Cosmological phase transitions

Mikko Laine

(University of Bielefeld, Germany)

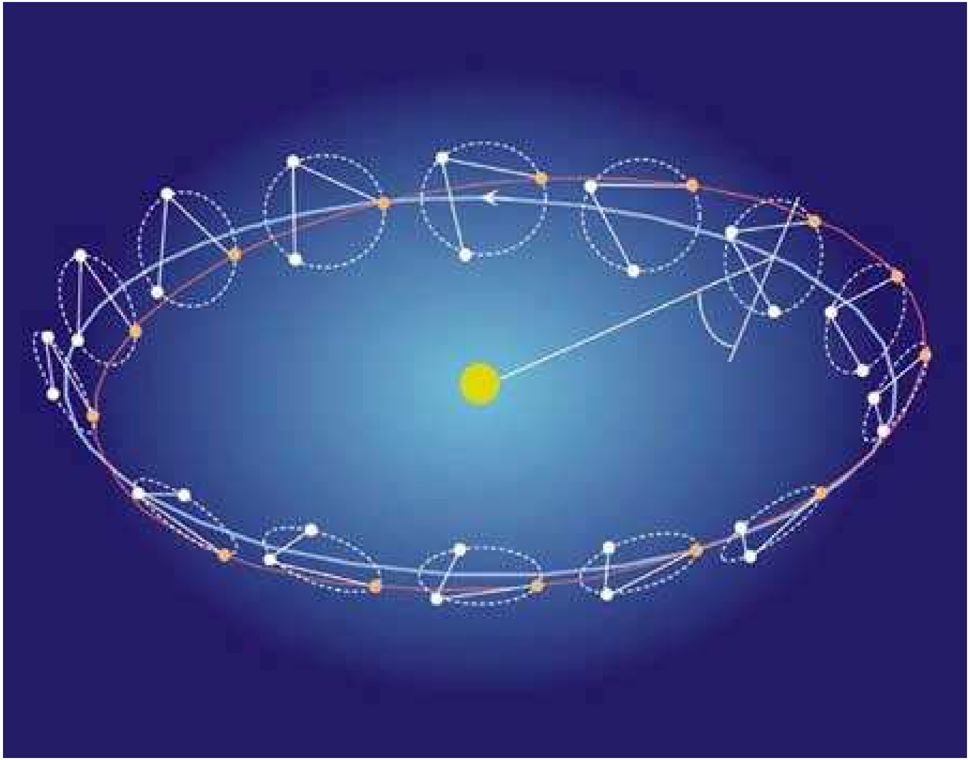
Why the topic?

JB: We would like to invite you to give ... a review on "cosmological phase transitions".

ML: I'd be happy to give a talk — however, if possible, I'd prefer to talk about something else than cosmological phase transitions, since there hasn't been much news on that since many years.

JB: A major reason why we need to review cosmological phase transitions is that the cosmologists recently got very interested in the electroweak transition because of the possibility of gravity wave production and possible signals for the LISA experiment.

Laser Interferometer Space Antenna [http://lisa.nasa.gov/]



Joint ESA/NASA mission, in operation in $\geq$ 2018?

Why is a space-based interferometer good here?

Horizon radius of electroweak epoch ($T \sim 100$ GeV) corresponds to 1 AU today; subhorizon physics leads to shorter wavelengths. This matches $f_{\text{LISA}} \sim 10^{-4} \ldots 10^{-2}$ Hz.

$\Rightarrow$ Direct experimental information about the cosmological electroweak phase transition?

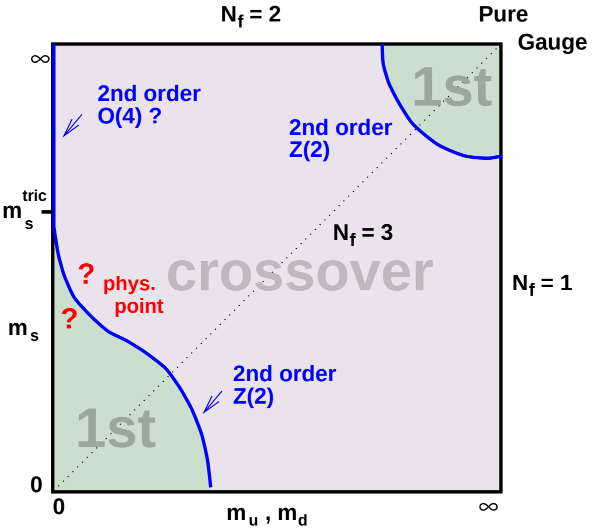
Recent work: Nicolis, *Relic gravitational waves from colliding bubbles and cosmic turbulence*, gr-qc/0303084; Grojean, Servant, *Gravitational Waves from Phase Transitions at the Electroweak Scale and Beyond*, hep-ph/0607107; Randall, Servant, *Gravitational waves from warped spacetime*, hep-ph/0607158; Huber, Konstandin, *Production of Gravitational Waves in the nMSSM*, 0709.2091; Delaunay, Grojean, Wells, *Dynamics of Non-renormalizable Electroweak Symmetry Breaking*, 0711.2511; Caprini, Durrer, Servant, *Gravitational wave generation from bubble collisions in first-order phase transitions: an analytic approach*, 0711.2593; . . . .

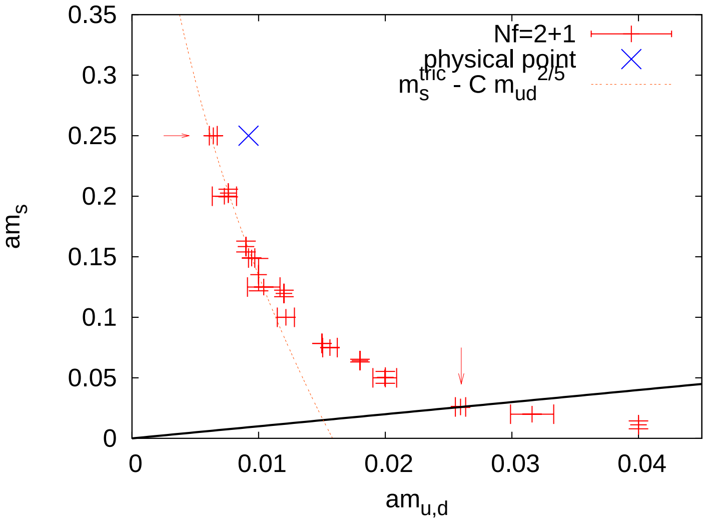
But first “cosm. phase transitions” more generally:

Thermal transition	Cosmological relic?
QCD crossover	⋆ imprint on gravity background ⋆ imprint on dark matter
EW in SM	⋆ imprint on baryon asymmetry
EW in MSSM and more exotic theories	⋆ gravitational background ⋆ baryon asymmetry (⋆ primordial magnetic fields)
(ISS model hep-th/0602239,...	⋆ SUSY breaking) hep-th/0610334,...
(GUT, . . .	⋆ topological defects)

QCD

Phase diagram



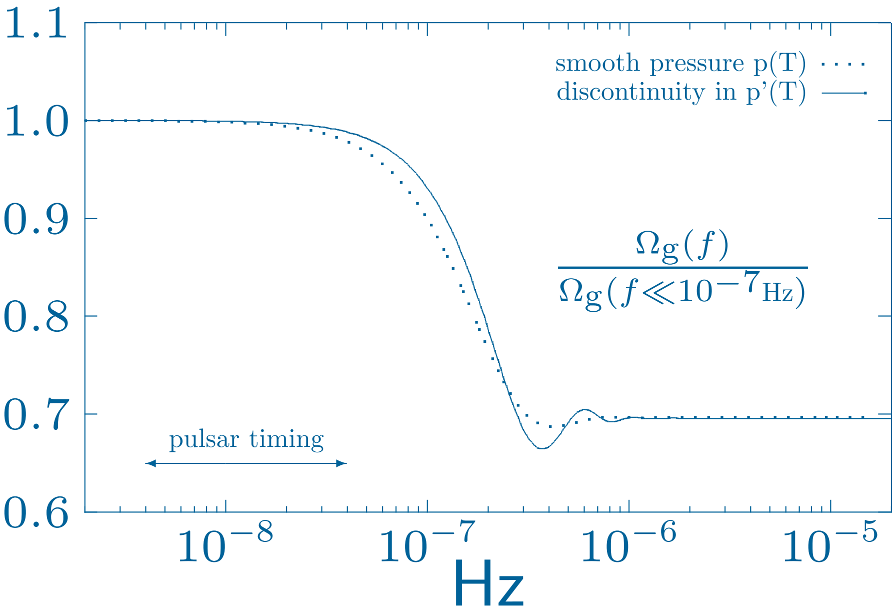


de Forcrand, Philipsen hep-lat/0607017
Aoki et al hep-lat/0611014

⇒ there is probably no actual singularity in physical QCD.

But there may still be indirect signatures

E.g., the frequency spectrum of primordial gravitational waves. Inflation generates a flat spectrum, but the amplitude decreases once a mode is within the horizon:



Schwarz gr-qc/9709027; Seto, Yokoyama gr-qc/0305096; Boyle, Steinhardt astro-ph/0512014

There is a QCD background effect also on the abundance of Cold Dark Matter (CDM)

WIMPs of mass m decouple at $T \sim m/25$. For $m =$ 10...1000 GeV, $T =$ 0.4...40 GeV. The equation of state in this range does affect the relic density.

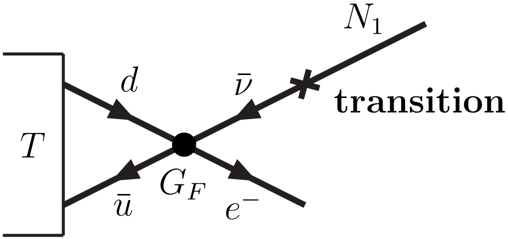
Srednicki, Watkins, Olive NPB 310 (1988) 693
Hindmarsh, Philipsen hep-ph/0501232

The dark matter relic density is supposedly determined up to few % by forthcoming CMB experiments, so even “QCD background” effects do play a role.

The effect is more significant for Warm Dark Matter (WDM)

Dodelson, Widrow hep-ph/9303287
Shi, Fuller astro-ph/9810076
Abazajian, Fuller astro-ph/0204293

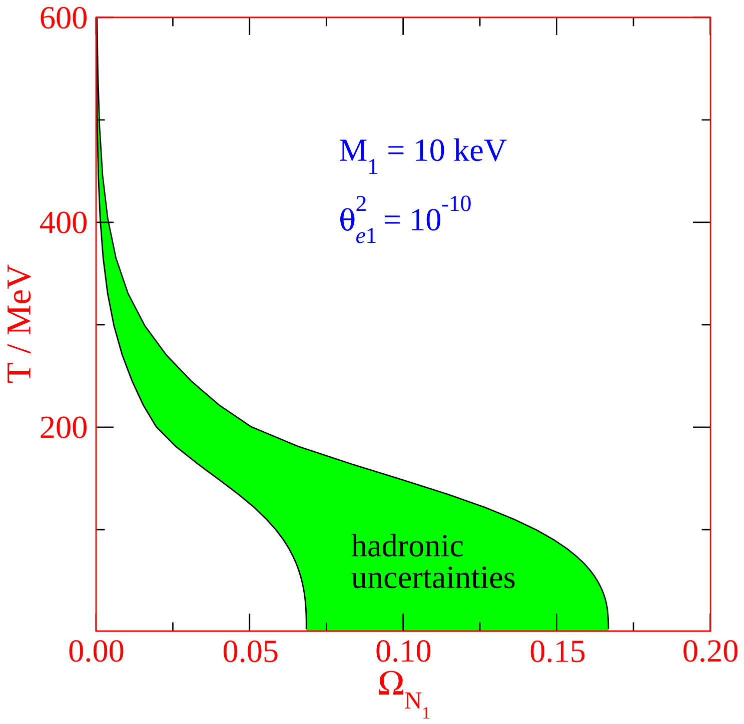
The observed neutrino masses suggest the existence of right-handed "sterile" neutrinos, but do not fix their masses M. If $M \sim 1 \ldots 50$ keV, they could act as WDM, produced through active-sterile oscillations.



Production peaks at $T \sim \left(\frac{M}{10\,G_F}\right)^{\frac{1}{3}} \sim 200$ MeV $\left(\frac{M}{1\,\mathrm{keV}}\right)^{\frac{1}{3}}$.

A concrete example:

Asaka, Laine, Shaposhnikov, hep-ph/0612182



⇒ Physics around QCD crossover does play an important role.

EW

The Standard Model case can be solved with high precision

Challenge: though the problem is more perturbative than in QCD, treatment is never fully perturbative.

Expansion parameter related to bosons ($p \sim \xi^{-1}$):

$$\epsilon_b \sim \frac{1}{\pi} g^2 n_b(p) = \frac{g^2}{\pi(e^{p/T} - 1)} \overset{p<T}{\sim} \frac{g^2 T}{\pi p}.$$

So for $p \lesssim g^2 T/\pi$, $\epsilon_b \gtrsim 1$, even if $g^2/\pi \ll 1$.

For fermions, on the contrary, no problem at $g^2/\pi \ll 1$:

$$\epsilon_f \sim \frac{1}{\pi} g^2 n_f(p) = \frac{g^2}{\pi(e^{p/T} + 1)} \lesssim \frac{g^2}{\pi}.$$

Effective theory approach

Kajantie et al hep-ph/9508379

Light degrees of freedom are Matsubara zero-modes of SU(2)×U(1) gauge fields and the Higgs boson.

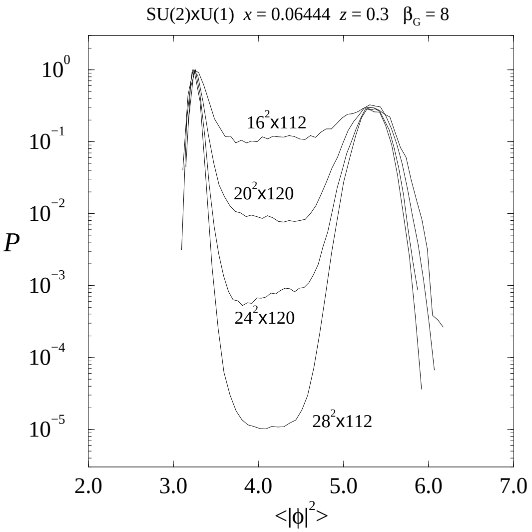
$$\mathcal{L}_{3\mathrm{d}} = \frac{1}{2}\mathrm{Tr}\,F_{ij}^2 + \frac{1}{4}B_{ij}^2 + (D_i\phi)^\dagger D_i\phi + m_3^2\phi^\dagger\phi + \lambda_3(\phi^\dagger\phi)^2,$$

$$Z = \mathrm{Tr}\exp(-\beta\hat{H}) = \int \mathcal{D}\Phi\exp[-\beta\int d^3x\mathcal{L}_{3\mathrm{d}}(\Phi)].$$

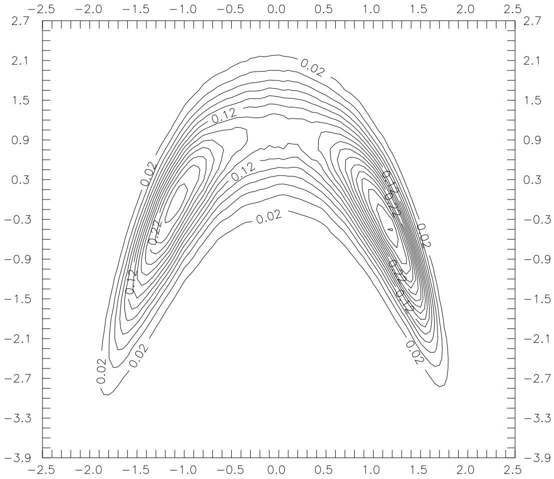
Information about other modes in effective couplings.

$$m_3^2 \sim -\frac{1}{2}m_H^2 + g^2T^2, \quad \frac{\lambda_3}{g_3^2} \approx \frac{1}{8}\frac{m_H^2}{m_W^2} + \mathcal{O}\left(\frac{g^2}{(4\pi)^2}\frac{m_{\mathrm{top}}^4}{m_W^4}\right).$$

Remaining dynamics can be studied with lattice simulations. Signals for a 1st order transition / 2nd order transition:

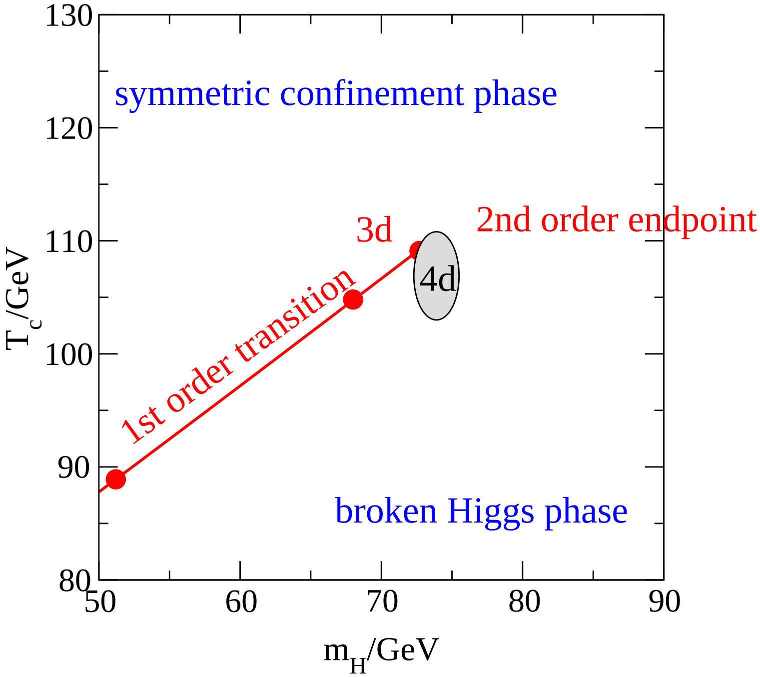


hep-lat/9612006



hep-lat/9805013

Phase diagram after infinite volume ($V \sim 20^3 \ldots 80^3$) and continuum ($g_3^2 a \sim 1 \ldots 0.2$) extrapolations:



3d lattice results:
[SU(2)×U(1)+Higgs+fermions]
Kajantie et al hep-ph/9605288,
hep-lat/9805013, hep-lat/9809045

4d lattice results:
[SU(2)+Higgs; relative
endpoint position conserved]
Csikor et al hep-ph/9809291

Again even a crossover may leave an imprint, e.g., on the baryon asymmetry generated in TeV-scale leptogenesis

Due to anomalous transitions,

$$B_{\text{present}} \simeq 4\Big(\frac{77T_{\text{ew}}^2 + 27v_{\text{ew}}^2}{869T_{\text{ew}}^2 + 333v_{\text{ew}}^2}\Big)(B-L)_{T_{\text{ew}}} \; ,$$

Khlebnikov, Shaposhnikov hep-ph/9607386

where $v_{\text{ew}}/T_{\text{ew}}$ is determined from

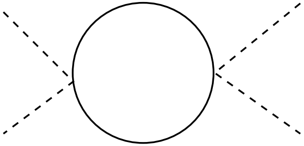
$$H(T_{\text{ew}}) \simeq \Gamma_{B+L}\left(\frac{v_{\text{ew}}}{T_{\text{ew}}}\right) \; ,$$

where H is the Hubble rate and Γ_{B+L} is the “sphaleron rate”. So, need to know this function across the crossover, in order to determine T_{ew} below which L is no longer converted to B.

Burnier et al hep-ph/0511246

But what if we do want a real transition?

Need some new degree of freedom which can decrease λ_3 by $\mathcal{O}(100\%)$. A strong effect can come from a bosonic zero mode:



$$\begin{aligned} \delta\lambda_3 &\sim -\frac{g_3^4}{8\pi m_3} \equiv -\mathcal{O}(\frac{g_3^2}{8}) \\ \Rightarrow m_3 &\sim \frac{1}{\pi} g_3^2 \approx \frac{1}{\pi} g^2 T \end{aligned}$$

⇒ The new degree of freedom should be non-perturbative, and take part in the transition, or be very close to doing so.

In fact strengthening appears to be quite generic

For instance, add a dimension-6 operator to the theory:

$$\delta V(\phi) \equiv \frac{1}{\Lambda^2}(\phi^\dagger \phi)^3 \ .$$

Zhang hep-ph/9301277, Zhang et al hep-ph/9406322; Grojean et al hep-ph/0407019; Bödeker et al hep-ph/0412366

Minimize potential keeping $m_H^2 = V''$ and $m_W = gv/2$ fixed, and solve for λ:

$$\lambda \approx \frac{g^2 m_H^2}{8 m_W^2}\left(1 - \frac{48}{g^4}\frac{m_W^4}{\Lambda^2 m_H^2}\right) \ .$$

$\Rightarrow$ λ decreases significantly even for $\Lambda \gg m_W$.

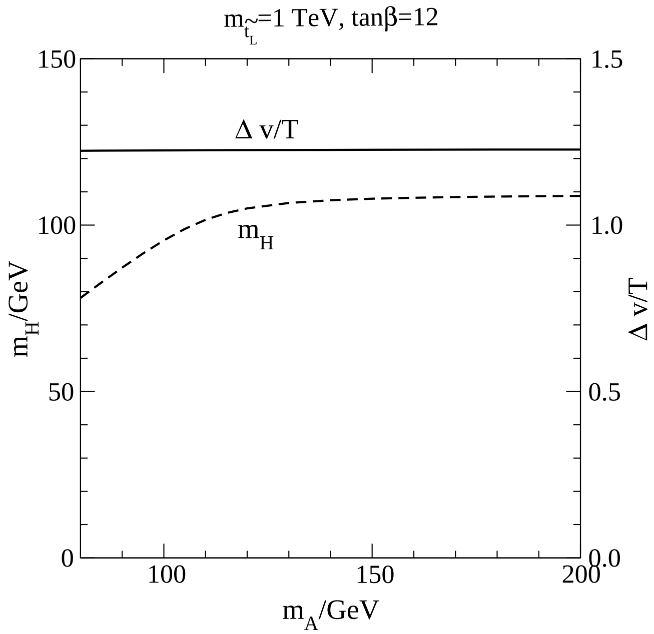
As a concrete example, consider the MSSM

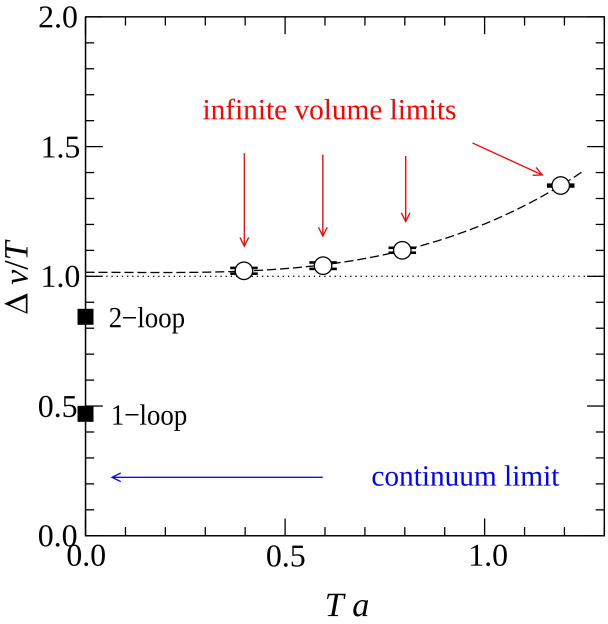
Has new strongly interacting light bosonic particles, the squarks. The prime example is a dominantly right-handed stop, with $m_{\tilde{t}_R} \sim \sqrt{m_U^2 + m_{\rm top}^2} < m_{\rm top}$, i.e. $m_U^2 \equiv -\tilde{m}_U^2 < 0$.

$$\delta\mathcal{L}_{\text{3d}} \sim h_t^2\, U^\dagger U \phi^\dagger \phi + (D_i^s U)^\dagger D_i^s U + \frac{1}{2} \mathrm{Tr}\, G_{ij}^s G_{ij}^s + \ldots$$

Integrating out U (whichever way) indicates that the transition does get stronger.

For a precise study, keep zero-mode of U dynamical and carry out 2-loop pert.theory as well as lattice simulations.



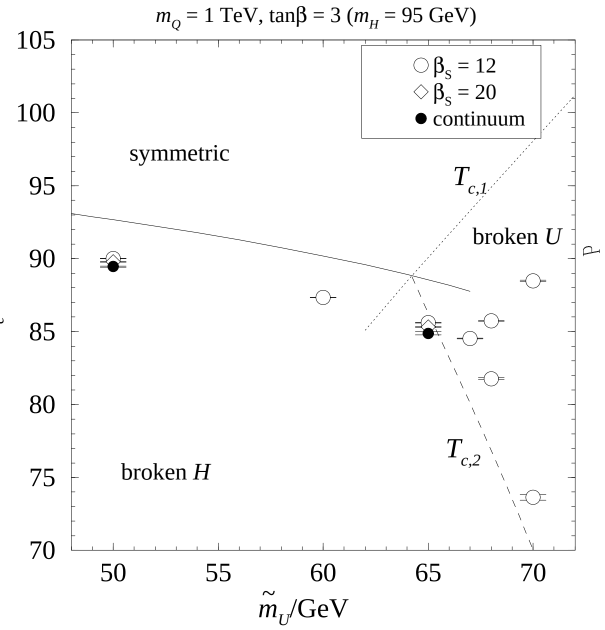


Pert.theory [hep-lat/9804019]

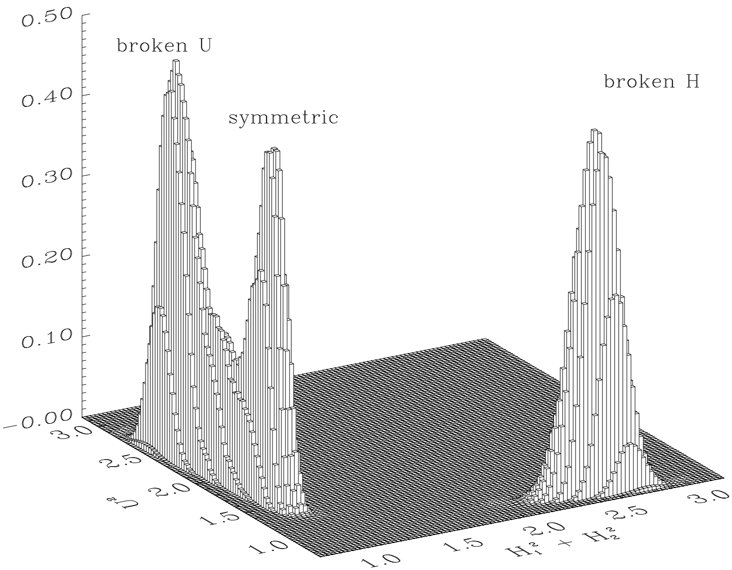
Check with simulation [hep-lat/0009025]

The most extreme case is a two-stage transition through a color-breaking phase

Bödeker et al hep-ph/9612364



hep-lat/9804019



Simulation at the triple point [hep-lat/0009025]

What could these transition do for cosmology?

After supercooling to $T_\mathrm{n} < T_\mathrm{c}$, bubbles nucleate (distance ℓ_B), expand, and release latent heat L.

Bubble collisions can lead to gravitational waves:

$$\Omega_\mathsf{GW} h^2 \lesssim 10^{-7} \left(\frac{L}{e}\right)^2 \left(\frac{\ell_B}{\ell_H}\right)^2 .$$

Witten, PRD 30 (1984) 272;
Hogan, MNRAS 218 (1986) 629;
Kamionkowski et al, astro-ph/9310044;
Caprino et al, 0711.2593.

The subsequent turbulent phase may lead to more:

$$\Omega_\mathsf{GW} h^2 \lesssim 5\times 10^{-6} \left(\frac{L}{e}\right)^{2...2.5} \left(\frac{\ell_B}{\ell_H}\right)^2 .$$

Dolgov et al, astro-ph/0206461;
Nicolis, gr-qc/0303084;
Caprini et al, astro-ph/0603476;
Gogoberidze et al, 0705.1733.

Detection threshold: $\Omega_\mathsf{GW} h^2 \sim 10^{-10}$. (Phinney 25 Feb 08: 3×10^{-12}.)

How to estimate the quantities needed?

Latent heat over energy density can be written as

$$\frac{L}{e} \simeq \frac{30\ L}{\pi^2 g_* T_{\rm c}^4} \simeq 0.03\,\frac{L}{T_{\rm c}^4}\ ,$$

where $g_* \gtrsim 110$ is the number of relativistic species.

Employing classical nucleation theory, the nucleation temperature can be expressed (universally and non-perturbatively) in terms of the surface tension σ and the latent heat L:

$$1 - \frac{T_{\sf n}}{T_{\rm c}} \simeq 0.34\,\frac{(\sigma/T_{\rm c}^3)^{\frac{3}{2}}}{(L/T_{\rm c}^4)} + \ldots$$

Non-perturbative check: Moore, Rummukainen, hep-ph/0009132

The bubble distances become

$$\frac{\ell_B}{\ell_H} \quad \simeq \quad 0.0035\,\frac{(\sigma/T_{\rm c}^3)^{\frac{3}{2}}}{(L/T_{\rm c}^4)}$$

Ignatius et al, hep-ph/9405336

So, need $L/T_{\rm c}^4 \gtrsim 1$ and $(\sigma/T_{\rm c}^3)^{3/2} \gtrsim (L/T_{\rm c}^4)$, but make sure that nucleation takes place, $1 - T_{\rm n}/T_{\rm c} \ll 1$.

Cline, Moore, Servant, hep-ph/9902220

On the other hand, baryogenesis requires $\Delta v_{\rm ew}/T \gtrsim 1$.

Shaposhnikov, NPB 287 (1987) 757

Some examples

	$\frac{L}{T_{\rm c}^4}$	$\frac{\sigma}{T_{\rm c}^3}$	$\frac{L}{e}$	$\frac{\ell_B}{\ell_H}$	$1-\frac{T_{\rm n}}{T_{\rm c}}$	$\frac{\Delta v_{\rm ew}}{T_{\rm c}}$
m_H	SM [hep-lat/9510020]					
51 GeV	0.124	0.0023	4×10^{-3}	3×10^{-6}	3×10^{-4}	0.689
68 GeV	0.08	0.0002	2×10^{-3}	1×10^{-7}	1×10^{-5}	0.575
$\tilde m_U$	“standard” MSSM [hep-lat/0009025]					
65 GeV	0.42	0.010	1×10^{-2}	8×10^{-6}	8×10^{-4}	1.02
$\tilde m_U$	“two-stage” MSSM [hep-lat/9804019]					
68 GeV	0.957	0.877	3×10^{-2}	3×10^{-3}	3×10^{-1}	2.67
70 GeV	1.402	1.426	4×10^{-2}	4×10^{-3}	4×10^{-1}	3.16

$\Rightarrow$ It is difficult to generate enough gravitational waves.

If succeed, transition is also strong enough for baryogenesis.

$\Rightarrow$ more drastic modifications are generically needed

NMSSM (Next-to-Minimal) with a gauge singlet scalar.

Pietroni, hep-ph/9207227; Davies, Froggatt, Moorhouse, hep-ph/9603388;
Huber, Schmidt, hep-ph/9809506; hep-ph/0003122; Bastero-Gil et al, hep-ph/0006198;
Kang et al, hep-ph/0402086; Funakubo et al, hep-ph/0501052.

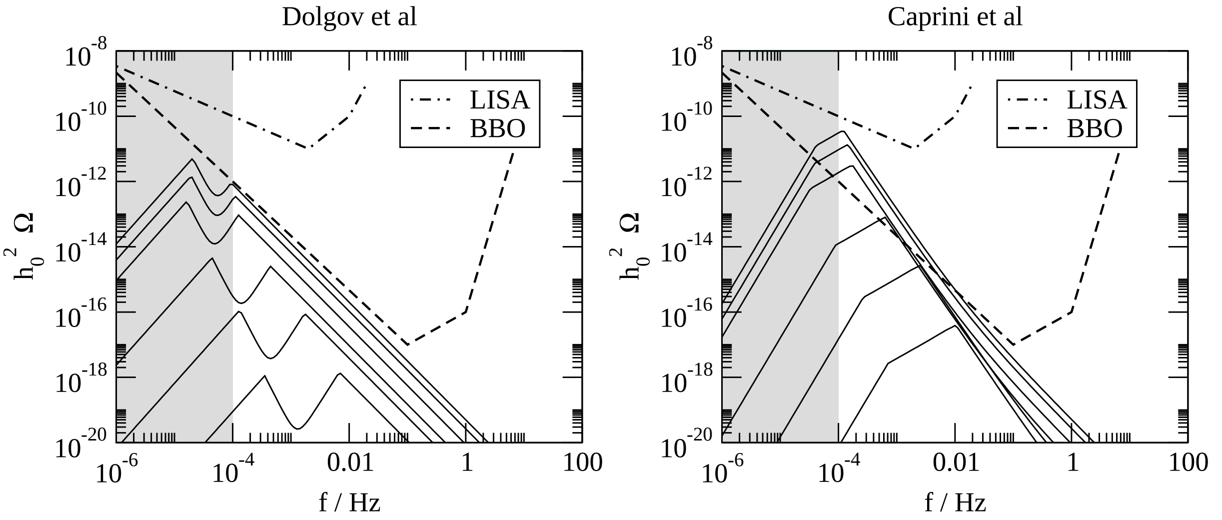
New fermions with large Yukawas; two Higgs doublets

Carena et al, hep-ph/0410352; Ham et al, hep-ph/0411012;
Fromme et al, hep-ph/0605242; Dine et al, 0707.0005; Hambye, Tytgat, 0707.0633.

nMSSM (nearly-Minimal): no S^3 in the superpotential (approximate R-symmetry creates a “small” singlet tadpole to evade the domain wall problem without destabilizing the EW minimum).

Menon et al, hep-ph/0404184; Huber et al, hep-ph/0606298; Profumo et al, 0705.2425;
Balazs et al, 0705.0431. **Gravitational waves: Huber, Konstandin 0709.2091.**

An explicit estimate of the spectrum (with two different treatments of the turbulent component):



Huber, Konstandin 0709.2091

New ideas 1/3: Higgs portal

Patt, Wilczek, hep-ph/0605188

Introduce new singlet degrees of freedom coupling directly only to the Higgs,

$$V(\phi) = V_{\mathsf{MSM}}(\phi) + \zeta^2 \phi^\dagger \phi \sum_{i=1}^{N} S_i^2 \ .$$

This is effectively the same as light stops in the MSSM (which is reproduced for $\zeta^2 = h_t^2/2$, $N = 6$), and can be made to work for baryon asymmetry by increasing ζ, N.

Patkós, Szép, hep-th/0612094;
Espinosa, Quirós, hep-ph/0701145.

New ideas 2/3: Technicolor

Technicolor $\overset{\text{here}}{\equiv}$ gauged linear sigma model

$\mathrm{SU}_\mathsf{L}(N_\mathrm{f})\times\mathrm{SU}_\mathsf{R}(N_\mathrm{f})\longrightarrow\mathrm{SU}_\mathsf{V}(N_\mathrm{f})$ chiral symmetry breaking is of first order for $N_\mathrm{f} > 3$ even without gauge interactions!

Pisarski, Wilczek, PRD 29 (1984) 338.

Let Φ be the corresponding scalar field; couple it minimally to SU(2)×U(1). $N_\mathrm{f} = 2 \Leftrightarrow$ SM. The $N_\mathrm{f}^2 - 1 - 3$ Goldstones made massive by breaking the symmetry explicitly. Yet the transition remains of 1st order if the explicit breaking is small enough.

Prediction: the existence of several fairly light (pseudo Nambu-Goldstone) scalars.

Kikukawa et al, 0709.2221.

New ideas 3/3: Warped extra dimensions

"EW symmetry breaking by orbifold boundary conditions" **or** "Composite Higgs as a holographic pseudo NG scalar"

In the latter case, physics is similar to that in technicolor, but realised in the AdS-CFT setup.

Like with two-stage, transition may even be too strong!

Creminelli, Nicolis, Rattazzi, *Holography and the electroweak phase transition*, hep-th/0107141; Panico, Serone, *The Electroweak phase transition on orbifolds with gauge-Higgs unification*, hep-ph/0502255; Maru, Takenaga, *Aspects of Phase Transition in Gauge-Higgs Unification at Finite Temperature*, hep-th/0505066; **Randall, Servant, *Gravitational waves from warped spacetime*, hep-ph/0607158**; Kaplan, Schuster, Toro, *Avoiding an empty universe in RS I models and large-N gauge theories,* hep-ph/0609012; Nardini, Quirós, Wulzer, *A Confining Strong First-Order Electroweak Phase Transition*, 0706.3388; Hassanain, March-Russell, Schvellinger, *Warped Deformed Throats have Faster (Electroweak) Phase Transitions*, 0708.2060.

Conclusions

QCD — theory known, but properties very hard to compute, requiring 4d lattice simulations with light dynamical fermions.

Yet does yield background effects for cosmology.

EW — theory not known, but many conceivable alternatives could be practically solved, with the help of relatively simple 3d lattice simulations.

Creating a gravitational wave signal is more demanding than creating baryon asymmetry. However, in principle a possibility exists (if manage to tunnel) even for probing stringy electroweak symmetry breaking both at LHC and at LISA.