A Matrix Model for Black Hole Thermalization

N. Iizuka & JP, arXiv 0801.3657
N. Iizuka, T. Okuda & JP, in progress

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Nonequilibrium Phenomena in Cosmology and
Particle Physics, KITP, Feb. 27, 2008

Many talks at this meeting:

black holes → nonequilibrium QFT

With a duality, information flows in both directions:

black holes ← nonequilibrium QFT

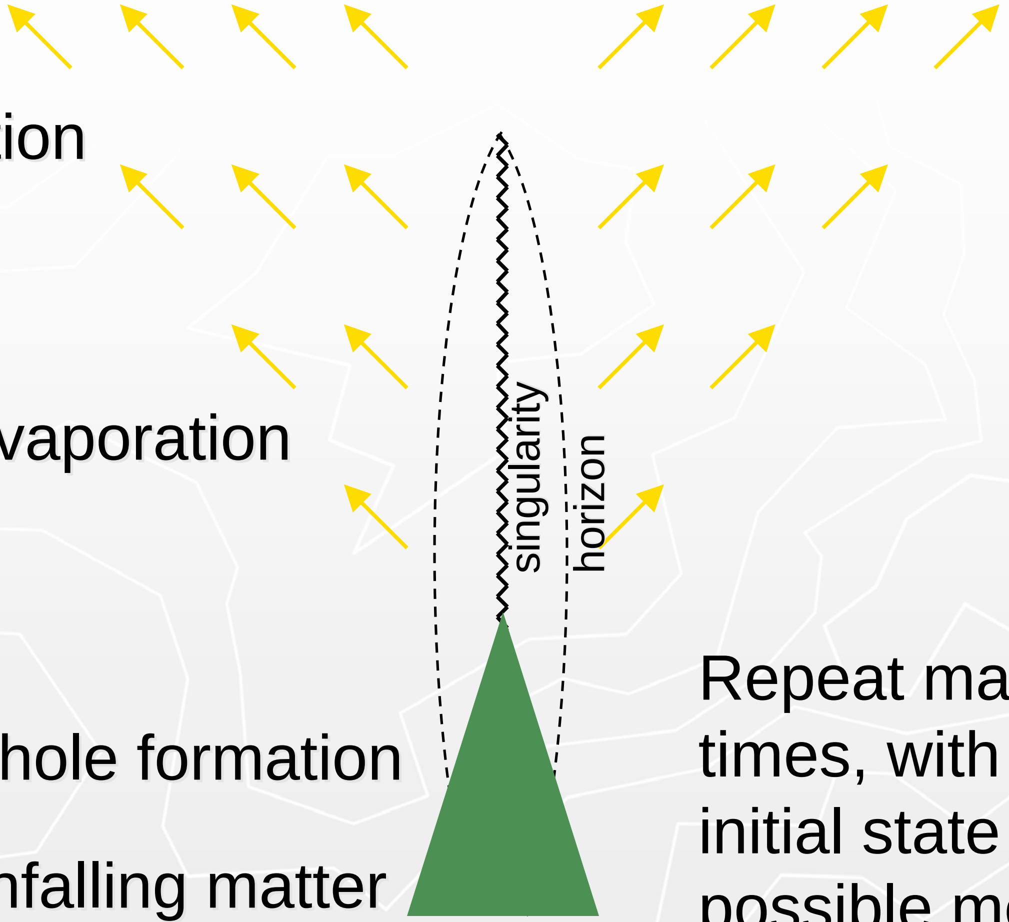
cf. talk by F. Cooper

cf. talk by D. Bak

Outline:

- A brief review of the information problem
- What has been learned, and what we would still like to understand
- A toy model for the black hole in the AdS/CFT correspondence

Hawking's thought experiment (1976):



4. Final state:
Hawking radiation

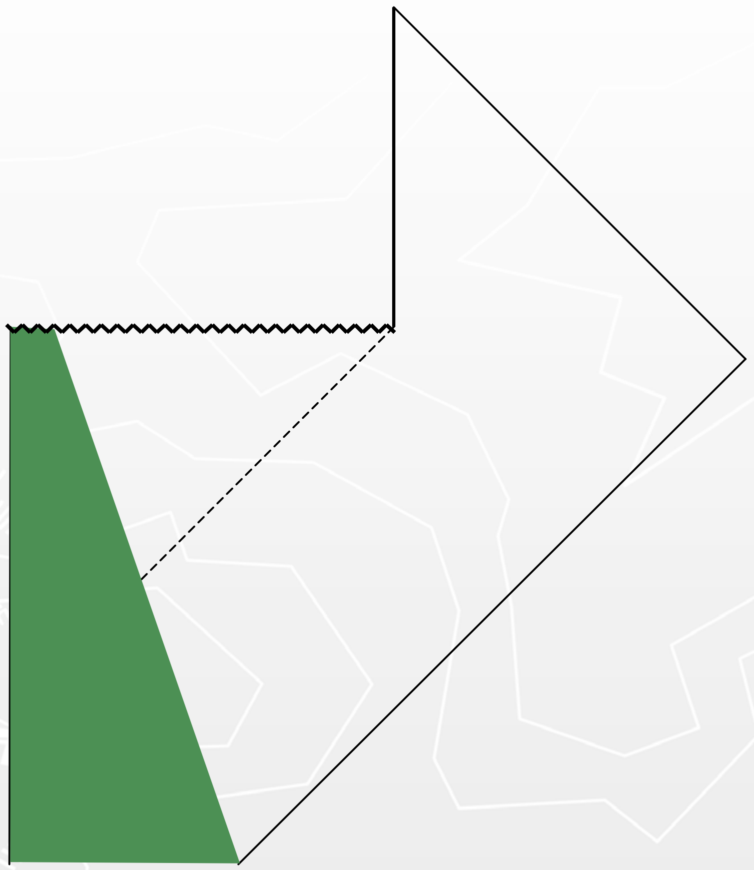
3. Black hole evaporation

2. Black hole formation

1. Initial state: infalling matter

Repeat many times, with same initial state and all possible measurements on the final state.

Conclusion: initial pure state must evolve to a density matrix. Information is lost.



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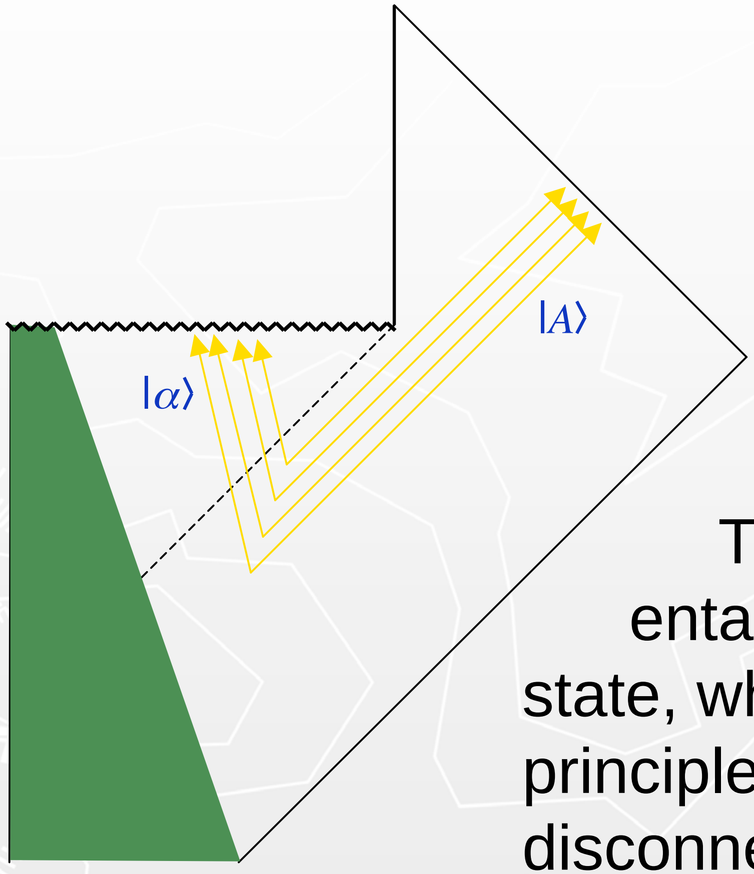

The external state is entangled with the internal state, which is unobservable in principle, because it is causally disconnected from the exterior.

"Not only does God play dice, but he sometimes throws them where they cannot be seen."

This thought experiment implies a breakdown of the ordinary rules of quantum mechanics, and this should be happening everywhere, all the time, via virtual black holes.

In addition to this *paradox* there is a related *puzzle*: black holes satisfy thermodynamic laws, with a Bekenstein-Hawking entropy $S = A/4l_P^2$. What is the underlying *statistical mechanics?* What *states* does this entropy count.

This talk will focus on the paradox.

Possible outcomes to black hole evaporation:

1. The state of the Hawking radiation is actually pure. The information (about what went into the black hole) is encoded in the Hawking radiation.

2. The state is indeed mixed. Information is lost.

3. The evaporation does not proceed to completion, but terminates in a stable remnant with a very large number of internal states.

4. A remnant which (slowly) decays, reemitting the information.

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Why can't information be lost?

This seems to imply energy non-conservation (Banks, Peskin, Susskind, 1984). Argument: a local time-evolution law that turns pure into mixed states can be written as evolution with a spacetime-dependent Hamiltonian (Ellis, Hagelin, Nanopoulos, Srednicki, 1984).

$$\mathcal{H}_{\text{eff}} = \sum_\alpha g_\alpha(t,x)\, O_\alpha(t,x)$$

Here, $O_\alpha(t,x)$ are some set of local operators, and $g_\alpha(t,x)$ is averaged incoherently. That is, *spacetime is dirty.*

Even if the *average* distribution of dirt is translation-invariant, any given distribution is not, and so momentum and energy conservation are lost.

Loopholes?

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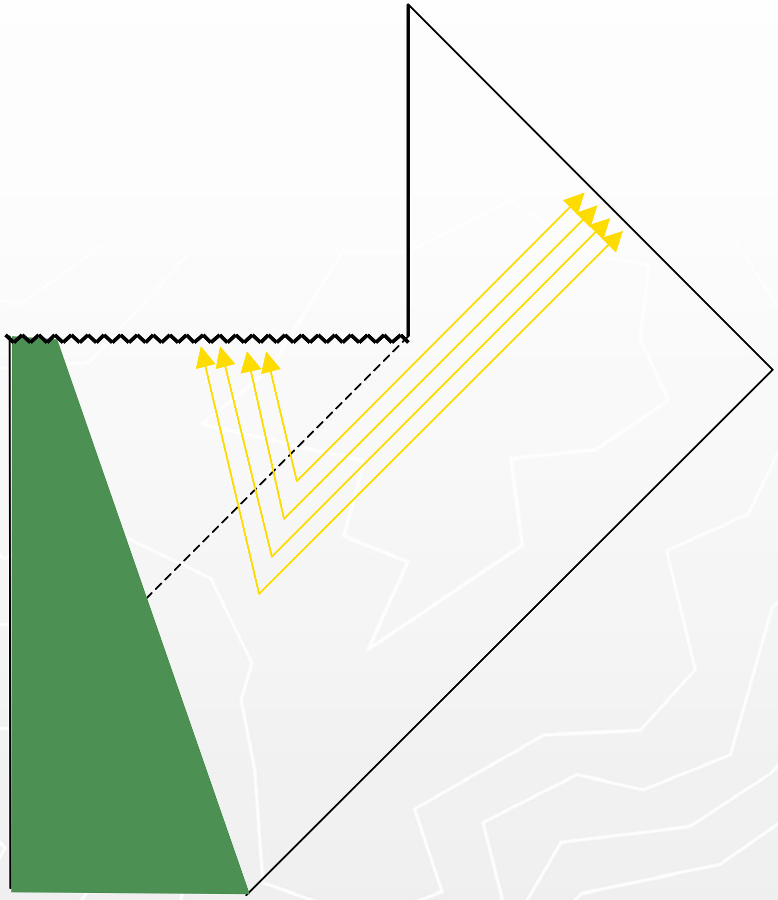
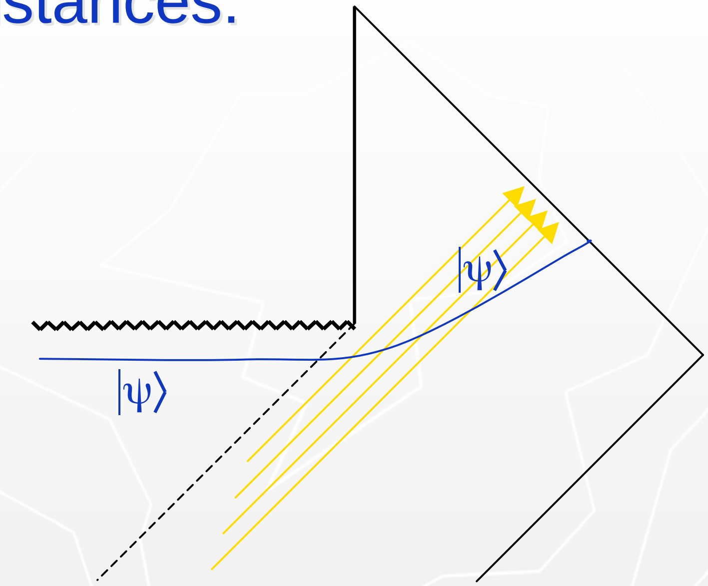


Thinking about the singularity leads back to the same set of possibilities

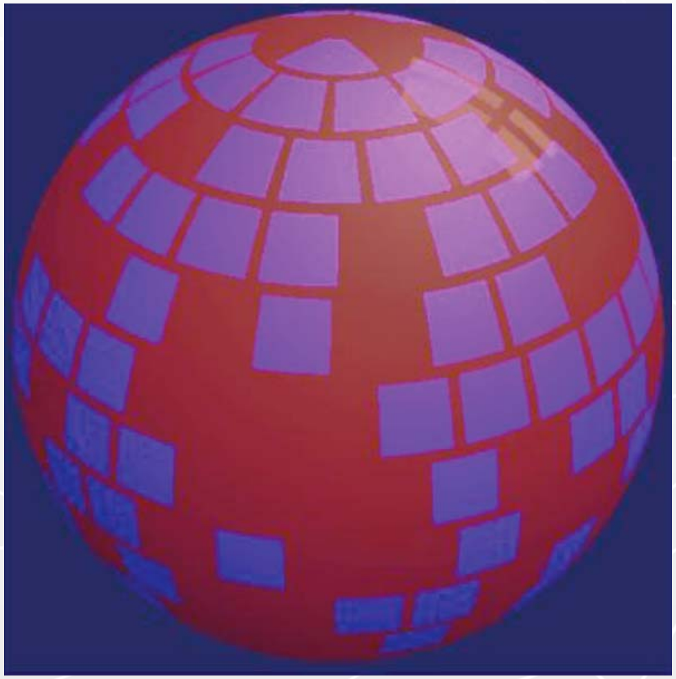
Lesson: In order for the information to be in the Hawking radiation, it must be transmitted over large spacelike distances:



Quantum xerox principle forbids duplication of bits.

Black hole complementarity (Susskind, 1993): these are the *same* bit as seen by two different observers --- radically nonlocal…

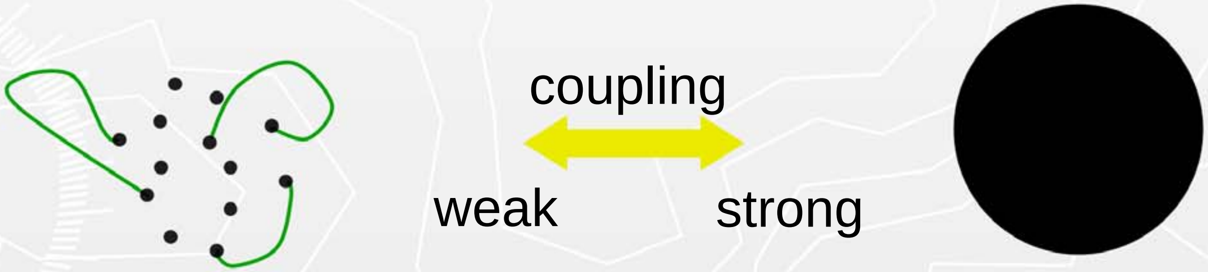
Another hint of radical nonlocality: the Bekenstein-Hawking entropy $S = A/4l_P^2$ suggests the *holographic principle* (‘t Hooft, Susskind, 1993), that quantum gravity in any space can be formulated in terms of degrees of freedom living on the *boundary* of the space.



From G. ‘t Hooft

Black hole entropy counting:

Strominger and Vafa (1996) argued that by turning down the coupling one could adiabatically turn some supersymmetric black holes into weakly coupled systems whose states can be explicitly counted, giving a statistical interpretation to the Bekenstein-Hawking entropy.



coupling

weak

strong

D-branes and strings

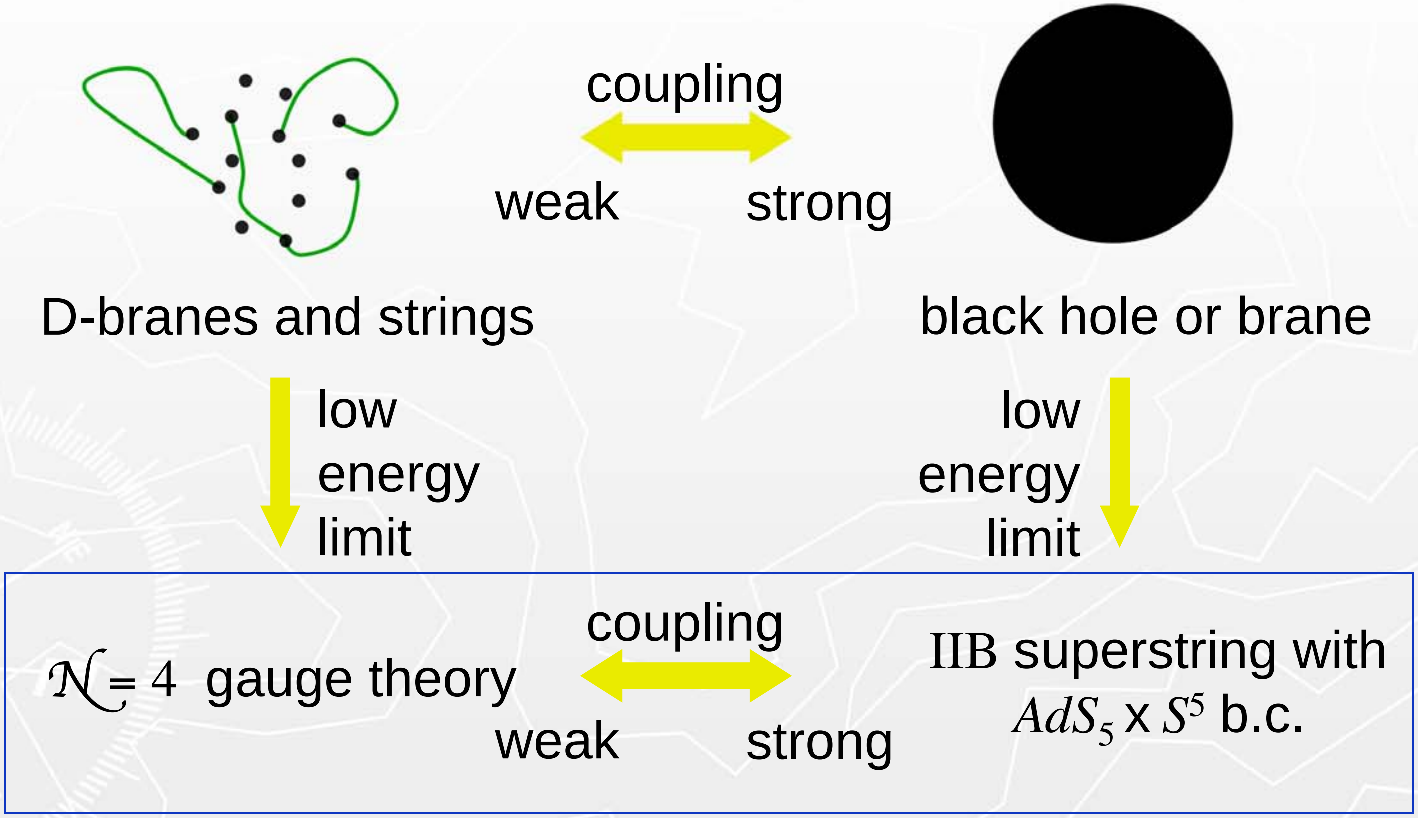
black hole or brane

Motivated by the information paradox, various groups studied dynamical properties of this system (scattering amplitudes, decays) and found surprising agreements between very different calculations:

Field theory loop graph

Gravitational tree amplitude in black hole background

Maldacena (1997) explained this in terms of a new duality:



D-branes and strings

coupling

weak strong

black hole or brane

low energy limit

low energy limit

$\mathcal{N} = 4$ gauge theory

coupling

weak strong

IIB superstring with AdS_5 x S^5 b.c.

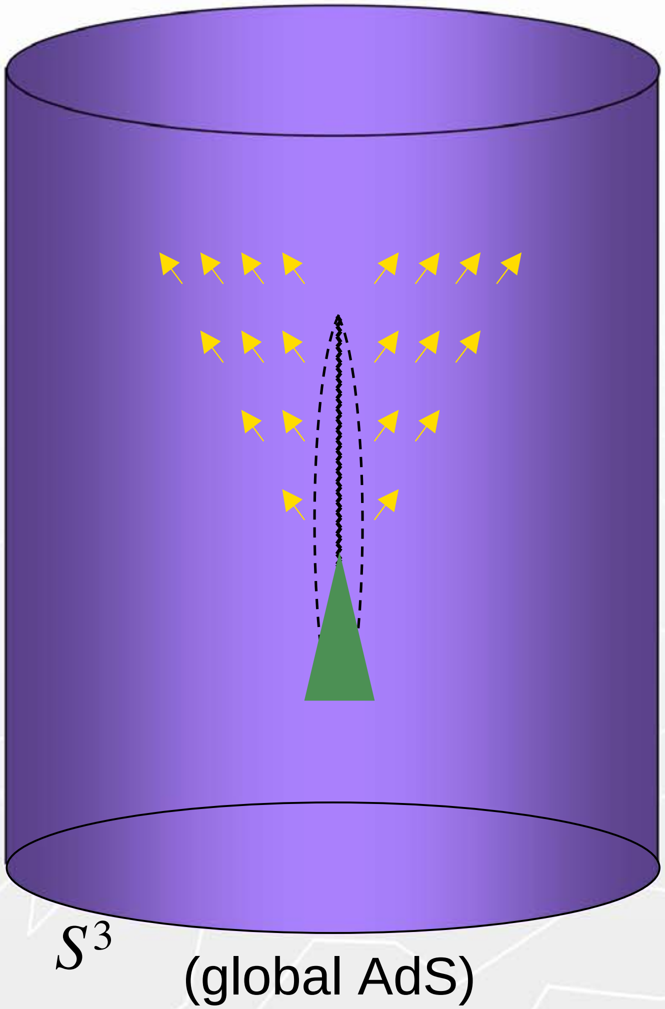
- This duality provides an *algorithmic* nonperturbative constuction of string theory with AdS boundary conditions: we could simulate the dual gauge theory on a (large enough) computer. (cf. Ken Wilson and QFT).

- It allows solution to some strongly coupled gauge theories.

- It tells us where the information goes.

We can repeat Hawking's thought experiment in an AdS box. The dual description is in an ordinary coherent system: information is preserved (option 1). A black hole is dual to a plasma of hot glue, not so different from a lump of coal.

The gauge theory variables are indeed strongly nonlocal, and holographic (the gauge theory lives on the boundary).



The future…

The problems of quantum gravity:

- UV divergences. ✓
- The cosmological constant. ✓
- Black hole entropy. ✓
- Black hole information. ✓
- Cosmological singularities; initial conditions.
- The interpretation of quantum mechanics, applied to the whole universe.

- The nonperturbative construction of string theory in a cosmological setting.

Is there more to learn from Hawking's paradox?

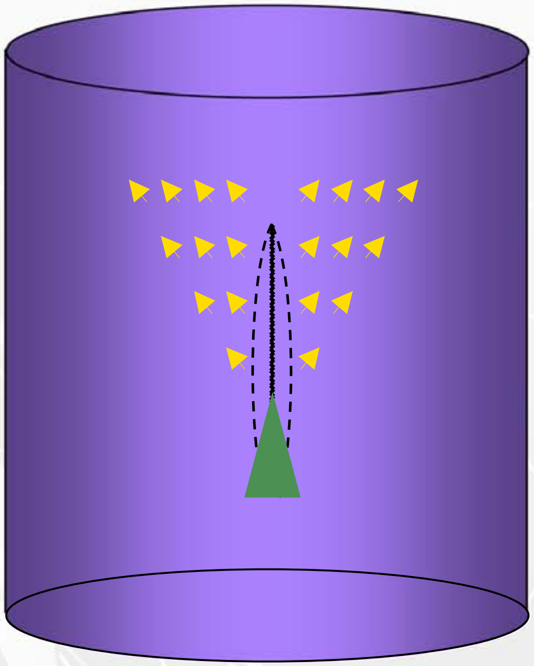
Answering one question raises a new one: where does the argument for information loss break down?

1. Perturbative gravitational corrections?
2. Semiclassical gravitational corrections?
3. Fully nonperturbative gravity?
4. Perturbative string corrections?

How does locality emerge, and how does it break down?

How do we calculate the black hole S-matrix?

In AdS/CFT:



final bulk state

duality

final CFT state

gauge
theory
evolution

initial bulk state

duality

initial CFT state

Can we short-circuit this?

First step: look for toy models

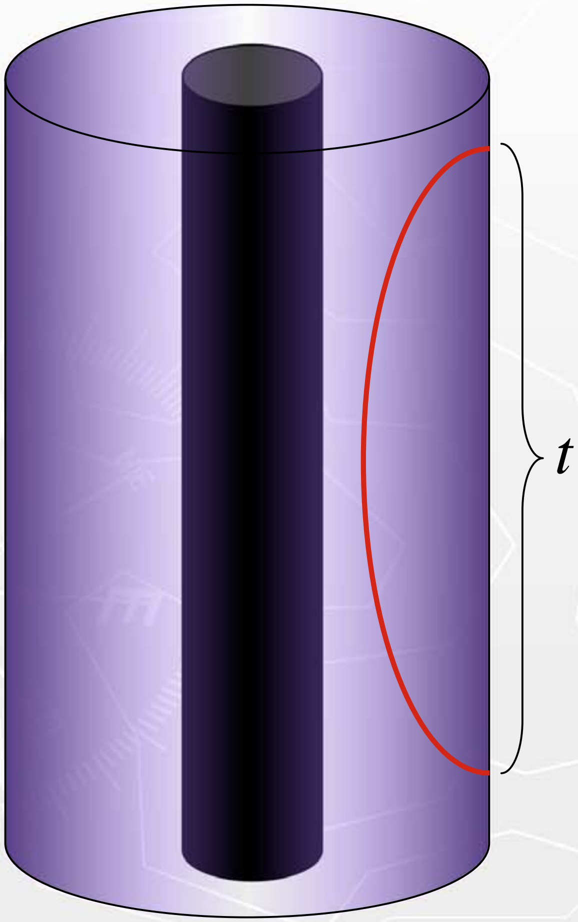
Looking for a toy model: Spacetime interpretation in AdS/CFT exists only at strong gauge coupling; at weak coupling, string is larger than curvature scale. Nevertheless, some properties of black holes persist to weak coupling:

- Hawking-Page/deconfinement transition
(Skagerstam 1993, Sundborg hep-th/9908001)

- Singularity (Festuccia & Liu hep-th/0506202)

- Information problem (Festuccia & Liu hep-th/0611098)

Maldacena’s version of the information problem: In AdS/CFT it is natural to consider *eternal* (non-evaporating) black holes, which correspond to thermal equilibrium in the gauge theory.



In bulk gravity theory: two-point function falls exponentially at late times because of the horizon (should hold for N and g^2N large but finite).

In the gauge theory, there must eventually be recurrences at finite N. Can we see this in $1/N$ expansion (= gravitational loop expansion).

Festuccia and Liu argue that this behavior, exponential decay in the planar limit, recurrences at any finite N, persists to weak coupling g^2N.

- Apparent problem: individual Feynman graphs do not have exponential decay, and g^2N is expected to converge in the planar limit.
- Resolution: radius of convergence goes to zero as t goes to infinity due to secular effects.

We would like to go further, and actually find the exponential decay analytically, to understand how it breaks down at finite N.

Note: at weak coupling there is no notion of bulk locality, so no sharp paradox, but the $1/N$ structure remains.

F&L actually reduce to quantum mechanics of two $N \times N$ matrices, with potential

$$m^2\,\mathrm{Tr}(X^2+Y^2) + g^2\,\mathrm{Tr}[X,Y]^2$$

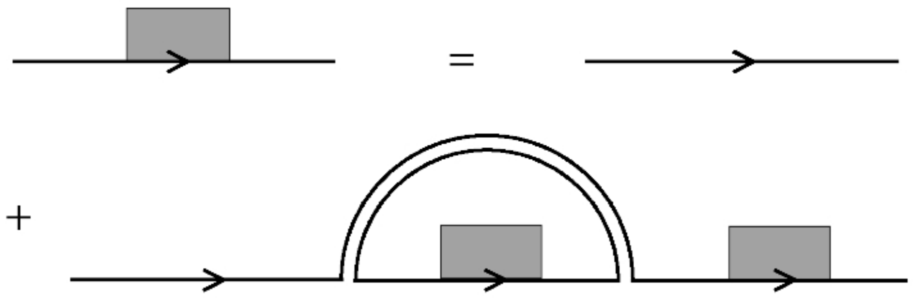
This is still a hard large-N problem, so simplify further to *One* matrix X and one fundamental ϕ and potential

$$m^2\,\mathrm{Tr}(X^2) + M^2\phi^\dagger\phi + g\,\phi^\dagger X\phi + g'(\phi^\dagger\phi)^2$$

X acts as a heat bath coupled to ϕ (for simplicity we can take $M >> T$ so there are no ϕ's in thermal eq.). Study $\phi^\dagger\,\phi$ two-point function.

$X \sim N$ D0 black hole, $\phi \sim$ D0 probe.

Graphical structure the same as 't Hooft's 2-D QCD:



$$\tilde{G}(\omega) = \tilde{G}_0(\omega) - \lambda \tilde{G}_0(\omega)\tilde{G}(\omega)\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\tilde{G}(\omega')\tilde{K}_0(\omega-\omega')$$

$$\tilde{G}_0(\omega) = \frac{i}{\omega + i\epsilon}\ , \quad \tilde{K}_0(\omega) = \frac{i}{\omega^2 - m^2 + i\epsilon}$$

($\tilde{G}_0(\omega)$ has been expanded near pole, and shifted, using M large)

Close contour, get

$$\tilde{G}(\omega) = \frac{i}{\omega}\left(1 - \frac{\lambda}{2m}\tilde{G}(\omega)\tilde{G}(\omega - m)\right)$$

At finite temperature we don’t need full Schwinger-Keldysh, because ensemble is free. Get same Schwinger-Dyson equation with thermal propagator

$$\tilde{K}_0(T,\omega) = \frac{i}{1-e^{-m/T}}\left(\frac{1}{\omega^2-m^2+i\epsilon} - \frac{e^{-m/T}}{\omega^2-m^2-i\epsilon}\right)$$

Slightly more complicated result:

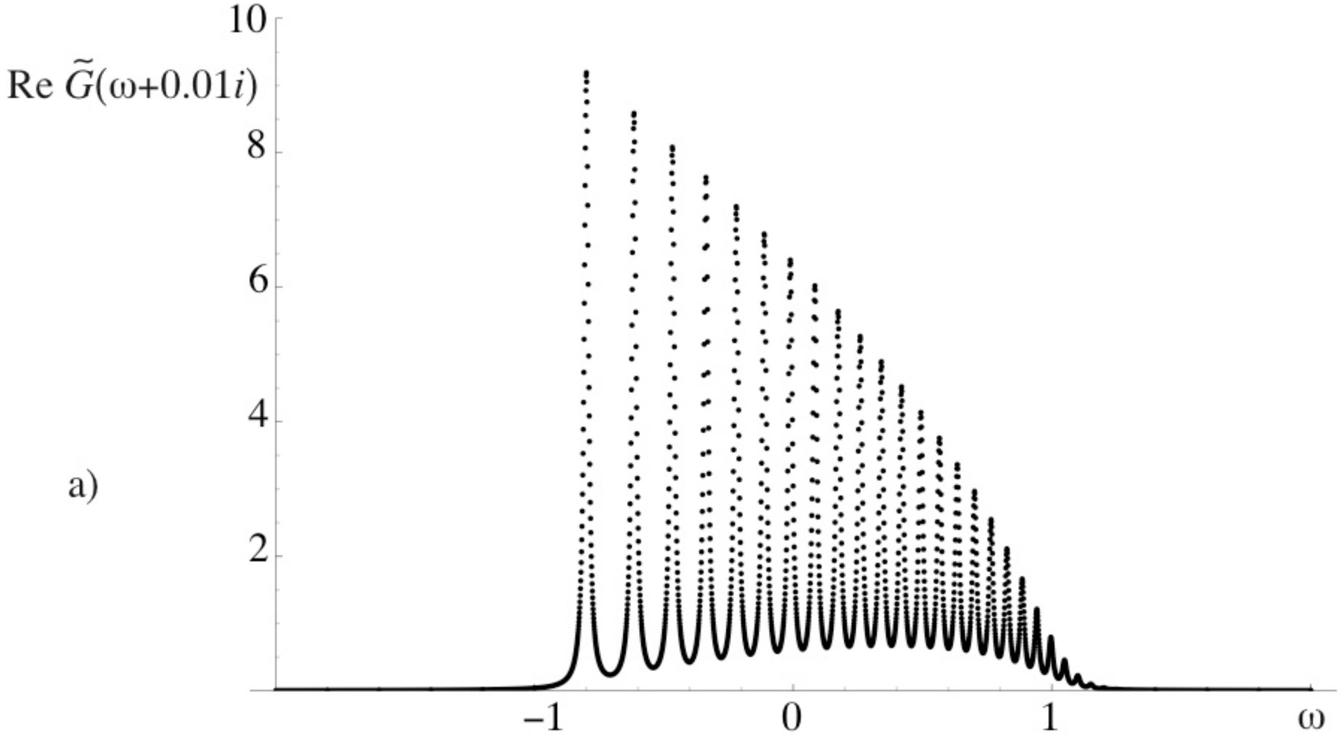
$$\tilde{G}(T,\omega) = \frac{i}{\omega}\left\{1 - \frac{\nu^2}{4(1-e^{-m/T})}\tilde{G}(T,\omega)\left[\tilde{G}(T,\omega-m) + e^{-m/T}\tilde{G}(T,\omega+m)\right]\right\}$$

$T = 0$: $$\tilde{G}(\omega) = \frac{i}{\omega}\left(1 - \frac{\lambda}{2m}\tilde{G}(\omega)\tilde{G}(\omega-m)\right)$$

One strategy: recurse from $\tilde{G}(T,\omega) \sim \frac{i}{\omega}$ at large $|\omega|$.

This works if recursion is stable, which is the case only for $T = 0$.

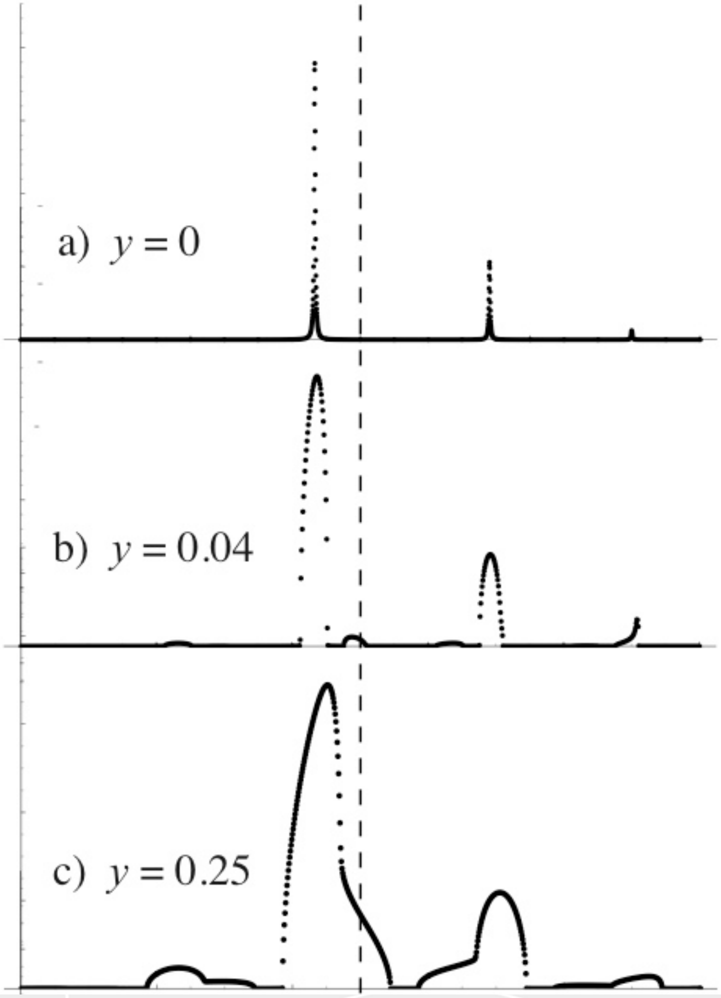
Zero temperature - poles on real axis:

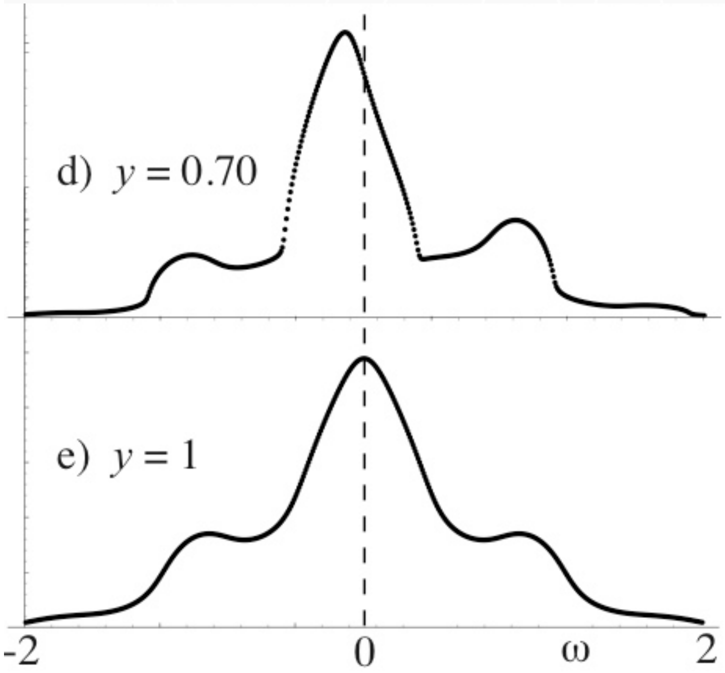


Finite temperature - poles widen into cuts, which then merge:

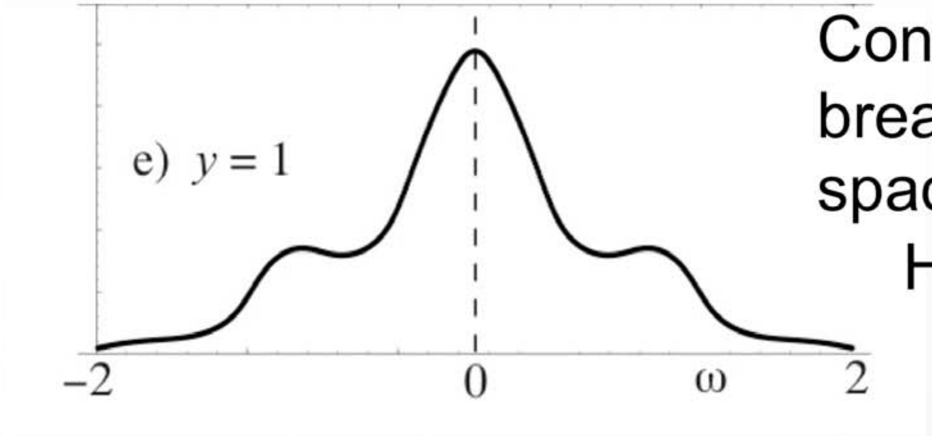
$$y = e^{-m/T}$$

coupling smaller than previous slide





What can we learn from this?

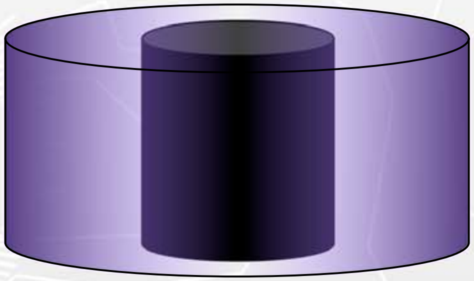


Continuous spectral weight breaks up into poles with spacing of order $\exp\{-O(N^2)\}$.

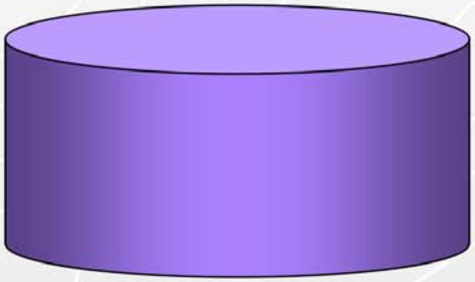
How do we see this in the $1/N$ expansion?

Conjecture of Maldacena (Hawking): additional Euclidean saddle, weight $\exp\{-O(N^2)\}$.

AdS black hole



thermal AdS



Problem (Barbon and Rabinovici): $\exp\{-O(N^2)\}$ do not have necessary secular growth.

Another conjecture…

Q: This model is so simple, what can remain?

A: The *stringy exclusion principle.* For $N \times N$ matrices, $\mathrm{Tr}(X^k)$ is not independent for $k > N$. This implies that the string Hilbert space is smaller than the naïve Fock space.

Conjecture: this is the same reduction as required by that required by black hole complementarity.

Ongoing work:

- Look for simpler model (Iizuka, Okuda, JP), would like to be able to solve for finite N. E.g. interaction

$$q_{kl}Q^{kl}$$

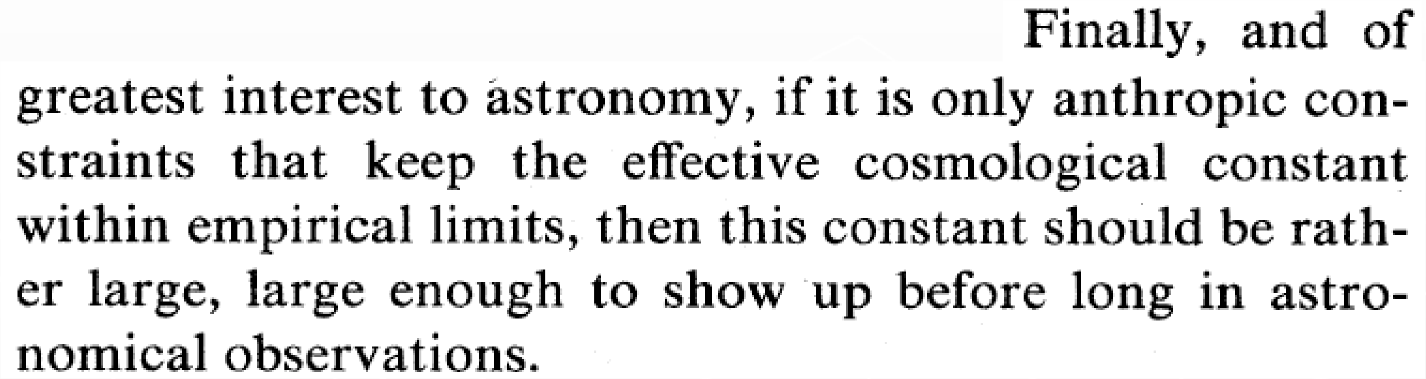
(fundamental, adjoint $U(N)$ generators).

- Translate into language of loop equations (~ gravitational/string variables).

Conclusions

The information problem has been a very fruitful thought experiment, there is likely more to be learned…


N
NE
E
SE
S
NW



Finally, and of greatest interest to astronomy, if it is only anthropic constraints that keep the effective cosmological constant within empirical limits, then this constant should be rather large, large enough to show up before long in astronomical observations.

From Weinberg (1989).