

EXPANDING PLASMA AT STRONG COUPLING

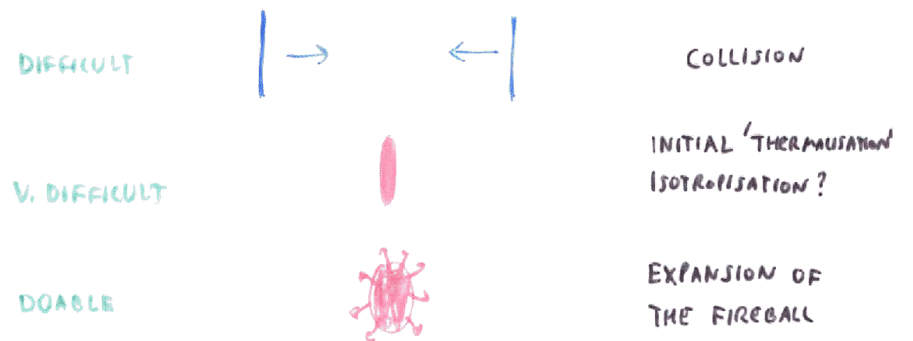
ROMUALD JANIK
JAGELLONIAN UNIVERSITY
KRAKÓW

- ① MOTIVATION
- ② ADS/CFT FRAMEWORK
-EXAMPLES
- ③ EXPANDING BOOST-INVARIANT PLASMA
- ④ 'THERMALIZATION' IN THE EXPANDING PLASMA SYSTEM
- ⑤ FLAVOURS — || — || — || —
- ⑥ PARTICLE PRODUCTION
- ⑦ ANISOTROPIC PLASMA AT STRONG COUPLING
- ⑧ SUMMARY/OUTLOOK

WORK WITH R. PESZANSKI
A. HELLER
J. GROSJE, P. SURODKA
A. BUCHEL, P. BEVINCASA
P. WITASZCZYK

6+7: WORK IN PROGRESS

AIM: USE ADS/CFT TO STUDY DYNAMICAL TIME-DEPENDENT PROCESSES FOR $N=4$ PLASMA



- STUDY PROPERTIES OF THE EXPANDING PLASMA SYSTEM
 - EQUILIBRATION TIME
 - INFLUENCE ON FUNDAMENTAL FLAVOURS
 - PARTICLE PRODUCTION ?
- STUDY PLASMA IN AN OUT OF EQUILIBRIUM SETTING
 - ANISOTROPIC $N=4$ PLASMA AT STRONG COUPLING

$N=4$ PLASMA VS. QCD PLASMA

SIMILARITIES :

- DECONFINED PHASE
- STRONGLY COUPLED

DIFFERENCES :

- NO RUNNING COUPLING
- CONFORMAL EQUATION OF STATE
- NO CONFINEMENT/DECONFINEMENT PHASE TRANSITION

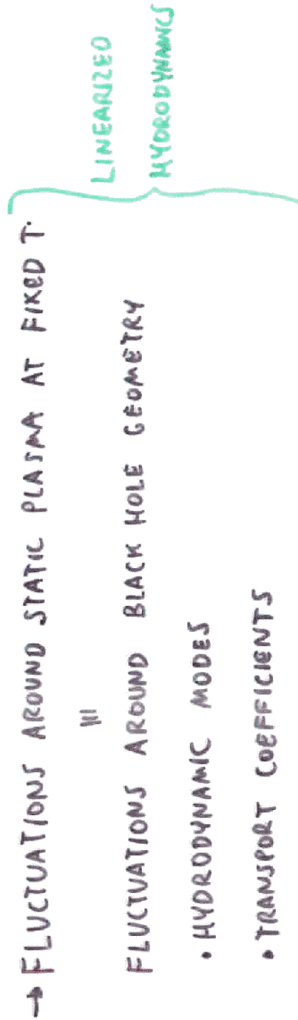
- PLASMA FIREBALL COOLS INDEFINITELY
- AT VERY HIGH ENERGY DENSITIES COUPLING REMAINS STRONG

....

BUT :

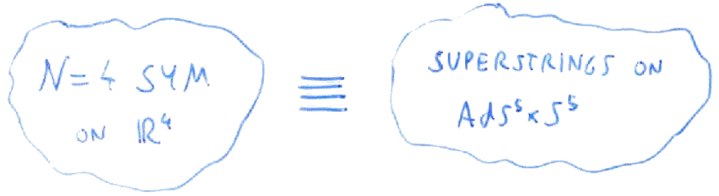
- TOY MODEL WHERE WE MAY COMPUTE FROM 'FIRST PRINCIPLES'
- DISCOVER SOME UNIVERSAL PROPERTIES ?
- FOR $N=4$ PLASMA ADS/CFT SETTING IS TECHNICALLY SIMPLEST
- EVENTUALLY ONE MAY CONSIDER MORE REALISTIC THEORIES WITH ADS/CFT DUALS...

COMPARISON WITH PREVIOUS CALCULATIONS OF VISCOSITY ETC.



HERE : • STUDY HYDRODYNAMICS IN THE NONLINEAR REGIME

- NONTRIVIAL CONSISTENCY CHECK
- STUDY CONFIGURATIONS FAR AWAY FROM BEING A THERMAL SYSTEM (OR FAR FROM HYDRODYNAMICS)



↑
LOOK FOR CONFIGURATIONS
CORRESPONDING TO A PLASMA
SYSTEM

EMPTY SPACE
(VACUUM)



$$ds^2 = \frac{\eta_{\mu\nu} dx^\mu dx^\nu + dz^2}{z^2}$$



- $\langle T_{\mu\nu} \rangle \neq 0$ ↔ GRAVITON $g_{\mu\nu}$
- $\langle \text{tr} F^2 \rangle \neq 0$ ↔ DILATON ϕ
- GENERIC OPERATOR ↔ GENERIC STRING STATE



DESCRIBED BY SOME BACKGROUND
GEOMETRY :

MACROSCOPIC SPACETIME
PROFILE OF ENERGY MOMENTUM
TENSOR

$$\langle T_{\mu\nu} \rangle = \dots \leftrightarrow ds^2 = \frac{g_{\mu\nu}(x^\mu, z) dx^\mu dx^\nu + dz^2}{z^2}$$

HOLOGRAPHIC RENORMALIZATION

DE HARO, SOLODUKHIN, SKENDERIS

① HOW TO READ OFF $\langle T_{\mu\nu} \rangle$ FROM A GIVEN (5D) GEOMETRY?

- ADOPT FEFFERMAN-GRAHAM COORDINATES

$$ds^2 = \frac{g_{\mu\nu} dx^\mu dx^\nu + dz^2}{z^2}$$

- LOOK AT THE EXPANSION NEAR THE BOUNDARY $z=0$

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + z^2 g_{\mu\nu}^{(2)} + z^4 g_{\mu\nu}^{(4)} + z^6 g_{\mu\nu}^{(6)} + \dots$$

↑
PHYSICAL 4D METRIC FOR THE GAUGE THEORY

($g_{\mu\nu}^{(0)} = \eta_{\mu\nu}$ HERE)

|| 0

↓

$$\langle T_{\mu\nu} \rangle = \frac{N_c^2}{2\pi^2} g_{\mu\nu}^{(4)}$$

② GIVEN SOME $\langle T_{\mu\nu} \rangle$ HOW TO RECONSTRUCT THE DUAL (5D) GEOMETRY?

- FIND SOLUTION OF EINSTEIN EQUATIONS

$$(*) \quad R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R + \underbrace{\Lambda}_{-6} = 0$$

WITH THE BOUNDARY CONDITIONS

$$g_{\mu\nu} = \eta_{\mu\nu} + z^4 g_{\mu\nu}^{(4)} + \dots$$

$\langle T_{\mu\nu} \rangle$

- (*) IMPLY $g_{\mu}^{\mu(4)} = 0$
 $D_\mu g^{(4)\mu\nu} = 0$

- FOR ANY SUCH $g_{\mu\nu}^{(4)}$ ONE CAN COMPUTE RECURSIVELY HIGHER ORDER TERMS IN $g_{\mu\nu}$!!

R. R. RESCHMANJSKI

OUR FRAMEWORK:

- FOR GENERIC PLASMA CONFIGURATIONS CONSTRUCT

DUAL

$$\langle T_{\mu\nu}(x^r) \rangle \quad \longleftrightarrow \quad g_{\mu\nu}(x^r, z)$$

- PHYSICAL CONFIGURATION SHOULD CORRESPOND

TO NONSINGULAR GEOMETRY

- STUDY PROPERTIES OF THE BACKGROUND....

EXAMPLE 1

CONSTANT $\langle T_{\mu\nu} \rangle = \begin{pmatrix} 3 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \cdot \text{const} = \frac{N_c^2}{2\pi^2}$

$\equiv \frac{1}{z_0^4}$

$= \begin{pmatrix} \epsilon & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix}$ WITH $\epsilon=3p$

EINSTEIN EQUATIONS

$$ds^2 = - \frac{(1 - z^4/z_0^4)^2}{1 + z^4/z_0^4} \frac{dt^2}{z^2} + (1 + \frac{z^4}{z_0^4}) \frac{dx^2}{z^2} + \frac{dz^2}{z^2}$$

$$\tilde{z} = \frac{z}{\sqrt{1 + \frac{z^4}{z_0^4}}}$$

STANDARD PLANAR ADS BLACK HOLE

$$ds^2 = - (1 - \frac{\tilde{z}^4}{z_0^4}) \frac{dt^2}{\tilde{z}^2} + \frac{d\tilde{x}^2}{\tilde{z}^2} + \frac{1}{1 - \frac{\tilde{z}^4}{z_0^4}} \frac{d\tilde{z}^2}{\tilde{z}^2}$$

$\tilde{z}_0 = \frac{z_0}{\sqrt{2}}$ ← LOCATION OF THE HORIZON

EXAMPLE 2 PLANAR SHOCKWAVE

- INTRODUCE LIGHT-CONE COORDINATES

$$x^- = t - y \quad x^+ = t + y$$

- PUT $T_{--} = \mu \delta(x^-)$

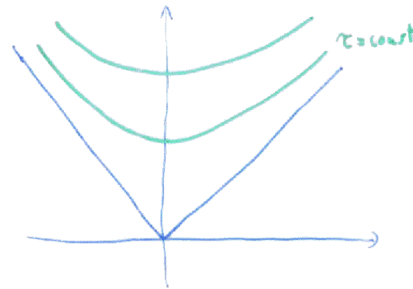
} EINSTEIN EQUATIONS
↓

$$ds^2 = \frac{-dx^- dx^+ + \mu z^4 \delta(x^-) dx^{-2} + dx_{\perp}^2}{z^2} + \frac{dz^2}{z^2}$$

- MODELS AN INFINITELY BOOSTED NUCLEUS
- STUDY COLLISIONS OF TWO SUCH SHOCKWAVES....
-
-
-

- BOOST INVARIANT PLASMA EXPANSION

/BORRKEN '83



ASSUME:

- NO DEPENDENCE ON RAPIDITY y
- NO DEPENDENCE ON TRANSVERSE COORDINATES \vec{x}_{\perp}

THEN

$$\left. \begin{aligned} \partial_{\mu} T^{\mu\nu} &= 0 \\ T^{\mu}_{\mu} &= 0 \end{aligned} \right\} \Rightarrow \begin{aligned} T_{\tau\tau} &= \epsilon(\tau) \\ T_{yy} &= -\tau^2 \left(\tau \frac{d\epsilon}{d\tau} + \epsilon \right) \\ T_{xx} &= \frac{1}{2} \tau \frac{d\epsilon}{d\tau} + \epsilon \end{aligned}$$

- $\epsilon(\tau)$ - ENERGY DENSITY IN THE LOCAL REST FRAME

Q: DETERMINE $\epsilon(\tau)$

- WEAK COUPLING → FREE STREAMING

$$\varepsilon(\tau) = \frac{1}{\tau}$$

- PERFECT FLUID ASSUMPTION

$$\varepsilon(\tau) = \frac{1}{\tau^{4/3}}$$

- FLUID WITH VISCOSITY $\eta = \frac{\eta_0}{\tau}$ $\parallel \eta \propto T^3$

$$\varepsilon(\tau) = \frac{1}{\tau^{4/3}} \left(1 - \frac{2\eta_0}{\tau^{4/3}} + \dots \right)$$

- SECOND ORDER VISCOUS HYDRODYNAMICS η, τ_π, \dots

$$\varepsilon(\tau) = \frac{1}{\tau^{4/3}} \left(1 - \frac{2\eta_0}{\tau^{4/3}} + \frac{\#}{\tau^{4/3}} + \dots \right)$$

↑
DEPENDS ON VERSION OF THE FORMALISM

HOW CAN THIS BEHAVIOR BE SEEN IN AdS/CFT?

- FOCUS ON LATE TIME ASYMPTOTICS

$$\varepsilon(\tau) = \frac{1}{\tau^5}$$

- CONSTRUCT DUAL GEOMETRY WITH SAME SYMMETRIES

$$ds^2 = \frac{1}{z^2} \left(-e^{a(z,\tau)} dt^2 + e^{b(z,\tau)} z^2 dy^2 + e^{c(z,\tau)} dx_i^2 \right) + \frac{dz^2}{z^2}$$

- LOOK AT

$$a(z,\tau) = -\frac{z^4}{\tau^5} + z^6 a_6(\tau) + z^8 a_8(\tau) + \dots$$

↑ ↑
PICK LARGE τ BEHAVIOUR

$$a(z,\tau) = a(V) + \dots$$

$\equiv \frac{1}{\tau^{4/3}}$

⇓
SOLVE EINSTEIN'S EQUATIONS (ODE'S!)

• SOLUTION:

$$a(v) = A(v) - 2m(v)$$

$$b(v) = A(v) + (2s-2)m(v)$$

$$c(v) = A(v) + (2-s)m(v)$$

WHERE

$$A(v) = \frac{1}{2} \left(\log(1+\Delta(s)v^4) + \log(1-\Delta(s)v^4) \right)$$

$$m(v) = \frac{1}{4\Delta(s)} \left(\log(1+\Delta(s)v^4) - \log(1-\Delta(s)v^4) \right)$$

AND

$$\Delta(s) = \sqrt{\frac{3s^2 - 8s + 8}{24}}$$

→ AGREES WITH SCALING LIMIT OF THE POWER SERIES SOLUTIONS

→ VALID ONLY FOR $\tau \rightarrow \infty$ BUT ANALYTIC

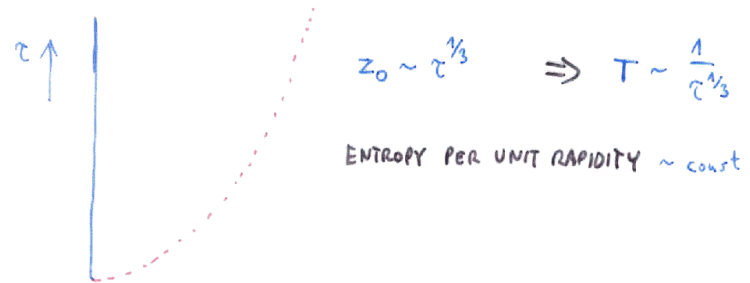
→ ONE CAN STUDY CURVATURE SINGULARITIES

COMPUTE

$$R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} = R_0(v) + \frac{1}{\tau^{4/3}} R_2(v) + \dots$$

NONSINGULAR ONLY FOR $s = \frac{4}{3}$

- LATE TIME DYNAMICS IS DESCRIBED BY PERFECT FLUID HYDRODYNAMICS
- GEOMETRY LOOKS LIKE A MOVING BLACK HOLE



$$ds^2 = \frac{1}{z^2} \left[- \frac{\left(1 - \frac{z^4}{3\tau^{4/3}}\right)^2}{1 + \frac{z^4}{3\tau^{4/3}}} dz^2 + \left(1 + \frac{z^4}{3\tau^{4/3}}\right) (\tau^2 dy^2 + dx_i^2) \right] + \frac{dz^2}{z^2}$$

ADVANCED QUALITATIVELY: SHUKRYA, SAMBASHIVAN, ZHANG

IS $\epsilon(z) = \frac{1}{z^{4/3}}$ EXACT?

$$R_{\text{pert}} R^{\text{pert}} = \underbrace{R_0(v)}_{\text{NONSINGULAR}} + \frac{1}{z^{4/3}} R_2(v) + \dots$$

↑ 4TH ORDER POLE SINGULARITY

• SET $\epsilon(z) = \frac{1}{z^{4/3}} \left(1 + \frac{\#}{z^\Gamma} + \dots \right)$

$$R_{\text{pert}} R^{\text{pert}} = R_0(v) + \frac{1}{z^\Gamma} R_1(v) + \frac{1}{z^{2\Gamma}} \tilde{R}_1(v) + \frac{1}{z^{4/3}} R_2(v) + \dots$$

NONSINGULAR NONSINGULAR SINGULAR SINGULAR

MAY CANCEL WITH EACH OTHER
IF $\Gamma = \frac{2}{3}$ AND # IS FIXED

THEN

$$\epsilon(z) = \frac{1}{z^{4/3}} \left(1 - \frac{2\eta_0}{z^{2/3}} + \dots \right)$$

↑ EXACTLY THE SUBLEADING CORRECTION ASSOCIATED WITH VISCOSITY

NAKAMURA JIR GEOMETRY $\frac{4}{z^{4/3}}$

η_0 FIXED TO $\underline{\eta_0 = 2^{-1/2} 3^{-2/3}}$ COMPATIBLE WITH $\frac{1}{5} = \frac{1}{4\pi}$ R)

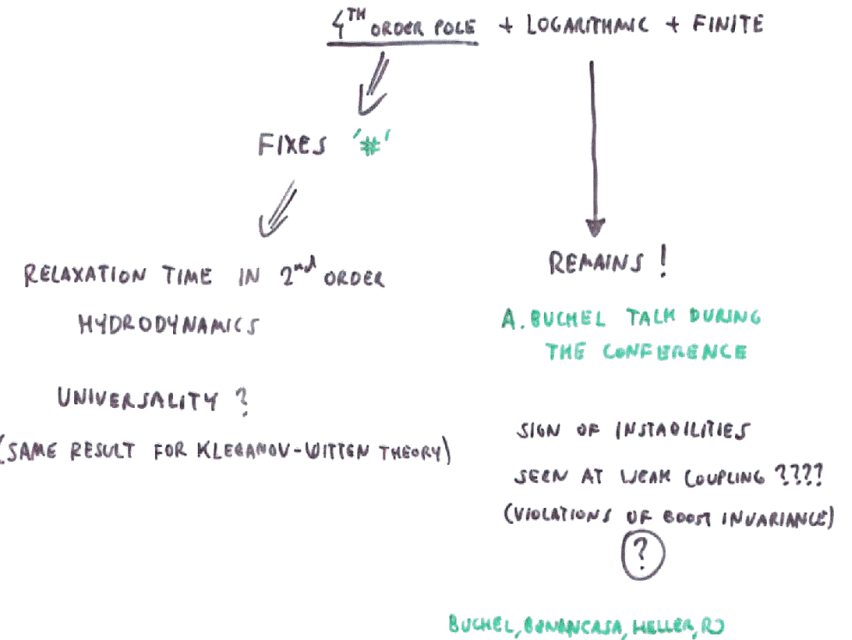
HELLER, RJ

• GO TO HIGHER ORDER

$$\epsilon(z) = \frac{1}{z^{4/3}} \left(1 - \frac{2\eta_0}{z^{2/3}} + \frac{\#}{z^{4/3}} + \dots \right)$$

• CURVATURE

$$R_{\text{pert}} R^{\text{pert}} = R_0(v) + \frac{1}{z^{2/3}} R_1(v) + \frac{1}{z^{4/3}} R_2(v) + \frac{1}{z^2} R_3(v) + \dots$$



'THERMALIZATION' IN THE EXPANDING PLASMA SYSTEM

Q: SUPPOSE WE PERTURB THE EXPANDING PLASMA
HOW WILL THE PERTURBATION BEHAVE WITH TIME?

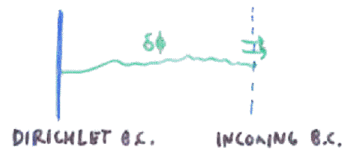
- CONSIDER STATIC PLASMA

HOROVITZ, MURPHY '99
STARINETS '02

$$\delta T_{xy} \longrightarrow \delta g_{xy}$$

$$\delta(B^2 - E^2) \longrightarrow \delta\phi$$

- QUASINORMAL MODES OF THE STATIC BLACK HOLE



- FLUCTUATIONS DECAY EXPONENTIALLY WITH TIME
LOWEST FREQUENCY (IN THE SCALAR CHANNEL):

$$e^{-i \omega_{\text{STATIC}}(T) t}$$

WHERE

$$\omega_{\text{STATIC}}(T) \approx (3.4194 - 2.747i) \cdot \pi T$$

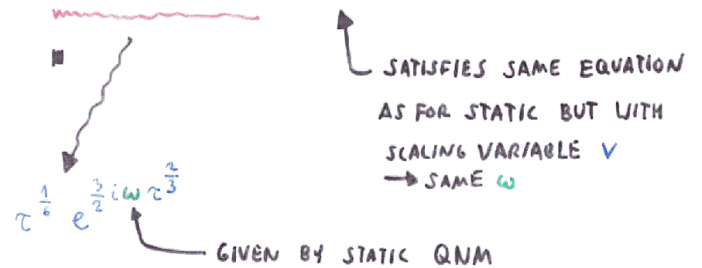
DOES THE EXPANDING PLASMA BEHAVE SIMILARLY?

R) R. PESCHANSKI

- SCALAR CHANNEL PERTURBATION OF THE PERFECT FLUID GEOMETRY

SEPARATION OF
VARIABLES FOR
LARGE τ

$$\sqrt{\tau} J_{\pm \frac{3}{4}} \left(\frac{3}{2} \omega \tau^{\frac{2}{3}} \right) \cdot \phi \left(v = \frac{z}{\tau^{\frac{1}{3}}} \right)$$



- DAMPING:

$$e^{-\frac{3}{2} 2.747 \cdot \tau^{\frac{2}{3}}}$$

UNITS:
 $z_0 = \tau^{1/3}$
IN F-G COORDINATES

- EXPANDING PLASMA IS STABLE UNDER PERTURBATIONS
- ATTRACTOR GEOMETRY?
- PERFECT FLUID GEOMETRY \rightarrow 'ADIABATIC' APPROXIMATION

$$\omega(\tau) t \sim \int \omega(\tau(z)) dz$$

- BUT NOTE POWER-LAW PREFACTORS + VISCOSITY CORRECTIONS...

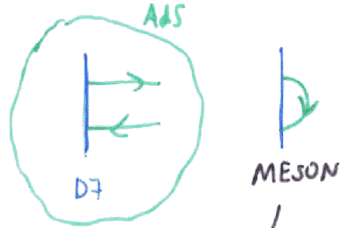
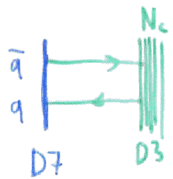
FUNDAMENTAL FLAVOURS IN THE EXPANDING PLASMA

- CONSIDER $N=4$ SYM + ONE ADDITIONAL FLAVOUR ($N=2$)

CAUTION: THIS THEORY DOES NOT HAVE CHIRAL SYMMETRY BREAKING..

- AdS/CFT DESCRIPTION

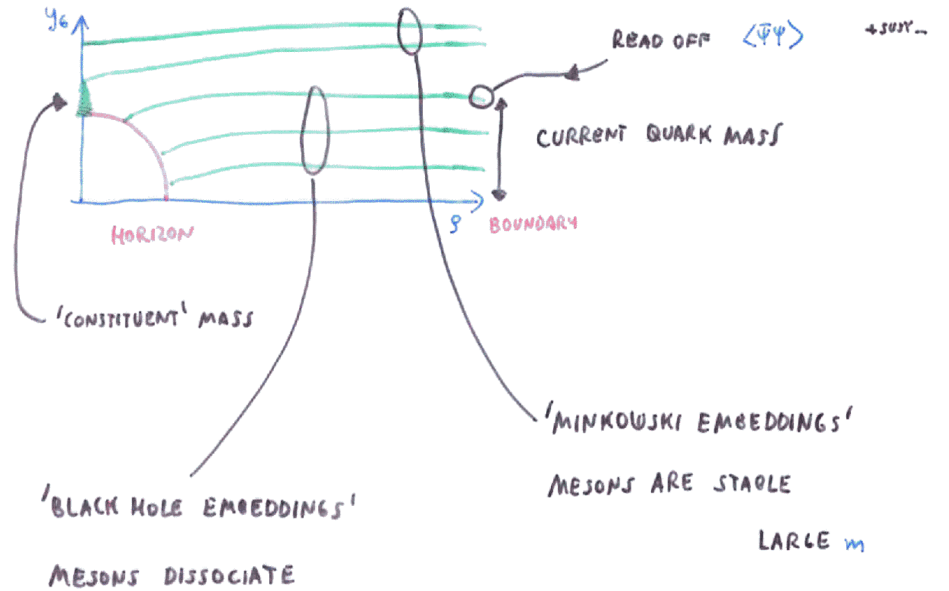
- EMBED A D7 BRANE IN THE GEOMETRY



LIGHTEST MESONS:
FLUCTUATIONS OF THE D7 BRANE EMBEDDING (OR D7 GAUGE FIELDS)

FINITE TEMPERATURE CASE (STATIC PLASMA)

- EMBED A D7 BRANE INTO THE BLACK HOLE GEOMETRY $g_6(g)$



- FIX m, T
- GET EMBEDDING
- READ OFF $\langle \bar{\psi}\psi \rangle$
- STUDY FLUCTUATIONS
- OBTAIN MESON MASSES

J. GROSJE, R. P. SUROWKA

QUESTION: HOW DOES THE EXPANDING PLASMA MODIFY FLAVOR PHYSICS?

- USE LATE TIME EXPANSION → 'MINKOWSKI EMBEDDING'
- EMBED D7 BRANE INTO PLASMA GEOMETRY (INCLUDING VISCOSITY)

$$y_6(g, \tau) = m + \left(-\frac{\#}{\tau^{4/3}} \left(1 - \frac{\#}{\tau^{4/3}} + \dots \right) \right) \cdot \frac{g^4 + 3g^2 u^2 + 3u^4}{(u^2 + g^4)^3} + \dots$$

- READ OFF THE CONDENSATE

$$\langle \bar{\psi} \psi \rangle \propto \# \frac{\tau^2(\tau)}{g^5}$$

- LOOK AT FLUCTUATIONS → MESONS

MAJOR COMPLICATION: LACK OF SEPARABILITY (PDE)

- ANSATZ FOR FLUCTUATIONS

$$\delta\phi = \sqrt{\frac{\text{Subtle}}{\omega(\tau)\tau}} \int_0^{\text{Subtle}} \int_{Y_0} \left(f_0(\tau) + \frac{1}{\tau^{4/3}} f_1(\tau) + \frac{1}{\tau^2} f_2(\tau) + \dots \right)$$

→ DETERMINE τ -DEPENDENT FREQUENCY $\omega(\tau)$

LOWEST MODE

$$\omega(\tau) = \frac{4\pi}{\Gamma\lambda} \left(m_g - \frac{3\lambda^2 \epsilon_0}{80\pi^2 \lambda^2} \frac{1}{\tau^{4/3}} \left(1 - \frac{2\lambda}{\epsilon_0} \frac{1}{\tau^{4/3}} + \dots \right) \right)$$

|| CAUTION: NOT REALISTIC MODEL OF QCD - NO QSB !

- HOW DO THESE MODES LOOK LIKE ON THE BOUNDARY?

$$\delta\phi_{4D}(x^\mu) = \lim_{g \rightarrow \infty} g^2 \delta\phi(x^\mu, g)$$

- THESE 4D MODES $\delta\phi_{40}(x^r)$ SATISFY

$$\boxed{\delta\phi_{40} + \omega^2(\tau)\delta\phi_{4A} = 0}$$

→ TYPICAL FORM AS FOR SCALAR FIELDS IN A COSMOLOGICAL SETTING!



EXPECT PARTICLE PRODUCTION

WORK IN PROGRESS

J. GROSJE, R.J., P. SURBOKA

ANISOTROPIC PLASMA AT STRONG COUPLING

MOTIVATION: PLASMA INSTABILITIES AT WEAK COUPLING

- ANISOTROPIC GLUON MOMENTUM DISTRIBUTION

→ SMALL FLUCTUATIONS (UNSTABLE MODES) ←

→ TIME EVOLUTION FROM INITIAL CONDITIONS

SETUP STUDY ANISOTROPIC PLASMA AT STRONG COUPLING

$$T_{\mu\nu} = \begin{pmatrix} \epsilon & & & \\ & P_{\parallel} & & \\ & & P_{\perp} & \\ & & & P_{\perp} \end{pmatrix}$$

$$P_{\parallel} + 2P_{\perp} = \epsilon$$

// MINKOWSKI COORDINATES!

CONSIDER STATIC CONFIGURATION

WORK IN PROGRESS: R.J., P. WITASZCZYK

FRAMEWORK

$$T_{\mu\nu} \longrightarrow ds^2 = \frac{1}{z^2} \left(-a(z) dt^2 + b(z) dx_{11}^2 + c(z) dx_{\perp}^2 \right) + \frac{dz^2}{z^2}$$

• EXACT SOLUTION

$$a(z) = (1 + Az^4)^{\frac{1}{2} - \frac{1}{4} \sqrt{36 - 2B^2}} (1 - Az^4)^{\frac{1}{2} + \frac{1}{4} \sqrt{36 - 2B^2}}$$

$$b(z) = (1 + Az^4)^{\frac{1}{2} - \frac{B}{3} + \frac{1}{42} \sqrt{36 - 2B^2}} (1 - Az^4)^{\frac{1}{2} + \frac{B}{3} - \frac{1}{42} \sqrt{36 - 2B^2}}$$

$$c(z) = (1 + Az^4)^{\frac{1}{2} + \frac{B}{6} + \frac{1}{42} \sqrt{36 - 2B^2}} (1 - Az^4)^{\frac{1}{2} - \frac{B}{6} - \frac{1}{42} \sqrt{36 - 2B^2}}$$

• A AND B RELATED TO ϵ , p_{11} , p_{\perp}

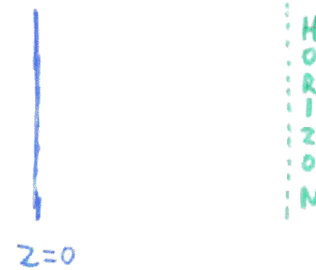
$$\epsilon = \frac{1}{2} \sqrt{36 - 2B^2} A$$

$$p_{11} = \left(\frac{1}{6} \sqrt{36 - 2B^2} - \frac{2}{3} B \right) A$$

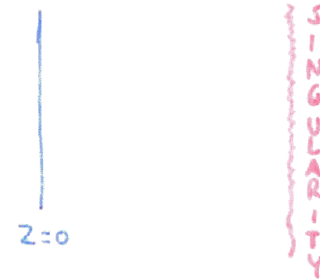
$$p_{\perp} = \left(\frac{1}{6} \sqrt{36 - 2B^2} + \frac{1}{3} B \right) A$$

→ B MEASURES THE ANISOTROPY

• B = 0 REDUCES TO STATIC BLACK HOLE



• B ≠ 0



• LOOK AT SMALL FLUCTUATIONS...

INSTABILITIES ???

WORK IN PROGRESS

SUMMARY & OUTLOOK

- VERY GENERAL FRAMEWORK FOR STUDYING TIME-DEPENDENT DYNAMICAL PROCESSES OR OUT-OF-EQUILIBRIUM CONFIGURATIONS
- ADSICFT \rightarrow HYDRODYNAMICAL EXPANSION WITH SPECIFIC TRANSPORT COEFFICIENTS
- LEFTOVER LOGARITHMIC SINGULARITIES(?)
- EXPANDING PLASMA SYSTEM IS STABLE AGAINST PERTURBATIONS
- TIME DEPENDENT MESONIC FREQUENCIES
- PARTICLE PRODUCTION ?
- ANISOTROPIC PLASMA AT STRONG COUPLING INSTABILITIES ??

MANY OPEN

QUESTIONS

- DEFINITIONS OF ENTROPY / TEMPERATURE
- GLOBAL STRUCTURE ?
- MEANING OF LOGARITHMIC SINGULARITIES
- INITIAL THERMALIZATION
- SCALE, PHASE TRANSITION ?