Plasma Instabilities in QCD and SYM(?)

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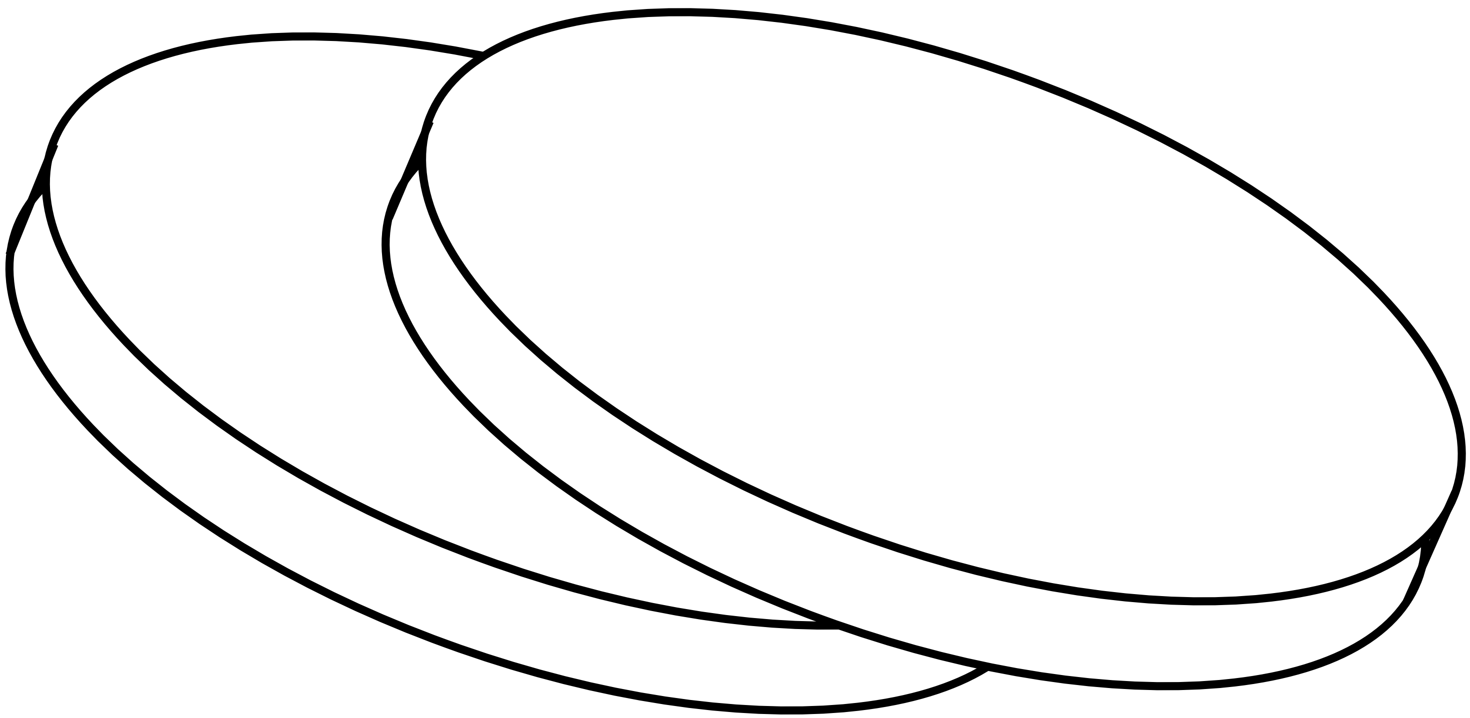
1. Why we expect anisotropy in heavy ion collisions
2. Anisotropy + weak coupling → Weibel instability
3. Parametric behavior, strong and weak anisotropy
4. Looking for gauge invariant indicators
5. Conclusions

Heavy ion collisions

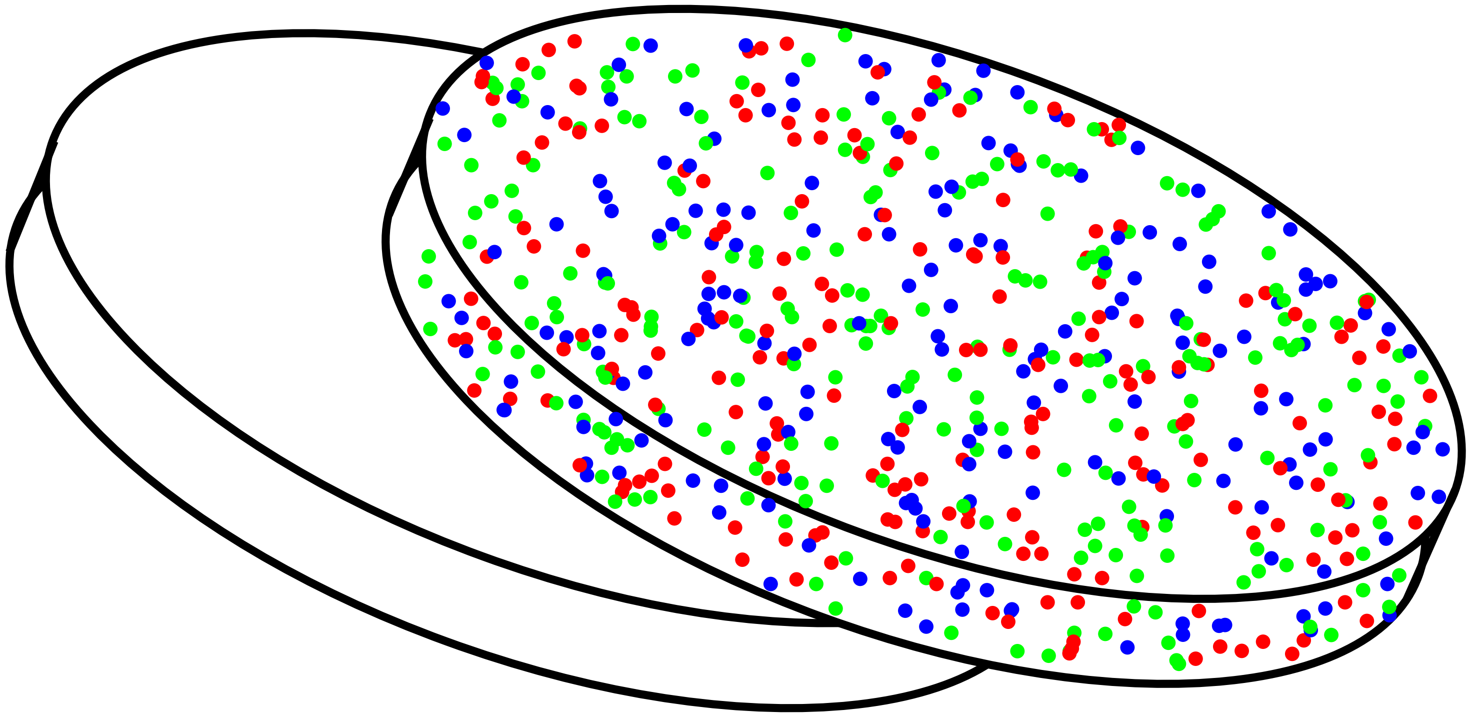
Accelerate two heavy nuclei to high energy, slam together.



Just before: Lorentz contracted nuclei

Heavy ion collisions

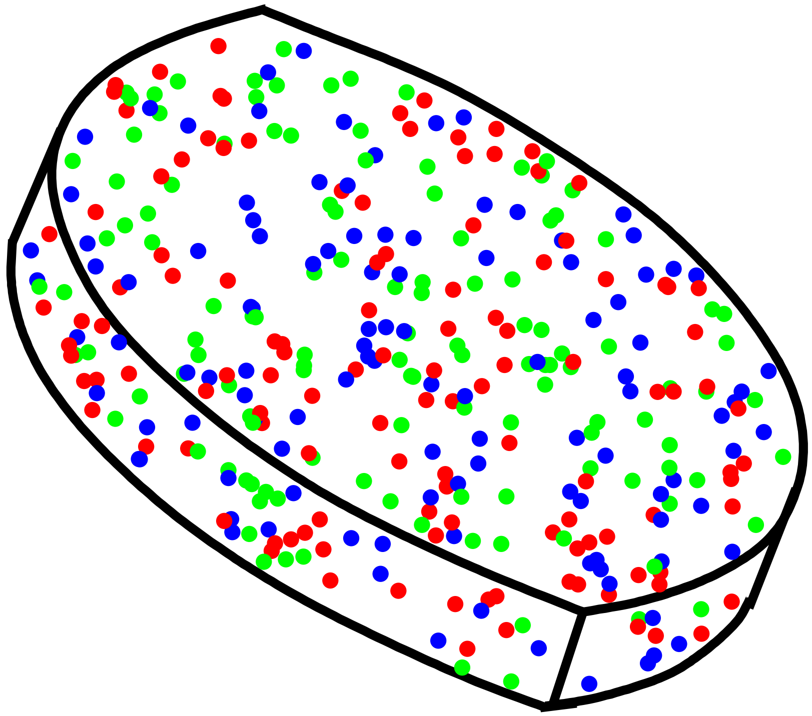
Each nucleus is $\sim 200\ p, n$, each built of $\sim 50\ q, \bar{q}, g$



It is the $q, \bar{q}, g$ which scatter.

After the scattering:

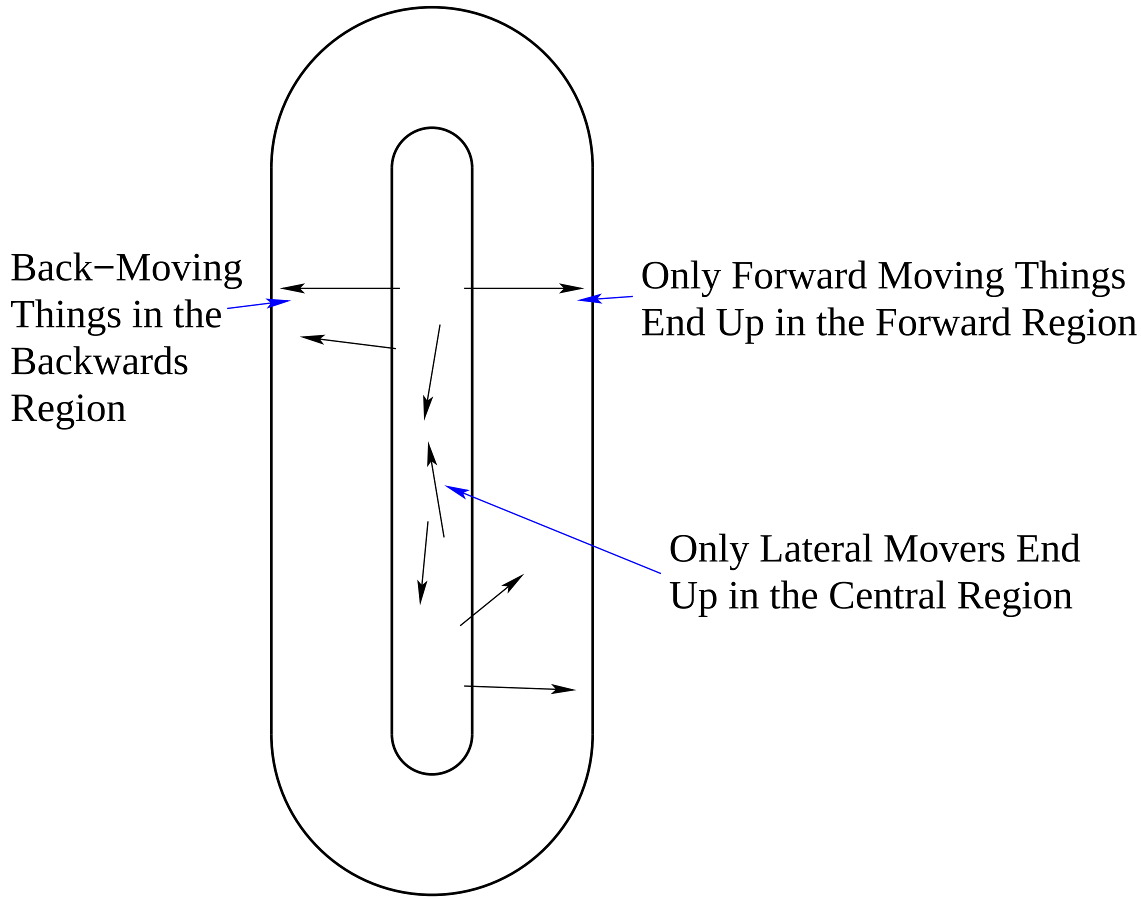
"Flat almond" shaped region of $q, \bar{q}, g$ which scattered.



Few thousand. random velocities. Quark-Gluon Plasma

Momentum Selection

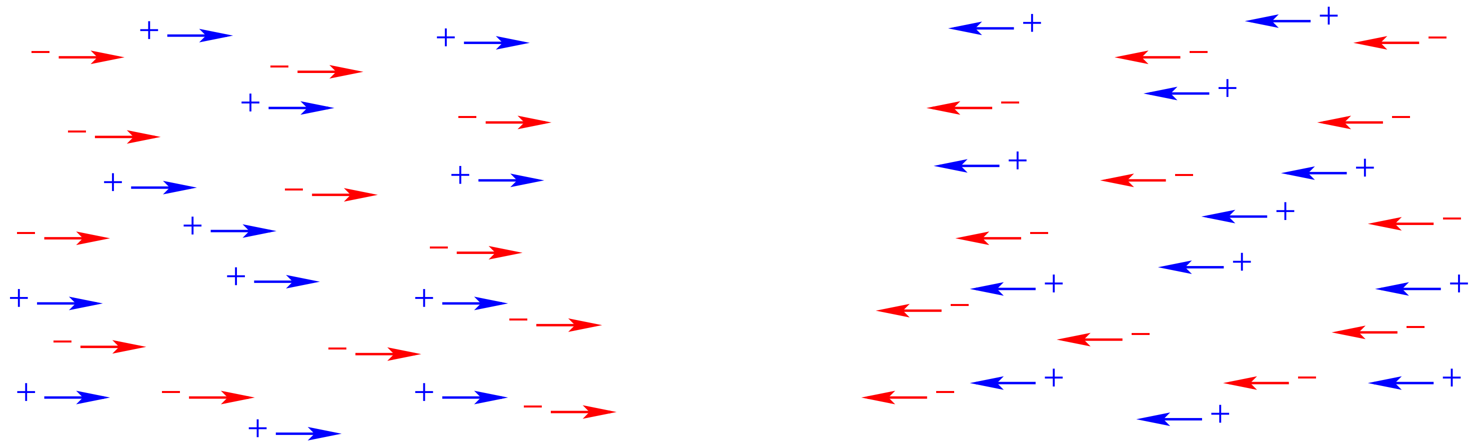
Side-on view of the flat almond as it expands



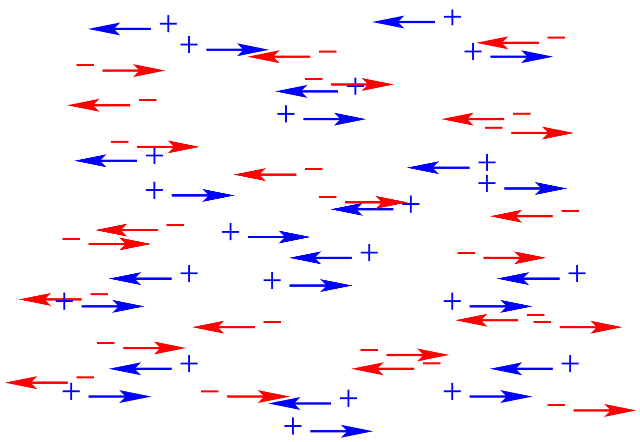
Automatically leads to strong anisotropy of “particle” flow

Simpler example of anisotropic particle flow:

interpenetrating beams of plasma



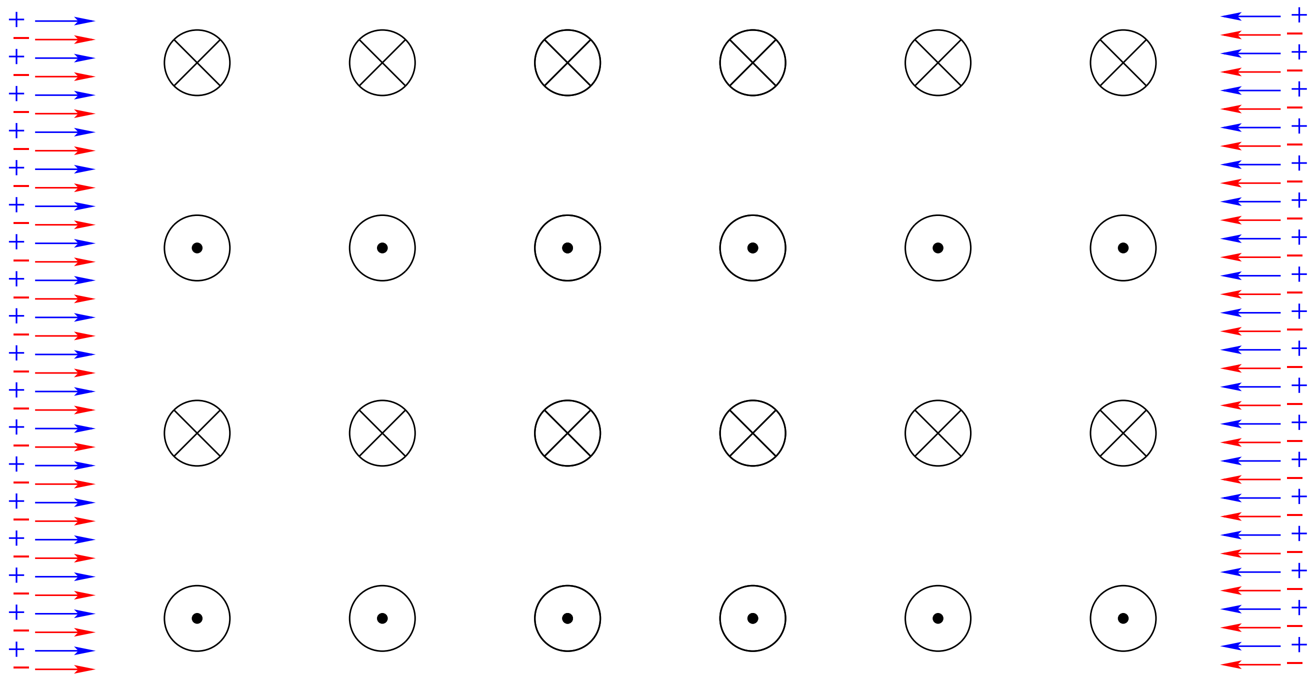
becomes



What happens?

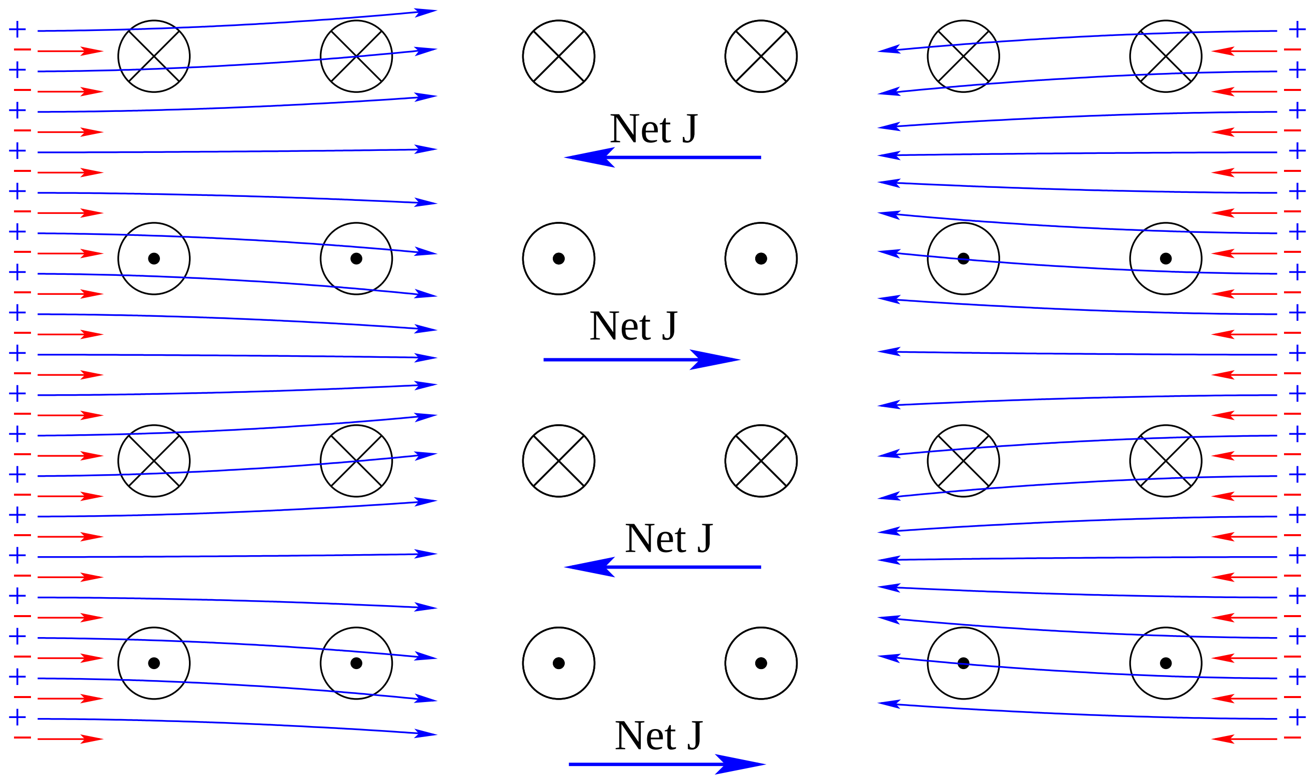
Magnetic field growth!

Consider the effects of a seed magnetic field $\hat{B} \cdot \hat{p} = 0$ and $\hat{k} \cdot \hat{p} = 0$



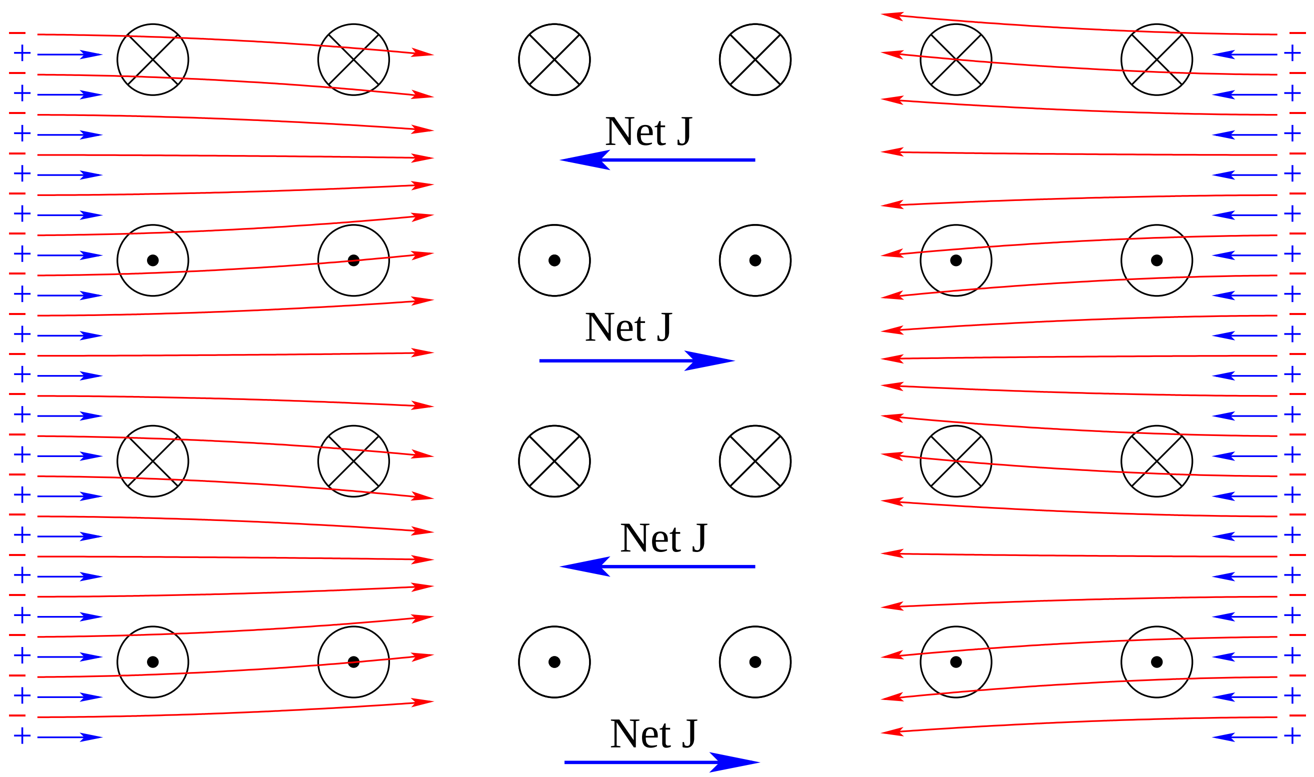
How do the particles deflect?

Positive charges:



No net ρ. Net current is induced as indicated.

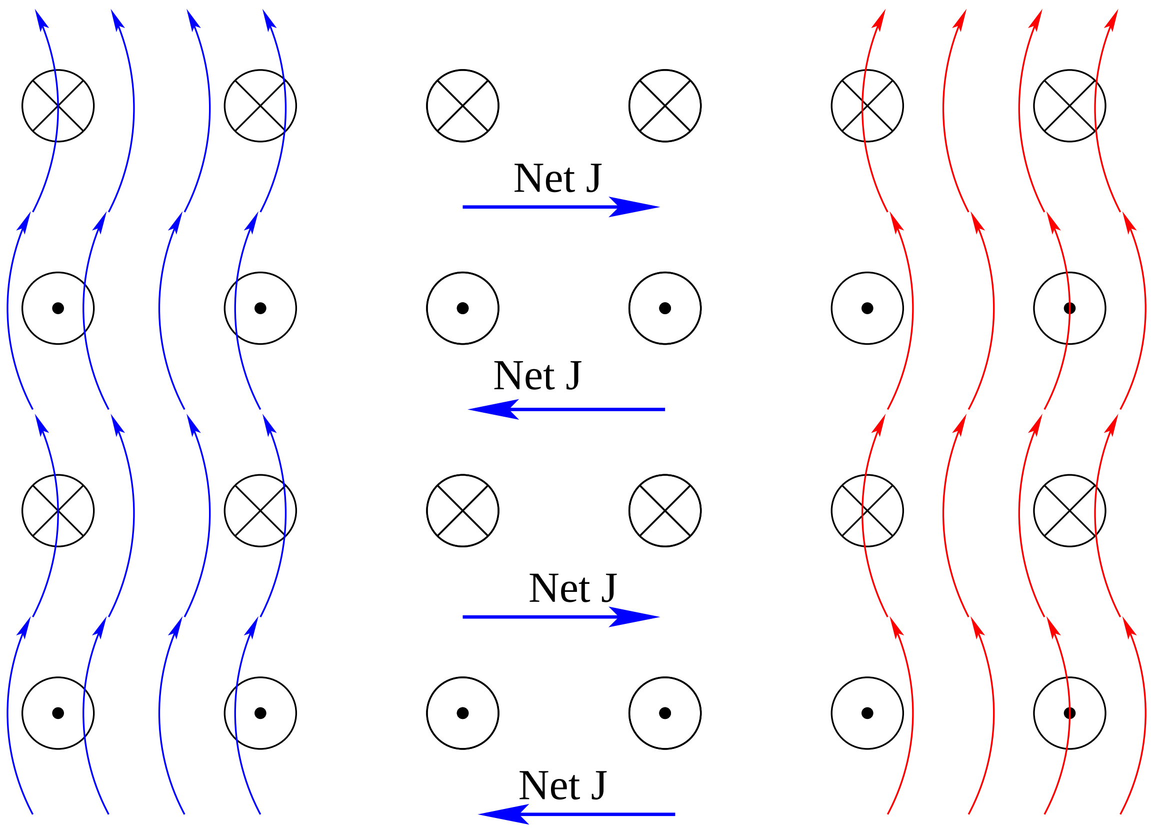
Negative charges: same-sign current contribution



Induced B *adds* to seed B. Exponential Weibel instability

Linearized analysis: B grows until bending angles become large.

Note: particles in other directions are stabilizing

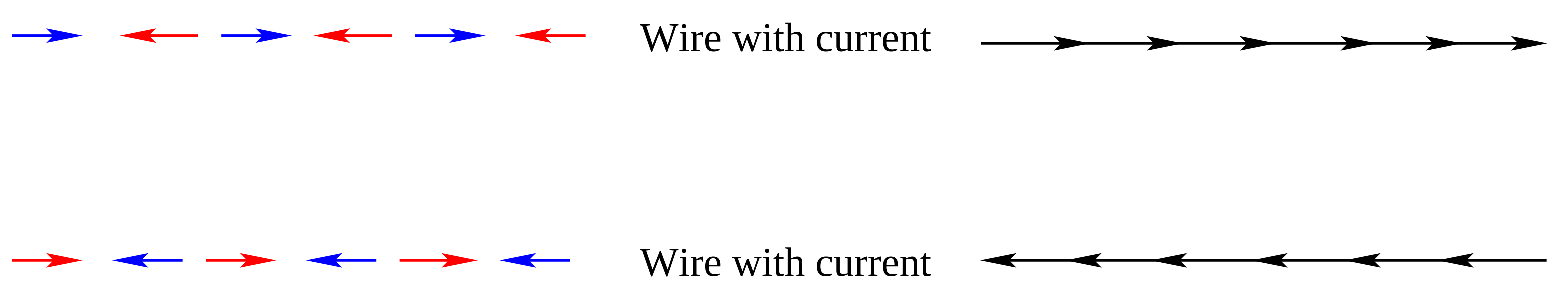


Sum of J from two signs weakens seed magnetic field.

Isotropy: effects from different directions cancel!

Another way of thinking about Weibel instability:

Think of charges as forming wires:

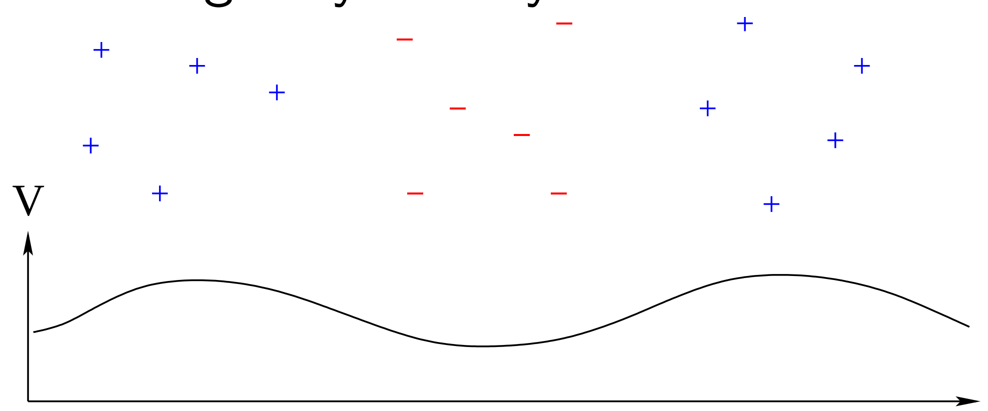


Parallel wires of like current attract. Opposite current repel.

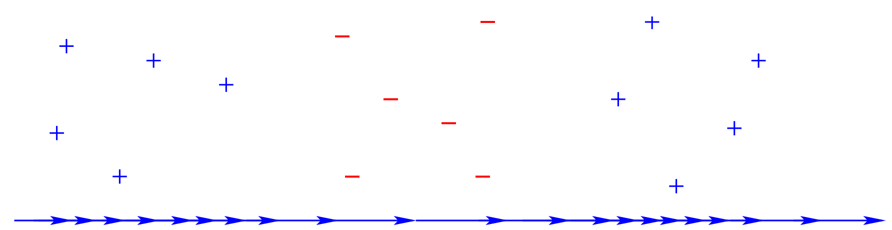
Unstable to bunching of like-sense wires (filimentation).

Many other plasma instabilities. Electric instability:

Consider a seed charge asymmetry:



$+$ charges move slow where V is high, fast where V is low:



$+$ charge spends more time where $+$ charges are.

Spontaneous charge clumping.

How? What about thermodynamics?

None of these occur for isotropic plasmas.

Anisotropic plasma–low entropy, large available free energy. Maximize entropy by deflecting charges to be more isotropic. E, B fields do exactly that–and are most efficient method leading to isotropy and equilibration.

Electric instability:

- faster than Weibel by c/v
- not generic–present for special plasma momentum distributions

Weibel instability: *Always* present for *any* anisotropy.

Hidden assumption: Weak coupling

Above arguments all based on

- Classical field picture of long-wavelength (low k) DOF
- Classical particle picture of short-wavelength (high k) DOF

Both require **weak coupling** $g^2 N_{\rm c} \ll 1$ to make sense.

- Classical field $\leftarrow$ high occupancy.
  $n_b(\omega) \sim 1/\alpha$ is strong coupled....
- dense quasiparticles, big α $\longrightarrow$ frequent scattering.
  Frequent enough; quasiparticle picture is nonsense
- Dilute quasiparticles, big α $\longrightarrow$ rapid fragmentation.
  Quickly ceases to be dilute

Can we reach weak coupling in QCD?

a moment of honesty

Higher energy nuclei $\longrightarrow$ denser QGP $\longrightarrow$ smaller g, but

- g gets smaller *logarithmically* with plasma density
- $q, \bar{q}, g$ get more transparent with energy $\longrightarrow$
  density only grows *logarithmically* with beam energy
  (but as $A^{1/3}$ with nucleus size)

g gets small very slowly.

May not "really" be small at current or even future facilities.

But we'll push ahead anyway

QED (setting $c = 1$): Maxwell Equations

$$\begin{aligned} \frac{d}{dt}\mathbf{E} + \nabla \times \mathbf{B} &= e\mathbf{j} \\ \mathbf{j} &= \sum_{\text{charges}} q\mathbf{v} \end{aligned}$$

QCD: Yang-Mills equations

$$\begin{aligned} \frac{d}{dt}\mathbf{E}^a + \nabla \times \mathbf{B}^a &= g\mathbf{j}^a - g f_{abc}(A_0^b \mathbf{E}^c + \mathbf{A}^b \times \mathbf{B}^c) \\ \mathbf{j}^a &= \sum_{\text{quarks}} q_k^\dagger T_{kl}^a q_l \mathbf{v} \end{aligned}$$

$\mathbf{A}$: gauge potential. g: strong coupling. f_{abc}, T^a_{kl}: known pure numbers. q_l: (3 component column) “color” of quark.

Complications in QCD:

- Eight fields ($r\bar{b}$, $b\bar{r}$, $r\bar{g}$, $\ldots$)
- Three colors of quarks (r, g, b)
- $g\mathbf{A} \times \mathbf{B}$ term: equations are *nonlinear*
- Color of a quark *changes* as quark moves!

$$\frac{d}{dt} q_k = -ig(-A_0^a + \mathbf{v} \cdot \mathbf{A}^a) T_{kl}^a q_l$$

Gauge field A^a acts as a "connection" for quark color.

Complications irrelevant if gA is small.

A minimal size set by quantum fluctuations $\rightarrow$ need g small

Weibel instabilities in QCD?

Yes! Provided

1. You can get a plasma of quarks and gluons

   That's what we do in heavy ion collisions!

2. You can get the coupling g to be small

   True in limit of high energies and large nuclei

3. You can get the plasma to be anisotropic

   Automatic in heavy ion context

4. Gauge field $\mathbf{A}$ is not too big.

   Problematic

In principle, we get "field saturated plasma instabilities".

Hard Loop Expansion

General method to treat plasma screening effects. Valid if

1. Weak coupling $\alpha_{\mathrm{s}} \ll 1$
2. Separation of scale $p \gg k$ (or $n \ll p^4/g^2$)
3. Hard modes homogeneous on scale $\geq 1/k$

In heavy ion setting, (2) and (3) follow from (1)

Does *NOT* assume $\sim k$ fields are perturbative.

Hard loop approach

Treat hard modes with *kinetic theory* (Vlasov equations)

$$[D_t + \mathbf{v}\cdot D_{\mathbf{x}}]\, f(\mathbf{p},\mathbf{x},t) = -\frac{1}{2}\left\{ g v^\mu F_{\mu i}\,,\ \frac{df(\mathbf{p},\mathbf{x},t)}{dp_i}\right\}$$

Perturb in effect of soft modes on hard modes

$$D_t + \mathbf{v}\cdot D_{\mathbf{x}} f^a_{\text{adj}} = -g v^\mu F^a_{\mu i} D_{p_i} f_{\text{singlet}}$$

Compute induced current and feed into Yang-Mills equations

$$D_\mu F^{\nu\mu}_a = J^\nu_a = g\int_{\mathbf{p}} v^\nu f^a_{\text{adj}}$$

Good news: $|\mathbf{v}| = 1$ so $|\mathbf{p}|$ is redundant, integrate it out.

Define (roughly)

$W(x,v) = \int_{|\mathbf{p}|} f_{\rm adj}/g$ Physically, net color of particles at x moving in v direction

Also write $$g^2 C_{\rm f} \int_{\mathbf{p}} f_{\rm singlet} \delta(\mathbf{p} - \mathbf{v}|\mathbf{p}|) \equiv m_\infty^2 \Omega(\mathbf{v})$$

Avg value $m^2 \equiv g^2 \int_{\mathbf{p}} f/p \sim g^2 T^2$ characteristic scale of HL's.

System of equations,

$$\begin{aligned} D_t W^a(x,\vec{v}) &= -\vec{v} \cdot \vec{D} W^a(x,\vec{v}) + m_\infty^2 \ \text{Source} \\ \text{Source} &= 2\Omega(\mathbf{v}) \vec{v} \cdot \vec{E} - \vec{E} \cdot \frac{\partial}{\partial \vec{v}} \Omega(\mathbf{v}) - F_{ij} v_i \frac{\partial \Omega(\mathbf{v})}{\partial v_j} \\ D_\mu F^{\nu\mu} &= J^\nu = \int_{\mathbf{v}} v^\nu \, W(\mathbf{v}) \end{aligned}$$

Lattice implementation

First: make v space discrete (otherwise, ∞ DOF/site!)

$Y_{\ell m}$ expansion, truncated at some $\ell_{\rm max}, m_{\rm max}$.

$W(\mathbf{v}) \rightarrow W_{\ell m}$ and $\int_{\mathbf{v}} \ldots$ turns into ℓm sums.

Gauge fields: link variables. F^2 term: standard. SU(2).

W on sites. J on link–average of W on two ends of link.

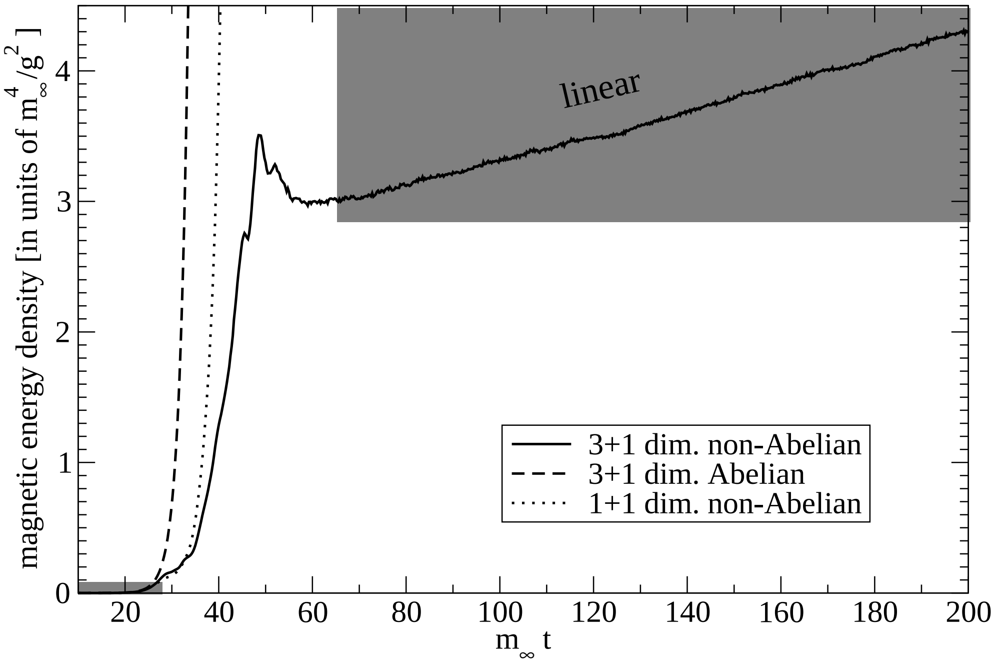
W Eq. 1'st order: doubling. W defined only on even spacetime points.

High ℓ: "infinite" energy sink. Mock up effect by applying weak damping to high ℓ (yes we checked...).

Consider 3 cases

- Generic departure from anisotropy
- Large anisotropy (early in heavy ion collision)
- Small anisotropy (relevant near equilibrium)

If gauge fields start with small fluctuations:



first: exponential growth. Fields get nonperturbatively large. Switches to linear growth in energy.

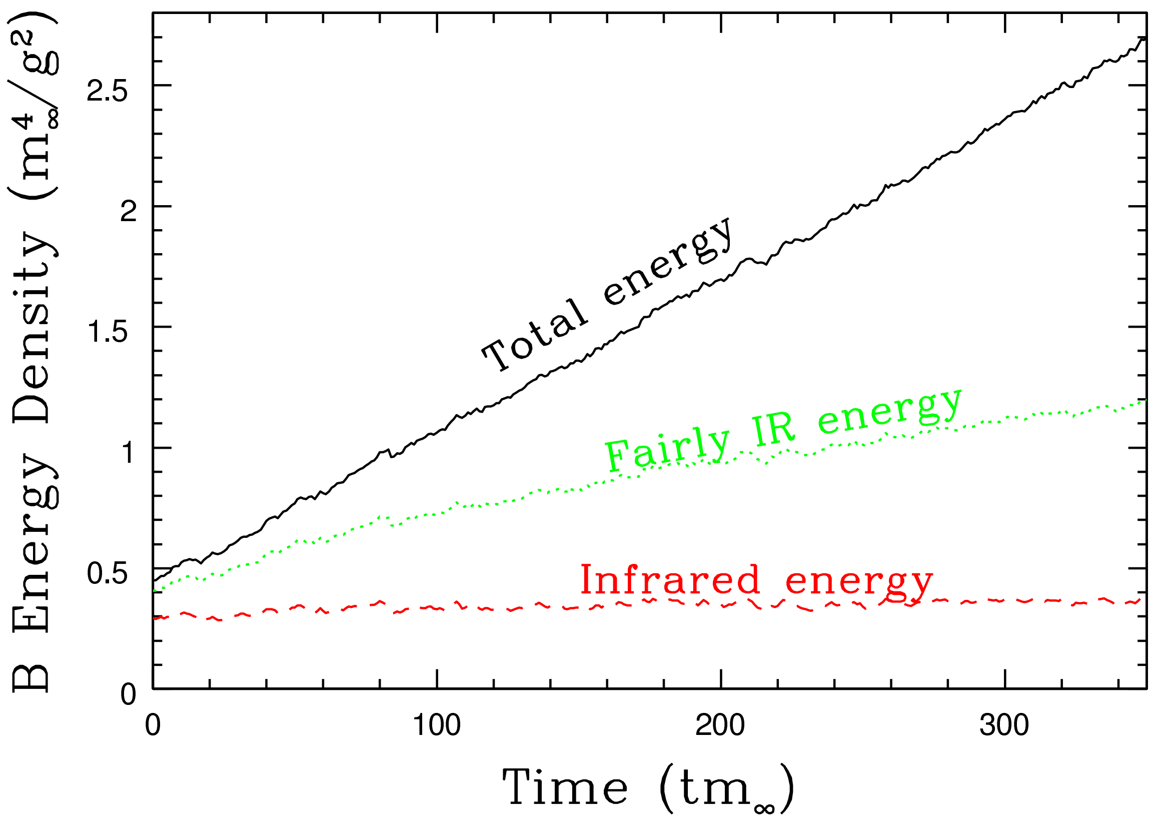
Time scales: exp. time shorter than system age.

all the time spent in the linear growth part.

Concentrate on nonperturbative linear part

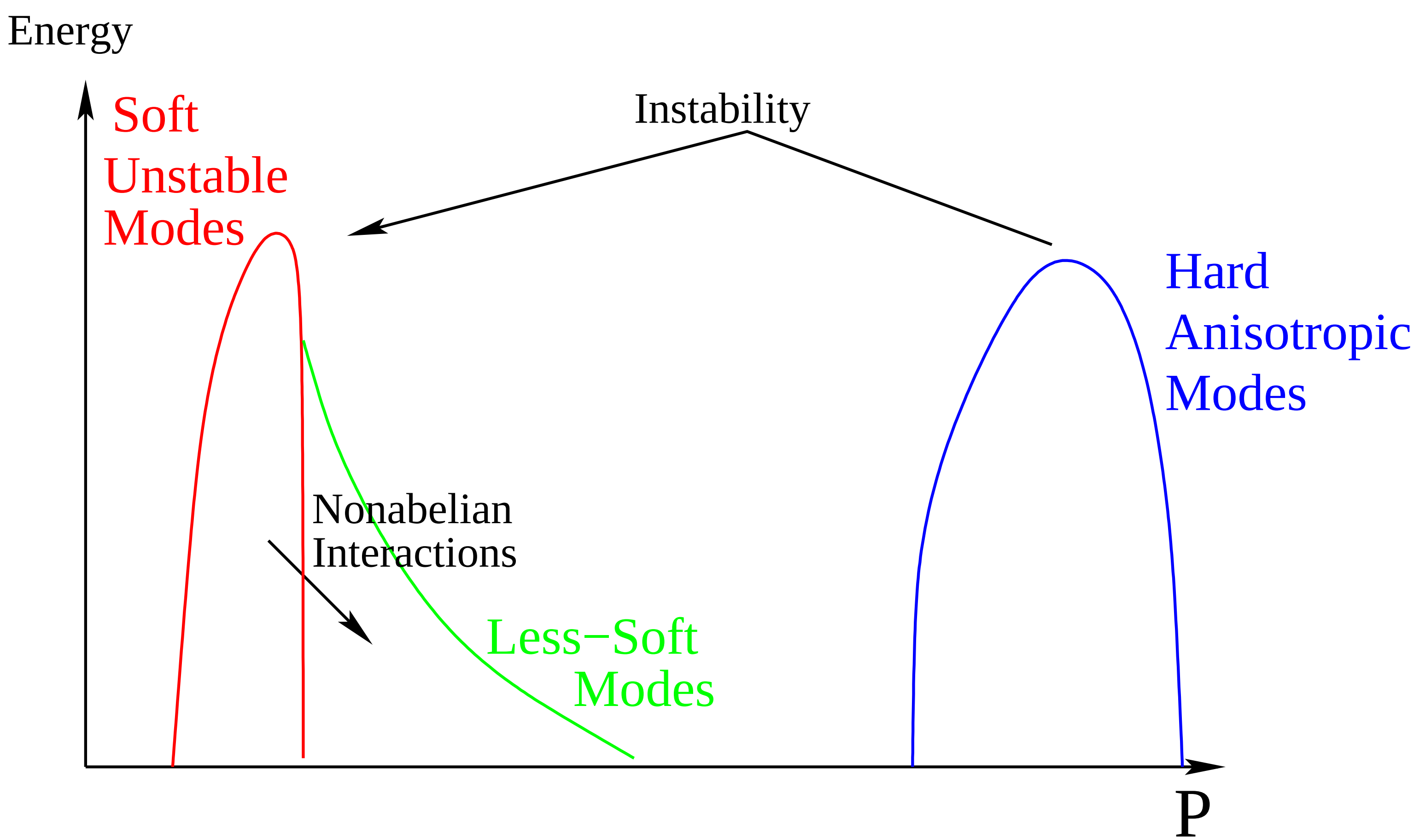
Energy in gauge fields grows linearly with time

Field smearing lets us see how much is very IR energy.

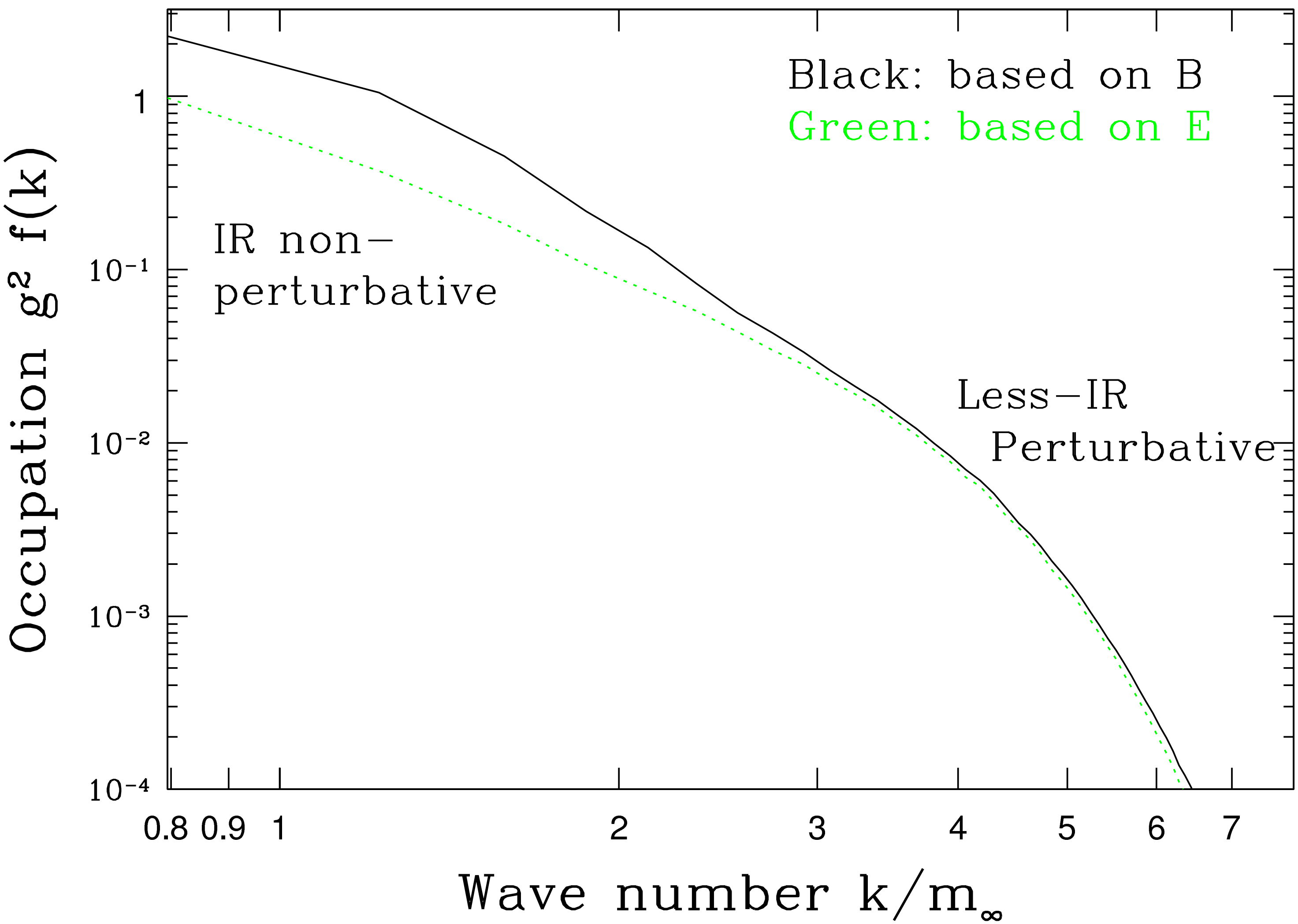


Very soft fields constant. Medium-soft grow slower.

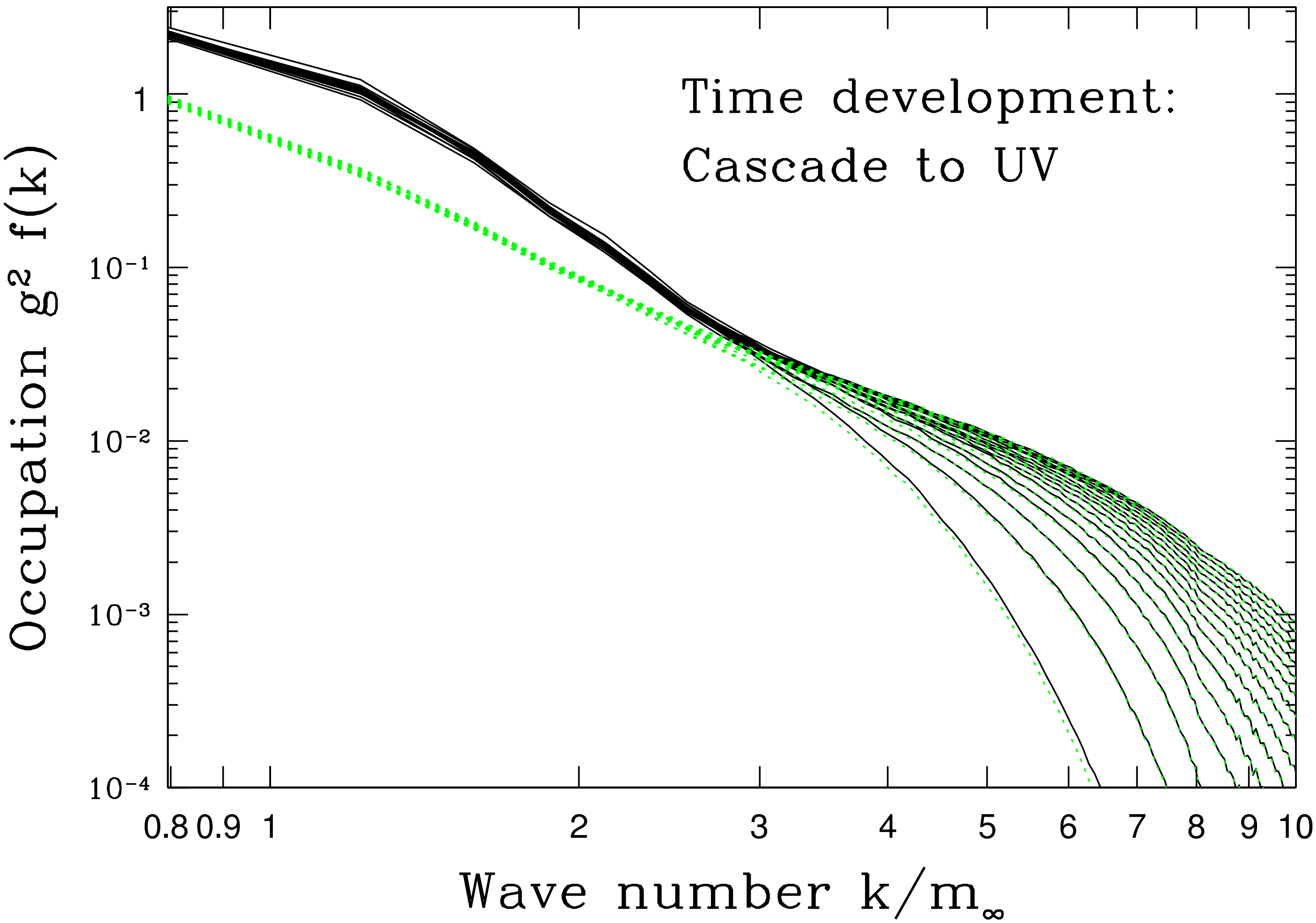
Instability pumps soft modes. Nonabelian interaction cascades energy into less-soft modes.



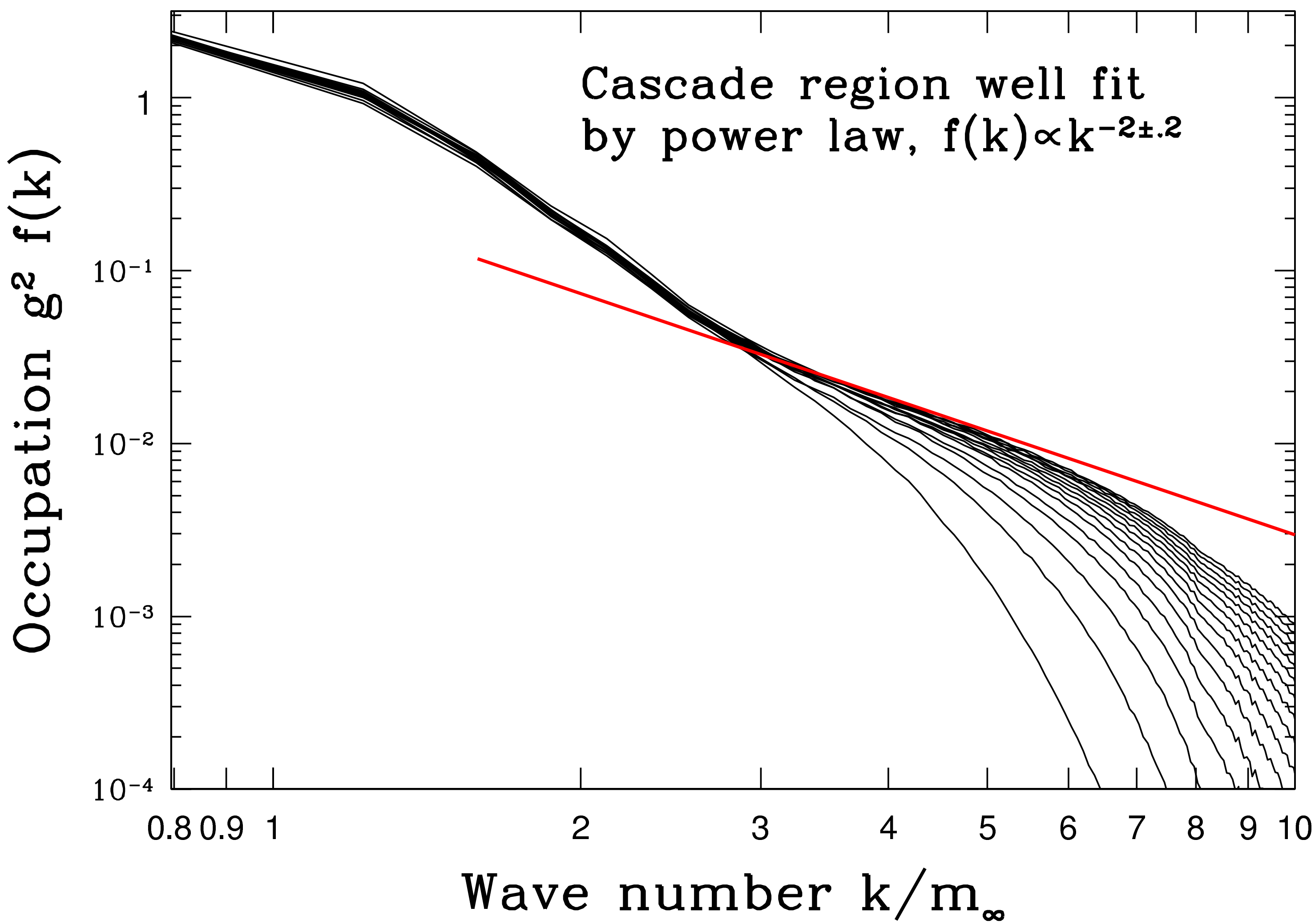
Coulomb gauge power spectrum: Initially



Time development of Coulomb gauge spectrum



Power-law behavior with moving cutoff



Strong anisotropy

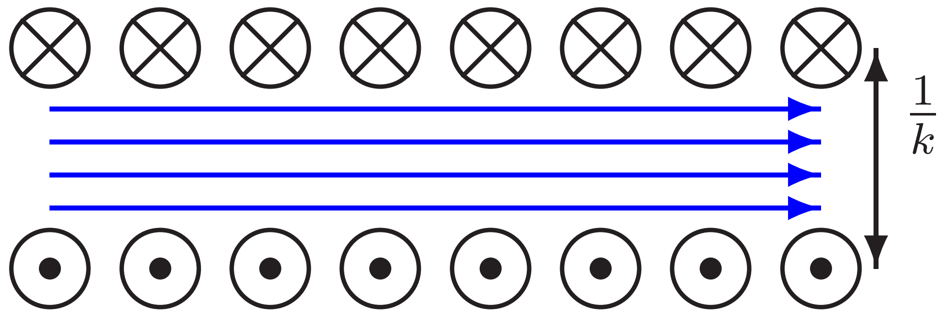
Hard particles mostly within narrow angle θ of in-plane.

Questions:

- What wave vectors k are unstable?
- How fast to they grow?
- How large is their energy when cascade sets in?
- Efficiency in deflecting hard modes

Scales: Physical role of scale m

m^{-1} is timescale for back-reaction to become important.



$$\delta\theta_{\mathbf{p}} \sim \frac{Bt}{p} \quad \delta z \sim \frac{Bt^2}{p} \quad j = g^2 n \delta z k \sim \frac{g^2 n}{p} t^2 k B = m^2 t^2 k B$$

Compare to the $D \times B$ term in Ampere's Law:

$$D \times B \sim kB \sim j \sim m^2 t^2 kB \quad \Rightarrow \quad t \sim 1/m$$

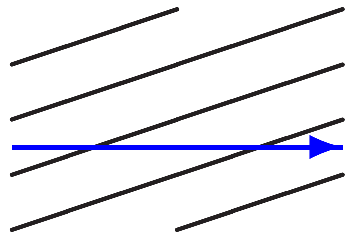
Who's unstable for HIGH anisotropy?

Last argument: growth timescale $\sim 1/m$

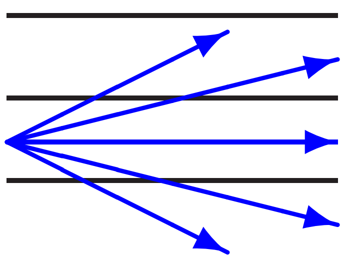
Particles must be coherently in same-sign B for $1/m$ time.

Gauge invariant–you should include Wilson line along particle trajectory

This can fail when

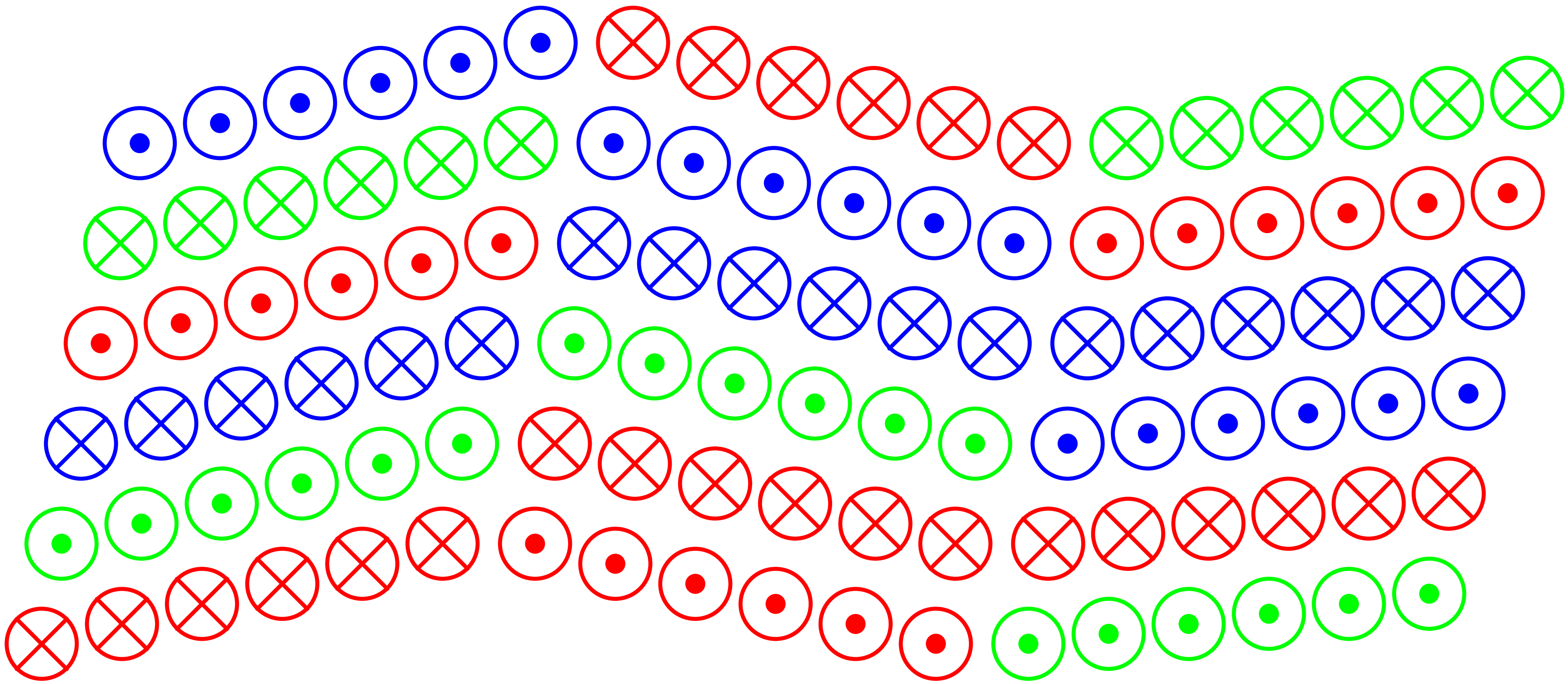
- B varies in plane:  $k_\perp > m$

- Range θ of particles' momenta too big:



$$k_z > m/\theta \equiv k_{\rm max}$$

Picture during pertubative growth: B pancakes



B varies in z with $\sim 1/k_{\rm max}$ but in x, y with $\sim 1/m$ coherence length.

What cuts off the growth?

Assume all unstable modes present, comparable amplitude.

Growth can stop due to

- Color randomization Particle color changes in length $< 1/m$, making J^a wrong component to grow B^a
- Nonabelian interaction $D_\perp = \partial_\perp - iA_\perp$ and large $A_\perp$ changes B evolution to take on nonvanishing $k_\perp \sim m$ components
- Nonabelian interaction: color rotation $A_\perp$ terms in YM equation can make A color precess so it's out of alignment with what J is doing

Which is which? Probably gauge dependent.

How large does B get?

Color randomization: Wilson line $U = \exp -iA \cdot dl$ far from $\mathbf{1}$ on length $1/m$: $A_\perp/m \sim 1$ or $A_\perp \sim m$

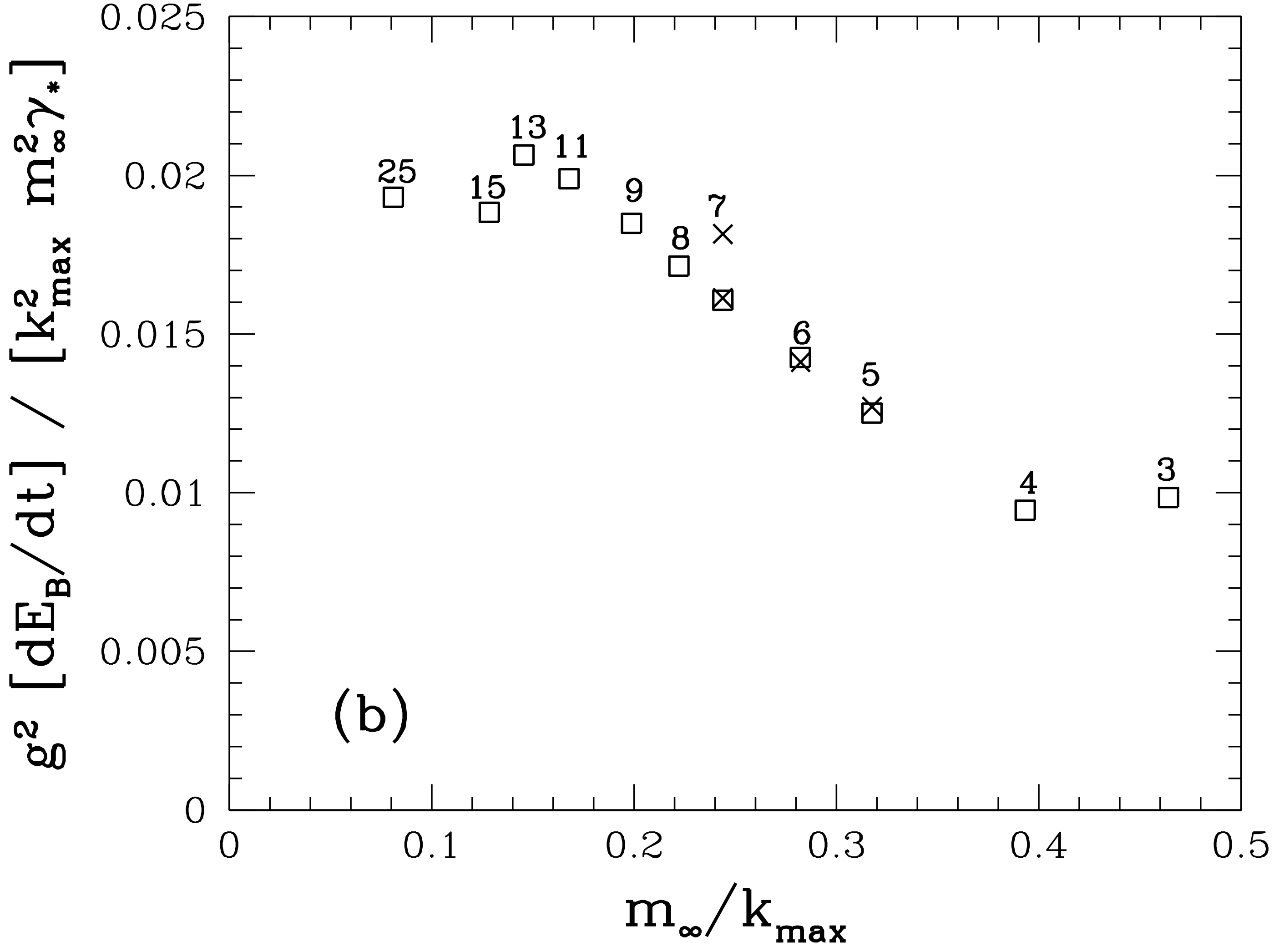
Other nonabelian effects also give $A_\perp \sim m$.

Magnetic energy

$$\frac{1}{g^2}B^2 = \frac{1}{g^2}(D \times A)^2 \sim \frac{1}{g^2}k_{\max}^2 m^2 = \frac{m^4}{g^2\theta^2}$$

Prediction: Energy grows with time as $\frac{d\epsilon}{dt} = \frac{m^5}{g^2\theta^2} \sim \frac{m^2 k_{\max}^2 \gamma}{g^2}$.

Prediction is correct



Weak instabilities

What happens under shear flow if $\nabla_i v_j$ is small?

Near equilibrium, $f(p) = f_0 + \delta f$, $f_0 = [\exp(\beta v_\mu p^\mu) \mp 1]^{-1}$

Departure δf *induced* by anisotropy

$$\frac{d(\delta f)}{dt} = f_0(1 \pm f_0)\frac{p_i p_j}{E}\left(\nabla_i v_j - \frac{\delta_{ij}}{3}\nabla_k v_k\right)$$

Departure *restored* by particle bending in resulting B.

[[In competition with plain old scattering events.]]

Need determine how much bending as function of δf.

Weak anisotropy $\delta f \ll f_0$: write $\epsilon \sim \frac{\delta f}{f_0}$.

Anisotropic modes: current due to δf particles must be enough to induce instability.

$$k_{\rm max}^2 \sim \epsilon m^2$$

These modes are Landau damped: growth rate γ

$$\frac{\gamma m^2}{k} \sim k^2 \to \gamma \sim \frac{k^3}{m^2} \sim \epsilon^{3/2} m$$

implies $B^2 \gg E^2$. Also, no cascade.

Maximum size B still the "nonpert. size" $B \sim k_{\rm max}^2$.

Deflection rate of particles

$$\hat{q} \equiv \frac{d(p_\perp^2)}{dt} \sim B^2 l_{\rm coh} \sim k_{\rm max}^3$$

Induced approach to equilibrium

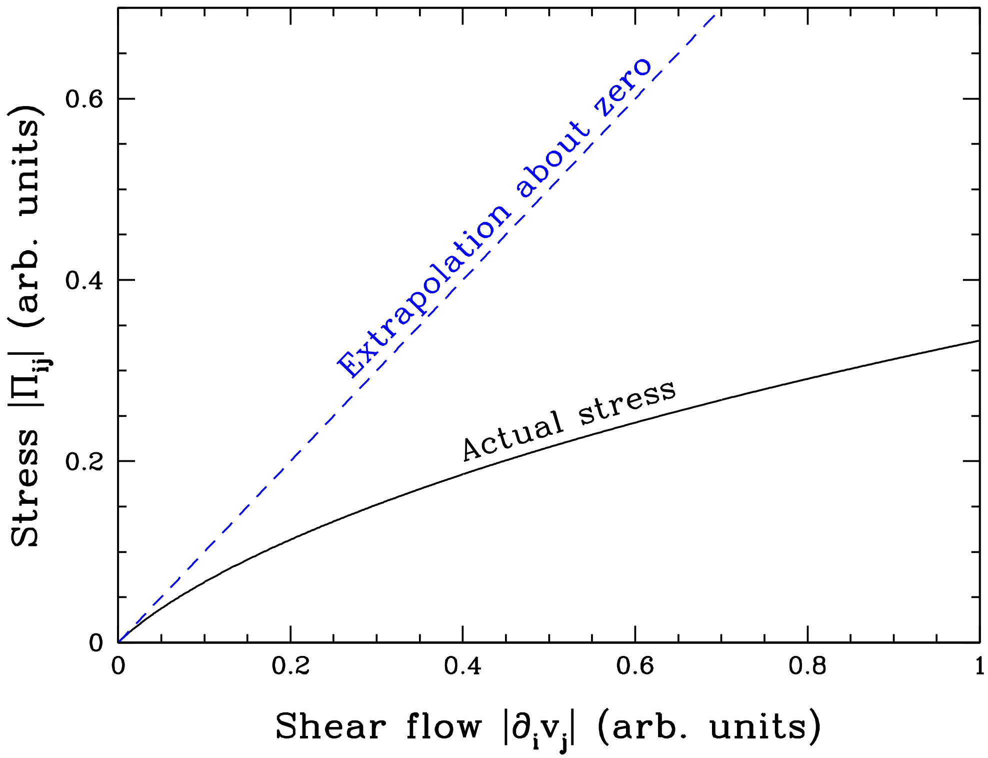
$$\frac{d(\delta f)}{dt} \sim \delta f \frac{p^2 \sim T^2}{\hat{q}} \sim \delta f^{5/2}/g^3$$

implies departure from equilibrium of

$$\delta f \sim \left(\frac{\partial_i v_j}{g^3 T^2} \right)^{2/5}$$

Weak anisotropy: prediction

Whenever δf above *smaller* than $\delta f_{\rm scatt} \sim \frac{\partial_i v_j}{g^4 T}$ plasma instabilities dominate. Parametrically $g^3 T \gg \partial_i v_j \gg g^{14/3} T$.



What about N=4 SYM?

None of previous special to QCD matter content

All of previous should arise at planar diagram level

Should all go through for $\mathcal{N}=4$ SYM replacing $g^2 \rightarrow g^2 N_{\rm c} = \lambda$

All of previous relied ESSENTIALLY on weak coupling

Gauge invariant ways to look if plasma instabilities happen?

Gauge invariant measurables and expected behavior:

- Behavior of T_{ij} versus $\partial_i v_j$
- Smeared field operator expectation values
- Features of Wilson lines (indicating anisotropic $\hat{q}$)

No guarantee plasma instabilities exist, or are even well defined, at strong coupling!

Behavior of T_{ij}

"Weak anisotropy" discussion predicts

$$T_{ij} \sim \partial_i v_j + (\partial v)^2 + (\partial v)^{5/2}$$

where first two arise from (boring) scattering
nonanalytic term arises from plasma instabilities.

Behavior through $(\partial_i v_j)^2$ known Baier Romatschke Son Starinets Stephanov.
Next order....

Smeared field operators

IR fields can be unambiguously extracted by defining

$$A_\mu[\tau]: \qquad A_\mu[0] = A_\mu\,, \quad \frac{dA_\mu}{d\tau} = D_j F_{j\mu}\,.$$

Build operators with $A[\tau]$: contain only $k^2 \sim \tau$ physics

Equivalent viewpoint: operators not fields are smeared.

Expect: under anisotropic flow, $B^2_{ij}[\tau]$ changes:
For Bjorken flow,

- large k: $B^2_{\|}$ larger, $B^2_{\perp}$ smaller
  (mostly planar particles with transverse B fields)
- small k: $B_{\perp}$ larger (unstable modes)

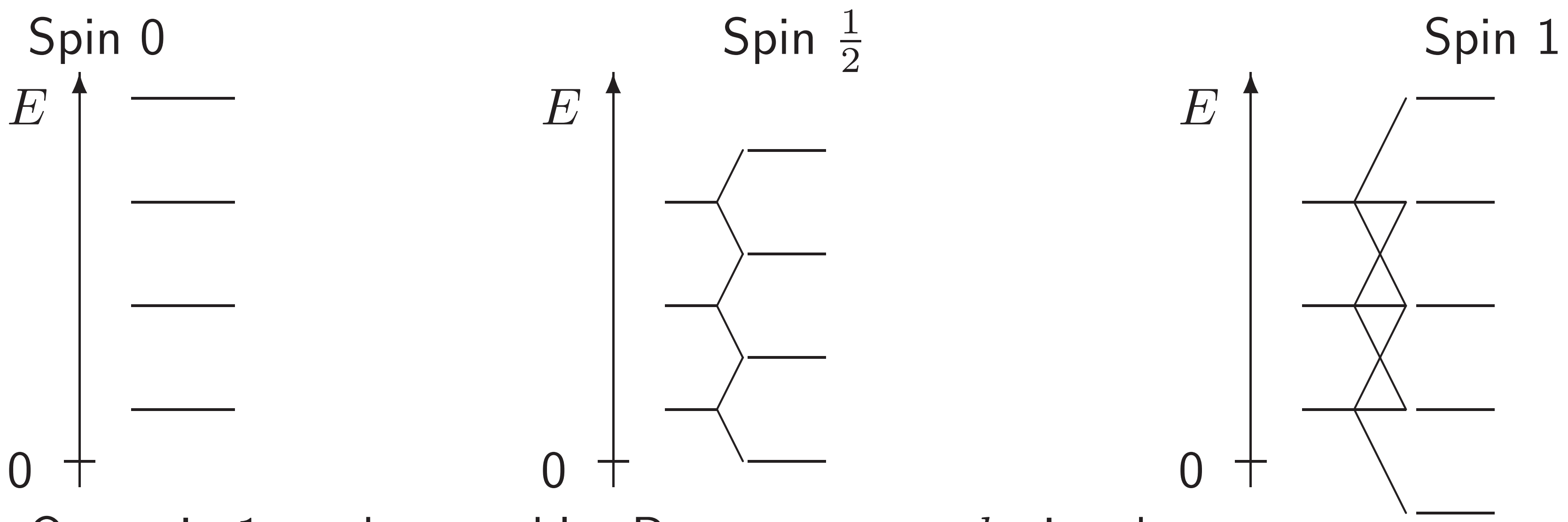
Conclusions

- Plasma instabilities are generic prediction of
  - weak coupling $+$
  - anisotropic flow
- Different scaling depending on degree of anisotropy.
  - Strong anisotropy: $B^2 \sim 1/\theta^2$, $\hat{q} \sim 1/\theta^2$
  - Weak anisotropy: $B^2 \sim \epsilon^2$, $\hat{q} \sim \epsilon^{3/2}$.
- Plas. instabilities should be there in $N{=}4$ SYM for $\lambda \ll 1$
- Strong coupling (either theory): may not even be well defined

Aside: Nielsen-Olsen Instability

What if fields are very "clean" with 1 mode + tiny fluctuations?

B field splits states into Landau levels. Split by $\vec{s} \cdot \vec{B}$.



One spin-1 mode unstable. But at nonzero k_z it takes

$$E^2 = k_z^2 + (s_z + \tfrac{1}{2})B < 0 \rightarrow B > k_{\max}^2, B^2/g^2 > m^4/g^2\theta^4$$