Gauge/String Duality

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Gauge/String duality or AdS/CFT

Solvable Toy Model(s) of
non-equilibrium
strong coupling dynamics.

"Finite temperature field theory
= Gravity with Black Hole"

Toy model needed, since:

Strong Coupling prevents perturbation theory from being applicable.

Real Time dynamics challenging for the lattice

But: Low energy effective theory works!

Effective Theory: hydrodynamics

We are looking for a description that is valid for:

long wavelength:

$$\lambda \gg l_{micr.}$$

Expand in powers of derivatives.

$l_{micr.} = 1/(g^2\ T)$ at weak coupling,

$l_{micr.} = 1/T$ at strong coupling!

Effective theory is more effective at strong coupling!

Hydrodynamics for global currents

Conserved charge density:

$$\rho = J_0$$

Conservation law:

$$\partial_\mu J^\mu = 0$$

Constitutive relation:

Diffusion Constant

$$J_i = D\, \partial_i \rho + \ldots$$

Matching: Kubo Formulas.

(from Arnold, Moore & Yaffe)

$$\eta = \frac{1}{20} \lim_{\omega\to 0} \frac{1}{\omega} \int d^4x \, e^{i\omega t} \, \langle [\pi_{lm}(t,\mathbf{x}), \, \pi_{lm}(0)] \rangle_{\rm eq} \, ,$$

$$\zeta = \frac{1}{2} \lim_{\omega\to 0} \frac{1}{\omega} \int d^4x \, e^{i\omega t} \, \langle [\mathscr{P}(t,\mathbf{x}), \, \mathscr{P}(0)] \rangle_{\rm eq} \, ,$$

$$\sigma = \frac{1}{6} \lim_{\omega\to 0} \frac{1}{\omega} \int d^4x \, e^{i\omega t} \, \langle [j_i^{\rm EM}(t,\mathbf{x}), \, j_i^{\rm EM}(0)] \rangle_{\rm eq} \, ,$$

$$D_{\alpha\beta} = \frac{1}{6} \lim_{\omega\to 0} \frac{1}{\omega} \int d^4x \, e^{i\omega t} \, \langle [j_i^{\alpha}(t,\mathbf{x}), \, j_i^{\gamma}(0)] \rangle_{\rm eq} \, \Xi^{-1}_{\gamma\beta} \, .$$

Diffusion constant

Current-current 2-pt function.

Need correlation functions in microscopic theory.

Do we need a toy model?

Hydrodynamics determines the non-equilibrium dynamics up to a few transport coefficients which in principle are determined by matching to microscopic physics but in practice have to be taken from experiment.

What good does it do to have a toy model where one can do that matching?

Solvable Toy model can

- test hydrodynamic formalism (you don't need AdS/CFT but it surely helps): 2nd order hydro, (2+1)d magento-hydro-dynamics
- give quantitative guidance (what are typical/limiting values?): η/s, Nernst, relaxation time
- indentify new dynamical mechanisms at strong coupling: energy loss, jets

Outline:

* Refinements of Hydro formalism
* Quantitative Guidance:
  Examples and Limitations
* AdS/CFT Basics
  How were these results obtained?
* New Dynamics at strong coupling
  (Recent results with Chesler and Jensen)

New refinements of the hydrodynamic formalism

Could have been obtained (and, knowing the answer, can be rederived) without any reference to AdS/CFT.

- 2^{nd} order hydro
- 2+1 d hydro in background magnetic field

Second Order Hydrodynamics:

Navier-Stokes equations are 0^{th} and 1^{st} order terms in derivative expansion for T_{ij} (that is: ideal fluid and viscous terms)

What is the structure of 2^{nd} order terms?

Enumerate all terms consistent with symmetry!

Special Case: 2^{nd} order conformal hydro

↑

(good approximation for RHIC fireball)

Muller-Israel-Stewart Theory:

Problem with standard 1st oder hydro:

$$\omega = Dk^2 \qquad\qquad v = \frac{d\omega}{dk} = Dk \xrightarrow{k\to\infty} \infty$$

Acausality at short distances (outside validity of hydro) is a big concern in numerical simulations.

MIS adds one particular 2nd order term to get finite v at short distances.

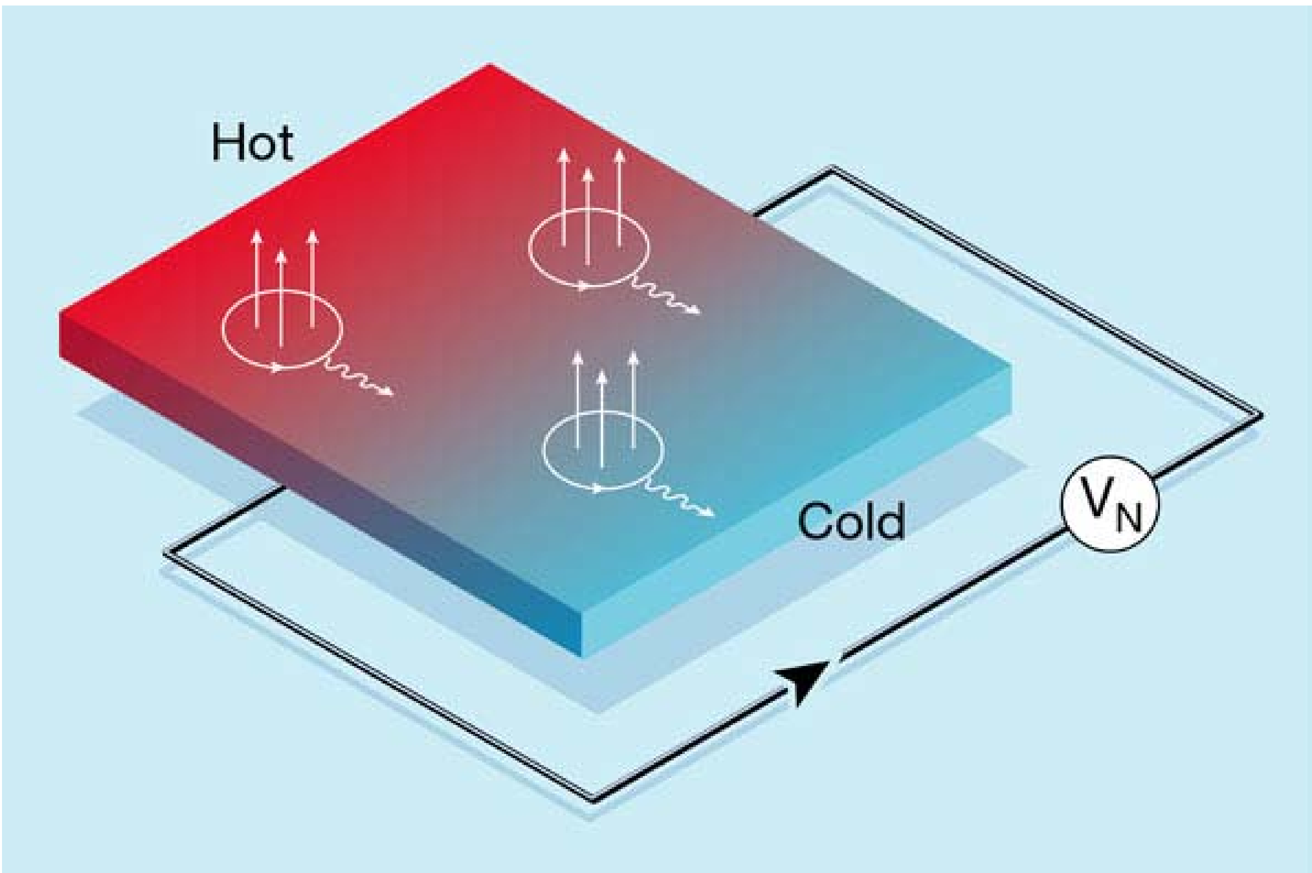
Second order conformal hydro:

MIS term is inconsistent with conformal invariance.

Conformal invariance allows 5 different 2nd order terms (not MIS). They have been first calculated in a particular AdS/CFT example. They are now used in (some) hydrodynamic simulations for RHIC.

(Baier, Romatschke, Son, Starinets, Stephanov;
Bhattacharyya, Hubeny, Minwalla, Rangamani)

The Nernst effect.



(image from Lee, Nature 2000)

Temperature gradient drives electric current

Nernst effect and hydrodynamics

Family of related transport coefficients:

(Hartnoll, Herzog; HKMS)

$$\begin{pmatrix} \vec{J} \\ \vec{Q} \end{pmatrix} = \begin{pmatrix} \sigma & \alpha \\ \alpha T & \bar{\kappa} \end{pmatrix} \begin{pmatrix} \vec{E} \\ -\vec{\nabla} T \end{pmatrix}$$

Claim: α and κ (as cpx. functions of ω) are completely determined in terms of a single function $\sigma(\omega)$

Nernst Effect from Hydro.

The relations between electrical conductivity, heat conductivity and Nernst coefficient have been obtained in a particular AdS/CFT realization.

They have been argued to hold generally as a consequence of Ward identities.

In addition AdS/CFT provides in a particular example the full functional form of σ. Useful?

Quantitative Guidance

Examples and Limitations of extracting quantitative data from the toy model.

Viscosity of N=4 plasma.

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

(Policastro, Son, Starinets)

Calculated either by directly plugging into Kubo formula using the AdS/CFT recipe for 2-pt-functions or by looking at damped quasi-normal modes and compare to attenuation of sound wave.

Viscosity of N=4 plasma:

Can equivalently be phrased as graviton absorption cross section by AdS black hole.

Suggests bound:

(Kovtun, Son, Starinets)

$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$

"Most perfect fluid"

Benchmark for RHIC.

All known substances within a factor of 10.

Universal strong coupling results

The viscosity to entropy ratio is so useful since at strong coupling it is independent of the coupling, dimension and similar details.

Similar universality has been reported for:

Current correlators in 2+1 d (HKSS)
Relaxation Time of certain non-hydro modes (HKKKY, Herzog)
Ratio of central charges defined from TT or s

The coupling constant in AdS/CFT:

What is the role of the coupling constant λ?

All tractable examples of gauge/gravity duality come with an intrinsic large dimensionless parameter.

- a marginal coupling constant (λ in N=4 SYM)
- ratio of flux tube tension to mass gap (~1 in QCD)
- and/or large number of colors N

The role of the coupling constant.

What is common to all those theories is that their dual is Einstein-Hilbert gravity (that is action with up to 2-derivatives only)

Scale of UV completion much larger than typical energies (which are ~ 1/curvature radius)

The role of the coupling constant

e.g. in N=4 SYM:

Flux tube tension: $$M_s \sim \lambda^{1/4} E$$

Planck scale: $$M_{Pl} \sim N^{2/3} E$$

Weakly coupled strings: $M_s < M_{pl}$

A new universality class?

The “universal” results are the ones that give the same answer in all theories whose dual is classical gravity coupled to matter.

P. Kovtun: “Universality without Symmetry”

Non-Universal Quantities:

Even in all theories with a gravity dual there are of course many quantities that are specific to any one of those.

Simple examples: Quantites that depend on N or λ even at large N and λ.

e.g: s, η (scale as N^2)
quark drag, jet quenching (scale as $\sqrt{\lambda}$)

The perfect dual?

For a given plasma one wants to model
(say QCD around the critical temperature),
is there a “perfect” dual for which one would
trust the answers quantitatively?

How to judge accuracy of a given model, size of errors?

(work with Bak and Yaffe)

Weak/weak comparison:

(Chesler, Moore, Yaffe, …)

Calculations comparing weakly coupled N=4 to weakly coupled QCD suggest that the two agree in their hydro if parameters are dialed so that the long distance correlation length of equilibrium fluctuations agree!

$$m_{gap}, \qquad m_{Debye}$$

$(\sim g^2\, T)$ $(\sim gT)$

The perfect dual.

This means that as long as you can calculate the long distance fall-off of equilibrium correlators in your favorite plasma (e.g. using the lattice for QCD) you can judge the quality of any holographic model and systematically look for a "perfect" one.

N=4 at infinite coupling:

$$m_{gap}, \qquad m_{Debye}$$

magnetic mass
falloff of generic correlator

electric mass
falloff of CT odd correlator

(Arnold and Yaffe)

QCD:	$\sim 4T$	$\sim 6T \sim 1.5\ m_{gap}$
N=4:	$\sim 7T$	$\sim 11T \sim 1.5\ m_{gap}$

Other holographic plasmas:

Suggests strategy to systematically look for a gauge theory with gravity dual that can serve as a good model for QCD.

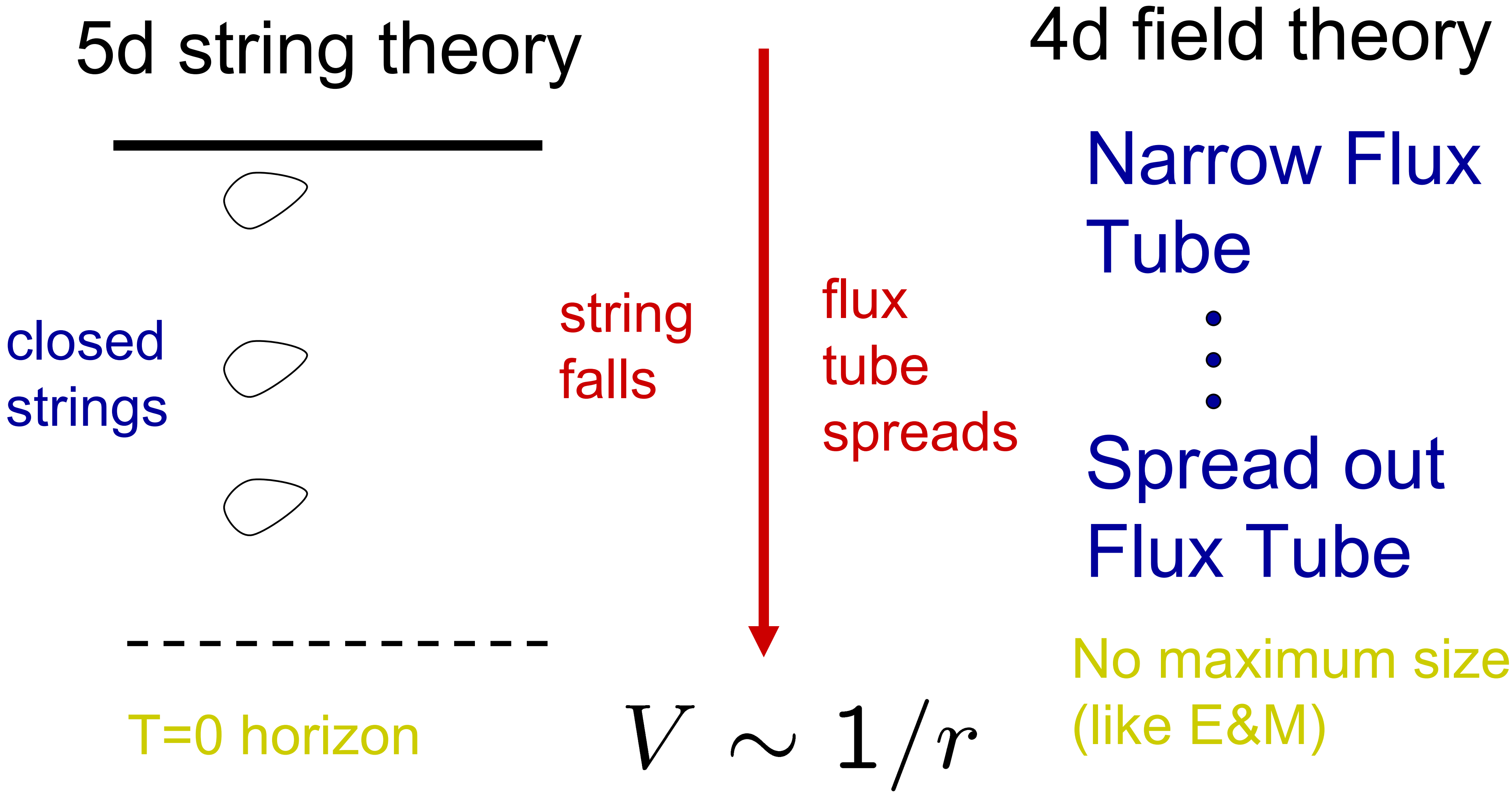
Witten black hole: 6 T and 10 T

How about KS? $N=2^*$? Numerical black hole solutions worked out recently. (Buchel, …)

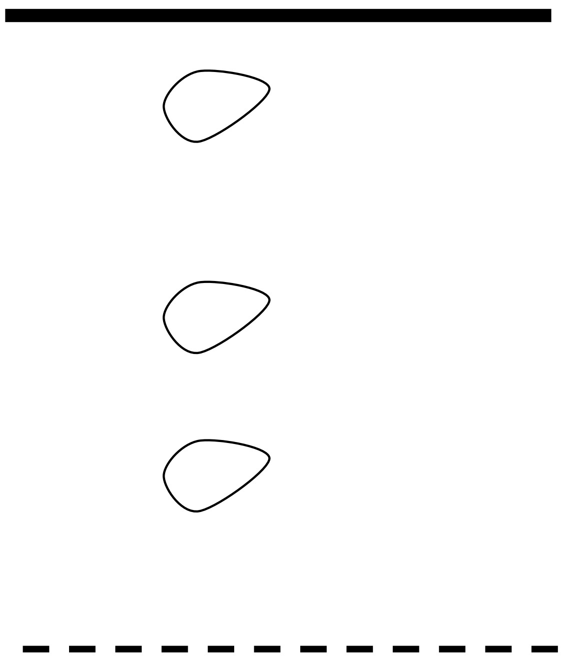
AdS/CFT Crash Course

How to obtain these results?

AdS/CFT


5d string theory
4d field theory
closed strings
string falls
flux tube spreads
Narrow Flux Tube
Spread out Flux Tube
T=0 horizon
$V \sim 1/r$
No maximum size (like E&M)

AdS/CFT at finite temperature



T $\neq$ 0 horizon

= Black Hole

- Conformal theory at finite T or confining theory at T>Λ
- Falling = Thermalizing.
- Thermal field theory =black hole thermo!

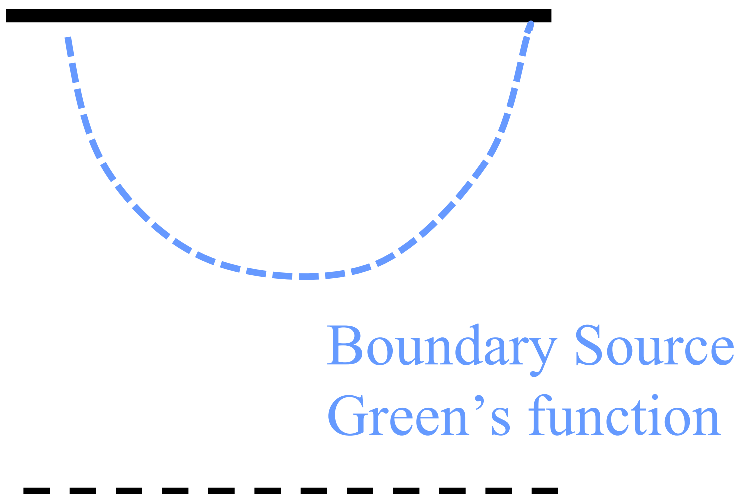
Correlation Functions:

The basic object to calculate in AdS/CFT are correlation functions.

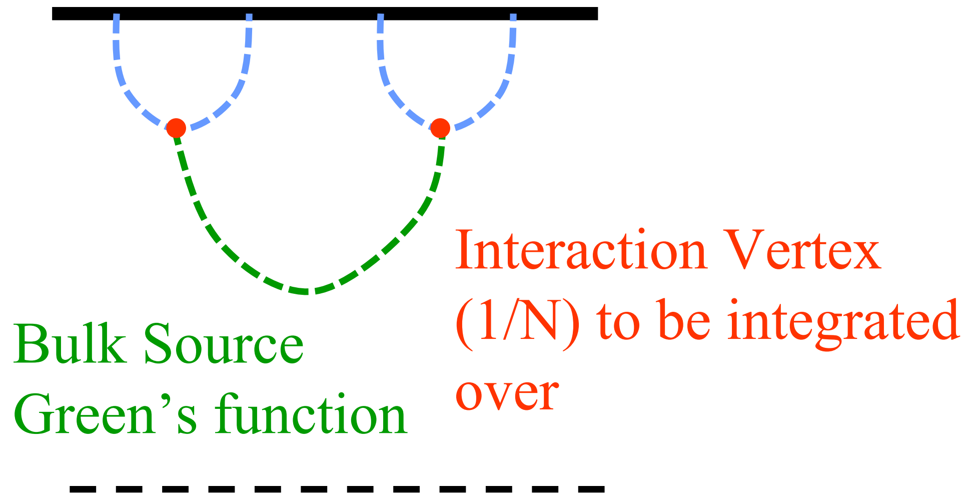
on shell action = generating functional

boundary value of field = source for the dual operator

Witten Diagrams:



$\langle OO \rangle$

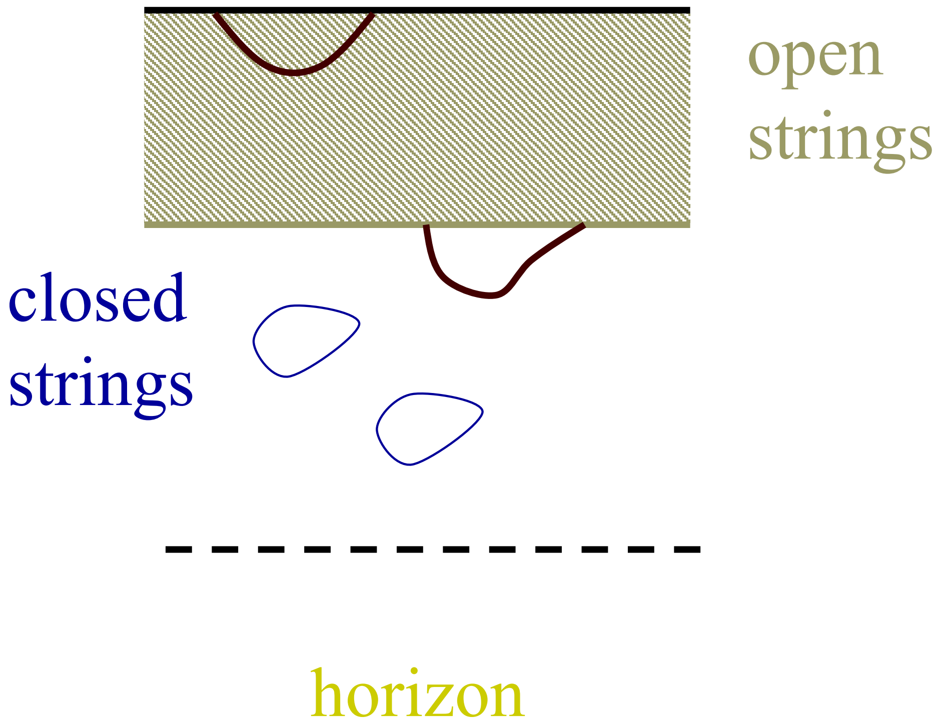


$\langle OOOO \rangle$

No loops! $1/N^2$ suppressed (classical gravity)

With additional bulk sources can also do 1-pt functions

Adding Flavor to AdS/CFT



(Karch and Katz)

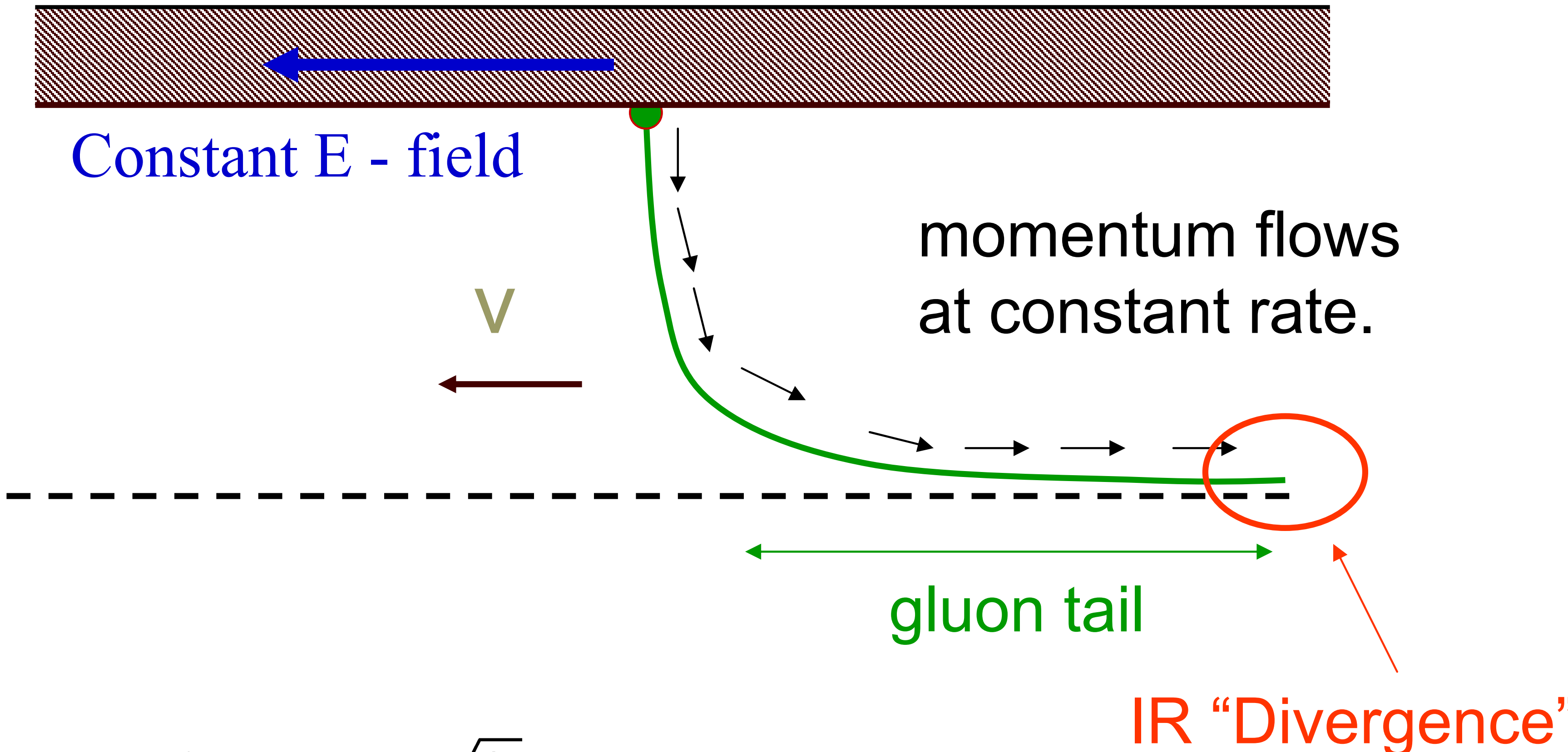
- Fundamental quarks can be simply incorporated into any background using flavor branes (D7 branes).
- Flavor brane terminates at a position determined by its mass.
- Open strings = Mesons
- Baryons = soliton.

New dynamical mechanisms at strong coupling

Example: Jets in N=4 SYM. Recent work with Paul Chesler and Kristan Jensen.

Heavy quark at finite T:

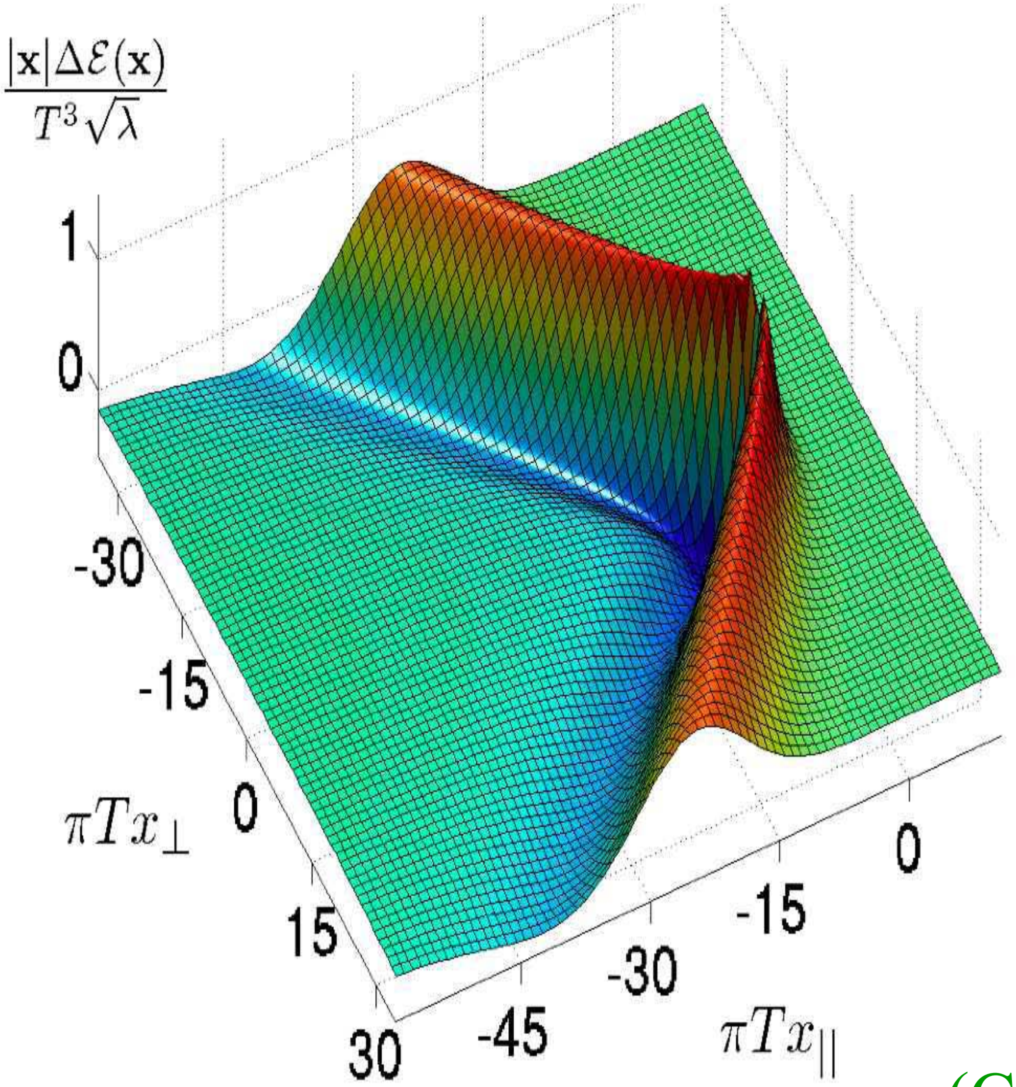
(HKKKY, Gubser)

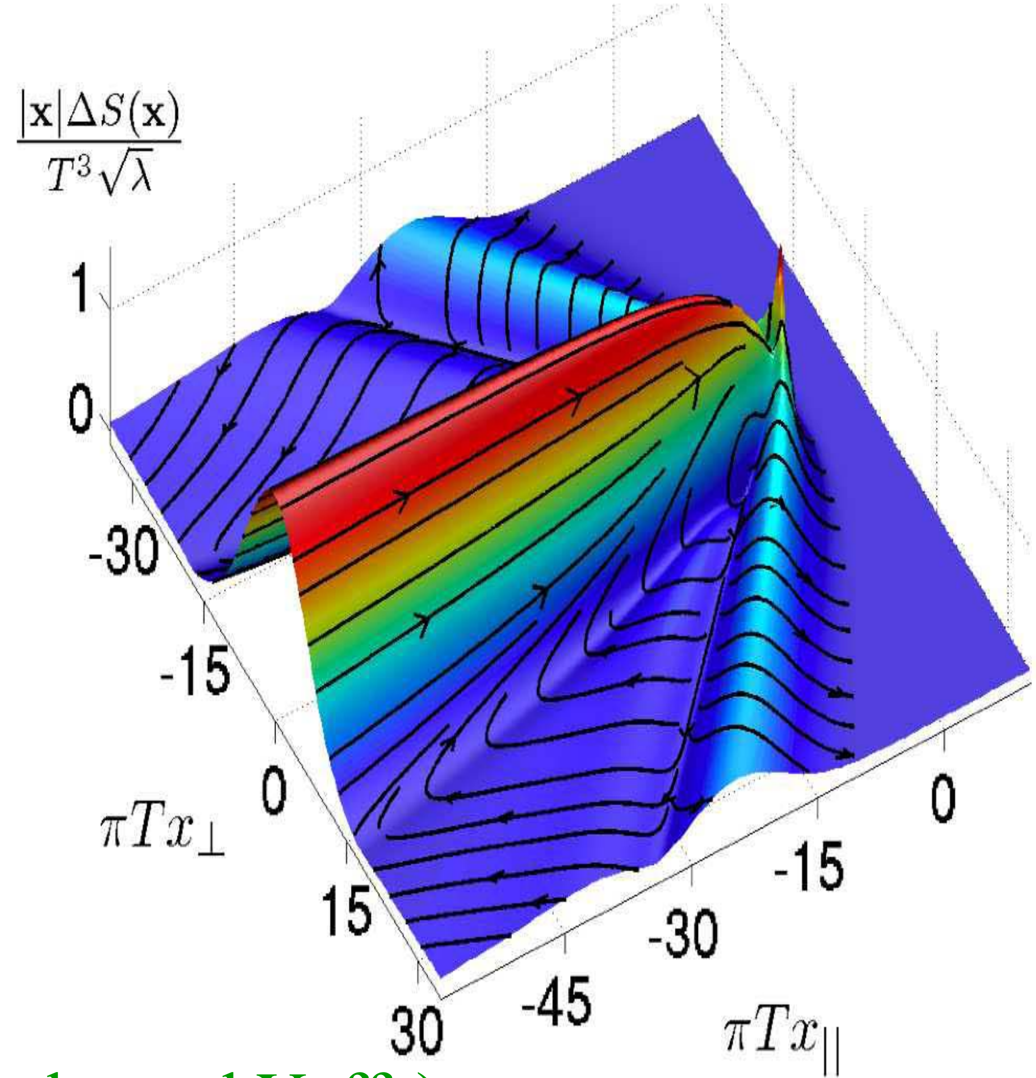


Loss rate: $$\frac{dP}{dt} = -\frac{\sqrt{\lambda}}{2\pi}\frac{v}{\sqrt{1-v^2}}(\pi T)^2$$

rate at which the external field does work.

Energy and Momentum Density





(Chesler and Yaffe)

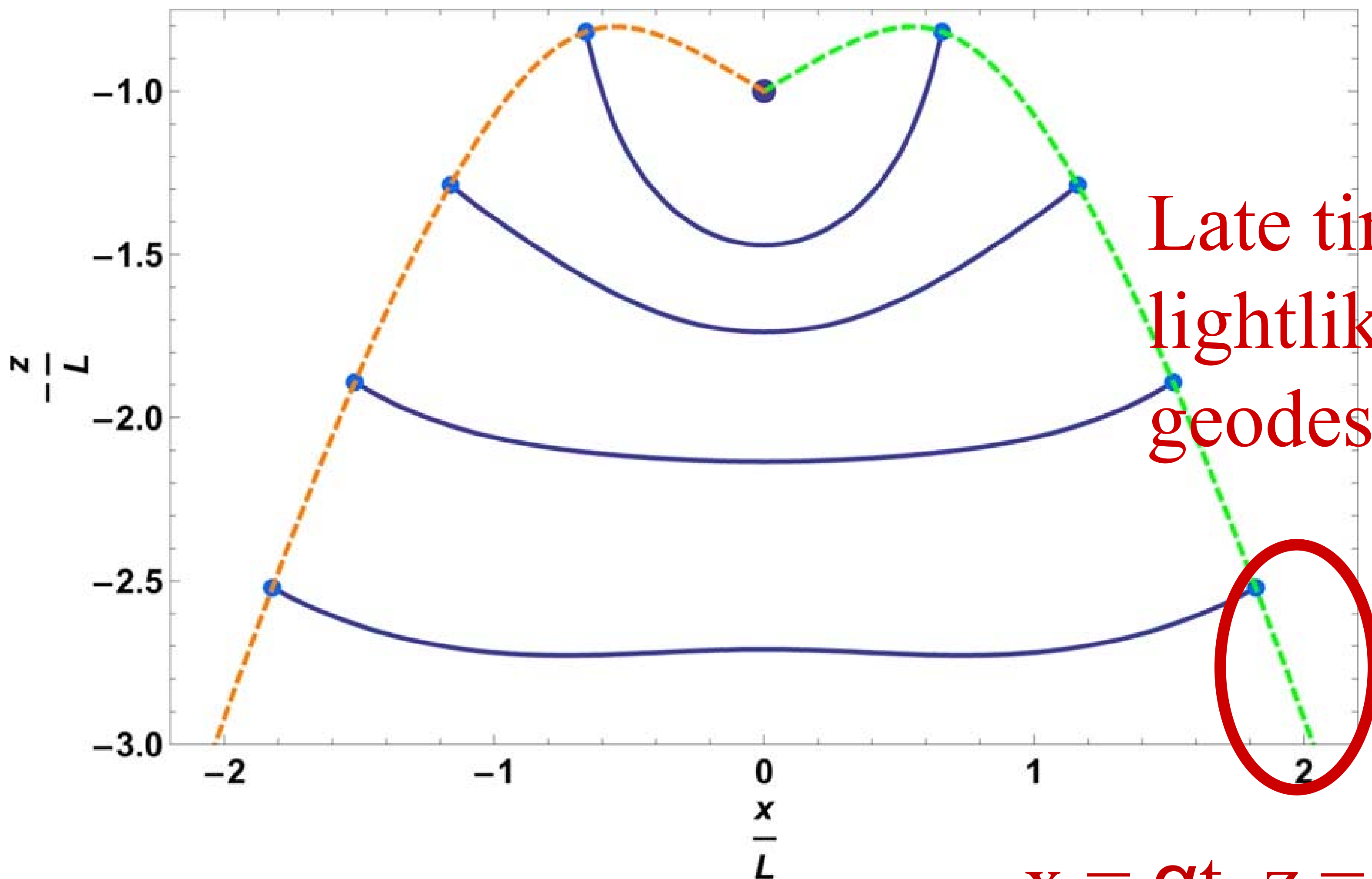
Energy Loss Mechanism

What is the dominant effect for energy loss?
Single gluon Bremsstrahlung (weak coupling)?
Glueball Emission (certainly not at large N)?

At strong coupling: Coherent gluon emission.

How about light flavor? Jets at zero T?

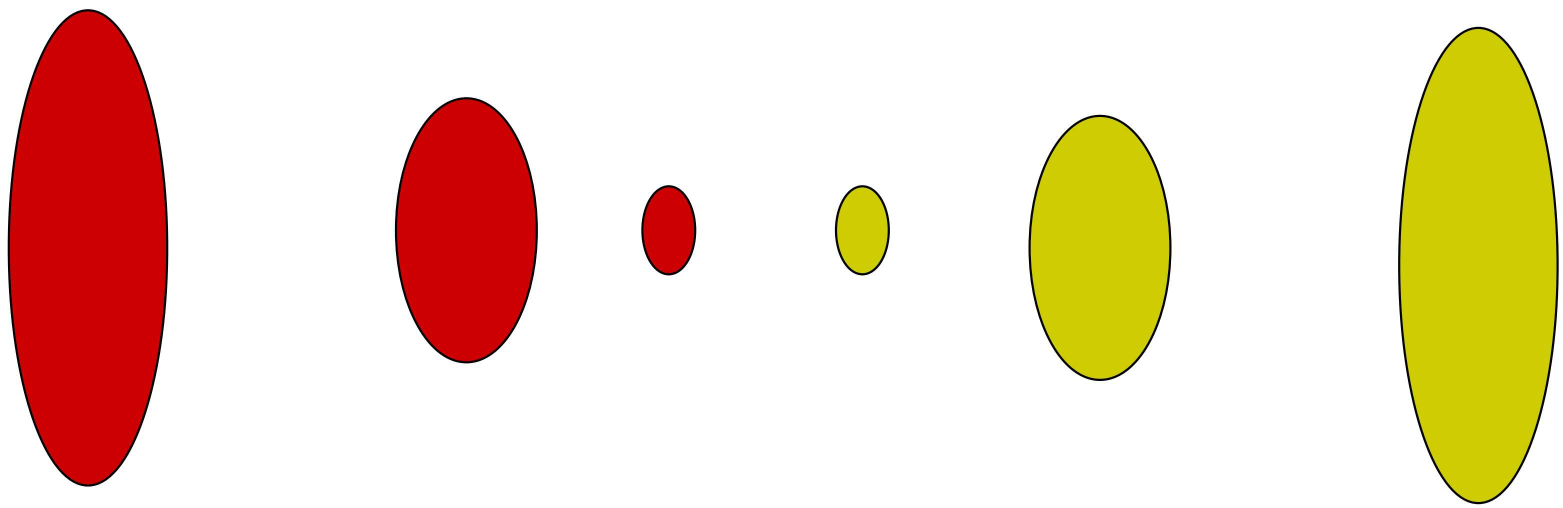
Falling String at zero temperature:



Late times: lightlike geodesic

$x = \alpha t,\ z = \beta t$

Holographic Image (zero T):



Two “blobs” of energy density / charge density rushing apart and expanding.

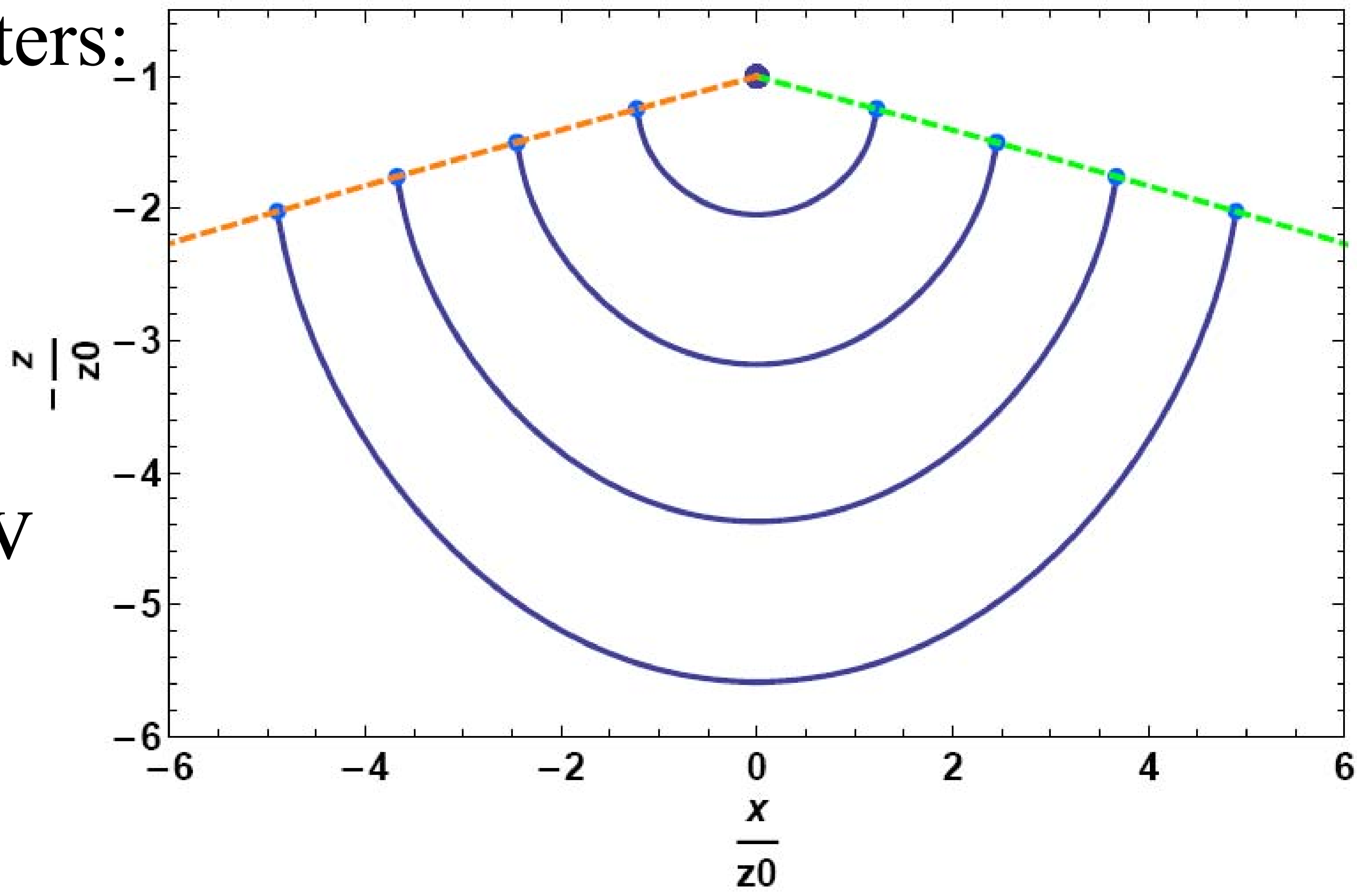
Universal endpoint behavior:

Two parameters:

Energy
+ angle (v)

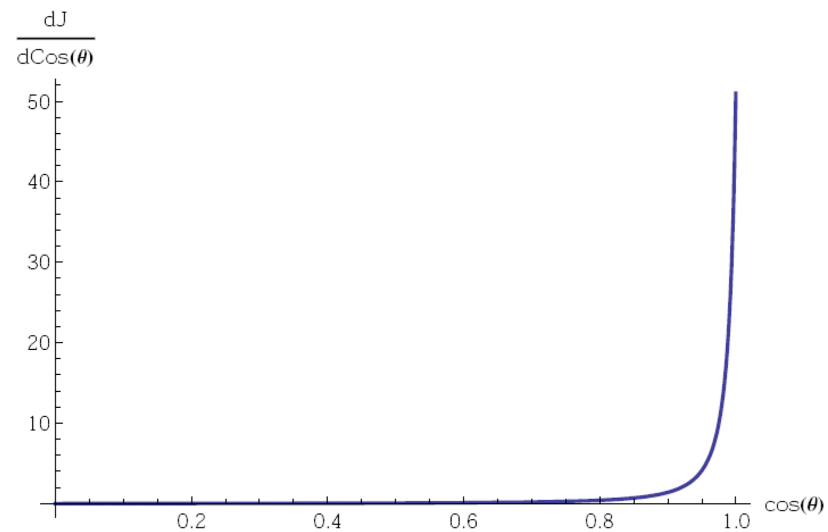
E/p ~ 1 / v

"jetmass"



Baryon Number Jet function

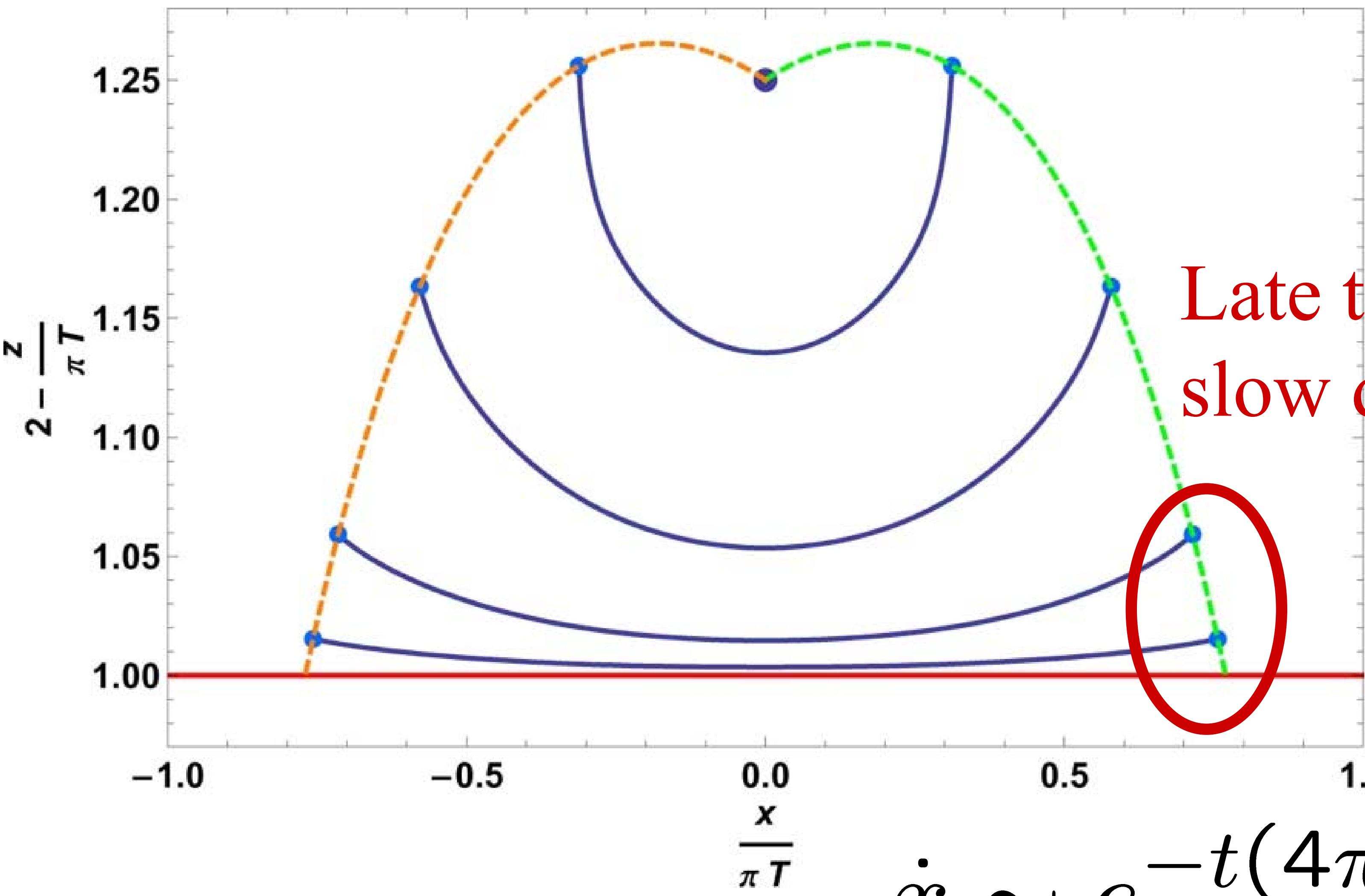
Baryon Number density as a function of angle in the cone.



for geodesic

$$\rho \propto 1/(1 - \mathrm{v}\cos(\theta))^2$$

Jets = Falling Strings (finite T)



Late time slow down

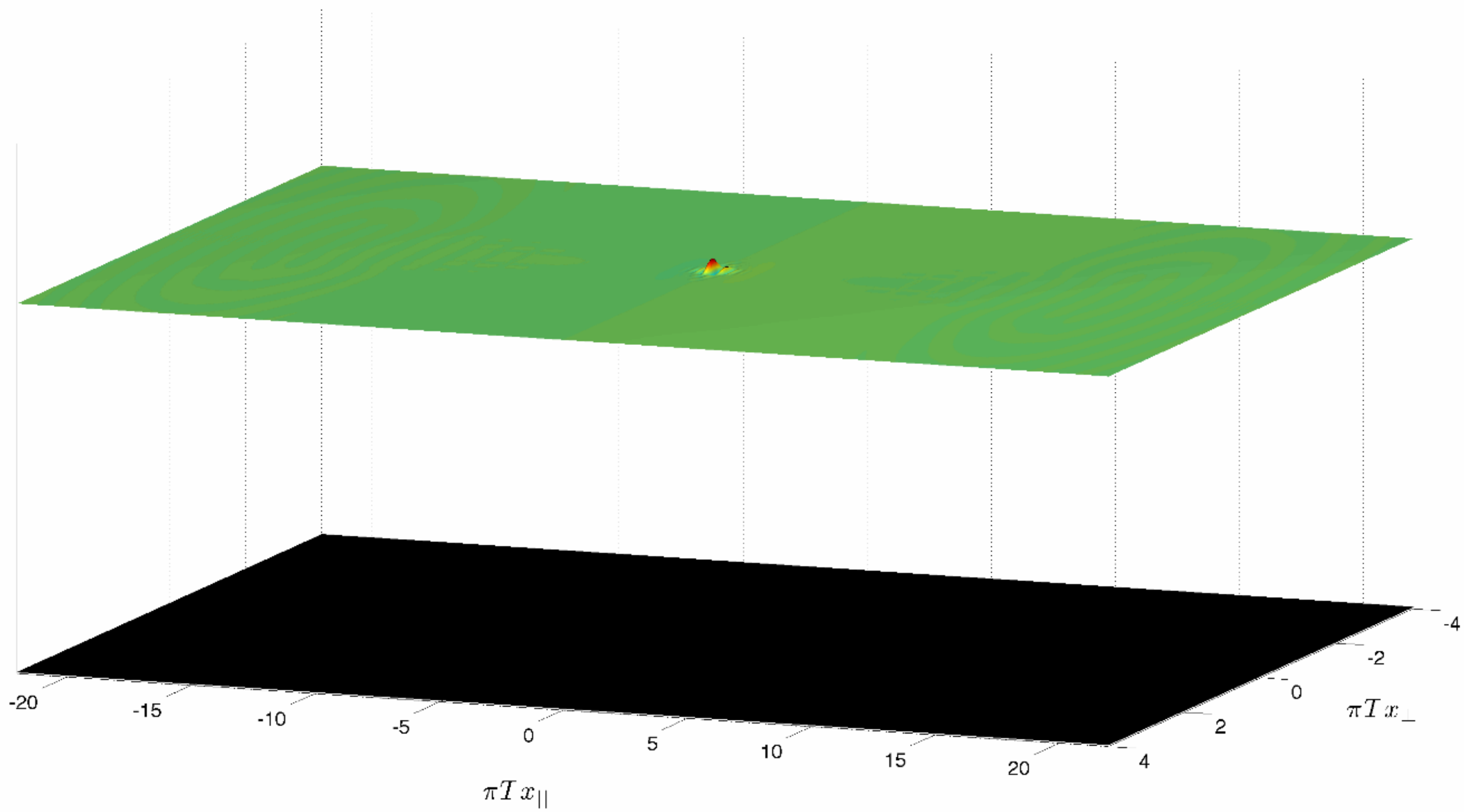
$$\dot{x} \sim e^{-t(4\pi T)}$$

Jets at finite T

At early times: like T = 0 jets as long as E >> T

At late times: diffusion + damping of non-hydro modes. Observed late time behavior agrees with time scales from quasinormal modes!

Jet in N=4 SYM:



Jet Dynamics:

- At weak coupling: emission of several individual gluons
- In Pythia: Flux Tubes breaking
- In strongly coupled N=4 SYM: coherent emission of soft radiation.
- Is this a good model for jets at LHC? Does substructure of jets help to distinguish new physics from QCD jets?

Conclusions:

Gauge/gravity duality is a great toy model to study non-equilibrium phenomena at strong coupling.