Impure AdS/CFT

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References

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Applications and Motivation

Quantum Phase Transition:

a phase transition between different quantum phases (phases of matter at $T = 0$). Quantum phase transitions can only be accessed by varying a physical parameter — such as magnetic field or pressure — at $T = 0$.

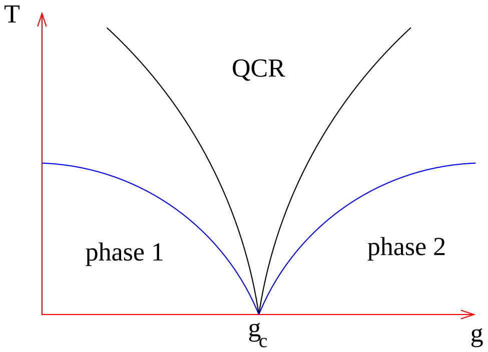


Figure: Phase diagram paradigm

Experimental relevance

Many important physical systems may have quantum critical points (QCPs). The QCP has an effective field theory description which continues to be valid at small "distances" away from the QCP. This quantum critical region may be in an experimentally accessible regime.
Examples:

- superfluid-insulator transition in thin films
- transitions between quantum Hall states
- high temperature, under-doped superconductors at $T > T_c$ and the Nernst effect

Thin Films

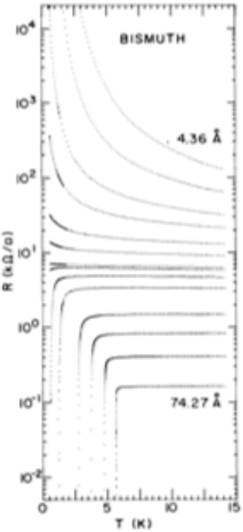


Figure: Haviland, Liu, Goldman, PRL, 62 (1989) 2180

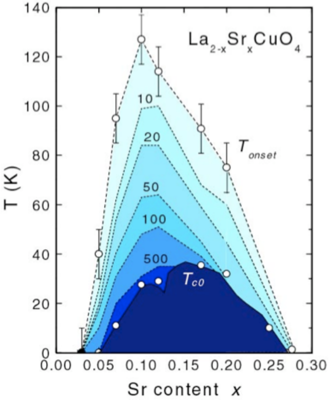
High T_c superconductors

- La_2CuO_4 is an antiferromagnetic insulator
- 2d physics: The Cu atoms arrange themselves into a square lattice on separated sheets.
- Hole doping: substitute some of the La with Sr, $La_{2-x}Sr_xCuO_4$

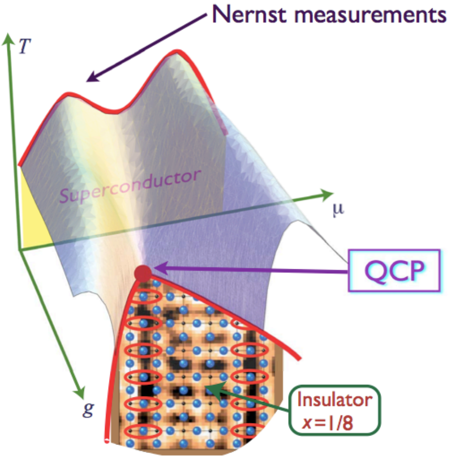
The Nernst effect

- Apply ∇T
- Apply $B \perp \nabla T$
- Measure $E \parallel B \times \nabla T$
- The Nernst coefficient is

$$\nu = \frac{E}{B|\nabla T|}$$



High T_c superconductors and quantum criticality



State of Theory

- There are many lattice models with quantum critical points — Boson-Hubbard model, quantum Ising and rotor models, etc.
- The effective field theory description of the fixed point is scale invariant.
- The field theory sometimes has a Lorentzian symmetry.

$$c \neq 3 \times 10^8 \text{ m/s}$$

- scale invariance + Lorentzian symmery $\implies$ conformal symmetry
- The description is often strongly interacting, e.g. a Wilson-Fisher fixed point

How do we analyze strongly interacting, Lorentzian conformal field theories?

The Sales Pitch

The AdS/CFT correspondence provides a tool to study a class of strongly interacting field theories with Lorentzian symmetry in d dimensions by mapping the field theories to classical gravity in $d+1$ dimensions.

- equation of state
- real time correlation functions
- transport properties — conductivities, diffusion constants, etc.

The ambitious program: There may be an example in this class of field theories which describes the quantum critical region of a real world material such as a high T_c superconductor.

The less ambitious program: By learning about this class of field theories, we may find universal features that could hold more generally for QCPs ($\eta/s = \hbar/4\pi k_B$).

Adding Dirt to AdS/CFT

Transport Coefficients

$$\begin{pmatrix} \vec{J} \\ \vec{Q} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\sigma} & \hat{\boldsymbol{\alpha}} T \\ \hat{\boldsymbol{\alpha}} T & \bar{\boldsymbol{\kappa}} T \end{pmatrix} \begin{pmatrix} \vec{E} \\ -(\vec{\nabla} T)/T \end{pmatrix}$$

Here $\vec{E}$ is electric field, T is temperature, $\vec{J}$ is charge current and the heat current $Q^\nu \equiv T^{0\nu} - \mu J^\nu$ where μ is the chemical potential.

The Nernst response is governed by

$$\vec{E} = -\boldsymbol{\theta} \vec{\nabla} T \qquad \text{where} \qquad \boldsymbol{\theta} = \boldsymbol{\sigma}^{-1} \hat{\boldsymbol{\alpha}}$$

The Nernst coefficient

$$\nu = \theta_{yx}/B \qquad (\text{Recall } \nu = E/B|\nabla T|)$$

Problematic Translation Invariance

- Imagine a material with translation invariance and a nonzero charge density.
- An electric field will accelerate the material rather than producing a steady state current.
- The dc conductivity σ is thus "infinite".
- The Nernst effect, $\nu \approx \hat{\alpha}/\sigma B$, will vanish.

Moral: Breaking translation invariance is important for modeling the Nernst effect in real world systems.

A Scattering Time

Add a weak random potential coupled to the most relevant scalar operator of dimension $\Delta_{\mathcal{O}}$:

$$\delta H = \int d^2 y \, V(y) \mathcal{O}(t, y)$$

Average over such potentials assuming

$$\langle V(x) \rangle = 0 \qquad \text{and} \qquad \langle V(x) V(y) \rangle = \bar{V}^2 \delta(x - y) \ .$$

By dimensional analysis to leading order in $\bar{V}$

$$\frac{1}{\tau_{\rm imp}} = \frac{\bar{V}^2}{T^{3-2\Delta_{\mathcal{O}}}} F\left(\frac{\rho}{T^2}, \frac{B}{T^2} \right) \ .$$

We used the AdS/CFT correspondence to calculate $1/\tau_{\rm imp}$ in a particular model — the M2 brane theory.

The Results

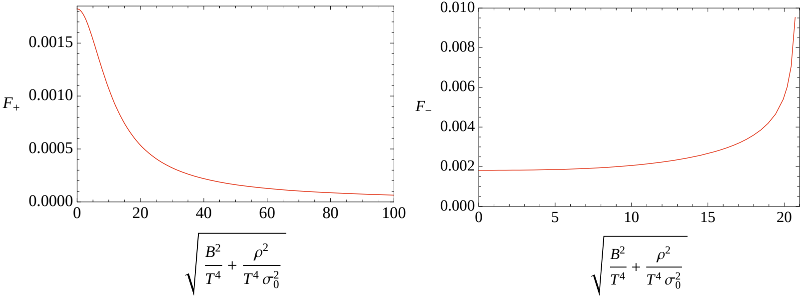


Figure: The function $F_{\pm} = T^{3-2\Delta_{\mathcal{O}}}/\bar{V}^2 \tau_{\pm}$ for the M2 brane theory for magnetic (+) and electric (-) impurities.

- The corresponding operators have the same discrete symmetries as the electric and magnetic field.
- The scattering time depends only on the above combination of the magnetic field B and charge density ρ.

An Instability in the Underlying Theory

- There is a divergence of F_- at $\sqrt{B^2 + \rho^2/\sigma_0^2} \sim 21T^2$.
- This divergence comes from an underlying instability in the translationally invariant theory.
- A pole in the retarded Green's function for the scalar operator $\mathcal{O}_-$ moves into the upper half plane.
- We have a dynamic instability without a corresponding thermodynamic instability, providing a counter-example to the Gubser-Mitra conjecture.

Observing a Cyclotron Resonance

The transport coefficients have a cyclotron resonance at $\omega = \omega_c - i\gamma - i/\tau_{\rm imp}$.

$$\sigma_+ = i\sigma_Q \frac{\omega + i/\tau_{\rm imp} + i\omega_c^2/\gamma + \omega_c}{\omega + i/\tau_{\rm imp} + i\gamma - \omega_c}$$

$$\omega_c = \frac{B\rho}{\epsilon + P} \; ; \quad \gamma = \frac{\sigma_Q B^2}{\epsilon + P} \; ; \quad \sigma_Q = \frac{(sT)^2}{(\epsilon + P)^2}\frac{1}{g^2} \; .$$

where

$$\sigma_\pm = \sigma_{xy} \pm i\sigma_{xx}$$

Originally it was thought that $\tau_{\rm imp}$ would make the resonance unobservable. However the behavior of F_+ at large ρ may change this story!

Impurities and the Nernst Effect

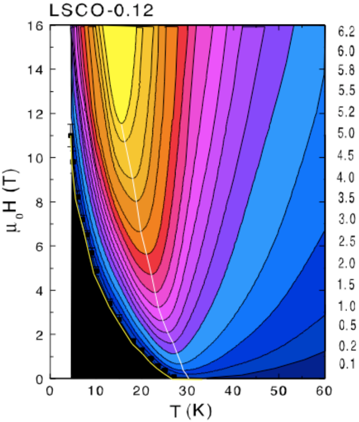
- To compare the Nernst effect with experiments, we have to add the effect of scattering from impurities, τ_{imp}

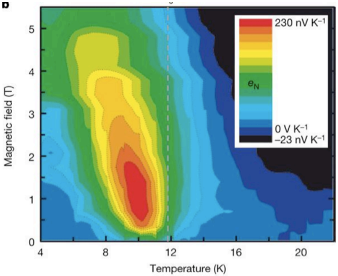
$$\nu = \frac{1}{T} \frac{1/\tau_{\mathrm{imp}}}{(\omega_c^2/\gamma + 1/\tau_{\mathrm{imp}})^2 + \omega_c^2}$$

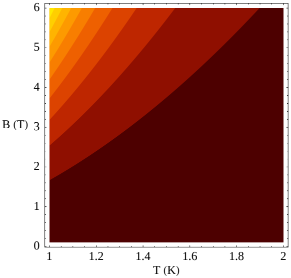
- When $\rho = 0$,

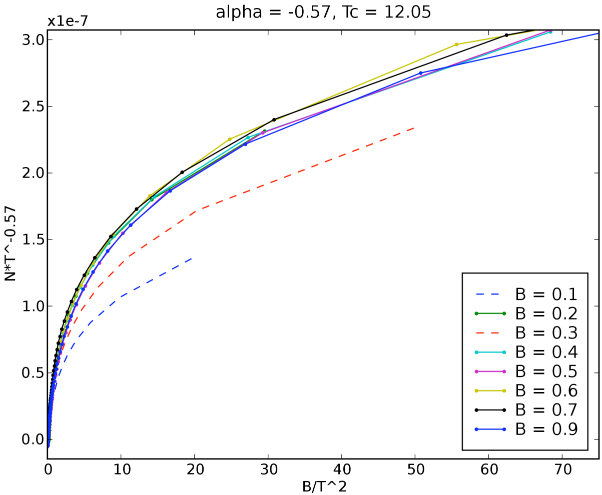
$$\nu = \frac{\tau_{\mathrm{imp}}}{T} = \frac{T^{2-2\Delta_{\mathcal{O}}}}{\bar{V}^2} F(B/T^2) \ .$$

"$\rho = 0$" experimental plots from Ong and Ardavan compared with our theoretical results.









(Plot provided by A. Ardavan.)

Warning: T here is really $T - T_c$.

Technical Details

- $1/\tau_{\rm imp}$ was computed using the memory function formalism (Götze and Wolfle, Forster, Giamarchi)
- $$\frac{1}{\tau_{\rm imp}} = \frac{\bar{V}^2}{2\chi_0} \lim_{\omega\to 0} \int \frac{d^2k}{(2\pi)^2} k^2 \frac{\operatorname{Im} G^R_{\mathcal{O}\mathcal{O}}(\omega, k)}{\omega}$$
  $$\chi_0 = \lim_{\omega\to 0} G^R_{\mathcal{P}\mathcal{P}}(\omega, 0)$$
  where $\mathcal{P} = n_i T^{0i}$.
- The scattering time reduces to a calculation of a Green's function of the operator $\mathcal{O}$ in the absence of impurities.
- This formalism resums a class of diagrams. There could be trouble in the limit $\omega^{2-\Delta_{\mathcal{O}}} \ll \bar{V}$. We believe T acts as an IR regulator and that the calculation is actually valid provided $\omega \ll T$.

Remarks and Plans for the Future

- Tried to convince you that AdS/CFT is a useful tool for studying strongly interacting field theories — equations of state, correlation functions, transport properties.
- The hope is that these field theories may be relevant for understand real world condensed matter systems.
- We saw today how to get away from a translationally invariant system and introduce impurities to AdS/CFT.
- Next on the list is getting away from the quantum critical point. Can we find supergravity solutions that correspond to deforming the effective field theory by a relevant operator?

Extra Slides

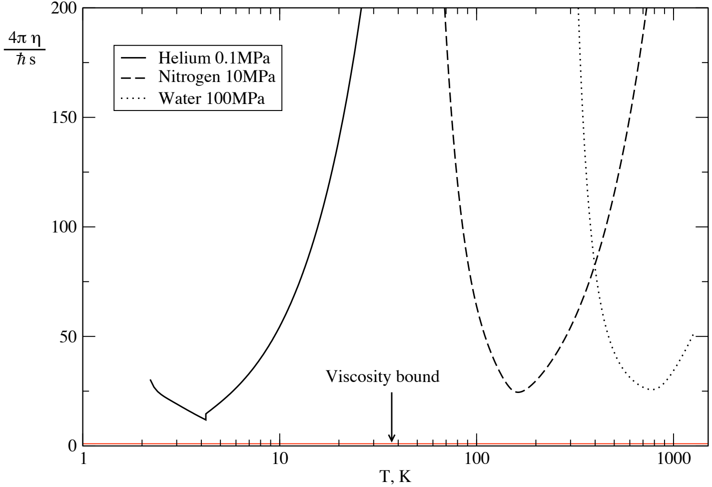


Figure: Viscosity to entropy density ratio

Current-current two-point functions at $B = \rho = 0$

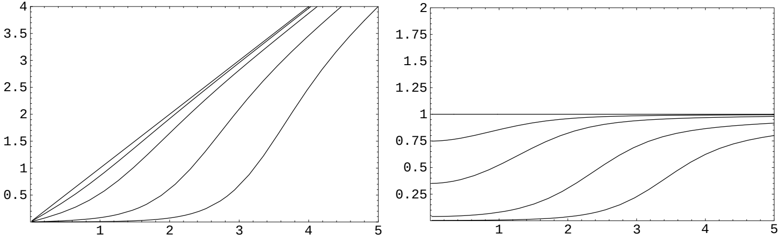


Figure: Imaginary part of the retarded function $C^{yy}(\omega, k)$, plotted in units of $(-\chi)$, as a function of dimensionless frequency $w \equiv 3\omega/(4\pi T)$, for several values of dimensionless momentum $q \equiv 3k/(4\pi T)$. Curves from left to right correspond to $q = 0, 0.5, 1.0, 2.0, 3.0$. Left: $\mathrm{Im}\, C^{yy}(w, q)$, Right: $\mathrm{Im}\, C^{yy}(w, q)/w$.

$\chi = 4\pi T/3g^2$

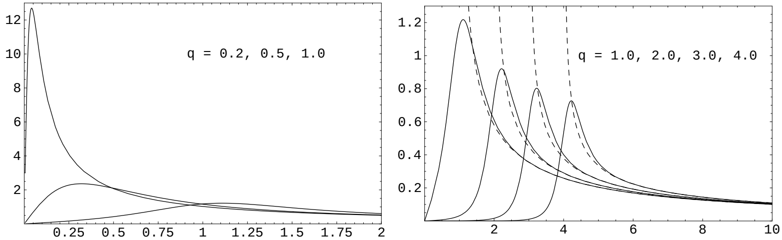


Figure: Imaginary part of the retarded function $C^{tt}(w,q)/q^2$, plotted in units of $(-\chi)$, as a function of dimensionless frequency $w \equiv 3\omega/(4\pi T)$, for several values of dimensionless momentum $q \equiv 3k/(4\pi T)$. Curves from left to right correspond to $q = 0.2, 0.5, 1.0$ (left panel), and $q = 1.0, 2.0, 3.0, 4.0$ (right panel). The dashed curves are plots of $1/\sqrt{w^2 - q^2}$.

$\chi = 4\pi T/3g^2$

small q: hydrodynamic peak at $w \sim q^2$

large q: collisionless peak at $w \sim q$

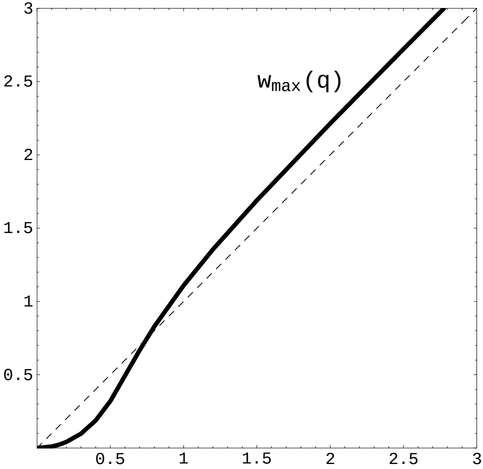
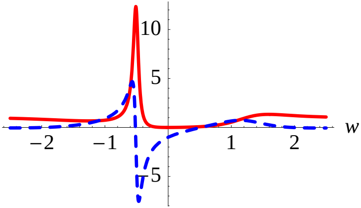
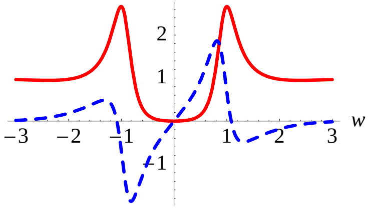


Figure: The position of the peak of the spectral function. The dashed line is $w = q$.

The cyclotron resonance



a)



b)

Figure: The dashed blue line is the $\mathrm{Im}(\sigma_+)$ while the solid red line is the $\mathrm{Re}(\sigma_+)$ as a function of w: a) $h = q = 1/\sqrt{2}$, b) $h = 1$ and $q = 0$.

Dyonic blackhole thermodynamics

$$T = \frac{\alpha(3 - h^2 - q^2)}{4\pi} .$$

$$B = h\alpha^2 , \quad m = -\frac{h\alpha}{g^2} , \quad \rho = -\frac{q\alpha^2}{g^2} , \quad \text{and} \quad \mu = -q\alpha \ .$$

$$s = \frac{\pi\alpha^2}{g^2} \ , \quad \epsilon = \frac{\alpha^3}{g^2}\frac{1}{2}(1 + h^2 + q^2) \ , \quad \text{and} \quad P = \epsilon/2 + mB \ .$$

$$\mathcal{P} = \langle T_{aa} \rangle = \epsilon/2 \ .$$

$$\frac{1}{g^2} = \frac{\sqrt{2}N^{3/2}}{6\pi} .$$