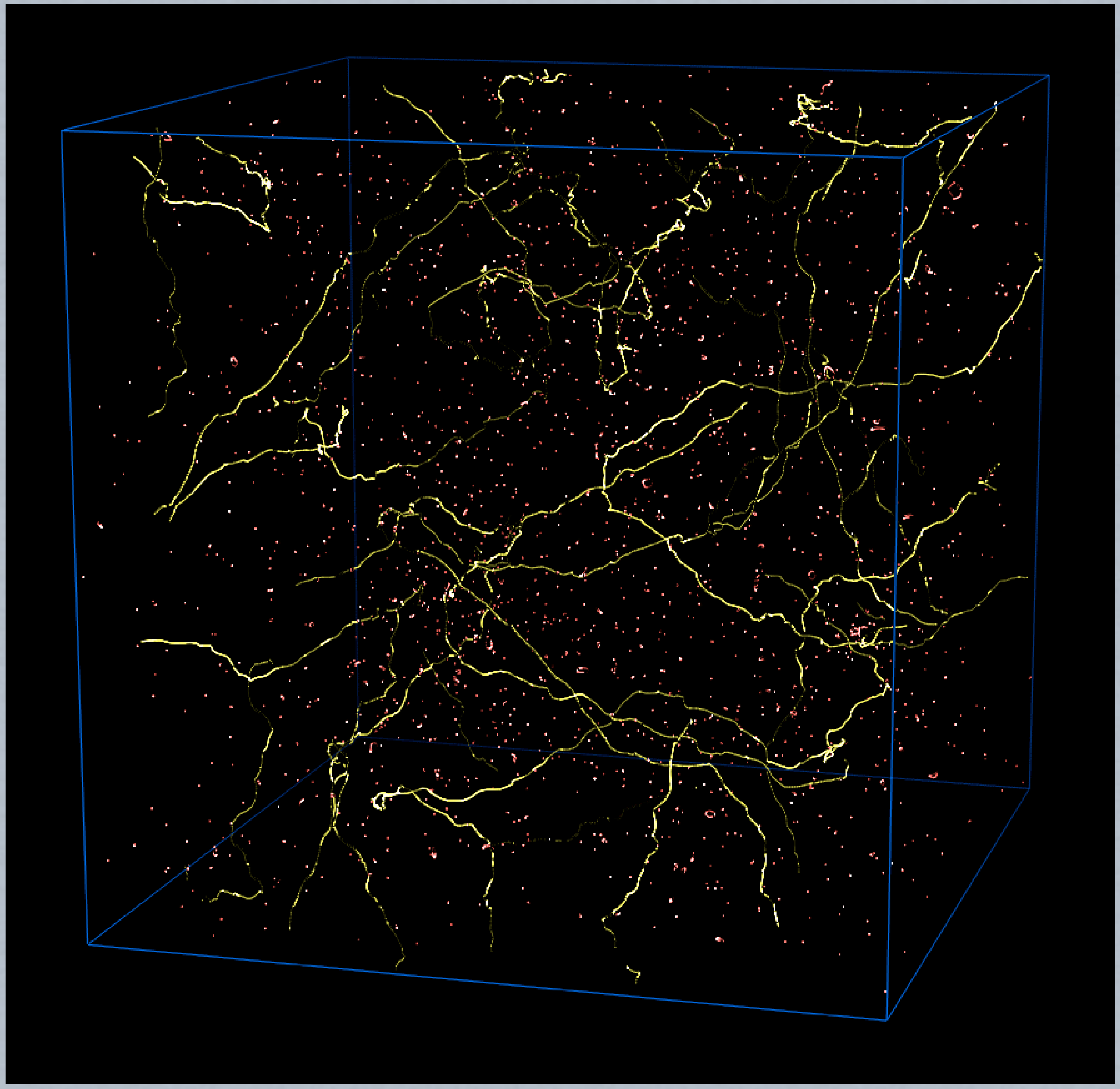
instability

turbulence

Khlebnikov 1996

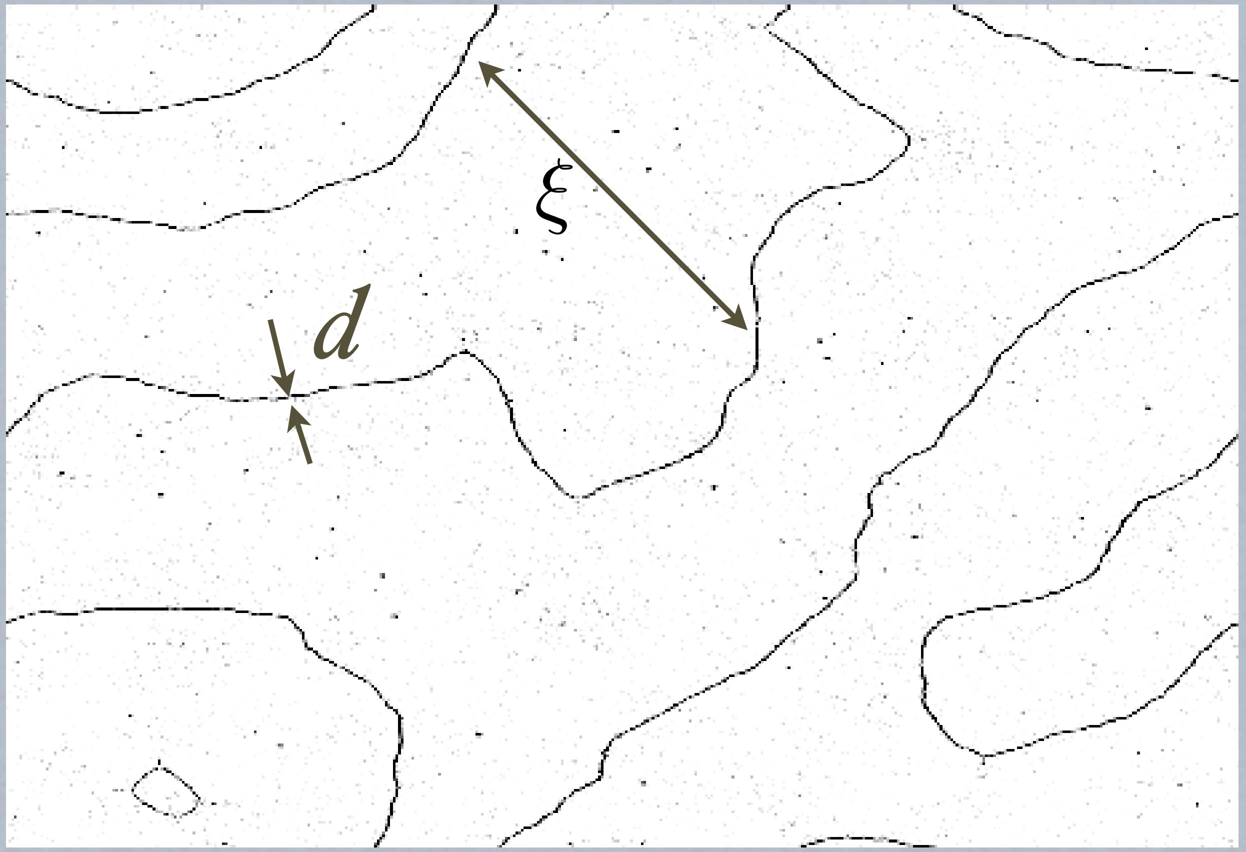
Tkachev,Misha 2004,... Arnold,Moore 2006

An other setting: defects



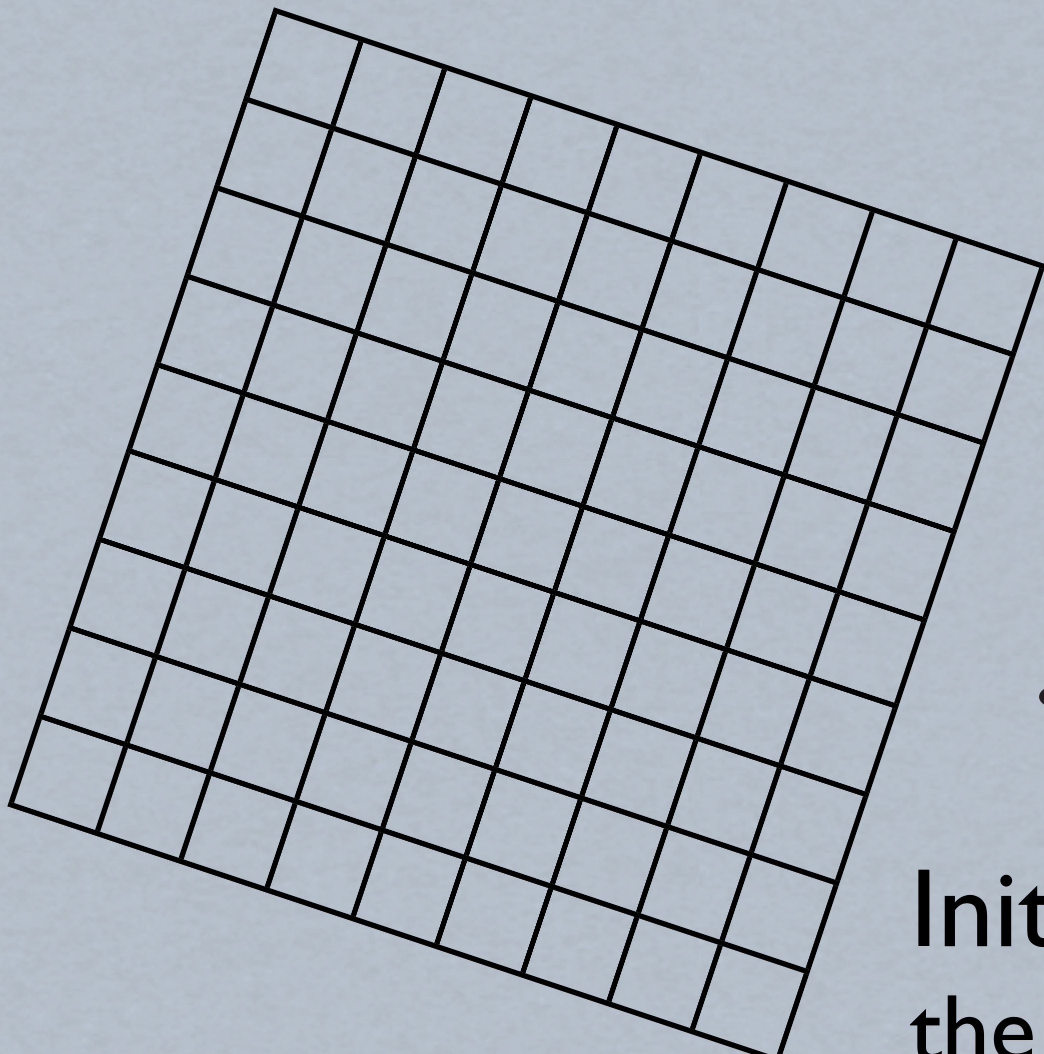
Strings appear as topological defects in field theory. Nonequilibrium QFT accounts for their decay.

$\xi \gg d$



see Mark Hindmarsh' talk

The classical approach



Define a lattice field theory,
Solve the 2nd order
Klein-Gordon (or Maxwell)
equations

$$\Phi_{\mathbf{n}}(t+a_t) + \Phi_{\mathbf{n}}(t-a_t) - 2\Phi_{\mathbf{n}}(t) - \frac{a_t^2}{a^2}\sum_i \left(\Phi_{\mathbf{n}+\hat{\mathbf{i}}}(t) + \Phi_{\mathbf{n}-\hat{\mathbf{i}}}(t) - 2\Phi_{\mathbf{n}}(t)\right) + a_t^2(-\Phi_{\mathbf{n}} + \Phi_{\mathbf{n}}^3 - h) = 0.$$

Initial condition:

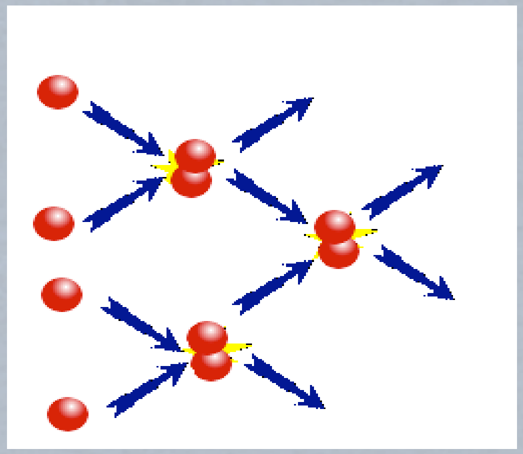
the field value is sampled from an ensemble that reproduces the n-point functions.
Evolution of the ensemble gives the n-point functions at a later time.
This evolution is *NONPERTURBATIVE* !

?

- continuum limit
- Bose-Einstein
- physical cutoff
- fermions

see Jan Smit's talk

The kinetic approach

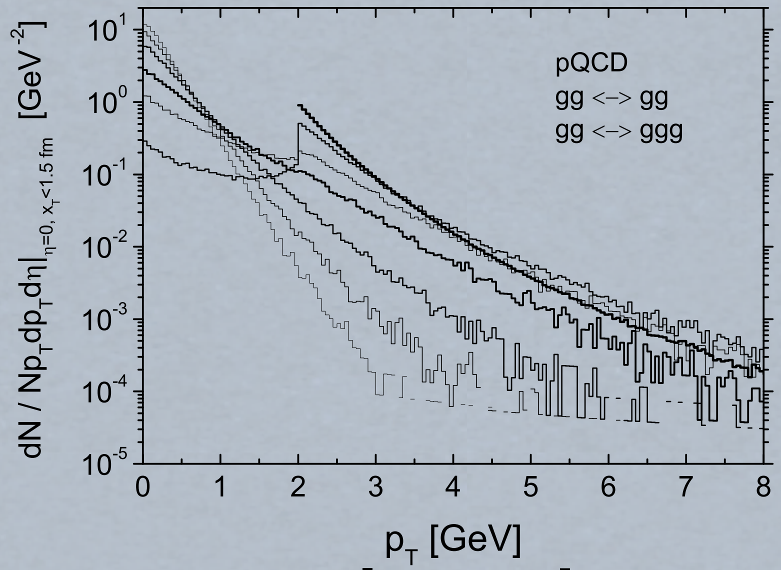


Particles (balls) collide and interact with a precalculated cross section.

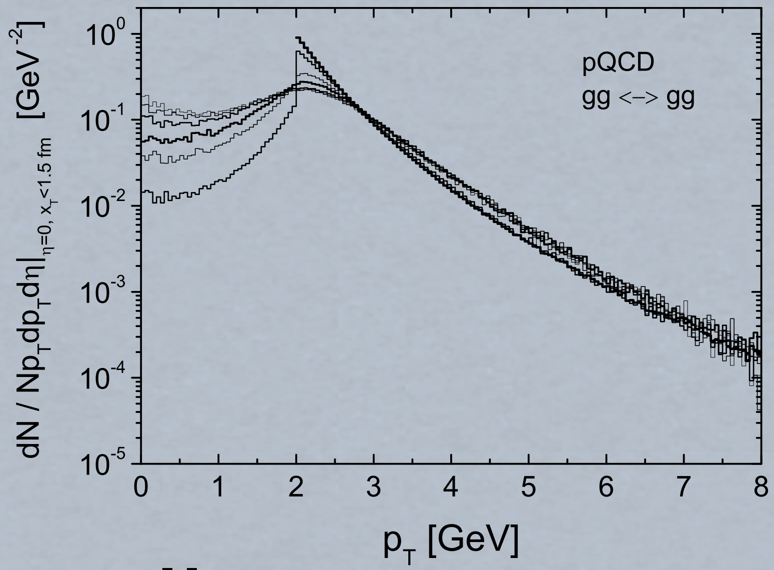
Example: Parton thermalisation

Greiner,Xu 2005

with gg→ggg



with gg→gg only



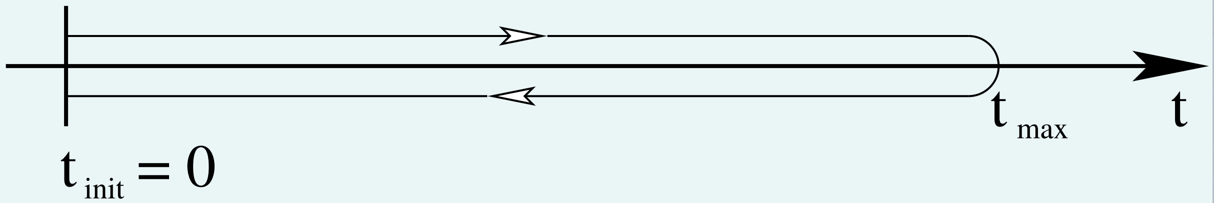
Coherence is lost between collisions.

Gradient expansion has been used. What does justify it?

see also MM Müller's talk

Initial value problem in QFT

Define path integral along the *closed time path* contour



$$\left\langle \hat{\mathcal{X}} \right\rangle (t) = \mathrm{Tr}\hat{\rho}(t)\hat{\mathcal{X}}(t_0) = \mathrm{Tr}\hat{\mathcal{U}}(t,t_0)\hat{\rho}(t_0)\hat{\mathcal{U}}^{-1}(t,t_0)\hat{\mathcal{X}}(t_0)$$

$$\hat{\mathcal{U}}(t,t') = \exp\left[-i\int_{t'}^{t}\hat{\mathcal{H}}(t'')dt''\right]$$

$$Z[J] = \int \mathcal{D}\phi e^{i\int_{\mathcal{C}} dx[\mathcal{L}(x)+J(x)\phi(x)]}$$

Propagators:

$$\begin{aligned}
G^{>}_{ij}(x,y) &= \langle\varphi_i(x)\varphi_j(y)\rangle \\
G^{<}_{ij}(x,y) &= \langle\varphi_j(y)\varphi_i(x)\rangle \\
G_{ij}(x,y) &= \langle\mathcal{T}_{\mathcal{C}}\varphi_i(x)\varphi_j(y)\rangle \\
iG_0 &= (\partial^2+m^2)^{-1} \\
F_{ij}(x,y) &= \frac{1}{2}\left(G^{>}_{ij}(x,y)+G^{<}_{ij}(x,y)\right) \\
\rho_{ij}(x,y) &= i\left(G^{>}_{ij}(x,y)-G^{<}_{ij}(x,y)\right),
\end{aligned}$$

Aarts, Berges 2001

$$G_{ij}(x,y) = F_{ij}(x,y) - \frac{i}{2}\rho_{ij}(x,y)\mathrm{sgn}_{\mathcal{C}}(x_0,y_0)$$

Is the dynamics irreversible?

Thermal equilibrium:

$$\hat{\rho} = e^{-\beta \hat{H}} / \mathrm{Tr} e^{-\beta \hat{H}} \qquad \left\langle \hat{X} \right\rangle = \mathrm{Tr} \hat{X} \hat{\rho}$$

Is thermalization possible in closed nonlinear system?

- Quantum Mechanics: Equilibrium is a fixed point of the evolution
- $\rho \not\longrightarrow e^{-\beta \hat{H}} / \mathrm{Tr} e^{-\beta \hat{H}}$ Unitarity!
- $\langle \hat{H} \rangle$=const. uniquely determines the equilibrium ensemble.

  But: $\langle \hat{H}^2 \rangle$, $\langle \hat{H}^3 \rangle$, $\ldots$ conserved *(initial conditions)*
- The quantum ensemble cannot converge to equilibrium!
- Still, the quantum average of some selected observables may converge to the equilibrium value:

$$\langle \Phi(x) \Phi(y) \rangle_{\text{noneq}} \longrightarrow \langle \Phi(x) \Phi(y) \rangle_{\text{thermal}} , \quad \text{as } x_0, y_0 \to \infty$$

Perturbation theory fails

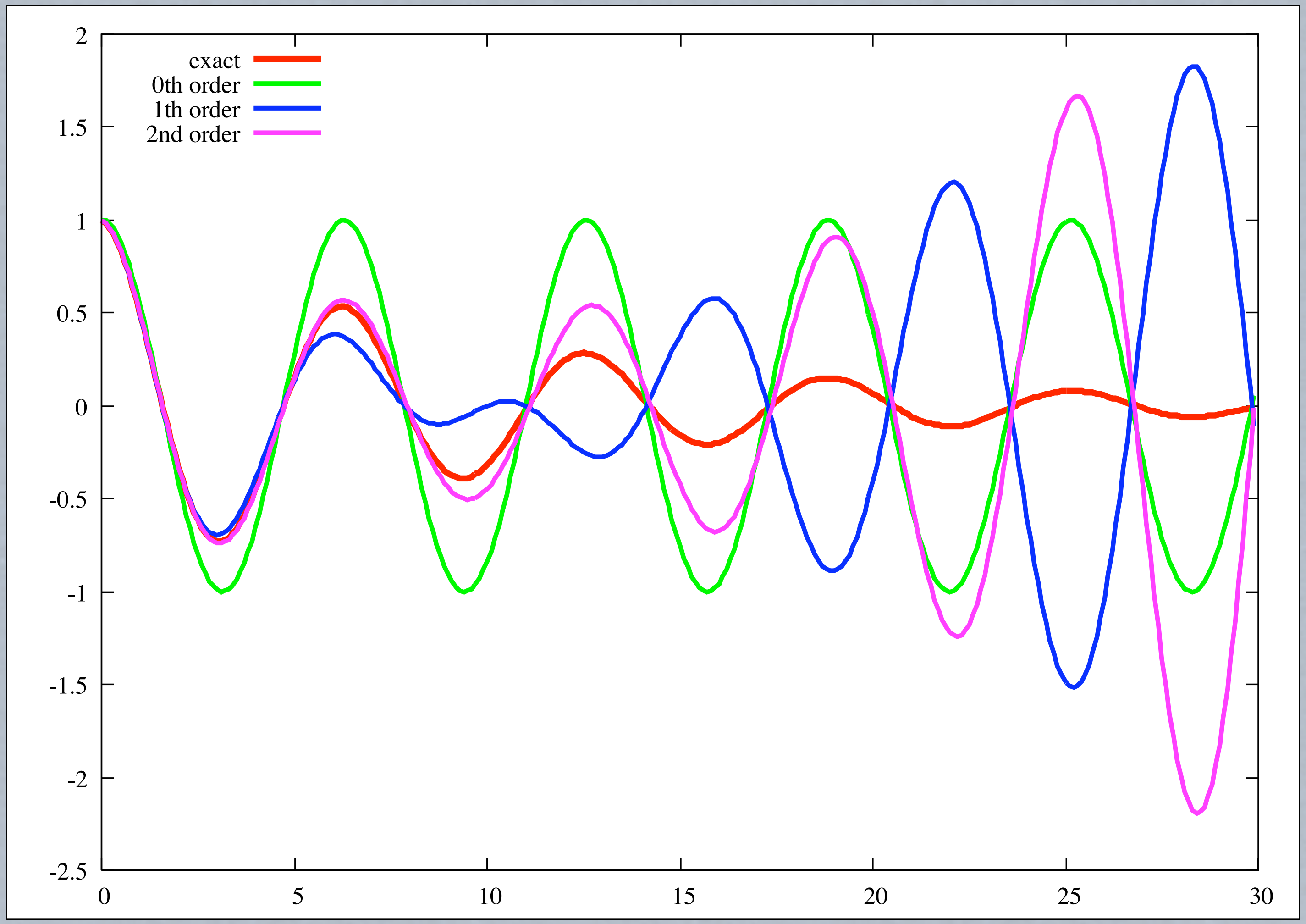
Example: a damped oscillator

$$\ddot{x}(t) + 2\gamma\dot{x}(t) + m^2 x(t) = 0$$

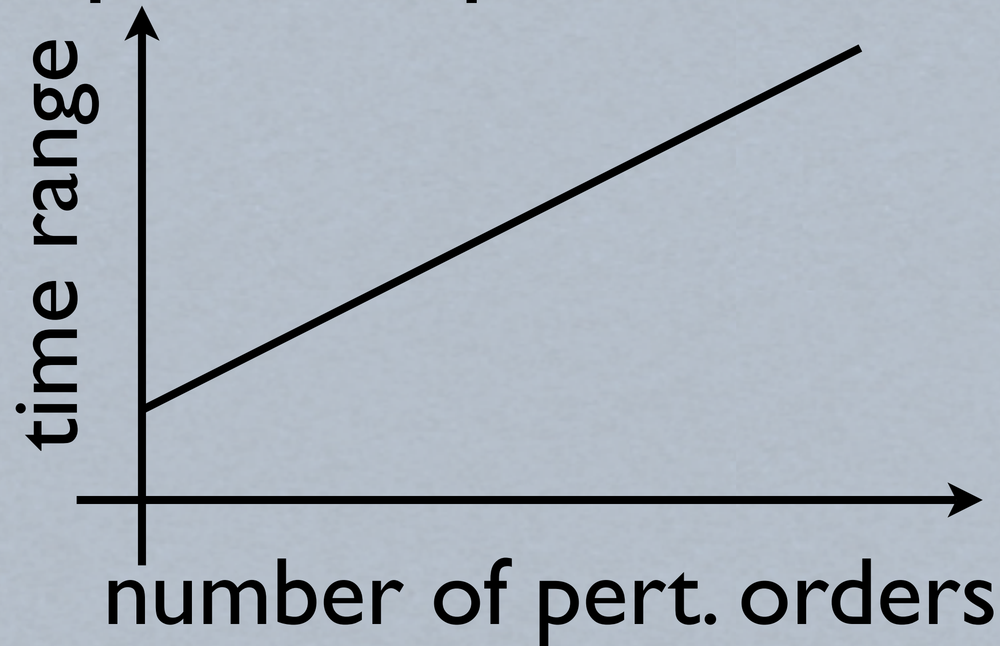
$$x(t) = A\sin(t\sqrt{m^2 - \gamma^2})e^{-t\gamma}$$

1st perturbative order:

$$x(t) = A\cos(tm)(1 - t\gamma)$$



Secular behaviour: the time is part of the expansion parameter!



A nonperturbative approach: Let's simulate on a lattice!

Euclidean Langevin equation:

$$\partial_\vartheta \phi(x,\vartheta) = -\frac{\delta S_E[\phi]}{\delta \phi(x,\vartheta)} + \eta(x,\vartheta)$$

$$\langle \eta(x_1,\vartheta_1)\eta(x_2,\vartheta_2)\rangle = 2\delta(\vartheta_1-\vartheta_2)\delta^{(4)}(x_1-x_2)$$

Parisi,Wu 1981

Langevin equation on the closed time path contour

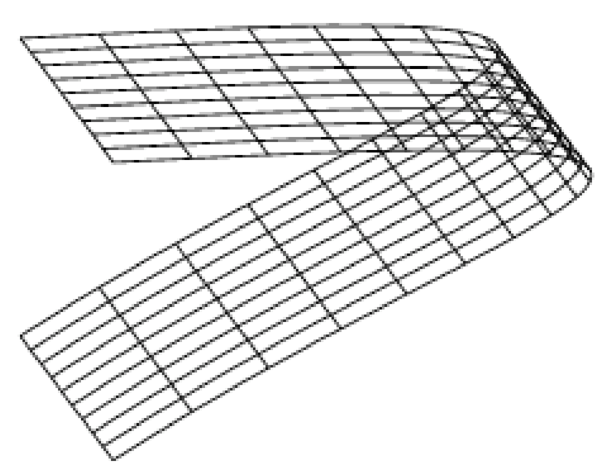
$$\frac{\partial \phi(C_j)}{\partial \vartheta} = i\frac{\partial S}{\partial \phi(C_j)} + \eta_j(\vartheta)$$

Contour points: C_j

see Dénes Sexty's talk

In this algorithm probabilities are never used.

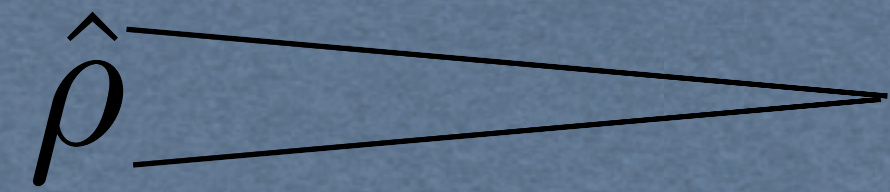
Reproduces the hierarchy of SD equations.



It really *does* converge in real time, too!

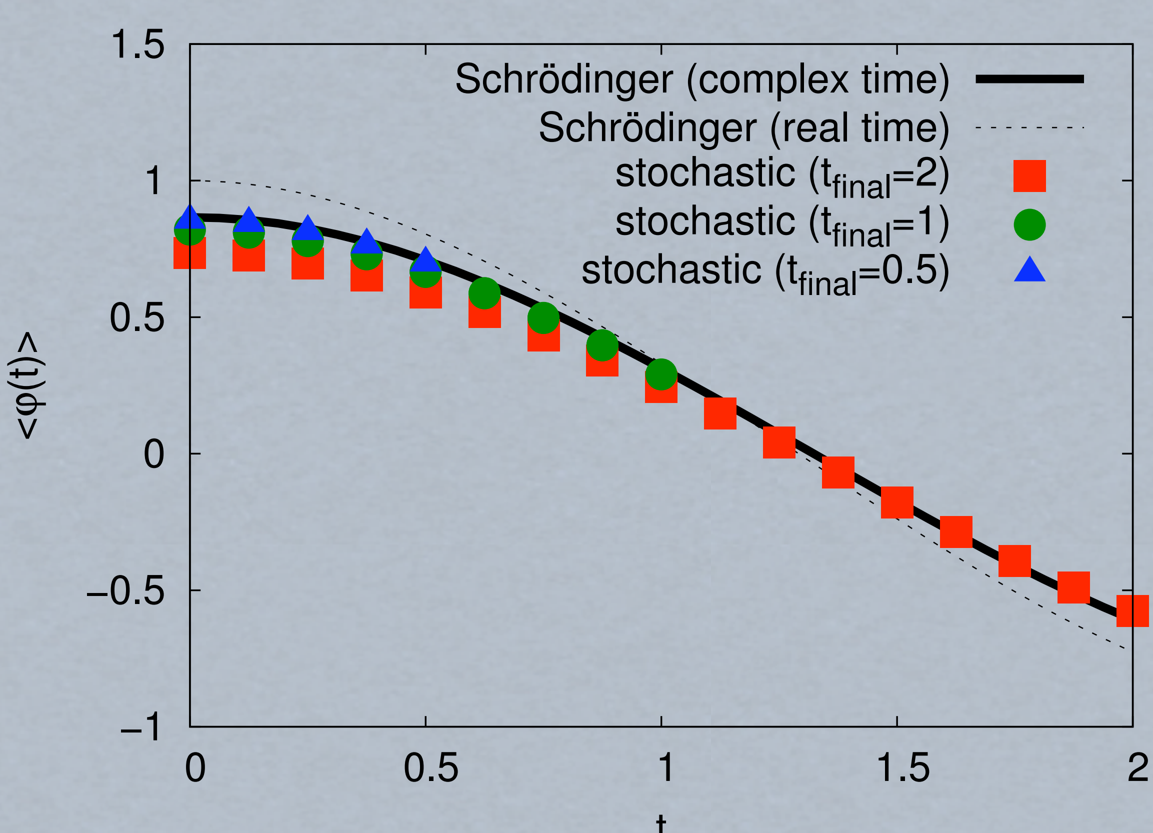
Toy model: anharmonic oscillator

Use the action with comlex Δt,
with two branches
and with $\hat{\rho}$ being part of the action.

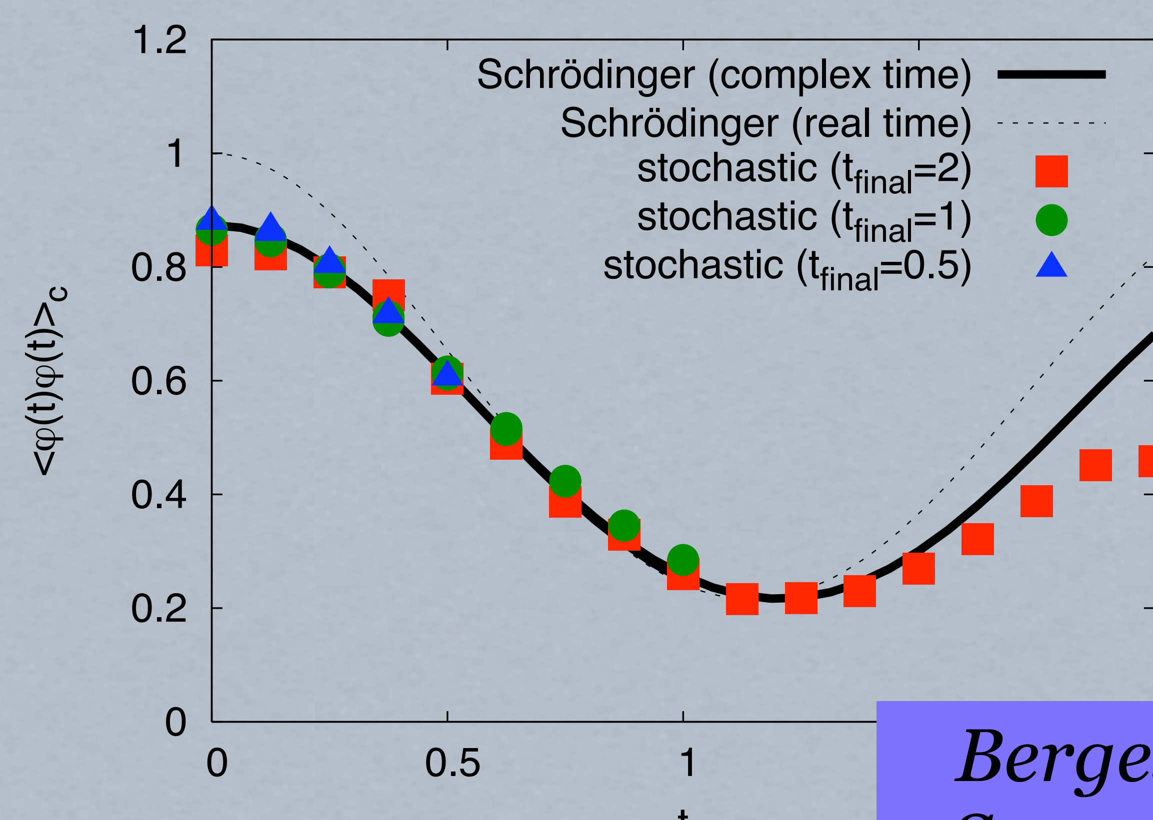


Comparison with Schrödinger's equation:

$\langle x(t) \rangle$



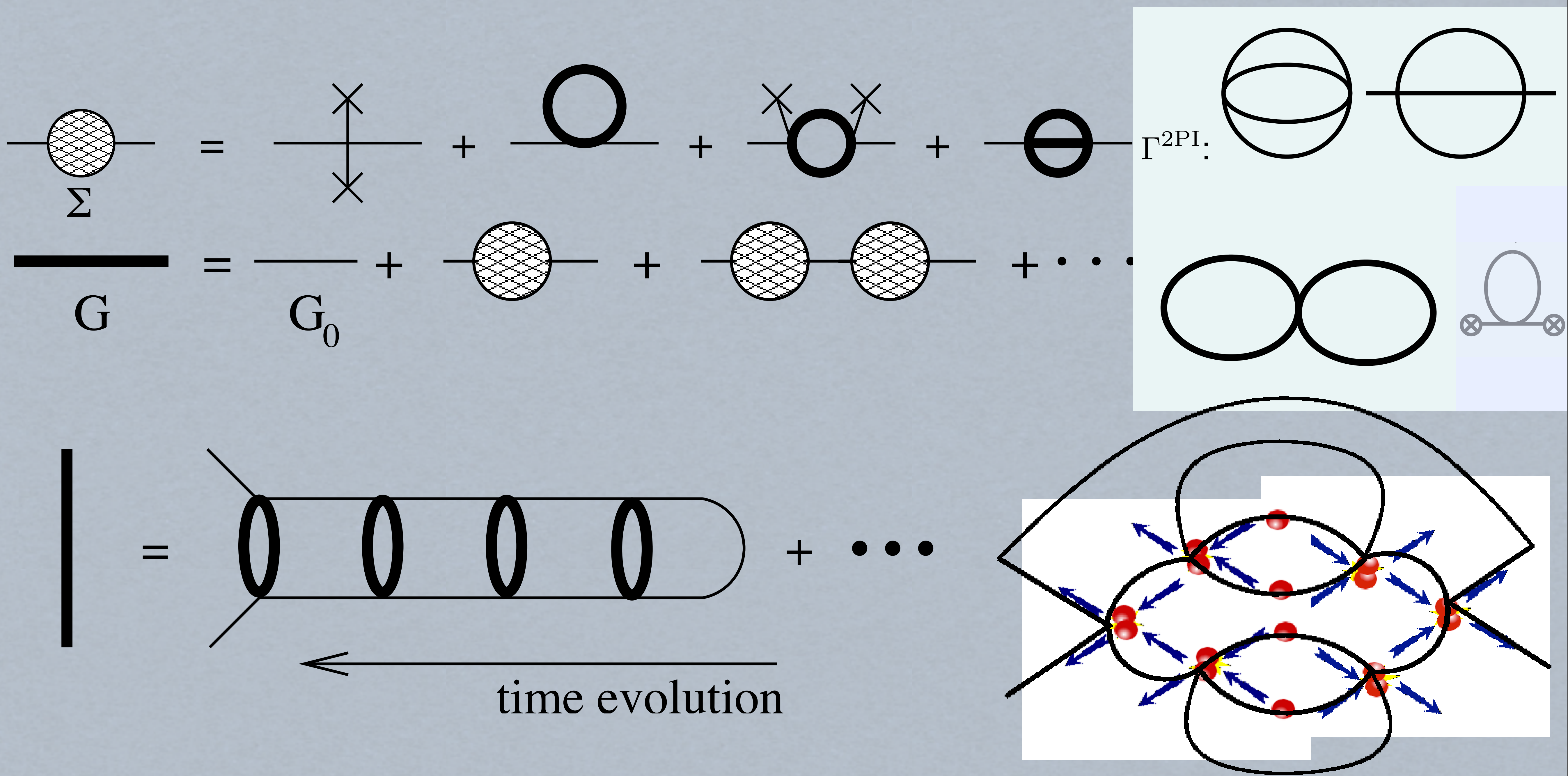
$\langle x(t)x(t) \rangle_c$



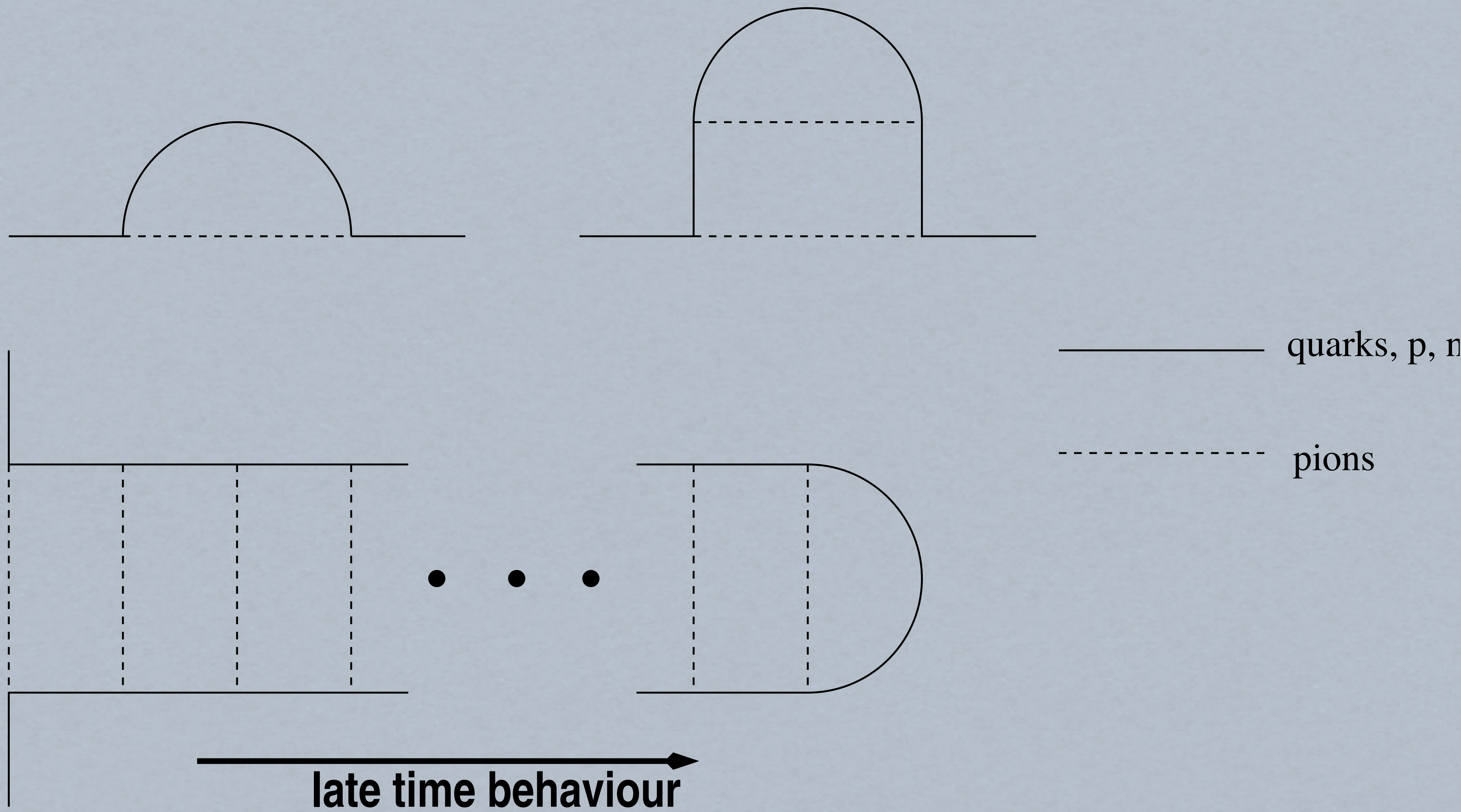
Challenge: Is the solution unique?

Berges, S.B, Stamatescu, Sexty 2005-...

A diagrammatic approach: the 2PI resummation



... or in a Yukawa theory:



The 2PI effective action

$$Z[J,K] = \int \prod_{c=1}^{N} \mathcal{D}\varphi_c(x) \exp\Big(i \int_{\mathcal{C}} \mathrm{d}^4 x \big[\mathcal{L}(x) + J_a(x)\varphi_a(x) \big] + \frac{i}{2} \int_{\mathcal{C}} \mathrm{d}^4 x \int_{\mathcal{C}} \mathrm{d}^4 y \left[\varphi_a(x) K_{ab}(x,y) \varphi_b(y) \right] \Big) ,$$

1st Legendre transform: effective action (1PI diagrams)

2nd Legendre transform: 2PI effective action (2PI diagrams)

$$W[J,K] = -i \log (Z[J,K]) \quad \delta W[J,K]/\delta J = \phi \quad \delta W[J,K]/\delta K = (\phi^2 - G)/2$$

$$\Gamma[\phi, G] = W[J,K] - \int_{\mathcal{C}} \mathrm{d}^4 x \left[J_a(x) \phi_a(x) \right] - \frac{1}{2} \int_{\mathcal{C}} \mathrm{d}^4 x \int_{\mathcal{C}} \mathrm{d}^4 y \Big[G_{ab}(x,y) K_{ab}(x,y) + \phi_a(x) K_{ab}(x,y) \phi_b(y) \Big]$$

Result: ladder resummation,
no overcounting

Cornwall,Jackiw, Tomboulis 1974,
Calzetta,Hu 1988;
Ivanov,Knoll,Voskresensky 1988
Cooper at al (2PI,BVA) 2000

Equations of Motion

are the stationarity conditions:

(a) $\frac{\delta\Gamma[\phi,G]}{\delta\phi_a(x)} = -J_a\left(x\right) - \int\limits_{\mathcal{C}} \mathrm{d}^4y\left[K_{ab}\left(x,y\right)\phi_b\left(y\right)\right] \stackrel{!}{=} 0$

(b) $\frac{\delta\Gamma[\phi,G]}{\delta G_{ab}(x,y)} = -\frac{1}{2}K_{ab}\left(x,y\right) \stackrel{!}{=} 0 \qquad \rightarrow \qquad G_{ab}(x,y;\phi) = \langle \mathcal{T}_{\mathcal{C}}\hat{\varphi}(x)\hat{\varphi}(y)\rangle_c$

Decomposition:

$$\Gamma_b\left[\phi,G\right] = S\left[\phi\right] + \frac{i}{2}\mathrm{tr}_{\mathcal{C}}\left[\log\left[G^{-1}\right]\right] + \frac{i}{2}\mathrm{tr}_{\mathcal{C}}\left[G_0^{-1}G\right] + \Gamma_{\mathrm{int}}\left[\phi,G\right] + \mathrm{const}$$

$$\Gamma_f\left[\psi,D\right] = S\left[\psi\right] - i\mathrm{tr}_{\mathcal{C}}\left[\log\left[D^{-1}\right]\right] - i\mathrm{tr}_{\mathcal{C}}\left[D_0^{-1}D\right] + \Gamma_{\mathrm{int}}\left[\psi,D\right] + \mathrm{const}$$

With $\quad \Sigma_f(x,y) \equiv 2i\frac{\delta\Gamma_{\mathrm{int}}[G]}{\delta G(y,x)} \qquad \Sigma_s(x,y) \equiv -i\frac{\delta\Gamma_{\mathrm{int}}[D]}{\delta D(y,x)}$

$$(\partial_x^2 + m^2)G_{ab}(x,y) = \int\limits_{\mathcal{C}} \mathrm{d}^4z\Sigma_{ab}\left(x,z;G,D\right)G_{bc}\left(z,y\right) + \delta_{\mathcal{C}}(x,y)\delta_{ab},$$

$$(\not{\partial}_x + im_f)D_{ij}(x,y) = \int\limits_{\mathcal{C}} \mathrm{d}^4z\Sigma_{ik}(x,z;G,D)D_{kj}(z,y) + \delta_{\mathcal{C}}^4(x,y)\delta_{ij}$$

$\leftarrow$ equivalent to Kadanoff–Baym equations

EoM: in terms of real time propagators:

$$F_{ij}(x,y) = \frac{1}{2}\left(G^{>}_{ij}(x,y) + G^{<}_{ij}(x,y)\right)$$
$$\rho_{ij}(x,y) = i\left(G^{>}_{ij}(x,y) - G^{<}_{ij}(x,y)\right),$$

(or with opposite signs for the fermions)

For fermionic fields:

$$\left(i\partial\!\!\!/ - m - \Sigma_0\right) F(x,y) = \int^{x_0} dz \Sigma^{\rho}(x,z) F(z,y) - \int^{y_0} dz \Sigma^{F}(x,z)\rho(z,y)$$

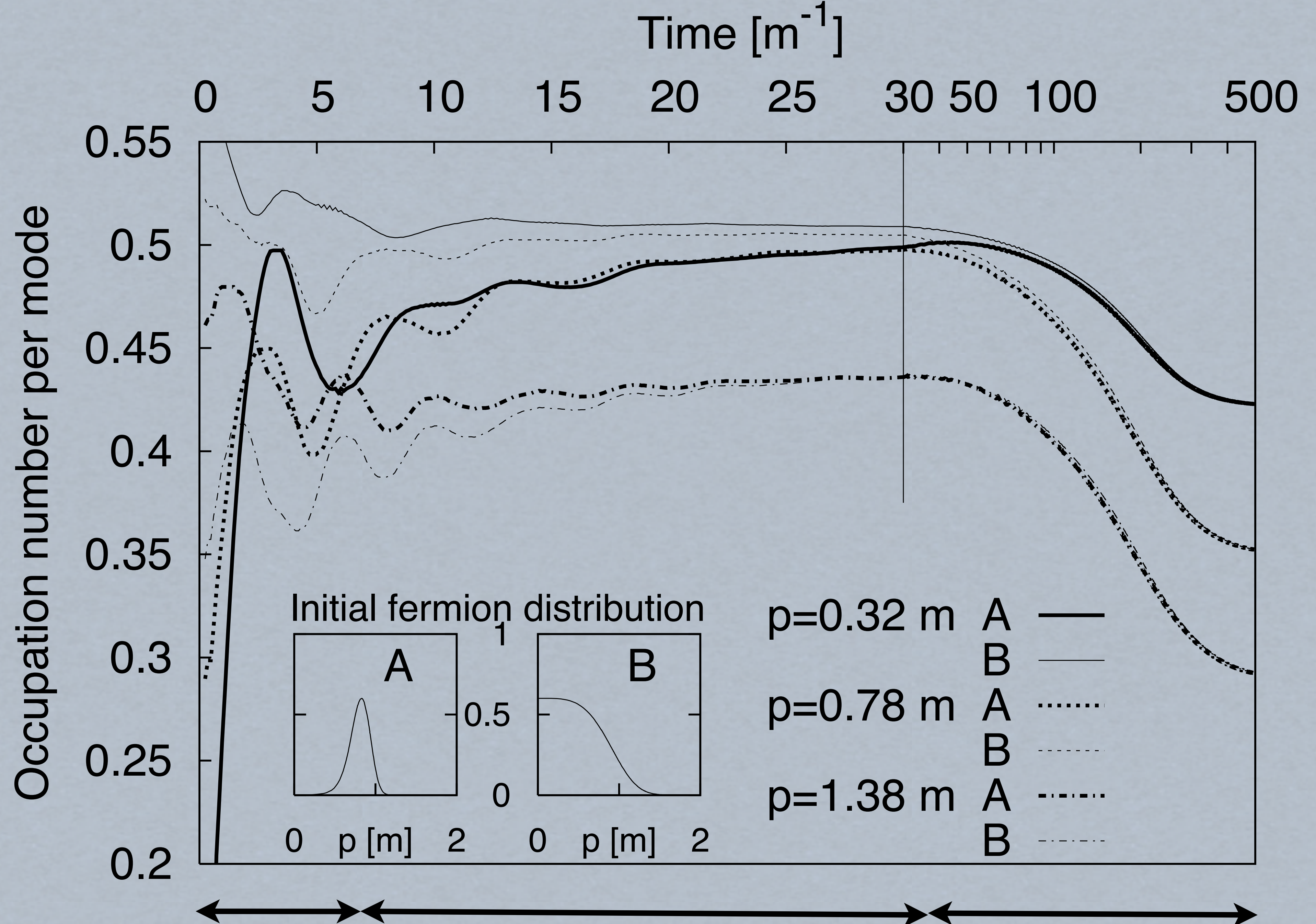
$$\left(i\partial\!\!\!/ - m - \Sigma_0\right) \rho(x,y) = \int_{y_0}^{x_0} dz \Sigma^{\rho}(x,z)\rho(z,y)$$

For scalar fields:

$$\left(\partial_x^2 + m^2 + \Sigma_{0,i}(x)\right) F_{ij}(x,y) = \int^{y_0} dz \Sigma^{F}_{ik}(x,z)\rho_{kj}(z,y) - \int^{x_0} dz \Sigma^{\rho}_{ik}(x,z) F_{kj}(z,y)$$

$$\left(\partial_x^2 + m^2 + \Sigma_{0,i}(x)\right) \rho_{ij}(x,y) = \int_{x_0}^{y_0} dz \Sigma^{\rho}_{ik}(x,z)\rho_{kj}(z,y)$$

Timescales of losing information



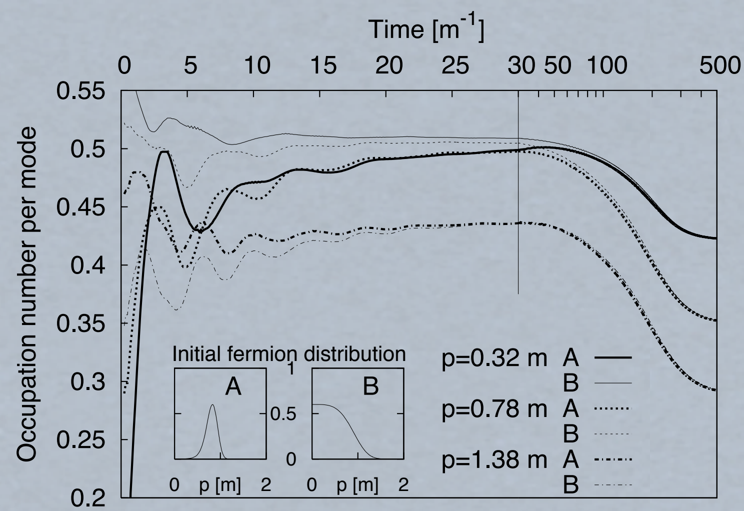
Prethermalisation

Damping, isotropisation

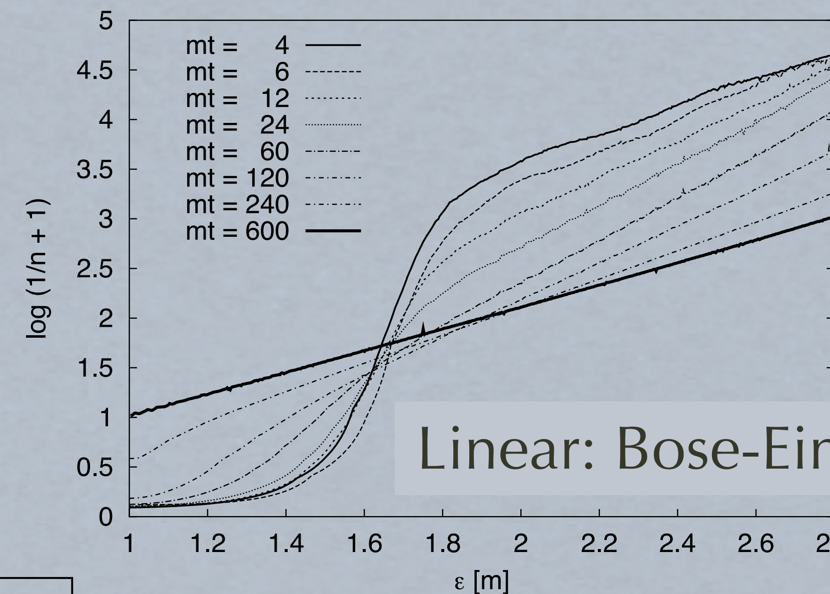
Equilibration

Berges, SB, Serreau, Wetterich 2003/4

evolution of the spectrum:

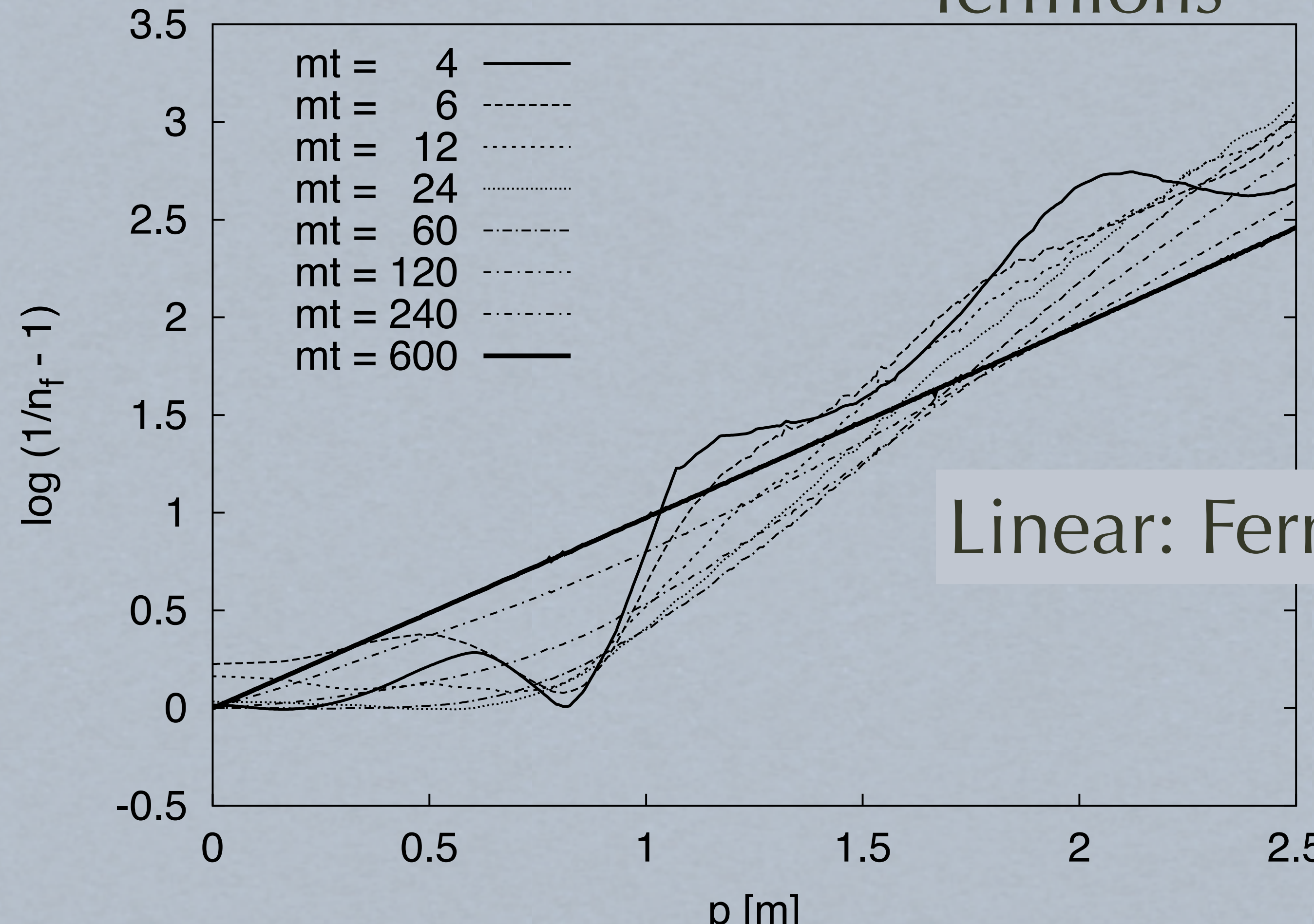


scalars



Linear: Bose-Einstein statistics

fermions



Linear: Fermi-Dirac statistics

A growing number of studies...

Scalars:

1+1dim: Cox&Berges2000, Aarts&Berges2001,2002 (vs exact, vs classical)
Blagoev&Cooper&Dawson&Mihaila 2001 (BVA)
Berges 2002 (O(N) resummation)
Gasenzer&Pawlowski 2007 (an RG approach)

2+1dim: Juhem&Cassing&Greinen 2001 (vs transport)

3+1dim: Danielewicz 1984 (nonrelativistic, vs. kinetic theory)
Berges&Borsanyi 2005 (isotropisation, vs. transport theory)
Muller&Lindner 2005 (vs. kinetic theory)
Berges&Serreau 2002 (parametric resonance)
Tranberg&Arrizabalaga&Smit 2004,2005 (bg field, tachionic instability)
Tranberg&Rajantie 2006 (looking for defects)
Aarts&Tranberg 2008 (inflationary)

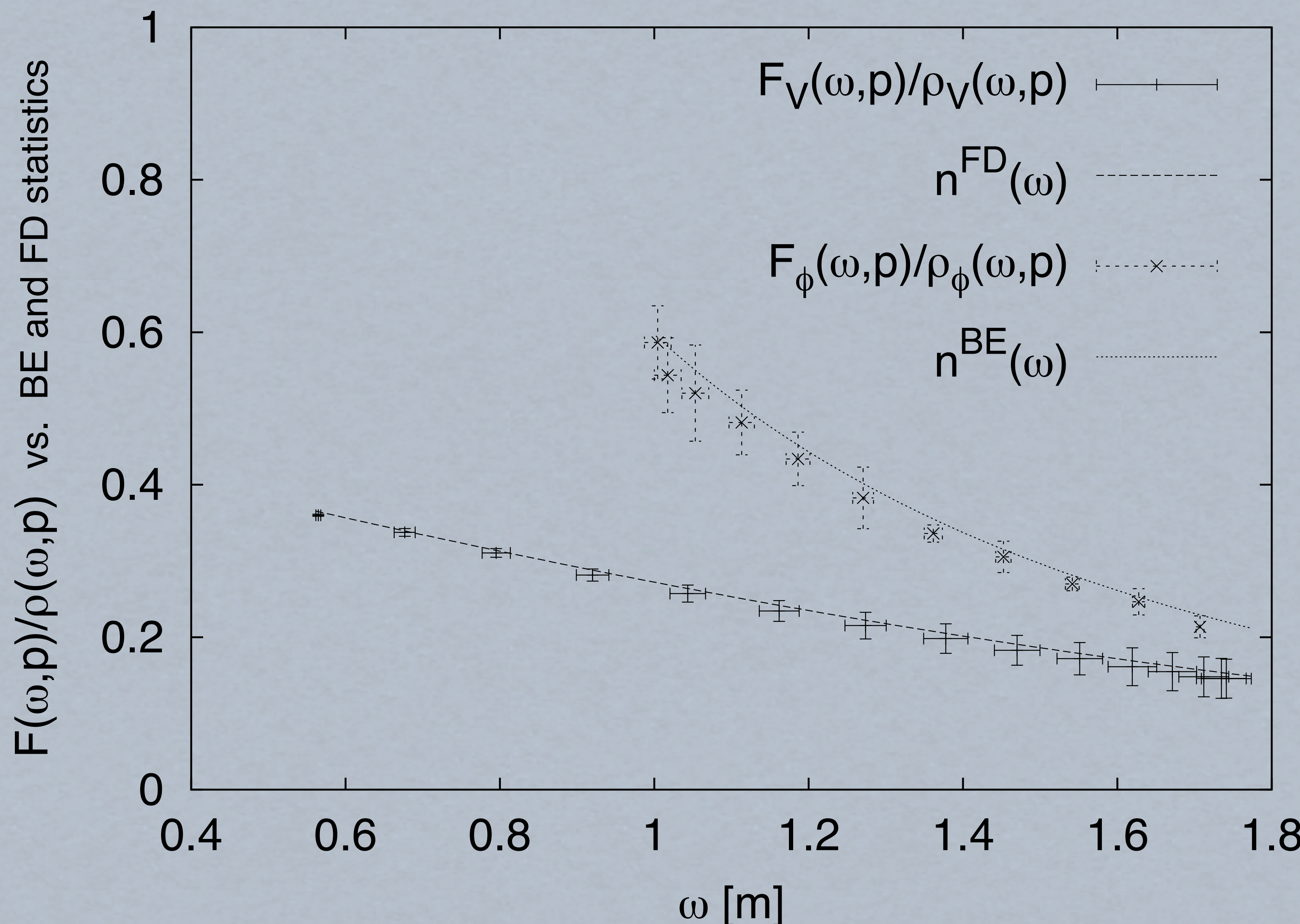
Yukawa:

3+1dim: Berges&Borsanyi&Serreau/Wettterich 2003
Muller&Lindner 2007 (vs. kinetic theory)

Cold atoms:

1+1dim: Berges&Gasenzer(&Seco&Schmidt) 2005,2007 (vs classical)
Gasenzer&Temme 2008 (inhomogeneous)
Braunschadel&Gasenzer 2008 (vs. transport)

The final state



The propagators become stationary, and the KMS condition becomes valid.

KMS condition:

$$G^{>}(\omega) = e^{\beta\omega} G^{<}(\omega)$$

$$F(\omega) = -i\left(\frac{1}{2} + n_B(\omega)\right)\rho(\omega)$$

↓ particle number

Boltzmann equation (if *n* is time dependent)

Before equilibration: F and ρ are related through *n*(*t*,ω).

Berges,SB,Serreau 2003, SB 2004

What is the stationary solution?

from analytics

$$
\begin{aligned}
(-p_0^2+\omega_p^2)\tilde{F}(p) &= \int \frac{d\omega}{2\pi}\left[\frac{\tilde{\Sigma}^F(p)\tilde{\rho}(\omega;\vec{p})}{i(p_0-\omega-i\epsilon)}+\frac{\tilde{\Sigma}^\rho(\omega;\vec{p})\tilde{F}(p)}{i(p_0-\omega+i\epsilon)}\right] \\
(-p_0^2+\omega_p^2)\tilde{\rho}(p) &= \int \frac{d\omega}{2\pi}\left[\frac{\tilde{\Sigma}^\rho(p)\tilde{\rho}(\omega;\vec{p})}{i(p_0-\omega-i\epsilon)}+\frac{\tilde{\Sigma}^\rho(\omega;\vec{p})\tilde{\rho}(p)}{i(p_0-\omega+i\epsilon)}\right]
\end{aligned}
$$

If $\tilde{F}(\omega) = -i\left(\frac{1}{2}+n_{BE}(\omega)\right)\tilde{\rho}(\omega) \longrightarrow \tilde{\Sigma}^F(\omega) = -i\left(\frac{1}{2}+n_{BE}(\omega)\right)\tilde{\Sigma}^\rho(\omega)$

then the two equations are equivalent.

There is a stationary solution that satisfies KMS.

This means late time thermalisation (if $\Sigma^{F/\rho}\neq 0$)

The sunset diagram (2→2)

$$\begin{aligned}
\Sigma^<(\omega,\vec{x}) &= -\frac{\lambda^2}{6}\int\frac{d\omega_1}{2\pi}\frac{d\omega_2}{2\pi}\frac{d\omega_3}{2\pi}\delta(\omega-\omega_1-\omega_2-\omega_3) \\
&\qquad G^<(\omega_1,\vec{x})G^<(\omega_2,\vec{x})G^<(\omega_3,\vec{x}) \\
&= -\frac{\lambda^2}{6}\int\frac{d\omega_1}{2\pi}\frac{d\omega_2}{2\pi}\frac{d\omega_3}{2\pi}\delta(\omega-\omega_1-\omega_2-\omega_3)e^{-\beta\omega_1-\beta\omega_2-\beta\omega_3} \\
&\qquad G^<(-\omega_1,\vec{x})G^<(-\omega_2,\vec{x})G^<(-\omega_3,\vec{x}) \\
&= -\frac{\lambda^2}{6}\int\frac{d\omega_1}{2\pi}\frac{d\omega_2}{2\pi}\frac{d\omega_3}{2\pi}\delta(\omega-\omega_1-\omega_2-\omega_3)e^{-\beta\omega} \\
&\qquad G^<(-\omega_1,\vec{x})G^<(-\omega_2,\vec{x})G^<(-\omega_3,\vec{x}) \\
&= -\frac{\lambda^2}{6}\int\frac{d\omega_1}{2\pi}\frac{d\omega_2}{2\pi}\frac{d\omega_3}{2\pi}\delta(-\omega-\omega_1-\omega_2-\omega_3)e^{-\beta\omega} \\
&\qquad G^<(\omega_1,\vec{x})G^<(\omega_2,\vec{x})G^<(\omega_3,\vec{x}) \\
&= \Sigma^<(-\omega,\vec{x})e^{-\beta\omega}
\end{aligned}$$

(similar argument for any two-loop diagram)

The self energy inherits the KMS condition from *G*.

What we see in numerics is a genuine thermalisation.

?

Late time is equilibrium

Euclidean

$$G_E(\tau;\vec{p}) = \int \frac{d\omega}{2\pi}\tilde{G}^{<}(\omega,\vec{p})e^{\tau\omega}$$
$$\Sigma_E(\tau;\vec{p}) = \int \frac{d\omega}{2\pi}\tilde{\Sigma}^{<}(\omega,\vec{p})e^{\tau\omega}\,.$$

and

$$(-\partial_\tau^2+\omega_p^2)G_E(\tau)-\delta(\tau) = -\int d\tau'\Sigma_E(\tau-\tau')G_E(\tau')\,.$$

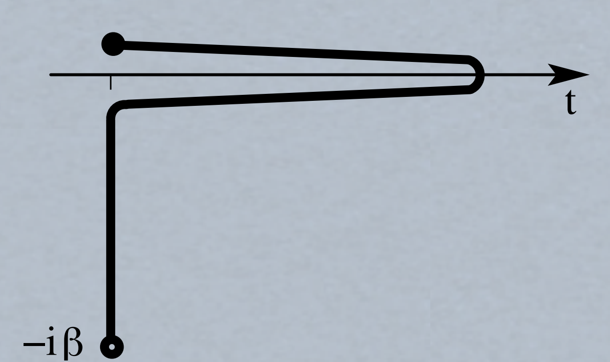
is equivalent to

late time

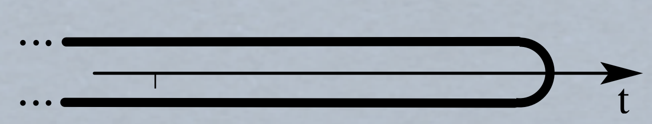
$$\tilde{F}(\omega) = -i\left(\frac{1}{2}+n_{BE}(\omega)\right)\tilde{\rho}(\omega)$$
$$\left.\frac{d\rho(t)}{dt}\right|_{t=0} = 1$$

and

$$\left(\partial_x^2+m^2+\Sigma_0\right)\rho(x) = -\int_0^{x_0} dz^4\Sigma^\rho(x-z)\rho(z)$$

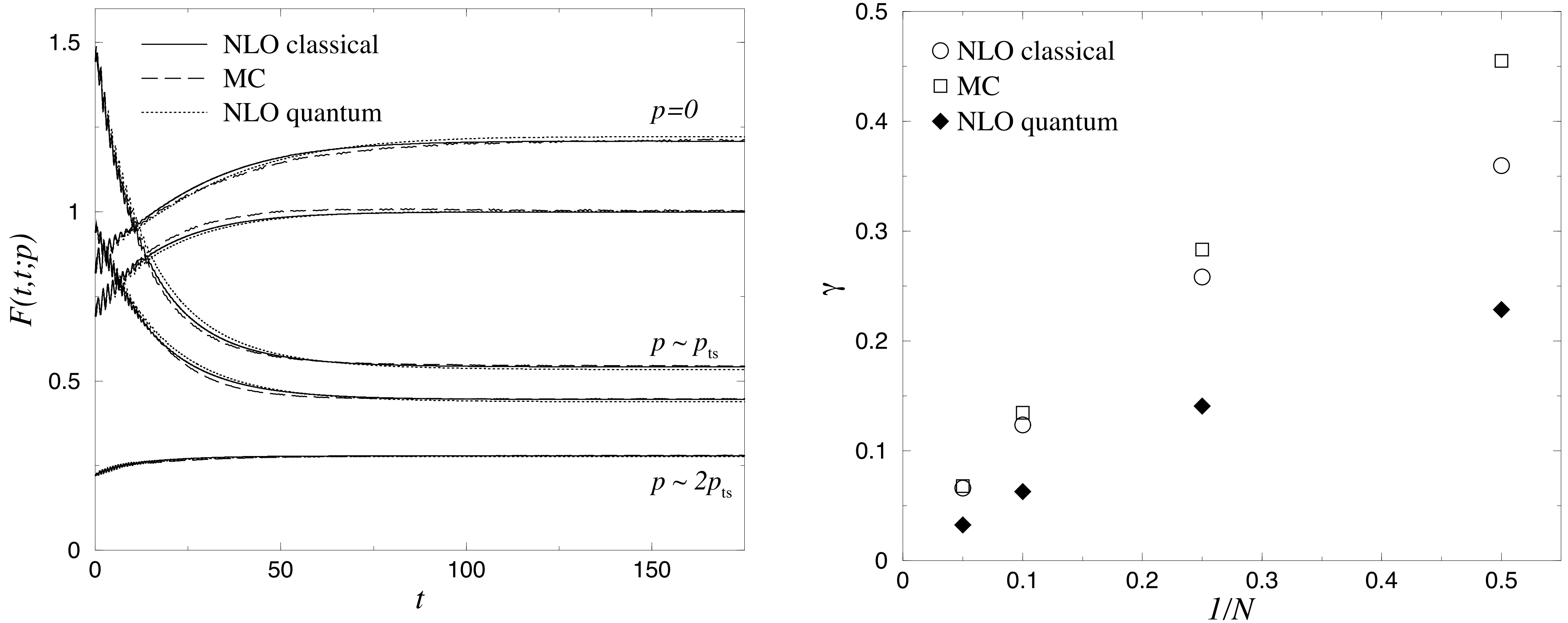


$\equiv$



Should we believe the dynamics?

Classical 2PI *vs* classical simulation.



1+1 d, O(N) NLO

Aarts,Berges 2001

Yes. In most cases.

(Small enough expansion parameter => exact dynamics)

Topological defects: *counterexample!* *Rajantie&Tranberg 2006*

Suppose you buy 2PI...

What should we think about other approaches?

Classical statistical field theory

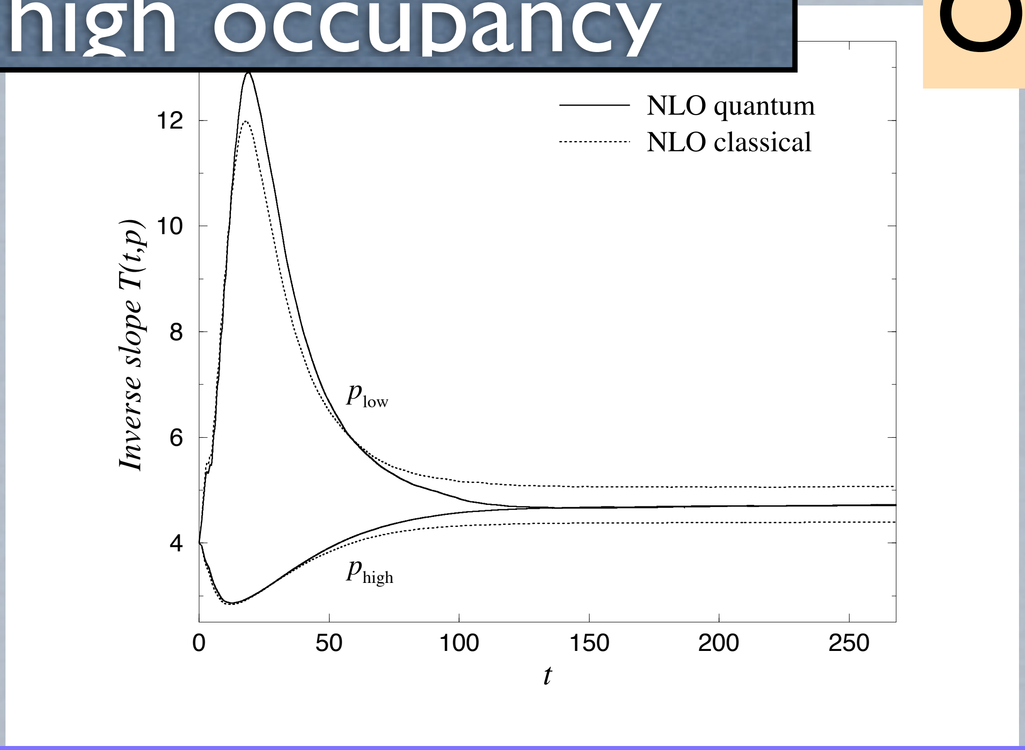
- Most modelling of Early Universe fields is based on classical methods: preheating, defects
- Do we have a classical - quantum comparison?

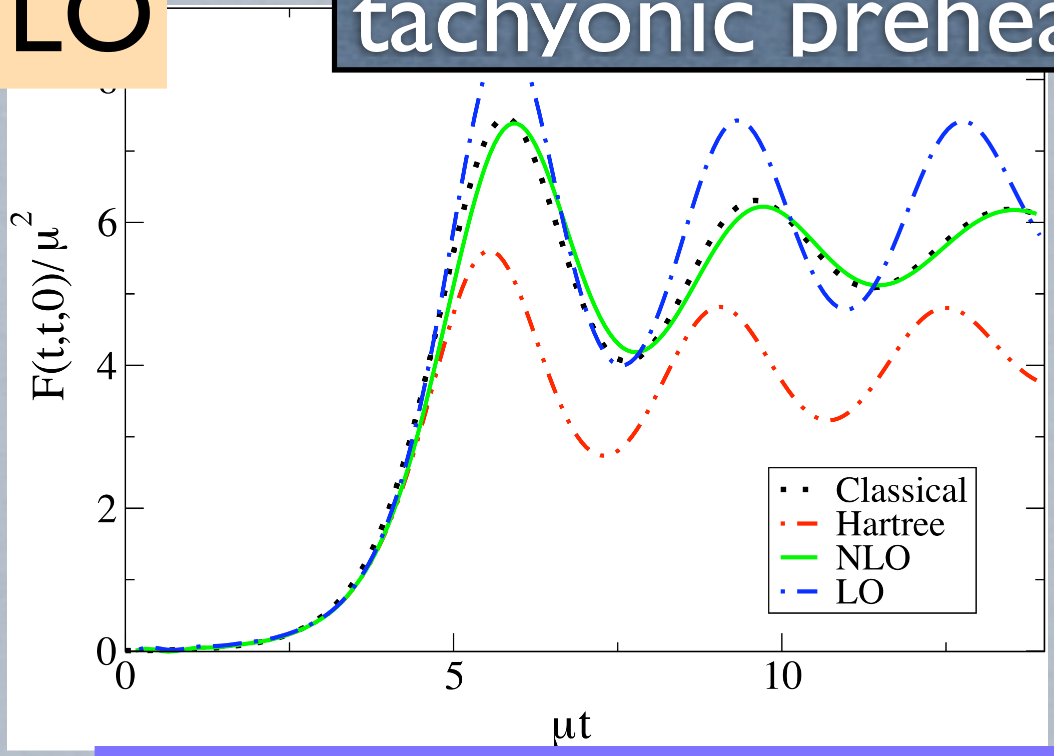
Classical vs quantum

high occupancy

O(N) NLO

tachyonic preheating



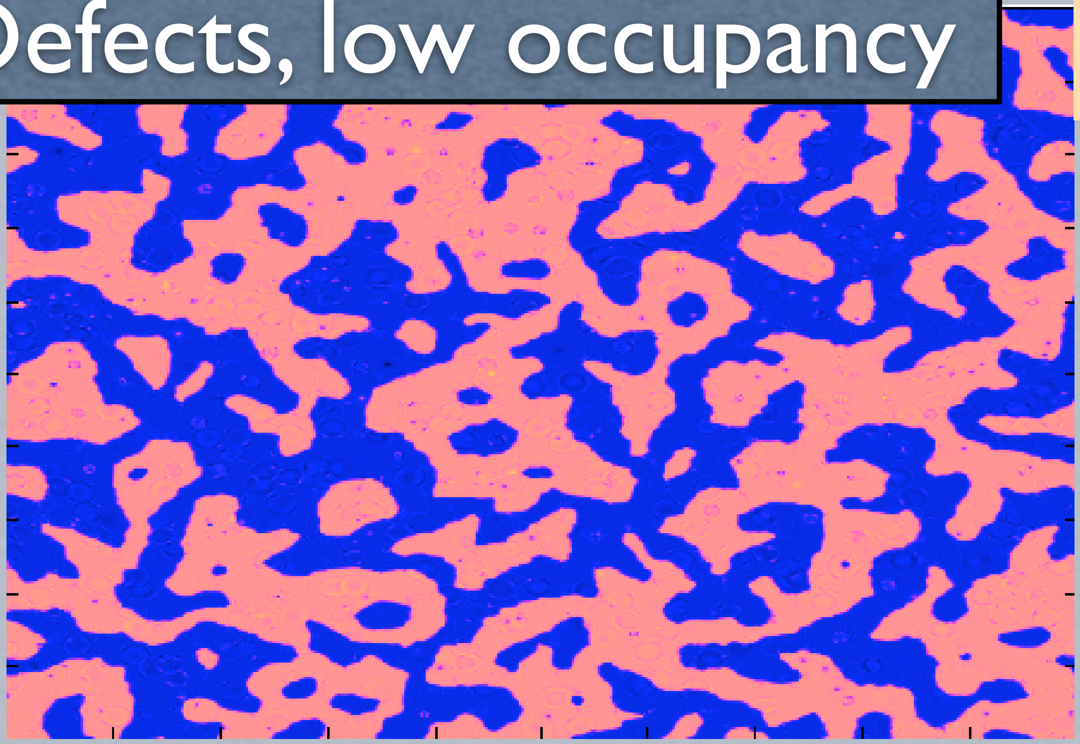


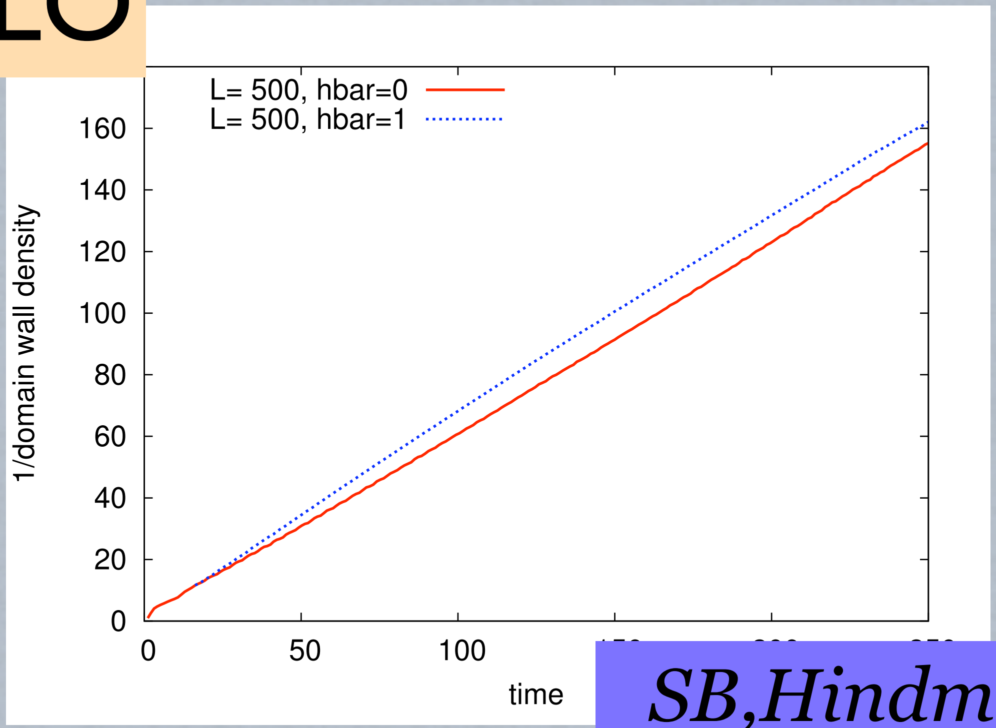
Aarts,Berges 2001

Arrizabalaga,Smit,Tranberg 2004

Defects, low occupancy

Z_2, LO





SB,Hindmarsh 2007

Suppose you buy 2PI...

What should we think about other approaches?

Classical statistical field theory

Most modelling of Early Universe fields is based on classical methods: preheating, defects

Do we have a classical - quantum comparison?

Transport theory

Boltzmann eq does the same resummations as 2PI.

Calzetta,Hu 1988

$$2p^{\mu}\partial^{x}_{\mu} i\bar{G}^{\gtrless} - \left\{\bar{\Sigma}^{\delta} + \mathrm{Re}\,\bar{\Sigma}^{R}, i\bar{G}^{\gtrless}\right\} - \left\{i\bar{\Sigma}^{\gtrless}, \mathrm{Re}\,\bar{G}^{R}\right\} = i\bar{\Sigma}^{<} i\bar{G}^{>} - i\bar{\Sigma}^{>} i\bar{G}^{<}$$

NLO

Lowest Order

Lowest order: 2-to-2 scattering *(scalar&setting-su*

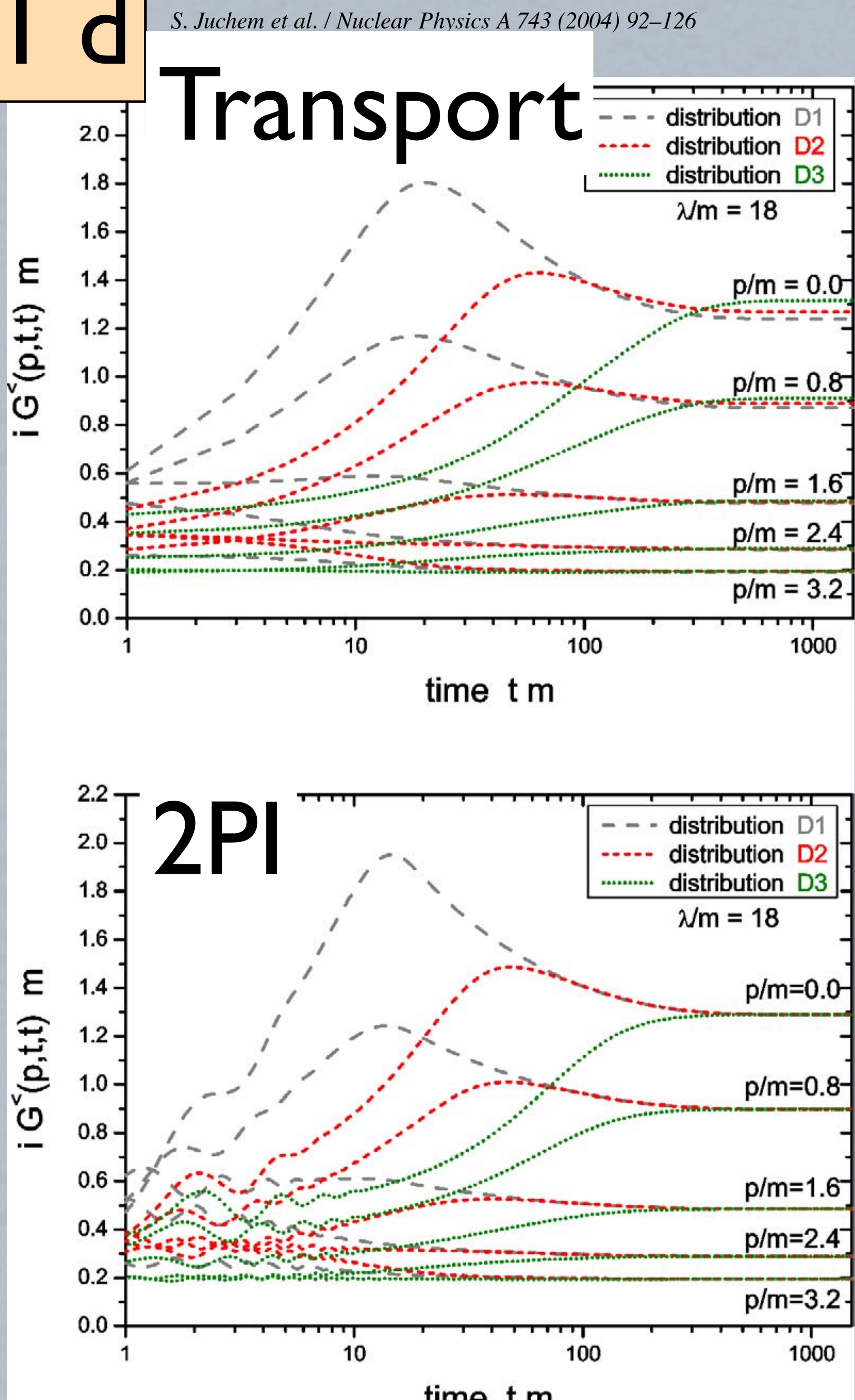
Particle number conservation

Muller,Lindner 2005

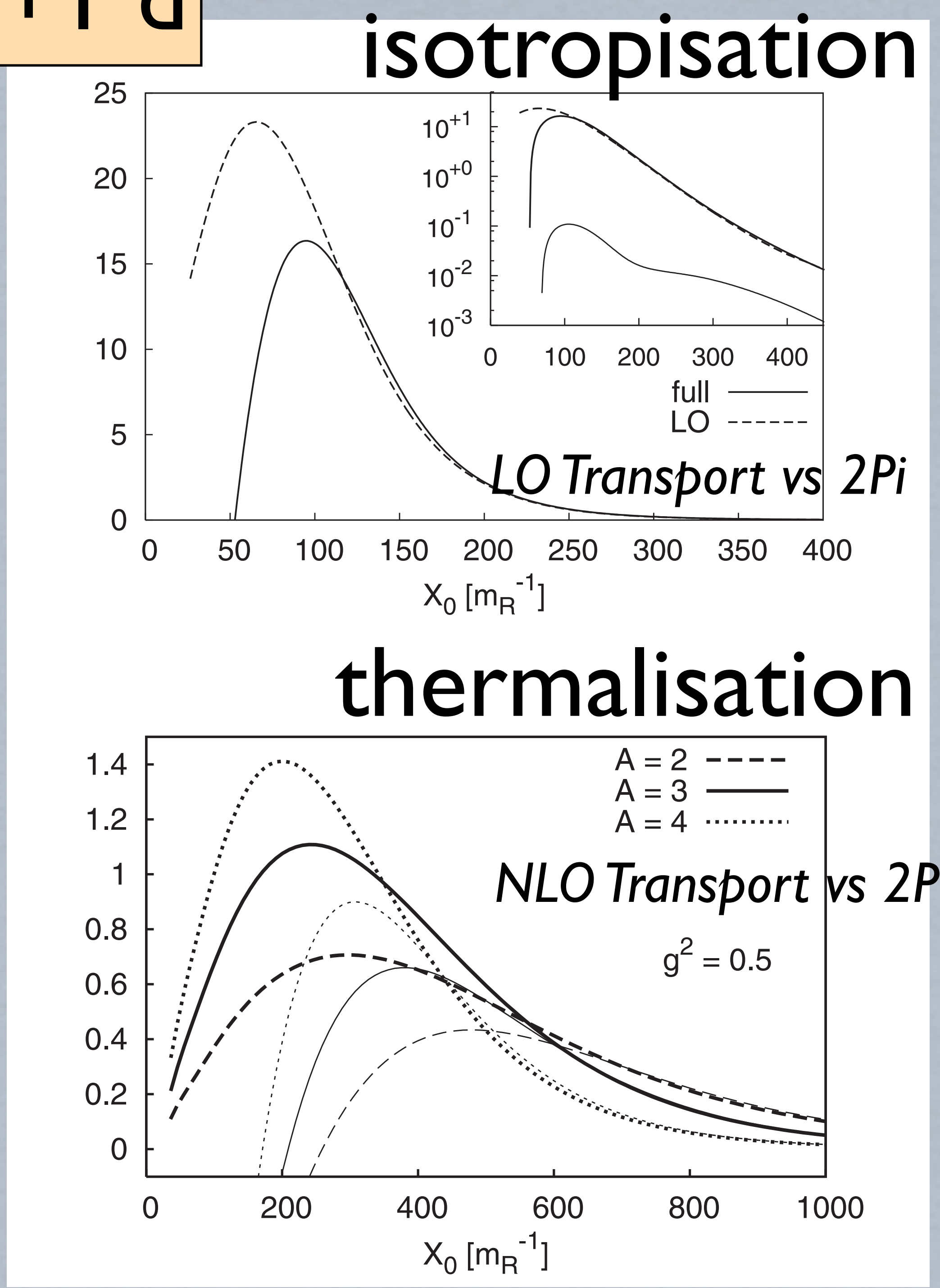
2PI equations → gradient expansion (LO or NLO?) → Transport eq.

NLO Transport *vs* 2PI

2+1 d



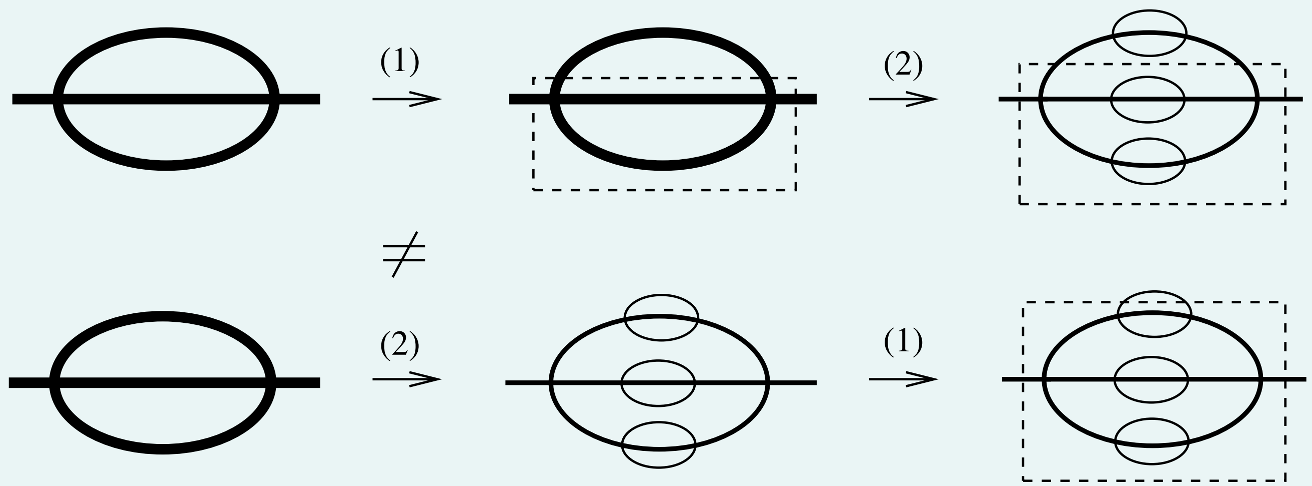
3+1 d

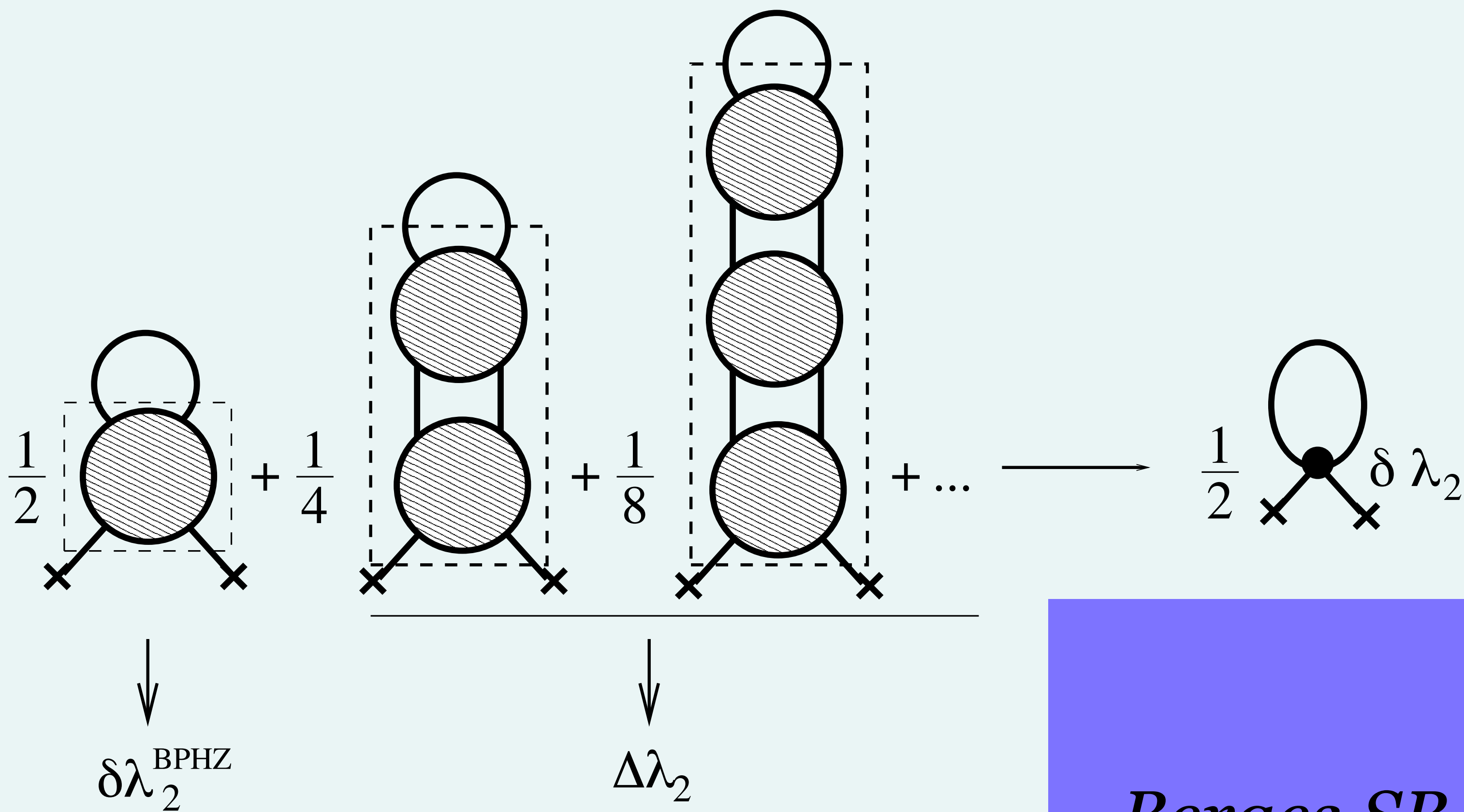


Juchem,Cassing,Greiner 2004

Berges, SB 2005

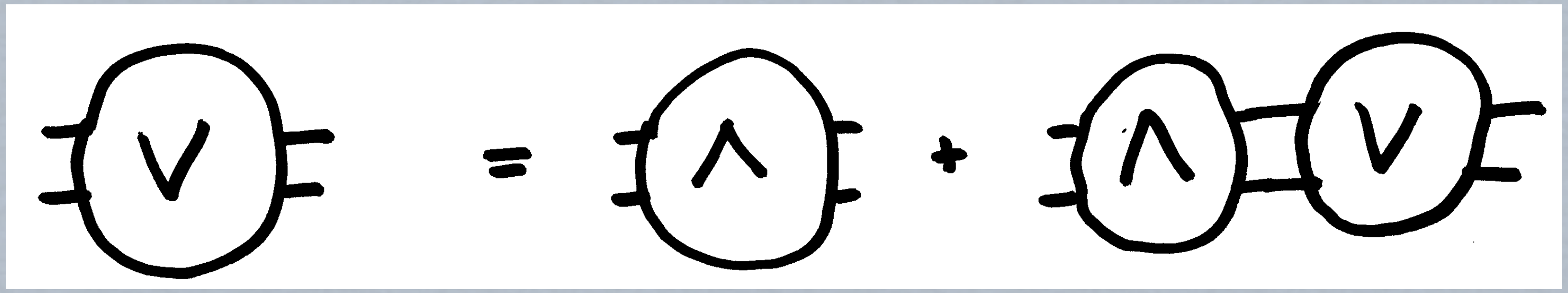
How to renormalize?





van Hees,Knoll 2001
Blaizot,Iancu,Reinosa 2003
Berges,SB,Reinosa,Serreau 2004,2005
Cooper,Mihaila,Dawson 2004,2006

the Bethe-Salpeter equation



$$\Lambda = 4\frac{\delta^2\Gamma_2[G,\Phi]}{\delta G \delta G}$$

implements a one-channel resummation of
the four-point function.

This is the *same resummation* as in the 2-point equation.

Renormalisation of this 4-point equation removes all
sub-divergences from the 2-point equation.

(a scalar example)

Renormalization: the lazy way

Renormalize at T_1 and T_2 independently $G^{-1} = G_0^{-1} - \Sigma$

The renormalization condition for Σ fixes $\delta m^2 + \delta\lambda$  , but not δm^2 and $\delta\lambda$ individually

keep $\delta\lambda_1 = 0$ $\delta\lambda_2 = 0$ and obtain $\delta m_1,$ δm_2 and the divergent tadpoles

$$\delta m_1^2 = m_T^2(T_1) + \delta m^2 + \delta\lambda$$  $_1$

$$\delta m_2^2 = m_T^2(T_2) + \delta m^2 + \delta\lambda$$  $_2$

2 equations,
2 unknowns: $\delta m^2 \quad \delta\lambda$

Matching: $m_T^2(T) \sim \lambda_R T^2$

Perturbative input: this defines the renormalized coupling.
Finiteness is not spoiled by the use of non-resummed input!

This realizes a renormalization condition like:

$$V|_{k^*} = \lambda_R + \mathcal{O}(\lambda_R^2)$$ (at leading order)

Instead of this one:

$$V|_{k^*} = \lambda_R$$

Proof of these statements: follows from the Bethe-Salpeter machinery

The 2PI propagator

The 2PI variational propagator: $\frac{\delta \Gamma_{2\mathrm{PI}}[\Phi, G]}{\delta G(x,y)} = 0$

$\Sigma = 2i \frac{\delta \Gamma_2}{\delta G}$

$$G_{2\mathrm{PI}}^{-1}[\Phi] = G_0^{-1}[\Phi] - \Sigma[\Phi, G]$$

Without truncation $G_{2\mathrm{PI}}$ is the full propagator.

If we do truncate at some order:

In the O(N) model $G_{2\mathrm{PI}}$ is gapless to given order only
In QED $G_{2\mathrm{PI}}$ is not transversal to given order only.

(This symmetry breaking effect appears at orders higher than the truncation of $\Gamma_{2\mathrm{PI}}$)

Reason for the apparent failure: only the s-channel was resummed or from the

standard effective action

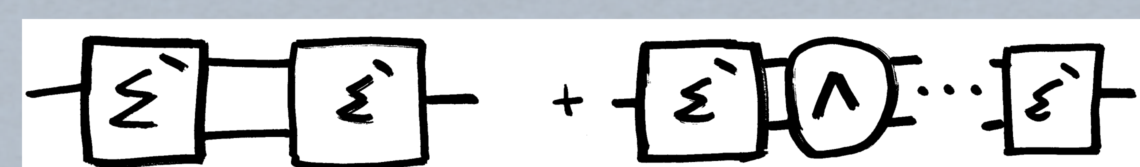
At vanishing sources: $\Gamma_{2\mathrm{PI}}[\Phi, G_{2\mathrm{PI}}[\Phi]] = \Gamma[\Phi]$

This is the resummed effective action (non-polynomial)

An alternative definiton of the propagator:

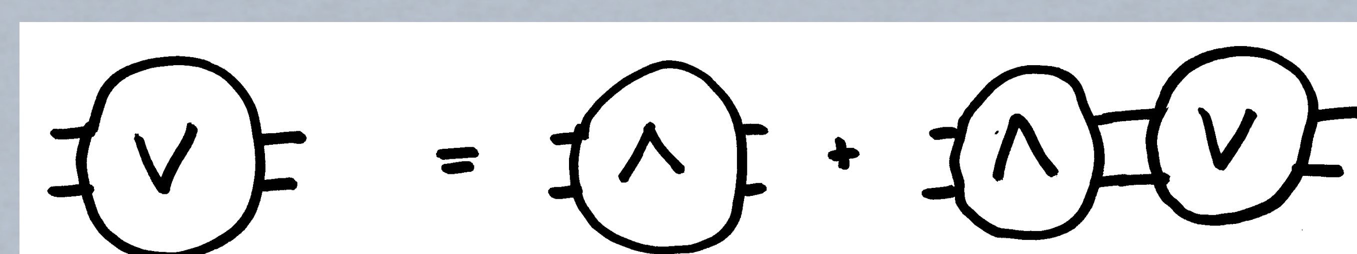
$$G^{-1}_{1\mathrm{PI}} = \frac{\delta^2 \Gamma[\Phi]}{\delta\Phi\delta\Phi} = \frac{\delta^2 \Gamma_{2\mathrm{PI}}[\Phi, G_{2\mathrm{PI}}[\Phi]]}{\delta\Phi^2}$$

$$G^{-1}_{\mathrm{proper}} = \frac{\delta^2 \Gamma_{2\mathrm{PI}}[\Phi, G_{2\mathrm{PI}}[\Phi]]}{\delta\Phi_x \delta\Phi_y} = G_0^{-1} - 2\frac{\delta^2 \Gamma_2}{\delta\Phi_x\delta\Phi_y} +$$

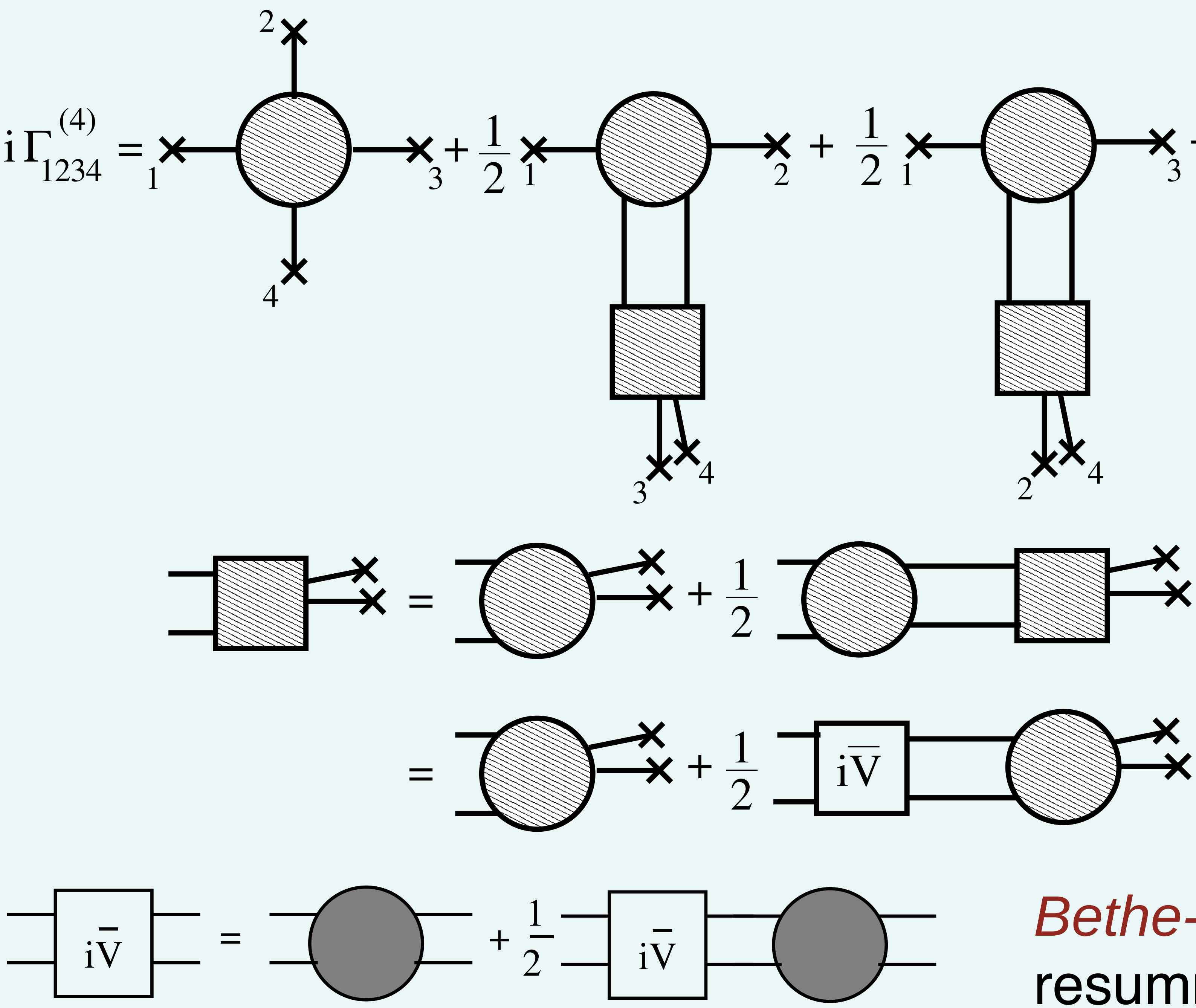


$\Sigma' = \frac{\delta\Sigma}{\delta\Phi}$ $\Lambda = 4\frac{\delta^2\Gamma_2[G,\Phi]}{\delta G \delta G}$

Bethe-Salpeter equation appears here naturally



Four point function from 2PI



All three channels are present

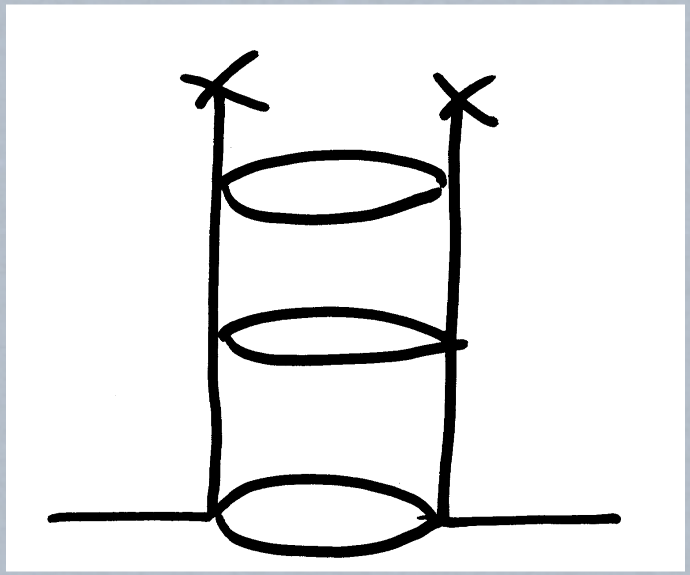
Bethe-Salpeter equation
resummation in one channel only

Berges,SB,Reinosa,Serreau 2004

restoration of the

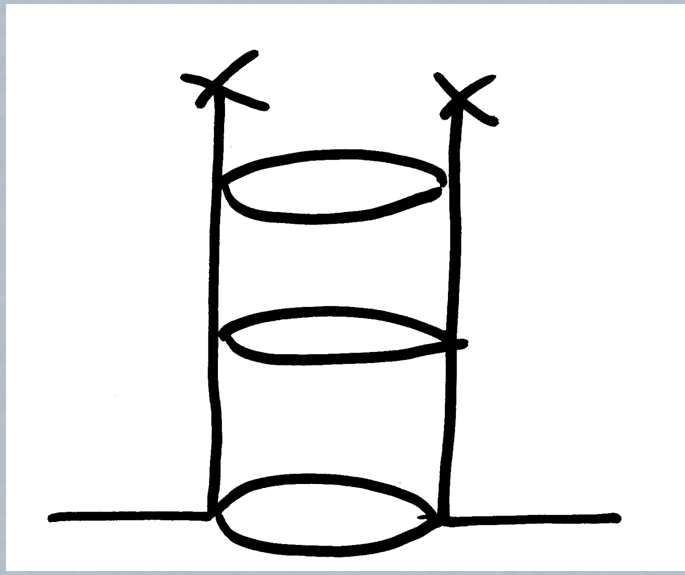
Goldstone theorem

G_{2PI} :

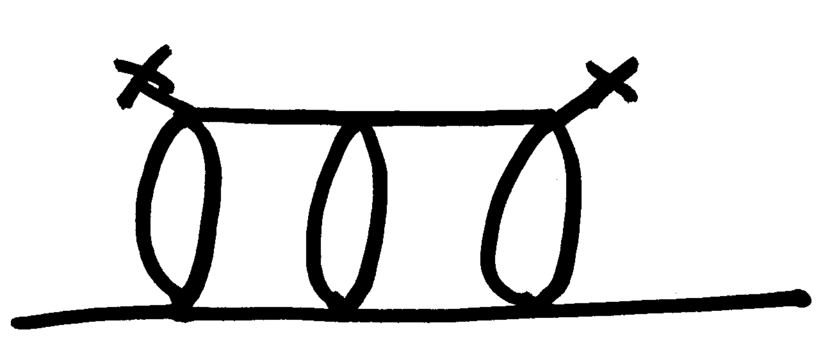


s channel only

G_{1PI} :



+

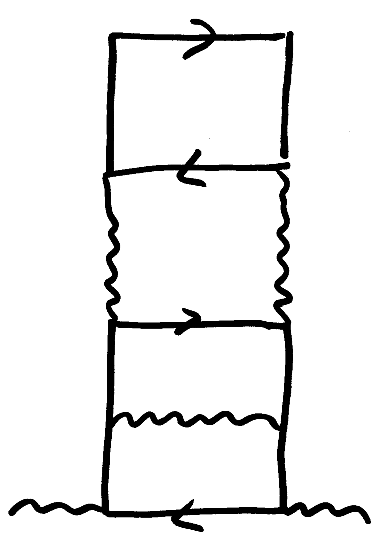


s+t+u channels

van Hees,Knoll 2001
Berges,SB,Reinosa,Serreau 2004

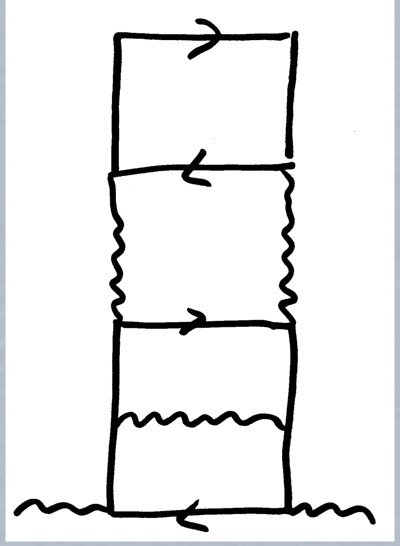
restoration of the Ward identities

$G_{2\mathrm{PI}}$:



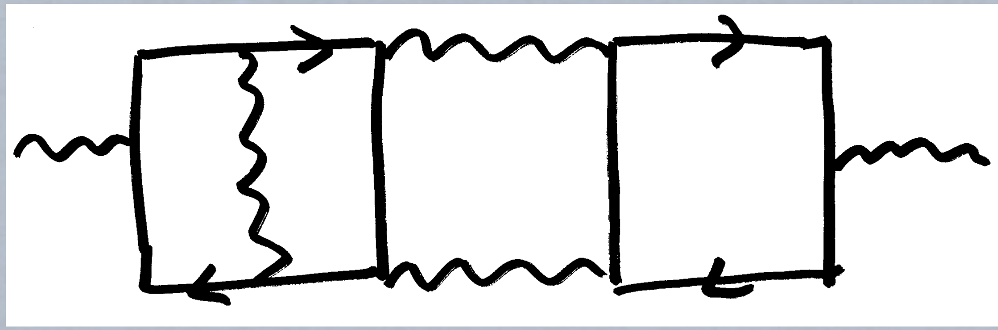
s channel only

$G_{1\mathrm{PI}}$:



s+t+u channels

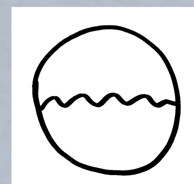
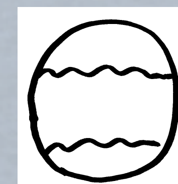
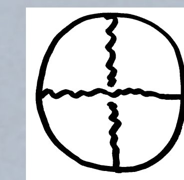
+



2PI effective action is just a means to ladder-resum the standard effective action

Reinosa,Serreau 2006-7
Carrington,Kovalchuk 2007

Can we do gauge fields?

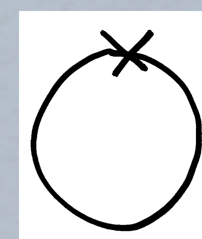
2-loop order  resummed:  3-loop order 

Broken gauge invariance: new counterterms appear. Reinosa, Serreau 2006

Gauge fixing: Covariant gauge: $\lambda = 1/\xi$

Usual counterterms:

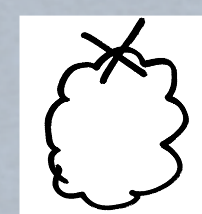
$(e^2 \log a) \sim$	$\delta Z_3, \quad \delta Z_2$

transversal photon
+ electron

New counterterms:

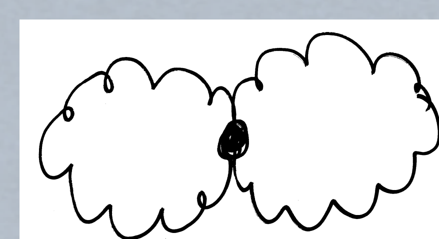
$(e^4 \log a) \sim$	$\delta\lambda G^{\mu\nu} k_\mu k_\nu$
$(e^4 a^2) \sim$	$\delta M^2 G^{\mu\nu} g_{\mu\nu}$
$(e^4 \log a) \sim$	$\delta g^a G_{\mu\nu} G^{\mu\nu}$
$(e^4 \log a) \sim$	$\delta g^b G^\mu_\mu G^\nu_\nu$



longitudial photon

$$\frac{\partial \Pi_L}{\partial k^2}\Big|_{k^*} = 0$$

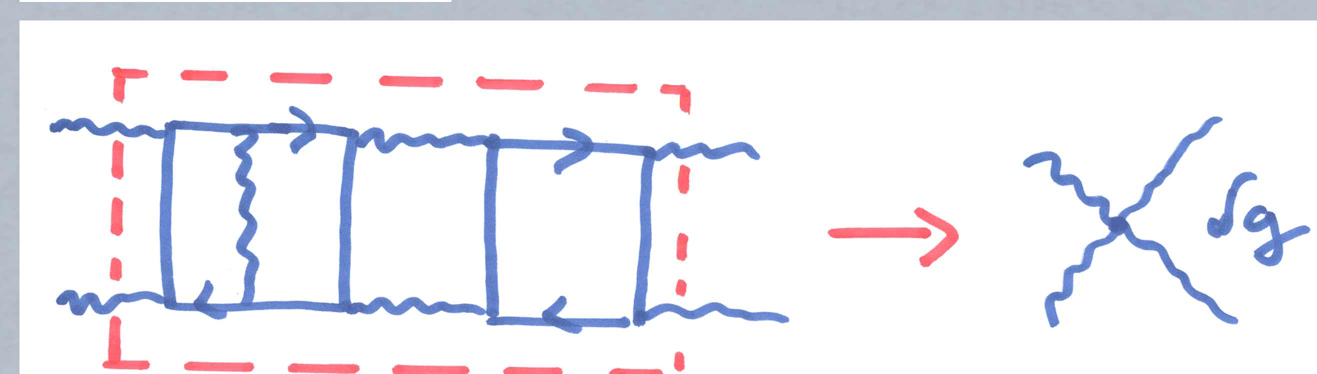
$$\Pi_L(k^*) = 0$$



photon self interaction

Bethe-Salpeter -> $V_L^{\mu\nu}|_{k^*} = 0$

Subdivergency in the ladder:
Calculated as the solution of
the Bethe-Salpeter equation



The 2PI pressure curve

Pressure is quartically divergent

-> we calculate $\frac{p(T_1)-p(T_2)}{T_1^4-T_2^4}$

$$P = \frac{T}{L_3} i\Gamma[\phi_0, D(\phi_0)]$$

$$\mathcal{S} = \frac{\partial P}{\partial T},$$

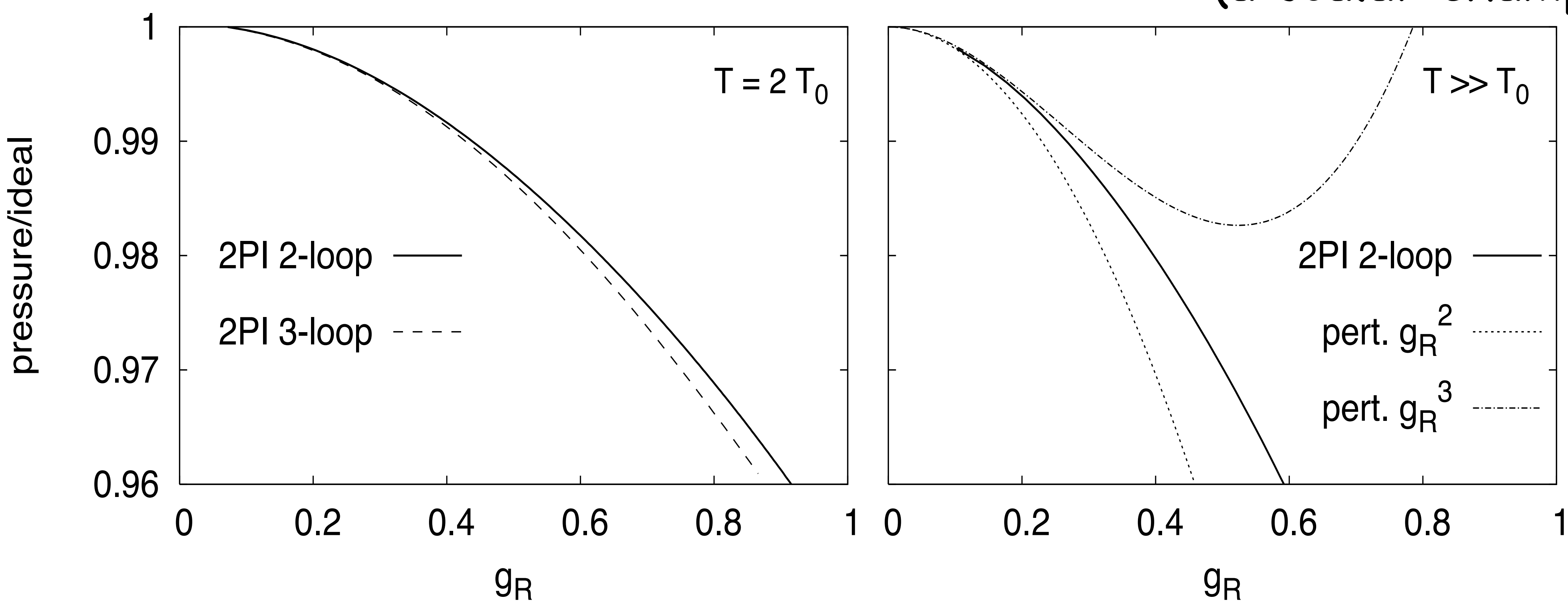
$$\mathcal{E} = -P + T\mathcal{S} = T^2 \frac{\partial}{\partial T}\left(\frac{P}{T}\right)$$

T_1 : find the counterterms, calculate the pressure

T_2 : use the counterterms, calculate the pressure

(The regularized equations are solved)

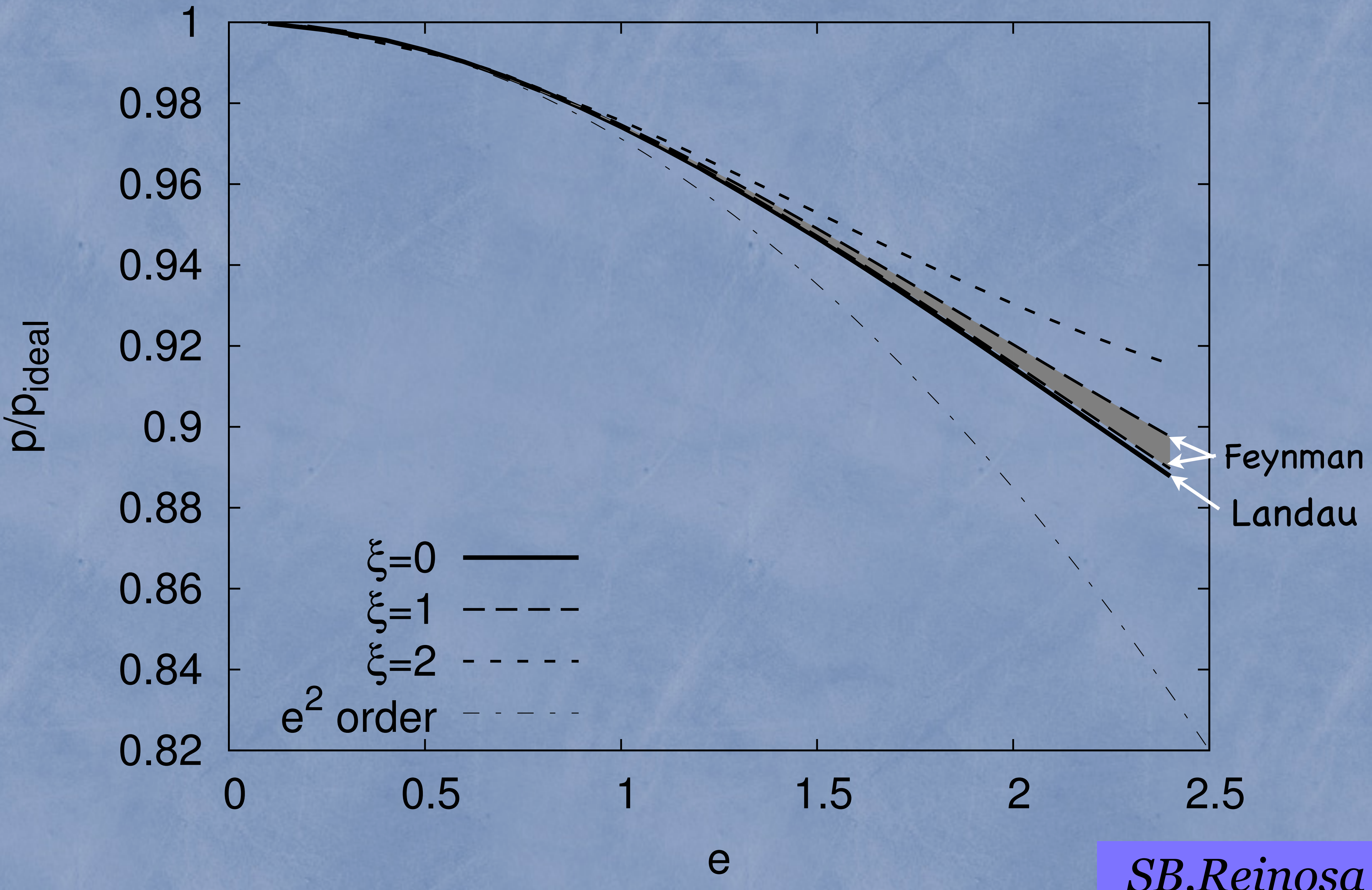
(a scalar example)



Berges,SB,Reinosa,Serreau 2004

The pressure curve: QED

(parameter: gauge fixing)



SB,Reinosa 2007

see also Andersen &Strickland 2005

Restoration of gauge parameter independence

Arrizabalaga,Smit 2002

"Strong" gauge parameter independence:

(e.g. Perturbation theory)

pressure(*N*,e,ξ) is ξ independent for any *N*

N: loop order, e couling

ξ: gauge parameter

"Weak" gauge parameter independence:

(e.g. 2PI effective action)

pressure(*N*,e,ξ) ξ dependence at $\mathcal{O}(e^{2N+2})$

N(e, ξ): order required for the required precision

$N_{2PI} < N_{pert}$, and $N_{2PI}(e, \xi=0) < N_{2PI}(e, \xi)$

Conclusion

Long live 2PI!

Self-consistent,
Cures secularity,
Renormalisable *(and we know how to renormalise)*
Gives a prescription for symmetry-respecting propagators
Gauge symmetry is restored as we increase the order in g
(all gauges are equal, but some gauges are more equal)

Orwell, Animal farm

We need:
more people,
more jobs,
more machines.