Comparison of Boltzmann Kinetics with Quantum Dynamics for Relativistic Quantum Fields

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Nonequilibrium Phenomena in Cosmology and Particle Physics

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Outline

Motivations for going into the subject

The situation

- Many interesting phenomena in particle physics and cosmology require the description of systems out of thermal equilibrium.
- Very often, such nonequilibrium situations are treated by means of (approximations to) Boltzmann equations.
- However, Boltzmann equations are only a classical approximation to the quantum thermalization process described by Kadanoff-Baym equations.

An obvious question

How reliable are Boltzmann equations as compared to Kadanoff-Baym equations?

Boltzmann Equation

for a spatially homogeneous system in the framework of a real scalar Φ^4 quantum field theory:

$$\partial_t n(t,\mathbf{k}) = \frac{\lambda^2\pi}{48}\int\frac{d^3p}{(2\pi)^3}\int\frac{d^3q}{(2\pi)^3}\int d^3r \Bigg[\frac{1}{E_k E_p E_q E_r} \times \delta(\mathbf{k}+\mathbf{p}-\mathbf{q}-\mathbf{r})\ \delta(E_k+E_p-E_q-E_r) \times \Big(\underbrace{(1+n_\mathbf{k})(1+n_\mathbf{p})\,n_\mathbf{q}n_\mathbf{r}}_{\text{gain term}} - \underbrace{n_\mathbf{k}n_\mathbf{p}(1+n_\mathbf{q})(1+n_\mathbf{r})}_{\text{loss term}}\Big)\Bigg]$$

Momentum conservation — Energy conservation

Isotropy: 9 dimensional integral $\Longrightarrow$ 2 dimensional integral.
Important for numerics! [Dolgov, Hansen, Semikoz (1997)]

Complete Schwinger-Keldysh Propagator

Definition

$$G(x,y) = \langle T_{\mathcal{C}} \{ \Phi(x) \Phi(y) \} \rangle$$

The index $\mathcal{C}$ denotes time ordering along the closed Schwinger-Keldysh real-time contour.

Decomposition [Aarts, Berges (2001)]

$$G(x,y) = G_F(x,y) - \frac{i}{2} \mathrm{sign}_{\mathcal{C}} \left(x^0 - y^0 \right) G_\varrho(x,y)$$

- Statistical propagator $\Longrightarrow$ effective particle number
- Spectral function $\Longrightarrow$ thermal mass, decay width

Effective Energy and Particle Number Densities

Free-field ansatz [Berges (2002)]

Effective kinetic energy density:

$$\omega^2(t,k) = \left(\frac{\partial_{x^0}\partial_{y^0} G_F(x^0, y^0, k)}{G_F(x^0, y^0, k)} \right)_{x^0 = y^0 = t}$$

Effective particle number density:

$$n(t,k) = \omega(t,k)\, G_F(t,t,k) - \frac{1}{2}$$

Advantages of these definitions

- They furnish a particle number density which thermalizes.
- They do not rely on any quasi-particle assumption.
- They comprise conserved charges, if present in the theory.

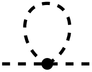
Kadanoff-Baym Equations

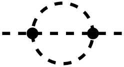
for a spatially homogeneous and isotropic system in the framework of a real scalar Φ^4 quantum field theory:

$$\left[\partial_{x^0}^2 + k^2 + M^2\left(x^0\right)\right] G_F\left(x^0, y^0, k\right)$$

$$= \int_0^{y^0} dz^0 \, \Pi_F\left(x^0, z^0, k\right) G_\varrho\left(z^0, y^0, k\right)$$

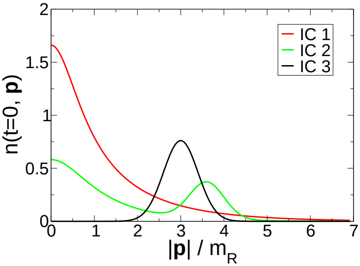
$$- \int_0^{x^0} dz^0 \, \Pi_\varrho\left(x^0, z^0, k\right) G_F\left(z^0, y^0, k\right)$$

Effective mass: $M^2\left(x^0\right) = m^2 +$ 

Nonlocal self-energy: $\Pi\left(x^0, z^0, k\right) =$ 

Internal lines represent the complete Schwinger-Keldysh propagator!

Initial Conditions

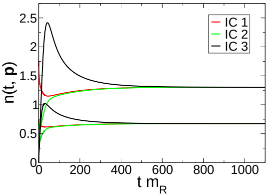


- All initial conditions correspond to the same (conserved) average energy density.
- The initial conditions IC1 and IC2 correspond to the same initial total particle number.

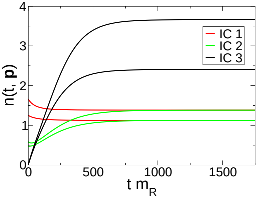
[Manfred Lindner, MMM (2006)]

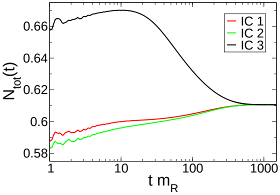
Universality

Kadanoff-Baym		Boltzmann

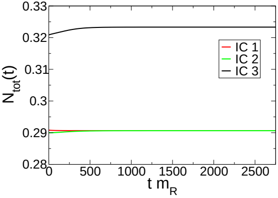


Evolution of the particle number densities





Evolution of the total particle numbers



Full Universality

Restricted Universality

[Manfred Lindner, MMM (2006)]

Chemical Equilibration

Boltzmann Kinetics vs. Quantum Dynamics

Markus Michael Müller

Motivations

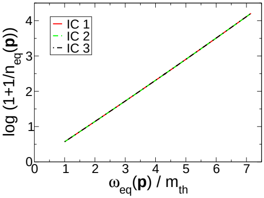
Boltzmann Kinetics

Quantum Dynamics

Comparison of Numerical Solutions

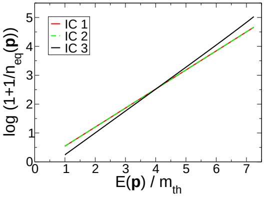
Conclusions and Outlook

Kadanoff-Baym



Equilibrium particle number densities

Boltzmann



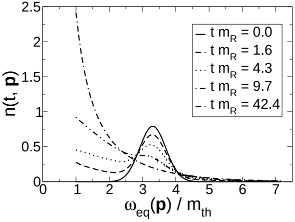
- Full Universality
- Chemical Equilibration

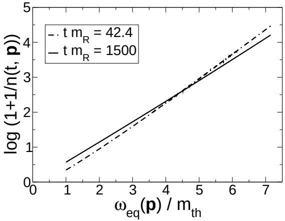
- Restricted Universality
- No Chemical Equilibration

[Manfred Lindner, MMM (2006)]

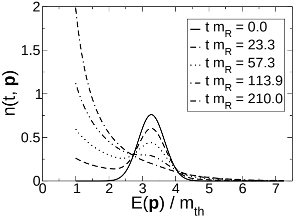
Separation of Time Scales

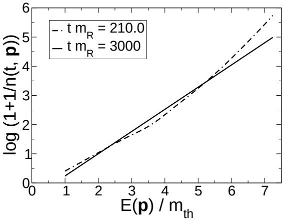
Kadanoff-Baym





Boltzmann





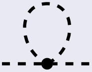
[Manfred Lindner, MMM (2006)]

Generalization to fermionic theories

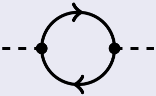
$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetric Yukawa model

$$\lambda \left(\Phi_a \Phi_a\right)^2 + i\eta \bar{\Psi} \Phi_a \left(\sigma_a P_R - \sigma_a^\dagger P_L\right) \Psi$$

Effective scalar mass:

$$M^2\left(x^0\right) = m^2 +$$ 

Nonlocal Self Energies:

scalars: $\Pi\left(x^0, z^0, k\right) =$ 

fermions: $\Sigma\left(x^0, z^0, k\right) =$ 

Generalization to fermionic theories
Cont.

Kadanoff-Baym Equations [Manfred Lindner, MMM (2008)]

- Full universality [Berges et al. (2003)]
- Quantum-chemical equilibration [Berges et al. (2003)]
- Prethermalization [Berges et al. (2004)]

Boltzmann equations [Manfred Lindner, MMM (2008); MMM (2006)]

- Restricted universality
- Classical, but no quantum-chemical equilibration
- No separation of time scales

Conclusions

Quantum Dynamics (Kadanoff-Baym equations)

- take memory and off-shell effects into account.
- respect full universality.
- include chemical equilibration.
- separate time scales between kinetic and chemical equilibration.

Classical Kinetics (Standard Boltzmann equations)

- do not take memory and off-shell effects into account (molecular chaos for quasi-particles).
- comprise fake constants of motion.
- respect only a restricted universality.
- do not include quantum chemical equilibration, and therefore
- cannot separate time scales between kinetic and chemical equilibration.

Outlook

Renormalization of the 2PI effective action for a real scalar $\lambda\Phi^4/4!$ theory at three-loop order

Standard approximate perturbative renormalization

A18: $\lambda = 18,\ m_B^2 = -6.87\ m_R^2$

A24: $\lambda = 24,\ m_B^2 = -9.49\ m_R^2$

Exact nonperturbative renormalization at zero temperature

E18: $\lambda_R = 18,\ \lambda_B = 37.18,\ m_B^2 = -14.39\ m_R^2$

E24: $\lambda_R = 24,\ \lambda_B = 63.43,\ m_B^2 = -25.14\ m_R^2$