Lattice Simulations of Preheating

Gary Felder
KITP February 2008

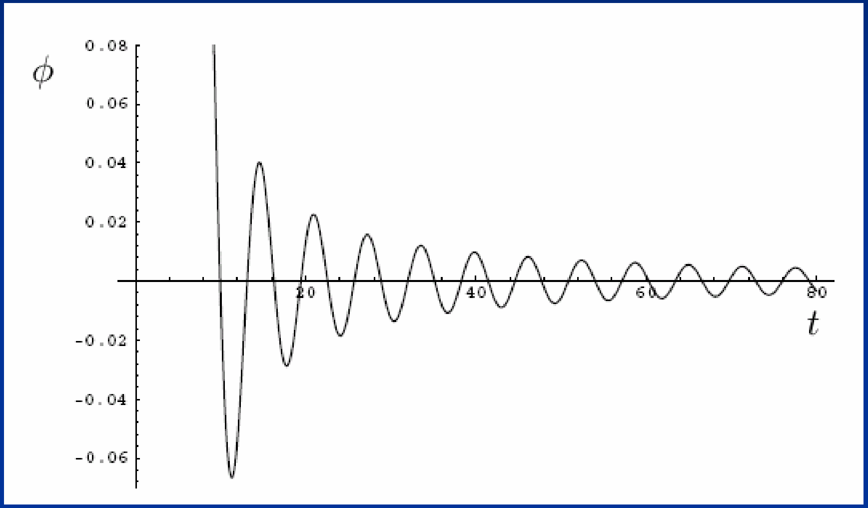
Outline

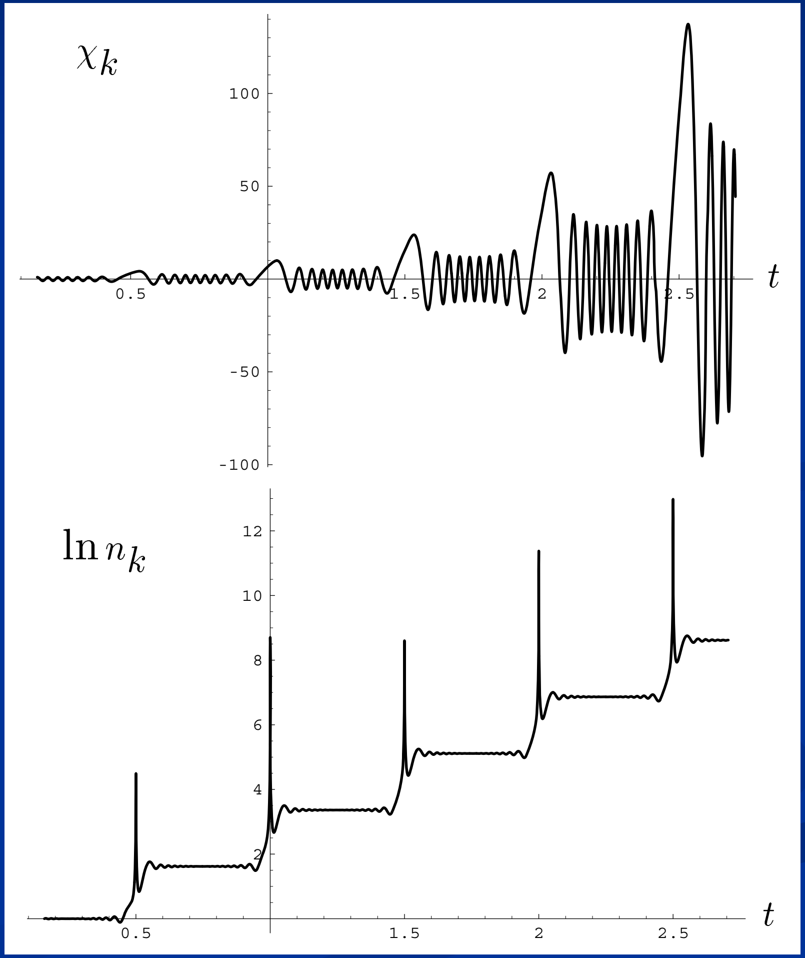
- Reheating and Preheating
- Lattice Simulations
- Gravity Waves from Preheating
- Conclusion

Reheating and Preheating

- "Reheating" is the decay of the homogeneous inflaton field.
- "Preheating" is a burst of exponentially rapid decay that occurs at the beginning of reheating in many models. (Kofman, Linde, and Starobinsky, 1994.)

Parametric Resonance





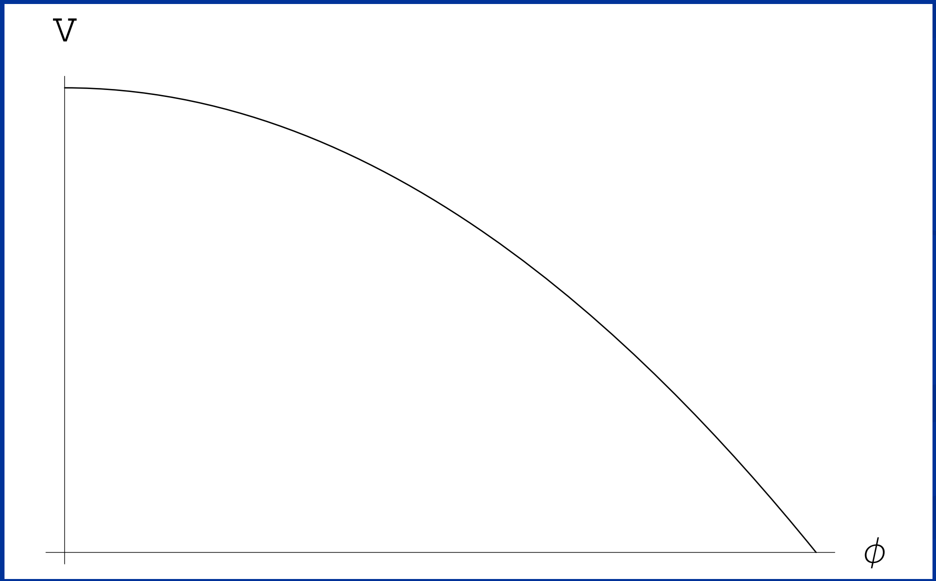
$$\ddot{\chi}_k + \omega_k^2 \chi_k = 0$$

$$\omega_k^2 \approx \frac{k^2}{a^2} + g^2 \phi^2$$

*Figure taken from Kofman, Linde, and Starobinsky, hep-ph/9704452.

Tachyonic Preheating

- In many models of inflation such as new inflation and hybrid inflation the scalar fields at the end of inflation fall down a slope with negative curvature.



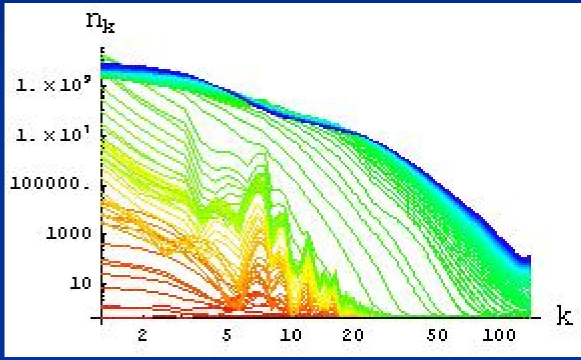
*G.F., et al, 2001.

Preheating in Inflationary Models

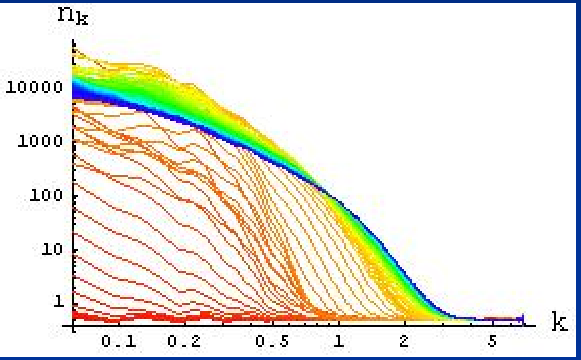
- Chaotic Inflation
  - Parametric Resonance, Tachyonic Resonance
- Hybrid Inflation
  - Tachyonic Preheating
- New Inflation
  - Parametric Resonance, Tachyonic Preheating, Tachyonic Resonance

The Results of Preheating

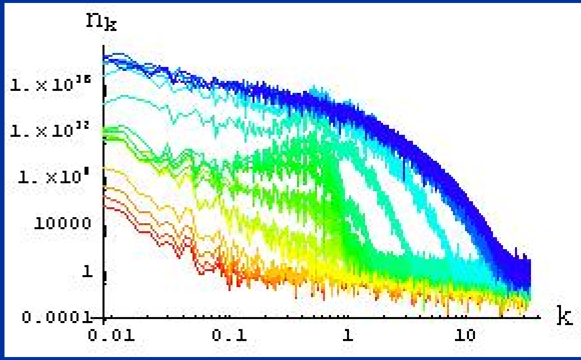
Parametric Resonance



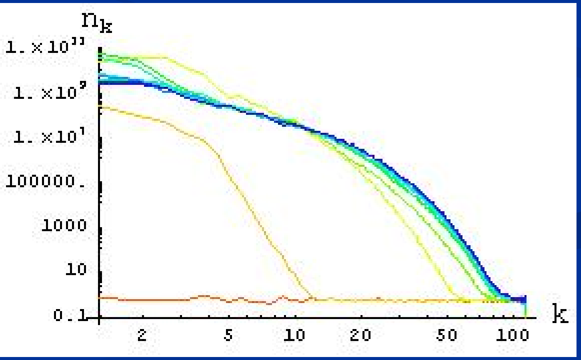
Tachyonic Preheating



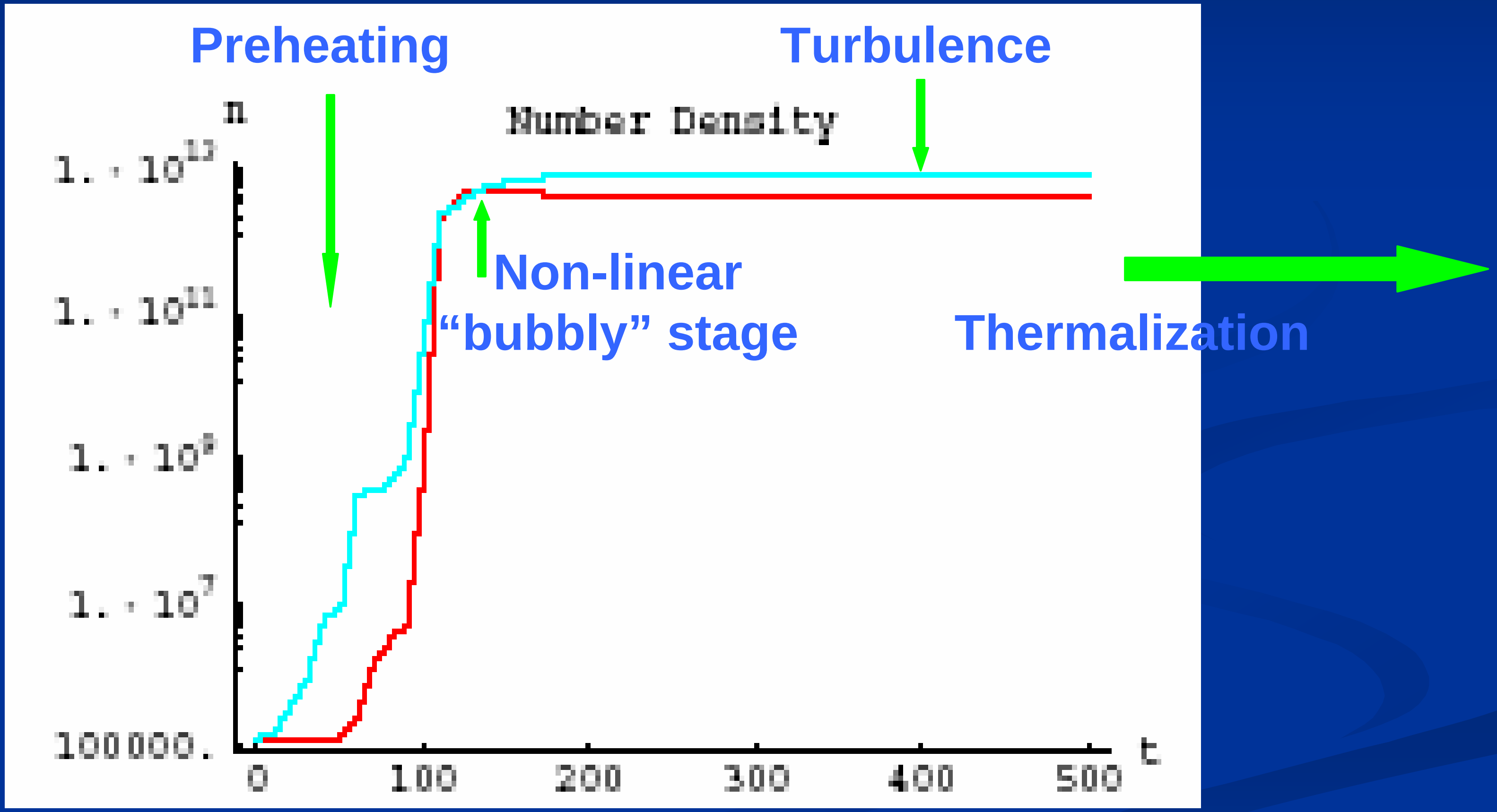
Parametric Resonance + Tachyonic Preheating



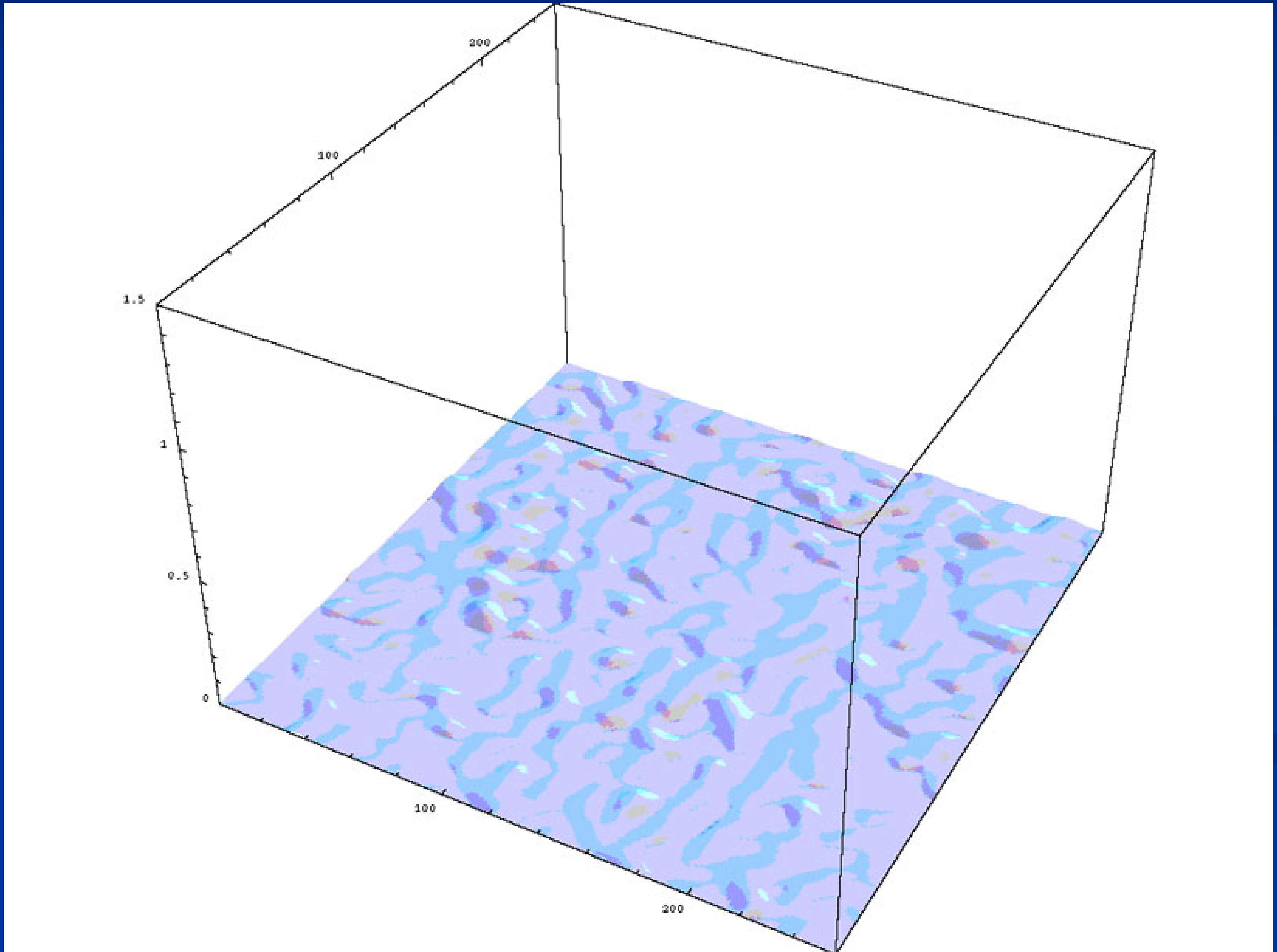
Tachyonic Resonance

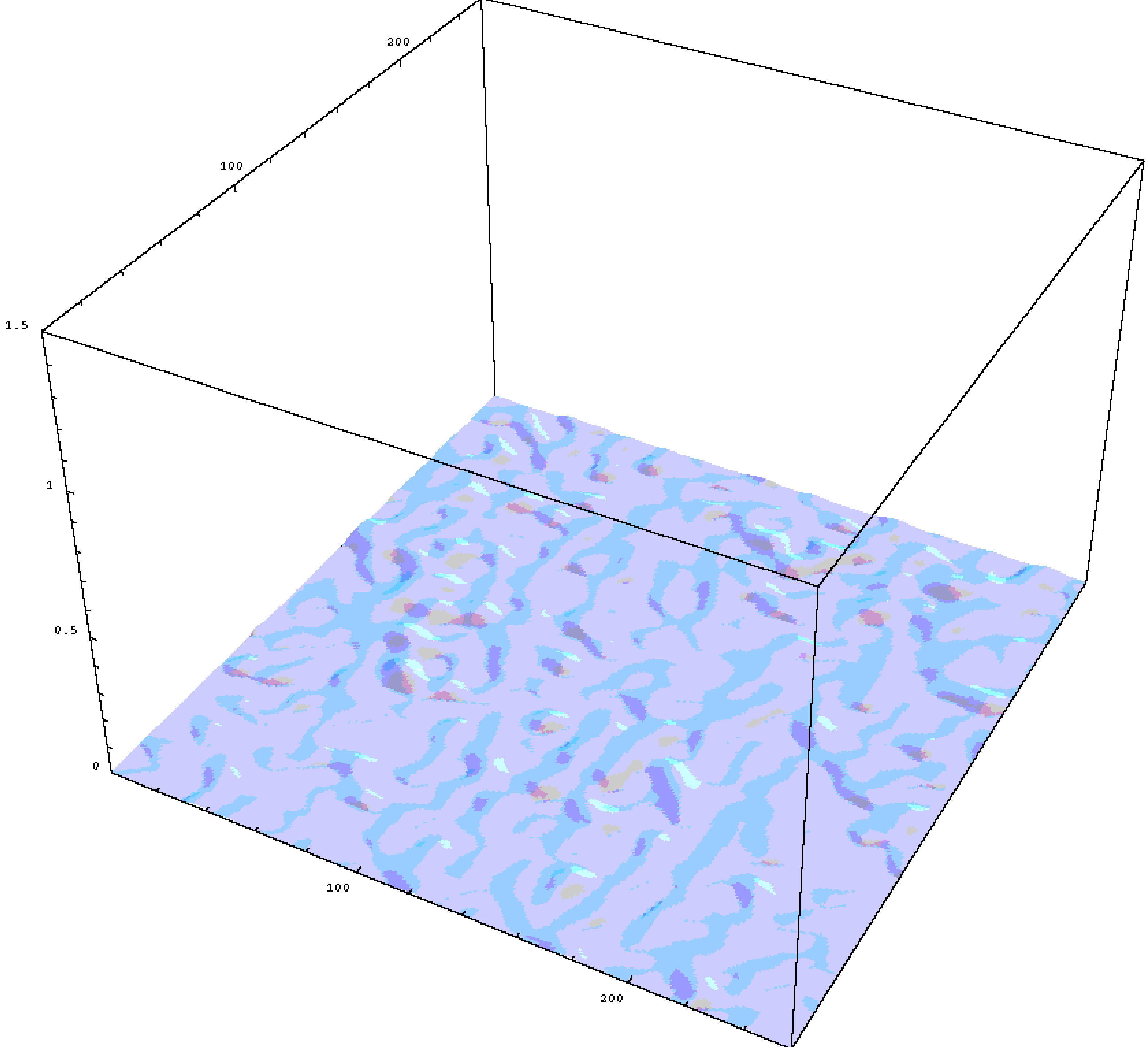


Stages of Reheating



Bubble Formation in Tachyonic Preheating




200
100
1.5
1
0.5
0
100
200

Lattice Simulations

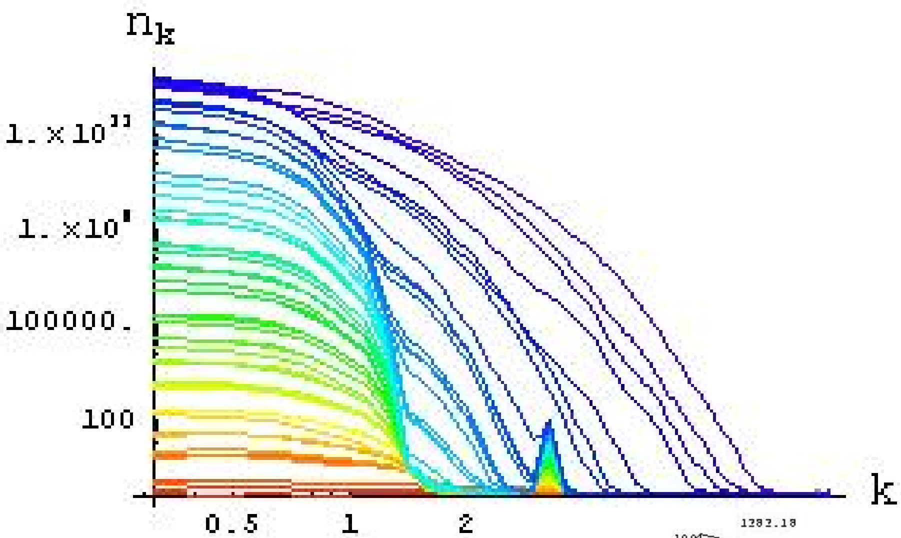
$$\ddot{\phi}_a + 3\frac{\dot{a}}{a}\dot{\phi}_a - \frac{1}{a^2}\nabla^2\phi_a + \frac{\partial V}{\partial \phi_a} = 0$$

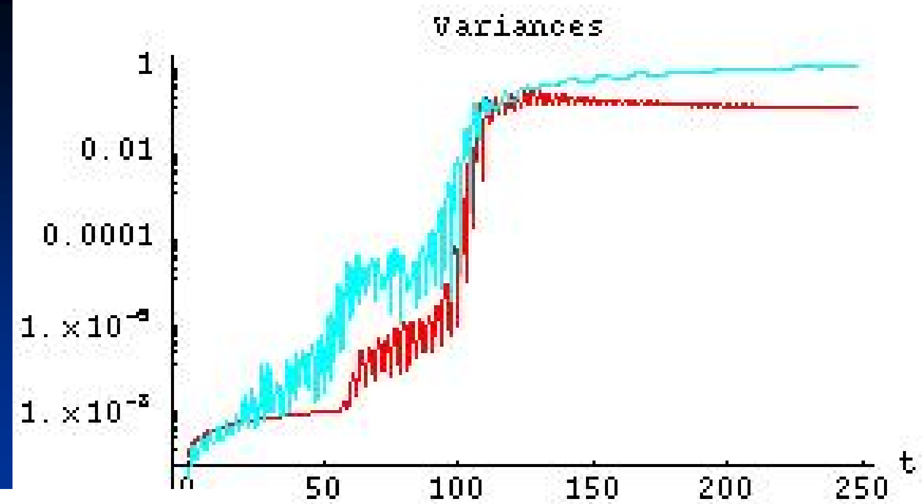
$$\ddot{a} + \frac{4\pi a}{3}(p + 3\rho) = 0$$

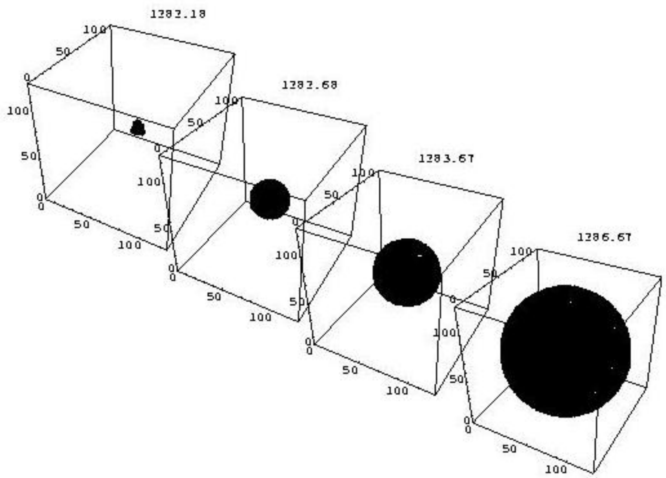
- During and shortly after preheating, the field dynamics is purely classical. (Khlebnikov and Tkachev, 1996; Prokopec and Roos, 1997; Tkachev, Khlebnikov, Kofman, and Linde, 1998; Berges, Rothkopf, and Schmidt, 2008.)

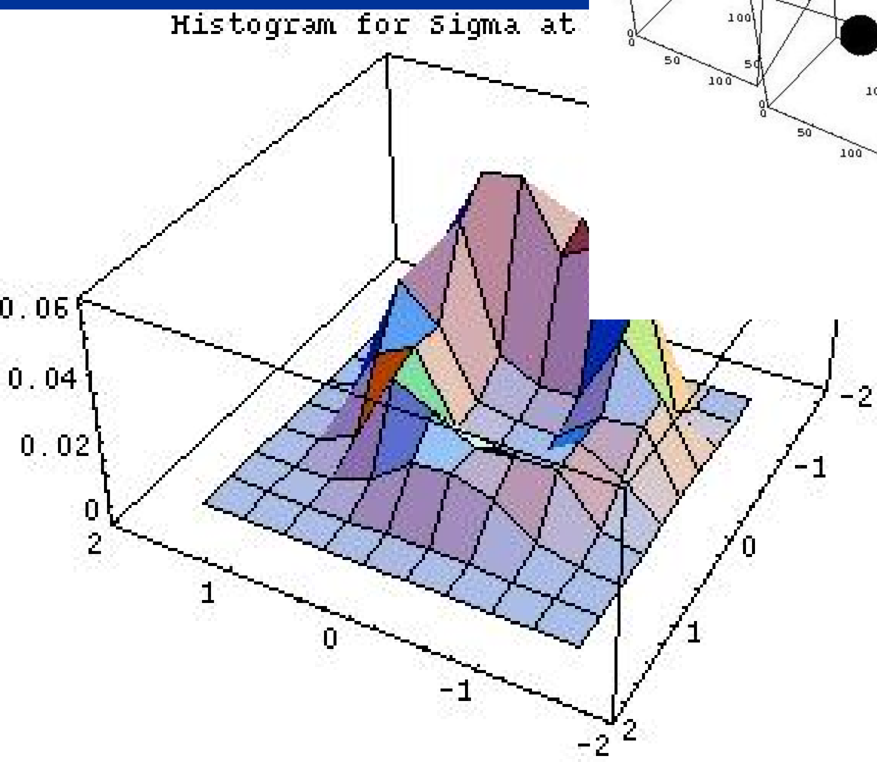
Lattice Simulations

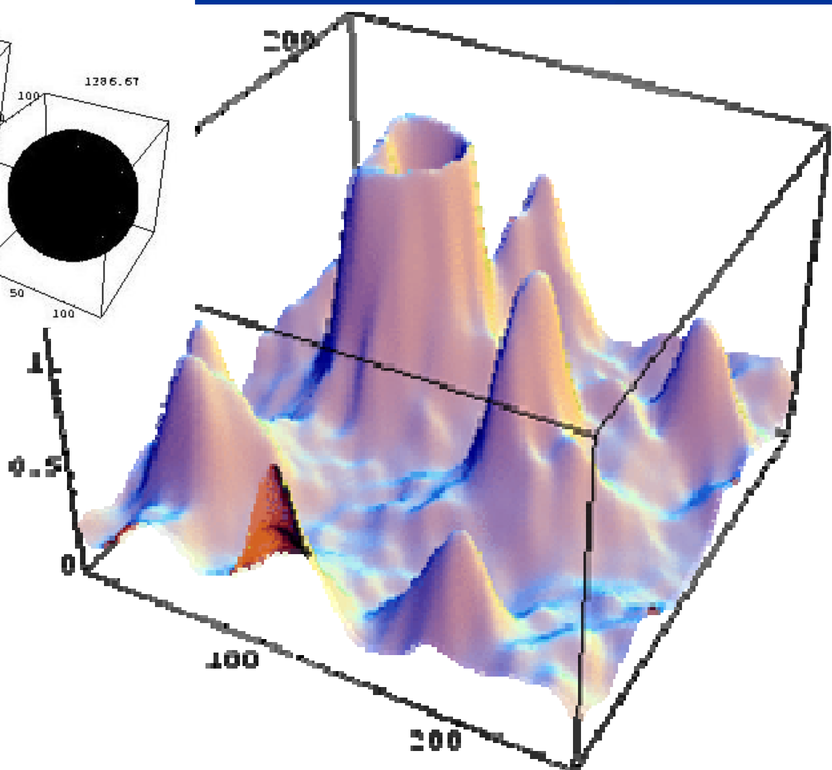
- LATTICEEASY - *A lattice simulation program for interacting scalar fields in an expanding universe.*
  - User enters potential and run parameters. LATTICEEASY solves evolution equations and generates outputs.
  - The only approximations are classicality and discretization.


n_k
1.×10^11
1.×10^8
100000.
100
0.5
1
2
k


Variances
1
0.01
0.0001
50
100
150
200
250
t


1282.18
1282.68
1283.67
1286.67


Histogram for Sigma at
0.06
0.04
0.02
0


1
0
100
200

LATTICEEASY Accuracy

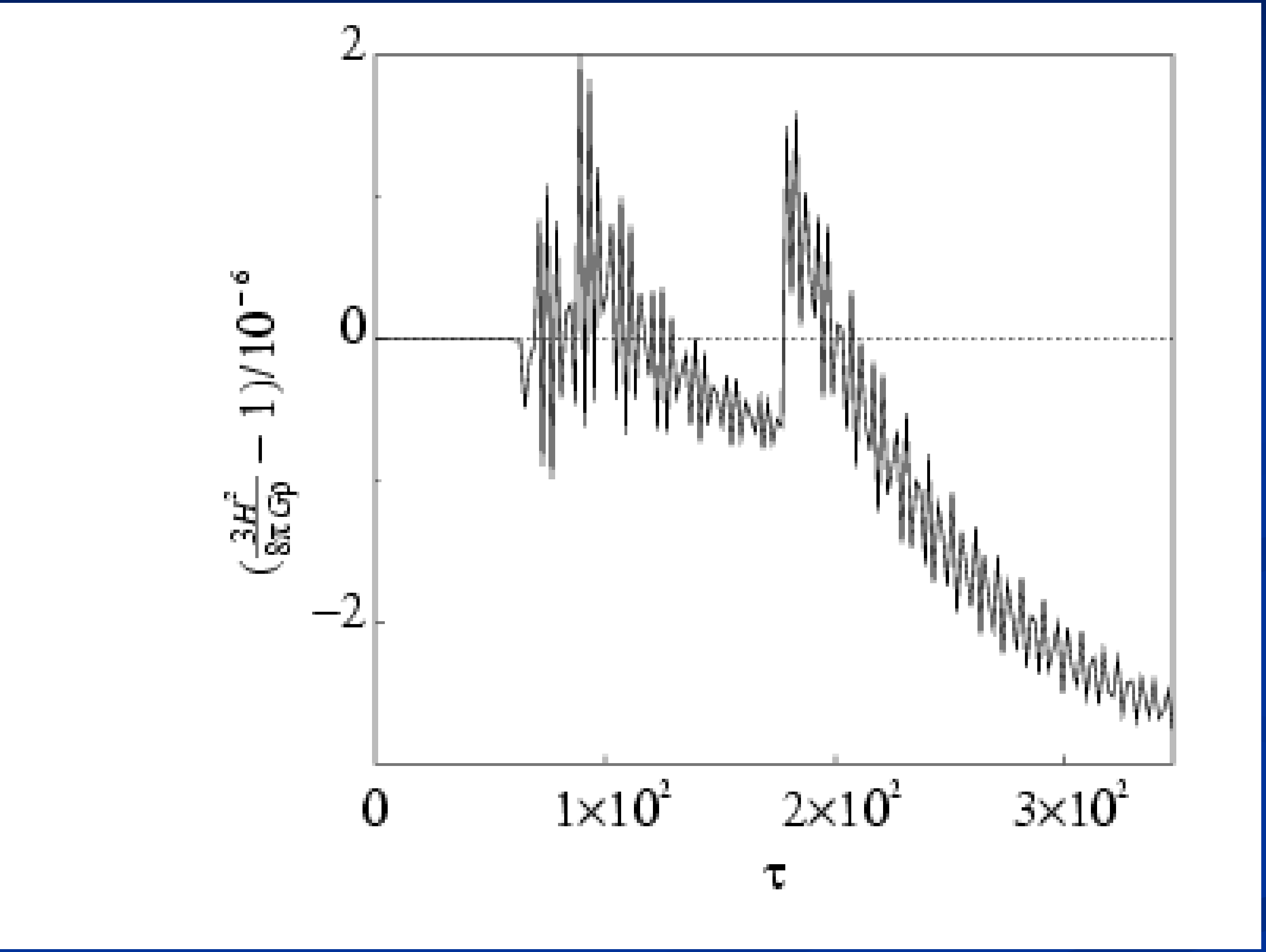


*Figure generated by Zhiqi Huang

Lattice Simulations So Far: A Small Sample

- Parametric Resonance (Kofman, Linde, and Starobinsky, 1994.) and Tachyonic Preheating (G.F., et al 2001.).
- Phase Transitions (Khlebnikov, Kofman, Linde, and Tkachev, 1998.) and Defect Formation (Tkachev, Khlebnikov, Kofman, and Linde, 1998.).
- Baryogenesis (Kolb, Riotto, and Tkachev, 1998.).

Lattice Simulations

- LATTICEEASY - *A lattice simulation program for interacting scalar fields in an expanding universe.*
- CLUSTEREASY – *The parallel programming version of LATTICEEASY.*
  - Has completed runs of up to 1024^3 points with nine fields.

Gravity Waves: Motivation

- Gravity waves decouple instantly and pass through the universe unimpeded.
- The intermediate stages from inflation to thermalization constitute the most poorly understood epoch in cosmological history.

Easther, Giblin, and Lim
Figueroa, Garcia-Bellido, and Sastre
Bergman, G.F., Dufaux, Kofman, Navros, and Uzan

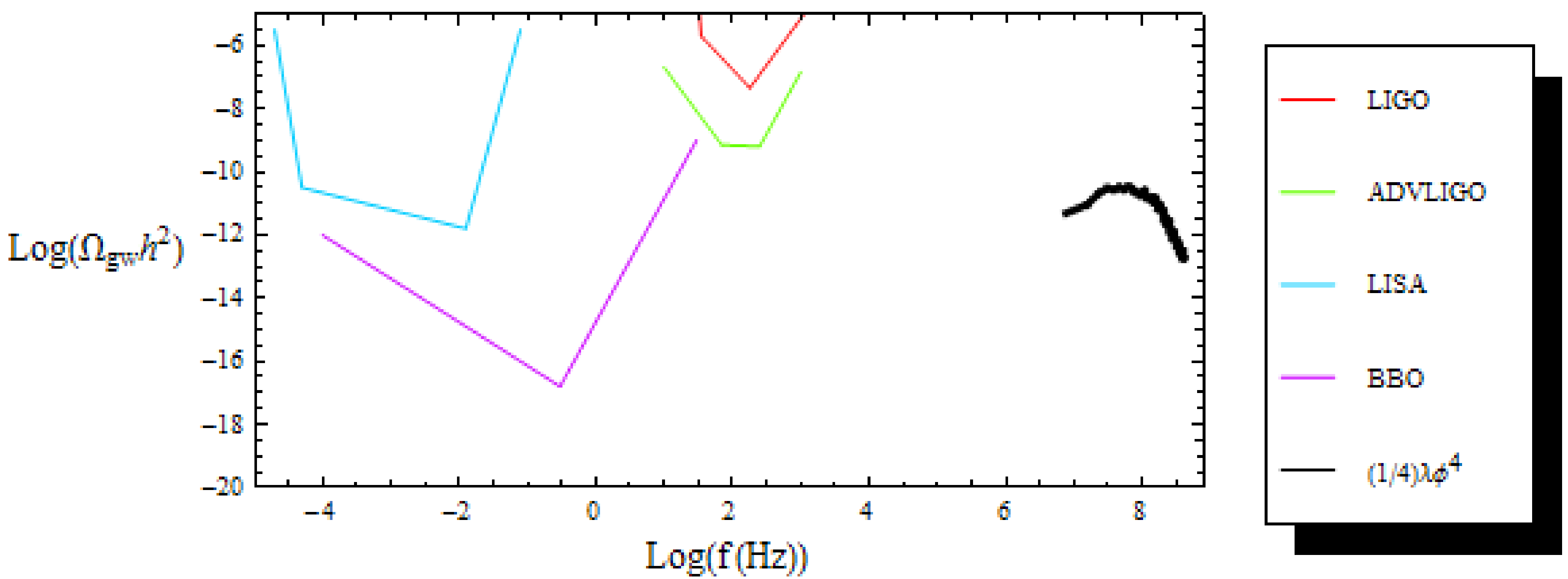
Calculation of Gravity Waves

$$\rho_{gw} = \frac{1}{32\pi G}\left\langle \dot{h}_{ij}\dot{h}_{ij} \right\rangle$$

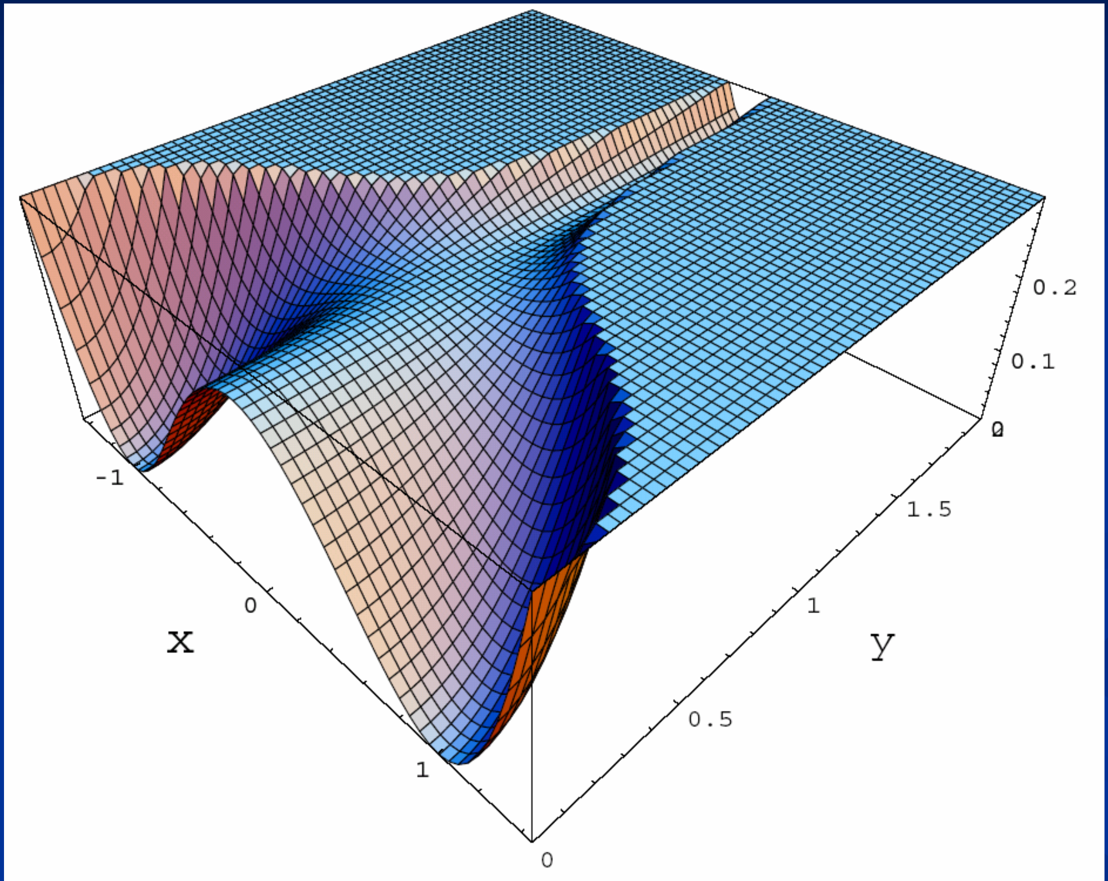
$$\ddot{h}_{ij} + 2\frac{\dot{a}}{a}\dot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{TT}$$

$$T_{ij}^{TT} = O_{ijlm} T_{lm}$$

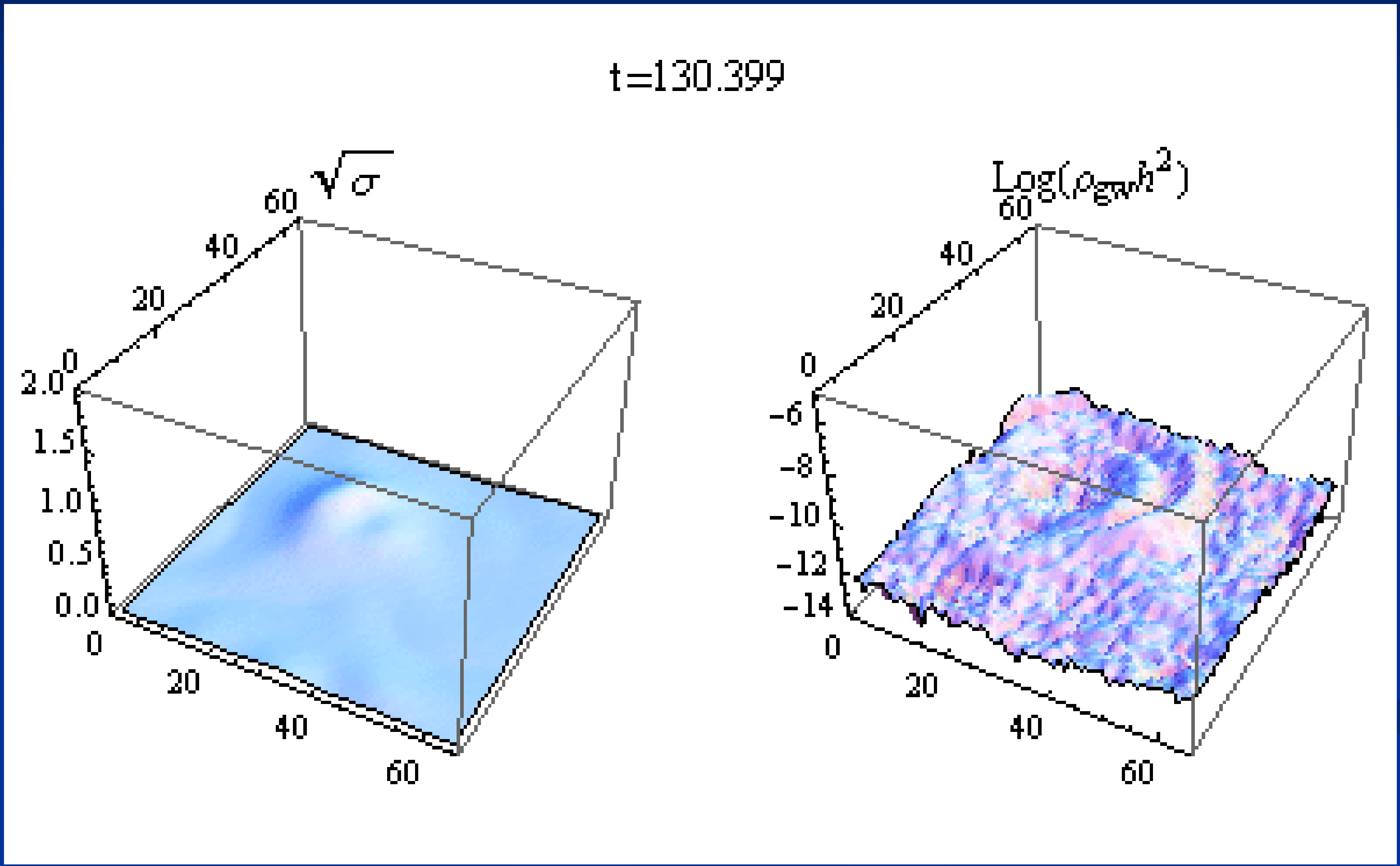
Results: Chaotic Inflation



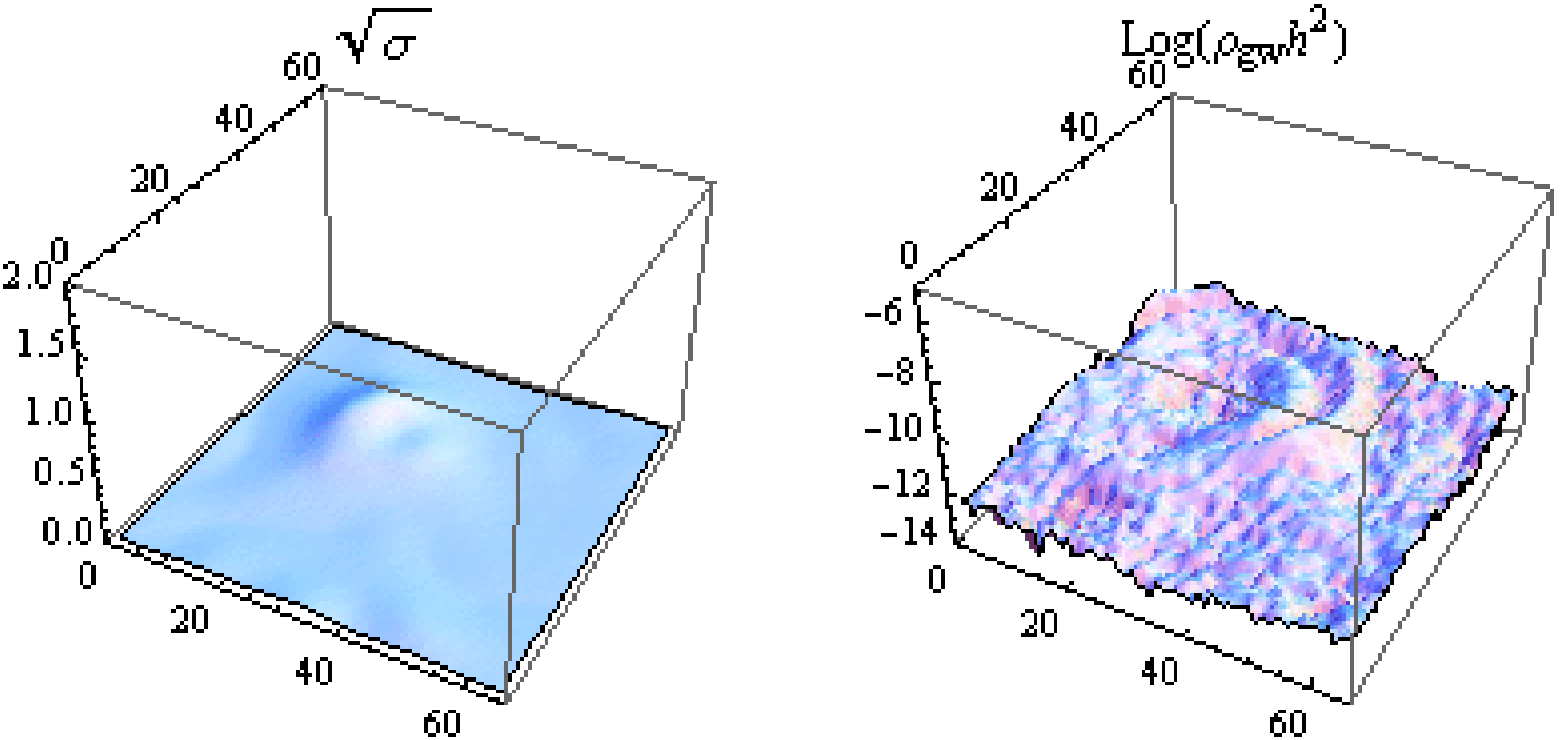
Hybrid Inflation



$$V(\phi,\sigma)=\frac{1}{4}\lambda(\sigma^2-v^2)^2+\frac{1}{2}g^2\phi^2\sigma^2+V_{\mathrm{inf}}(\phi)$$


t=130.399
$\sqrt{\sigma}$
$\mathrm{Log}(\rho_{gw} h^2)$
0
20
40
60
2.0
1.5
1.0
0.5
0.0
-6
-8
-10
-12
-14

t=130.399



$\sqrt{\sigma}$
60
40
20
0
2.0
1.5
1.0
0.5
0.0
0
20
40
60
$\mathrm{Log}(\rho_{gw}h^2)$
60
40
20
0
-6
-8
-10
-12
-14
0
20
40
60

Hybrid Inflation

$$V(\phi,\sigma) = \frac{1}{4}\lambda(\sigma^2 - v^2)^2 + \frac{1}{2}g^2\phi^2\sigma^2 + V_{\inf}(\phi)$$

$$f_{*,\min}(\lambda) \approx \lambda^{3/4}(10^{12}\,Hz)$$

Future Prospects

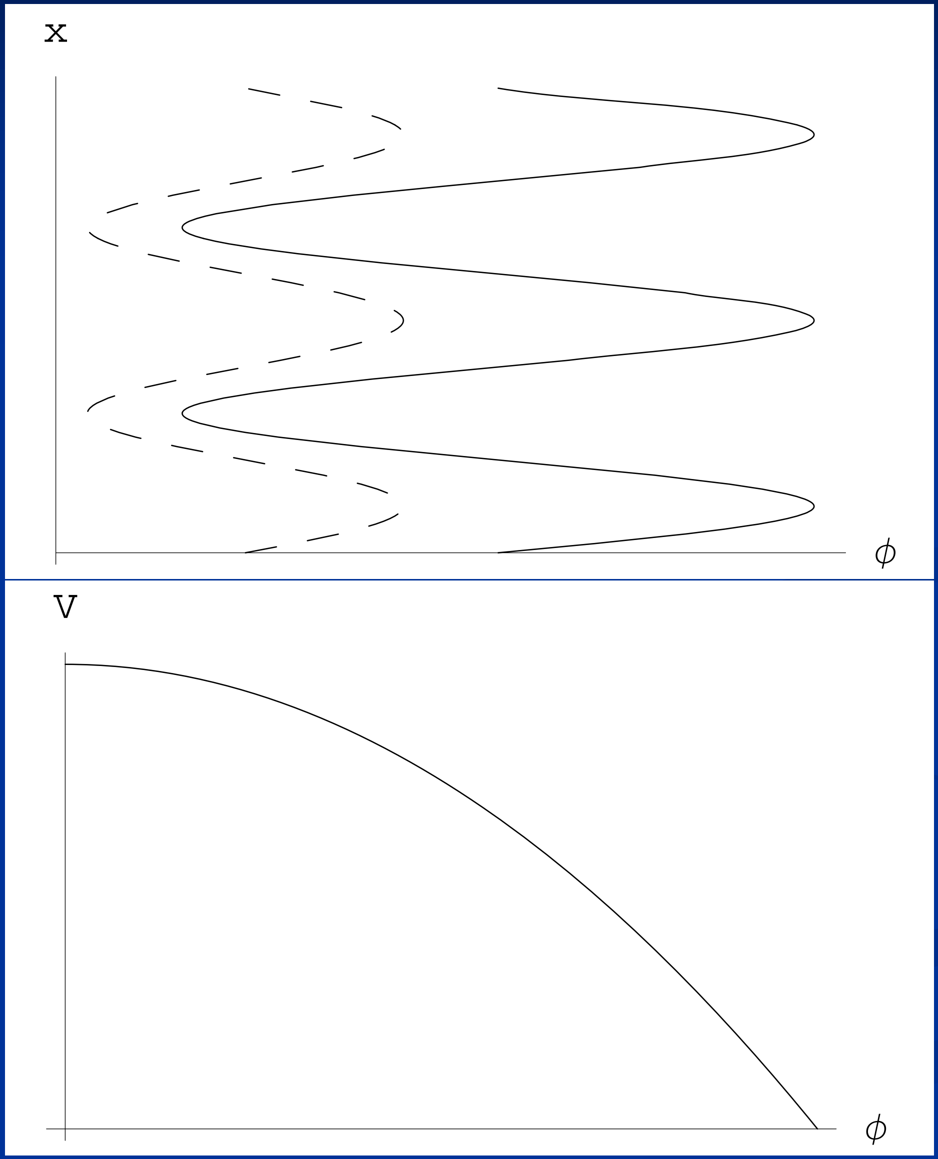
- These same techniques can be applied to many other scenarios:
  - Early structure formation
  - Phase transitions
  - Defect formation
  - …

Conclusions

- The epoch from inflation to thermalization is highly important and poorly understood.
- In many models of inflation that epoch begins with a stage of preheating.
- During and shortly after preheating the universe can be accurately modeled by classical lattice simulations.

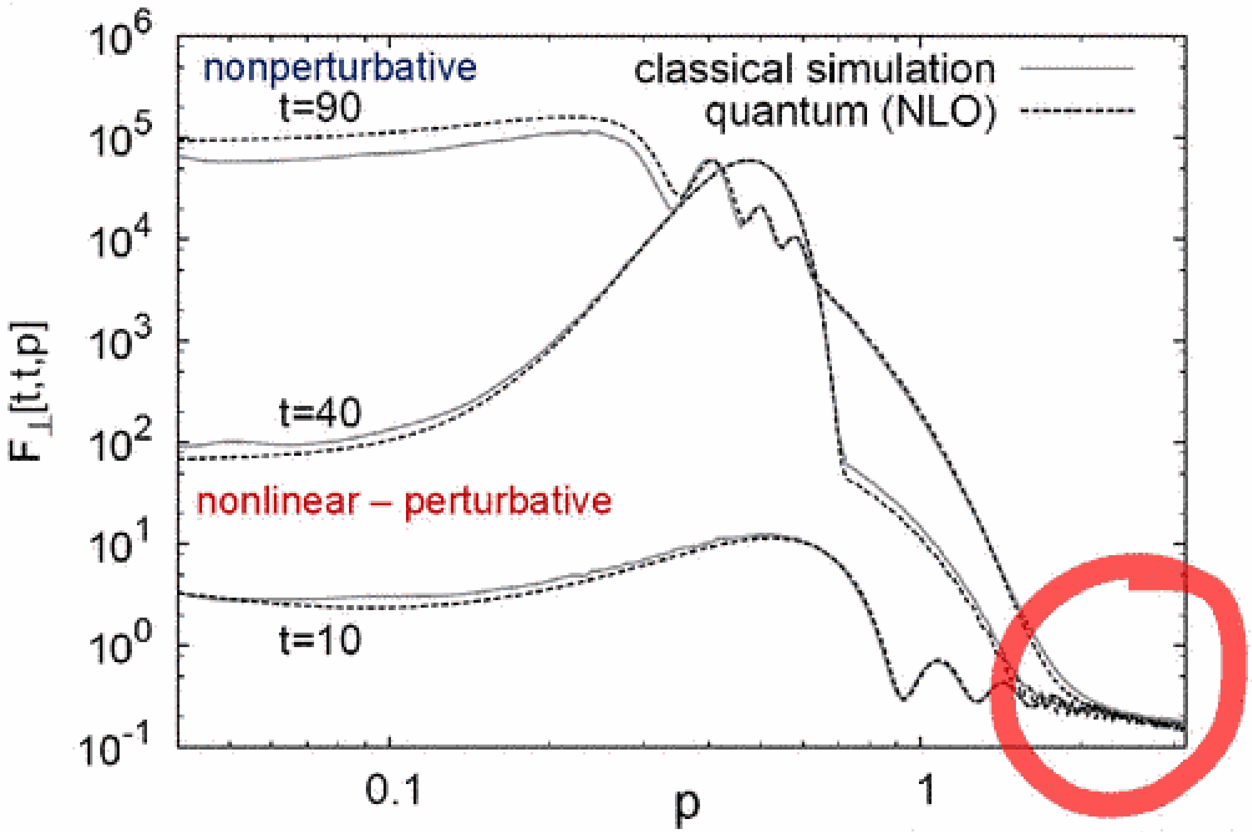
Additional Slides

Tachyonic Preheating, cont.



Comparison quantum/classical dynamics

Classical-statistical simulations: Khlebnikov, Tkachev '96; Prokopec,Roos '97; Tkachev, Khlebnikov, Kofman, Linde '98; ...



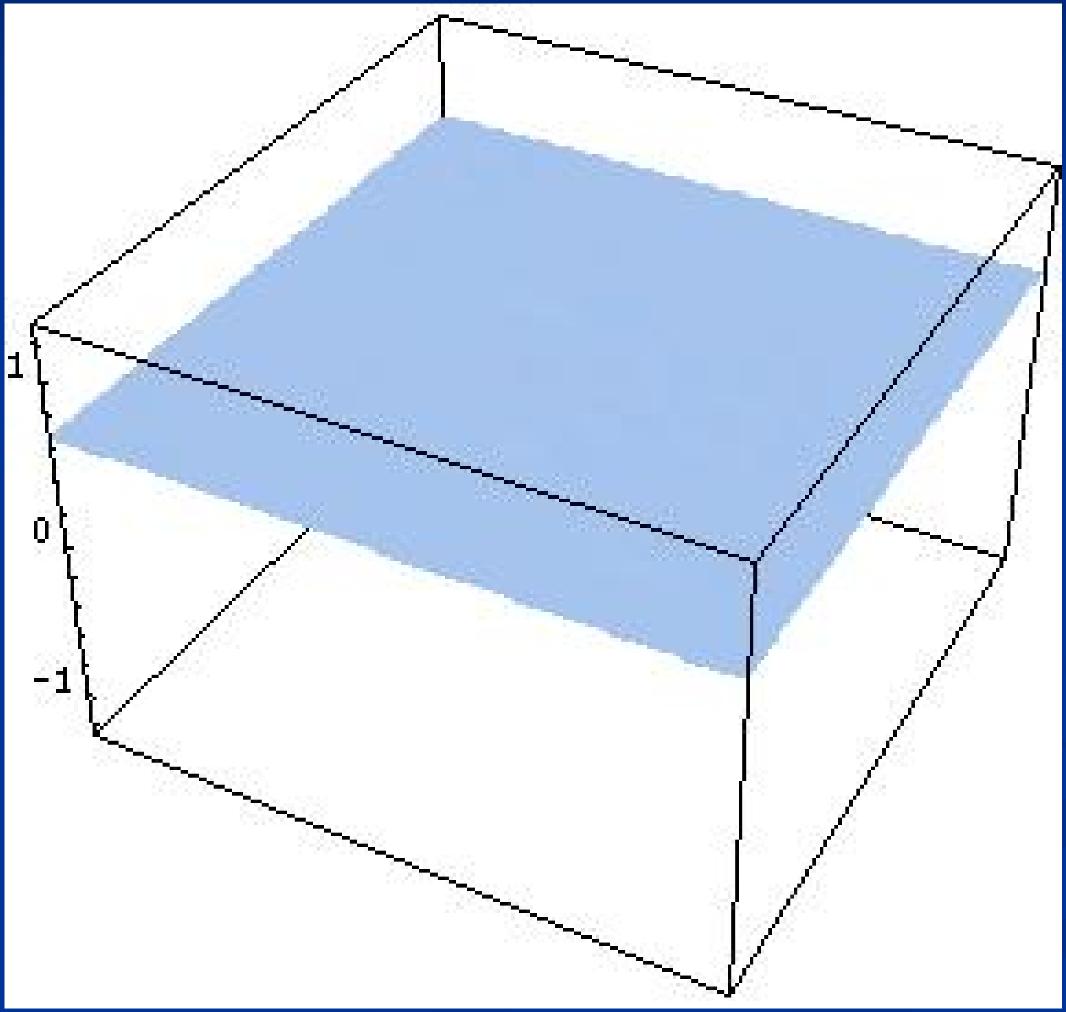
Berges, Rothkopf, Schmidt '08

Practically no quantum corrections at the end of preheating

Accurate nonperturbative description by 2PI 1/*N* to NLO

*Slide taken from Juergen Berges

A Closer Look at Preheating: Parametric Resonance



Gravity Waves on the Lattice

$$\ddot{\phi}_a + 3\frac{\dot{a}}{a}\dot{\phi}_a - \frac{1}{a^2}\nabla^2\phi_a + \frac{\partial V}{\partial \phi_a} = 0$$

$$\ddot{a} + \frac{4\pi a}{3}(p + 3\rho) = 0$$

$$\ddot{h}_{ij} + 2\frac{\dot{a}}{a}\dot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{TT}$$

Hybrid Inflation

$$V(\phi,\sigma) = \frac{1}{4}\lambda(\sigma^2 - v^2)^2 + \frac{1}{2}g^2\phi^2\sigma^2 + V_{\inf}(\phi)$$

$$Free\ Parameters: \lambda, g^2, v, \dot{\phi}_0$$

$$g^2 \sim \lambda$$

$$\dot{\Phi}_0 \equiv \frac{\dot{\phi}_0}{\phi_{cr}}\sqrt{\lambda}v$$

$$f_* \propto \lambda^{1/4}\dot{\Phi}_0^{1/3}, \quad \Omega_{gw}h^2 \propto \frac{v^2}{\dot{\Phi}_0^{2/3}}$$

$$f_{*,\min}(\lambda) \approx \lambda^{3/4}(10^{12}\,Hz)$$