Ultracool dynamics far from equilibrium

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Overview

- Preface
  Ultracold gases out of equilibrium
- Non-equilibrium quantum field theory
  Functional RG approach
- Equilibration of a 1D Bose gas
  s-Channel approximation and 2PI NLO 1/N

Preface

ULTRAcool...

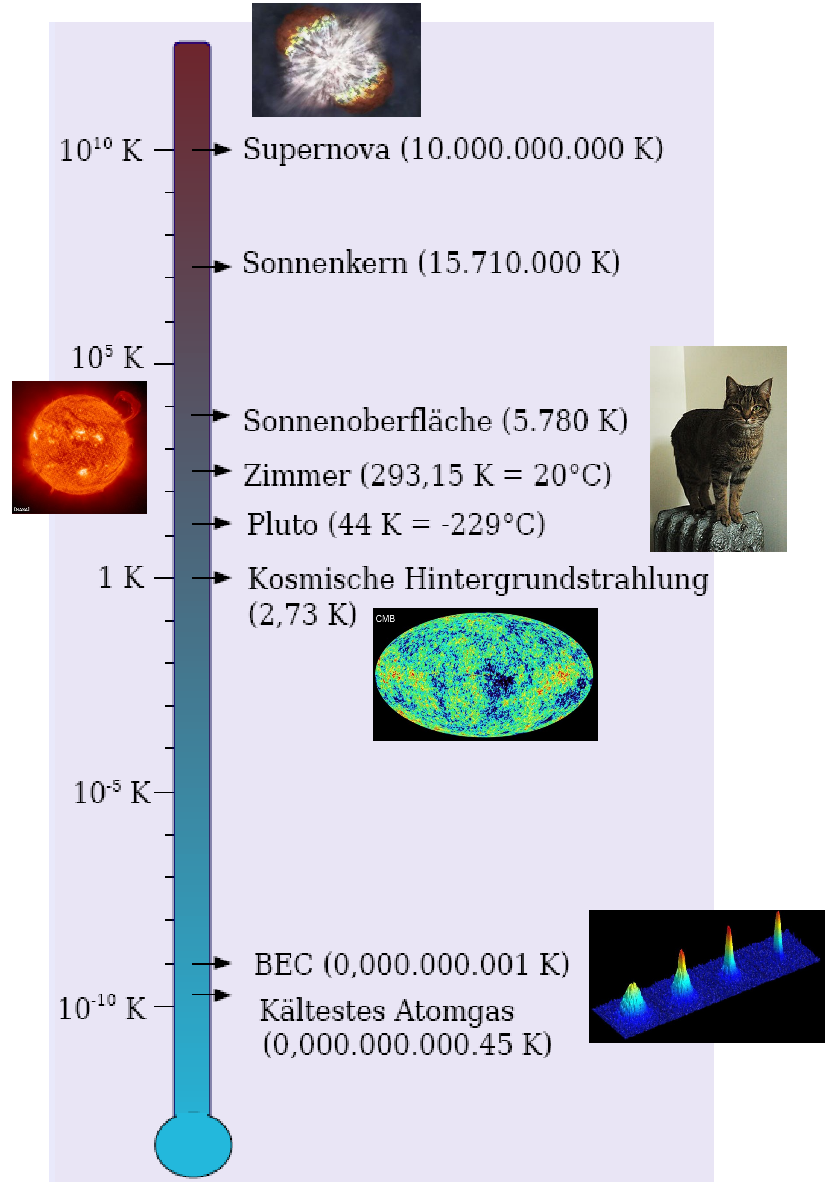
... atoms @ nanokelvins –

trapped only a few mm away from

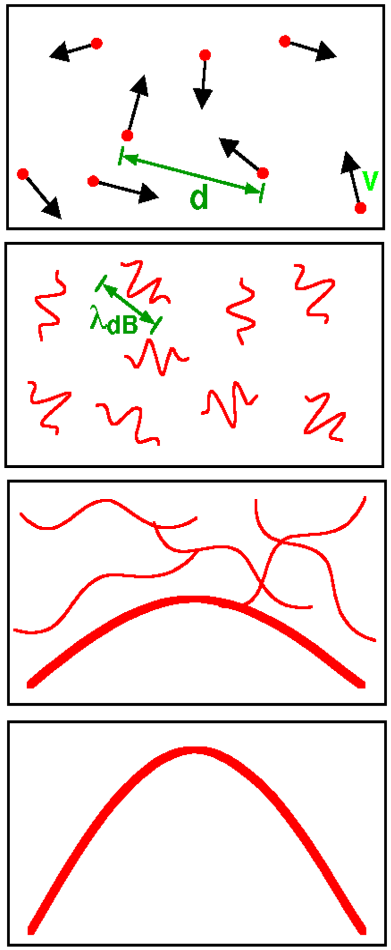
glass cell @ room temperature

(vacuum of 10^{-12} Torr,
i.e. 10^{-15} bar,
or 10^{-10} Pa,

≈ atmospheric
pressure on the moon)



Bose-Einstein condensation



High Temperature T:
thermal velocity v
density d^{-3}
"Billiard balls"

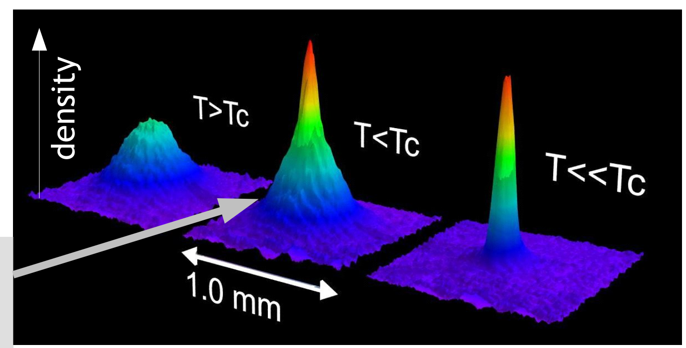
Low Temperature T:
De Broglie wavelength
$\lambda_{dB} = h/mv \propto T^{-1/2}$
"Wave packets"

$T = T_{crit}$: Bose-Einstein Condensation
$\lambda_{dB} \approx d$
"Matter wave overlap"

T=0: Pure Bose condensate
"Giant matter wave"

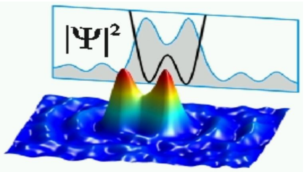
Experimental picture after free expansion of the trapped cloud:

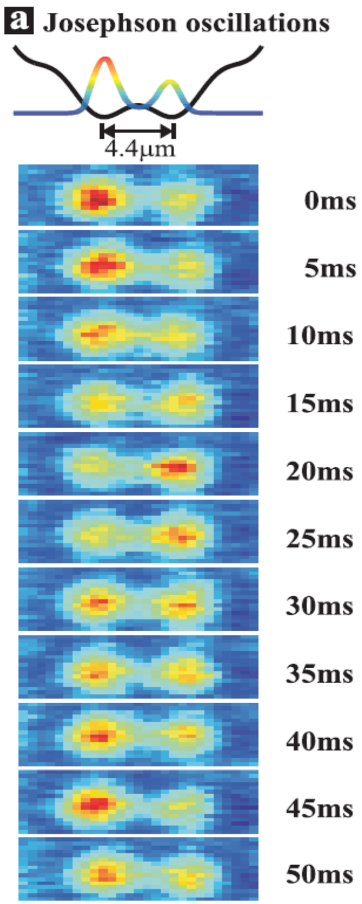
Bose-Einstein condensation (BEC)



bimodal distribution

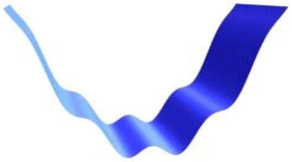
Cold-gases livestream on TV

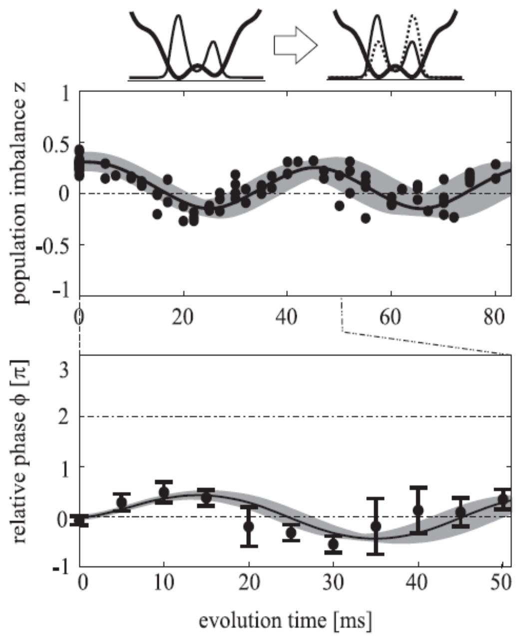




Experimenters can now...

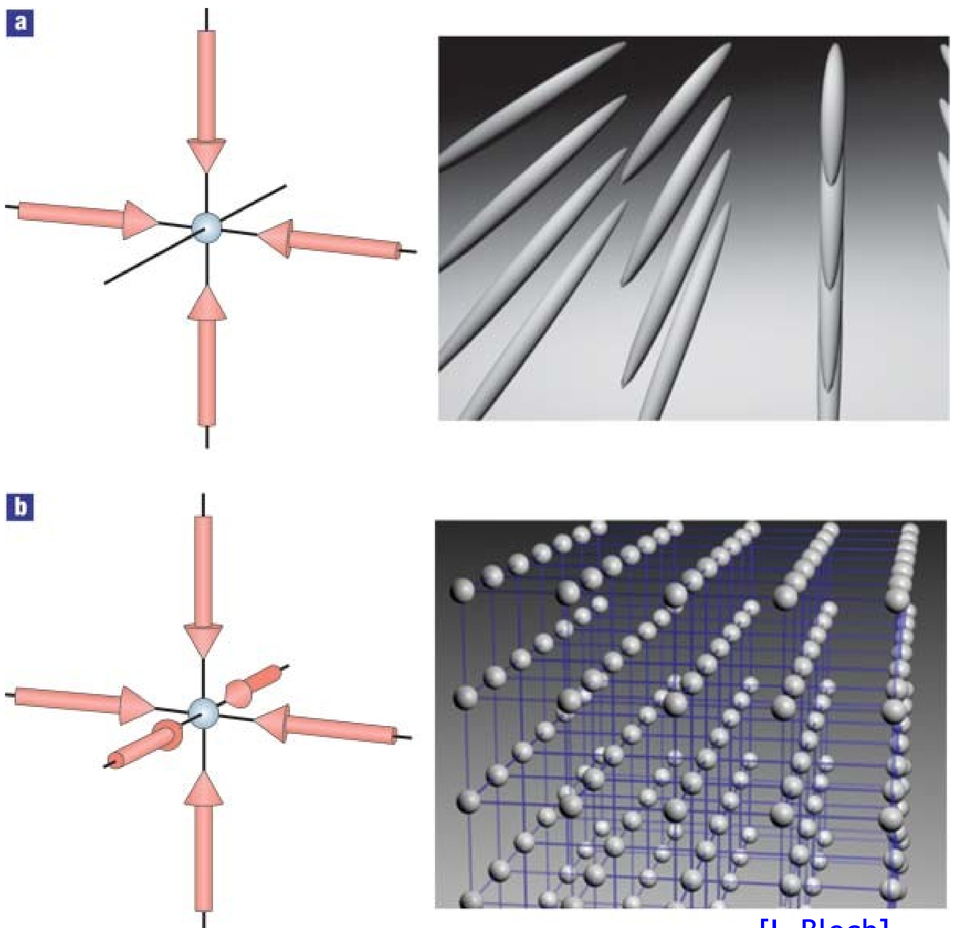
- ✔ observe evolution in real time
- ✔ model freely initial state
- ✔ change boundary conditions
- ✔ measure mean densities, phases, fluctuations
- ✔ reduce atom numbers to a few hundreds & less





Oberthaler Labs (Heidelberg)

1D traps and lattices

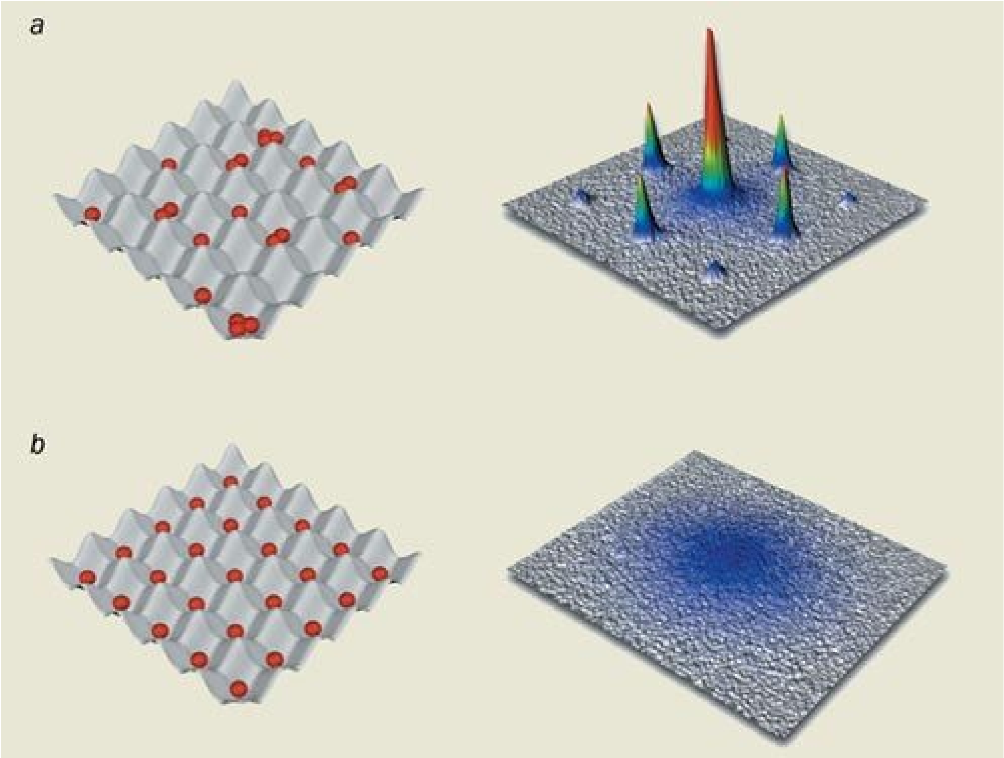


[I. Bloch]

Lasers allow to create lower dimensional traps and lattices

No restrictions to magnetic low-field seeking hyperfine states!

Optical lattices



Superfluid - Mott-insulator quantum phase transition

[M. Greiner et al., Nature 415 (02)]

Optical lattices allow

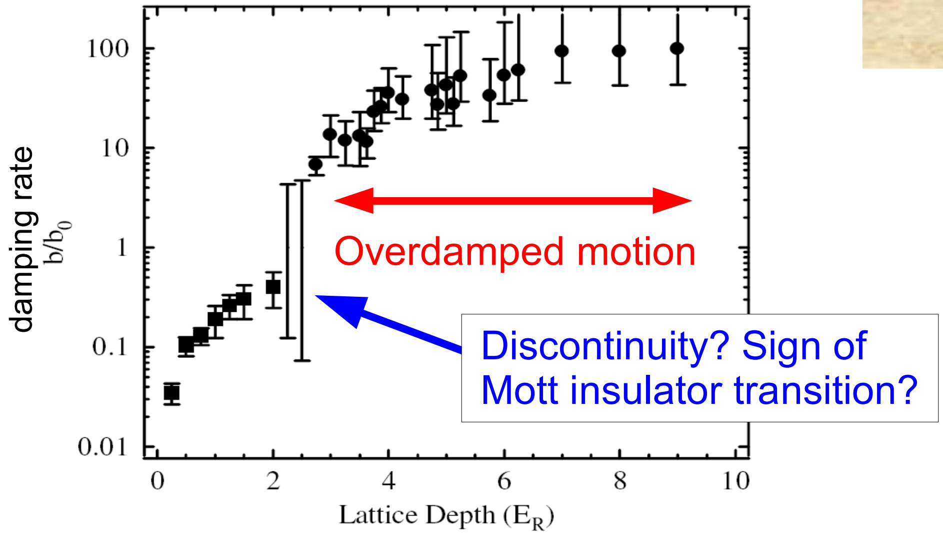
- simulation of solid state systems,
- study of quantum phase transitions,
- fast changes in long-range correlations

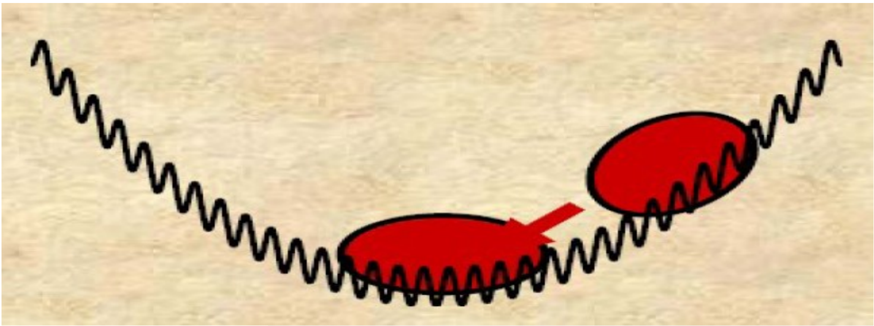
Nonequilibrium dynamics in lattices

[with P. Struck]

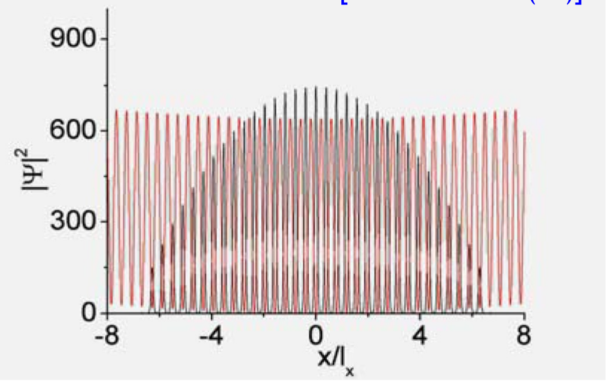
Dipole oscillations in lattices:
Damping rates?

[Expt. @ NIST: Fertig et al. PRL94 (05)]

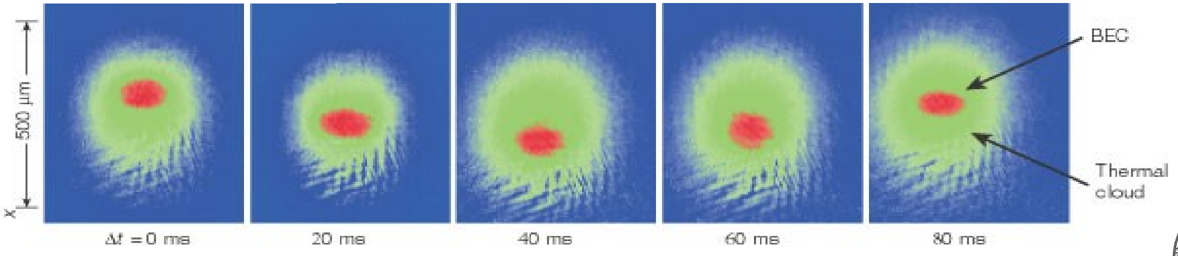




[J. Ruostekoski (07)]



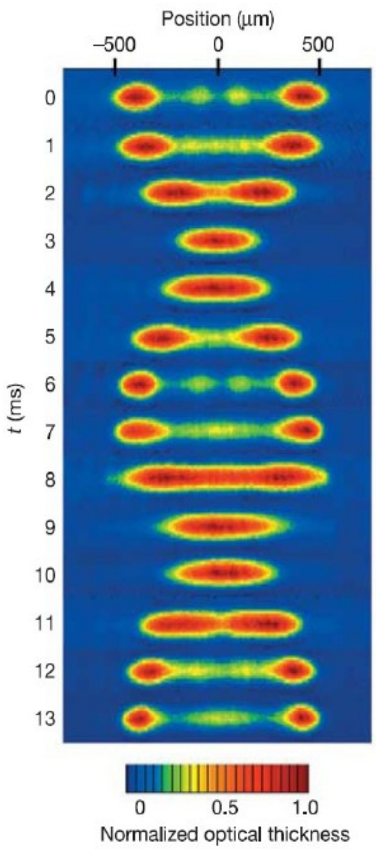
[Anglin & Ketterle (02)]



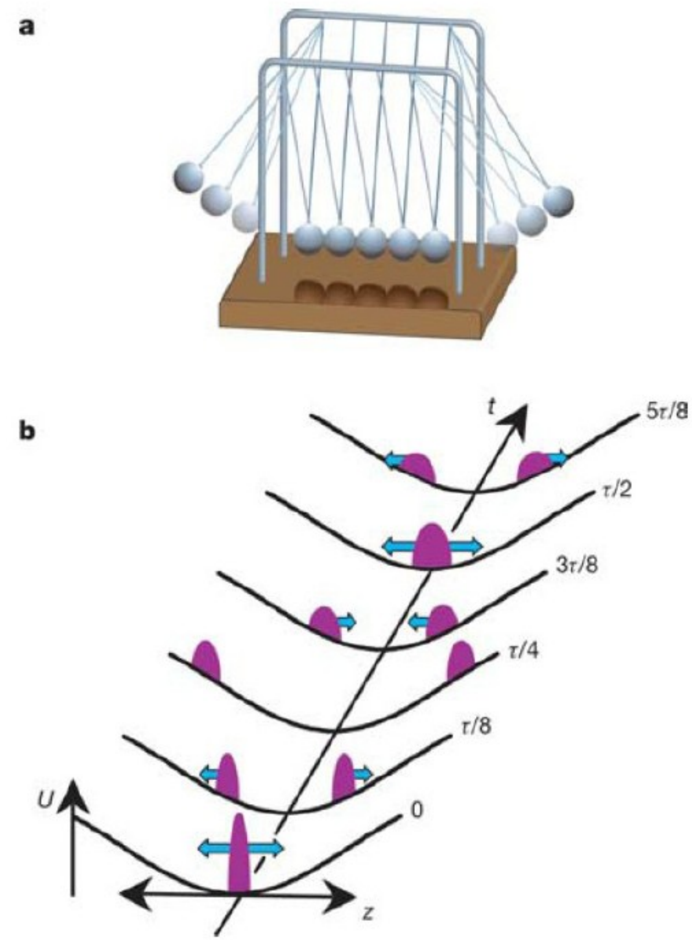
Long-time dynamics of ultracold gases

A quantum Newton's cradle.

[T. Kinoshita et al. Nature 440 (06)]



Indication for strong suppression of damping



Theory?

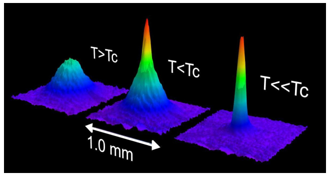
Available methods:

- Mean-field theories [Gross-Pitaevskii, Hartree-Fock(-Bogoliubov)]
- Kinetic approaches [Quantum Boltzmann, ...]
- (Semi-)classical simulations [Truncated Wigner Approximation, ...]
- Exactly solvable models
  [Lieb & Liniger, Girardeau, McGuire, Gaudin, Minguzzi, Buljan, ...]
- tDMRG, MPS/PEPS
  [Vidal, Kollath, Schollwöck, White, Feiguin, Manmana, Muramatsu, Wolf, Cirac, ...]
- Quantum Monte Carlo, stochastic Quantisation
  [Mak, Egger, Berges & Stamatescu, ...]

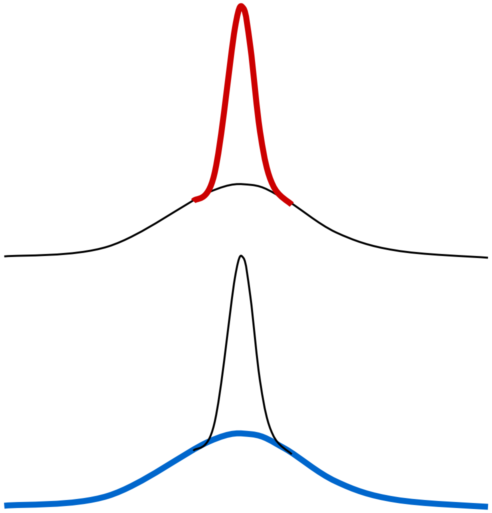
- **Functional QFT**

Non-equilibrium evolution in Quantum Field Theory

How to describe a condensate?



For **bosons**: $[\hat{\Phi}_{t,\mathrm{x}}, \hat{\Phi}^\dagger_{t,\mathrm{x}'}] = \delta(\mathrm{x}-\mathrm{x}')$



- Matter wave **mean field** $[x = (x_0, \mathrm{x}) = (t, \mathrm{x})]$

$$\phi_{\boldsymbol{x}} = \langle \hat{\Phi}_{\boldsymbol{x}} \rangle, \qquad |\phi_{\boldsymbol{x}}|^2 = n_{\mathrm{c}}(x) = \text{condensate density},$$

- Density of **non-condensed atoms** $(\hat{\Phi} = \phi + \widetilde{\Phi},\ \phi = \langle \hat{\Phi} \rangle)$

$$\langle \widetilde{\Phi}^\dagger_{\boldsymbol{x}} \widetilde{\Phi}_{\boldsymbol{x}} \rangle = n_{\mathrm{nc}}(x) \equiv n(x) - n_{\mathrm{c}}(x),$$

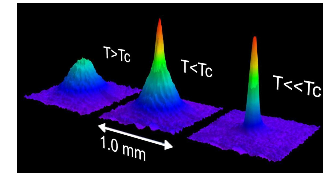
- Total one-body **density matrix**

$$G_{11}(x,y) = \langle \widetilde{\Phi}^\dagger_{\boldsymbol{x}} \widetilde{\Phi}_{\boldsymbol{y}} \rangle \Rightarrow \text{spatial Fourier transform: momentum distribution } n(\mathrm{p}, t)$$
$$\Rightarrow 1^{\mathrm{st}}\text{-order phase coherence}$$

- **Anomalous one-body density** matrix

$$G_{12}(x,y) = \langle \widetilde{\Phi}_{\boldsymbol{x}} \widetilde{\Phi}_{\boldsymbol{y}} \rangle \Rightarrow \text{e.g., number of Bose-condensed bound pairs (molecules)}$$

How to describe a condensate?



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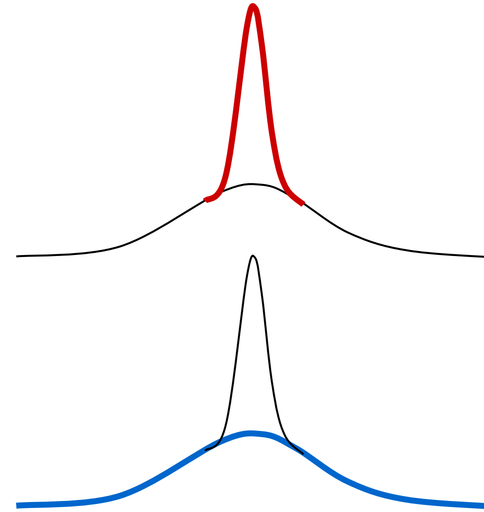

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Dynamical Field Theory

We will be interested, in particular, in the explicit time dependence of the lowest-order correlation functions:

$$\phi_a(x) = \langle \Phi_a(x) \rangle$$ (mean field)

$$G_{ab}(x,y) = \langle \mathcal{T} \Phi_a(x) \Phi_b(y) \rangle_c$$ (density matrix, 2-point function, propagator)

connected, i.e., $= \langle \mathcal{T} \Phi_a \Phi_b \rangle - \phi_a \phi_b$

where $x = (\mathbf{x}, t)$

Initial value problems...

...require the Schwinger-Keldysh closed time path (CTP):

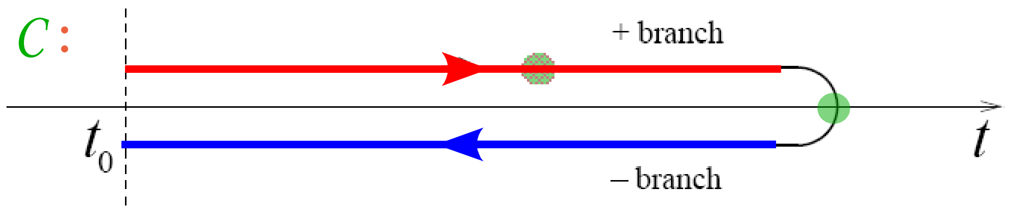
$$\langle t\,|O|\,t\,\rangle = \langle t_0\,|U^\dagger(t) O\, U(t)|\,t_0\,\rangle$$
$$= \mathrm{Tr}[\rho(t_0)\, U^\dagger(t) O\, U(t)\,]$$

path ordering along CTP $\mathcal{C}$

e.g.

$$G_{ab}(x,y) = \mathrm{Tr}[\rho(t_0)\, \mathcal{T}_{\mathcal{C}}\, U^\dagger(t)\Phi_a(x)\Phi_b(y)U(t)\,] - \text{disc.}$$

CTP $\mathcal{C}$:

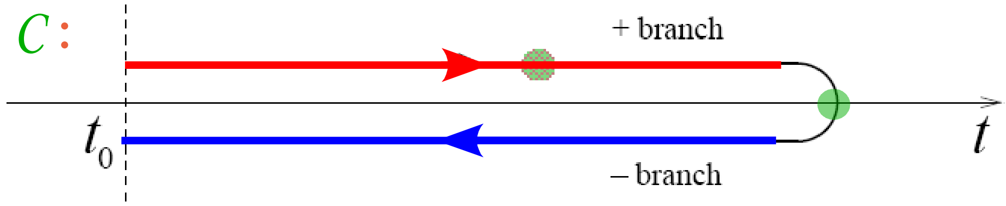


Initial value problems...

...require the Schwinger-Keldysh closed time path:

$$\langle t \,|O|\, t\rangle = \langle t_0 \,|U^\dagger(t) O\, U(t)|\, t_0\rangle$$
$$= \int \mathcal{D}\varphi_0 \mathcal{D}\varphi_0 \,\rho[\varphi_0,\varphi_0] \int \mathcal{D}\varphi' \mathcal{D}\varphi' \; O \; e^{i(S[\varphi]-S[\varphi])/\hbar}$$

CTP C:



Functional RG approach

Renormalisation-group approach

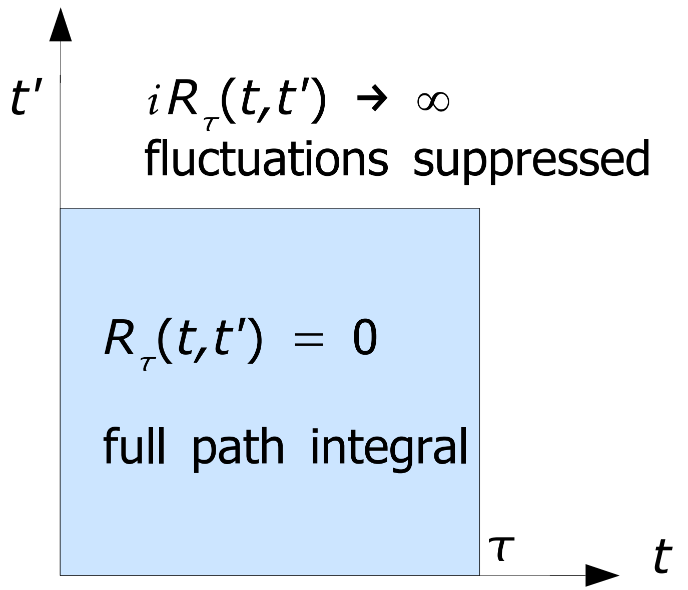
to far-from-equilibrium dynamics

[TG & J.M. Pawlowski, cond-mat/0710.4627]

- Regularise generating functional

$$Z_\tau = \exp\left\{ i \int_{x,y;\mathcal{C}} \frac{\delta}{\delta J_a(x)} R_{\tau,ab}(x,y) \frac{\delta}{\delta J_b(y)} \right\} Z$$

$$Z[J;\rho_0] = \int \mathcal{D}\varphi\, \rho_0 \exp\left\{ iS[\varphi] + i \int_{x,\mathcal{C}} J_a \varphi_a \right\},$$



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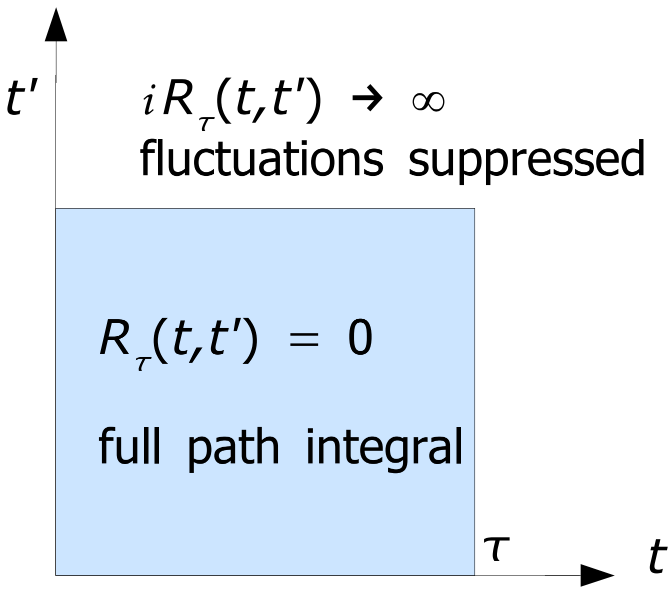


- Functional RG equation [C. Wetterich (92)]

$$\partial_\tau \Gamma_\tau = \frac{i}{2} \int_{\mathcal{C}} \left[\frac{1}{\Gamma_\tau^{(2)} + R_\tau} \right]_{ab} \partial_\tau R_{\tau,ab}$$

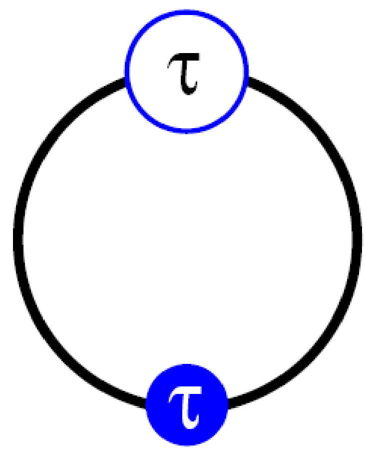
$$\Gamma_\tau[\phi, R_\tau] = W_\tau[J,\rho_0] - \int_{\mathcal{C}} J_a \phi_a - \frac{1}{2} \int_{\mathcal{C}} \phi_a R_{\tau,ab} \phi_b$$

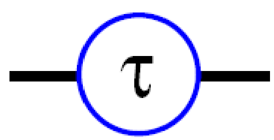
Real-time Functional RG equation

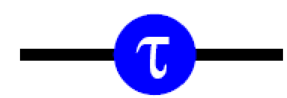
graphically

[TG & J.M. Pawlowski, cond-mat/0710.4627]

$$\partial_\tau \Gamma_\tau = \frac{i}{2}\int_{\mathcal{C}} \left[\frac{1}{\Gamma_\tau^{(2)} + R_\tau}\right]_{ab} \partial_\tau R_{\tau,ab}$$

$$\partial_\tau \Gamma_\tau = \frac{i}{2}$$ 

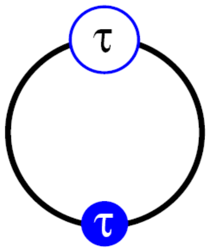
$\dot{R}_{\tau,ab} =$ 

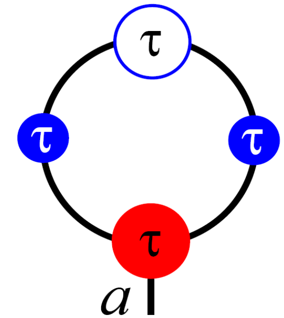
$G_{\tau,ab} =$ 

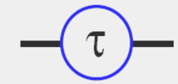
Exact flow equations

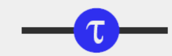
for moments $\Gamma^{(n)}$

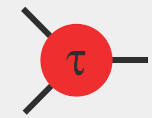
[TG & J.M. Pawlowski, cond-mat/0710.4627]

$\partial_\tau \Gamma_\tau = \frac{i}{2}$ 

$\partial_\tau \Gamma^{(1)}_{\tau,a} = \frac{i}{2}$ 

$\dot{R}_{\tau,ab} =$ 

$G_{\tau,ab} =$ 

$\Gamma^{(3)}_{\tau,abc} =$ 

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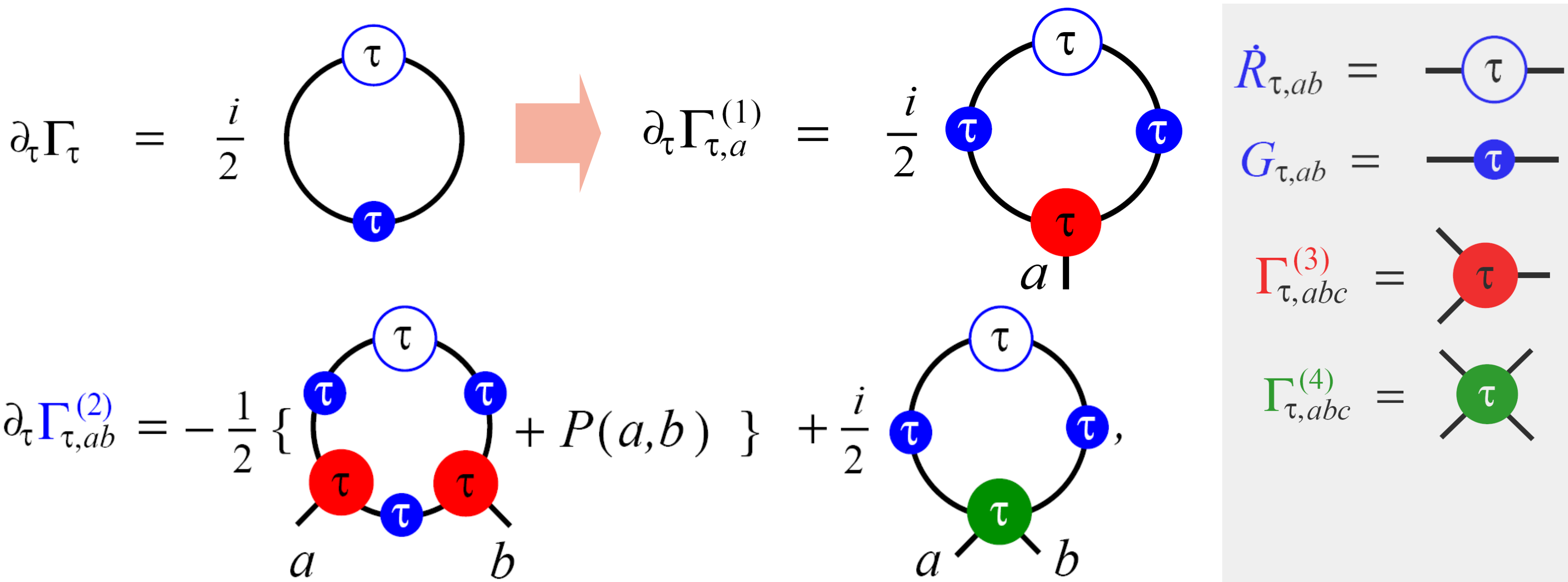

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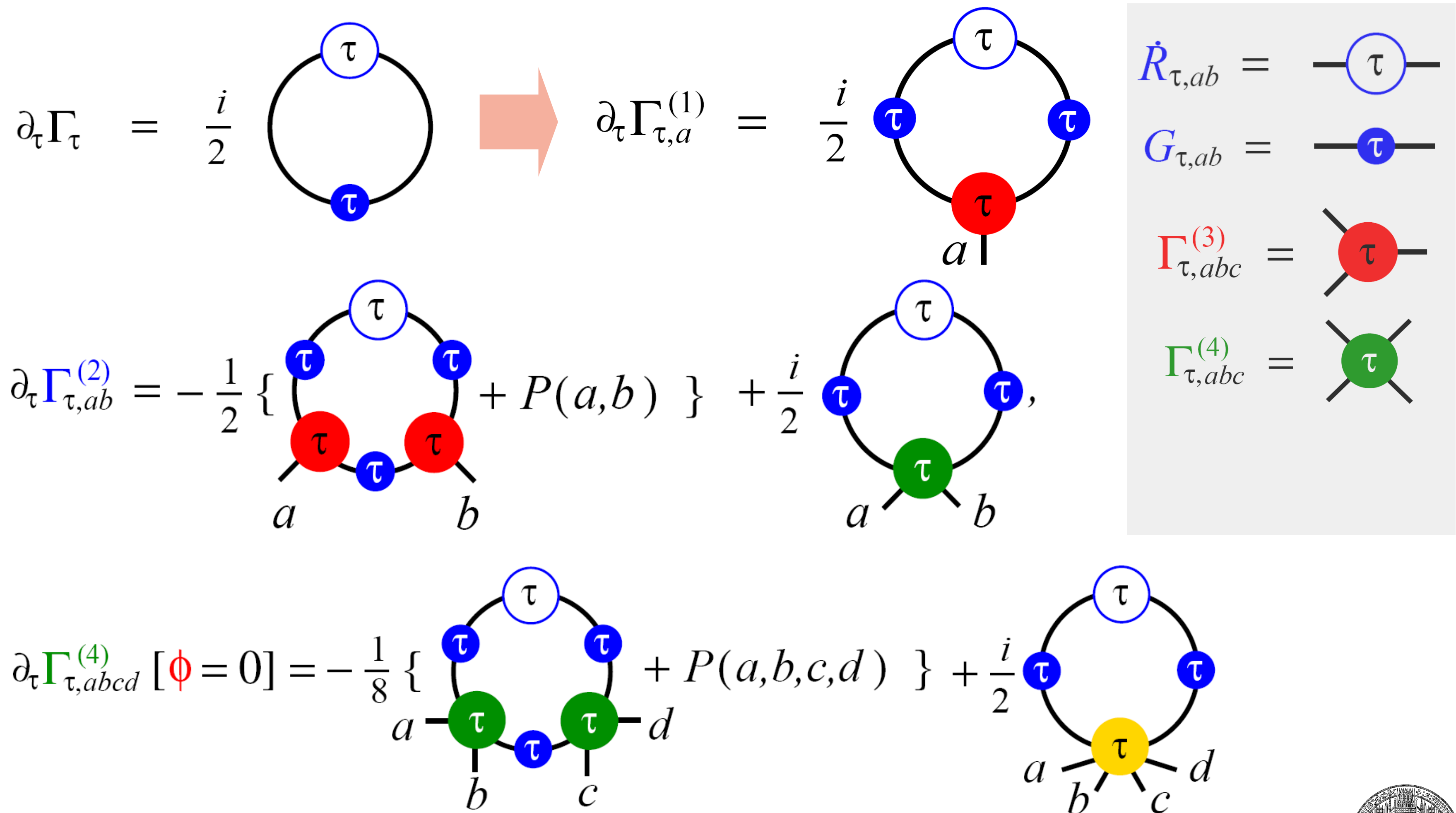
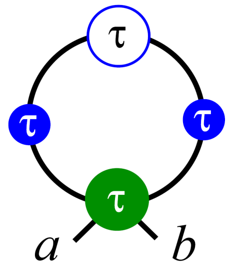


Integrated flows

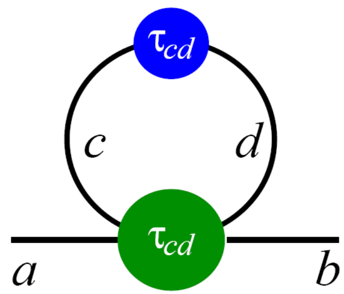
of moments $\Gamma^{(n)}[\phi \equiv 0]$

[TG & J.M. Pawlowski, cond-mat/0710.4627]

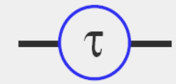
$$\partial_\tau \Gamma^{(2)}_{\tau,ab} = \frac{i}{2}$$



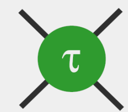
$$\Gamma^{(2)}_{\tau_{ab},ab} = \Gamma^{(2)}_{t_0,\,ab} + \frac{1}{2}$$



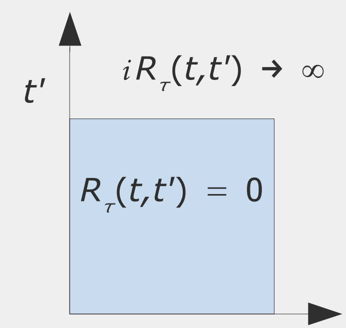
$t_c, t_d = t_0 \ldots \tau_{ab}$

$\dot{R}_{\tau,ab} =$ 

$G_{\tau,ab} =$ 

$\Gamma^{(4)}_{\tau,abc} =$ 

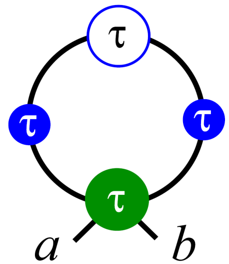
$$\tau_{ab\ldots} = max(t_a, t_b, \ldots)$$

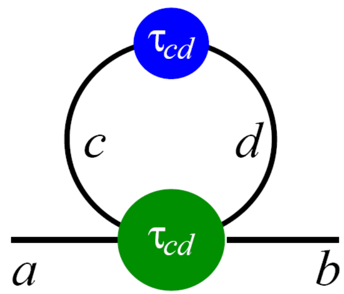


Integrated flows

of moments $\Gamma^{(n)}[\phi \equiv 0]$

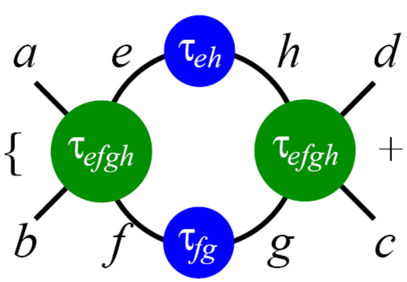
[TG & J.M. Pawlowski, cond-mat/0710.4627]

$\partial_\tau \Gamma^{(2)}_{\tau,ab} = \frac{i}{2}$ 

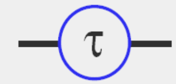
$\Rightarrow$ $\Gamma^{(2)}_{\tau_{ab},ab} = \Gamma^{(2)}_{t_0,\,ab} + \frac{1}{2}$ 

$\Big|_{t_c, t_d = t_0 \dots \tau_{ab}}$

similarly:

$\Gamma^{(4)}_{t,\,abcd} = \Gamma^{(4)}_{t_0,abcd} + \frac{i}{2}\{$ 

$+ P(a,b,c,d)\}\Big|_{t_{e\dots h} = t_0 \dots t}$

$\dot{R}_{\tau,ab} =$ 

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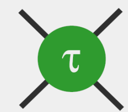
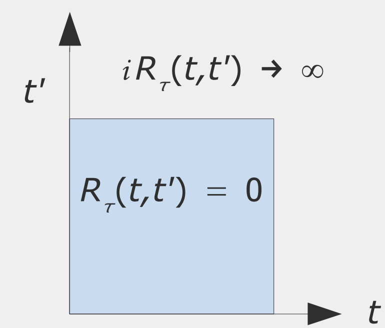
$\Gamma^{(4)}_{\tau,abc} =$ 

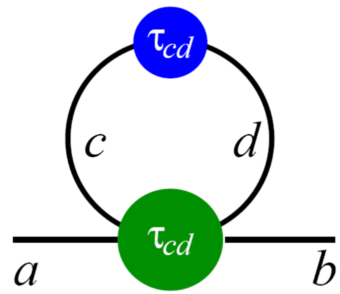
$\tau_{ab\dots} = max(t_a, t_b, \dots)$



Dynamic equations

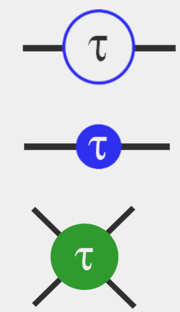
[TG & J.M. Pawlowski, cond-mat/0710.4627]

$$\Gamma^{(2)}_{\tau_{ab},ab} = \Gamma^{(2)}_{t_0,\,ab} + \frac{1}{2}$$

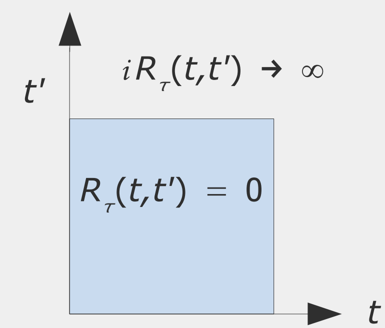


$t_c, t_d = t_0 \dots \tau_{ab}$

$\dot{R}_{\tau,ab} =$; $G_{\tau,ab} =$; $\Gamma^{(4)}_{\tau,abc} =$



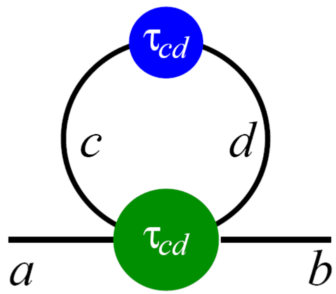
$\tau_{ab\dots} = max(t_a, t_b, \dots)$



Dynamic equations

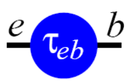
[TG & J.M. Pawlowski, cond-mat/0710.4627]

$$\Gamma^{(2)}_{\tau_{ab},ab} = \Gamma^{(2)}_{t_0,\,ab} + \frac{1}{2}$$

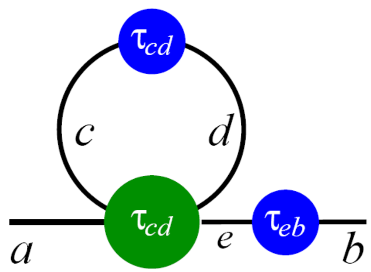


$t_c, t_d = t_0 \dots \tau_{ab}$

$G^{-1}_{0\;ae}$

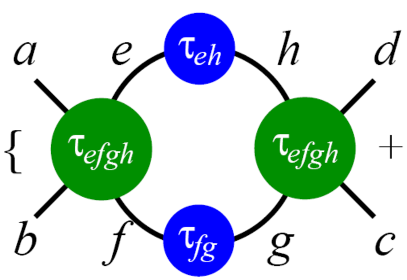


$= \delta_{ab} - \frac{1}{2}$



$t_c, t_d = t_0 \dots \tau_{ae}$

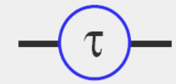
$$\Gamma^{(4)}_{t,\,abcd} = \Gamma^{(4)}_{t_0,abcd} + \frac{i}{2}\Big\{$$



$$+ P(a,b,c,d)\Big\}$$

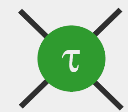
$t_{e\dots h} = t_0 \dots t$

$\dot{R}_{\tau,ab} =$

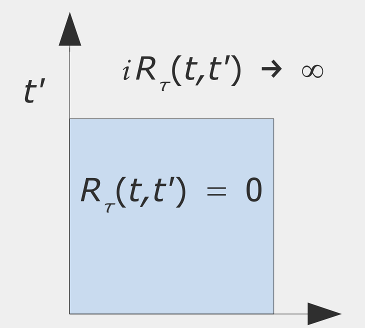


$G_{\tau,ab} =$

$\Gamma^{(4)}_{\tau,abc} =$



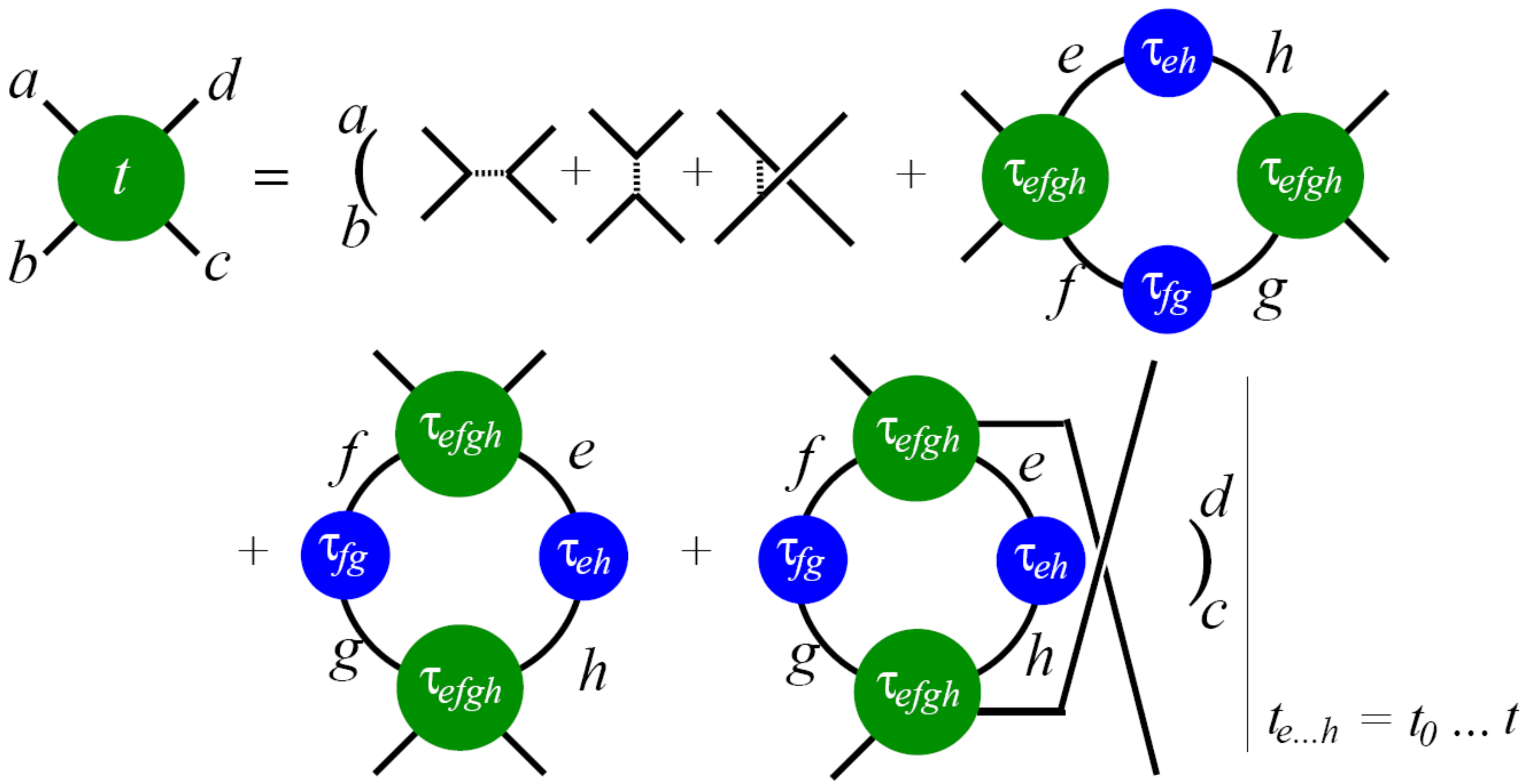
$$\tau_{ab\dots} = max(t_a, t_b, \dots)$$



Integrated flow

of 4-point function $\Gamma^{(4)}[\phi \equiv 0]$

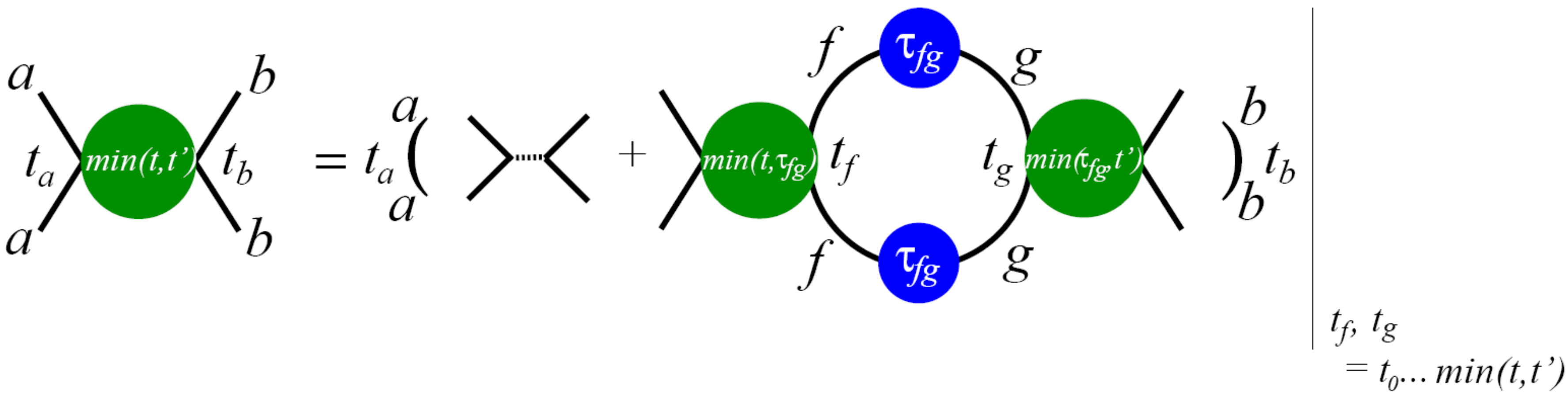
[TG & J.M. Pawlowski, cond-mat/0710.4627]



Integrated flow

in s-channel approximation

[TG & J.M. Pawlowski, cond-mat/0710.4627]



Renormalisation-group approach
to far-from-equilibrium dynamics

[TG & J.M. Pawlowski, cond-mat/0710.4627]

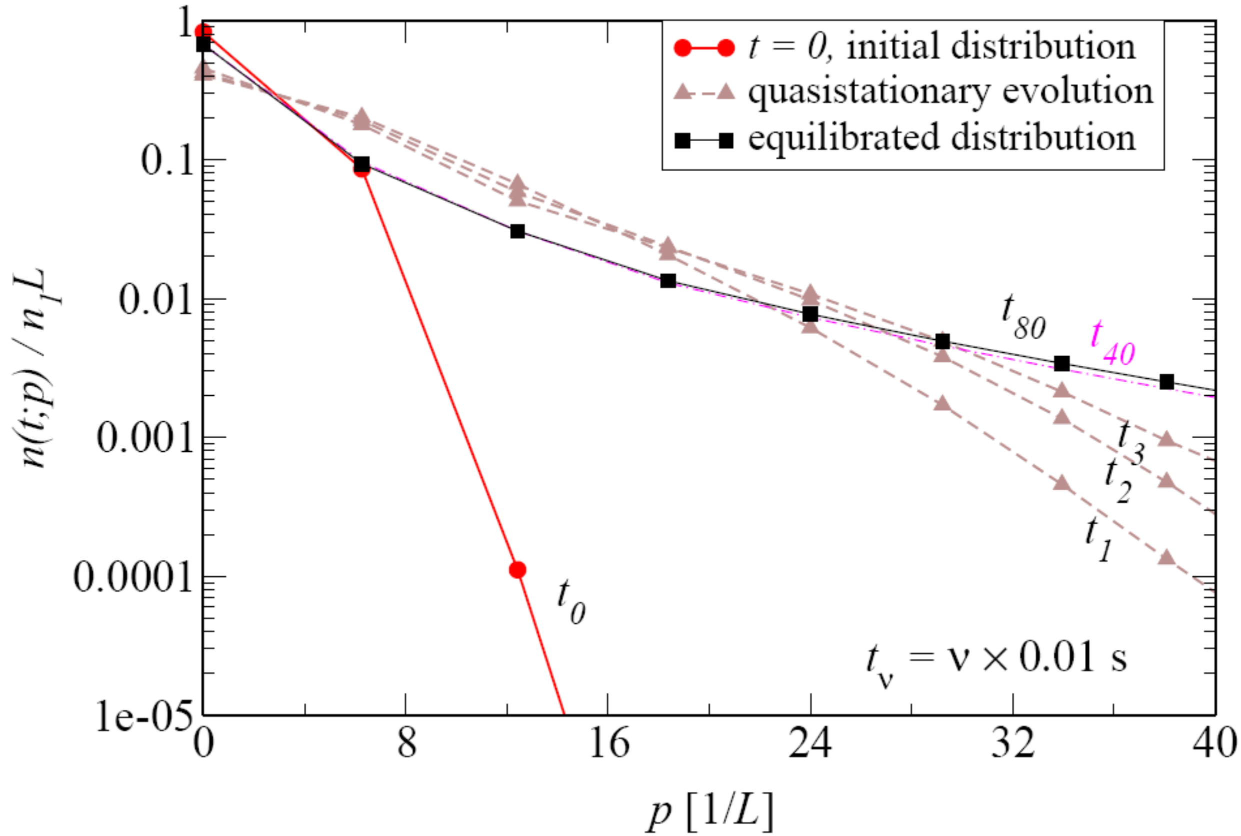
The dynamic equations derived from the Functional RG equation...

- ... can be solved iteratively in time,
- ... provide a resummed 4-vertex beyond 2PI NLO $1/\mathcal{N}$,
- ... allow non-perturbative truncations neglecting higher *n*-vertices,
- ... provide handle to study the quality of the truncation.

Equilibration of a 1D Bose gas

Equilibration of a 1D Bose gas

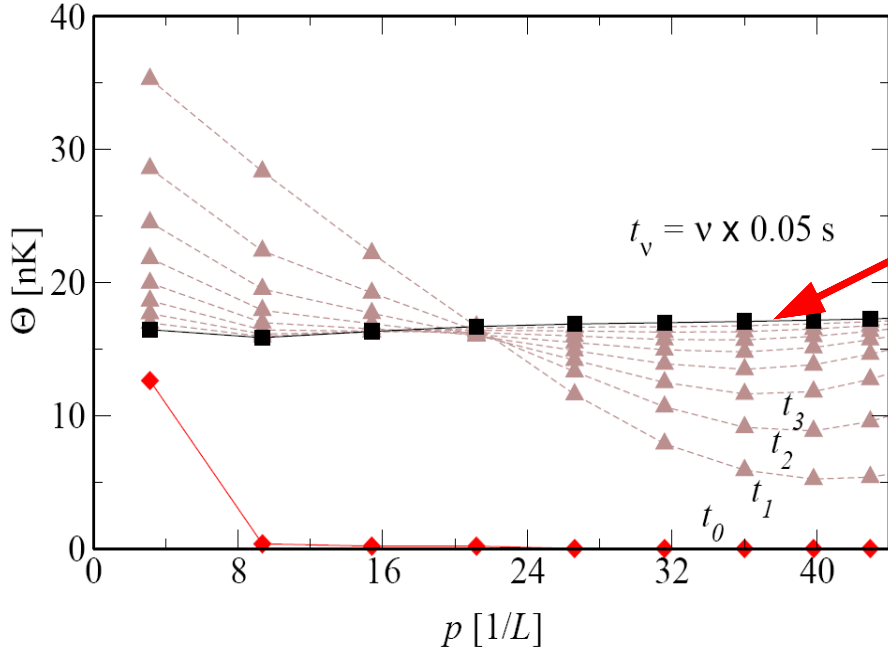
Momentum distribution for different times:



[TG, J. Berges, M. Seco & M.G.Schmidt, PRA **72** (05); J. Berges & TG, PRA **76** (07)]

Temperature appears

‘Temperature’ parameter $\Theta(p)$ at t_n



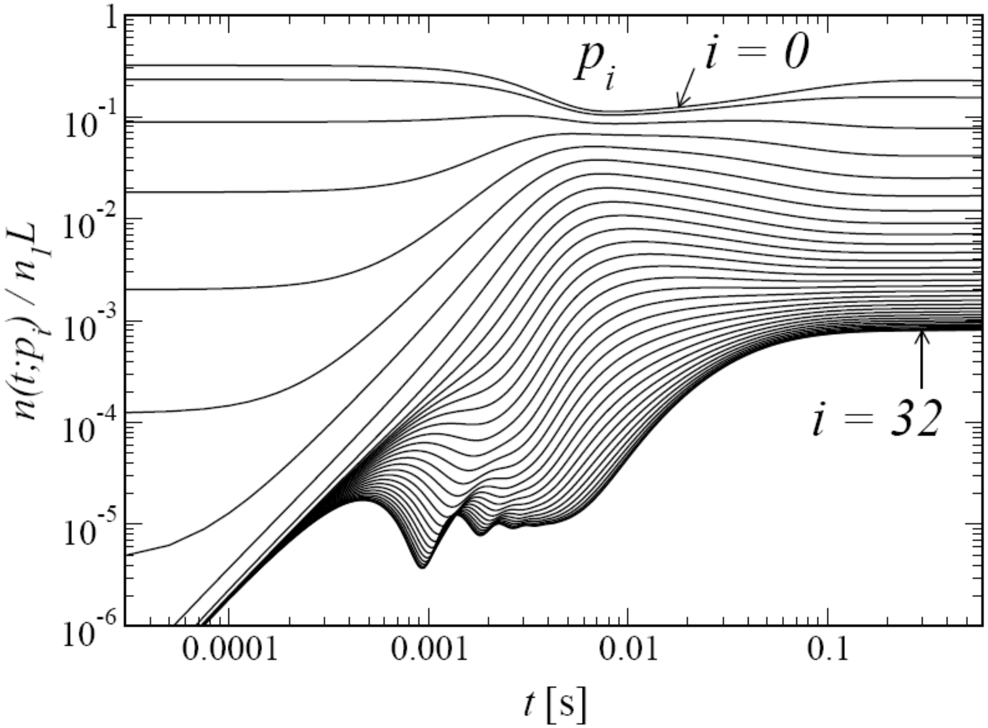
flat = temperature

[J. Berges & TG, PRA **76** (07)]

$$n(t;p) = \frac{1}{e^{\frac{1}{k_B \Theta(p)}(\frac{p^2}{2m}-\mu)} - 1}$$

Far-from-equilibrium evolution

Time evolution of mode occupation no^s:



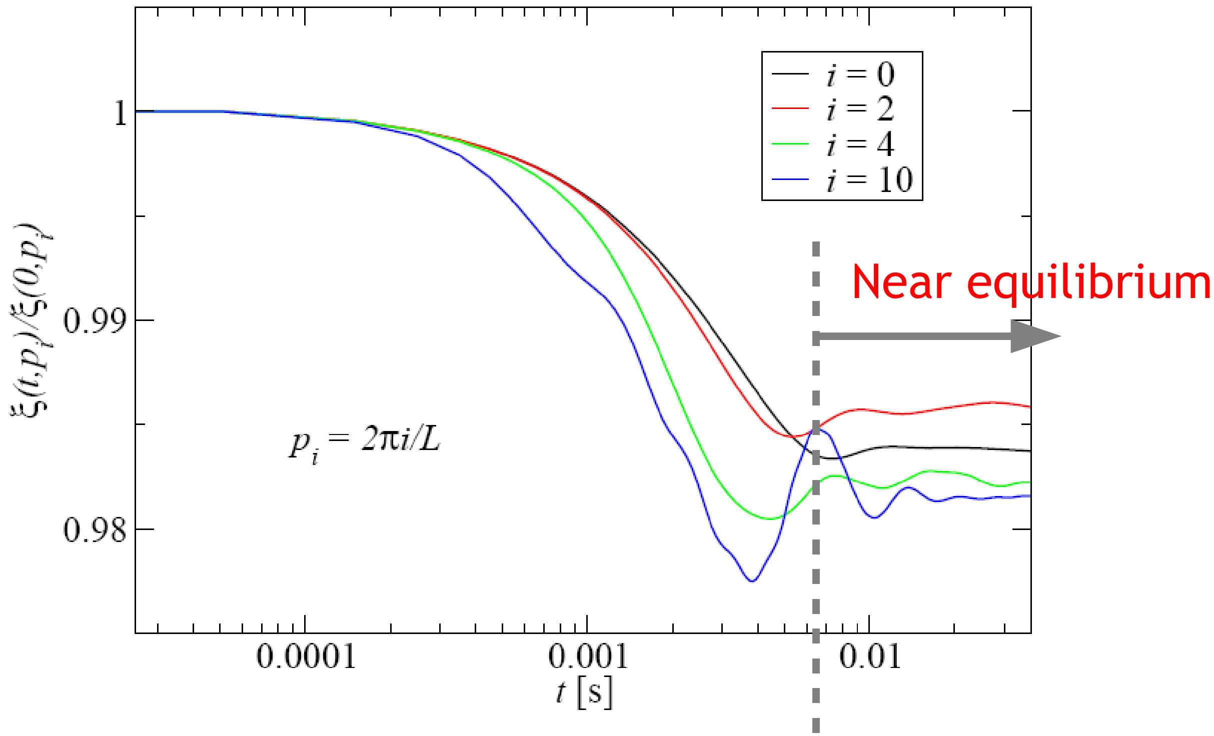
initial state:

- ^{23}Na atoms in 1D, $n_1 = 10^7\ \mathrm{m}^{-1}$
- interaction parameter $\gamma = \lambda m/(\hbar^2 n_1) = 7.5 \cdot 10^{-4}$
- Gaussian momentum distribution

[J. Berges & TG, PRA **76** (07)]

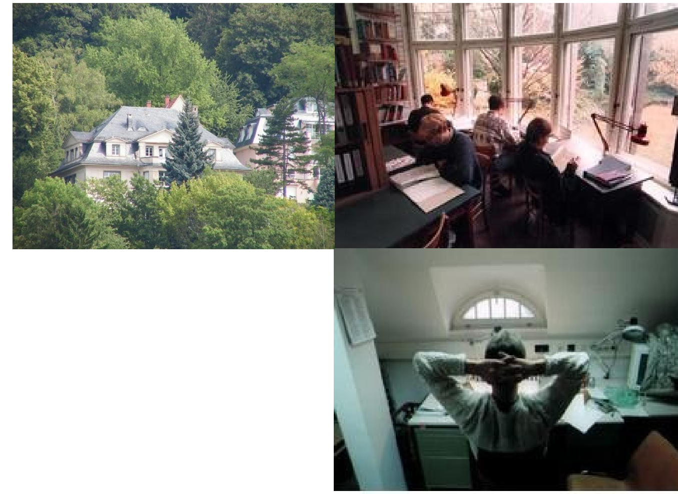
Onset of near-equilibrium evolution

Time evolution of temporal correlations $\xi(t,p) = F(t,0;p)/\rho(t,0;p)$:



(Fluctuation-Dissipation rel.: $F^{(\mathrm{eq})}_{\omega\mathbf{p}} = -i\left(n(\omega,T)+\frac{1}{2}\right)\rho^{(\mathrm{eq})}_{\omega\mathbf{p}}$)

Thanks & credits to...



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