Partons in Phase Space

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Outline

Introduction

Large systems such as Au+Au at RHIC: final state a result of history where different processes are localized in phase space.

?Can such a perspective be developed for elementary processes?

?Elementary processes embedded in a large system?

– Perturbative processes
– Partons: folding in cross sections
– Generalized Wigner representation:

$$A(x,p) = \int d^4q \, A\left(p + \frac{q}{2}\right) \, A^*\left(p - \frac{q}{2}\right) \, e^{-iqx}$$

where $A(p)$ is an amplitude

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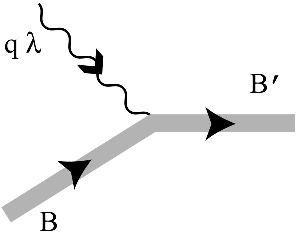
γ Absorption



Transition Amplitude

$$S_{\gamma B \rightarrow B'} = \int d^4x \, \langle 0 | A^{\mu}(x) | \vec{q}, \lambda \rangle \, \langle B' | j_{\mu}(x) | B \rangle$$

$$\text{Transition No} = |S_{\gamma B \rightarrow B'}|^2 = \int d^4x \, d^4k \, \delta^{(4)}(q-k) \, \epsilon_{\mu} \, \epsilon_{\nu}^{*} \, J_{BB'}^{\mu\nu}(x\,k)$$

$$\equiv \int d^4x \, \mathcal{W}_{\gamma B \rightarrow B'}(x\,q)$$

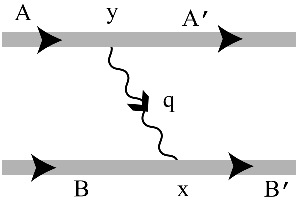
where

$$J_{BB'}^{\mu\nu}(x\,k) \equiv \int d^4\tilde{k} \, e^{-i\tilde{k}x} \, \langle B | j^{\dagger\nu}(k - \tilde{k}/2) | B' \rangle \, \langle B' | j^{\mu}(k + \tilde{k}/2) | B \rangle$$

$\Rightarrow$ Wigner transform of transition current

$\equiv$ Phase-Space density for γ absorption/emission

γ-Exchange



Amplitude

$$S_{AB\to A'B'} = \int \mathrm{d}^4x\,\mathrm{d}^4y\,\langle B'|j^{B\nu}(x)|B\rangle \times D_{\nu\mu}(x-y)\,\langle A'|j^{A\mu}(y)|A\rangle$$

Transition No $= |S_{AB\to A'B'}|^2$

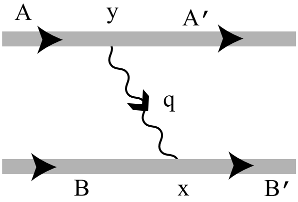
$$= \int \mathrm{d}^4y\,\mathrm{d}^4x\,\mathrm{d}^4q\,J^{\mu\nu}_{AA'}(y\,q)\,D_{\mu\nu\mu'\nu'}(y-x,q)\,J^{\mu'\nu'}_{BB'}(x\,q)$$

where the Wigner transform of a propagator is

$$D_{\mu\nu\mu'\nu'}(x,q) = \int \mathrm{d}^4\tilde{x}\,e^{iq\tilde{x}}\,D_{\mu\nu}(x+\tilde{x}/2)\,D_{\mu'\nu'}(x-\tilde{x}/2)$$

In Phase Space:
$\rightarrow$ A emits γ of momentum q at y
$\rightarrow$ γ propagates from y to x
$\rightarrow$ B absorbs γ at x

γ-Exchange



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- $\to$ γ propagates from y to x
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NSCL

Towards Weizsäcker-Williams Interpretation

?Vector potential of object A?

$$A^\mu(x) = \int d^4y \, D_{\mu\nu}(x-y) \, J_A^\nu(y)$$

Wigner transform of the vector potential:

$$A_{\mu\nu}(x\,q) = \int d^4\tilde{x} \, e^{i\tilde{x}q} \, A_\mu(x+\tilde{x}/2) \, A_\nu(x-\tilde{x}/2)$$
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Source-Propagator Form: the current of A creates γ with momentum q at position y and the γ propagates to x

For γ-exchange, the WW interpretation applied in phase space:

$$\text{Transition No} = |S_{AB\to A'B'}|^2 = \int d^4x \, d^4q \, A_{\mu\nu}(xq) \, J_B^{\mu\nu}(xq)$$
$$\equiv \int d^4x \, d^4q \, \frac{d\,n_\gamma}{d^3x \, d^4q} \, \mathcal{W}_{\gamma B\to B'}(xq)$$

In standard WW: γ distribution in energy *only*

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Issue of Current in WW

In standard WW approximation, A-body current classical:

$$j_\mu(x) = Z \, \mathrm{e} \, v_\mu \, \delta^3(\vec{x} - x^0 \, \vec{v})$$

Then

$$J^{\mu\nu}_{\text{classical}}(xq) = \int \mathrm{d}^4\tilde{x} \, \mathrm{e}^{i\tilde{x}q} \, j_\mu(x + \tilde{x}/2) \, j^\dagger_\nu(x - \tilde{x}/2)$$
$$= Z^2 \, \mathrm{e}^2 \, v_\mu v_\nu \, \delta(q\,v) \, \delta^{(3)}(\vec{x} - x^0 \, \vec{v})$$

$\delta(q\,v)$ stems from current conservation, photons purely spacelike in emitter frame; $q_0 = q_L$, $v_L \simeq q_L$ in lab frame

---

Us: good approximation for $q \ll p$, when one can construct an A-packet such that $q \ll (\Delta x_A)^{-1} \ll p_A$

The current of body A in terms of Wigner functions:

$$J^{\mu\nu}_{AA'}(xq) = \int \mathrm{d}^4\tilde{q} \, \mathrm{e}^{-i\tilde{q}x} \, \langle A | j^{\dagger\nu}(q - \tilde{q}/2) | A' \rangle \, \langle A' | j^\mu(q + \tilde{q}/2) | A \rangle$$
$$= \int \mathrm{d}^4p_i \, \mathrm{d}^4p_f \, \delta^{(4)}(p_i - p_f - q) \, \Gamma^{\mu\nu}(q, p_i, p_f) \, f_A(x\,p_i) \, f_{A'}(x\,p_f)$$

For $q \ll (\Delta x_A)^{-1} \ll p_A$: $\Gamma_{\mu\nu}(q, p_i, p_f) \simeq Z^2 \, \mathrm{e}^2 \, (p_i + p_f)_\mu \, (p_i + p_f)_\nu$

$\Rightarrow$ If packet localized on q^{-1} scale, we could take $J^{\mu\nu} \simeq J^{\mu\nu}_{\text{classical}}$

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Propagator in γ-Exchange

$$A^\mu(x) = \int \mathrm{d}^4 y \, D_{\mu\nu}(x-y) \, J_A^\nu(y)$$

Wigner transform of what propagator ?

→ Feynman doable analytically but acausal!

→ Retarded – the same physics results for the same asymptotic states but simpler to interpret

Scalar propagators:

$$G^+(p) = \frac{1}{p^2 - m^2 + i\epsilon p_0} \qquad G^c(p) = \frac{1}{p^2 - m^2 + i\epsilon}$$

Propagator in phase space:

$$G(x,q) = \int \mathrm{d}^4\tilde{x} \, \mathrm{e}^{iq\tilde{x}} \, G(x+\tilde{x}/2) \, G(x-\tilde{x}/2)$$

Massless scalar

$$G^+(x\,p) = \theta(x_0)\,\theta(x^2)\,\theta(\lambda^2)\,\frac{\sin\left(2\sqrt{\lambda^2}\right)}{\sqrt{\lambda^2}}$$

where $\lambda^2 = (x\,p)^2 - x^2 p^2$. $\rightarrow D^+_{\mu\nu\mu'\nu'}(x\,q) = g_{\mu\nu}\, g_{\mu'\nu'}\, G^+(x\,q)$

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Features of Retarded Phase-Space Propagator

– causal, propagation inside light cone only
– ptcles do not propagate beyond $0 \leq \lambda^2 \lesssim 1$

q^2-Case	Special Frame		Moving Frame $(q^0, q^L, \vec{0}^T)$
> 0	$(q^0, \vec{0})$	$\lvert\vec{x}\rvert \lesssim x^0 < \frac{1}{\lvert q^0\rvert}$	$x^0 \lesssim \frac{1}{\lvert q^0\rvert}$, $\lvert x^L\rvert \lesssim \frac{1}{\lvert q^L\rvert}$ $\lvert\vec{x}^T\rvert \lesssim \frac{1}{\sqrt{q^2}}$
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0			$x^0 - \lvert x^L\rvert \lesssim \frac{1}{\lvert q^0\rvert}$, $\lvert\vec{x}^T\rvert \lesssim \sqrt{\frac{x^0}{\lvert q^0\rvert}}$

Off-shell propagation limited, from emission point,
by the inverse of energy, momentum or mass
On-shell propagation limited, from the classical trajectory,
by the inverse of energy

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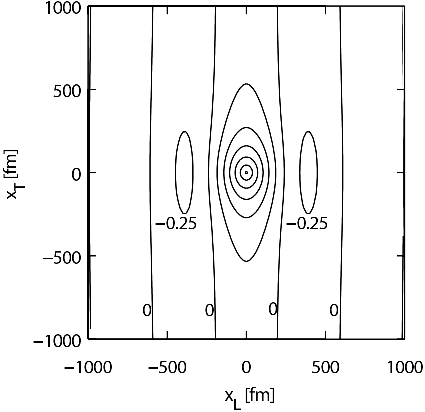
γ Phase-Space Distribution for Static Charge

Putting current + propagation together yields γ distribution:

$$A^{\mu\nu}(x\,q) = Z^2\, e^2\, v^\mu\, v^\nu\, \frac{\delta(qv)}{\sqrt{-q^2}}\, \mathcal{A}$$

where $\mathcal{A}$ is dimensionless, semi-analytic and gives shape:

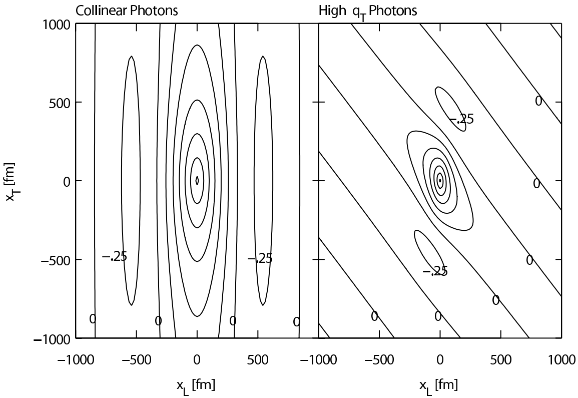


photon 4-momentum
$q_\mu = (0, 0.788, \vec{0}_T)$ MeV/c

Lorentz contraction
in the direction of
photon momentum

γ Phase-Space Distribution for Moving Charge

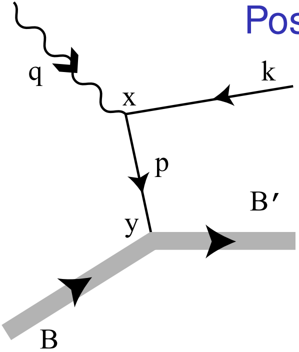
Charge moving at $\vec{v} = (0.9c, \vec{0}_T)$



Photon $q^\mu = (m_e, m_e/v_L, \vec{0}_T)$ & $q^\mu = (m_e, m_e/v_L, 0.56\ \text{MeV/c}, 0)$

Again contraction in the *photon* direction

Positron Production



$$\gamma + B \to \overline{e} + B'$$

Process amplitude

$$S_{\gamma B \to \bar{e} B'} = \int d^4x\, d^4y\, A_\mu(x)\, \psi_{\bar{e}}(x\, s) \times e\, \gamma^\mu\, S(x-y)\, \mathcal{V}_{Be \to B'}(y)$$

$$\text{Transition No} = |S_{\gamma B \to \bar{e} B'}|^2 = e^2 \int d^4y\, d^4p\, d^4x\, d^4q\, \frac{d^3k}{|k^0|}\, A_{\mu\nu}(x\, q) \times G(y-x,p)\, \delta^{(4)}(k+q-p)\, \mathrm{Tr}\{\mathcal{V}_{Be \to B'}(y,p)\, (\not{p} - \ldots) \ldots\}$$

Distribution of virtual electrons for a photon:

$$\frac{dn_{\bar{e}}}{d^3y\, d^3p\, dp^2} \propto e^2 \int d^4x\, \frac{d^3k}{|k^0|}\, G(y-x,p)\, A_{\mu\nu}(x, p-k)$$

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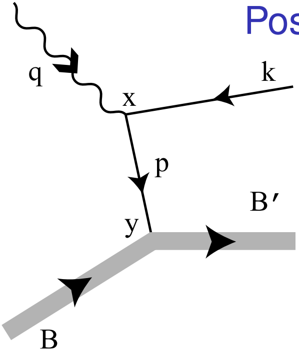

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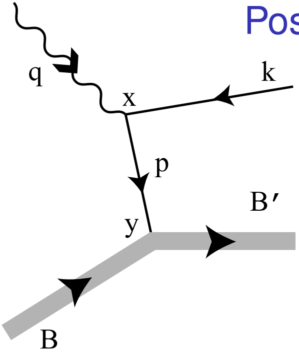

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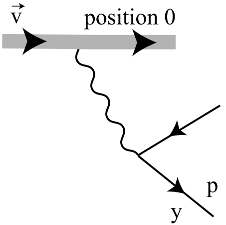
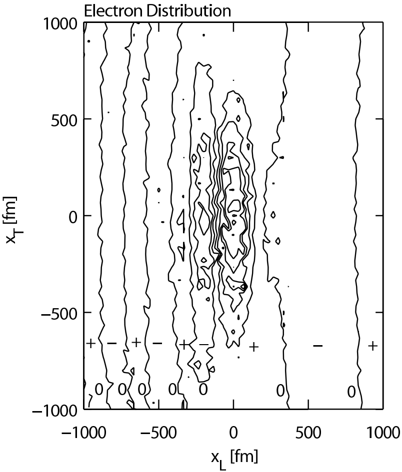
Virtual Electron Distribution

Point charge, moving at 0.9c to the right, is the source of photons that are the source of virtual electrons

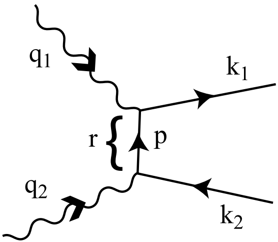


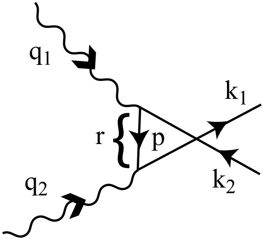


Electron momentum
$p^\mu = (2.0, 2.05, \vec{0})$ MeV/c

Forward-backward asymmetry due to ē recoil

Interference in 2-γ Interaction





$$\text{Transition No} = |S_{12\to 1'2'e\bar{e}}|^2 = \int d^4R\, d^4r \ldots$$

$$\times\, G(r\,p)\,\delta^{(4)}(q_1 + q_2 - k_1 + k_2)$$

$$\times\, \{A_1^{\mu\mu'}(R - r/2, q_1)\, A_2^{\nu\nu'}(R + r/2, q_2)$$

$$\times\, \delta^{(4)}(q_1 + p - k_1) + A_1^{\nu\nu'}(R + r/2, q_1)$$

$$\times\, A_2^{\mu\mu'}(R - r/2, q_2)\,\delta^{(4)}(q_1 + k_2 - p)$$

$$+ \int d^4\tilde{r}\, 2\cos\left[\tilde{r}\cdot(-p + \frac{k_1 + k_2}{2}) - r\cdot(q_1 - q_2)\right]$$

$$\times\, A_1^{\nu\mu'}(R - \tilde{r}/4, q_1)\, A_2^{\mu\nu'}(R + \tilde{r}/4, q_2)\}$$

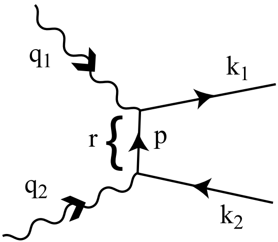
2 direct terms + HBT-like interference term

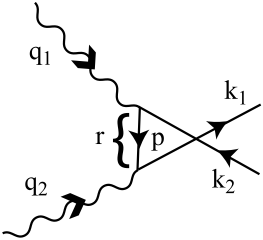
The direct terms factorize into e-density $\times$ absorption.

The interference term does not.

$\Rightarrow$ Scale separation needed to achieve a factorization.

Interference in 2-γ Interaction





$$\text{Transition No} = |S_{12\to 1'2'e\bar{e}}|^2 = \int \mathrm{d}^4 R\, \mathrm{d}^4 r \ldots$$

$$\times\, G(r\,p)\,\delta^{(4)}(q_1 + q_2 - k_1 + k_2)$$

$$\times\, \{ A_1^{\mu\mu'}(R - r/2, q_1)\, A_2^{\nu\nu'}(R + r/2, q_2)$$

$$\times\, \delta^{(4)}(q_1 + p - k_1) + A_1^{\nu\nu'}(R + r/2, q_1)$$

$$\times\, A_2^{\mu\mu'}(R - r/2, q_2)\,\delta^{(4)}(q_1 + k_2 - p)$$

$$+ \int \mathrm{d}^4\tilde{r}\, 2\cos\left[\tilde{r}\cdot(-p + \frac{k_1 + k_2}{2}) - r\cdot(q_1 - q_2)\right]$$

$$\times\, A_1^{\nu\mu'}(R - \tilde{r}/4, q_1)\, A_2^{\mu\nu'}(R + \tilde{r}/4, q_2)\}$$

2 direct terms + HBT-like interference term

The direct terms factorize into e-density $\times$ absorption.

The interference term does not.

$\Rightarrow$ Scale separation needed to achieve a factorization.

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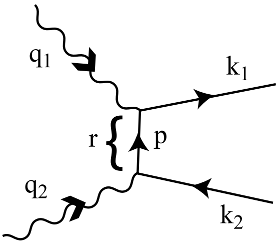

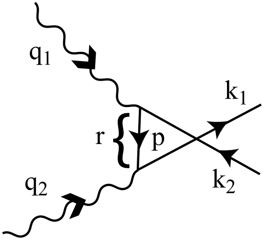

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$$\times \left\{ A_1^{\mu\mu'}(R - r/2, q_1)\, A_2^{\nu\nu'}(R + r/2, q_2) \right.$$
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Propagation with Mass and in Medium

?Massive propagator?

$$G^+(x\,p) = G_0^+(x\,p) - \frac{2m}{\pi}\,\theta(x^0)\,\theta(x^2) \times \int_0^{\sqrt{x^2}} \mathrm{d}\xi\, J_1(2m\xi)\,\frac{\sin\left(2\sqrt{\lambda^2+\xi^2 p^2}\right)}{\sqrt{\lambda^2+\xi^2p^2}}$$

G_0^+ – massless propagator; the correction removes pieces close to the light cone...

For an on-shell ptcle, as before $|x_0 - \vec{v}\cdot\vec{x}| \lesssim \frac{1}{|p_0|}$

In medium, fluctuation-dissipation theorem: $G^{\gtrless} = G^+\,\Sigma^{\gtrless}\,G^-$

In Wigner representation:

$$G^{\gtrless}(x\,p) = \int \mathrm{d}^4y\,\mathrm{d}^4p'\,G^+(x\,p;y\,p')\,\Sigma^{\gtrless}(y\,p') \simeq \int \mathrm{d}^4y\,G^+(x-y,p)\,\Sigma^{\gtrless}(y\,p')$$

$G^{\gtrless}$ – density, $\Sigma^{\gtrless}$ – source

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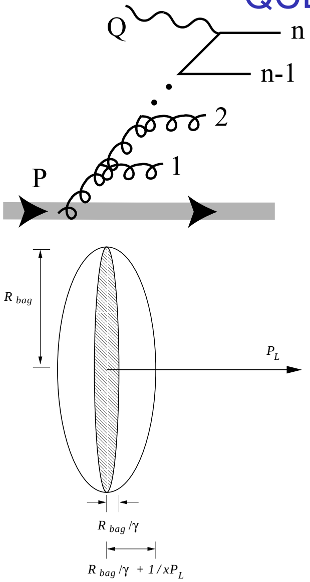
QCD DGLAP Cloud



Large Q^2 & $-q_1^2 \ll -q_2^2 \ldots \ll -q_n^2$
increasing virtuality

decreasing portion of beam momentum
$1 \le x_1 \le \ldots \le x_{n-1} \le x_n$

Each parton a source for the subsequent one. Propagation determines size of virtual cloud.

First source localized to $\left(\frac{R_{bag}}{\gamma}, R_{bag}^{\perp}\right)$.

Propagation increases $R^{\perp}$ by $\sim \frac{1}{\sqrt{-q_n^2}}$.

Because of large virtuality $R_n^{\perp} \sim R_{\text{bag}}^{\perp}$.

In $\|$ direction size increases by $\frac{1}{|q_{n0}|}$,

i.e. $R_n^{\|} \sim \frac{R_{\text{bag}}}{\gamma} + \frac{1}{x_n P}$

If $\frac{1}{x_n P} > \frac{R_{\text{nuc}}}{\gamma}$ partons delocalized beyond the nucleus. . .

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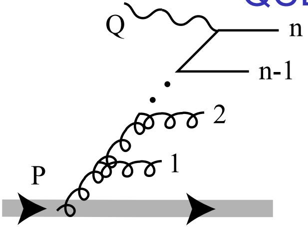

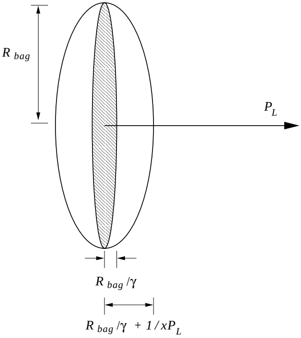

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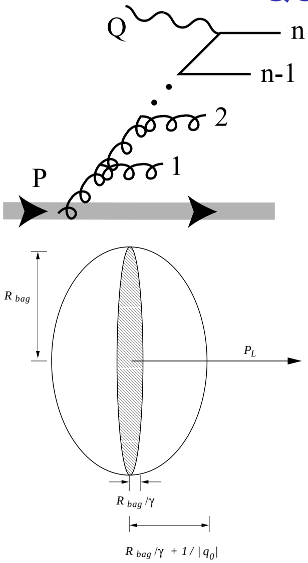
QCD BFKL Cloud



$1 \gg x_1 \gg \ldots \gg x_{n-1} \gg x_n$
strongly ordered low-x

No constraints on $q_\perp^2$;
in practice $q_{\perp 1}^2, \ldots, q_{\perp n}^2 \gg 1/R_{\text{bag}}$.

If $\frac{1}{x_n P} > \frac{R_{nuc}}{\gamma}$ partons delocalized beyond the nucleus.

Small-x partons can travel beyond the longitudinal nuclear size and then see color charge of any other nucleon in a longitudinal tube centered on the parent nucleon.

$\Rightarrow$ Support for McLerran-Venugopalan

Summary

- Without interference, probabilities from tree-type diagrams can be expressed in terms of transparent phase-space convolutions of source-propagator form.
- Massless phase-space propagators in analytic form; massive involve 1D integral.
- Interferences produce HBT-type terms in probability.
- Phase-space propagation is causal for the retarded propagators. Deviations from classical motion are limited by energy, momentum or mass.
- Interference terms will get suppressed when there is space-momentum scale separation and that is the case in different limits where the partonic model is employed.
- Wee partons extend farthest from a nucleus and should be first to interact.

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