# On SUGRA description of boost-invariant conformal plasma at strong coupling

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Original work: **R. A. Janik**, $\cdots$

Gauge theory/string theory (Maldacena correspondence)

$$\Downarrow$$

Consider $\mathcal{N} = 4\ SU(N)$ SYM in the planar ('t Hooft) limit:

- $g_{YM}^2 N \ll 1$ (weak effective coupling) $\Longrightarrow$ perturbative gauge theory description
- $g_{YM}^2 N \gg 1$ (strong effective coupling) $\Longrightarrow$ IIB string theory on $AdS_5 \times S^5$

## Motivation:

Use *String Theory* in a context of Maldacena correspondence as a guiding principle in constructing *Non-equilibrium Quantum Field Theory*

$\Rightarrow$use string theory to formulate dissipative relativistic theory of conformal fluids

# Outline of the talk:

- Boost-invariant expansion of the conformal fluids (phenomenological theory)

$\Rightarrow$Ideal CFT fluid dynamics

$\Rightarrow$First order dissipative CFT fluid dynamics

$\Rightarrow$Second order dissipative CFT fluid dynamics (why it is needed?)

$\Rightarrow n$th-order dissipative CFT fluid dynamics

- Some aspects of AdS/CFT correspondence

$\Rightarrow$Nonsingularity of the background geometry as a guiding principle to determine correct physics

- Janik's proposal for string theory dual to boost invariant expansion

$\Rightarrow$Successes of the proposal (equation of state, shear viscosity, relaxation time)

$\Rightarrow$Singularities in the supergravity approximation

- Interpretation of singularities and future directions

We study expansion of the CFT fluid (gauge theory plasma) in boost invariant frame

$\Rightarrow$Widely expected to be a correct description of central region of QGP produced in ultra-relativistic collisions of heavy nuclei

Convert Minkowski frame

$$ds_4^2 = -dx_0^2 + dx_\perp^2 + dx_3^2$$

into a frame with boost-invariance along $x_3$ direction

$$x_0 = \tau\ \cosh y\,, \qquad x_3 = \tau\ \sinh y$$

$$ds_4^2 = -d\tau^2 + \tau^2\ dy^2 + dx_\perp^2$$

Assume

$$\epsilon = \epsilon(\tau)\,, \qquad p = p(\tau)$$

for local energy density $\epsilon$ and pressure $p$ in the fluid

## Ideal CFT fluid

Stress energy tensor:

$$T_{\mu\nu} \equiv T_{\mu\nu}^{equilibrium} = (\epsilon + p) u_\mu u_\nu + p \eta_{\mu\nu}$$

where $u^\mu$ is local 4-velocity of the fluid, $u^2 = -1$.

From conformal invariance

$$T_\mu^\mu = 0 \qquad \Rightarrow \qquad \epsilon = 3P$$

Conservation law in boost-invariant frame:

$$\partial_\mu T^{\mu\nu} = 0 \qquad \Rightarrow \qquad \partial_\tau \epsilon = -\frac{4}{3}\frac{\epsilon}{\tau}$$

Scaling of $\epsilon$, $s$ (entropy density), $\eta$ (shear viscosity), $T$ (temperature), $\tau_\pi$ (relaxation time)

$$\epsilon \propto \tau^{-4/3}\,, \qquad T \propto \epsilon^{1/4} \propto \tau^{-1/3}\,, \qquad \eta \propto s \propto T^3 \propto \tau^{-1}$$

$$\tau_\pi \propto T^{-1} \propto \tau^{1/3}$$

## First-order dissipative CFT fluid dynamics

Stress energy tensor:

$$T_{\mu\nu} = T_{\mu\nu}^{equilibrium} + \tau_{\mu\nu}\,, \qquad \tau_{\mu\nu} \propto \eta\left(\nabla_\mu u_\nu + \nabla_\nu u_\mu - \mathrm{trace}\right)$$

$\Rightarrow$

$$\partial_\tau \epsilon = -\frac{4}{3}\frac{\epsilon}{\tau} + \frac{4\eta}{3\tau^2}$$

From scaling, viscous correction becomes subdominant as $\tau \to \infty$:

$$\frac{\epsilon}{\tau} \sim \frac{\tau^{-4/3}}{\tau} \sim \tau^{-7/3}\,, \qquad \frac{\eta}{\tau^2} \sim \frac{\tau^{-1}}{\tau^2} \sim \tau^{-9/3}$$

Thus we expect approach to equilibrium in boost-invariant frame to correspond to late-time dynamics

**Why go to second order?**

$\Rightarrow$ first order hydro allow for acausal signal propagation

## Second-order dissipative CFT fluid dynamics

From Müller-Israel-Stewart theory:

$$0 = \frac{d\epsilon}{d\tau} + \frac{\epsilon + p}{\tau} - \frac{1}{\tau}\Phi$$

$$0 = \frac{d\Phi}{d\tau} + \frac{\Phi}{\tau_\pi} + \frac{1}{2}\Phi\left(\frac{1}{\tau} + \frac{1}{\beta_2}T\frac{d}{d\tau}\left(\frac{\beta_2}{T}\right)\right) - \frac{2}{3}\frac{1}{\beta_2}\frac{1}{\tau}$$

where $\tau_\pi$ is the relaxation time, $\Phi$ is related to the dissipative part of the energy-momentum, and

$$\beta_2 = \frac{\tau_\pi}{2\eta}$$

From scaling, $\tau \longrightarrow \infty$ limit corresponds effectively to $\tau_\pi \longrightarrow 0$ and second-order hydro is reduced to a first order hydro

$\Rightarrow$Clearly, as in this limit relaxation is *instantaneous*, it is not surprising that causality is violated

## Second-order dissipative $\mathcal{N} = 4$ SYM plasma

$$\epsilon(\tau) = \frac{3}{8}\pi^2 N^2\, T(\tau)^4\,, \qquad p(\tau) = \frac{1}{3}\epsilon(\tau)\,, \qquad \eta(\tau) = A\, s(\tau) = A\,\frac{1}{2}\pi^2 N^2\, T(\tau)^3$$

$$\tau_\pi(\tau) = r\,\tau_\pi^{\mathrm{Boltzmann}}(\tau) = r\,\frac{3\eta(\tau)}{2p(\tau)}$$

where $A$ is the ratio of shear viscosity to entropy density, $r$ is the relaxation time in units Boltzmann relaxation time.

From Müller-Israel-Stewart equations as $\tau \to \infty$:

$$T(\tau) = \frac{\Lambda}{\tau^{1/3}}\left(1 + \sum_{k=1}^{\infty} \frac{t_k}{(\Lambda\tau^{2/3})^k}\right)\,, \qquad \Phi(\tau) = \frac{2}{3}\pi^2 N^2\, A\,\frac{\Lambda^3}{\tau^2}\left(1 + \sum_{k=1}^{\infty} \frac{f_k}{(\Lambda\tau^{2/3})^k}\right)$$

where $\Lambda$ is an arbitrary scale and

$$t_k = t_k(A, r)\,, \qquad f_k = f_k(A, r)$$

## $n$th-order dissipative CFT fluid dynamics

???

$\Uparrow$

Use gauge/string theory correspondence of Maldacena

General formulation of relativistic hydrodynamics might be useful in astrophysics!

## Some aspects of AdS/CFT correspondence

⇒ Maldacena correspondence is a duality between a gauge theory and a full String Theory

# HOWEVER:

⇒ the correspondence is useful when it is computationally tractable; typically this implies truncation of the full String Theory to it’s low-energy supergravity approximation

# HOWEVER:

⇒ such a truncation is not always consistent!

$(A) \Rightarrow$ In some cases singularities of the supergravity backgrounds are simply an indication that (further) Kaluza-Klein truncation of the supergravity is incorrect, and including a finite number of SUGRA modes (doing consistent KK truncation) one obtains a smooth geometry (example: black hole solution on the singular conifold with self-dual fluxes)

$(B) \Rightarrow$ In some cases singularities of the supergravity backgrounds are expected to be resolved by including and infinite set of String Theory $\alpha'$ corrections — from the gauge theory perspective this would imply that infinite set of gauge theory operators (of increasingly high dimension) would develop a vacuum expectation value at strong coupling

$(C) \Rightarrow$ In some cases the singularities of the supergravity truncation are not expected to be resolved within full String Theory, as this would falsify gauge/string correspondence — string theory would predict a gauge theory phase, which can not be realized physically (example: singularity of the Klebanov-Tseytlin geometry is not expected to be resolved in string theory preserving both the supersymmetry and the chiral symmetry)

$(A) \Rightarrow$ consistency of supergravity would help determine additional operators on the gauge theory side that would develop a VEV at strong coupling (example: SUGRA is smooth once a $U(1)$ fiber inside $T^{1,1}$ is warped $\Leftrightarrow$ a dim-6 operator of the thermal gauge theory plasma develops a VEV)

$(B) \Rightarrow$ SUGRA truncation is not useful

$(C) \Rightarrow$ A phase of the gauge theory with prescribed symmetries simply does not exists (Klebanov-Tseytlin solution is replaced with a smooth Klebanov-Strassler solution, where the chiral symmetry is broken)

## Janik's proposal for the SUGRA dual to boost-invariant $\mathcal{N} = 4$ SYM dynamics

Given symmetries of the problem, most general truncation of type IIB SUGRA takes form

$$ds_{10} = e^{-2\alpha(\tau,z)} \left\{ \frac{1}{z^2} \left[ -e^{2a(\tau,z)} d\tau^2 + e^{2b(\tau,z)} \tau^2 dy^2 + e^{2c(\tau,z)} dx_\perp^2 \right] + \frac{dz^2}{z^2} \right\}$$

$$+ e^{6/5\alpha(\tau,z)} \left( dS^5 \right)^2$$

for the Einstein frame metric;

$$F_5 \;=\; \mathcal{F}_5 + \star\mathcal{F}_5 \,, \qquad \mathcal{F}_5 \;=\; -4Q\, \omega_{S^5} \,, \qquad \phi = \phi(\tau, z)$$

for the 5-form ($Q$ is constant related to the rank of the gauge group) and the dilaton

$$Q = 1 \qquad \Leftrightarrow \qquad R_{AdS} = 1$$

Asymptotically as $z \longrightarrow 0$

$$\{a, b, c, \alpha, \phi\} \longrightarrow 0$$

however,

$$a(\tau, z) \sim \mathcal{O}\left(z^4\right) \neq 0$$

⇒We try to construct a nonsingular geometry everywhere in the bulk, subject to the above boundary conditions

⇒ evaluate stress-energy tensor one-point correlation function

$$\langle T_{\mu\nu}(\tau)\rangle = \frac{N_c^2}{2\pi} \lim_{z\to 0} \frac{g^{(5)}_{\mu\nu}(\tau) - \eta_{\mu\nu}}{z^4}$$

⇒ extract from $\langle T_{\mu\nu}(\tau)\rangle$

$$\epsilon(\tau)\,, \qquad p(\tau)$$

and interpret results in the framework of dissipative relativistic fluid dynamics

$\Rightarrow$ We saw before that near equilibrium hydrodynamics corresponds to late-time asymptotic expansion of the boost invariant CFT plasma

$$\Downarrow$$

$\Rightarrow$Janik's proposal:

$$a(\tau, z) = a\left(\tau\,, v \equiv \frac{z}{\tau^s}\right)$$

as well as for the remaining SUGRA modes; then study background geometry as asymptotic expansion in $\tau$, while keeping the scaling variable $v$ finite

$\Rightarrow$ to leading order as $\tau \longrightarrow \infty$, the absence of singularities in

$$\mathcal{I}^{[2]} \equiv \mathcal{R}_{\mu\nu\rho\lambda}\mathcal{R}^{\mu\nu\rho\lambda}\,, \qquad v^4 \to 3_-$$

requires

$$s = \frac{1}{3}$$

Given the value of $s$, the asymptotic expansion for the 5-dim geometry takes form

$$a(\tau,v)=a_0(v)+\frac{1}{\tau^{2/3}}a_1(v)+\frac{1}{\tau^{4/3}}a_2(v)+\frac{1}{\tau^{2}}a_3(v)+\mathcal{O}(\tau^{-8/3})$$

$$b(\tau,v)=b_0(v)+\frac{1}{\tau^{2/3}}b_1(v)+\frac{1}{\tau^{4/3}}b_2(v)+\frac{1}{\tau^{2}}b_3(v)+\mathcal{O}(\tau^{-8/3})$$

$$c(\tau,v)=c_0(v)+\frac{1}{\tau^{2/3}}c_1(v)+\frac{1}{\tau^{4/3}}c_2(v)+\frac{1}{\tau^{2}}c_3(v)+\mathcal{O}(\tau^{-8/3})$$

$$\Downarrow$$

$$\mathcal{I}^{[2]}=\mathcal{I}_0^{[2]}(v)+\frac{1}{\tau^{2/3}}\mathcal{I}_1^{[2]}(v)+\frac{1}{\tau^{4/3}}\mathcal{I}_2^{[2]}(v)+\frac{1}{\tau^{2}}\mathcal{I}_3^{[2]}(v)+\mathcal{O}(\tau^{-8/3})$$

Assume first

$$\alpha(\tau,v)\equiv 0\,,\qquad \phi(\tau,v)\equiv 0$$

which on the gauge theory side implies that neither $\left\langle \mathrm{Tr}\, F^2(\tau)\right\rangle$ (dual to a dilaton) nor the dim-8 operator (dual to SUGRA scalar $\alpha$) develop a VEV

( **Emphasize**: this is an assumption which might or might not be correct —we use nonsingularity condition of the dual string (supergravity) description to test this)

$\Rightarrow$ we find (up to second subleading order)

$$\epsilon(\tau)=\left(\frac{N^2}{2\pi^2}\right)\ \frac{1}{\tau^{4/3}}\left\{1-\frac{2\eta_0}{\tau^{2/3}}+\left(\frac{10}{3}\eta_0^2+\frac{C}{36}\right)\frac{1}{\tau^{4/3}}+\cdots\right\}$$

Matching the gauge theory expansion for the energy density with that of the dual gravitational description we find

$$\Lambda=\frac{\sqrt{2}}{3^{1/4}\pi}\,,\qquad A=\frac{3^{3/4}}{2^{3/2}\pi}\,\eta_0\,,\qquad r=-\frac{11}{18}-\frac{1}{108}\ \frac{C}{\eta_0^2}$$

**NOTE:** further expansions on the SUGRA side will **define** higher order dissipative relativistic dynamics!

## Successes of Janik's proposal

For generic values of $\{\eta_0, C\}$:

$$\left\{\mathcal{I}_2^{[2]}(v)\,,\mathcal{I}_3^{[2]}(v)\right\} = \mathcal{O}\left(\frac{1}{(3-v^4)^4}\right)\,, \qquad v^4 \to 3_-$$

Tuning

$$\eta_0 = \frac{1}{2^{1/2}3^{3/4}}\,, \qquad C = 2\sqrt{3}\ \ln 2 - \frac{17}{\sqrt{3}}$$

*all* pole singularities in $\{\mathcal{I}_2^{[2]}(v)\,,\mathcal{I}_3^{[2]}(v)\}$ are removed.

$$\Downarrow$$

$$A = \frac{1}{4\pi}\,, \qquad r = \frac{1}{3}(1-\ln 2)$$

in agreement with computations from equilibrium higher point correlation functions !!!

However:

$$\mathcal{I}_3^{[2]} = \text{finite} \, + \left(8\, 2^{1/2}\, 3^{3/4}\right)\, \ln(3 - v^4)\,, \qquad v \to 3_-^{1/4}$$

$\Rightarrow$ it appears inconsistent to set $\alpha$ and/or the dilaton to zero; in fact weak coupling analysis suggests that there are instabilities in expanding plasma generating VEV’s of various operators, in particular $\left\langle \mathrm{Tr}\, F^2(\tau)\right\rangle$.

$\Rightarrow$ A careful analysis show that without introducing pole curvature additional singularities one can turn on only the $\alpha$ mode to relevant order

$$\alpha(\tau, v) = \frac{1}{\tau^2}\, \alpha_3(v) + \mathcal{O}\left(\tau^{-8/3}\right)$$

$$\alpha_3 = \alpha_{3,0} \left( \left( \frac{1}{96 v^4} + \frac{v^4}{864} \right)\, \ln \frac{3 + v^4}{3 - v^4} - \frac{1}{144} \right)$$

where $\alpha_{3,0}$ is a normalizable mode, related to the VEV of dim-8 operator in $\mathcal{N} = 4$ SYM plasma

We find:

$$\mathcal{I}_3^{[2]} = \text{finite} \, + \left( 8\, 2^{1/2}\, 3^{3/4} + \frac{14}{3}\, \alpha_{3,0} \right) \, \ln(3 - v^4)\,, \qquad v \to 3_-^{1/4}$$

but

$$\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} = \text{finite} + \frac{1}{\tau^2}\, \frac{40}{3}\, \alpha_{3,0}\, \ln(3 - v^4)\,, \qquad v \to 3_-^{1/4}$$

$\Rightarrow$ Logarithmic singularity can not be canceled within the SUGRA approximation (there are no SUGRA modes consistent with symmetry of the problem that can be “turned on”)

We consider other models of CFT plasma (Klebanov-Witten plasma) with an additional SUGRA mode (less symmetry) and showed that logarithmic singularities both in

$$\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} \qquad \text{and} \qquad \mathcal{R}_{\mu\nu\rho\lambda}\mathcal{R}^{\mu\nu\rho\lambda}$$

at the third subleading order can be canceled (Ricci scalar is nonsingular)

However, new logarithmic singularities at the third order in higher curvature invariants such as

$$\mathcal{R}_{\mu_1\nu_1\lambda_1\rho_1}\mathcal{R}^{\mu_1\nu_1\lambda_2\rho_2}\mathcal{R}_{\mu_2\nu_2}{}^{\lambda_1\rho_1}\mathcal{R}^{\mu_2\nu_2}{}_{\lambda_2\rho_2}$$

as well as logarithmic singularities with different coefficients in

$$(\mathcal{R}_{....})^8\,, \qquad (\mathcal{R}_{....})^{16}\,,$$

and so on

$\Rightarrow$ One needs an infinite set of fields to cancel singularities in gravitational description, corresponding to infinite set of gauge invariant operators develop a VEV during boost-invariant expansion

Conclusion: SUGRA truncation of a string dual to boost-invariant conformal plasma is inconsistent

$(A) \Rightarrow$ is not realized

$(B) \Rightarrow$ though SUGRA truncation is inconsistent, maybe the requirement of the cancellation of the pole singularities at low orders is a correct prescription to extract second order transport coefficients (which are of relevance to RHIC); tantalizingly, we see hints of the universality of the relaxation time — further study of non-conformal models is needed

$(C) \Rightarrow$ SUGRA singularity might be indication of the genuine singularity in full string theory description — search for onset of instabilities in expanding plasma? turbulence?