INTRO TO NON-EQUILIBRIUM 2PI EFFECTIVE ACTION TECHNIQUES

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INTRODUCTION

nonequilibrium quantum field theory:

- framework with many applications
- in early universe:
  inflation, baryon asymmetry, phase transitions, ...
- in relativistic heavy ion collisions
  probing strongly interacting matter/extreme QCD
- in atomic physics, BEC, plasma physics, ...

INTRODUCTION

in this lecture:

- emphasis on methods
- relativistic quantum fields
- a few illustrations

REFERENCES

- I'll discuss work of many (relativistic) people, not properly inserting references throughout

Berges, Cox (2000)
Aarts, Berges, + Ahrensmeier, Baier, Serreau
Berges + Borsanyi, Serreau, Wetterich, + Reinosa
Cooper, Dawson, Mihaila
Juchem, Cassing, Greiner
Müller, Lindner
Arrizabalaga, Smit, Tranberg
Rajantie, Tranberg
Aarts + Bonini, Wetterich, + Martinez Resco, + Tranberg
Jeon, Yaffe
Calzetta, Hu
Carrington *et al*
...

Outline

- what is nonequilibrium field theory?
- mean field theory
- 2PI effective action
- a few selected applications
- transport

QUANTUM DYNAMICS

GENERAL FORMULATION

well-defined problem:

- initial conditions: density matrix ρ_D
- time evolution: Heisenberg e.o.m. $\mathcal{O}(t) = e^{iHt}\mathcal{O}e^{-iHt}$
- observables:

$$\langle\mathcal{O}(t)\rangle = \mathrm{Tr}\,\rho_D\,\mathcal{O}(t) \qquad \langle\mathcal{O}(t)\mathcal{O}(t')\rangle = \mathrm{Tr}\,\rho_D\,\mathcal{O}(t)\mathcal{O}(t') \qquad \text{etc.}$$

- in equilibrium: $\rho_D \sim e^{-H/T}$, commutes with the evolution operator
- time translation invariance:

$$\langle\mathcal{O}(t)\rangle = \langle\mathcal{O}(0)\rangle \qquad \langle\mathcal{O}(t)\mathcal{O}(t')\rangle = G(t-t')$$

QUANTUM DYNAMICS

GENERAL FORMULATION

out of equilibrium:

$$\langle \mathcal{O}(t) \rangle = \mathrm{Tr}\, \rho_D\, \mathcal{O}(t) \qquad \langle \mathcal{O}(t)\mathcal{O}(t') \rangle = \mathrm{Tr}\, \rho_D\, \mathcal{O}(t)\mathcal{O}(t') \qquad \text{etc.}$$

- density matrix ρ_D arbitrary ($[H, \rho_D] \neq 0$)
- initial value problem: start at $t = t_0$
- time translation invariance is broken:

$$\langle \mathcal{O}(t) \rangle = G(t - t_0) \qquad \langle \mathcal{O}(t)\mathcal{O}(t') \rangle = G(t - t_0, t' - t_0)$$

- relation to the initial conditions: memory
- effective independence of t_0 as $t \to \infty$?

NONEQUILIBRIUM DYNAMICS

MAIN OBSTRUCTION

no exact solution method available

- use approximation methods
- language of unequal-time correlation functions
- n-point functions: hierarchy of coupled equations

approximation: truncate hierarchy

- problem not specific for quantum dynamics
- fluctuations: quantum and/or statistical

$\Rightarrow$ consider also classical statistical field theory

NONEQUILIBRIUM DYNAMICS

CORRELATION FUNCTIONS

- quantum field theory

$$\langle\phi(x)\phi(y)\rangle = \underbrace{\int d\phi' d\phi'' \langle\phi'|\rho_D|\phi''\rangle}_{\text{initial cond.}} \underbrace{\int \mathcal{D}\phi\, e^{iS}\phi(x)\phi(y)}_{\text{quantum evolution (path integral)}}$$

initial cond. quantum evolution (path integral)

- classical statistical field theory

$$\langle\phi(x)\phi(y)\rangle_{\rm cl} = \int \underbrace{D\pi D\phi\, \rho_{\rm cl}[\pi,\phi]}_{\text{initial prob. distribution and phase space integral}} \underbrace{\phi(x)\phi(y) \;+\; \text{equations of motion}}_{\text{classical evolution}}$$

initial prob. distribution and phase space integral — classical evolution

NONEQUILIBRIUM DYNAMICS

ASIDE

in classical statistical field theory:

- “exact” evolution can be found numerically

$$\langle\phi(x)\phi(y)\rangle_{\rm cl} = \int D\pi D\phi\, \rho_{\rm cl}[\pi,\phi]\, \phi(x)\phi(y) \quad + \quad \text{e.o.m.}$$

- sample initial conditions from $\rho_{\rm cl}[\pi,\phi]$
- solve e.o.m. for each of them
- average over initial conditions

note:

- classical thermal statistics: $n_{\rm cl}(\omega) = T/\omega$
- Rayleigh-Jeans divergence
- careful with interpretation at late times

Mean field approximations

SIMPLEST ATTEMPT

- equation of motion: $(\Box + m^2)\phi = -\frac{\lambda}{6}\phi^3$
- expectation values:

$$\langle\phi(x)\rangle \quad \text{coupled to} \quad \langle\phi^3(x)\rangle$$

$$G(x,y) = \langle\phi(x)\phi(y)\rangle \quad \text{coupled to} \quad \langle\phi^3(x)\phi(y)\rangle$$

etc.

- mean field/Gaussian/Hartree approximation: replace

$$\phi^3 \to 3\langle\phi^2\rangle\phi \qquad (\langle\phi\rangle = 0 \text{ for simplicity})$$

- self-consistent equation for two-point function

$$\left[\Box + m^2 + \frac{\lambda}{2}G(x,x)\right]G(x,y) = 0$$

Mean field approximations

SIMPLEST ATTEMPT

- successfully truncated hierarchy of correlation functions
- Gaussian approximation for $G(x,y) = \langle \phi(x)\phi(y) \rangle$
- same in quantum and classical theory

alas:

- approximation has a nonthermal fixed point
- best seen using equal-time correlation functions

$$G_{\phi\phi}(x-y,t) = \langle \phi(x,t)\phi(y,t) \rangle$$
$$G_{\pi\pi}(x-y,t) = \langle \pi(x,t)\pi(y,t) \rangle$$
$$G_{\pi\phi}(x-y,t) = \frac{1}{2}\langle \pi(x,t)\phi(y,t) + \phi(x,t)\pi(y,t) \rangle$$

MEAN FIELD APPROXIMATIONS

SIMPLEST ATTEMPT

- Gaussian approximation:

$$\partial_t G_{\phi\phi}(p,t) = 2G_{\pi\phi}(p,t)$$
$$\partial_t G_{\pi\phi}(p,t) = -\bar{\omega}_p^2 G_{\phi\phi}(p,t) + G_{\pi\pi}(p,t)$$
$$\partial_t G_{\pi\pi}(p,t) = -2\bar{\omega}_p^2 G_{\pi\phi}(p,t)$$

with $\bar{\omega}_p^2 = p^2 + m^2 + \frac{\lambda}{2}\langle\phi^2\rangle$

- conserved quantity for every momentum mode p

$$C^2(p) = G_{\phi\phi}(p,t)G_{\pi\pi}(p,t) - G_{\pi\phi}^2(p,t)$$
$$\partial_t C(p) = 0$$

MEAN FIELD APPROXIMATIONS

SIMPLEST ATTEMPT

- nonthermal fixed point:

$$G^*_{\pi\pi}(p) = \bar{\omega}_p^2 G^*_{\phi\phi}(p)$$

$$G^*_{\pi\phi}(p) = 0$$

$$C^2(p) = G^*_{\phi\phi}(p) G^*_{\pi\pi}(p)$$ fixed by initial ensemble

$$\bar{\omega}_p^{*2} = p^2 + m^2 + \frac{\lambda}{2}\langle\phi^2\rangle^*$$

explicit solution:

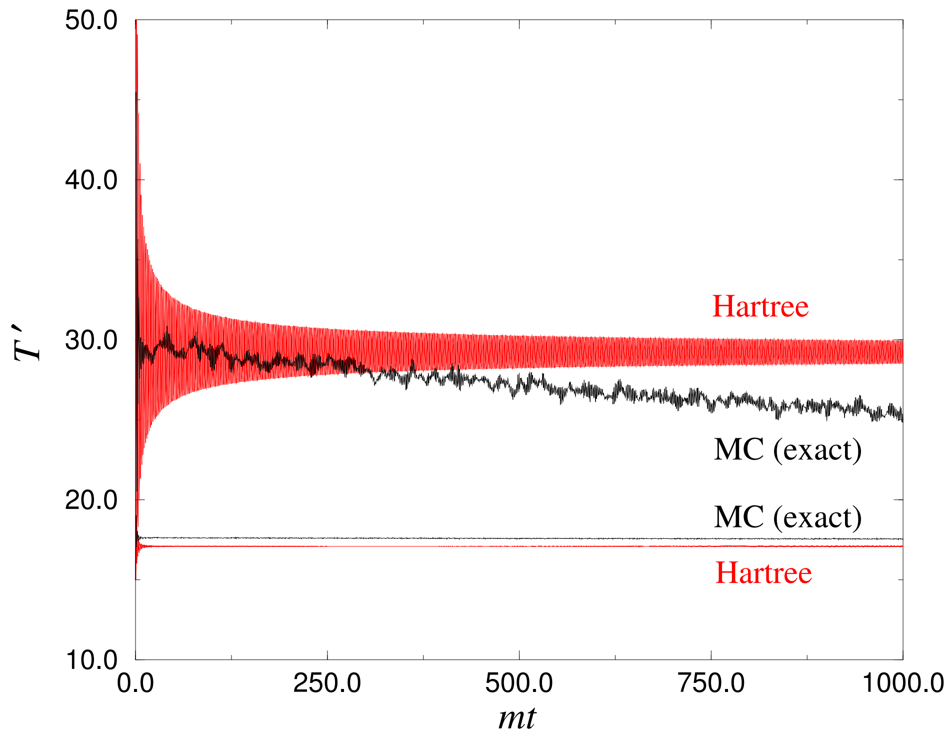
- $\langle\phi^2\rangle^*$ determined by gap equation
- $G^*_{\pi\pi}(p) = C(p)\bar{\omega}_p^*$ $\qquad G^*_{\phi\phi}(p) = C(p)/\bar{\omega}_p^*$

fixed point relevant for actual nonperturbative dynamics?

NONTHERMAL FIXED POINTS

G.A., BONINI AND WETTERICH

- classical test in $1+1$ dimensions



classical mode temperature:

$$T(p,t) = G_{\pi\pi}(p,t)$$

in classical thermal equilibrium:

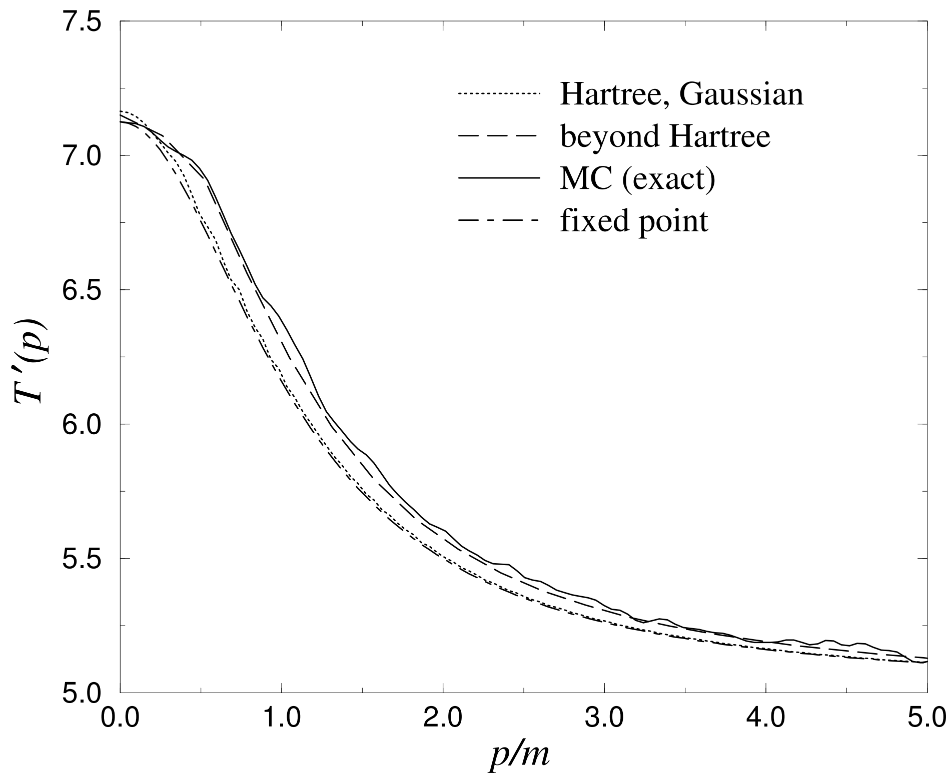
$$T(p,t) = T$$

- Hartree approximation: oscillating around nonthermal fixed point

Nonthermal fixed points

G.A., Bonini and Wetterich

- momentum-dependent "temperature" profile



classical mode temperature:

$$T(p,t) = G_{\pi\pi}(p,t)$$

fixed point:

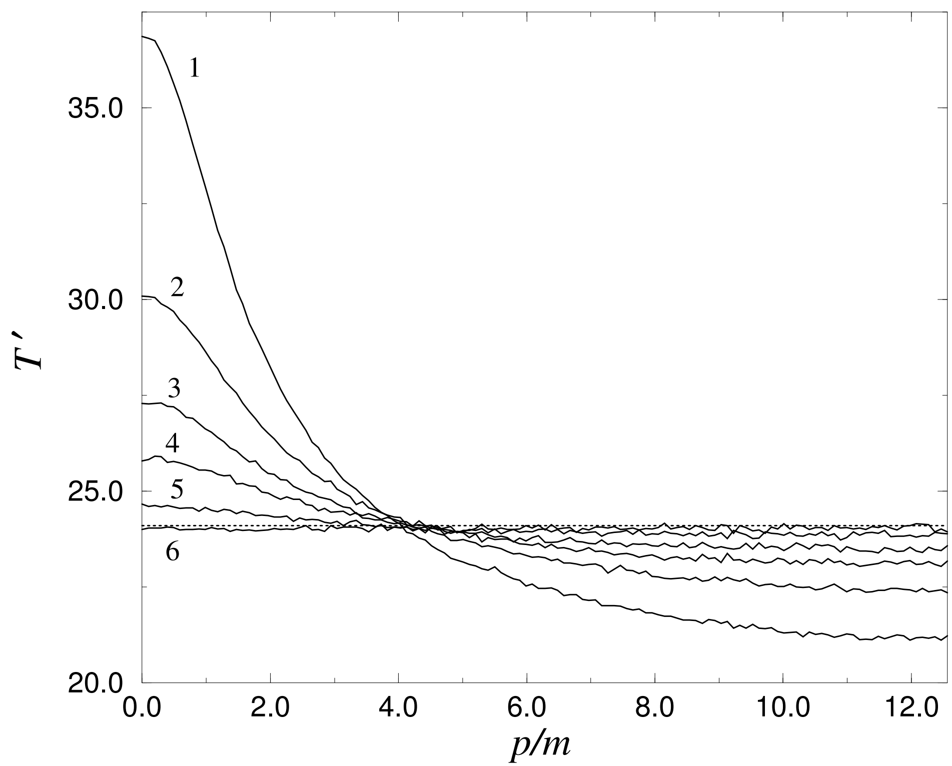
$$T^*(p) = T_0 \left[1 + \frac{\lambda}{2} \frac{\langle \phi^2 \rangle^*}{p^2 + m^2} \right]^{1/2}$$

- initial response determined by nonthermal fixed point, also for exact (MC) evolution

NONTHERMAL FIXED POINTS

G.A., BONINI AND WETTERICH

- momentum-dependent “temperature” profile



classical mode temperature:

$$T(p,t) = G_{\pi\pi}(p,t) \to T$$

$$t_1 < t_2 < \ldots < t_6$$

- fixed point relevant at early times
- exact (MC) evolution eventually thermalizes: all modes the same temperature

NONEQUILIBRIUM QUANTUM FIELDS?

WISH LIST

- mean field approximation (dramatically) inadequate
- need to include scattering

want:

- stable time evolution
  - nontrivial due to secularity: many schemes break down when $t \sim 1/$(expansion parameter)
- connection with well-established approaches, e.g. kinetic theory
- dynamics at very late times: conservation laws and hydrodynamics, transport
- ...

KINETIC THEORY

CONNECTION WITH ESTABLISHED METHODS

example:

- Boltzmann equation: $(\partial_t + \mathbf{v_p} \cdot \partial_X)\, f(\mathbf{p}, X) = C[f]$

$$X = (t, \mathbf{x}) \qquad \mathbf{v_p} = \mathbf{p}/E_\mathbf{p} \qquad p^0 = E_\mathbf{p} \text{ (onshell)}$$

- real particles undergo isolated collisions
- collision kernel for two-to-two scattering processes:

$$C[f] = \frac{1}{2}\int_{\mathbf{p'kk'}} |\mathcal{M}|^2 (2\pi)^4 \delta^4(p + p' - k - k')$$
$$\left[(1 \pm f_\mathbf{p})\,(1 \pm f_\mathbf{p'})\, f_\mathbf{k} f_\mathbf{k'} - f_\mathbf{p} f_\mathbf{p'}\,(1 \pm f_\mathbf{k})\,(1 \pm f_\mathbf{k'})\right]$$

- stationary solution: $f(\mathbf{p}, X) \to n(E_\mathbf{p}) = 1/[e^{E_\mathbf{p}/T} \mp 1]$

KINETIC THEORY

BEYOND KINETIC THEORY?

assumptions:

- onshell particles: phase space distribution
- isolated collisions, well separated in space and time
- 'slowly varying', gradient expansion

relax these assumptions:

quantum field theory

$\Rightarrow$ dynamics of correlation functions, in particular two-point functions

KINETIC THEORY

TWO-POINT FUNCTIONS

- Wightman functions:

$$G^{>}(x,y) = \langle \phi(x)\phi(y) \rangle = G^{<}(y,x)$$

- spectral function:

$$\rho(x,y) = i\langle [\phi(x), \phi(y)] \rangle = i\left(G^{>}(x,y) - G^{<}(x,y)\right)$$

- statistical function:

$$F(x,y) = \frac{1}{2}\langle [\phi(x), \phi(y)]_{+} \rangle = \frac{1}{2}\left(G^{>}(x,y) + G^{<}(x,y)\right)$$

two-point functions closely related to particle distribution functions, after series of manipulations

KINETIC THEORY

TWO-POINT FUNCTIONS

- separation of slow and fast variables: Wigner transform

$$X = \frac{1}{2}(x+y) \qquad (x-y) \to p \qquad \Rightarrow \qquad G^>(x,y) \to G^>(p,X)$$

- in equilibrium: Kubo-Martin-Schwinger (KMS) condition periodicity of the trace (X independent)

$$G^>(x,y) \sim \mathrm{Tr} e^{-H/T}\phi(x)\phi(y) \quad \Rightarrow \quad G^>(\omega,\mathbf{p}) = e^{\omega/T} G^<(\omega,\mathbf{p})$$

- all 2-point functions related to the spectral density

$$G^>(\omega,\mathbf{p}) = [n_B(\omega)+1]\,\rho(\omega,\mathbf{p}) \qquad G^<(\omega,\mathbf{p}) = n_B(\omega)\rho(\omega,\mathbf{p})$$

- noneq. distr. function: $G^<(p,X) = f(p,X)\rho(p,X)$
- onshell approximation $f(\mathbf{p},X)$, with $p^0 = E_{\mathbf{p}}(X)$

2PI EFFECTIVE ACTION

FIELD THEORY APPROACH

therefore:

- two-point function important role
- obeys Dyson equation: $G^{-1} = G_0^{-1} - \Sigma$
- what is self energy Σ?

- formalize: action principle

two-particle irreducible effective action

or

Φ-derivable approach

Luttinger/Ward, Baym, Cornwall/Jackiw/Tomboulis, ...

2PI EFFECTIVE ACTION

FIELD THEORY APPROACH

- generating functional with local and bilocal sources

$$Z[J,K] = e^{iW[J,K]} = \int \mathcal{D}\varphi\, e^{i\left(S[\varphi]+J_i\varphi^i+\frac{1}{2}\varphi^i K_{ij}\varphi^j\right)}$$

- Legendre transform: $\frac{\delta W}{\delta J_i} = \phi^i$, $\frac{\delta W}{\delta K_{ij}} = \phi^i\phi^j + G^{ij}$

$$\Gamma[\phi, G] = W[J,K] - J_i\phi^i - \frac{1}{2}K_{ij}\left(G^{ij} + \phi^i\phi^j\right)$$

- effective action can be written as

$$\Gamma[\phi, G] = S[\phi] + \frac{i}{2}\mathrm{Tr}\ln G^{-1} + \frac{i}{2}\mathrm{Tr}\, G_0^{-1}\left(G - G_0\right) + \Gamma_2[\phi, G]$$

- variational principe (in absence of sources)

$$\frac{\delta\Gamma}{\delta\phi} = 0, \quad \frac{\delta\Gamma}{\delta G} = 0 \quad \Rightarrow \quad G^{-1} = G_0^{-1} - \Sigma[G], \quad \Sigma = 2i\frac{\delta\Gamma_2}{\delta G}$$

2PI EFFECTIVE ACTION

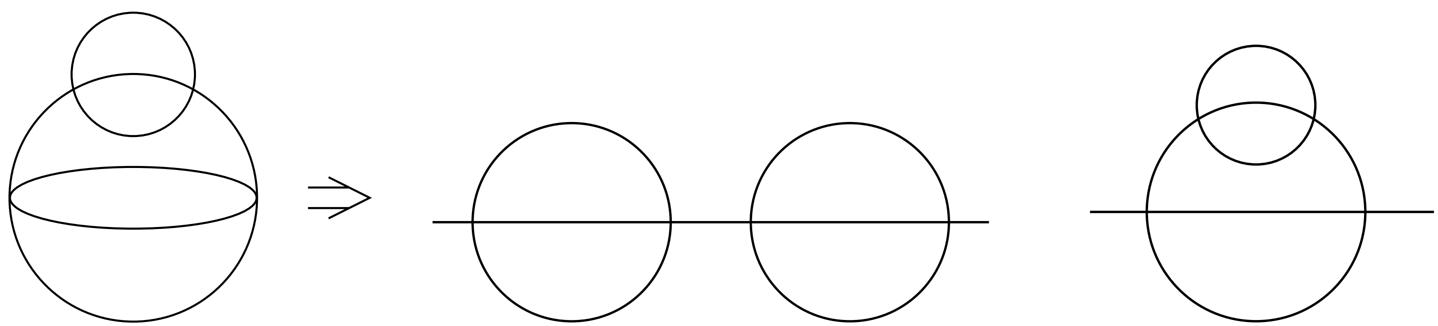
FIELD THEORY APPROACH

- action principle, at the extremum

$$\left(\Box + V'[\phi]\right)\phi + \frac{\delta\Gamma_2[\phi, G]}{\delta\phi} = 0 \qquad G^{-1} = G_0^{-1}[\phi] - \Sigma[\phi, G]$$

- prescription for the self energy $\Sigma = 2i\delta\Gamma_2/\delta G$
- Γ_2 is 2PI $\Leftrightarrow$ Σ is 1PI, depends on full G

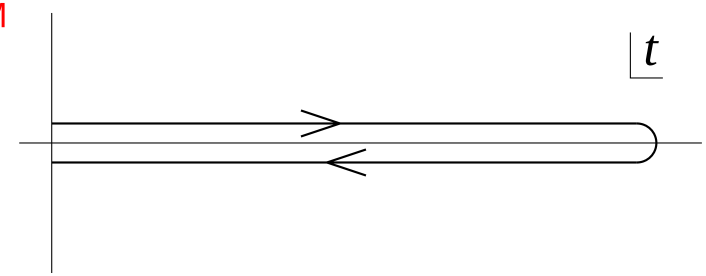
example:



- avoid overcounting

NONEQUILIBRIUM DYNAMICS

INITIAL VALUE PROBLEM



solve equations in real time:

$$\langle \phi(x)\phi(y)\rangle = \underbrace{\int d\phi' d\phi'' \langle \phi' | \rho_D | \phi'' \rangle}_{\text{initial cond.}} \underbrace{\int \mathcal{D}\phi \, e^{iS} \phi(x)\phi(y)}_{\text{quantum evolution (path integral)}}$$

use Schwinger-Keldysh contour for initial value problems

$$i\left(\Box_x + m^2\right) G(x,y) = \int_{\mathcal{C}} dz \, \Sigma(x,z) G(z,y) + \delta_{\mathcal{C}}(x-y)$$

action principle along complex-time path $\mathcal{C}$

NONEQUILIBRIUM DYNAMICS

INITIAL VALUE PROBLEM

- Green functions: $G^>$, $G^<$ etc.
- minimal choice
- decompose contour propagator in real and imaginary parts:

$$G(x,y) = F(x,y) - \frac{i}{2}\text{sign}(x^0 - y^0)\rho(x,y)$$

statistical function
even, anti-commutator

spectral function
odd, commutator

- spectral function is a commutator:

$$\rho(x,y)\big|_{x^0=y^0} = 0, \qquad \partial_{x^0}\rho(x,y)\big|_{x^0=y^0} = \delta(\mathbf{x}-\mathbf{y})$$

NONEQUILIBRIUM DYNAMICS

INITIAL VALUE PROBLEM

manifestly real and causal equations

$$\left[\Box_x + m^2\right] F(x,y) = -\int_0^{x^0} dz^0 \int d\mathbf{z}\; \Sigma_\rho(x,z) F(z,y) + \int_0^{y^0} dz^0 \int d\mathbf{z}\; \Sigma_F(x,z) \rho(z,y)$$

$$\left[\Box_x + m^2\right] \rho(x,y) = -\int_{y^0}^{x^0} dz^0 \int d\mathbf{z}\; \Sigma_\rho(x,z) \rho(z,y)$$

with $\Sigma_{F,\rho}$ given in terms of F and ρ

predicting the future = remembering the past

NONEQUILIBRIUM DYNAMICS

INITIAL VALUE PROBLEM

- action principle
- conserved energy ($\langle\phi\rangle = 0$):

$$E = \int d^3x \, \frac{1}{2} \left[\partial_{x^0}\partial_{y^0} + \partial_{x^i}\partial_{y^i} + m^2\right] F(x,y)\Big|_{x=y}$$

$$+\frac{1}{4}\int d^3x \int_0^{x^0} dz^0 \int d^3z \left[\Sigma_\rho(x,z)F(z,x) - \Sigma_F(x,z)\rho(z,x)\right]$$

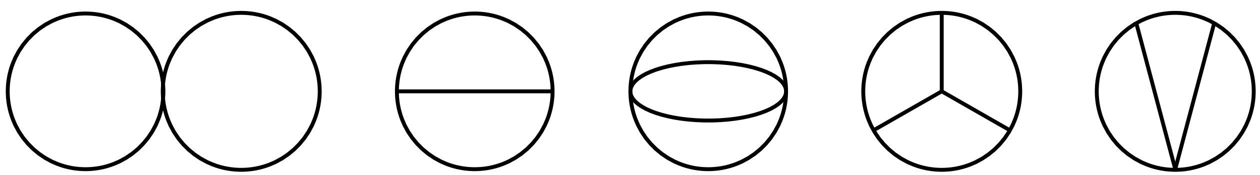
- conserved for every truncation

2PI TRUNCATIONS

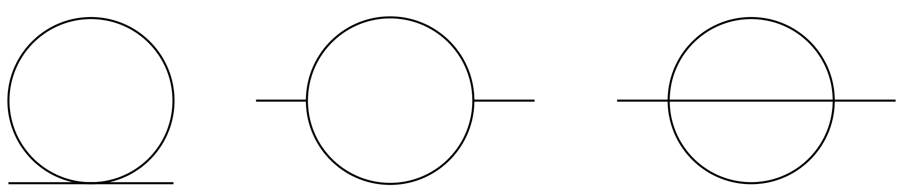
LOOP AND $1/N$ EXPANSIONS TO NEXT-TO-LEADING ORDER

- so far exact, approximation enters via truncation of Γ_2
- systematic, in practice loop and $1/N$ expansions

- three-loop expansion ($\langle\phi\rangle = 0$)



- diagrams 1, 2, 3 well-studied (no internal vertices)
- self energies:

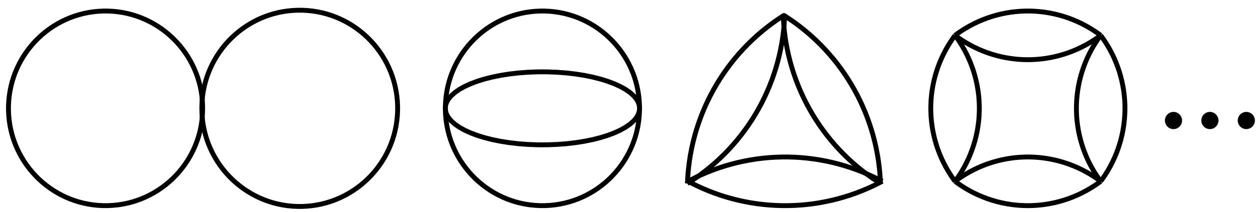


2PI TRUNCATIONS

LOOP AND $1/N$ EXPANSIONS TO NEXT-TO-LEADING ORDER

large N expansion:

- $O(N)$ model, vertex $\sim 1/N$ (with $\langle\phi\rangle = 0$ for simplicity)



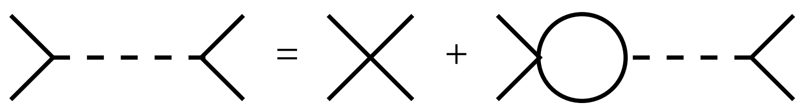
$\sim N$ ~ 1 $\sim 1/N$

2PI TRUNCATIONS

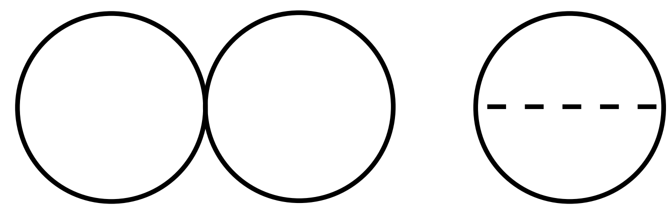
LOOP AND $1/N$ EXPANSIONS TO NEXT-TO-LEADING ORDER

large N expansion:

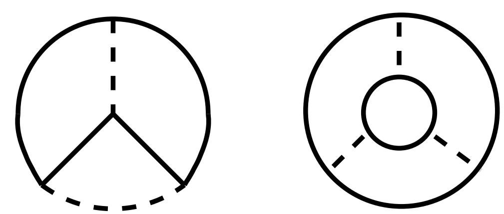
- $O(N)$ model, vertex $\sim 1/N$ (with $\langle\phi\rangle = 0$ for simplicity)
- efficient formulation: use chain of bubbles



$\Rightarrow$ effective two-loop approximation



- NNLO contribution ($\sim 1/N$):



2PI TRUNCATIONS

LOOP AND $1/N$ EXPANSIONS TO NEXT-TO-LEADING ORDER

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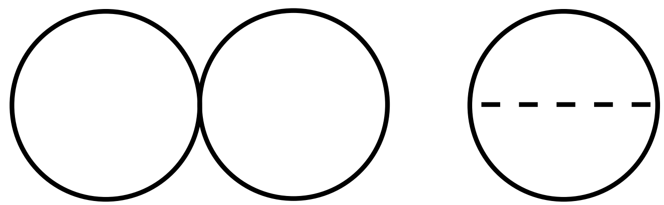
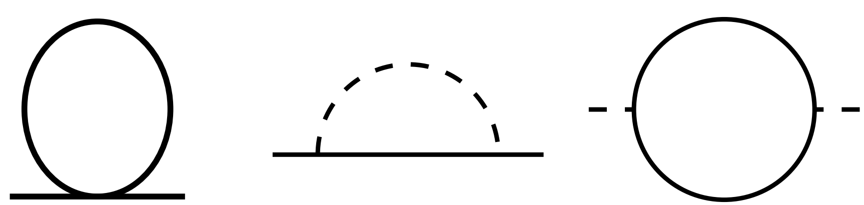


- dressed propagators:

$$G^{-1} = G_0^{-1} - \Sigma \qquad D^{-1} = D_0^{-1} - \Pi$$

- self energies:



2PI TRUNCATIONS

LOOP AND $1/N$ EXPANSIONS TO NEXT-TO-LEADING ORDER

- closed set of self-consistent equations:

$$\left[\Box_x + M^2(x)\right] F(x,y) = -\int_0^{x^0} dz^0 \int d\mathbf{z}\; \Sigma_\rho(x,z) F(z,y) + \int_0^{y^0} dz^0 \int d\mathbf{z}\; \Sigma_F(x,z) \rho(z,y)$$

$$\left[\Box_x + M^2(x)\right] \rho(x,y) = -\int_{y^0}^{x^0} dz^0 \int d\mathbf{z}\; \Sigma_\rho(x,z) \rho(z,y)$$

with

$$\Sigma_F(x,y) = -\frac{\lambda}{3N}\left[F(x,y) D_F(x,y) - \frac{1}{4}\rho(x,y) D_\rho(x,y)\right]$$

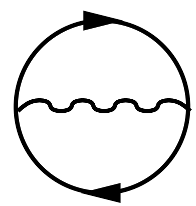
$$\Sigma_\rho(x,y) = -\frac{\lambda}{3N}\left[\rho(x,y) D_F(x,y) + F(x,y) D_\rho(x,y)\right]$$

2PI TRUNCATIONS

LOOP AND $1/N$ EXPANSIONS TO NEXT-TO-LEADING ORDER

large N expansion:

- large N_f gauge theory, vertex $e^2 \sim 1/N$



- dressed propagators:

$$G^{-1} = G_0^{-1} - \Sigma \qquad D^{-1} = D_0^{-1} - \Pi$$

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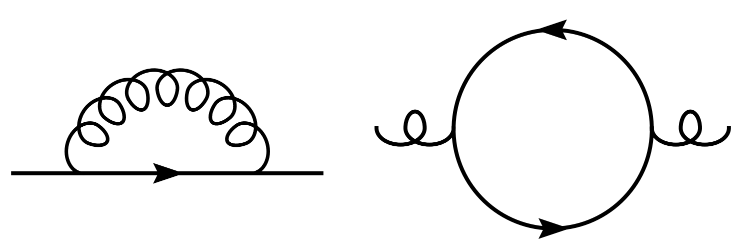


SOLUTIONS

NUMERICAL

- solve integro-differential equations on a spacetime lattice
- straightforward discretization, no further approximation
- expensive numerically due to “memory kernel”

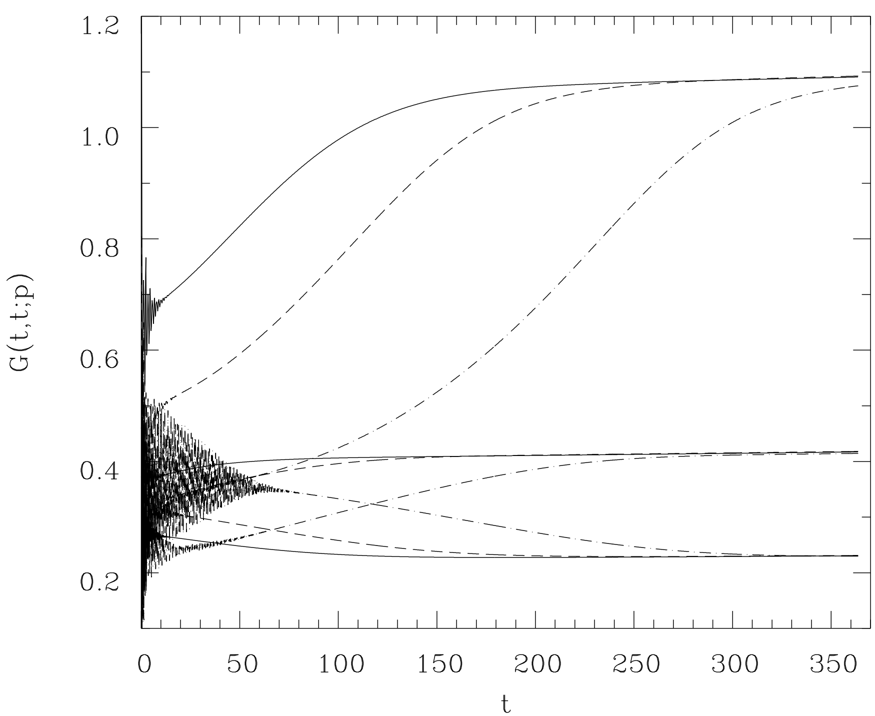
some applications

LOSS OF MEMORY

THERMALIZATION

first results by Berges and Cox (2000):

- take different initial conditions (or density matrices) with the total energy density identical
- independence of initial conditions at late times



3-loop expansion in $\lambda\phi^4$ in $1+1$ dimensions

time evolution of different momentum modes $F(t,t;p)$

PRECISION TESTS

CLASSICAL 2PI APPROXIMATION

- 2PI approach in classical statistical field theory
- possibility to compare with “exact” solution
- sampling of initial conditions $+$ numerical integration of classical equation of motion

example of classical limit: three-loop approximation

$$\Sigma_\rho(x,z) = -\frac{\lambda^2}{2}\rho(x,z)\left[F^2(x,z) - \frac{1}{12}\rho^2(x,z)\right],$$

$$\Sigma_F(x,z) = -\frac{\lambda^2}{6}F(x,z)\left[F^2(x,z) - \frac{3}{4}\rho^2(x,z)\right]$$

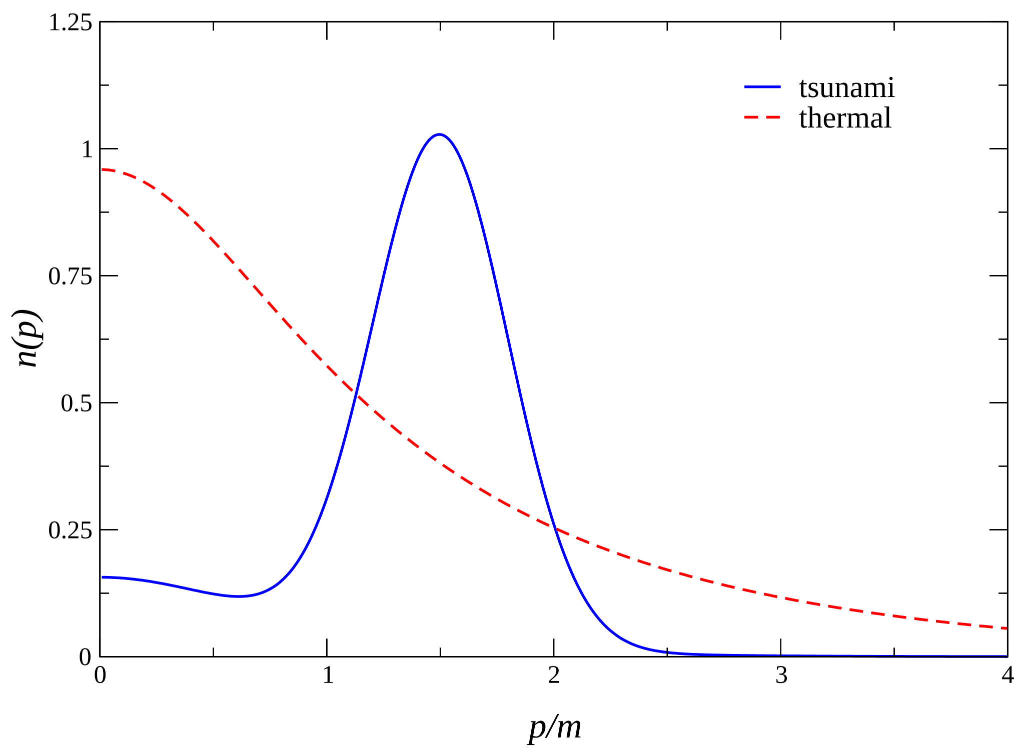
classically:

$$\Sigma_\rho^{\mathrm{cl}}(x,z) = -\frac{\lambda^2}{2}\rho(x,z)F^2(x,z) \qquad \Sigma_F^{\mathrm{cl}}(x,z) = -\frac{\lambda^2}{6}F^3(x,z)$$

NONEQUILIBRIUM INITIAL CONDITIONS

TSUNAMI

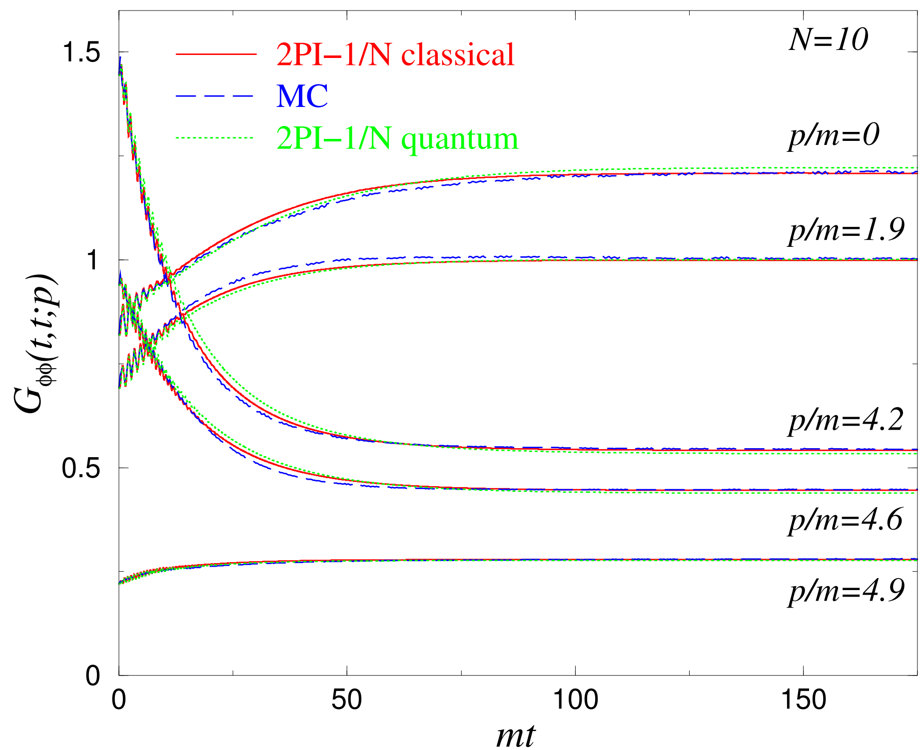
- Gaussian initial conditions far from equilibrium
- specify $F(t,t';\mathbf{p}), \partial_t F(t,t';\mathbf{p}), \partial_t \partial_{t'} F(t,t';\mathbf{p})$ at $t = t' = 0$ in terms of initial particle number $n(\mathbf{p})$



- easily implemented in exact and 2PI dynamics

PRECISION TESTS

G.A. & BERGES

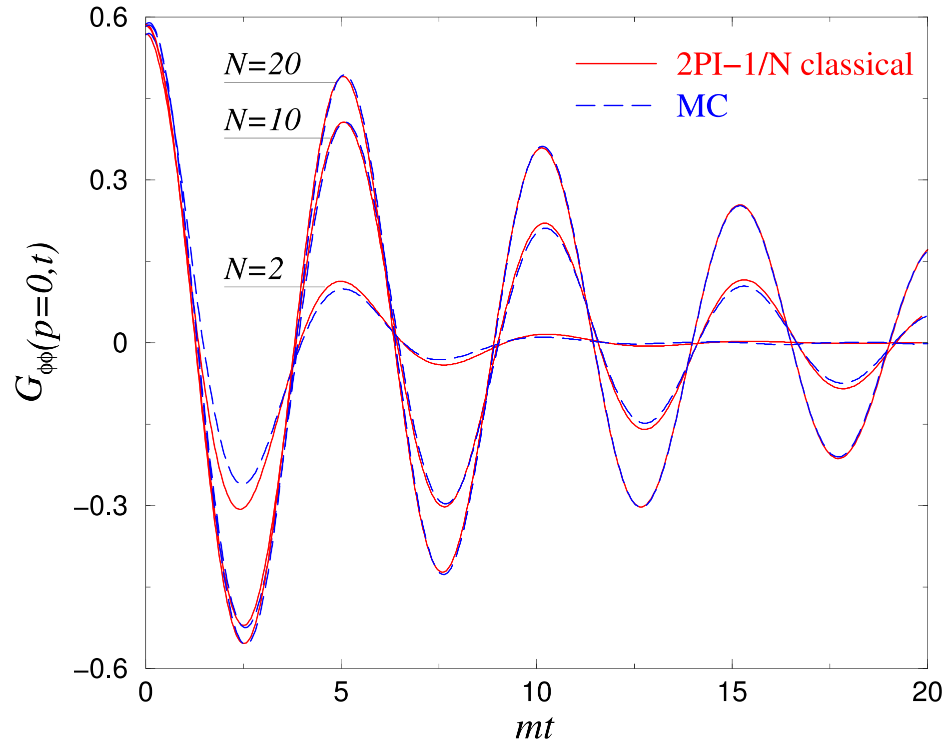


- tsunami initial conditions
- equal-time correlation function: 'particle number'
- high energy density: compare quantum and classical evolution

- evolution from 2PI-$1/N$ expansion in agreement with 'exact' evolution, also for late times.
- reliable description of both early and late times
- capable of describing equilibration

PRECISION TESTS

G.A. & BERGES



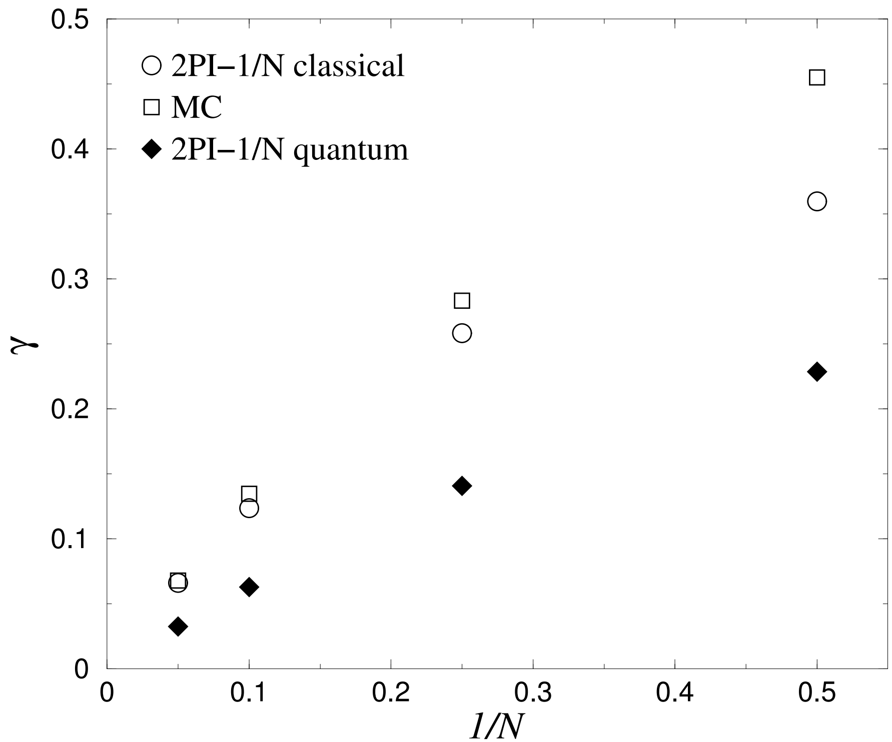
2PI-$1/N$ expansion

unequal-time correlation function

- Monte Carlo: sample of 80.000 initial conditions
- 2PI-$1/N$: one (expensive) numerical solution
- quantitative agreement for larger N

Precision tests

G.A. & Berges



2PI-$1/N$ expansion

assume ansatz

$$G(t,0;\mathbf{p}) \sim e^{-\gamma t} \cos mt$$

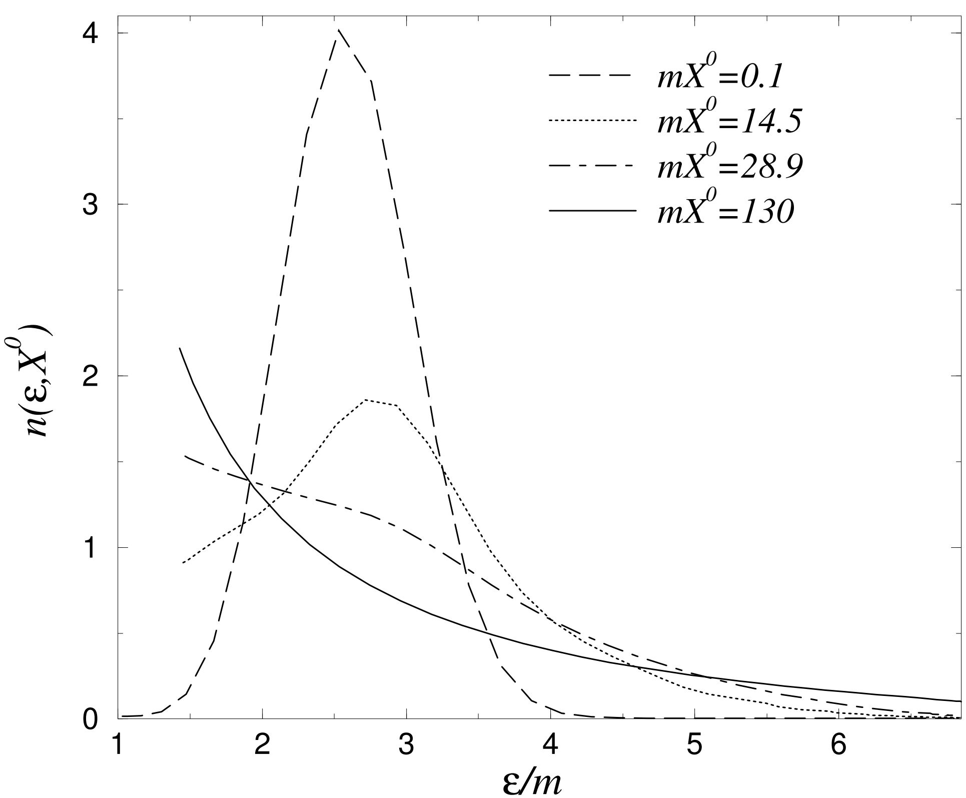
fit γ and m

- compare classical 2PI with classical exact
- quantitative agreement for larger N
- compare classical 2PI with quantum 2PI
- quantum $\neq$ classical!

(NOT) KINETIC THEORY

G.A. & BERGES

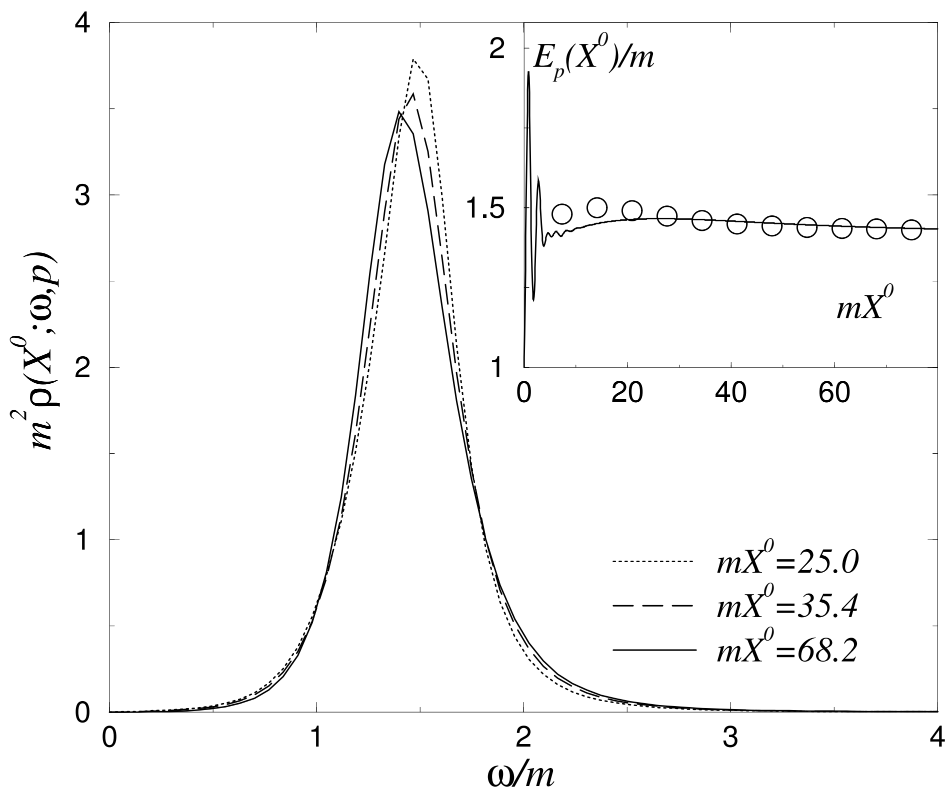
- separation of fast and slow variables
- effective particle number distribution is evolving fast and wildly



(NOT) KINETIC THEORY

G.A. & BERGES

- self-consistent evolution of the spectral function $\rho(t, t'; \mathbf{p})$
- no quasiparticle approximation
- Wigner transform: $\rho(t, t'; \mathbf{p}) \to \rho(\omega, \mathbf{p}; X^0)$



$$X^0 = (t + t')/2$$

- quasiparticle peak
- non-zero width
- slowly evolving

ET CETERA

much more work has been done:

- quick establishment of equation of state (prethermalization)
- fermions
- momentum anisotropy
- (tachyonic) preheating
- warm inflation
- renormalization
- ...

- cold atoms

TRANSPORT

FINAL STAGES

unified picture:

- dynamics far from equilibrium with 2PI truncations
- system will (eventually) equilibrate and thermalize

precise question:

- which scattering processes are certainly included?
- which scattering processes are certainly not included?

TRANSPORT

FINAL STAGES

- final stages of evolution
- dynamics of nearly conserved quantities
- hydrodynamic modes are slowest

- energy-momentum
- charges
- ...

evolve according to “low-energy effective field theory”

$$=$$

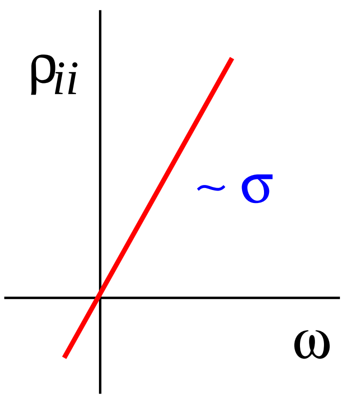
hydrodynamics

NEAR EQUILIBRIUM: TRANSPORT COEFFICIENTS

KUBO RELATIONS AND LINEAR RESPONSE

electrical conductivity: $$\sigma = \frac{1}{6}\frac{\partial}{\partial\omega}\rho^{ii}(\omega,\mathbf{0})\Big|_{\omega=0}$$

shear viscosity: $$\eta = \frac{1}{20}\frac{\partial}{\partial\omega}\rho_{\pi\pi}(\omega,\mathbf{0})\Big|_{\omega=0}$$



spectral densities:

$$\rho^{\mu\nu}(\omega,\mathbf{p}) = \int d^4x\, e^{ipx}\langle[j^\mu(x), j^\nu(0)]\rangle_{\rm eq}$$

$$\rho_{\pi\pi}(\omega,\mathbf{p}) = \int d^4x\, e^{ipx}\langle[\pi_{ij}(x), \pi_{ij}(0)]\rangle_{\rm eq}$$

with $j^\mu = \bar\psi\gamma^\mu\psi$, $\pi_{ij} = T_{ij} - \frac{1}{3}\delta_{ij}T_k^k = \partial_i\phi\partial_j\phi - \frac{1}{3}\delta_{ij}\partial_k\phi\partial_k\phi$

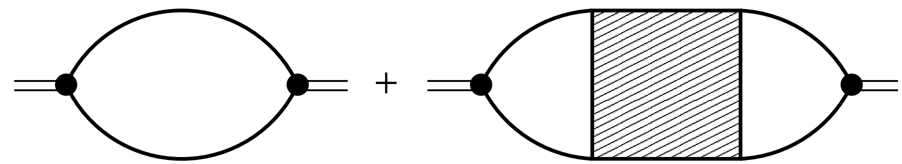
transport coefficients $\sim$ slope of current-current spectral functions at $\omega = 0$

NEAR EQUILIBRIUM: TRANSPORT COEFFICIENTS

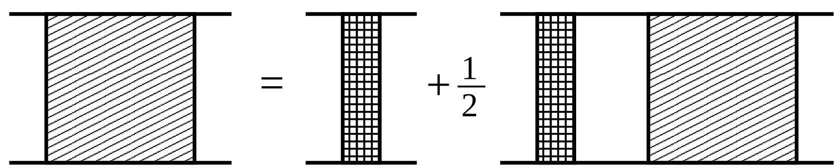
G. A. AND J. M. MARTINEZ RESCO

imaginary part of correlators of bilocal operators

2PI effective action
as generating functional:



generates ladder diagrams:



with kernel or rung:

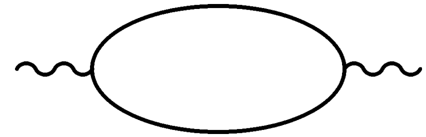
$$\Lambda_{ij;kl} = 4i \frac{\delta^2 \Gamma_2}{\delta G^{ij} \delta G^{kl}} = 2 \frac{\delta \Sigma_{ij}}{\delta G^{kl}}$$

kernel (and self energy) determined by Γ_2

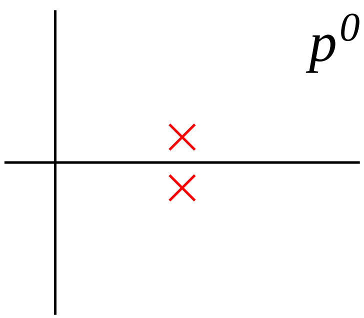
NEAR EQUILIBRIUM: TRANSPORT COEFFICIENTS

DRESSED PROPAGATORS

- pinching poles:



$$\lim_{\omega\to 0}\rho^{ii}(\omega,\mathbf{0}) = 4e^2\omega\int\frac{d^4p}{(2\pi)^4}\,n'_F(p^0)G_R(p)G_A(p)$$



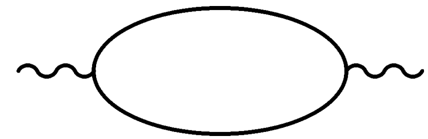
propagators in the loop carry the same momentum, product of retarded (R) and advanced (A) propagators

  - with bare propagators ill-defined

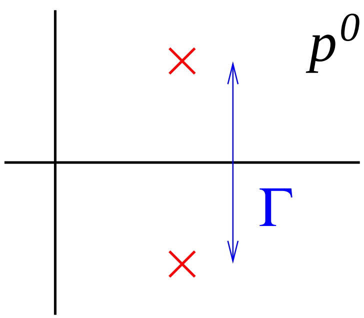
NEAR EQUILIBRIUM: TRANSPORT COEFFICIENTS

DRESSED PROPAGATORS

- pinching poles:



$$\lim_{\omega \to 0} \rho^{ii}(\omega, \mathbf{0}) = 4e^2\omega \int \frac{d^4p}{(2\pi)^4}\, n'_F(p^0) G_R(p) G_A(p)$$



propagators in the loop carry the same momentum, product of retarded (R) and advanced (A) propagators

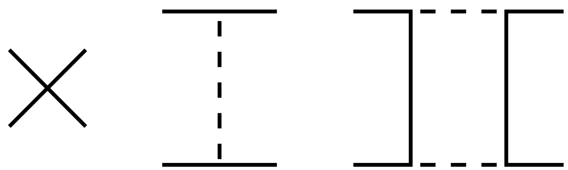
- inclusion of thermal width $\Gamma \sim 1/N$ required

$\Rightarrow$ finite collision time/mean free path in a medium resummed nonperturbatively

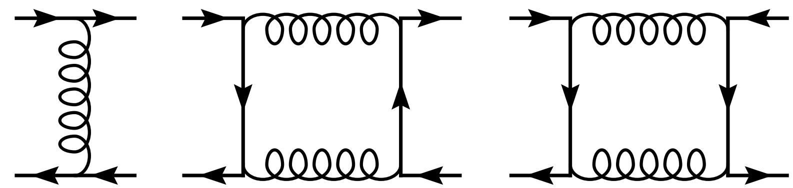
Near equilibrium: transport coefficients

RUNGS AND LADDER DIAGRAMS

- $O(N)$ model



- large N_f QED/QCD

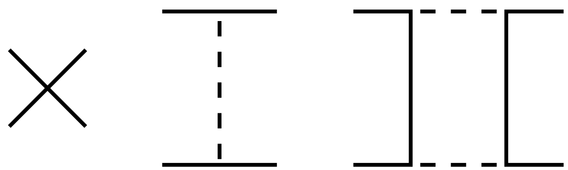


$+$ subleading terms in the $1/N$ expansion

NEAR EQUILIBRIUM: TRANSPORT COEFFICIENTS

RUNGS AND LADDER DIAGRAMS

- $O(N)$ model



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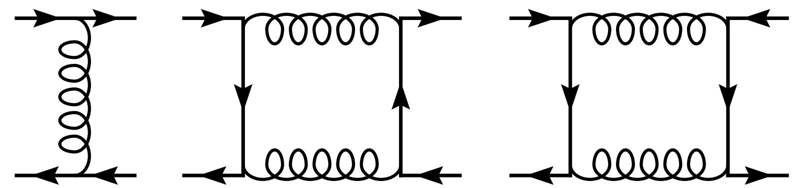
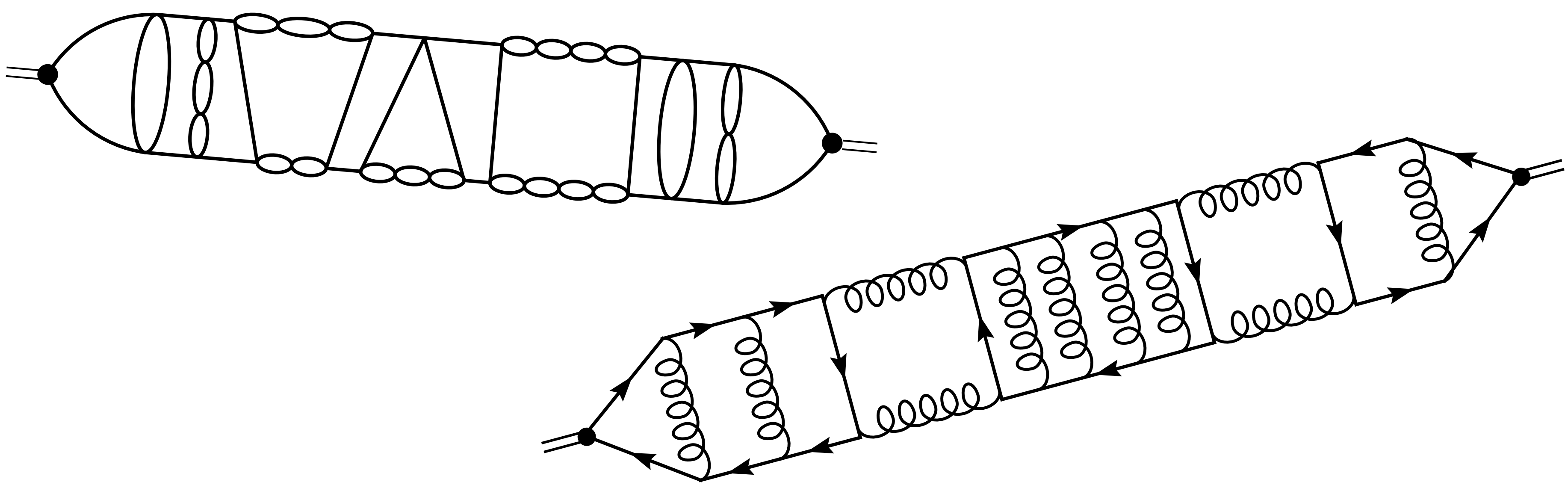


$+$ subleading terms in the $1/N$ expansion

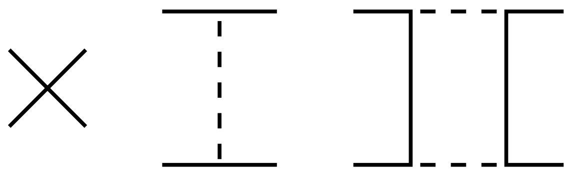
typical ladder diagrams:



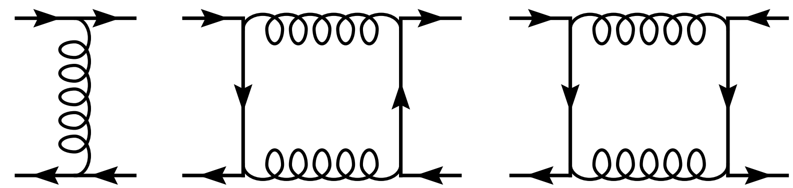
NEAR EQUILIBRIUM: TRANSPORT COEFFICIENTS

RUNGS AND LADDER DIAGRAMS

- $O(N)$ model



- large N_f QED/QCD



$+$ subleading terms in the $1/N$ expansion

- power counting
  - positive powers of N: closed scalar or fermion loops and pairs of propagators with pinching poles
  - negative powers of N: vertices

all contributions to LO in $1/N$ expansion

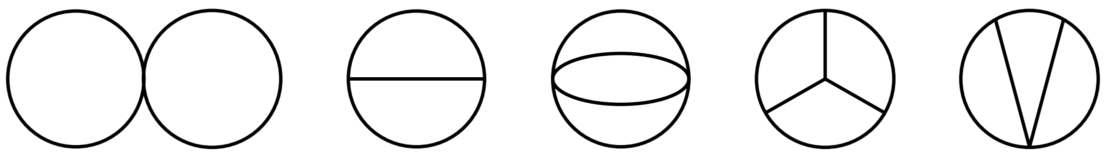
- subleading terms: cannot be neglected for self-consistent dynamics far from equilibrium

TRANSPORT

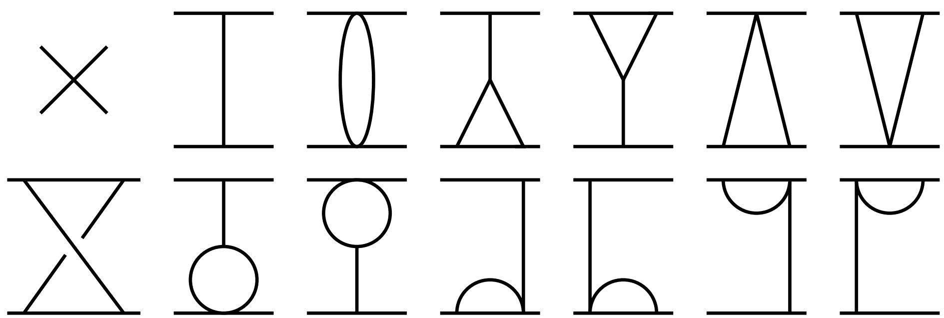
PRECISE QUESTION

which scattering processes are (not) included?

- 3-loop expansion in $g\phi^3 + \lambda\phi^4$ theory



- kernel

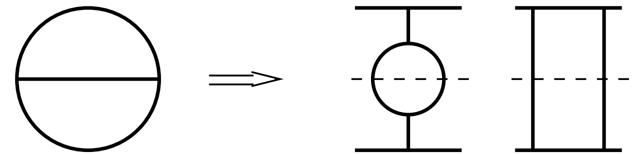


- a lot of scattering processes

TRANSPORT

PRECISE QUESTION

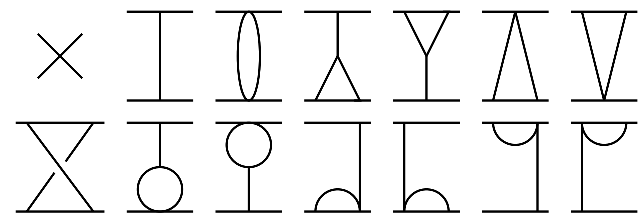
- 2-loop approximation: (iterated)



- sum of squares of $2 \to 2$ scattering processes

$$|\mathcal{M}|^2 \sim g^4 \left[|G(s)|^2 + |G(t)|^2 + |G(u)|^2\right]$$

- 3-loop approximation:



- square of sum of $2 \to 2$ scattering processes (+ subleading vertex corrections)

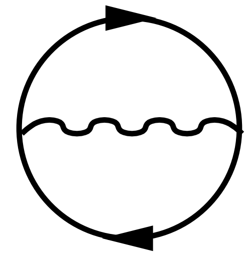
$$|\mathcal{M}|^2 \sim \left|\lambda + g^2 \left[G(s) + G(t) + G(u)\right]\right|^2$$

- interference included

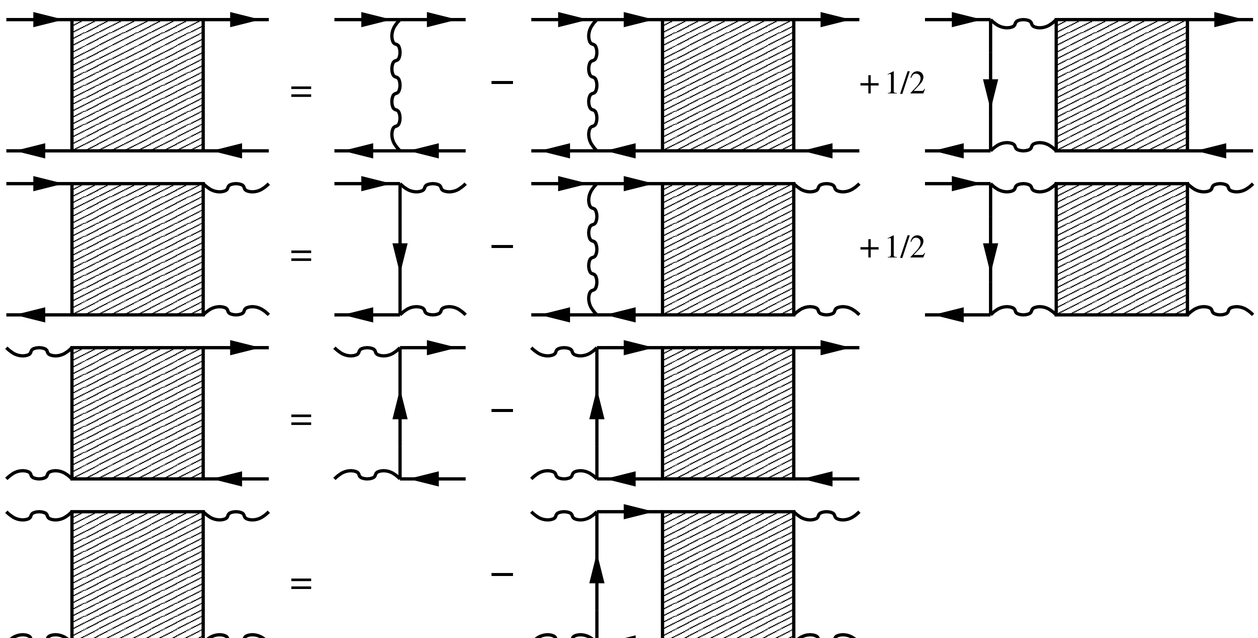
TRANSPORT

PRECISE QUESTION

- 2-loop or large N_f expansion in QED



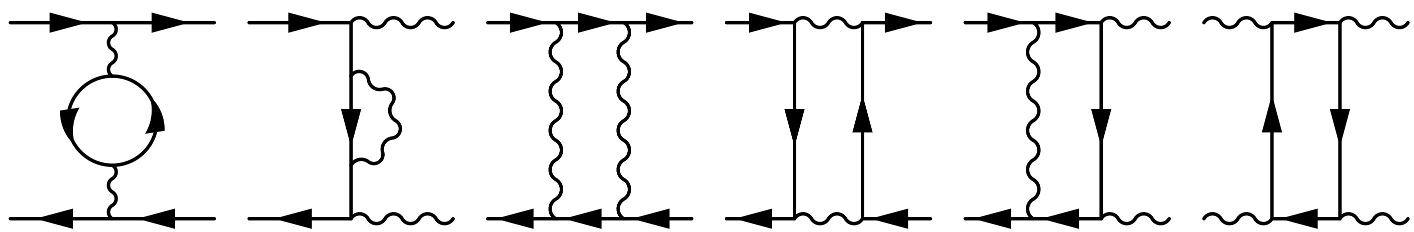
- coupled integral equations:



TRANSPORT

PRECISE QUESTION

- scattering kernel

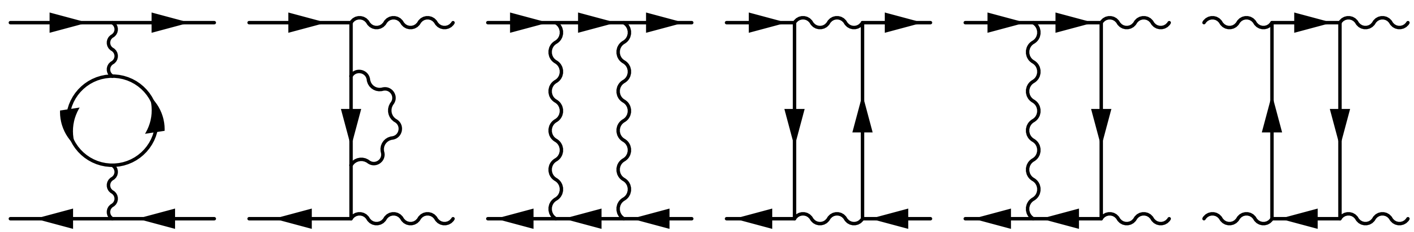


- weak coupling in the leading log approximation (transport coefficient $\sim 1/e^4 \ln 1/e$)
  - rung 1 $\Rightarrow$ t-channel Coulomb scattering
  - rung 2 $\Rightarrow$ Compton scattering, pair annihilation

TRANSPORT

PRECISE QUESTION

- scattering kernel



- weak coupling in the leading log approximation (transport coefficient $\sim 1/e^4 \ln 1/e$)
  - rung 1 $\Rightarrow$ t-channel Coulomb scattering
  - rung 2 $\Rightarrow$ Compton scattering, pair annihilation
- leading order large N_f QED:
  - rung 1 and 4 $\Rightarrow$ Coulomb scattering in all channels (no interference)

TRANSPORT

SUMMARY

scalars/fermions (with current truncations):

- most transport coefficients correct to LO
- notable exception: bulk viscosity `Calzetta and Hu`

gauge theories (two loop truncations):

- correct to leading log
- correct at leading order in large N_f
- full leading order requires use of 3PI effective action `(Carrington et al)`

OUTLOOK

done:

- scalars/fermions: most formal aspects studied
- some applications
- gauge theories: formal developments in progress

to do:

- more applications for scalars/fermions possible
- gauge theories: more formal developments
- gauge theories: numerical implementation and tests