

signatures from Preheating

KITP Seminar
UC Santa Barbara
24th January 2008

Juan García-Bellido
Inst. Física Teórica
UAM Madrid

Gravitational Waves and Magnetic Fields: A new window into the early Universe

J. G.-B.
Daniel G. Figueroa
Alfonso Sastre

PRL98, 061302 (2007)
[astro-ph/0701014] +
arXiv:0707.0839 [hep-
ph]

Andres Diaz-Gil, J. G.-B.

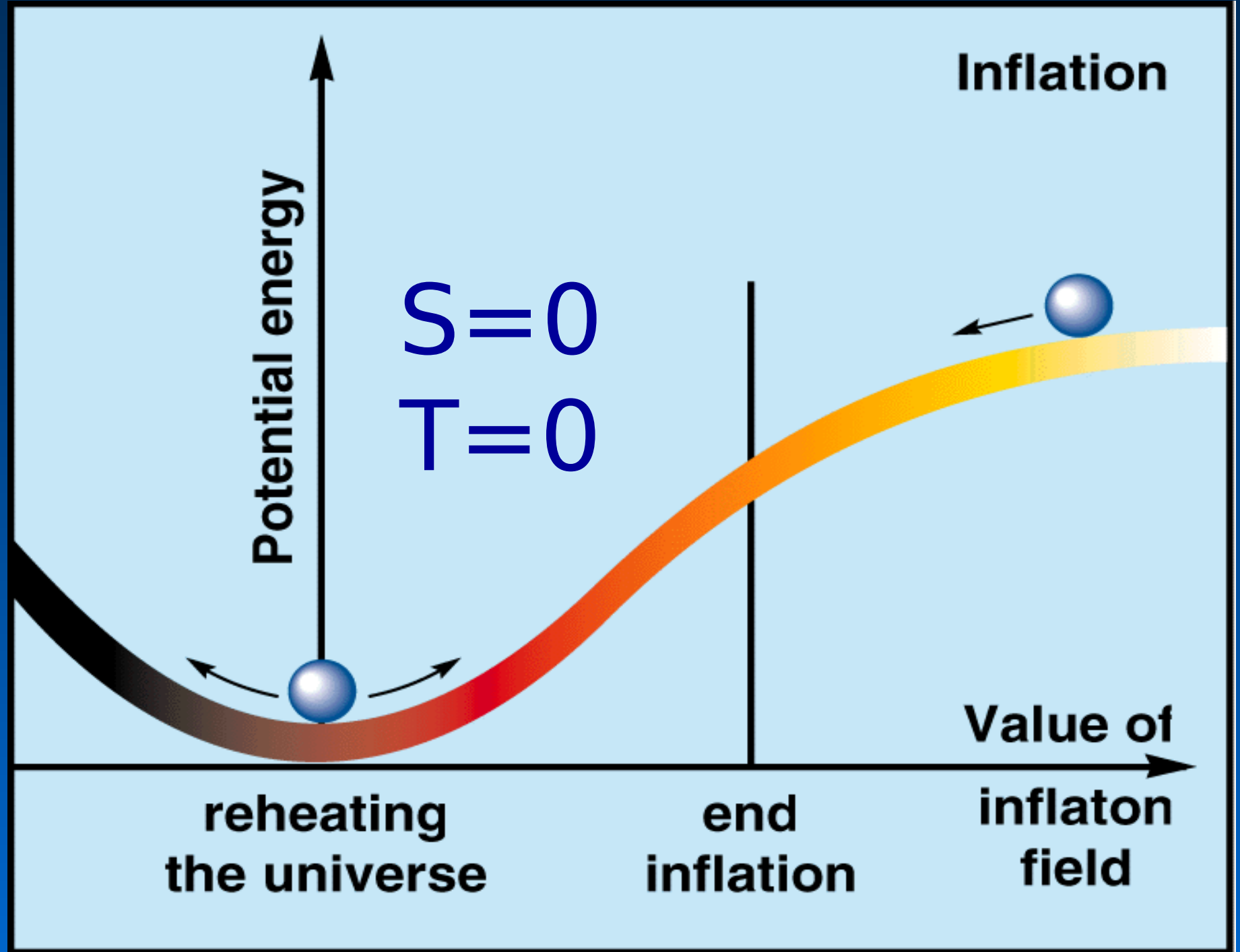
hep-lat/0509094
arXiv:0710.0580 [hep-
lat]

Margarita Garcia Perez

Antonio Gonzalez

Yi 0712.4262 [hep-
th]

The origin of matter and radiation



Preheating

very rich phenomenology after inflation

- Non-thermal production of particles (CDM)
- Production of topological defects (strings)
- EW baryogenesis & leptogenesis
- Production of gravitational waves
- Production of primordial magnetic fields
- etc.

Tachyonic preheating

JGB, Linde

PRD57, 6075 (1998)

Felder, JGB, Kofman,
Linde, Tkachev

PRL87, 011601
(2001)

JGB, García-Perez,
González-Arroyo

PRD64, 123517
PRD67, 103501
(2001)
(2003)

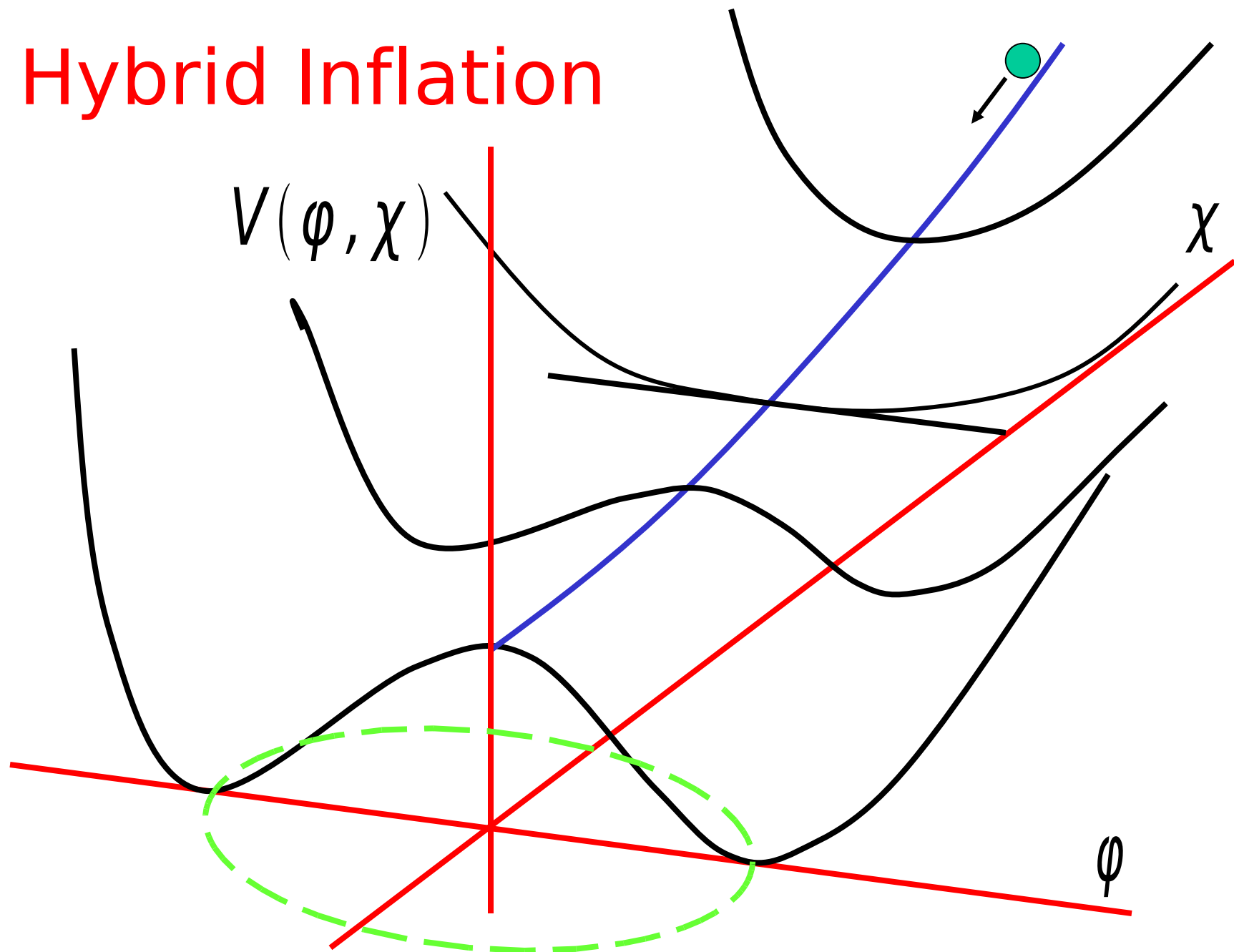
PRD66

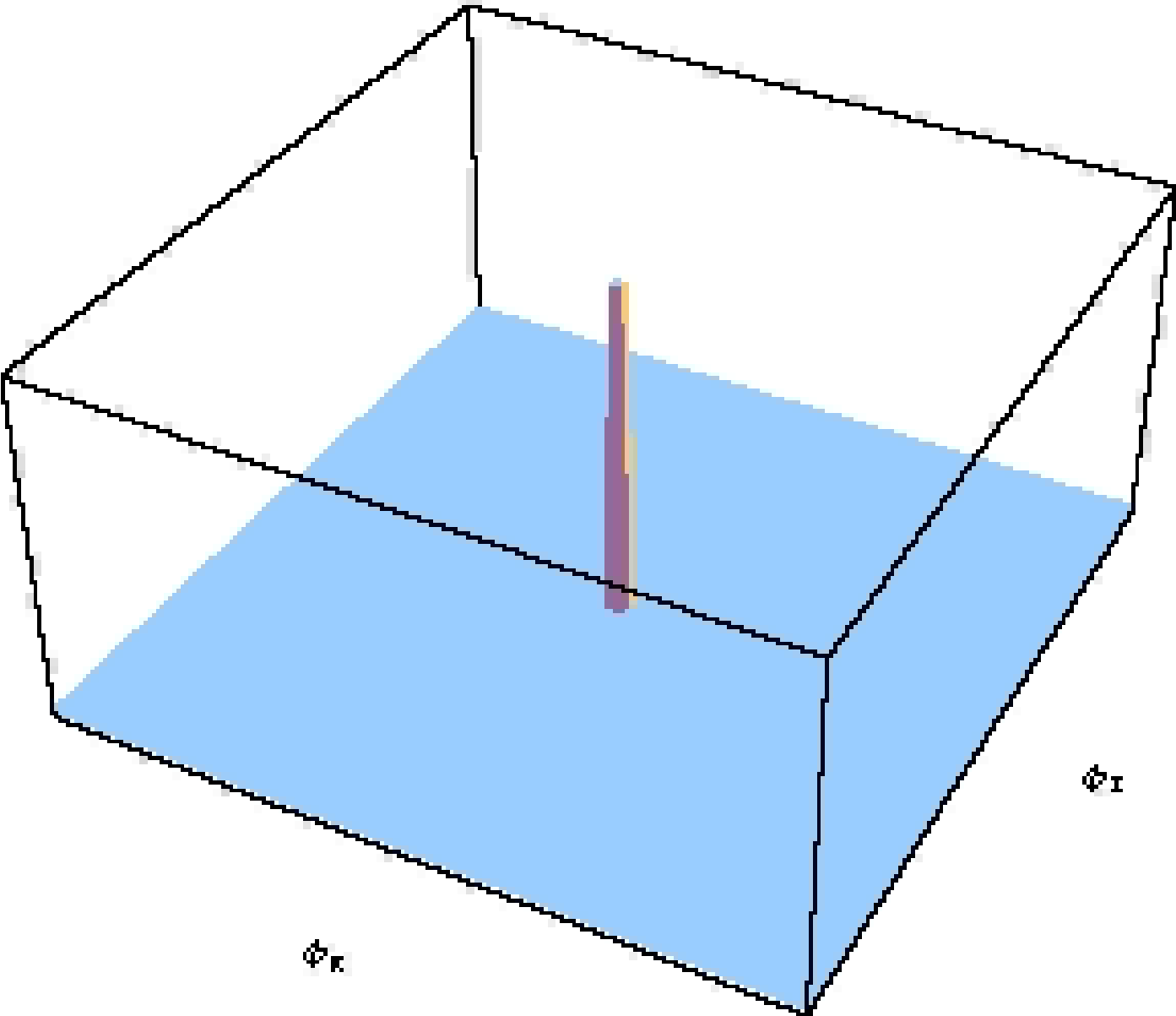
Tachyonic preheating

Spinodal growth of long wave Higgs modes

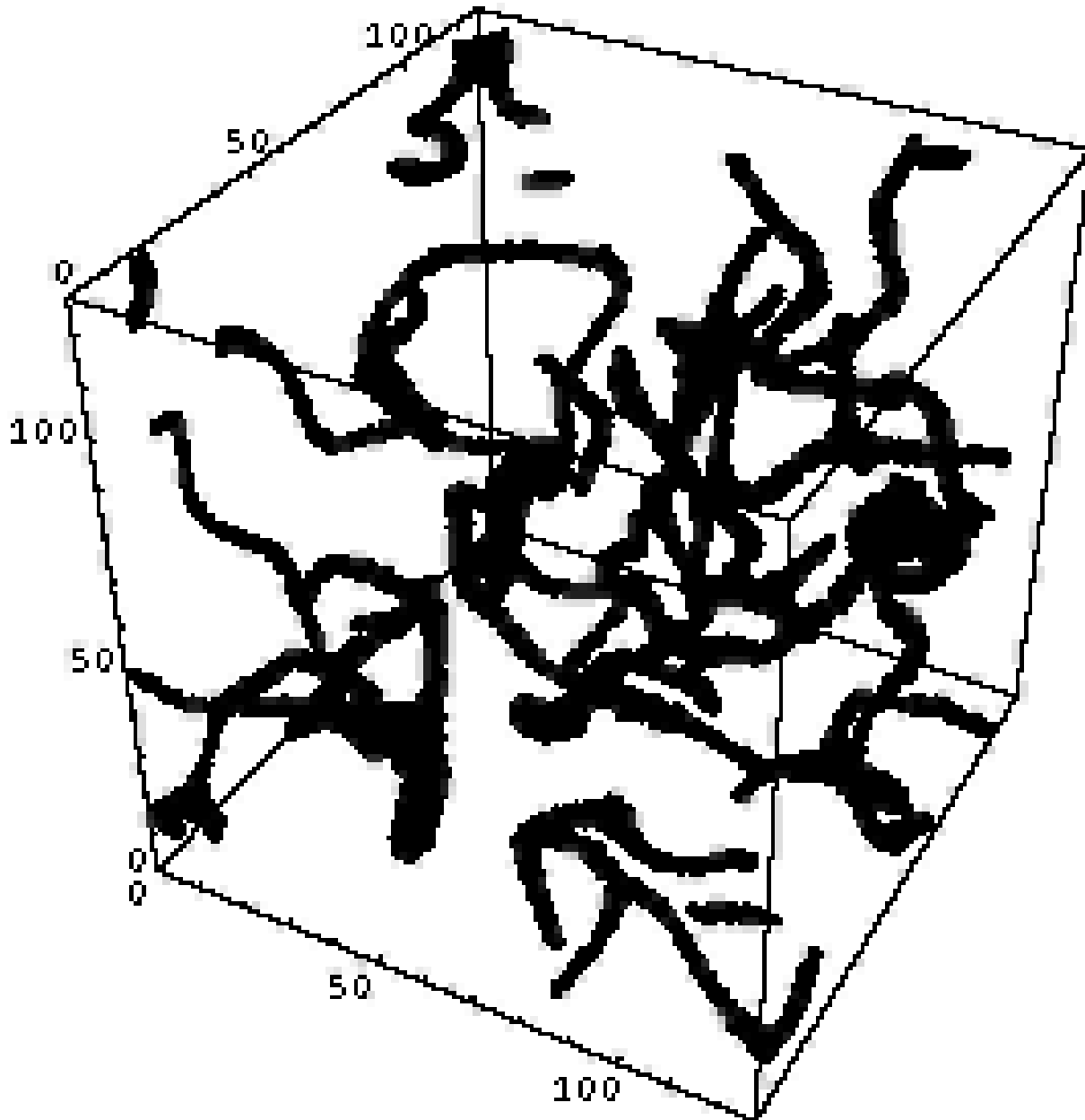
- At the end of Hybrid Inflation
- Higgs couples to gauge fields
- Strong production of fermions
- Production of cosmic strings

Hybrid Inflation



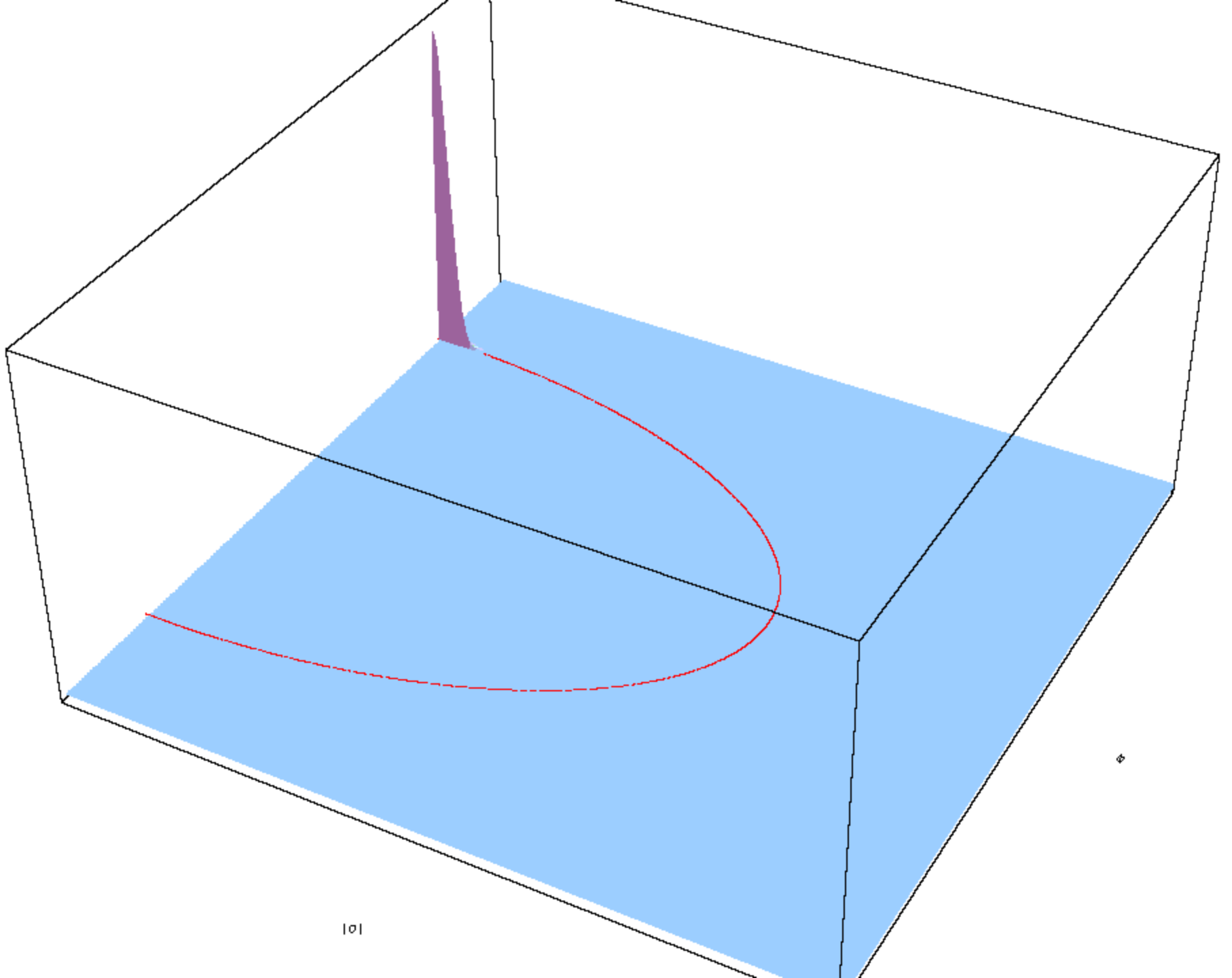


9.000111



$$\varphi \in U(1)$$

String
production
@ end
hybrid
inflation



The Higgs Evolution

$$m_{\phi}^2 = m^2 \left(\frac{\chi^2}{\chi_c^2} - 1 \right) \approx -2Vm^3(t-t_c)$$
$$= -M^3(t-t_c) = -M^2\tau$$

$$H = \frac{1}{2} \int d^3k \left[p_k(\tau) p_k^+(\tau) + (k^2 - \tau) y_k(\tau) y_k^+(\tau) \right]$$

$$\left[y_k(\tau), p_{k'}(\tau) \right] = i\hbar \delta^3(k - k')$$

Higgs Quantum Field

$$y_k(\tau) = f_k(\tau) a_k(\tau_0) + f_k^{\dot{i}}(\tau) a_{-k}^+(\tau_0)$$

$$p_k(\tau) = -i \left[g_k(\tau) a_k(\tau_0) - g_k^{\dot{i}}(\tau) a_{-k}^+(\tau_0) \right]$$

$$f_k'' + (k^2 - \tau) f_k = 0 \qquad g_k = i f_k'$$

Airy function

$$\Omega_k(\tau) = \frac{g_k^{\dot{i}}(\tau)}{f_k^{\dot{i}}(\tau)} = \frac{1 - 2iF_k(\tau)}{2|f_k(\tau)|^2}$$

$$F_k(\tau) = \text{Im}(f_k^{\dot{i}} g_k)$$

Quantum Initial Conditions

$$\forall k \quad a_k(\tau_0)|0, \tau_0\rangle = 0 \Rightarrow \Psi_0(\tau_0) = N_0 e^{-k|y_k^0|^2}$$

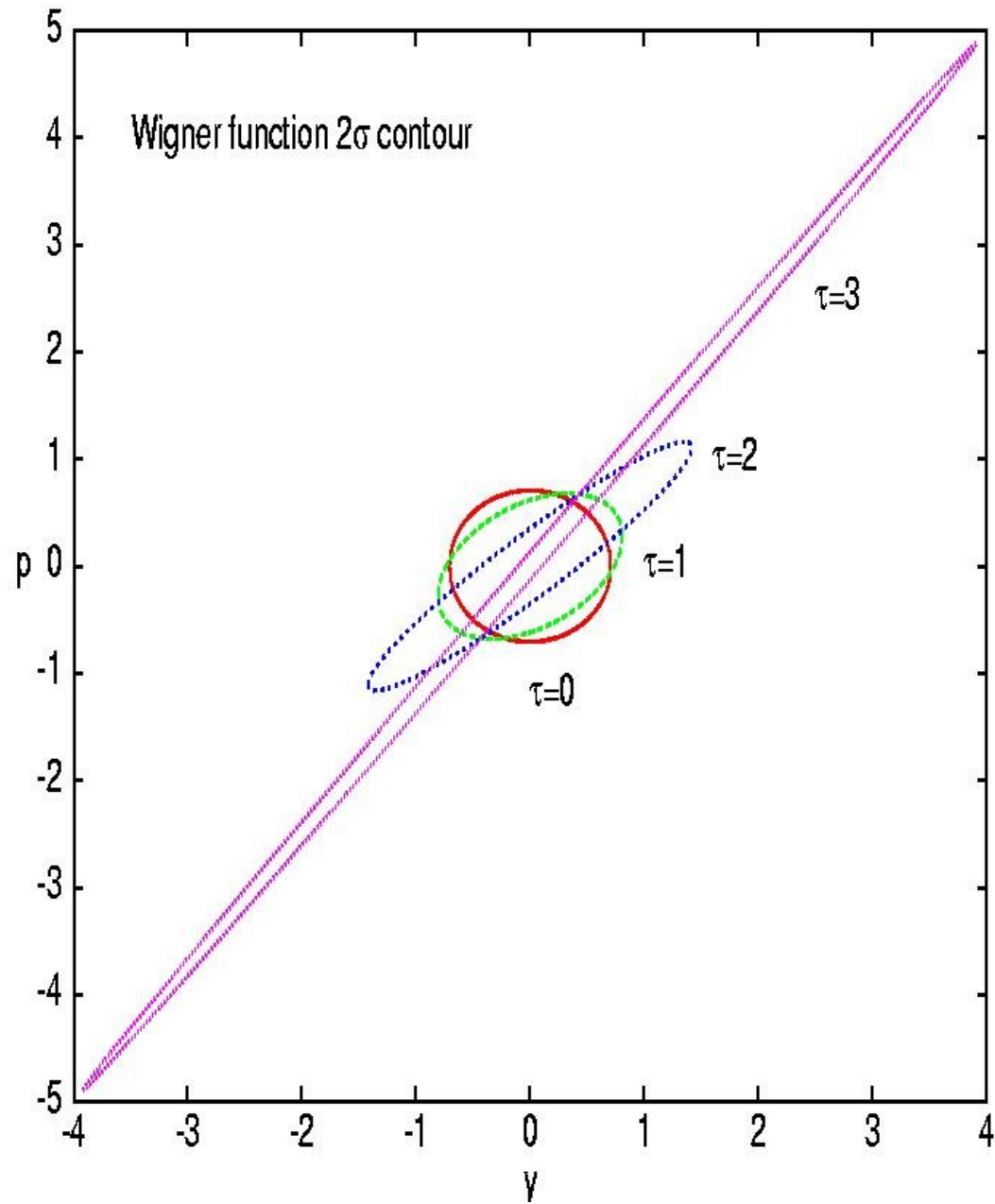
Unitary Evolution

$$|0, \tau\rangle = U|0, \tau_0\rangle \Rightarrow \Psi_0(\tau) = \frac{1}{\sqrt{\pi}|f_k|} e^{-\Omega_k(\tau)|y_k^0|^2}$$

Occupation number of mode k

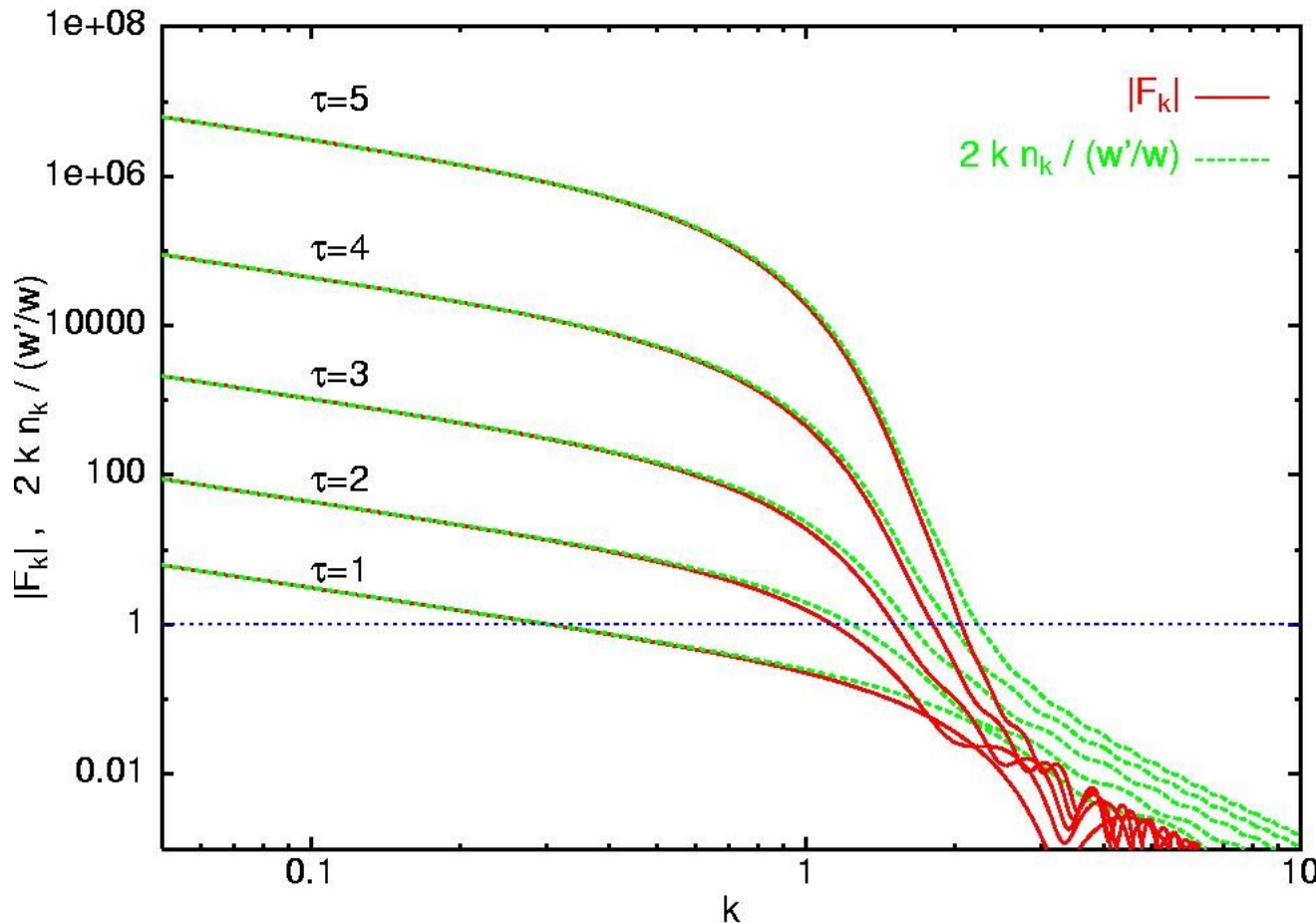
$$n_k(\tau) = \langle 0, \tau | N_k(\tau_0) | 0, \tau \rangle = \frac{1}{2k} |g_k|^2 + \frac{k}{2} |f_k|^2 - \frac{1}{2}$$

Wigner function



Quantum to Classical Transition

$$\langle 0, \tau | G(\hat{y}, \hat{p}) | 0, \tau \rangle \approx \langle G_0(y, p) \rangle_{\text{gaussian}}$$



$$|F_k(\tau)| \gg 1$$



Semiclassical
(diagonal
density
functional)

Quantum to Classical Transition

For $k < \sqrt{\tau}$ (longwave modes)

Power spectrum (approximation):

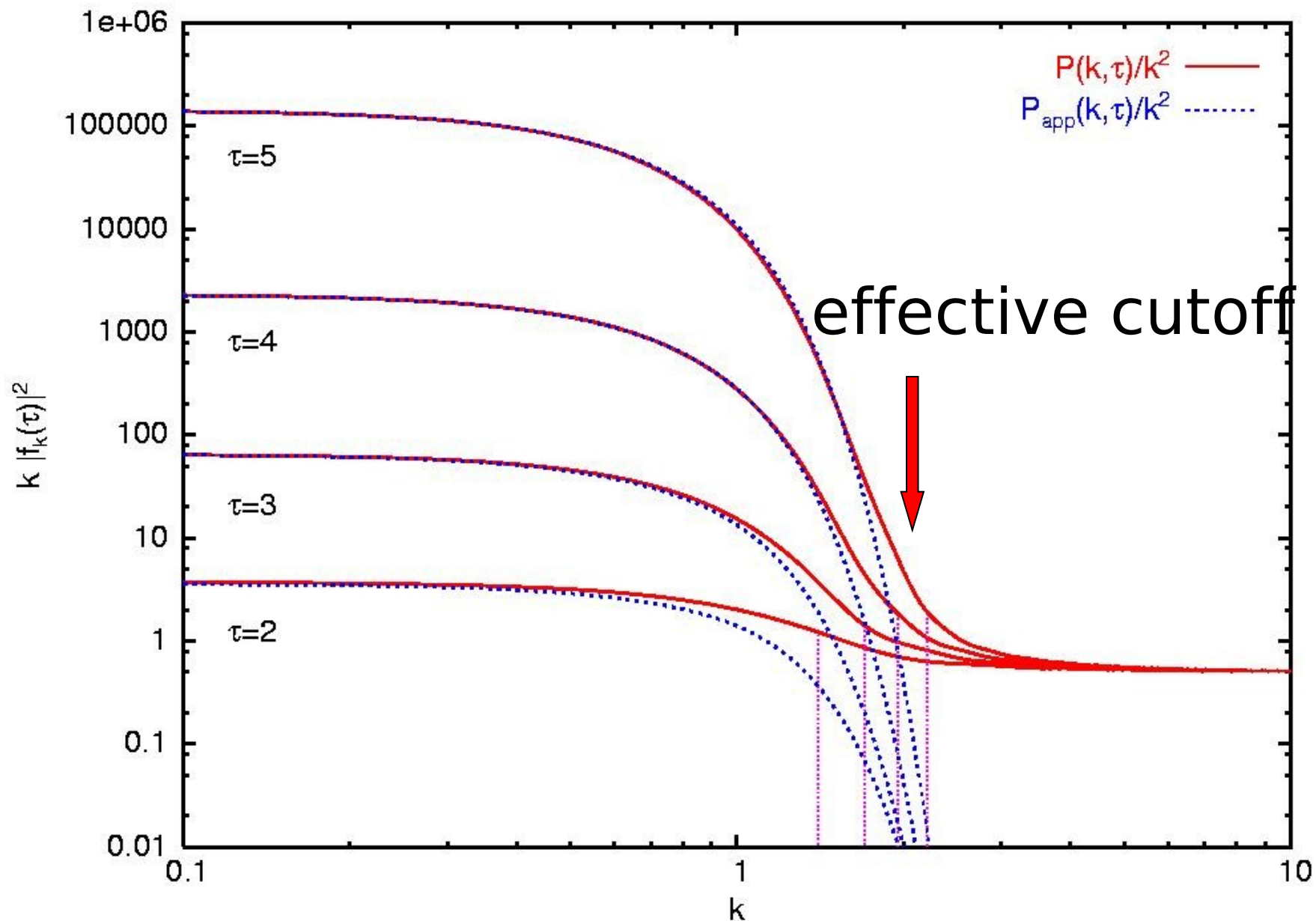
$$P_{app}(k, \tau) = k^3 |f_k(\tau)|^2 \approx A(\tau) k^2 e^{-B(\tau)k^2}$$

$$A(\tau) = A_0 Bi^2(\tau) \approx \frac{A_0}{\pi \sqrt{\tau}} e^{\frac{4}{3}\tau^{3/2}}$$

$$B(\tau) = 2\sqrt{\tau}$$

Airy function

Power spectrum of longwave modes



Lattice Simulations

Quantum averages = Gaussian ensemble averages

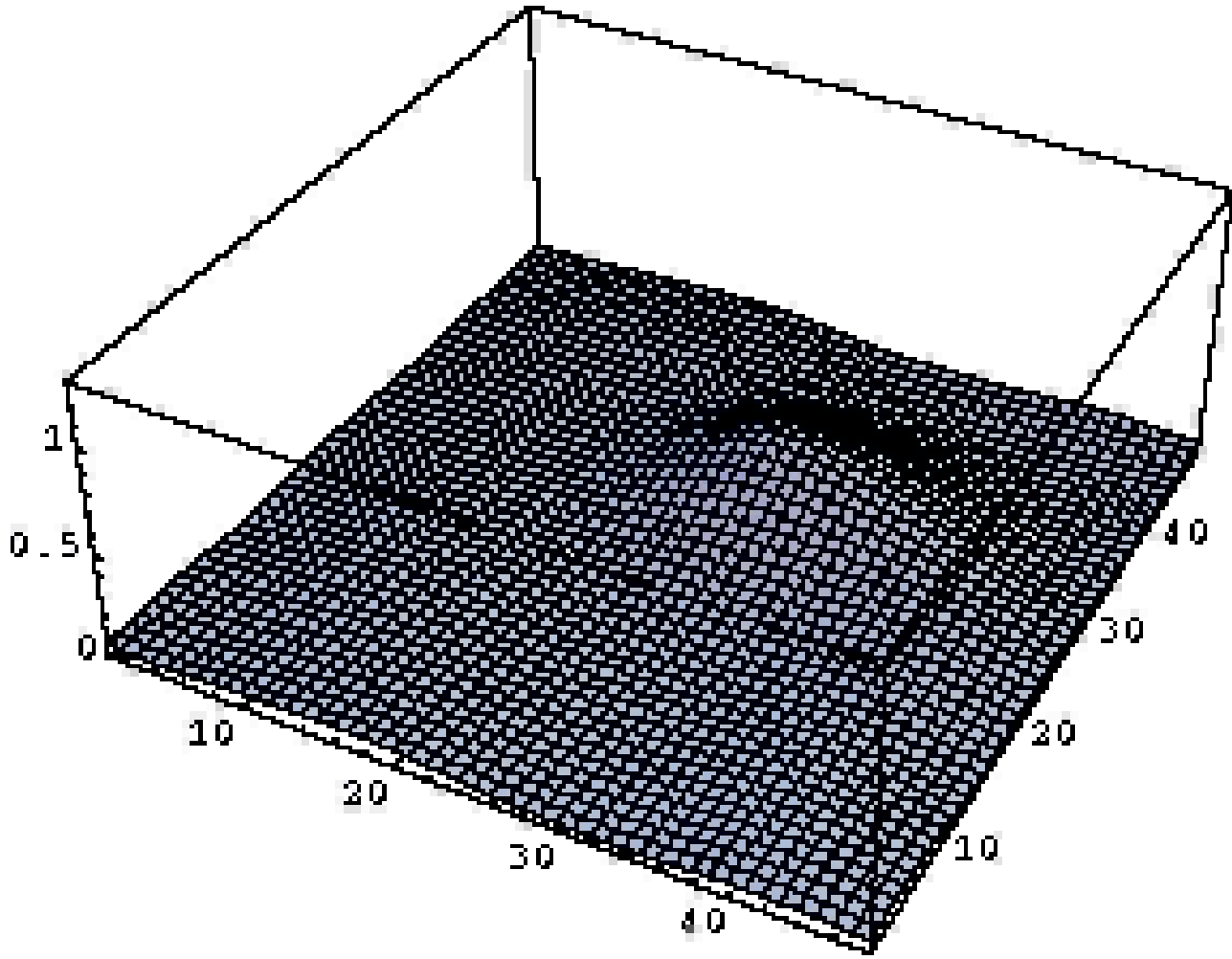
Initial conditions: Highly occupied modes

$$|0, \tau\rangle = U|0, \tau_0\rangle \Rightarrow \psi_0(\tau) = \frac{1}{\sqrt{\pi}|f_k|} e^{-\Omega_k(\tau)|y_k^0|^2}$$

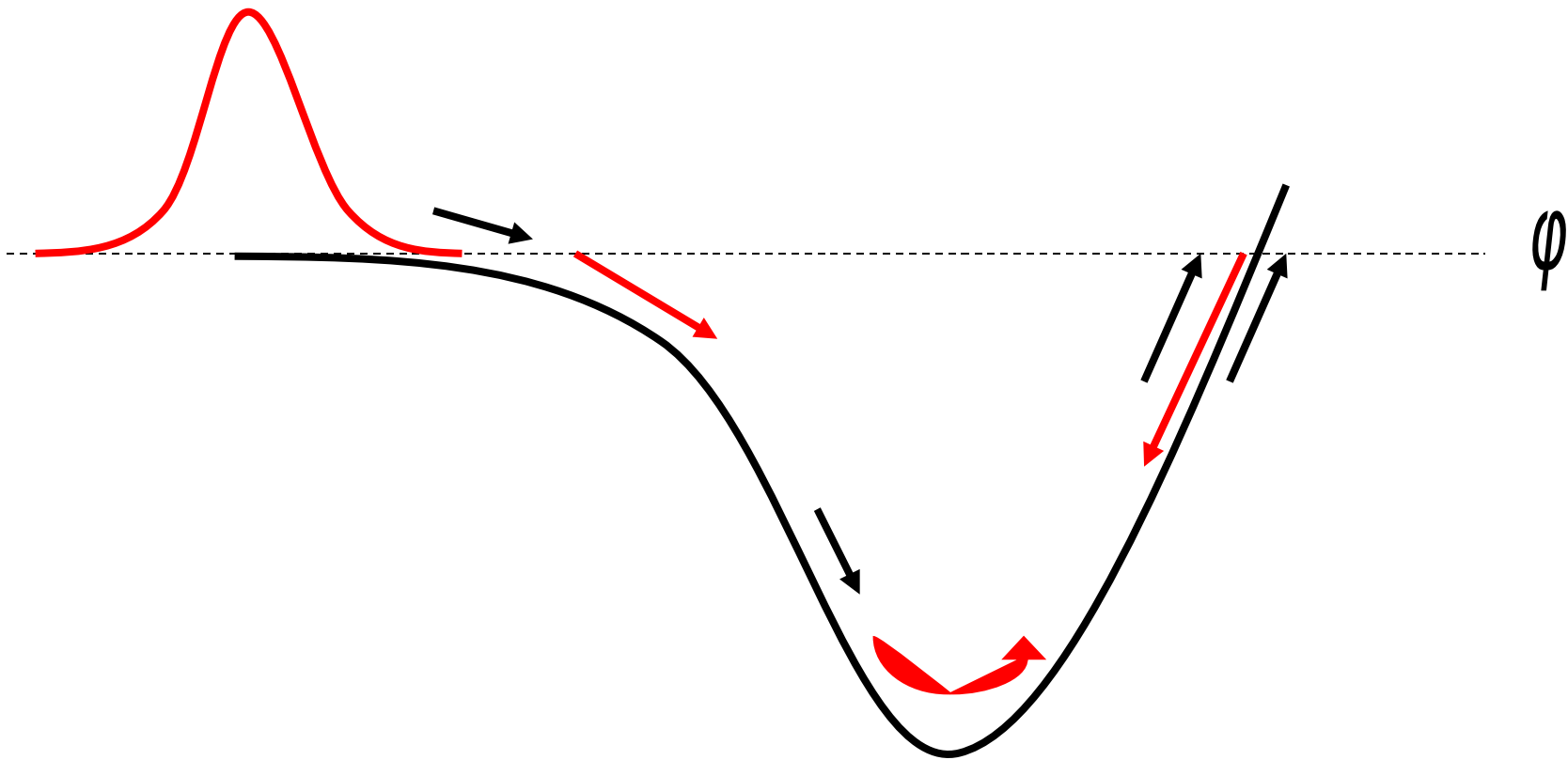
Rayleigh
distribution:

$$P_\psi(|\varphi_k|) d|\varphi_k| d\theta_k = e^{-\frac{|\varphi_k|^2}{|f_k|^2}} \frac{d|\varphi_k|^2}{|f_k|^2} \frac{d\theta_k}{2\pi}$$

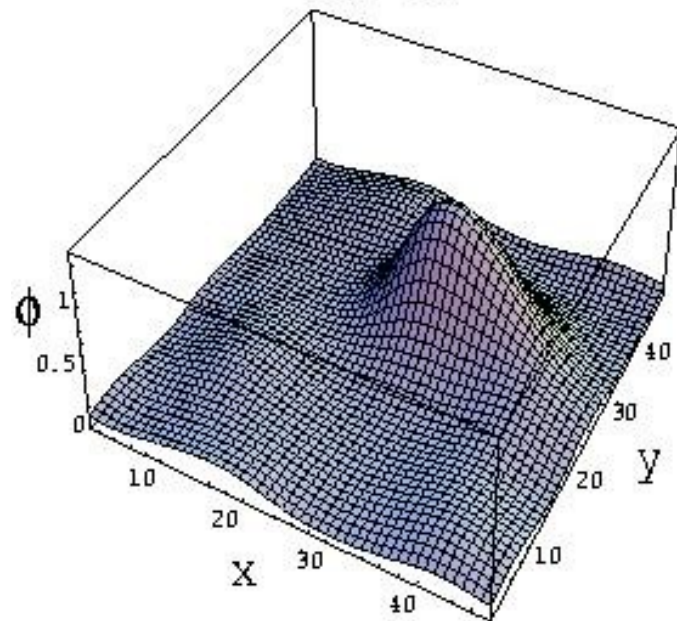
High peaks of Higgs field



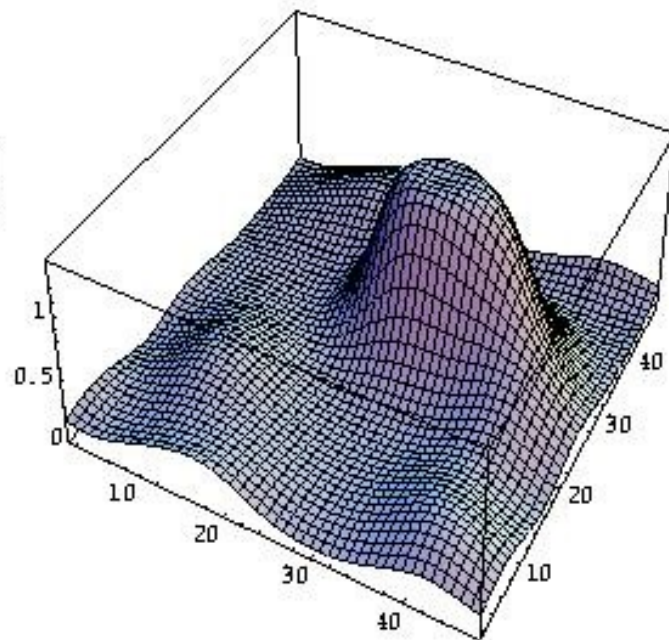
High peaks



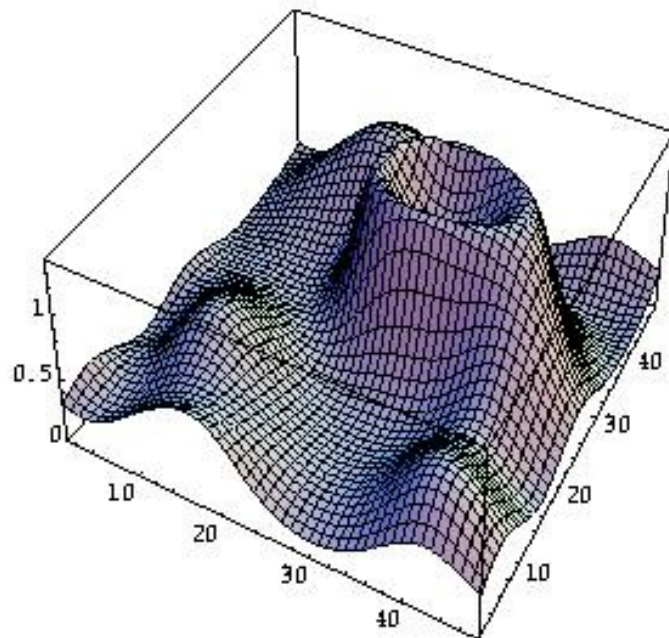
mt = 23



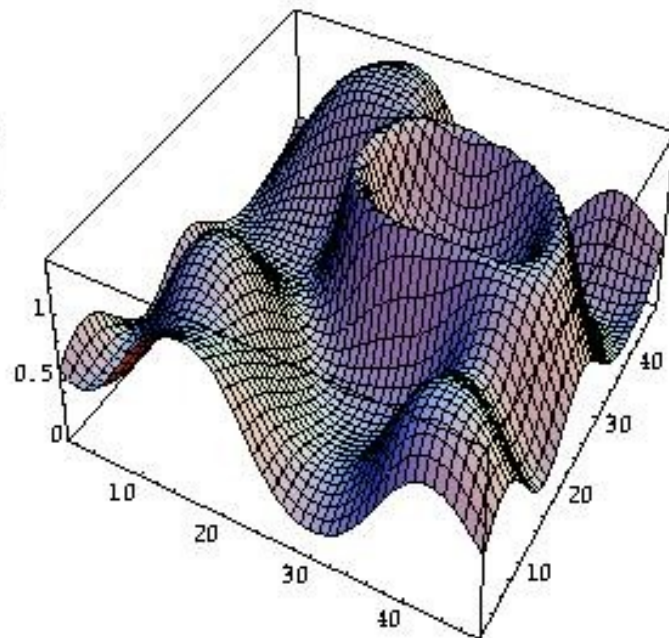
mt = 24



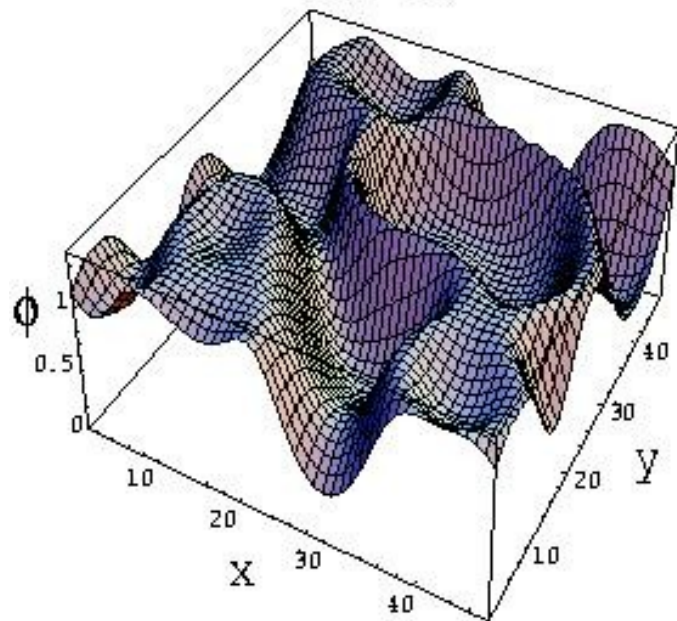
mt = 25



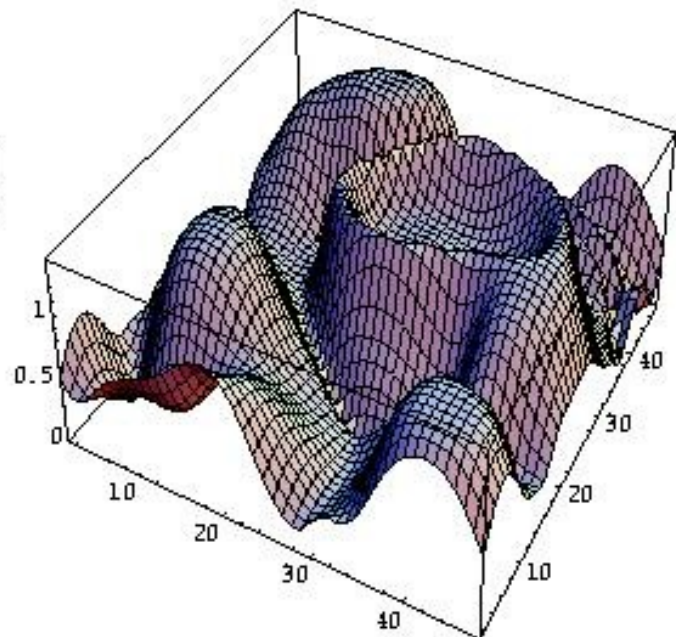
mt = 26



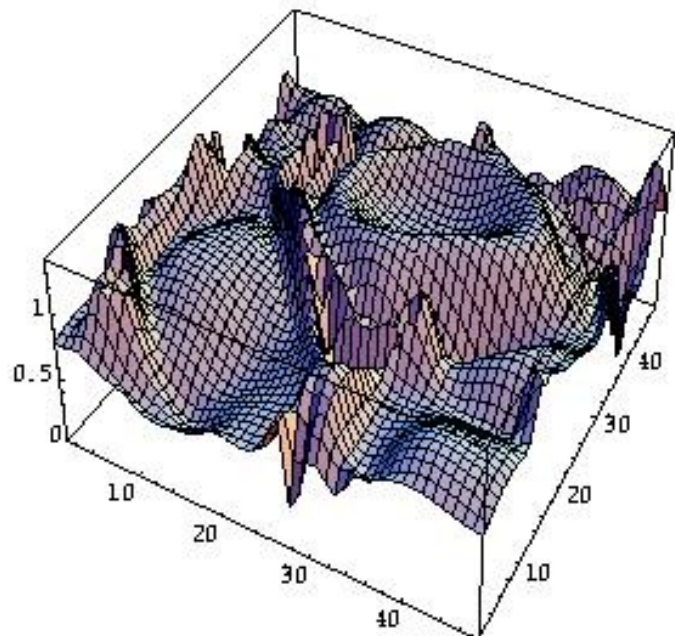
mt = 27



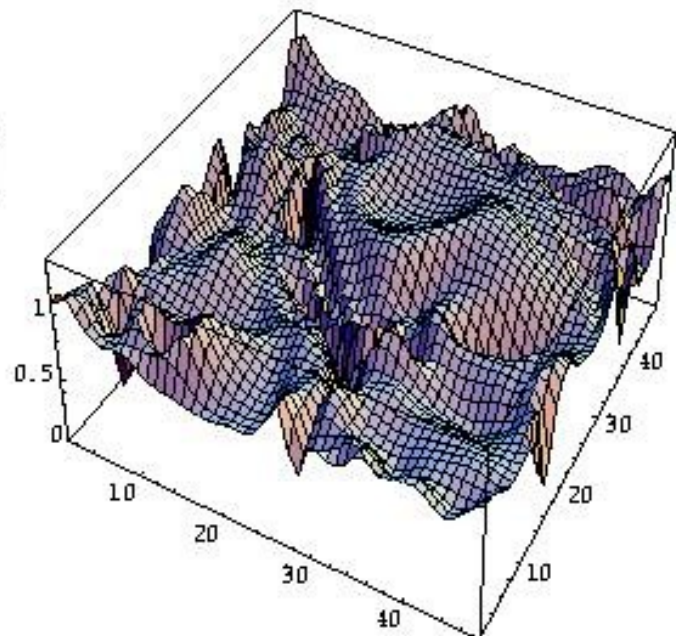
mt = 32



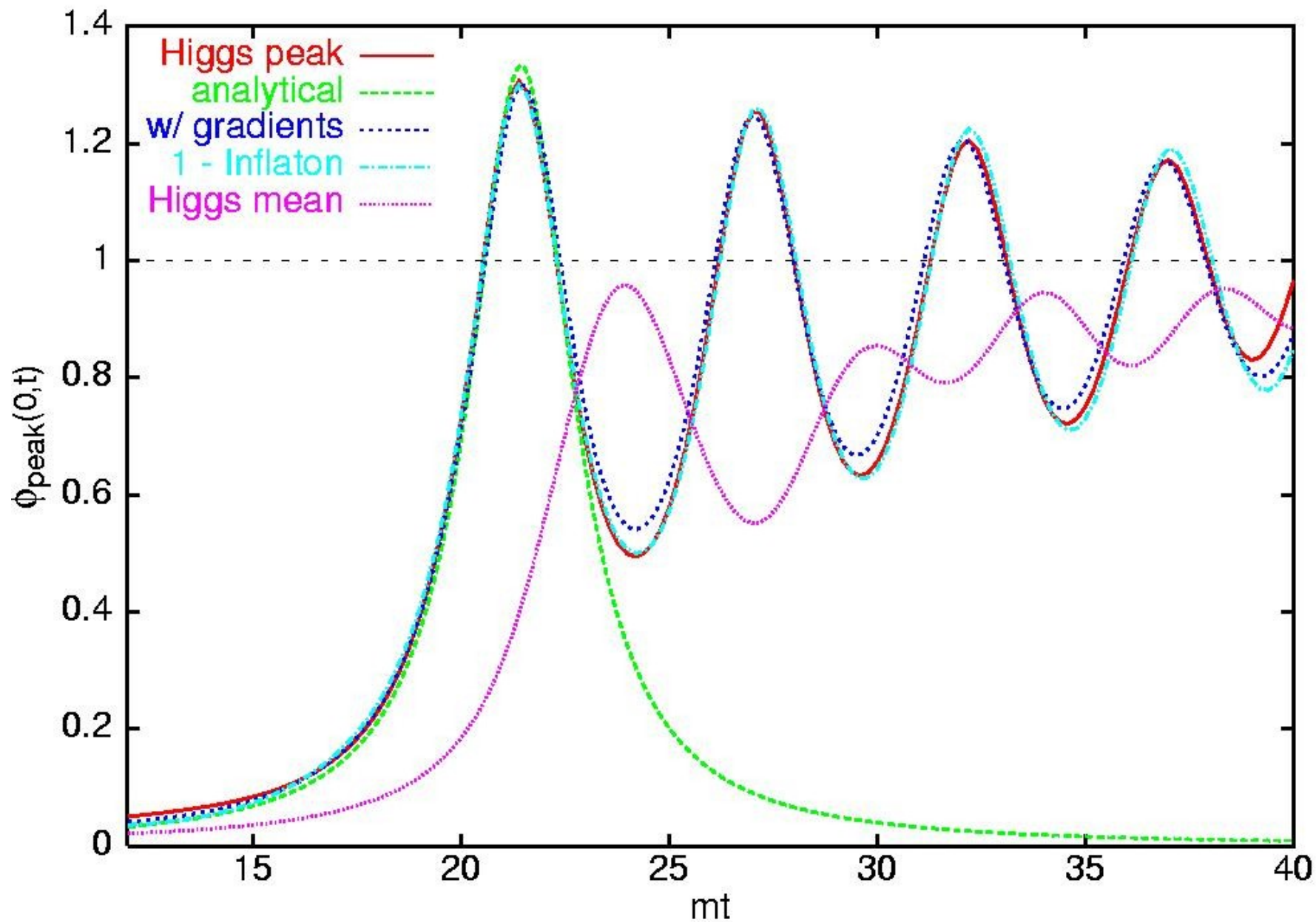
mt = 36



mt = 40



High peaks and mean of Higgs field



Stochastic background gravitational waves

J. G.-B.

Daniel G. Figueroa

+Alfonso Sastre

PRL98, 061302 (2007)

arXiv:0707.0839
[hep-ph]

The Higgs-Inflaton model

$$L = \text{Tr}[(\partial_\mu \Phi)^\dagger \partial^\mu \Phi] + \frac{1}{2}(\partial_\mu \chi)^2 - V(\Phi, \chi)$$

$$\text{Tr}[\Phi^\dagger \Phi] = \frac{1}{2}(\varphi_0^2 + \varphi^a \varphi_a) \equiv \frac{1}{2}\varphi^2$$

$$V(\varphi, \chi) = \frac{\lambda}{4}(\varphi^2 - v^2)^2 + \frac{g^2}{2}\varphi^2 \chi^2 + \frac{1}{2}m^2 \chi^2$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

backreaction

$$g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi = (\partial_0 \varphi)^2 - (\nabla \varphi)^2 - h^{ij} \nabla_i \varphi \nabla_j \varphi$$

Gravity waves evolution equation

$$\partial_0^2 h_{ij} - \nabla^2 h_{ij} = 16\pi G \Pi_{ij} \quad \text{anisotropic stress tensor}$$

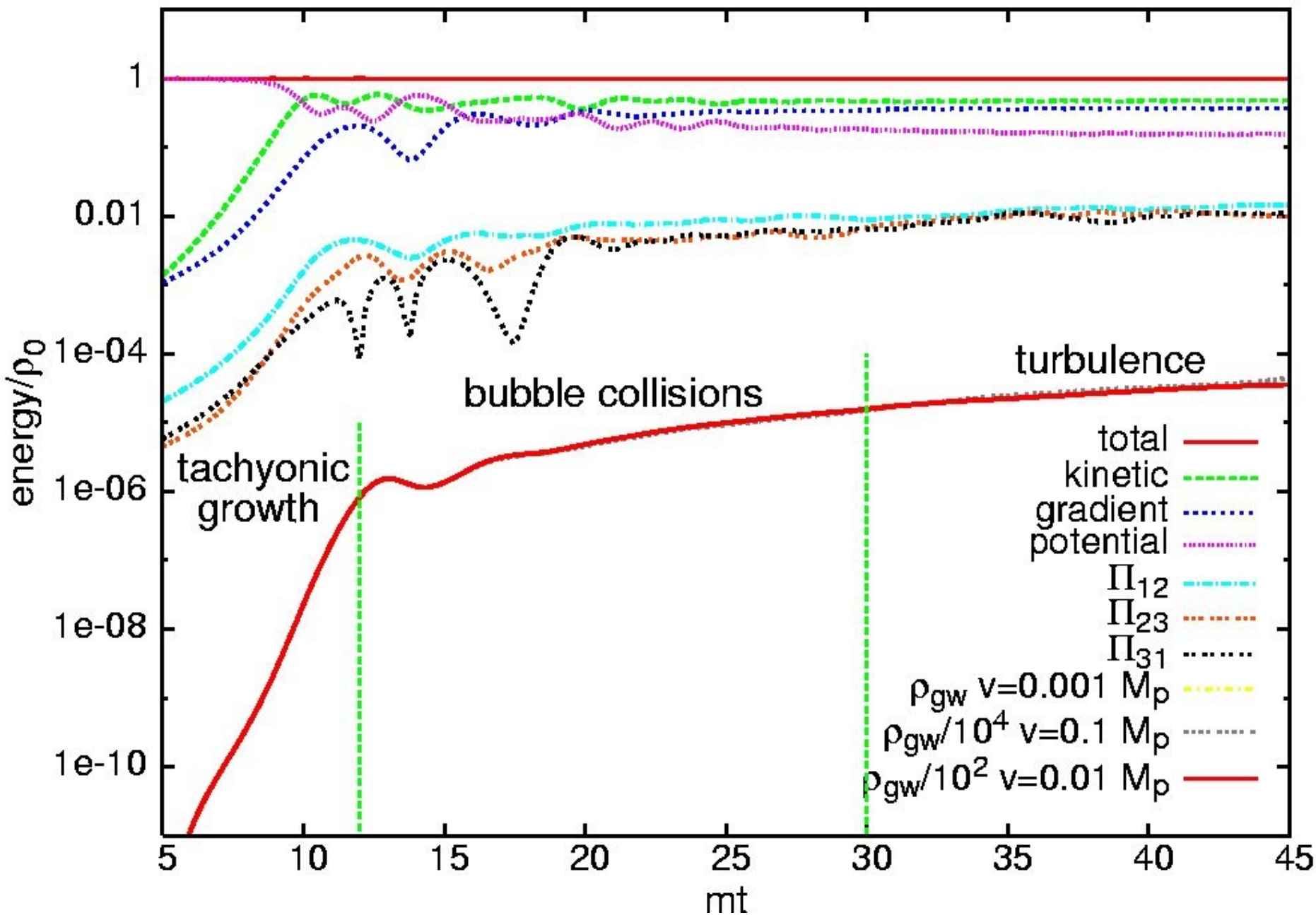
$$\Pi_{ij} = \nabla_i \varphi \nabla_j \varphi - \frac{1}{3} \delta_{ij} (\nabla \varphi)^2 + \nabla_i \chi \nabla_j \chi - \frac{1}{3} \delta_{ij} (\nabla \chi)^2$$

$$t_{\mu\nu} = \frac{1}{32\pi G} \langle \partial_\mu h_{ij}^{\text{TT}} \partial_\nu h_{ij}^{\text{TT}} \rangle \quad \text{energy density}$$

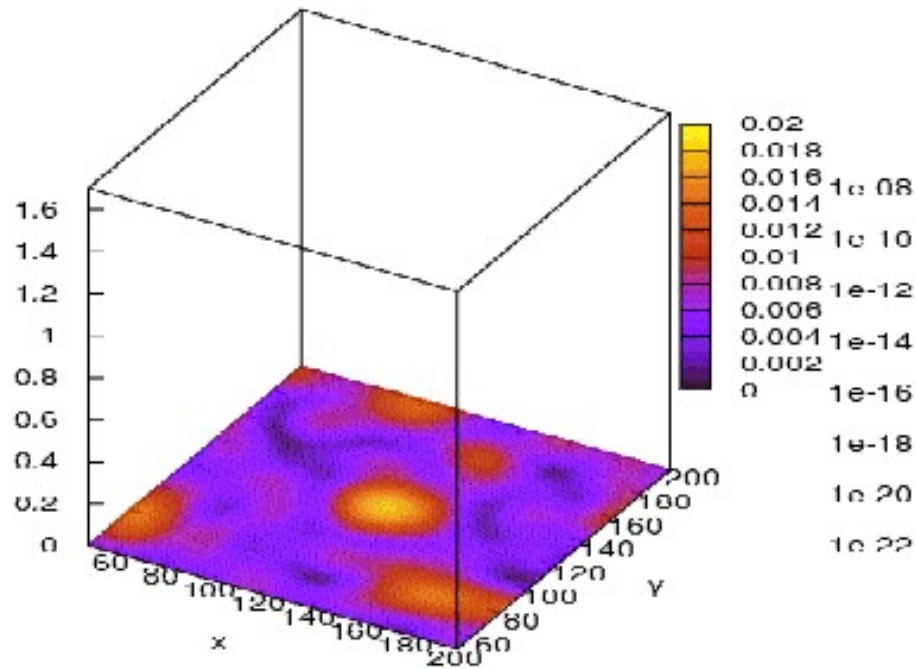
$$\frac{\rho_{gw}}{\rho_0} = \frac{1}{8\pi G v^2 m^2} \langle \partial_0 h_{ij}^{\text{TT}} \partial_0 h_{ij}^{\text{TT}} \rangle = \frac{2}{5} \frac{1}{8\pi G v^2 m^2} \langle \partial_0 h_{ij} \partial_0 h_{ij} \rangle$$

$$\Omega_{gw} = \int \frac{df}{f} \Omega_{gw}(f) = \int \frac{dk}{k} \frac{k^3}{2\pi^2} \frac{\rho_{gw}(k)}{\rho_0} \frac{\rho_{rad}}{\rho_c}$$

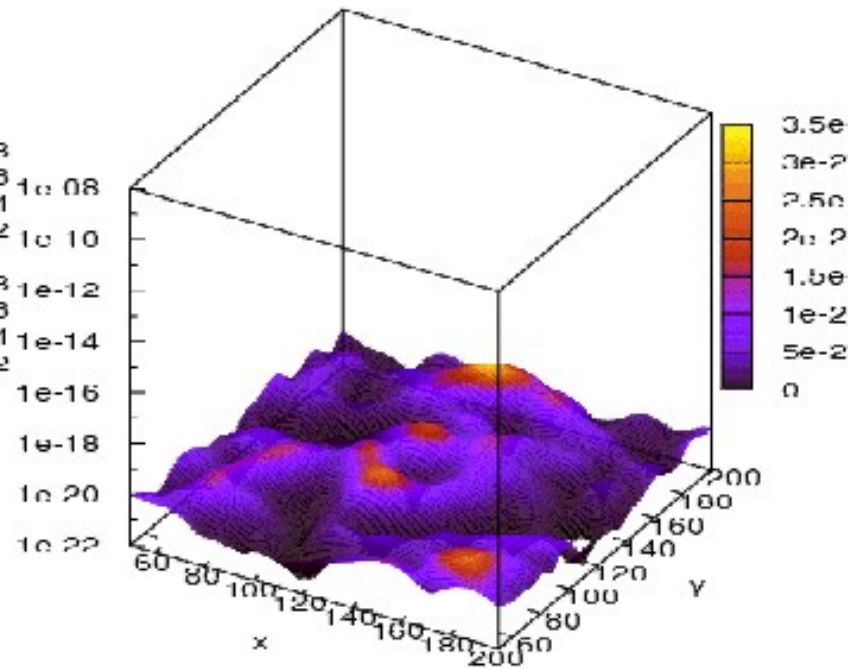
Time evolution after inflation

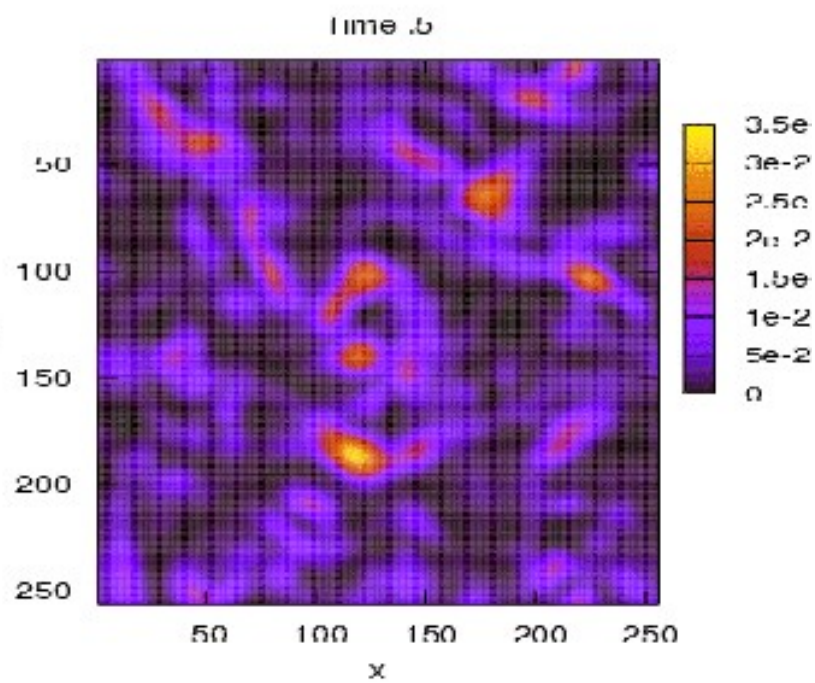
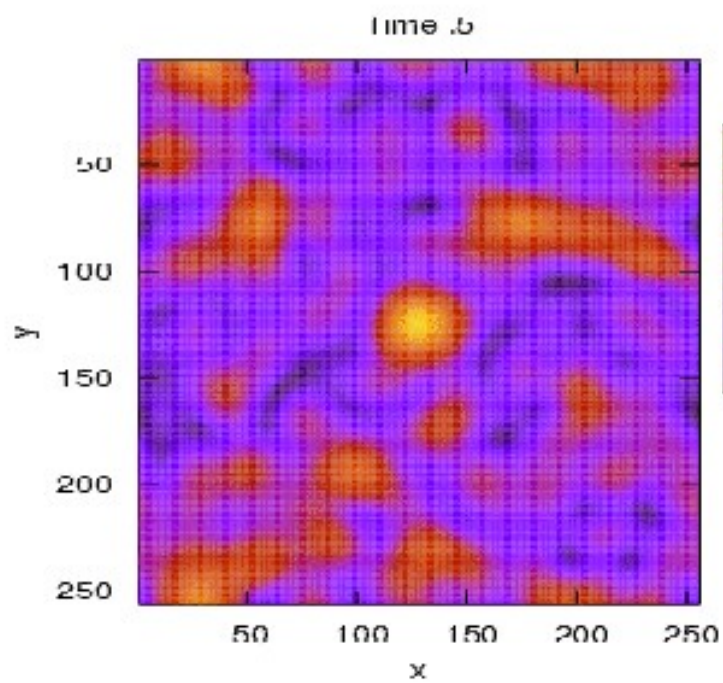


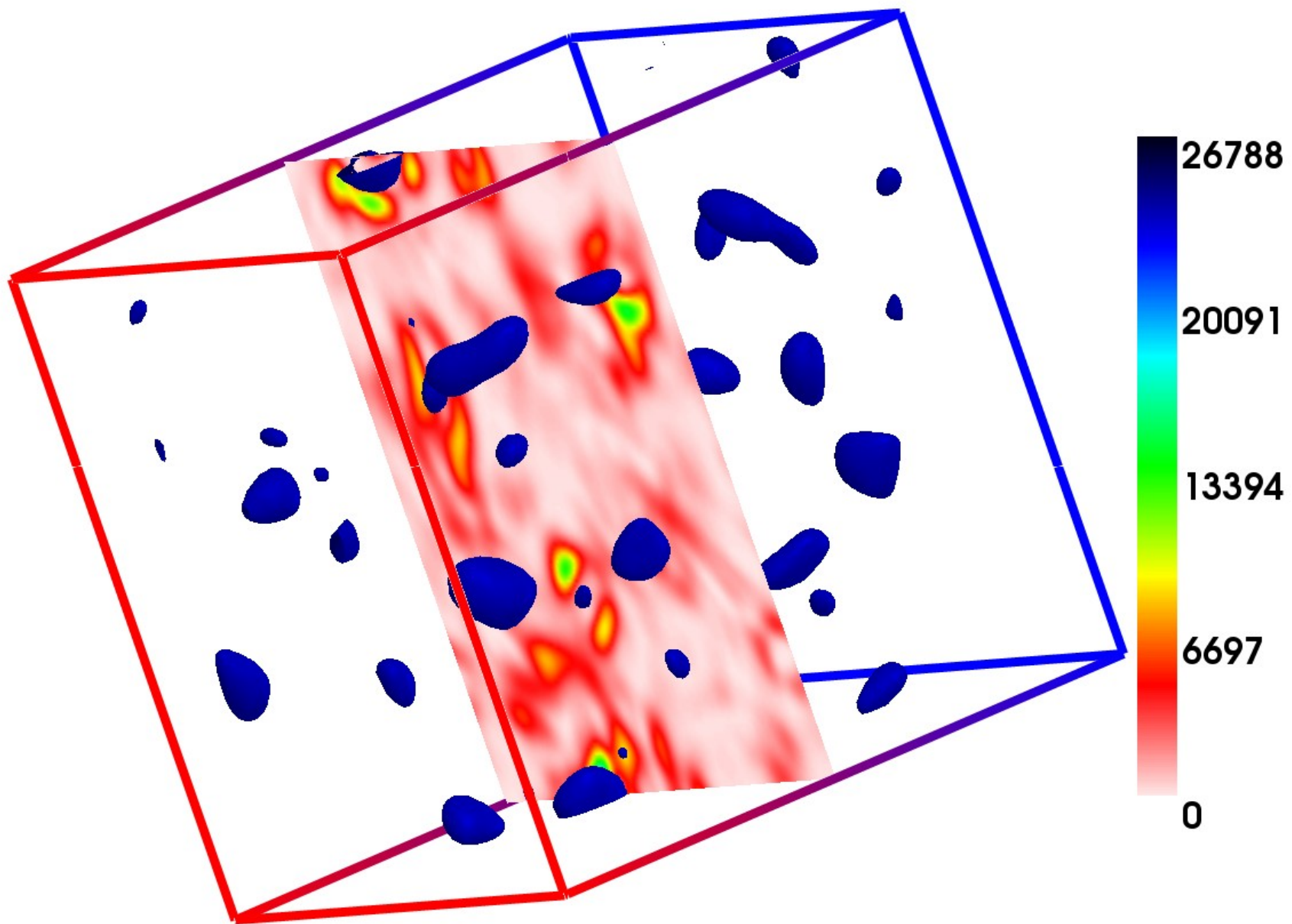
time .5

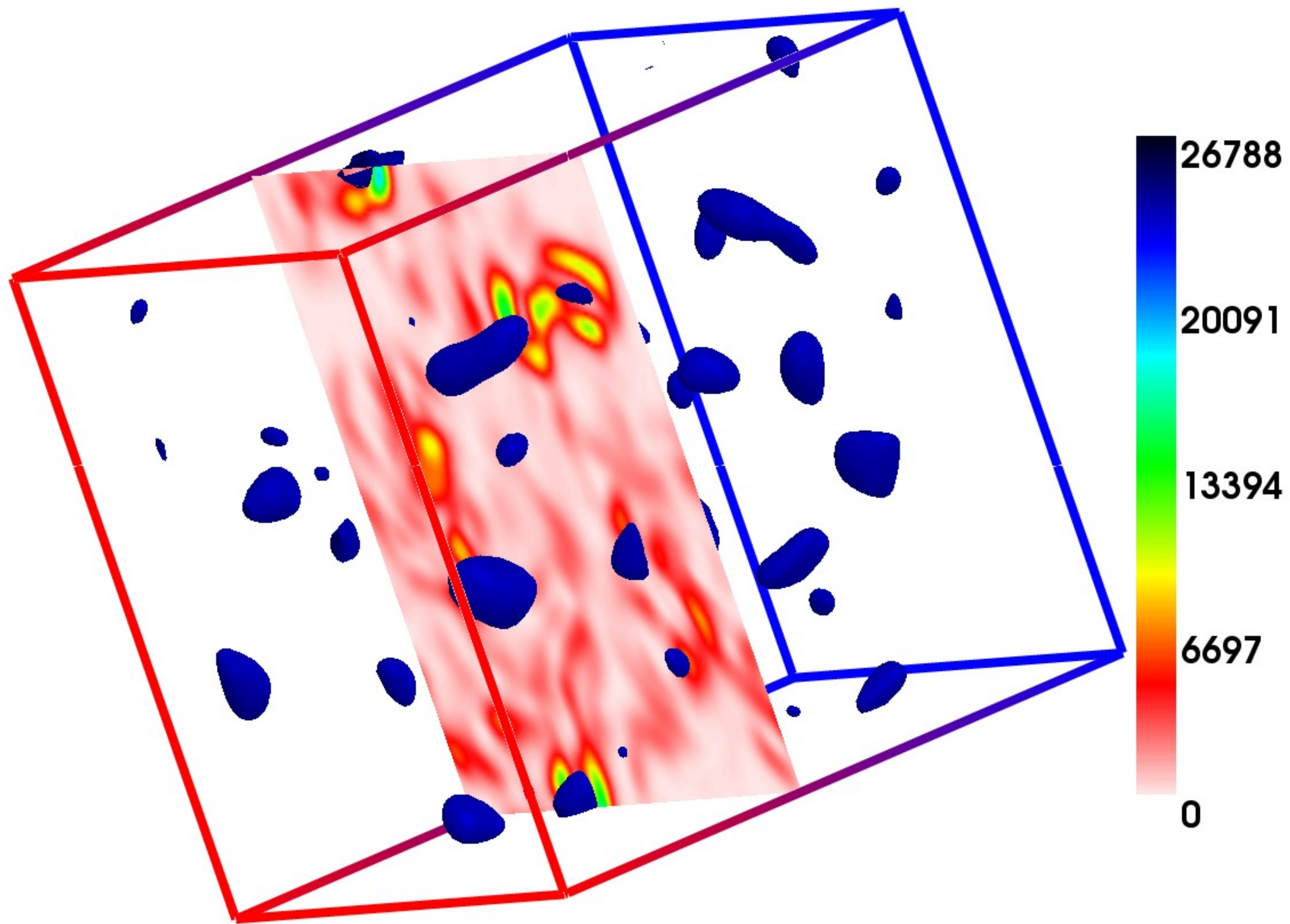


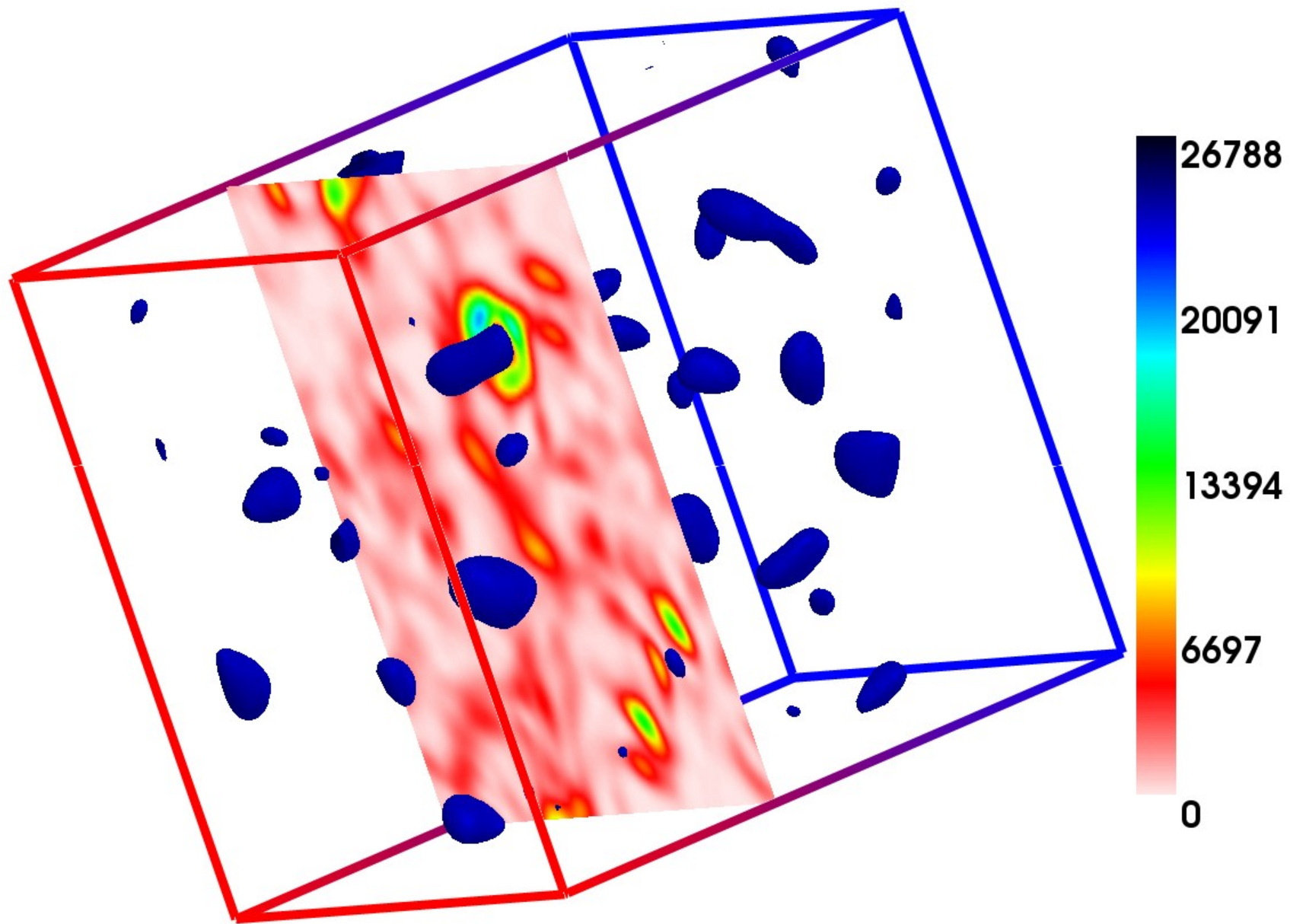
time .5

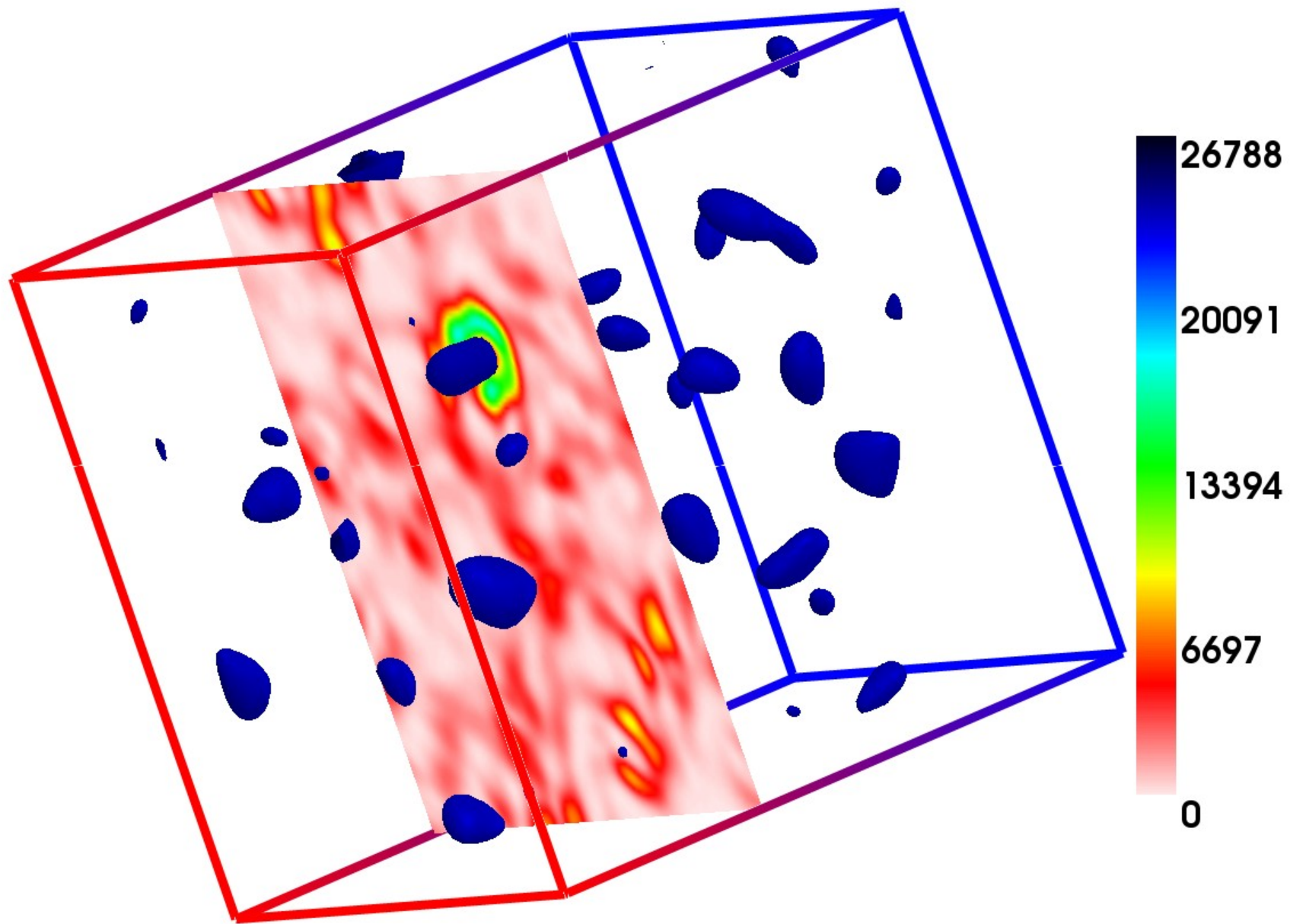


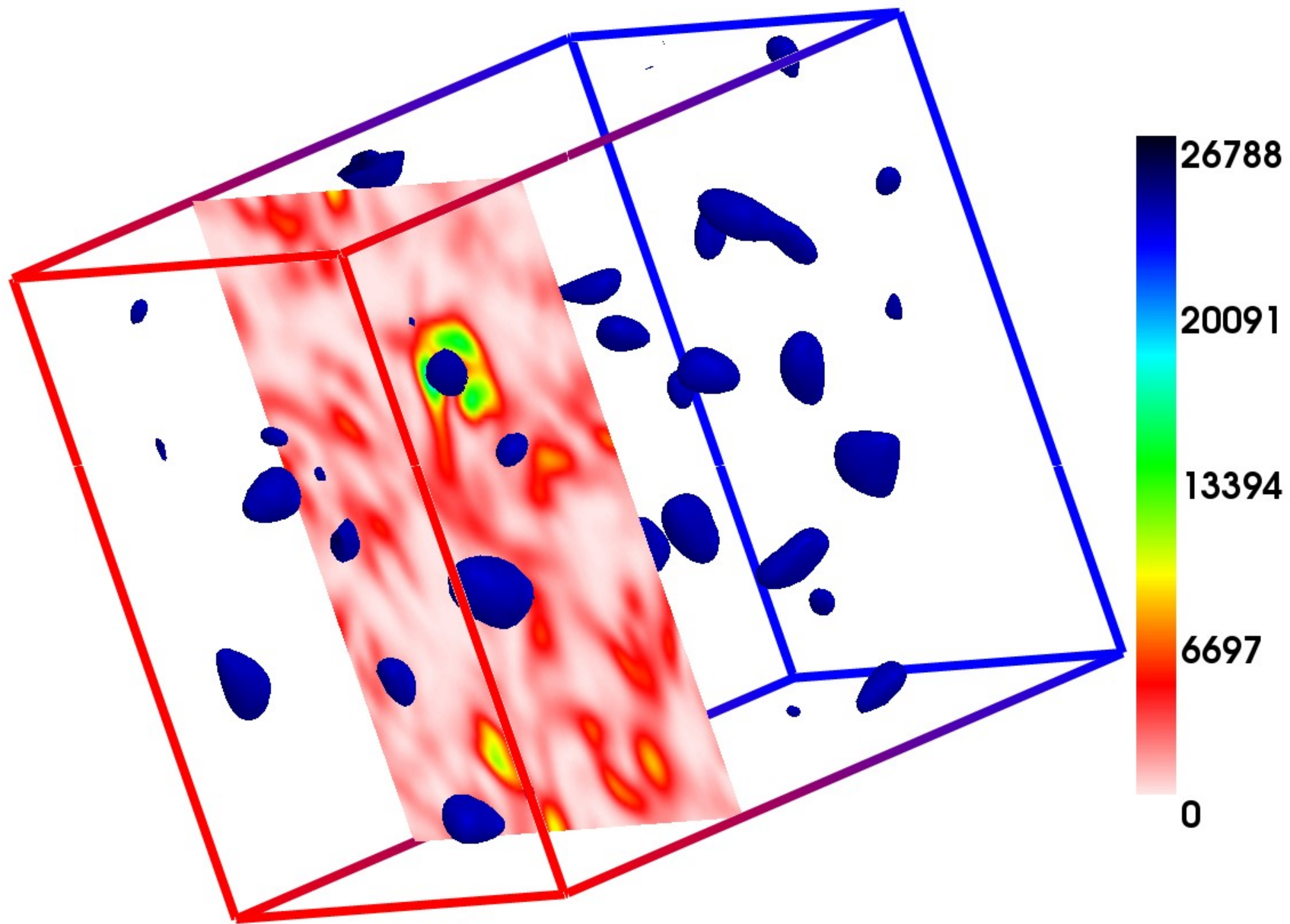


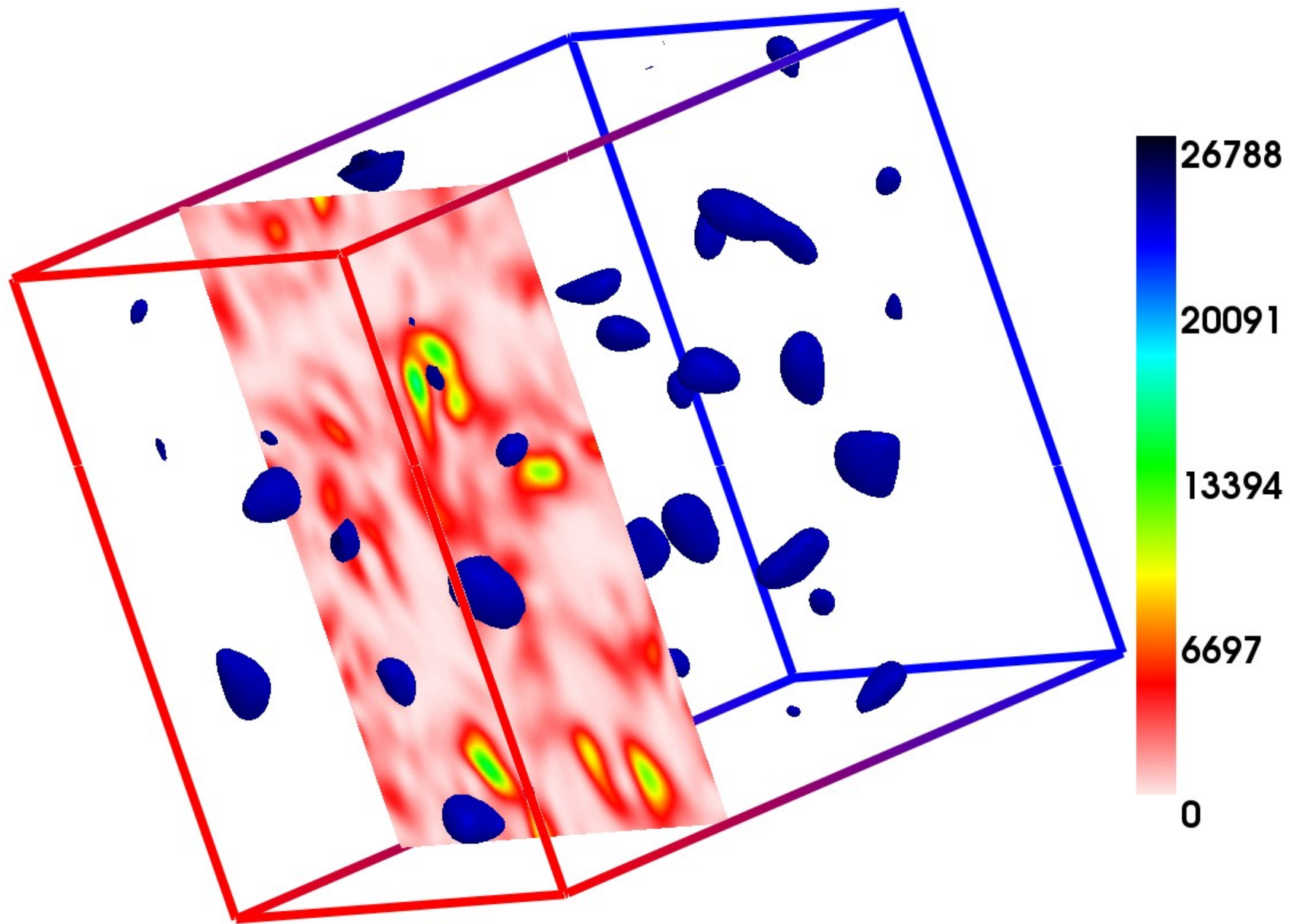


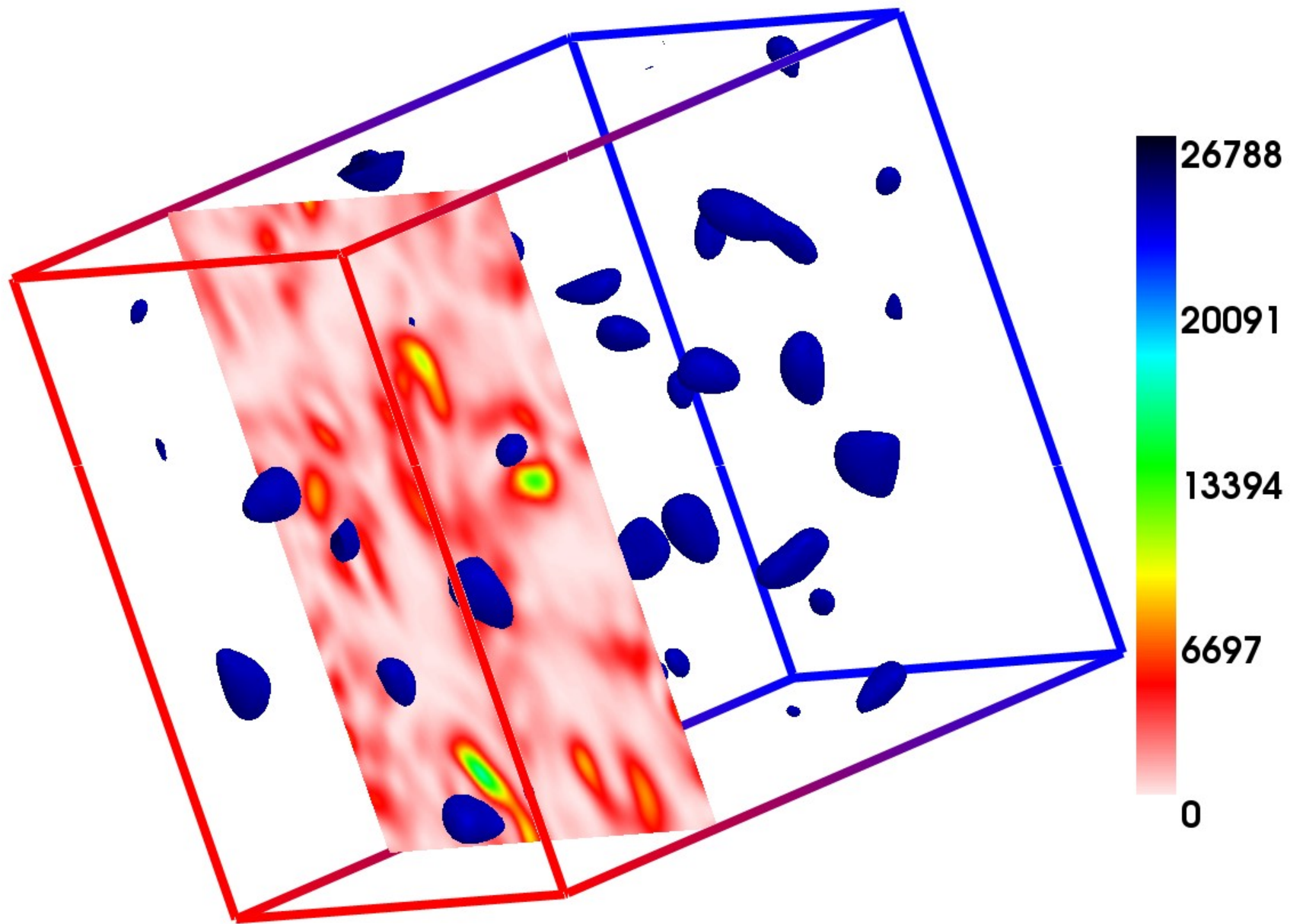




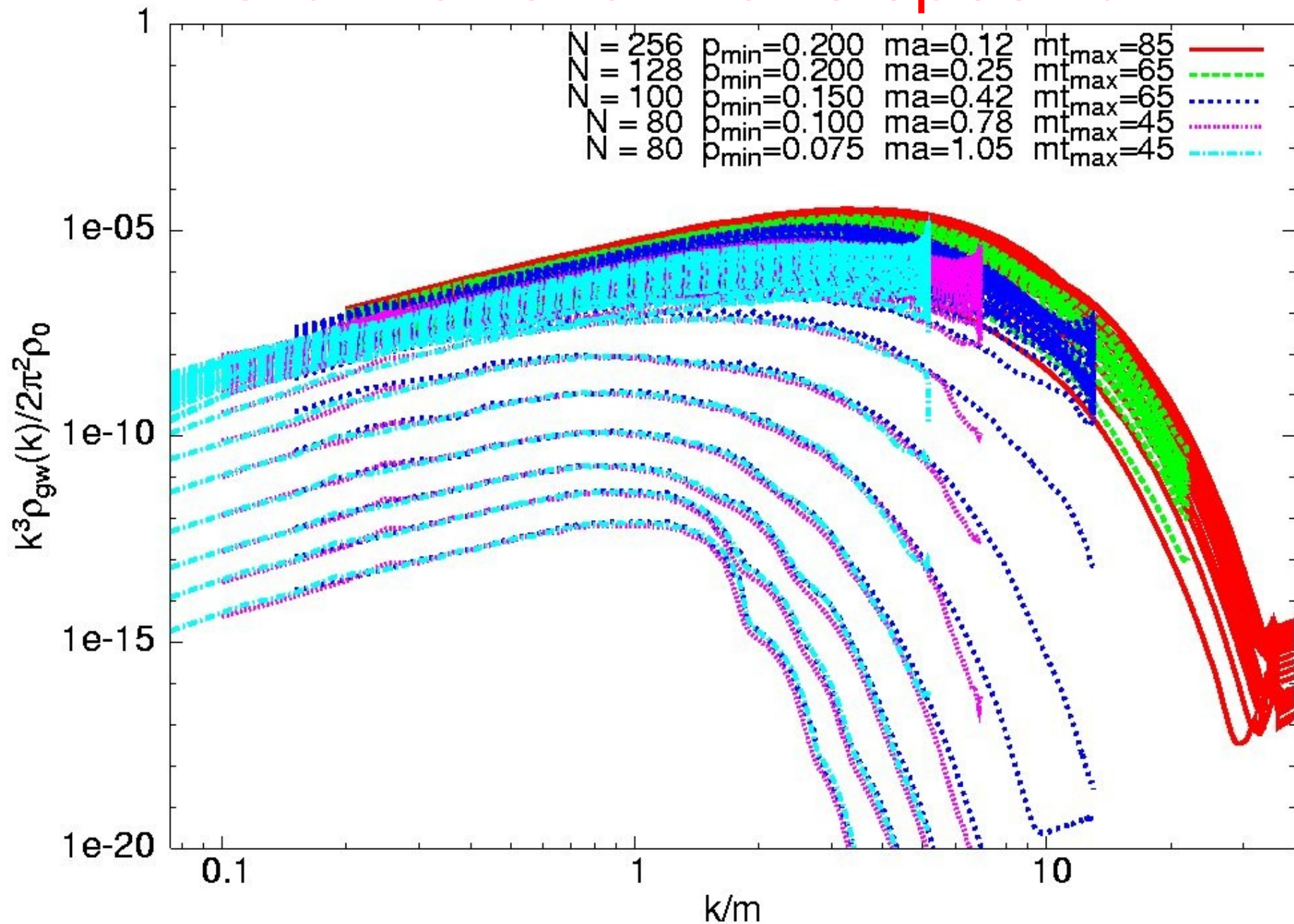




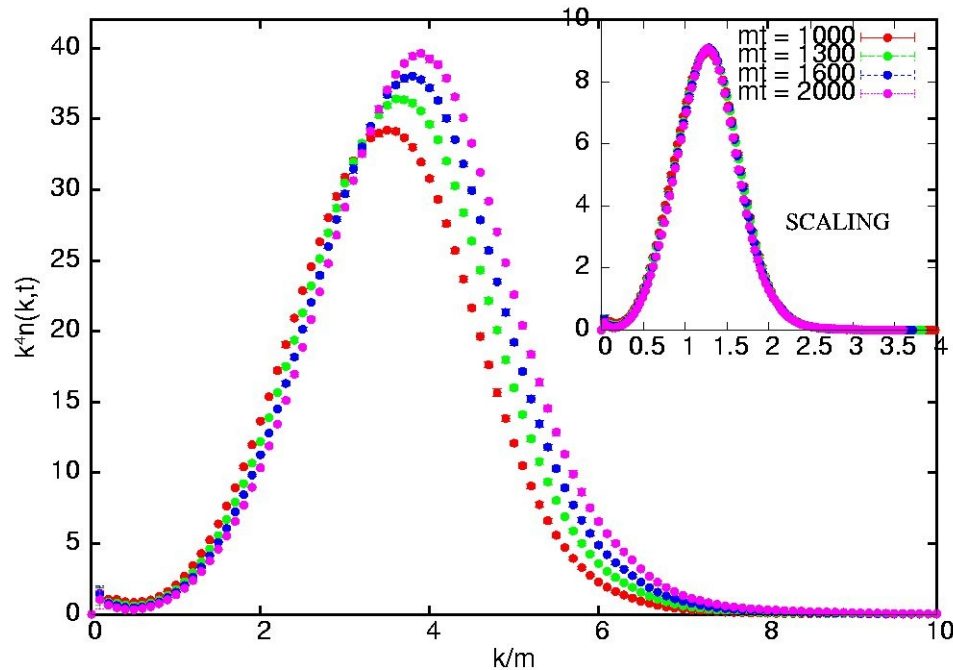




Gravitational wave spectrum



Kinetic Turbulence &



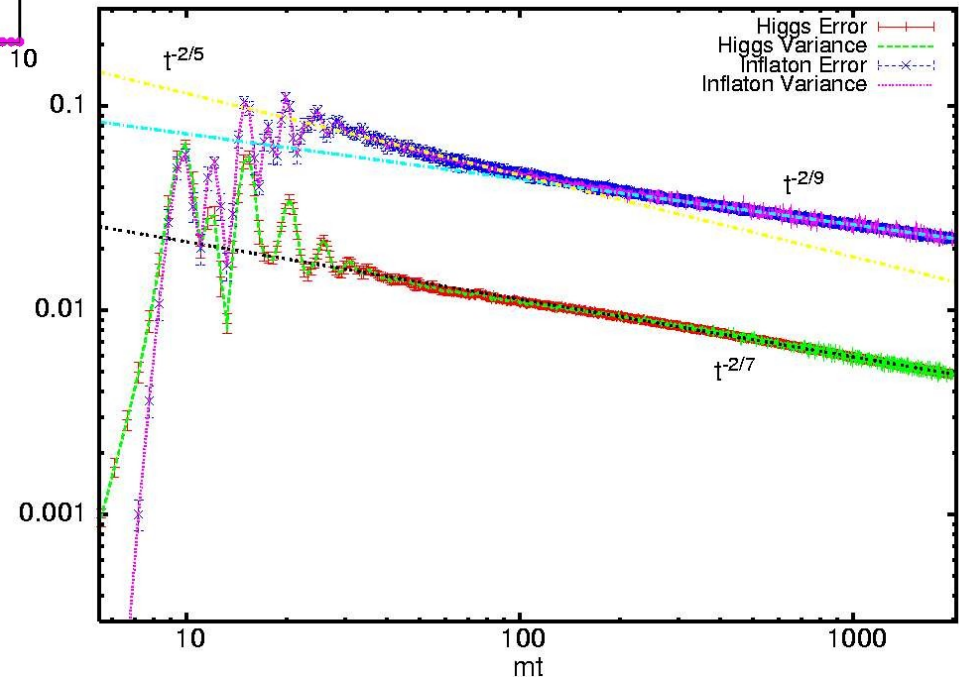
$$n(k, t) = t^{-q} n_0(kt^{-p})$$

$$q = 3.5p$$

$$p = \frac{1}{2m-1}$$

$$\Delta\phi^2 = \langle \phi^2 \rangle - \langle \phi \rangle^2 \propto t^{-\nu}$$

$$\nu = \frac{2}{2m-1}$$



GW spectrum during turbulence

$$\frac{k^3 \rho_{gw}(k, t)}{2\pi^2 \rho_0} = 0.002 Gv^2 t^{1.78} k^2 \exp(-0.32 t^{-2/9} k^2)$$

instantaneous spectrum:

$$\Omega_{gw}(t) = \int \frac{dk}{k} \frac{k^3 \rho_{gw}(k, t)}{2\pi^2 a^4 \rho_c} = 0.002 \Omega_{rad} \frac{Gv^2 t^2}{a^4}$$

integrated spectrum after end of turbulence (t^*):

$$\Omega_{gw} = \int dt \Omega_{gw}(t) \approx \Omega_{gw}(t_1) (mt)^\alpha \quad \alpha = 1, \frac{1}{3}$$

Gravitational Waves are produced directly at the Big Bang

BIG BANG

End of Inflation
(Big Bang
 10^{-35} Seconds)

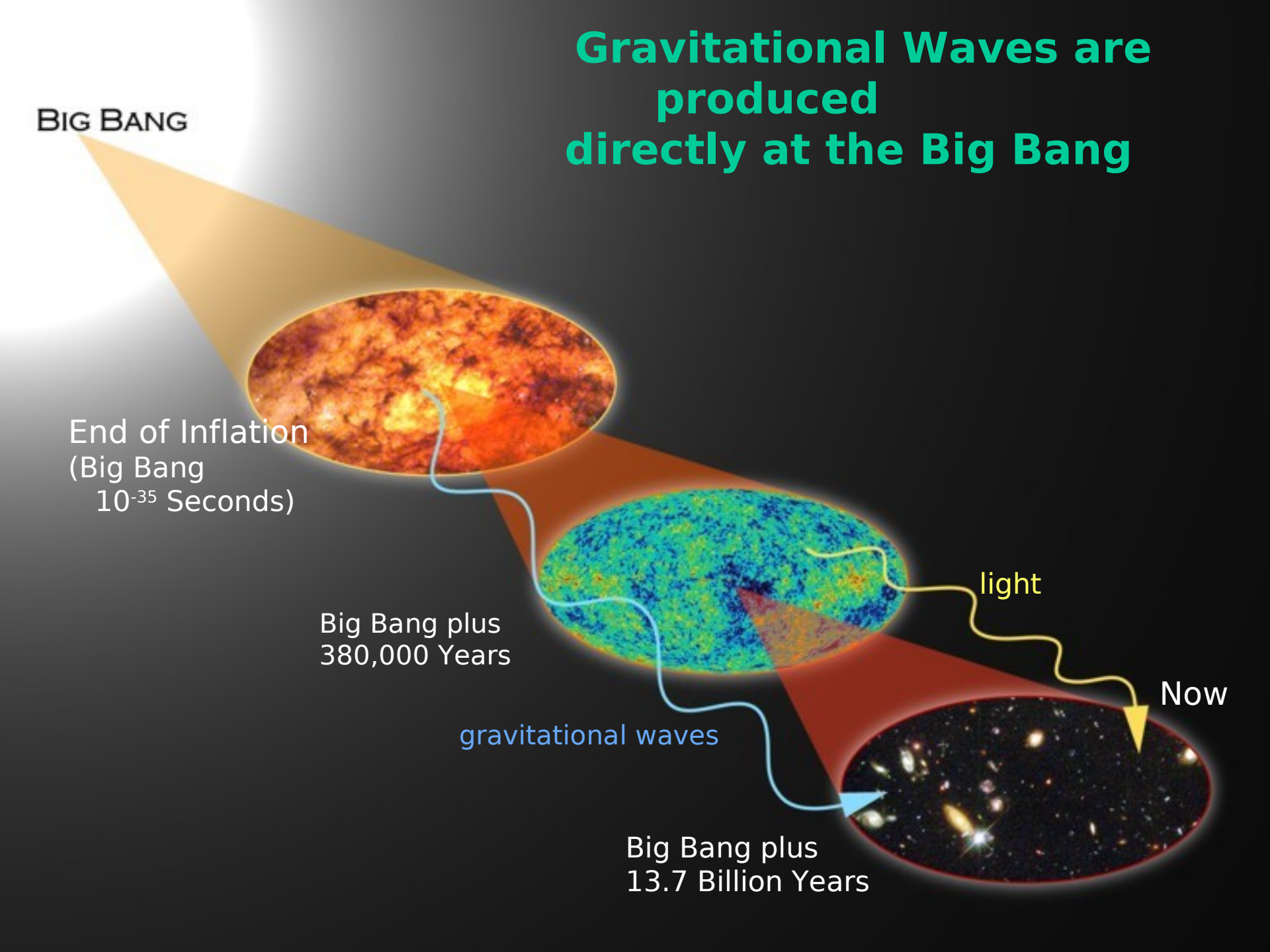
Big Bang plus
380,000 Years

gravitational waves

light

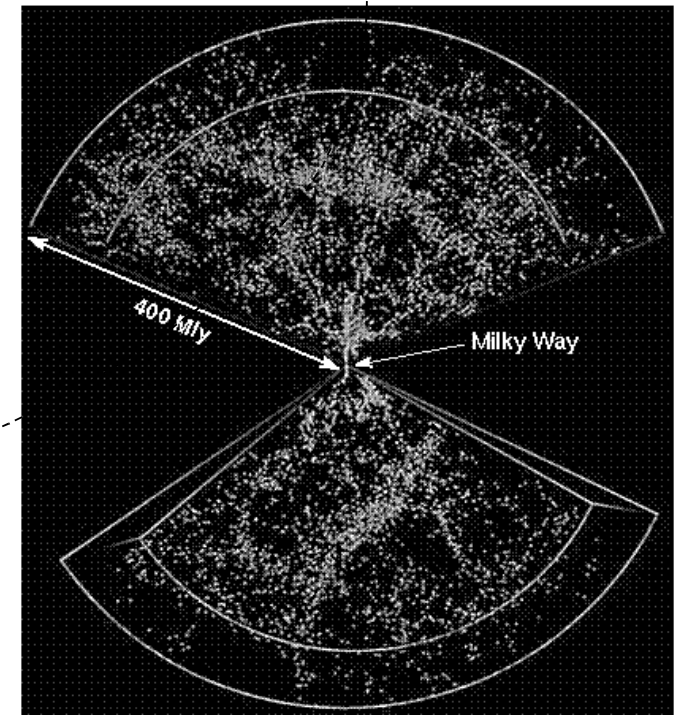
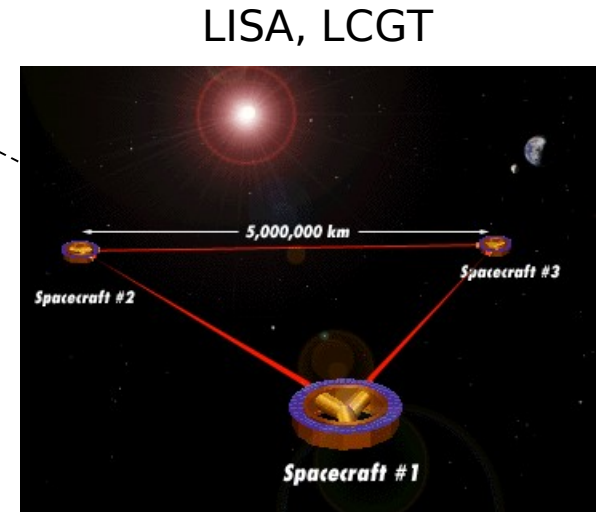
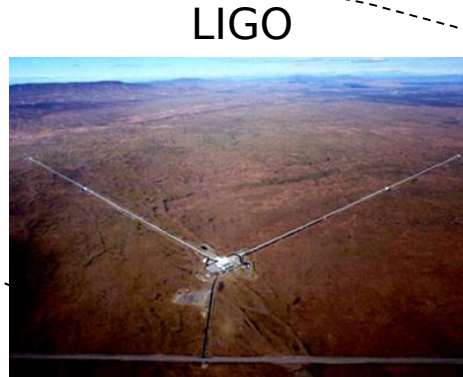
Now

Big Bang plus
13.7 Billion Years



Detection Gravitational al Waves

Ranges of Gravitational Wave Detectors in the World



1 Mpc

Andromeda

20 Mpc

Virgo cluster

200 Mpc

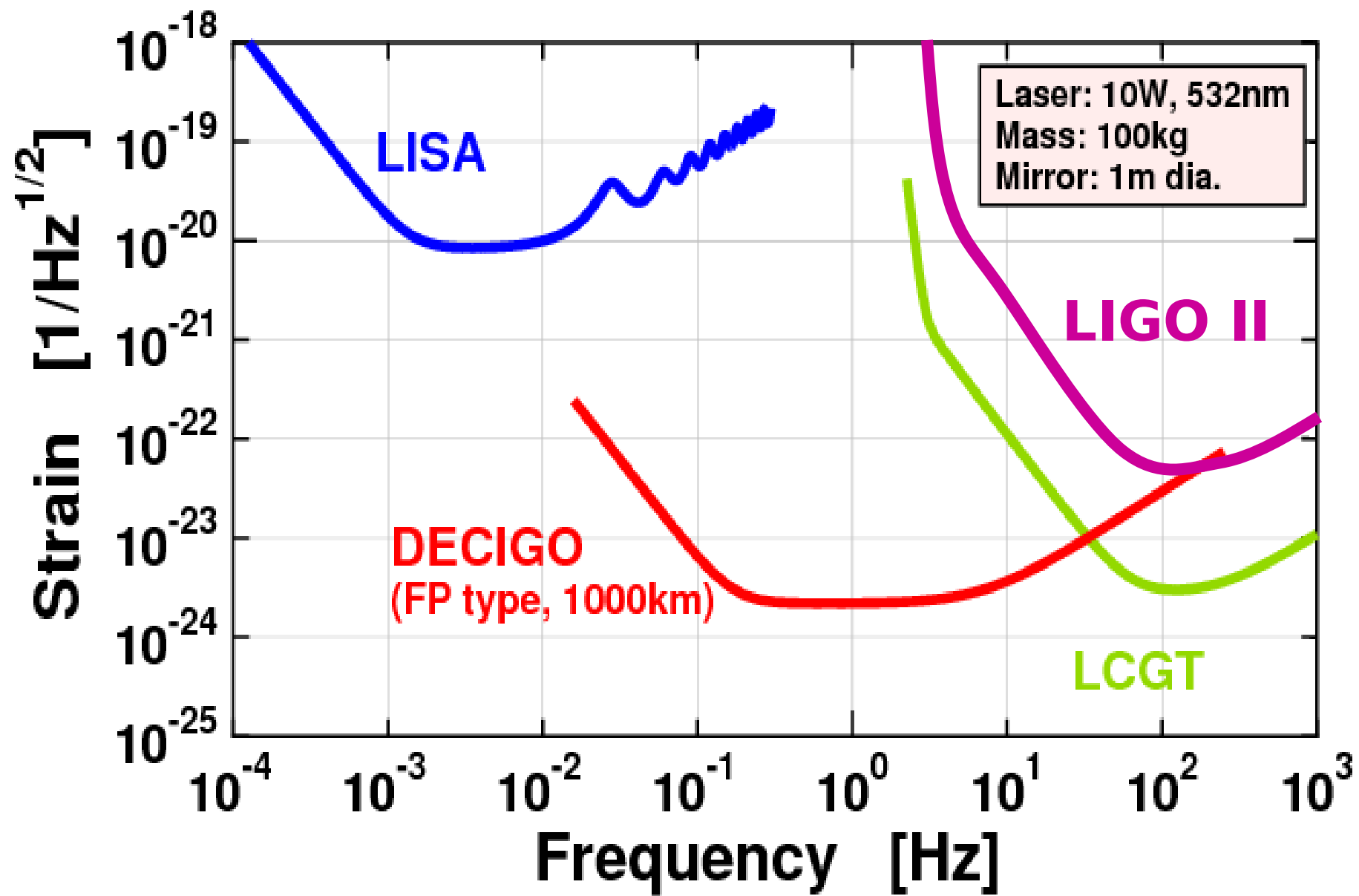
Hercules cluster

Dimensionless stress amplitude

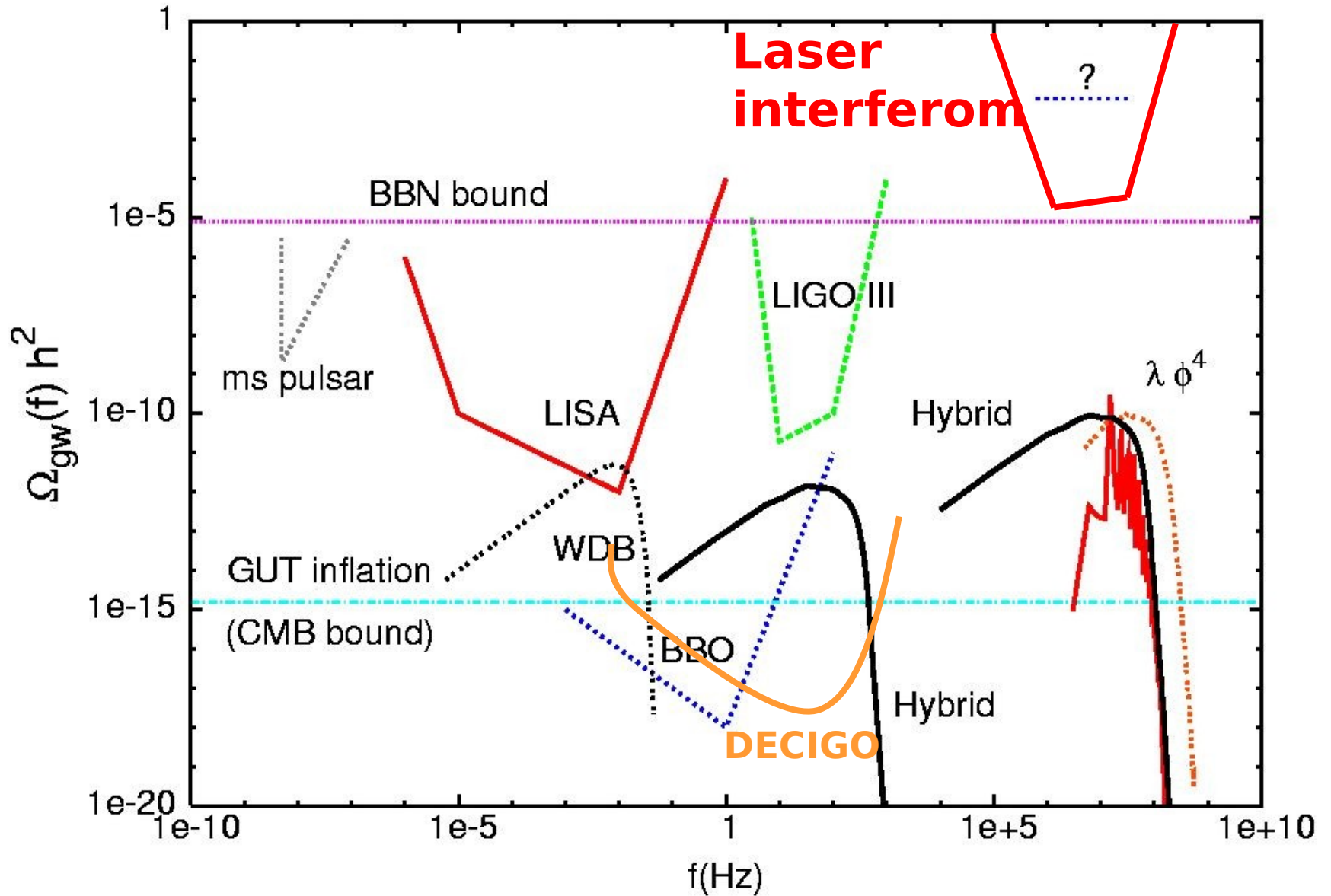
$$\langle h_{ij}(t) h^{ij}(t) \rangle = 2 \int_0^{\infty} \frac{df}{f} h_c^2(f)$$

$$\Omega_{gw}(f) = \frac{f d\rho_{gw}}{\rho_c df} = \frac{2\pi^2}{3H_0^2} f^2 h_c^2(f)$$

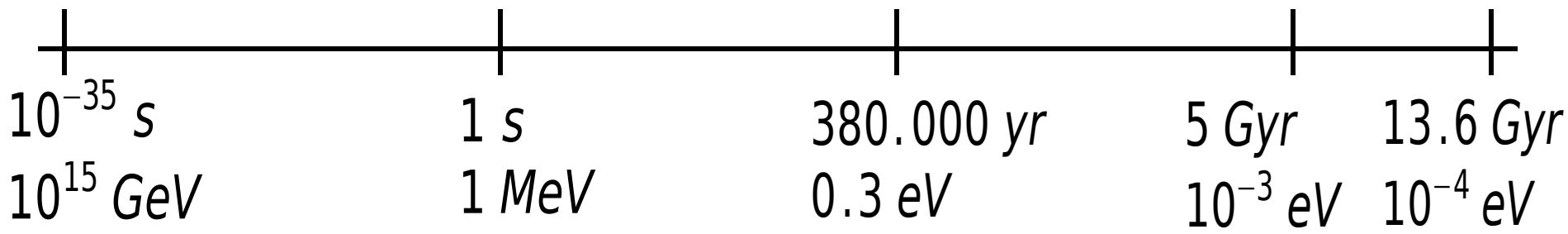
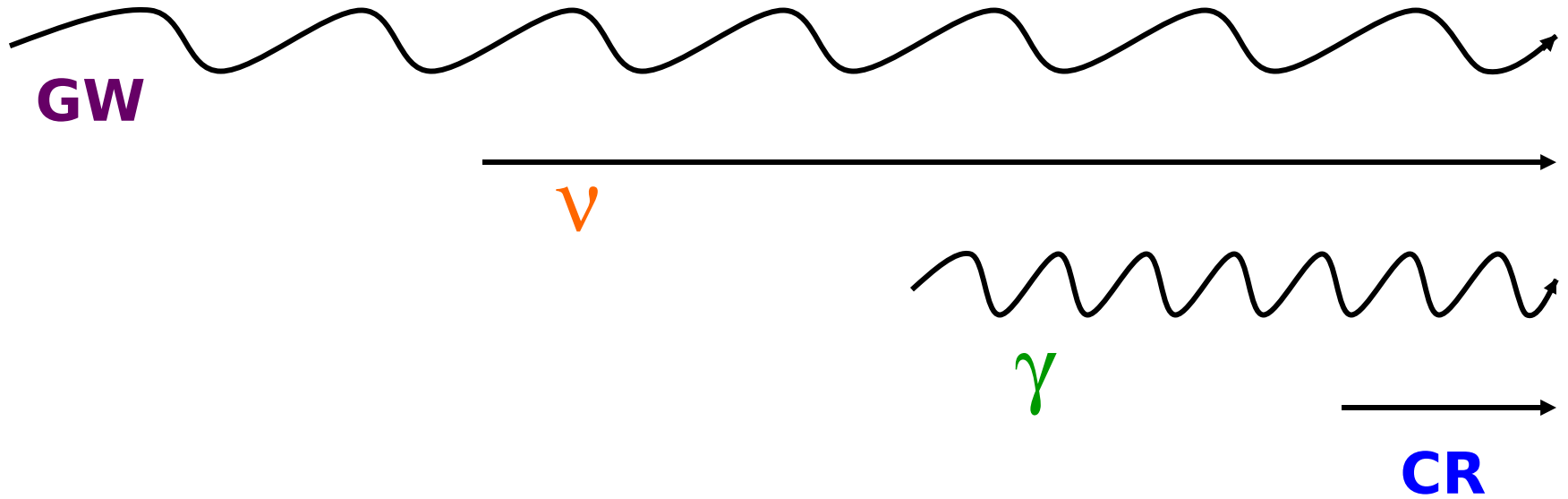
$$h_c(f) = 1.3 \times 10^{-18} \left(\frac{1 \text{ Hz}}{f} \right) \sqrt{\Omega_{gw}(f) h_0^2}$$



Backgrounds, Bounds & Sensitivity



Cosmic Messengers



Big Bang

Neutrino
decoupling

Photon
decoupling

GRB
AGN

Today

<i>Telescope</i>	<i>Person</i>	<i>Date</i>	<i>Objective</i>	<i>Discovery</i>
Optical	Galileo	1608	Navigation	Jupiter's moons
Geiger	Hess	1912	Geothermal	Cosmic Rays
Optical	Hubble	1929	Nebulae	Universe Expansion
Radio	Jansky	1932	Atmos. Noise	Radio Galaxies
Microwaves	Penzias, Wilson	1964	Telecommunications	Backgr. Radiation
X Rays	Giacconi	1965	Sun, Moon	Neutron Stars
Radio	Hewish, Bell	1967	Ionosphere	Pulsars
Rays	military	1960s	Nuclear Tests	Gamma Ray Bursts
Radio	Hulse, Taylor	1974	Binary Pulsar	Gravitational Waves
Cerenkov	Koshiaba	1998	Proton Decay	Sol./Atm. Neutrinos
Optical	Kirschner Perlmutter	1998	Supernovae	Universe Acceleration
Laser Interferom.	?	2020?	Gravitational Waves	Big Bang, Inflation?

Conclusions

- CMB anisotropies suggest inflation
- The end of inflation is our local Big Bang
- It is extremely violent at preheating
- Production of gravitational waves at Big Bang
- New detectors of GW are under construction

Primordial Magnetic Fields

J. G.-B.
Andres Diaz-Gil
Margarita Garcia-Perez
Antonio Gonzalez-
Arroyo

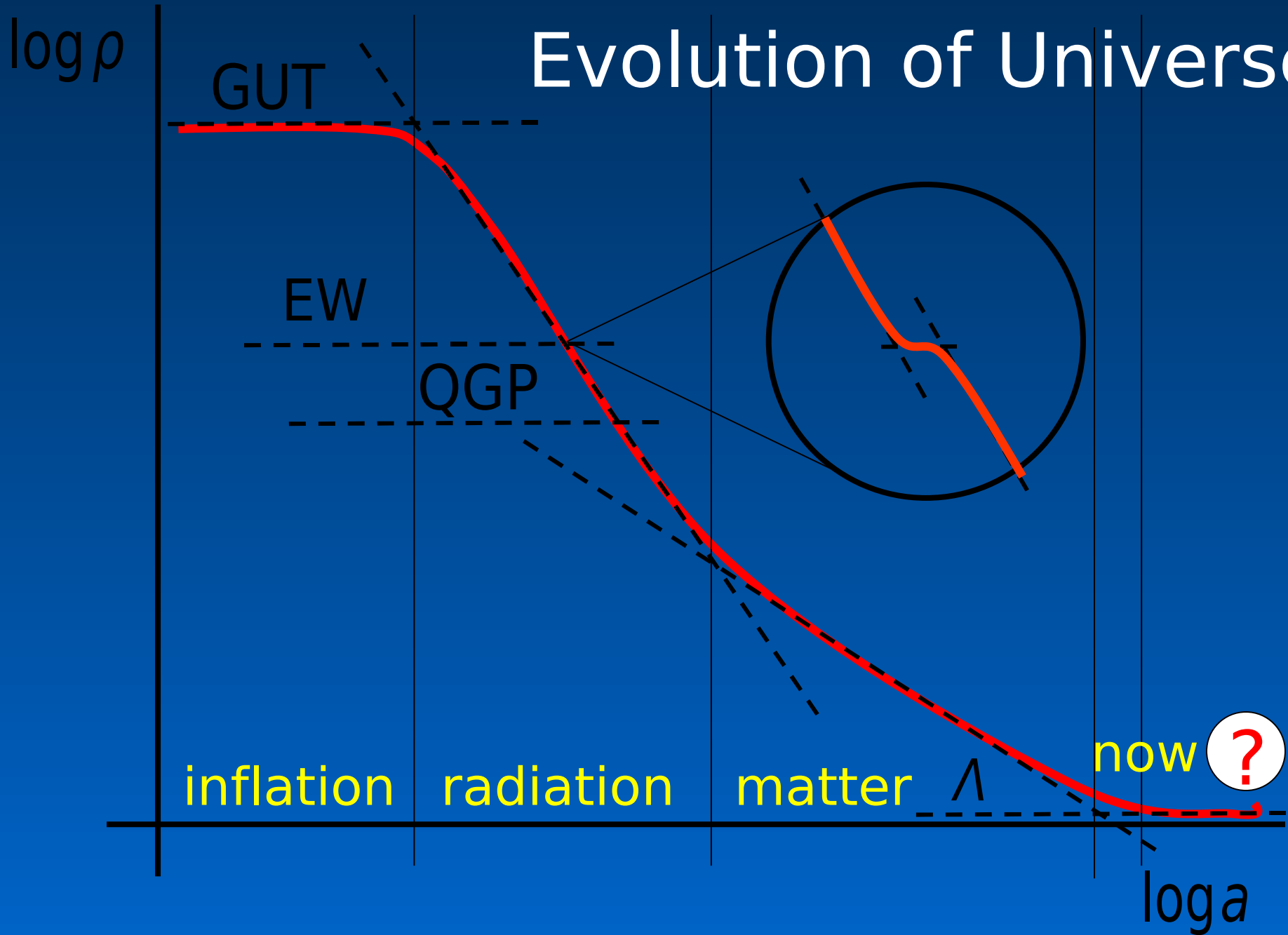
[hep-lat/0509094](https://arxiv.org/abs/hep-lat/0509094)
[arXiv:0710.0580 \[hep-
lat\]](https://arxiv.org/abs/0710.0580)
[arXiv:0712.4263 \[hep-
ph\]](https://arxiv.org/abs/0712.4263)

EW Tachyonic Preheating

Spinodal growth of long wave Higgs modes

- At the end of EW Hybrid Inflation
- Inflaton couples to Higgs
- Higgs couples to SM fields
- Strong production of fermions and gauge fields

Evolution of Universe



The SU(2)xU(1) Higgs-Inflaton mode

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} + \text{Tr}[(D_\mu \Phi)^\dagger D^\mu \Phi]$$

$$D_\mu = \partial_\mu - \frac{i}{2} g_w A_\mu^a \tau_a - \frac{i}{2} g_Y B_\mu \tau_3 + \frac{1}{2} (\partial_\mu \chi)^2 - V(\Phi, \chi)$$

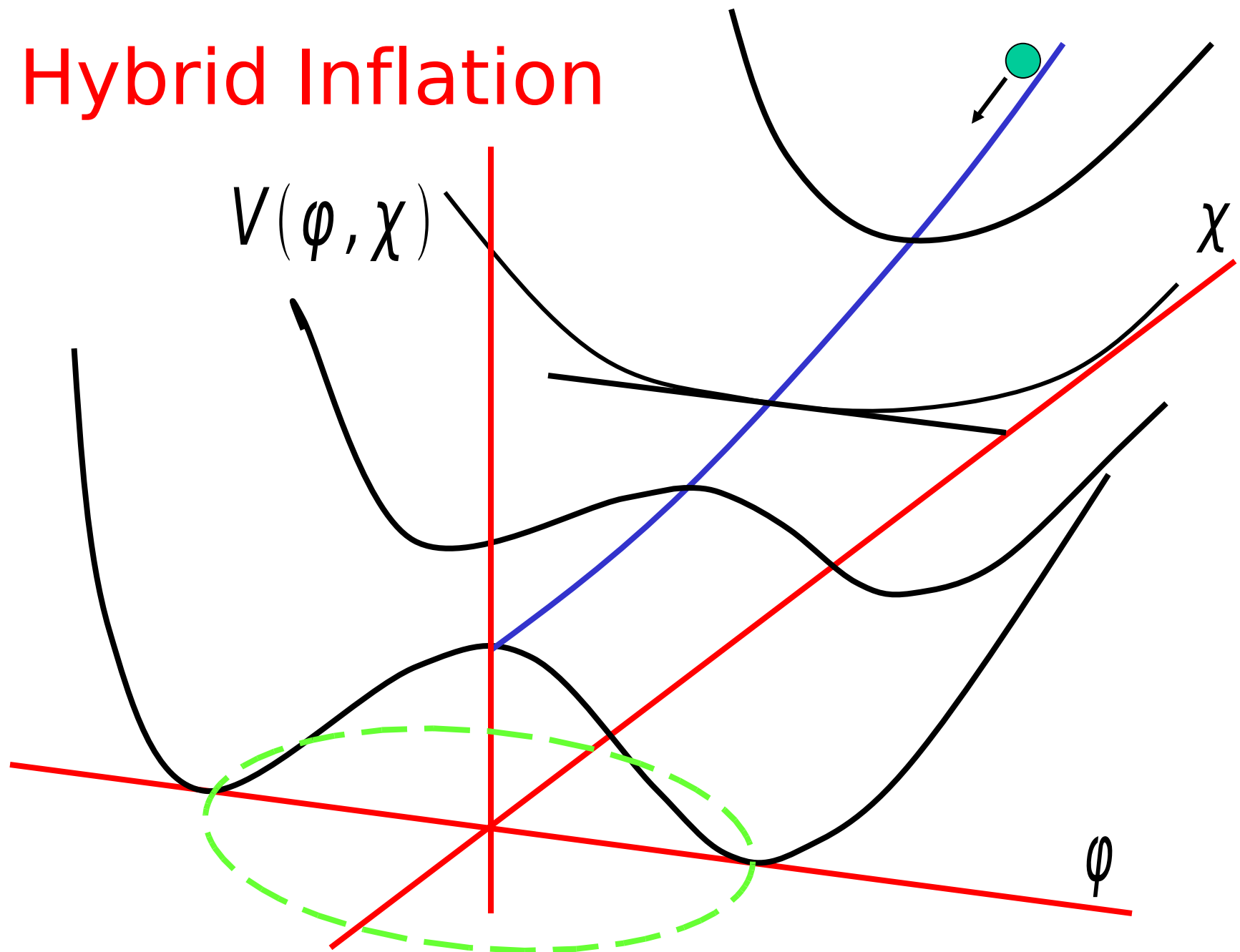
$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_w \varepsilon^{abc} A_\mu^b A_\nu^c$$

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

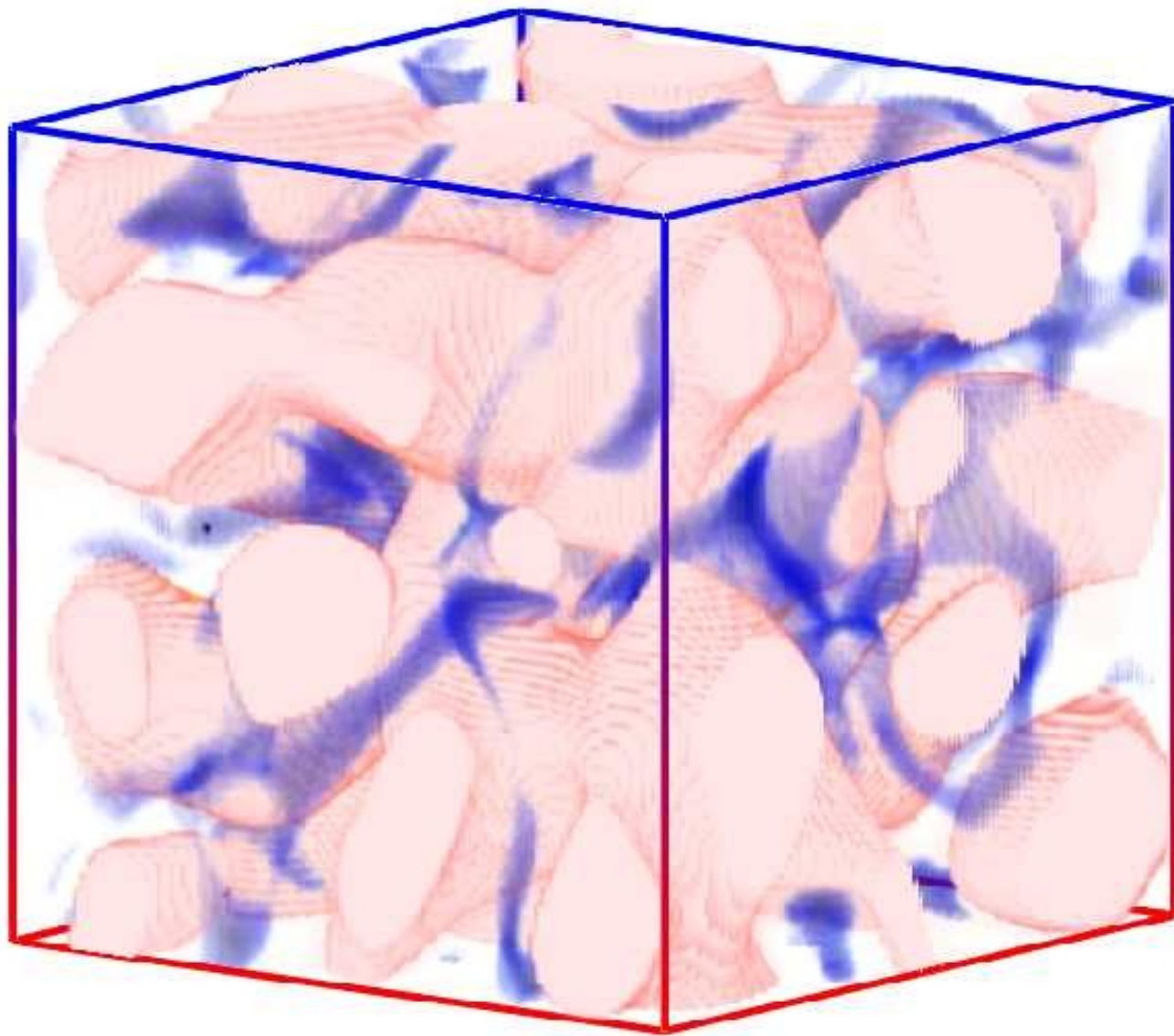
$$\text{Tr}[\Phi^\dagger \Phi] = \frac{1}{2} (\varphi_0^2 + \varphi^a \varphi_a) \equiv \frac{1}{2} \varphi^2$$

$$V(\varphi, \chi) = \frac{\lambda}{4} (\varphi^2 - v^2)^2 + \frac{g^2}{2} \varphi^2 \chi^2 + \frac{1}{2} m^2 \chi^2$$

Hybrid Inflation

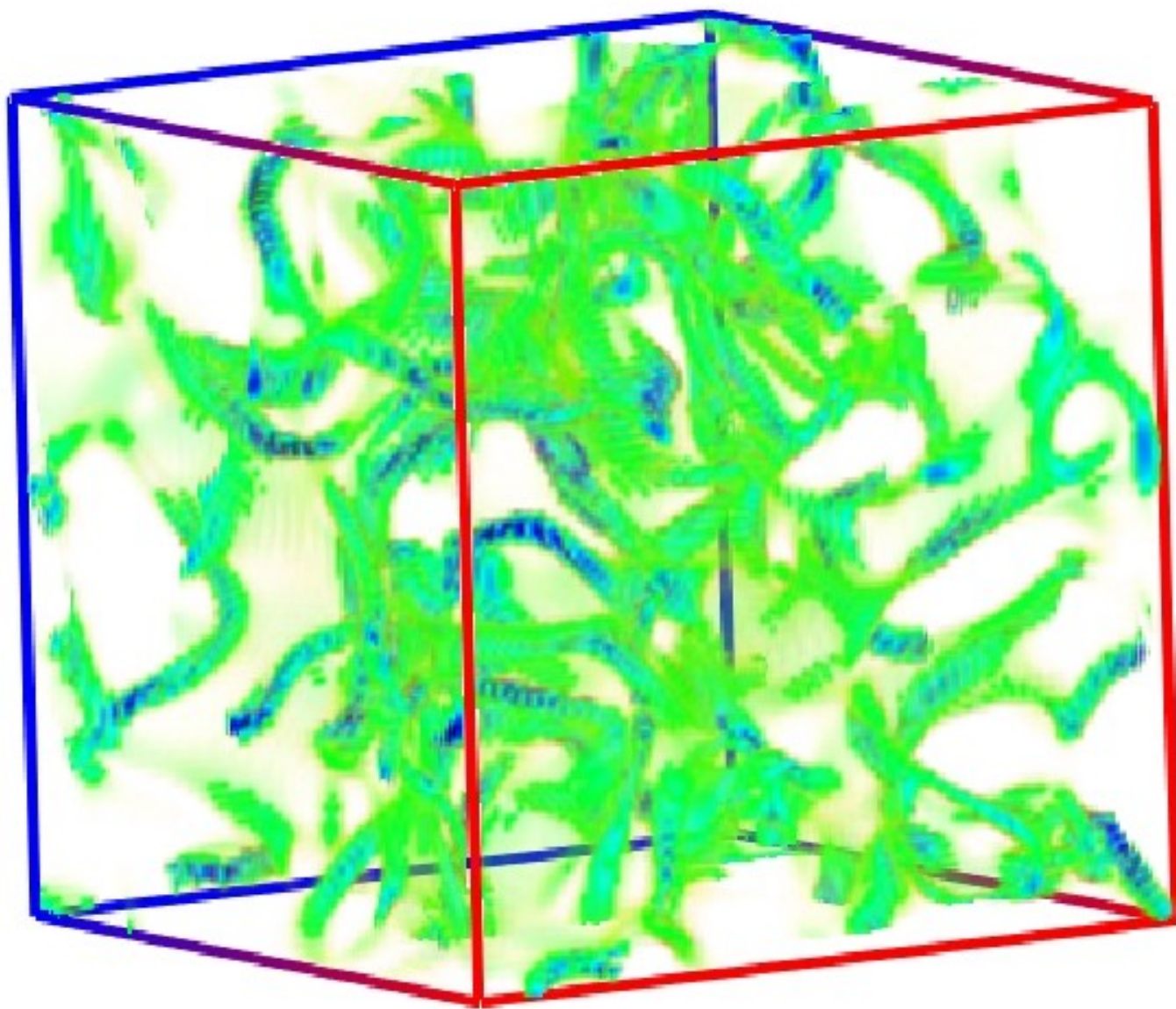


\emptyset

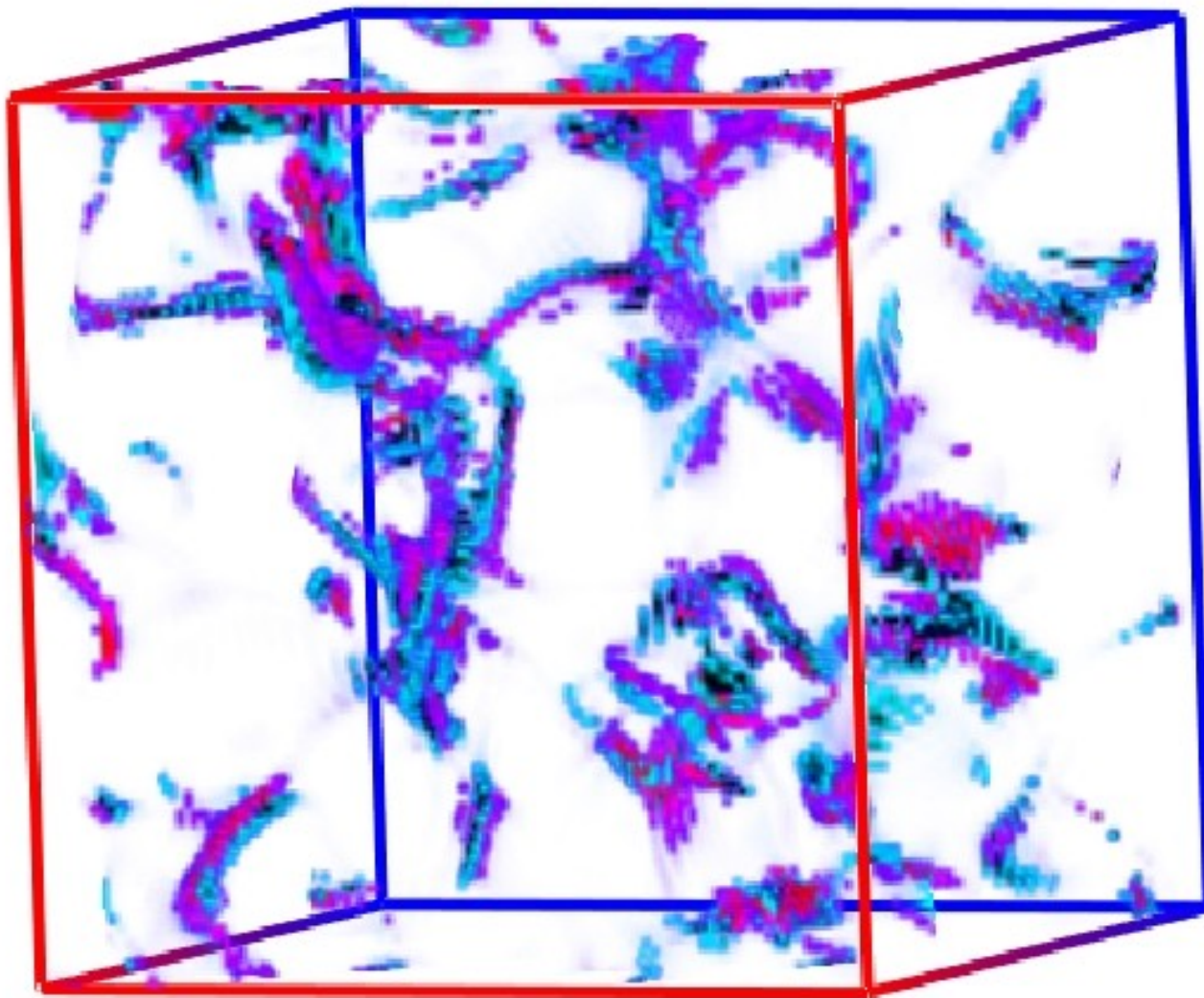


B

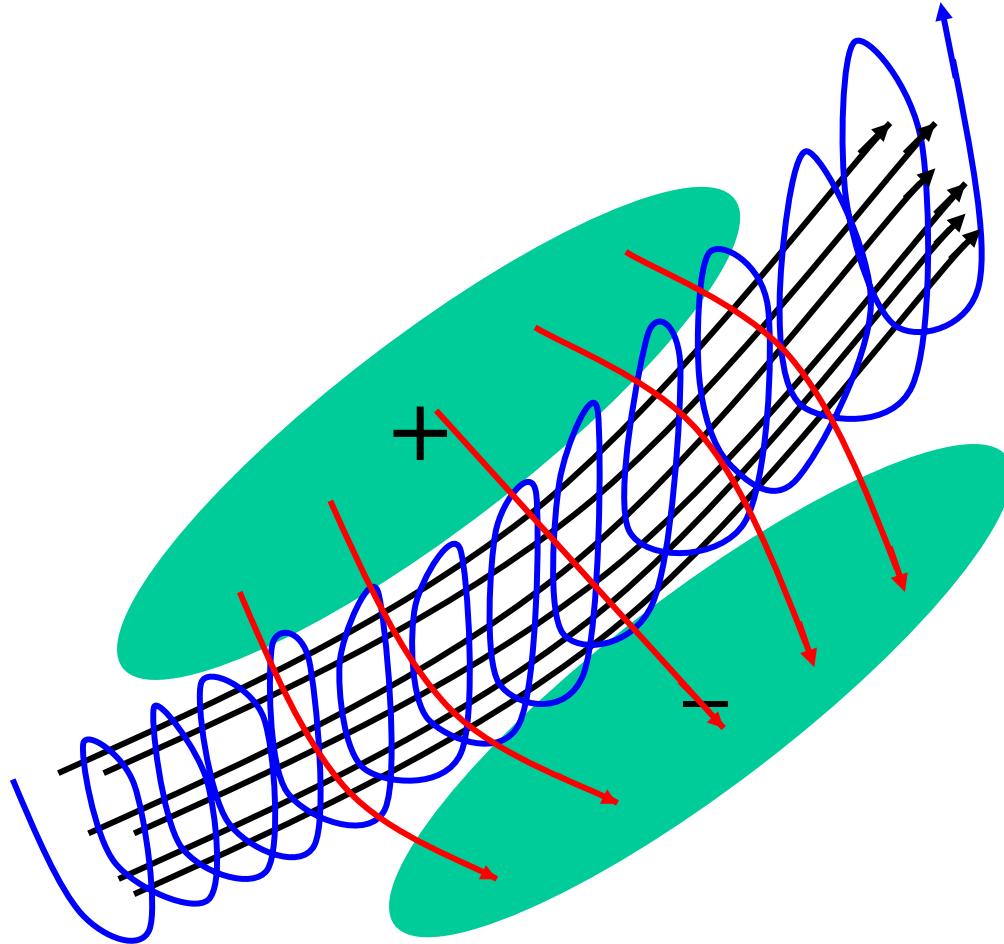
B



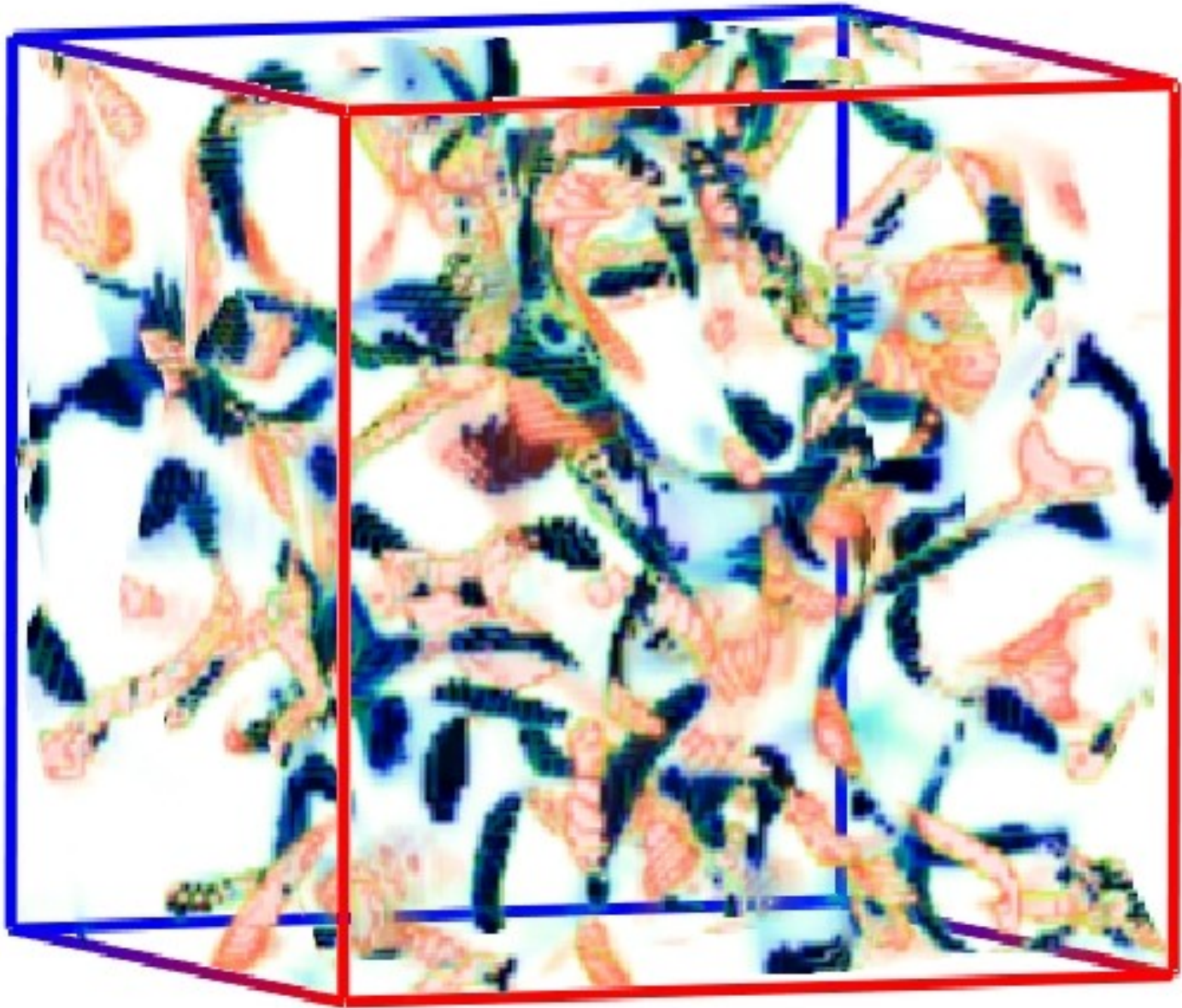
Charges in W^+_-



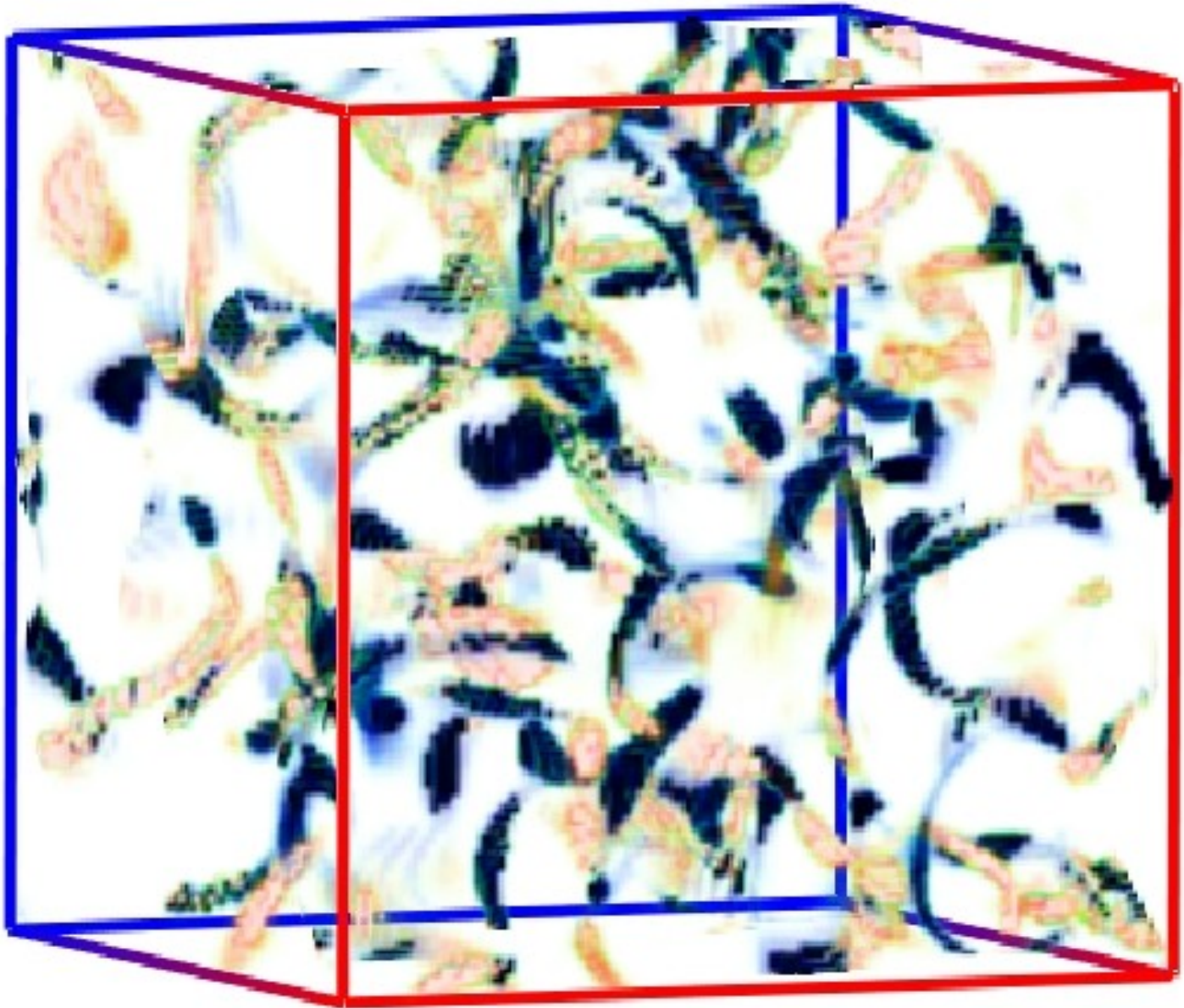
Charged plasma

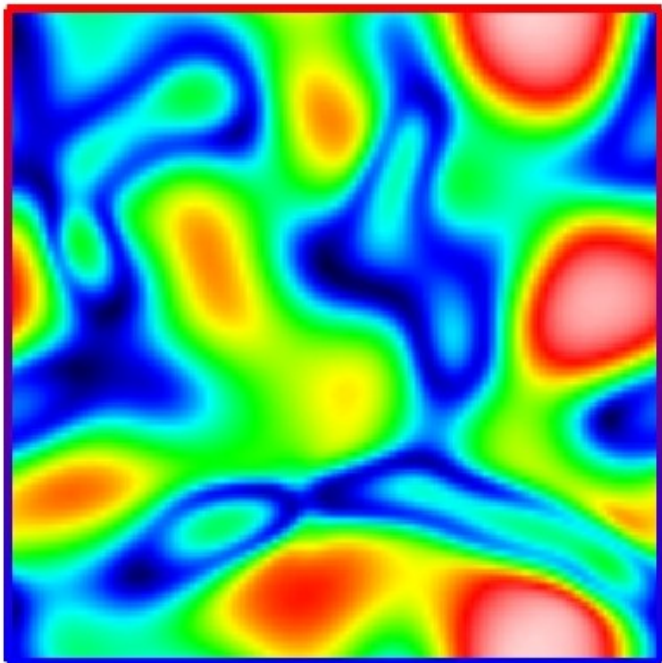


H_B

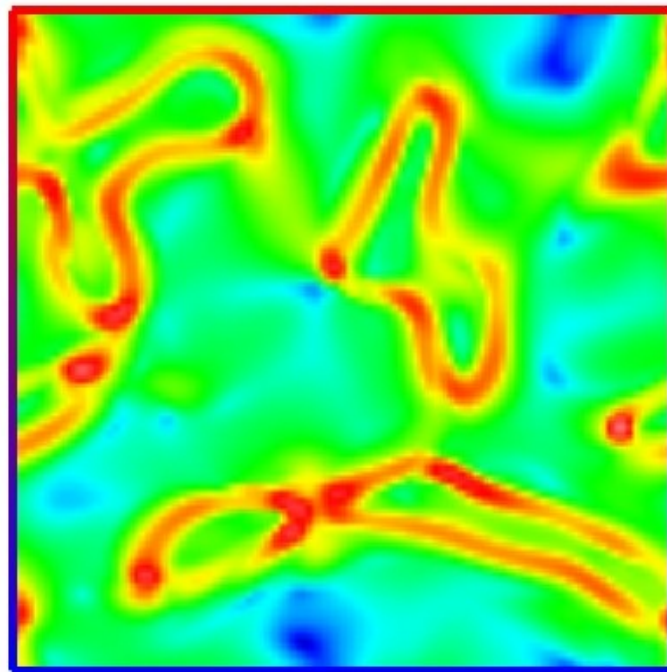
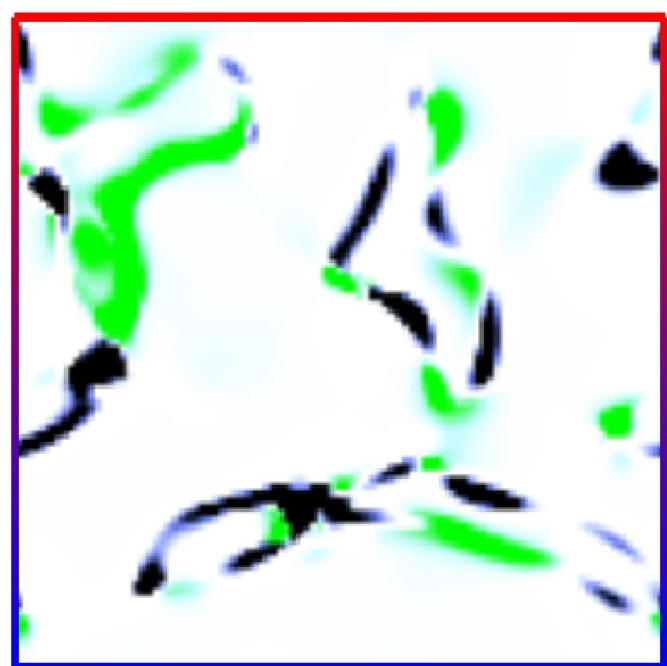


H_z

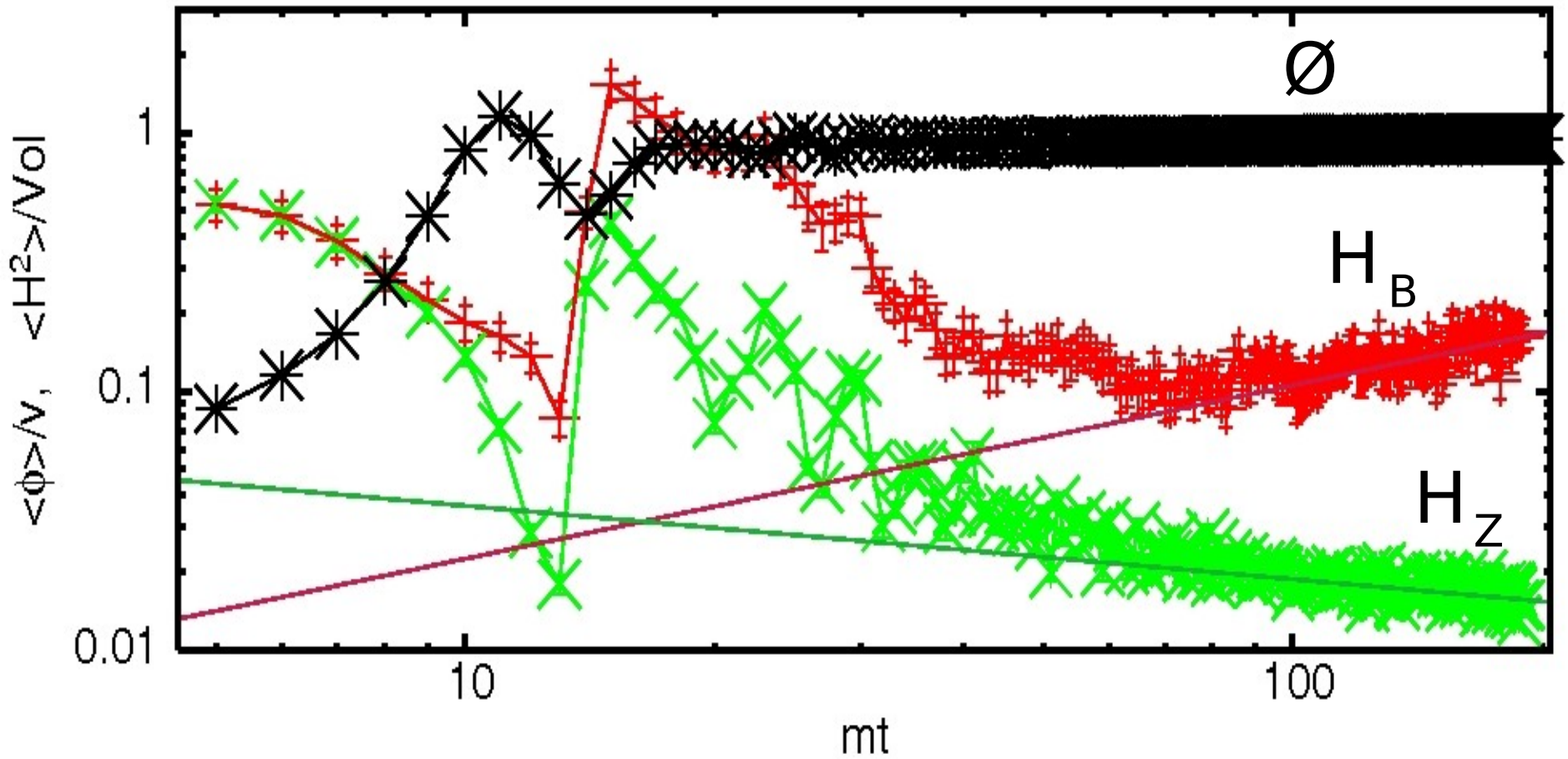


\emptyset 

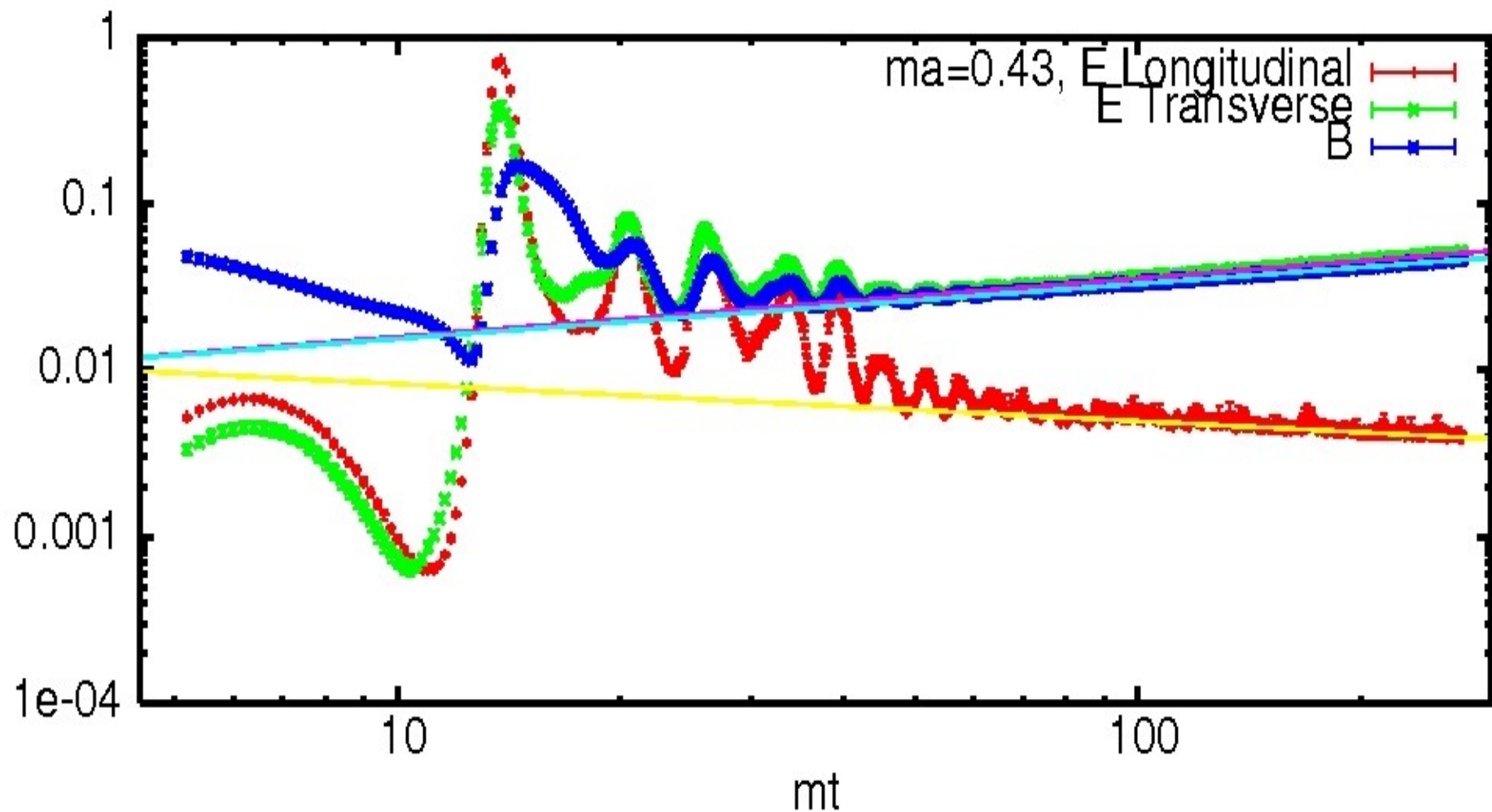
B

 H_Z  H_B 

Time evolution



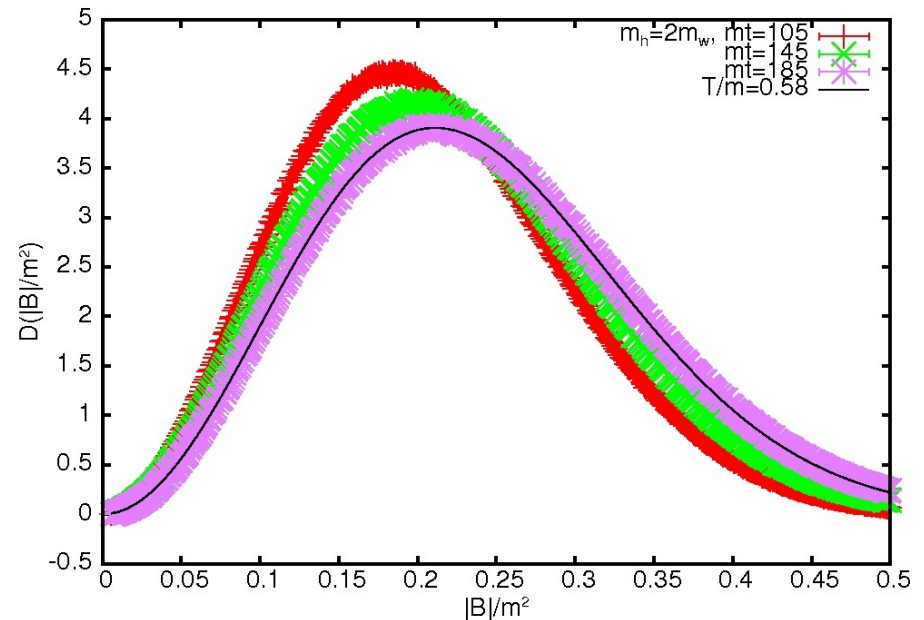
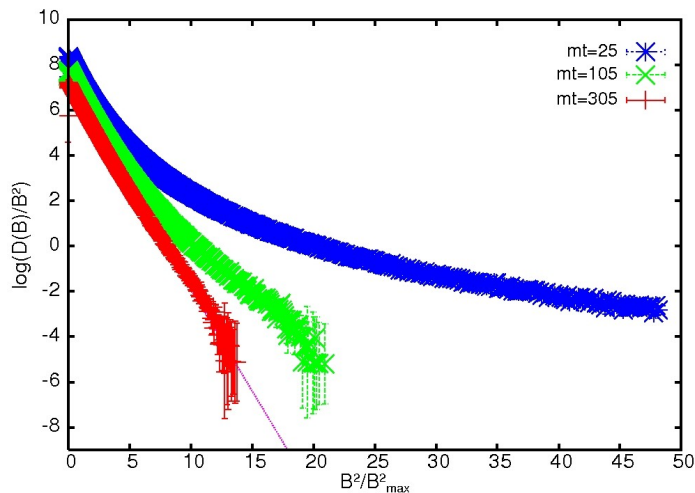
Time evolution



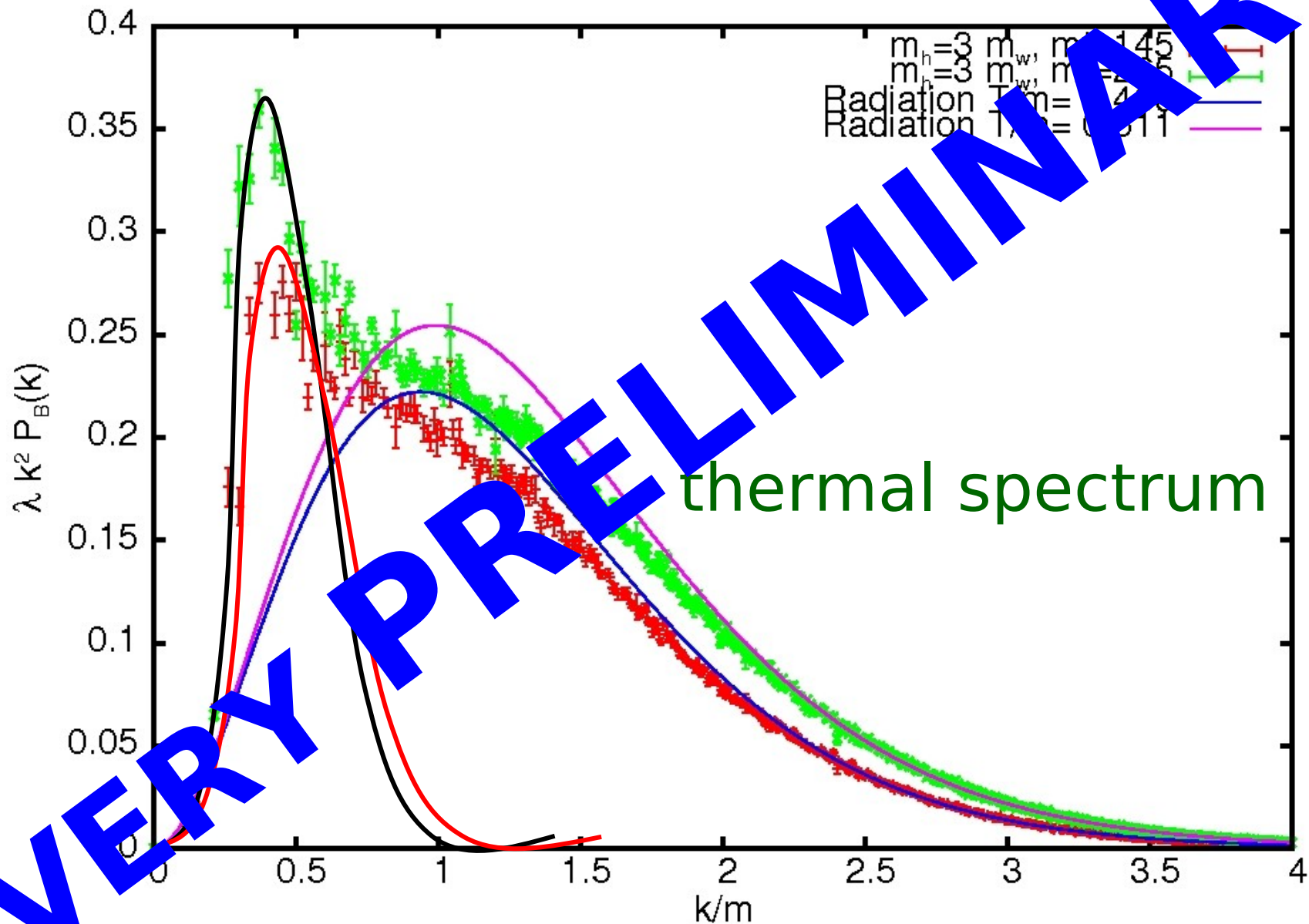
Boltzman-Maxwell distribution

$$P(B) = B^2 \exp\left\{-\frac{3B^2}{2\langle B^2 \rangle}\right\} \quad \langle B^2 \rangle = \frac{\pi^2}{15} T^4$$

$$T \approx 0.4 - 0.6 m$$



Inverse Cascade



Spatial averages

$$B_{(1)}(L) = \frac{1}{L} \int_C \vec{B} \cdot d\vec{x}$$

Linear average

$$B_{(2)}(L) = \frac{1}{L^2} \int_S \vec{B} \cdot d\vec{S}$$

Magnetic flux

$$B_{(3)}(L) = \frac{1}{L^3} \int_V \vec{B} \cdot d^3\vec{x}$$

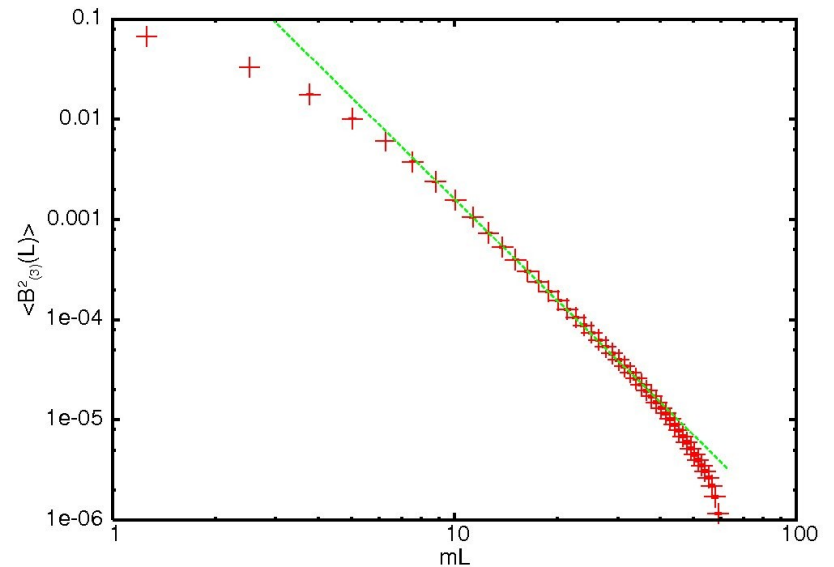
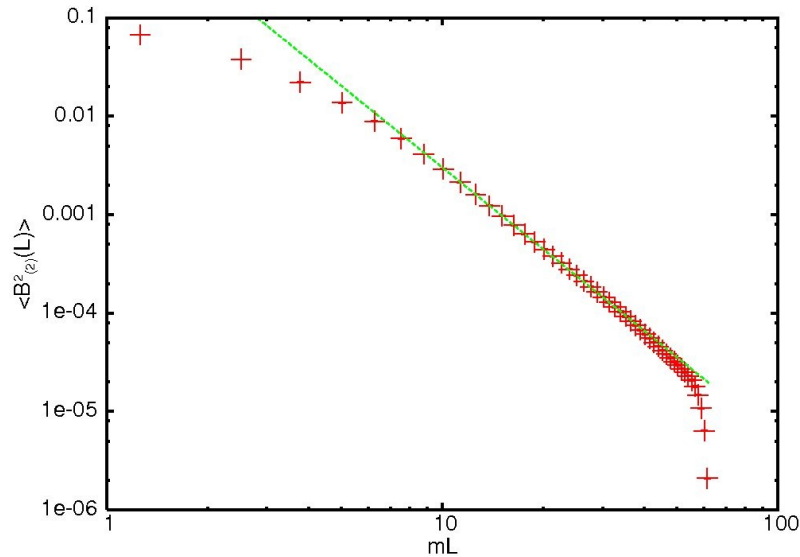
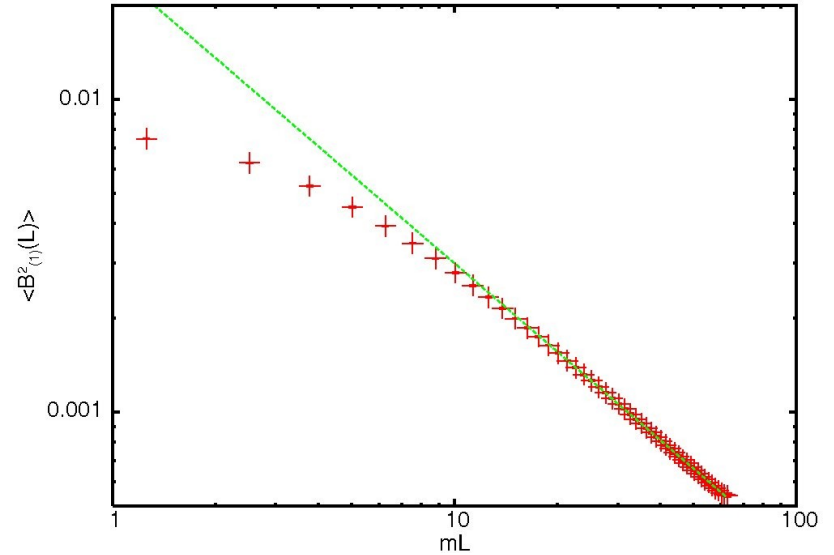
Volume average

Spatial averages

$$\langle B_{(1)}^2(L) \rangle \approx B_{\xi}^2 \left(\frac{\xi}{L} \right)$$

$$\langle B_{(2)}^2(L) \rangle \approx B_{\xi}^2 \left(\frac{\xi}{L} \right)^2$$

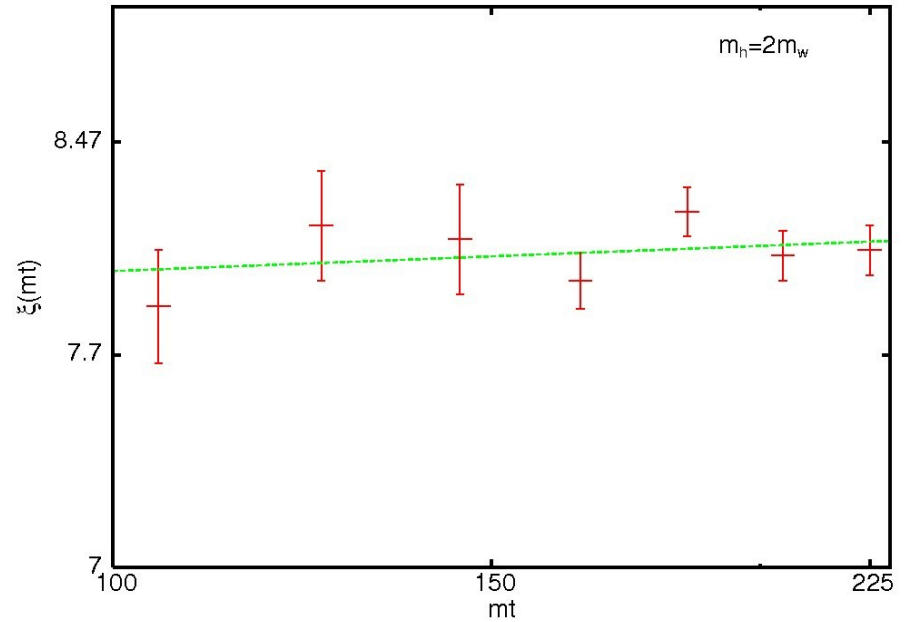
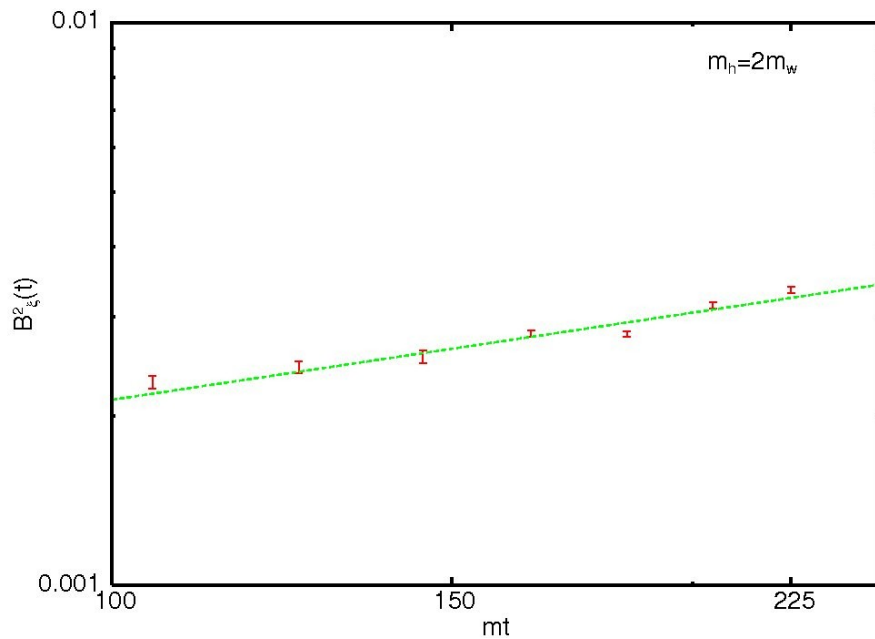
$$\langle B_{(3)}^2(L) \rangle \approx B_{\xi}^2 \left(\frac{\xi}{L} \right)^{2p+3}$$



Spatial averages

$$B_{\xi}^2 \approx 3 \times 10^{-3} t^{0.5} \rho_0$$

$$\xi \approx 8 t^{0.02} m^{-1}$$



The amplitude of magnetic fields

$$\rho_{mag} \leq 10^{-3} \rho_0 \approx 10^{-4} m_H^2 v^2 = (10 \text{ GeV})^4$$
$$\rho_{mag}^{(0)} = \left(\frac{T_0}{T_{EW}} \right) \rho_{mag} \approx (0.5 \mu G)^2 / 8\pi$$
$$\frac{1}{8\pi} \text{Gauss}^2 = 1.39 \times 10^{-42} \text{GeV}^4 \quad \text{Conversion factor}$$

The coherence scale of magnetic field

$$\xi \propto t$$

During inverse cascade

$$\xi \propto a(t)$$

After photon decoupling

$$\xi_0 \approx 3 \text{ cm} \left(\frac{a_{dec}}{a_{EW}} \right)^2 \left(\frac{a_0}{a_{dec}} \right) \approx 20 \text{ Mpc}$$

Observatio

n

Magnetic

Fields

Coherent Magnetic Fields

$B \approx 50 \mu G$ at $L < 5 kpc$	}	galaxies
$B \approx 5 - 10 \mu G$ at $L \approx 10 kpc$		
$B \approx 1 \mu G$ at $L \approx 1 Mpc$		clusters
$B < 10^{-2} - 10^{-3} \mu G$ at $L \approx 1 - 50 Mpc$		supercluster
$B < 10^{-3} - 10^{-5} \mu G$ at $L > 100 Mpc$		ξ_{CMB}
$B < 10^{11} G$ at $T = 10^9 K$		BBN

Coherent Magnetic Field in M31

$$B \approx 1 - 3 \mu G$$

$$l \approx 10 \text{ kpc}$$

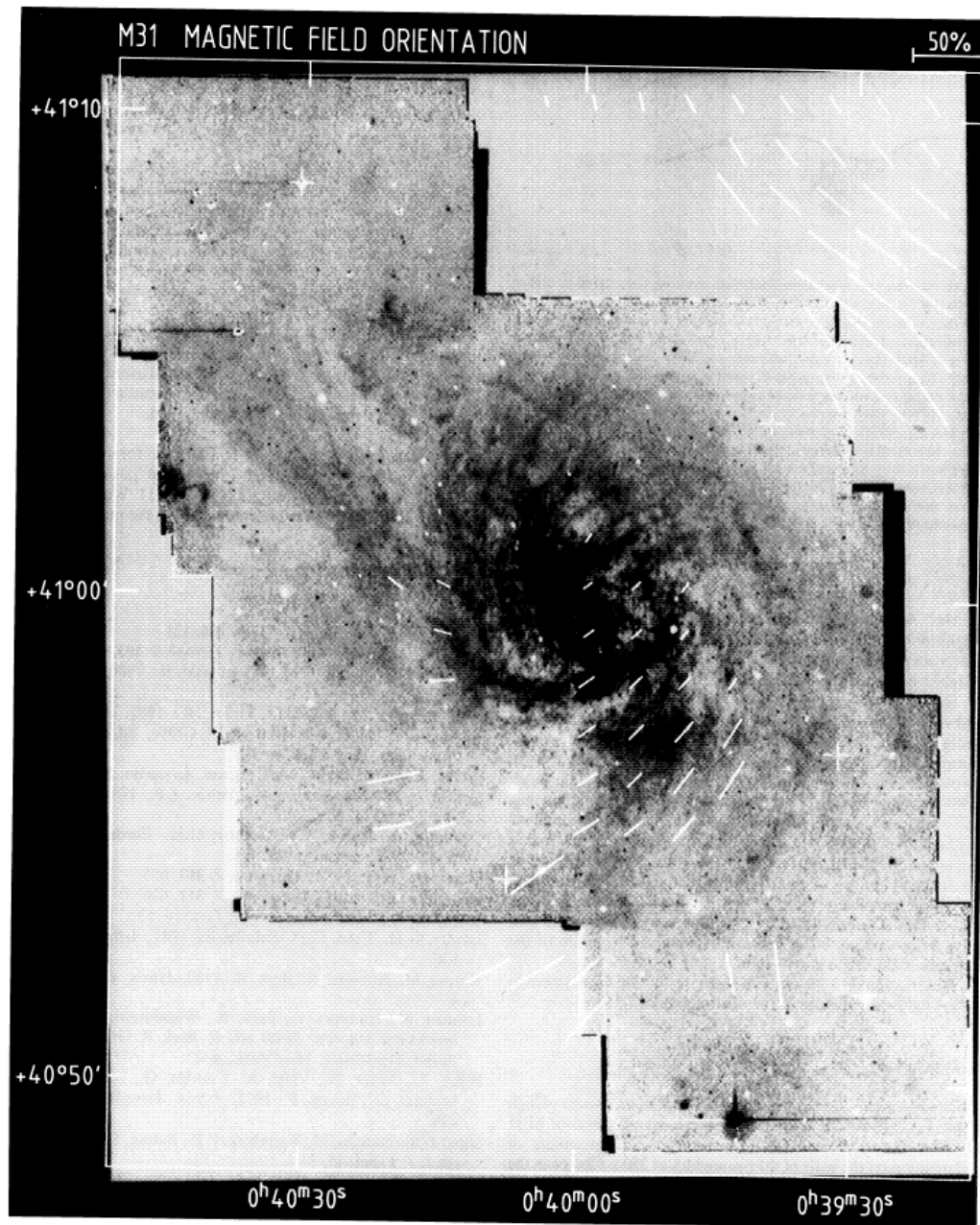
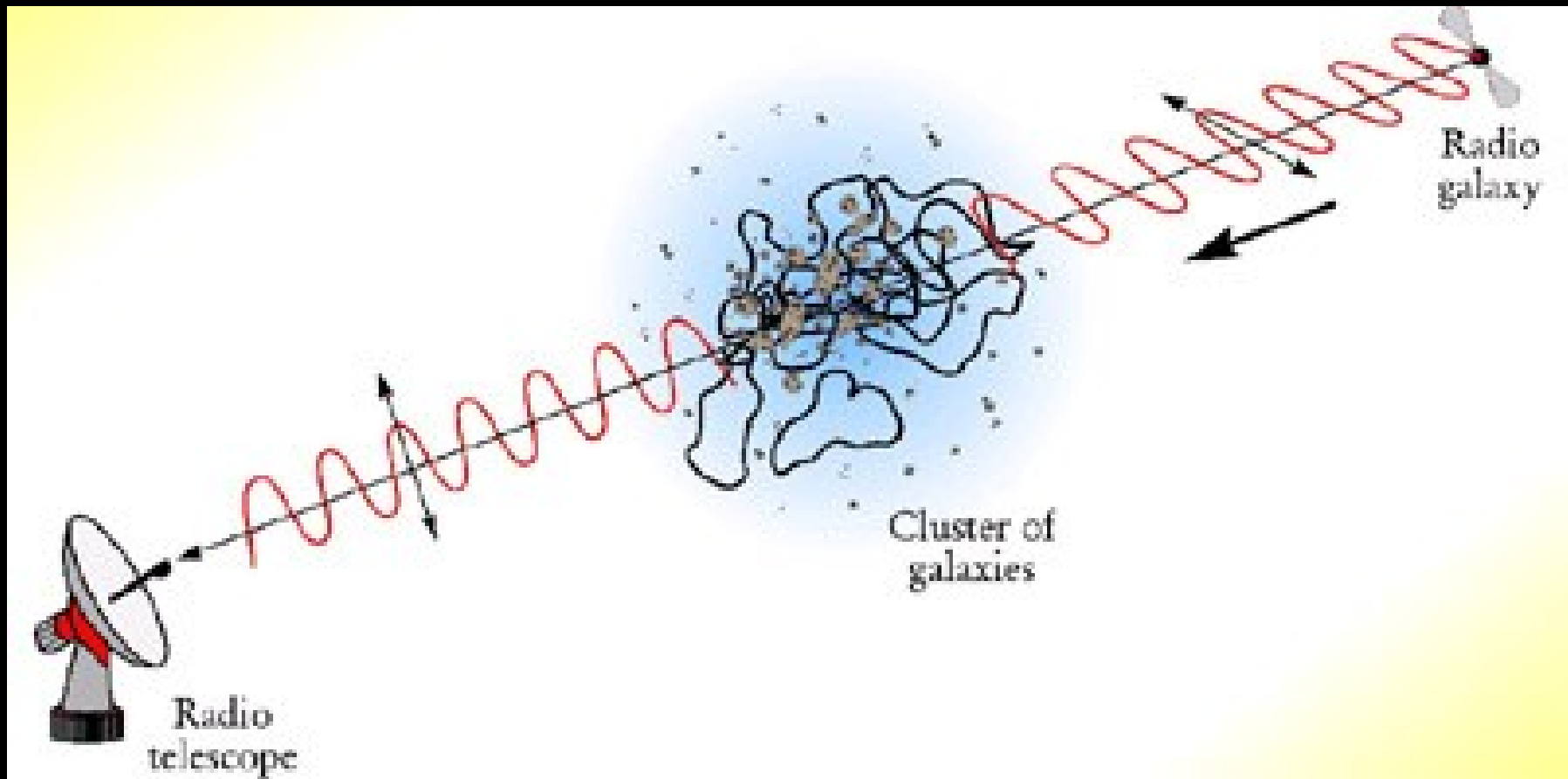


Fig. 11. Orientation of the magnetic field (z_b) in the central region of M31 overlaid onto the $H\alpha$ photograph of Ciardullo et al. (1988). The lengths of the vectors indicate the degree of linear polarization at $\lambda 6.3 \text{ cm}$

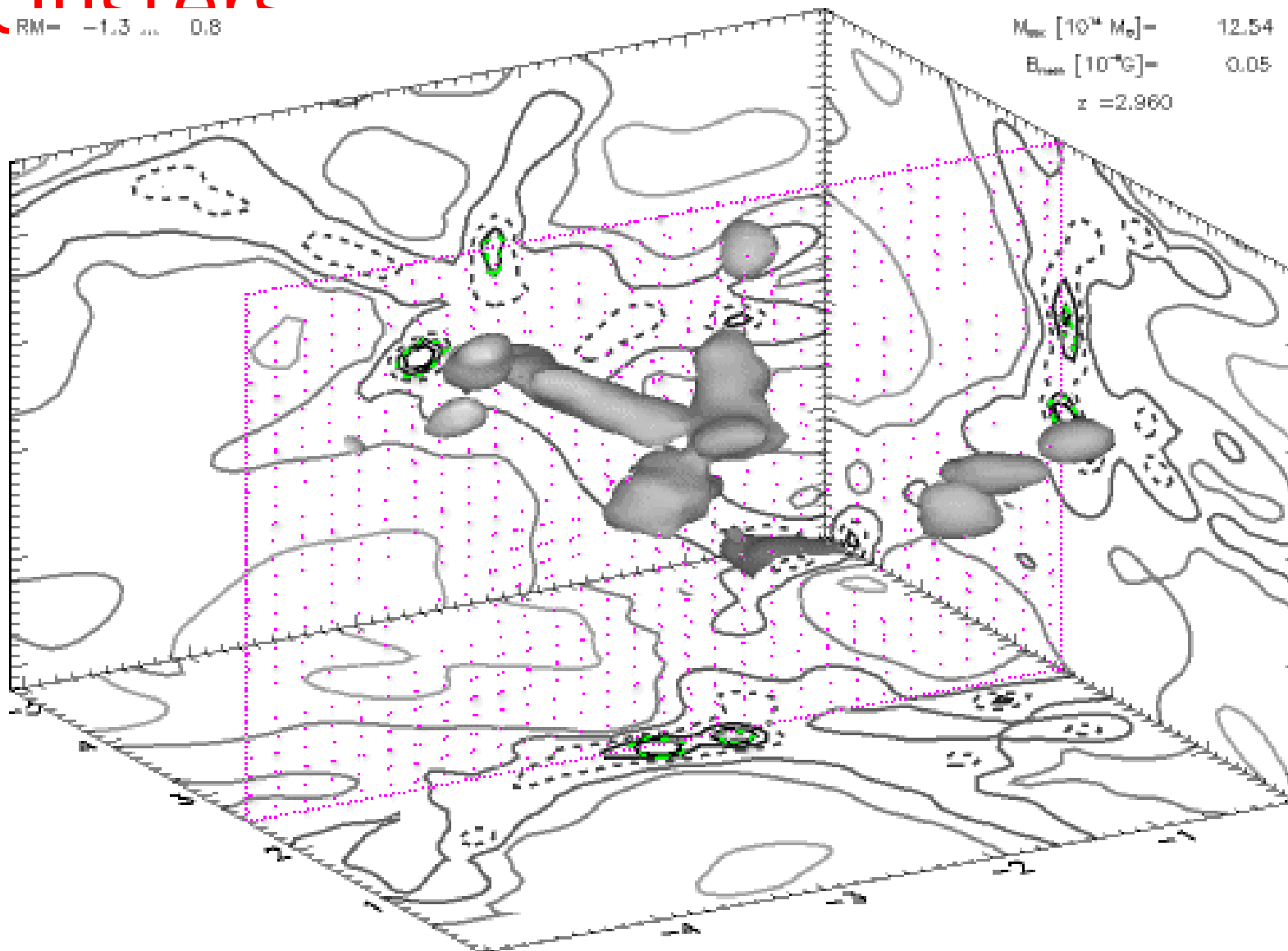
Faraday rotation by cluster galaxies



Coherent Magnetic Fields in clusters

RM = -1.3 ... 0.8

$M_{\text{tot}} [10^{14} M_{\odot}] = 12.54$
 $B_{\text{tot}} [10^{-6} \text{G}] = 0.05$
 $z = 2.960$



EW Symmetry Breaking can lead to the production of primordial magnetic fields at tachyonic preheating after hybrid inflation

The right amplitude and scale of magnetic fields depends on the extent of kinetic turbulence

initial conditions for magneto-hydrodynamic simulations