Towards new relativistic hydrodynamcis from AdS/CFT

Michael Lublinsky

Stony Brook
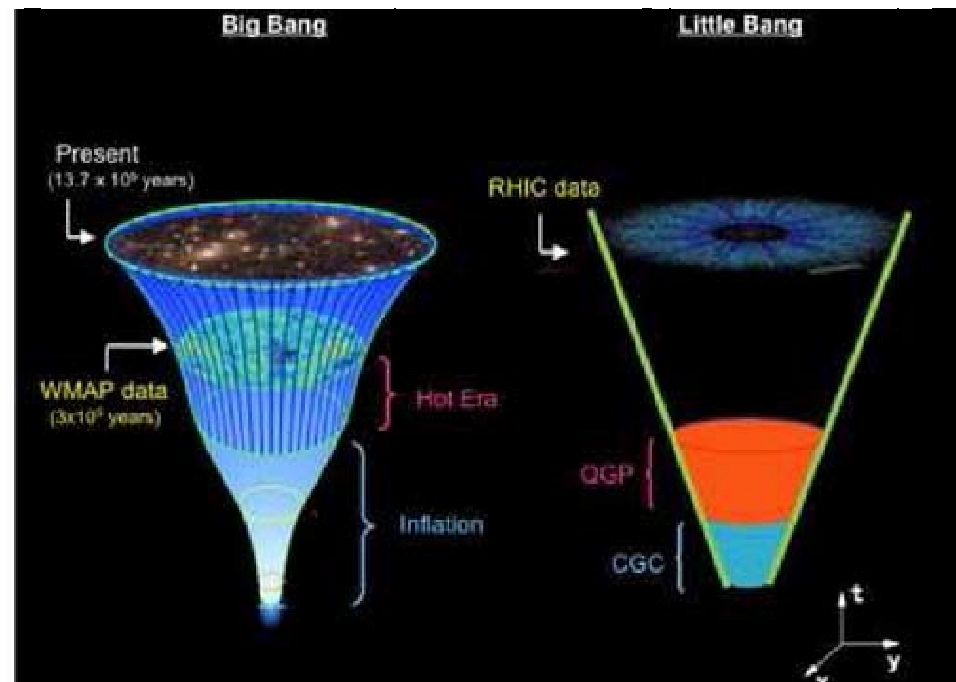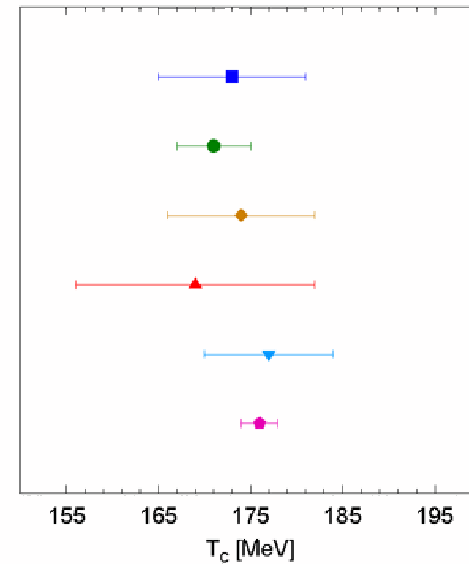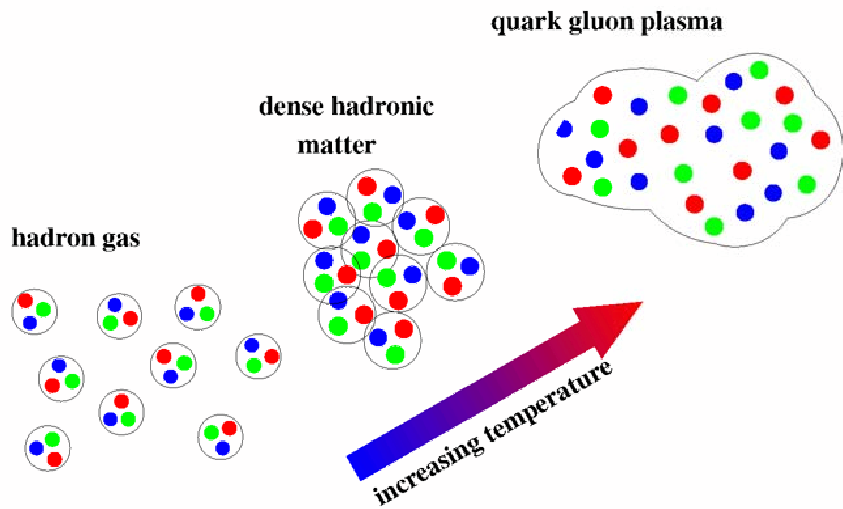
Summary of lattice results on the transition temperature T_c take

- **QGP is Deconfined**

- **QGP is strongly coupled (sQGP)**
  behaves "almost" like a perfect liquid (Navier-Stokes with very small viscosity)
  $$\eta \sim mean \ free \ path \sim 1/\sigma$$

QCD $\longrightarrow \mathcal{N}=4$ SYM (CFT)

Strong coupling (and large N_c) $\rightarrow$ AdS/CFT $\rightarrow$ SUGRA on AdS$_5$

CFT at finite Temperature $\leftrightarrow$ AdS Black Hole

Outline of the talk

- **Why do we think QGP is strongly coupled**

- **Relativistic hydrodynamics**

- **Black Hole AdS/CFT**

- **All order hydro: momenta dependent viscosity**

Motivation: Experiments (RHIC) probe systems with finite gradients.

Introduce higher viscosity terms in the gradient expansion of $T^{\mu\nu}$

Extract momenta dependent viscosities by matching two-point correlation functions of stress energy tensor with correlation functions computed from BH AdS/CFT.

(when applying to QCD we hope for some universality for transport coefficients)

We propose to use this hydro as a "nonlinear model" for real simulations at RHIC

(ideally hydro equations should have no fitting parameters in them)

CGC Initial Singularity Glasma sQGP Hadron Gas

$z = -t$ t $z = t$

Quark Gluon Plasma → **Perfect Fluid Hadronization** $\tau \sim 1 - 10$ fm/c

Glasma → **Topological Excitations Density Fluctuations, Thermalization** $\tau \sim 0.1 - 1$ fm/c

Initial Singularity → **Event Horizon Quantum Fluctuations** $\tau \sim 0 - 0.1$ fm/c

z

Initial Nuclei as CGC → **Coherent, High-density Gluons**
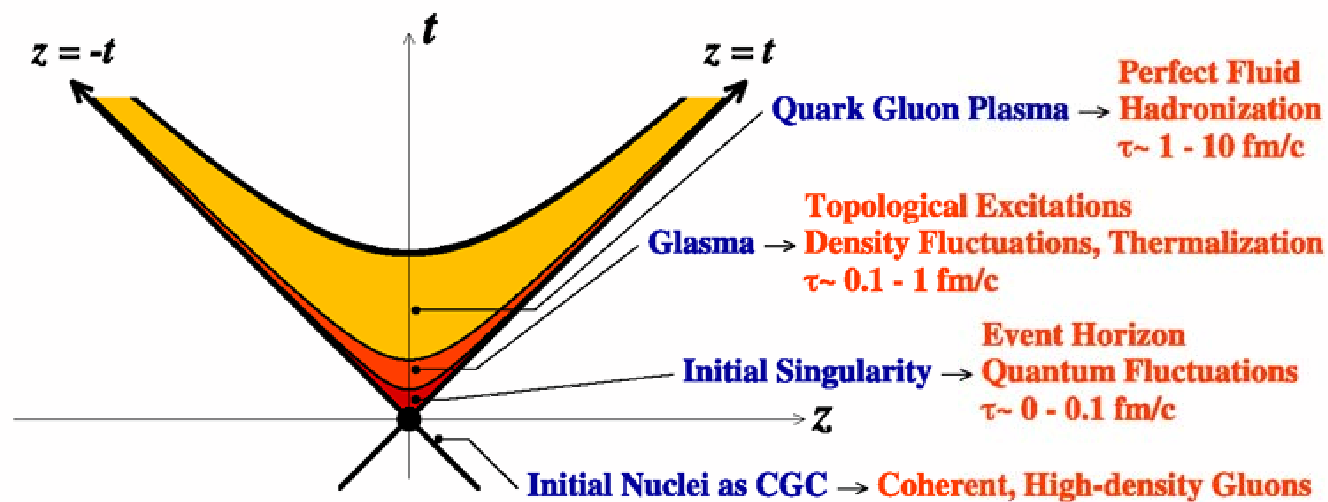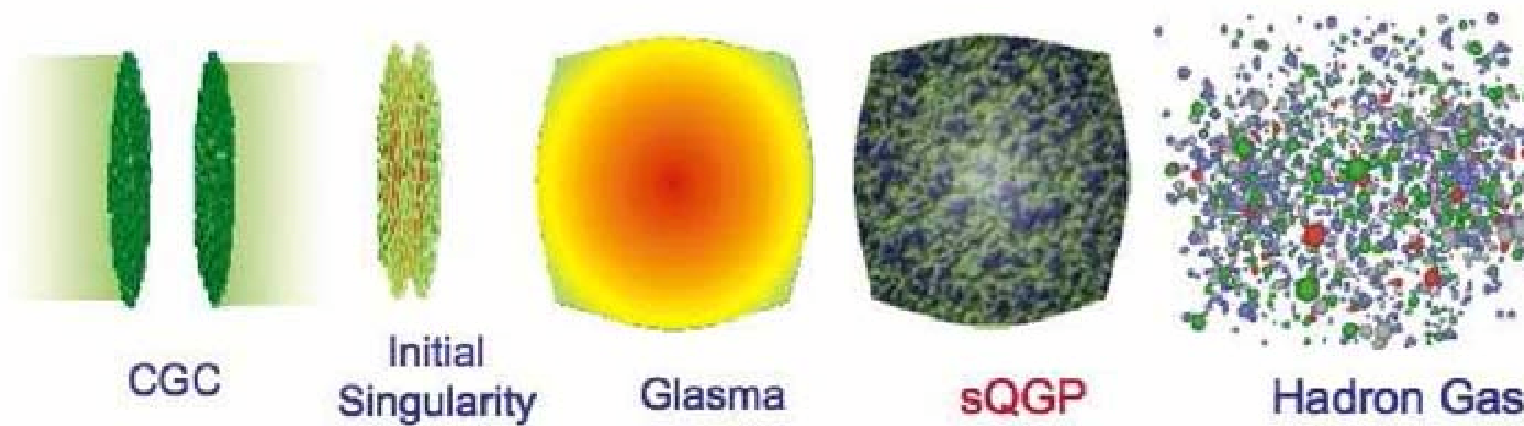
Medium induced jet suppression and two particle correlations
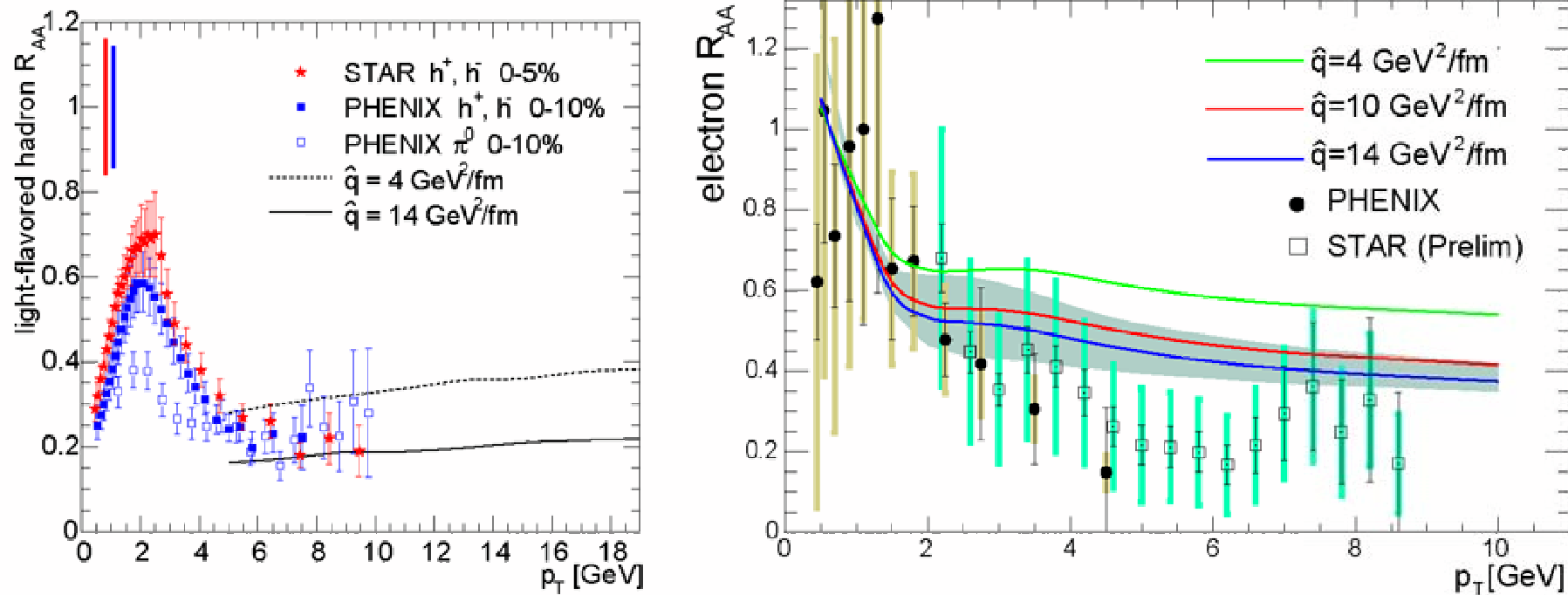

FIGURE 1. Central Au–Au collisions at $\sqrt{s_{NN}} = 200$ GeV: R_{AA} for light-flavored hadrons (left, adapted from [9]) and for heavy-flavor decay electrons (right, from [12])
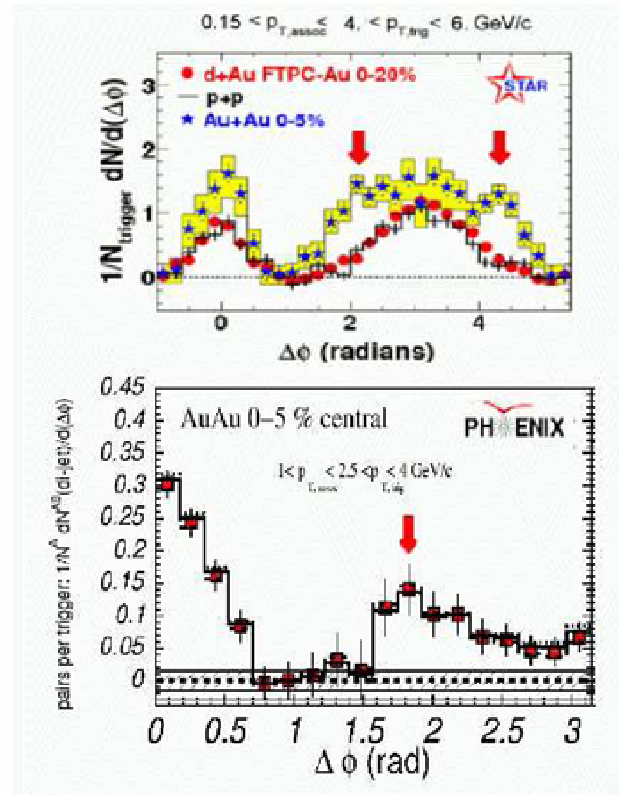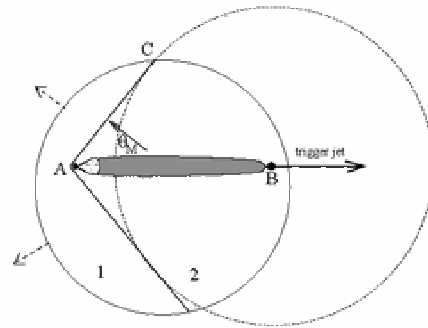
Fig. 1. (a) A schematic picture of flow created by a jet going through the fireball. The trigger jet is going to the right from the origination point B. The companion quenched jet is moving to the left, heating the matter (in shadowed area) and producing a shock cone with a flow normal to it, at the Mach angle $cos\theta_M = v/c_s$, where v, c_s are jet and sound velocities. (b)The background subtracted correlation functions from STAR and PHENIX experiments, a distribution in azimuthal angle $\Delta\phi$ between the trigger jet and associated particle. Unlike in pp and dAu collisions where the decay of the companion jet create a peak at $\Delta\phi = \pi$ (STAR plot), central $AuAu$ collisions show a minimum at that angle and a maximum corresponding to the Mach angle (downward arrows).

ALICE

Alice and the soup of quarks and gluons
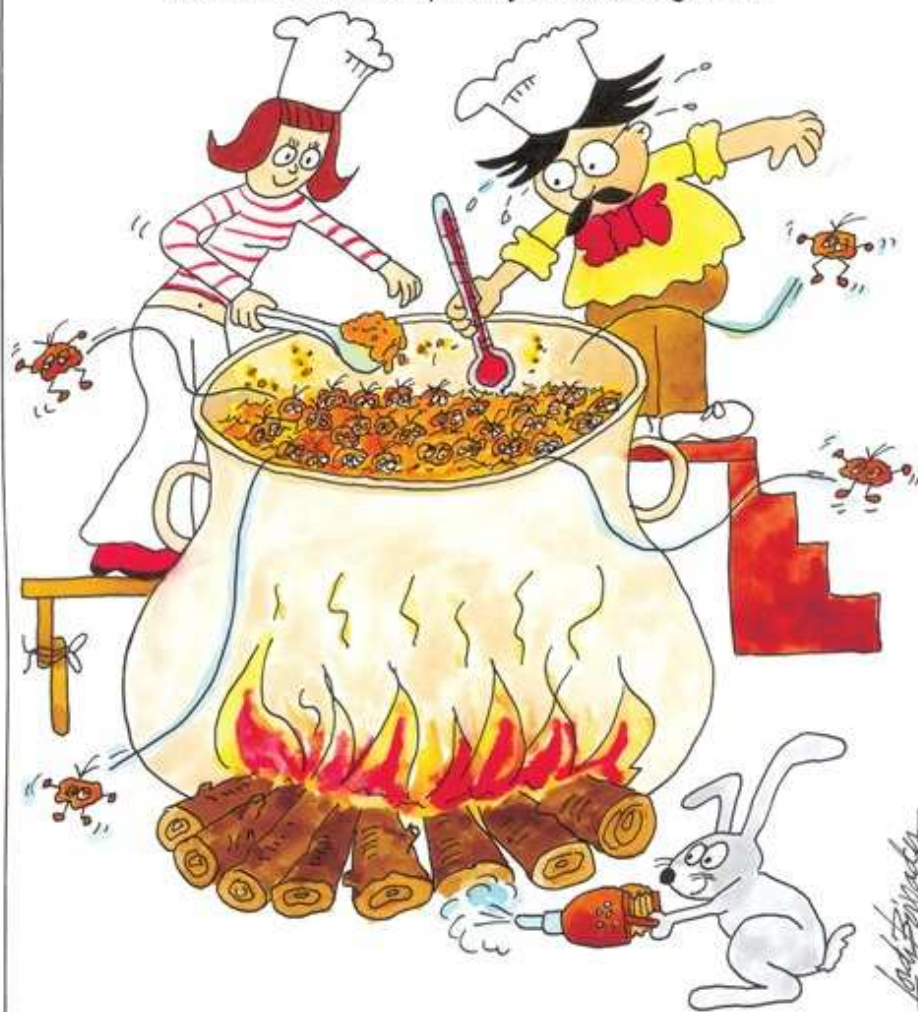
Relativistic Hydrodynamics

Energy momentum tensor

$$T^{\mu\nu} = (\epsilon + P)\, u^\mu\, u^\nu + P\, g^{\mu\nu} + \Pi^{\mu\nu}$$

u - **velocity field of the fluid** $\qquad u^2 = -1$ $\qquad\qquad$ P - **Pressure**

$\Pi^{\mu\nu}$ - **tensor of dissipations (ideal fluid:** $\Pi^{\mu\nu} = 0$**)**

$u_\mu\, \Pi^{\mu\,\nu} = 0 \qquad$ - $\qquad$ **no dissipation in the local rest frame**

Navier Stokes term (expanding in the velocity gradient)

$$\Pi^{\mu\nu} = -\eta\,(\Delta^{\mu\lambda}\, \nabla_\lambda\, u^\nu + \Delta^{\nu\lambda}\, \nabla_\lambda\, u^\mu - \frac{2}{3}\Delta^{\mu\nu}\, \nabla_\lambda\, u^\lambda) - \xi\,\Delta^{\mu\nu}\, \nabla_\lambda\, u^\lambda$$

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu\, u^\nu$$

Energy - momentum conservation:

$$\nabla_\mu \, T^{\mu\nu} \; = \; 0 \qquad \longrightarrow \qquad \mathrm{Navier - Stokes\ Eq.}$$

Conformal invariance

$$T^\mu_\mu \; = \; 0 \qquad \longrightarrow \qquad \epsilon \; = \; 3\,P \quad \mathrm{and} \quad \xi \; = \; 0$$

Entropy density and EoS

$$s \; = \; \frac{\epsilon \; + \; P}{T} \; = \; 4\,k_{SB}\,T^3$$

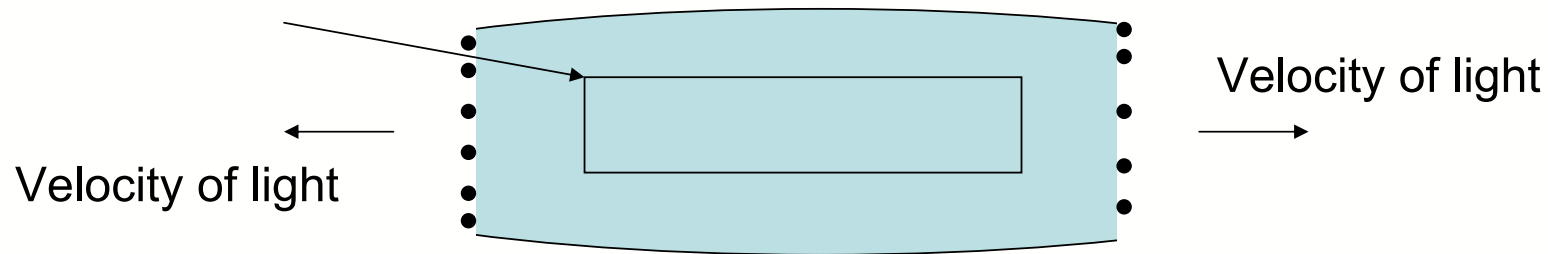**No dissipation no entropy production:**

$$\frac{ds}{dt} \; = \; 0 \qquad if \qquad \Pi^{\mu\nu} \; = \; 0$$

RHIC and Bjorken set up
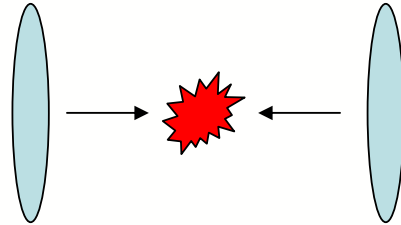
Relativistically accelerated heavy nuclei

Central Rapidity Region (CRR)

Velocity of light

Velocity of light

After collision

- **one-dimensional expansion**: **Boost Invariant Formulation**

Local rest frame $u = (1, 0, 0, 0)$ $(x^0, x^1, x_\perp) \rightarrow (\tau, y, x_\perp)$

τ **- proper time,** y **- spacetime rapidity**

$$x^0 = \tau \, ch(y) \qquad\qquad x^1 = \tau \, sh(y)$$

The metric (1d Hubble expansion)

$$ds^2 = - d^2\tau + \tau^2 \, d^2y + d^2x_\perp$$

Hydro eq. simplify dramatically:

$$\partial_\tau \, \epsilon(\tau) = - \frac{4\,\epsilon}{3\,\tau} + \frac{4\,\eta}{3\,\tau^2}$$

Solution for $\eta = 0$:   **Bjorken (1986)**

$$\epsilon \sim \frac{1}{\tau^{4/3}} \qquad\qquad T \sim \frac{1}{\tau^3} \qquad\qquad \partial_\tau \, (s\,\tau) = 0$$

Sound waves in Relativistic Hydrodynamics

Plane wave perturbation:

$$\delta u = \delta u_0 \, e^{-i\,\omega\,t \,+\, i\,q\,x} \qquad\qquad \delta P = \delta P_0 \, e^{-i\,\omega\,t \,+\, i\,q\,x}$$

Linearized Hydro leads to the dispersion relation

$$\omega = c\,q \;-\; i\,\frac{2\,\eta}{s\,T}\,q^2$$

Sound velocity $\quad c = 1/\sqrt{3}$ $\qquad\qquad$ **Sound attenuation** $\quad \sim \eta$

Spectral functions in the sound and shear channels ($2\,\pi\,T = 1$ **and** $\bar{\eta} \equiv 4\,\pi\,\eta/s$)

$$\chi^L = \frac{2\,\omega\,c^2\,q^4\,\bar{\eta}}{(\omega^2 - c^2\,q^2)^2 + 4\,\omega^2\,c^2\,q^4\,\bar{\eta}^2} \qquad\qquad \chi^T = \frac{\omega\,q^2\,\bar{\eta}/2}{\omega^2 + q^2\,\bar{\eta}^2/4}$$

Solves causality problems encoded in Navier-Stokes

Add extra term in the gradient expansion + non-linear terms in (∇u)

$$\Pi^{\mu\nu} \;=\; (1 \;-\; \tau_R \, u_\lambda \, \nabla^\lambda) \;\; \Pi^{\mu\nu}_{NS}$$

Iterate the equation

$$(1 \;+\; \tau_R \, u_\lambda \, \nabla^\lambda) \;\; \Pi^{\mu\nu} \;=\; \Pi^{\mu\nu}_{NS}$$

When thinking about small perturbations $\;\; u_\lambda \, \nabla^\lambda \;\to\; \nabla_t \;\to\; -\,i\,\omega$
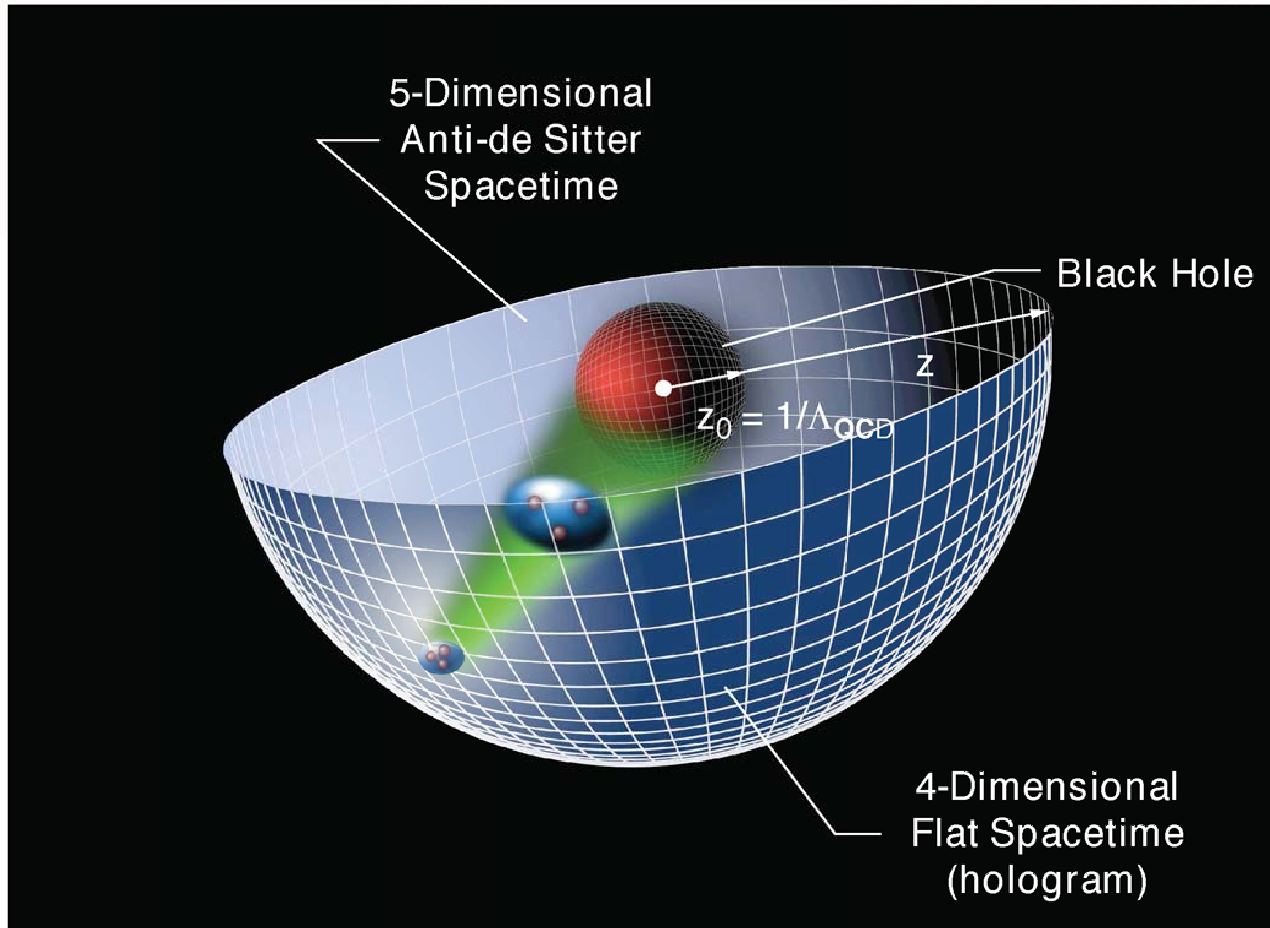
The second order hydro is equivalent (in the linear approximation) to

$$\eta \;\to\; \frac{\eta}{1 \;-\; i\,\tau_R\,\omega}$$

Sound dispersion

$$\omega \;=\; c\,q \left[1 \;+\; \bar{\eta}\, c^2\, q^2\, (2\,\tau_R \;-\; \bar{\eta}) \right] \;-\; i\, c^2\, \bar{\eta}\, q^2 \left[1 \;+\; c^2\, q^2\, \bar{\eta}\, \tau_R\, (2\,\bar{\eta} \;-\; \tau_R) \right]$$

picture due to Stan Brodsky

AdS/CFT correspondence: weakly coupled super-gravity in $AdS_5 \times S_5$ is "dual" to strongly coupled $\mathcal{N} = 4$ **SYM gauge theory in 4d**

AdS_5 **Schwarzschild BH metric**

$$ds^2 = \frac{\rho^2}{L^2} \left[-\left(1 - \frac{\rho_0^4}{\rho^4}\right) dt^2 + dx^2 + dy^2 + dz^2 \right] + \frac{L^2}{\rho^2 (1 - \rho_0^4/\rho^4)} d\rho^2$$

BH Horizon at $\rho = \rho_0$

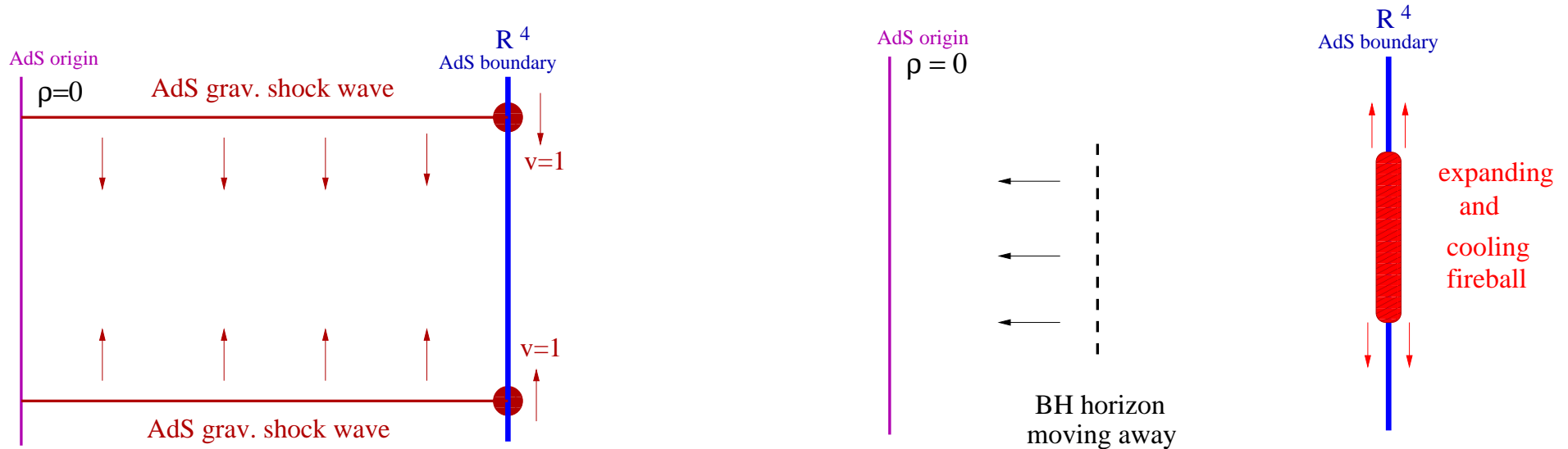**AdS "boundary"** $\rho \to \infty$ **is Minkowski space (t,x,y,z)**

Gauge theory at the boundary is $\mathcal{N} = 4$ **SYM static plasma at finite temperature.**

The Hawking temperature is

$$\mathbf{T} = \frac{\rho_0}{\pi \, \mathbf{L^2}}$$

Gravity Dual of RHIC collisions: the Vision

E. Shuryak, I. Zahed, S.J. Sin ...

AdS origin
$\rho=0$

R^4
AdS boundary

AdS grav. shock wave

v=1

v=1

AdS grav. shock wave

AdS origin
$\rho = 0$

R^4
AdS boundary

BH horizon
moving away

expanding
and
cooling
fireball

AdS Black Hole (black brane) is formed at the collision. The BH is NOT static! The horizon moves away from the boundary and this corresponds to cooling of the fireball.

No realization of the "Vision" yet, but a lot of work is in progress:

Gravity dual of Bjorken hydro   R. Janik, PRL 98, 022302 (2007)

Y. V. Kovchegov, A. Taliotis, Phys.Rev.C76:014905,2007.

Gravity dual of spherically symmetric adiabatically expanding hydro

J. J. Friess, S. S. Gubser, G. Michalogiorgakis and S. S. Pufu, JHEP 0704:080,2007

Retarded correlators and Viscosity from AdS BH

Retarded correlators:

$$G_R^{\mu\nu\mu'\nu'}(\omega,\,q) \,=\, -\,i\,\int_0^\infty dt\,\int dx\,e^{-\,i\,\omega\,t\,+\,i\,q\,x}\,\langle[T^{\mu\nu}(t,x)\,,\,T^{\mu'\nu'}(0,0)]\rangle$$

AdS/CFT: **energy-momentum tensor $T_{\mu\nu}$ couples at the boundary to metric perturbations (gravitons). Solve linearized GR in 5d with absorptive boundary conditions at the horizon.**

Shear viscosity

$$\eta \,=\, \lim_{\omega\to 0}\frac{1}{2\,\omega}\int dt\,dx\,e^{-\,i\,\omega\,t}\,\langle[T^{xy}(t,x)\,,\,T^{xy}(0,0)]\rangle$$

P. Kovtun, D. T. Son and A. O. Starinets, PRL 94, 111601 (2005)

$$\frac{\eta}{s} \,=\, \frac{1}{4\,\pi}$$

Imposing also Dirichlet boundary conditions at the AdS boundary leads to quantization: quasi-normal modes.

Is anything wrong with viscous hydro?

The phenomenologically proffered value for η/s is very small
Viscosity kills the elliptic flow!
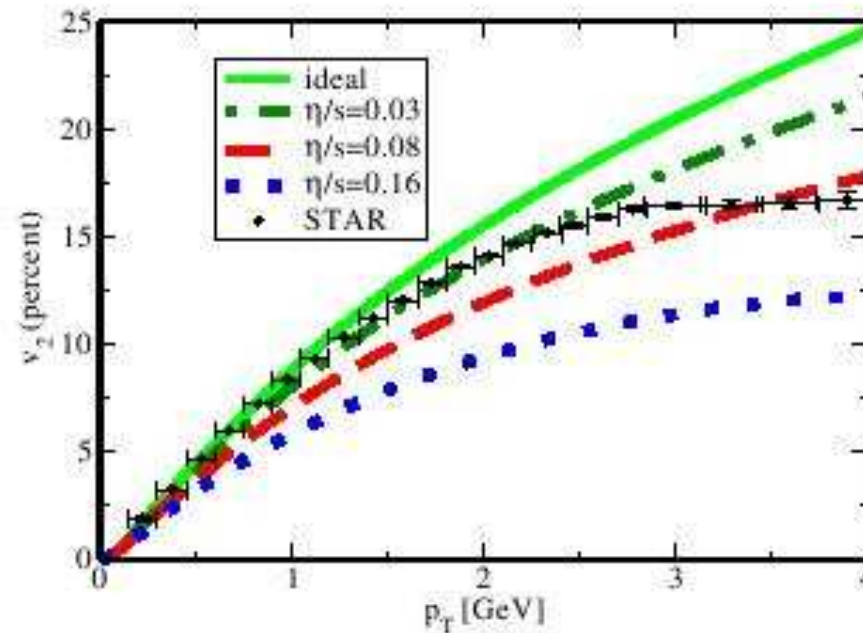
P. Romatschke, U. Romatschke, Phys.Rev.Lett.99:172301,2007

Largely supported by H. Song, U. W Heinz, .arXiv:0712.3715
**But disagrees with** K. Dusling, D. Teaney arXiv:0710.5932

At RHIC Hydro seems to start at very early time $\tau_0 \sim 0.5 - 1\, fm.$
This is needed to explain the elliptic flow and other data.

The hydro phase is long (10 fm) $\rightarrow$ too much entropy is produced by the hydro phase.
That seems to contradict the RHIC data on produced particle multiplicities.

Entropy production in the Bjorken (1d) Hydro

$$\partial_\tau (s\, \tau) = \frac{4\, s}{3} \; \frac{\eta}{s} \; \frac{1}{T\, \tau}$$

A. Dumitru , E. Molnar , Y. Nara, Phys.Rev.C76:024910,2007.
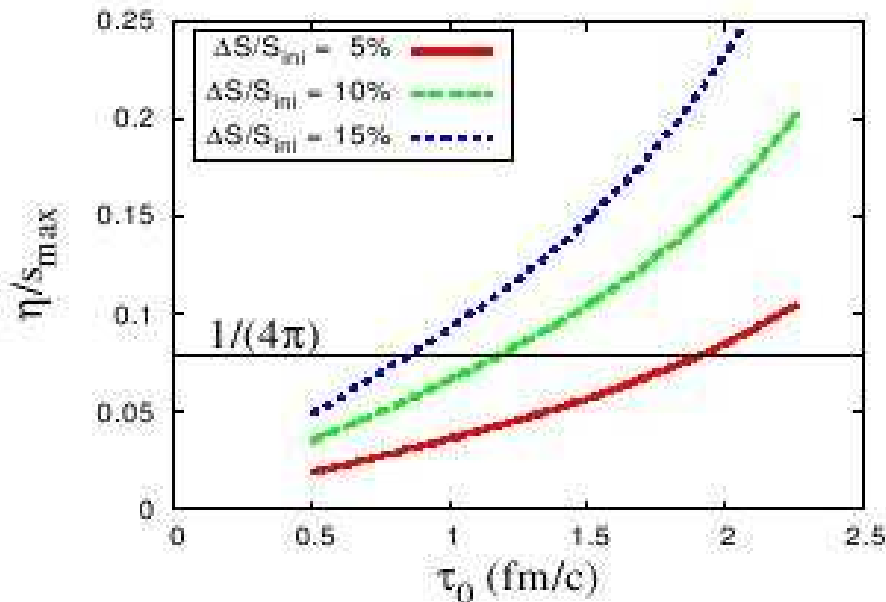
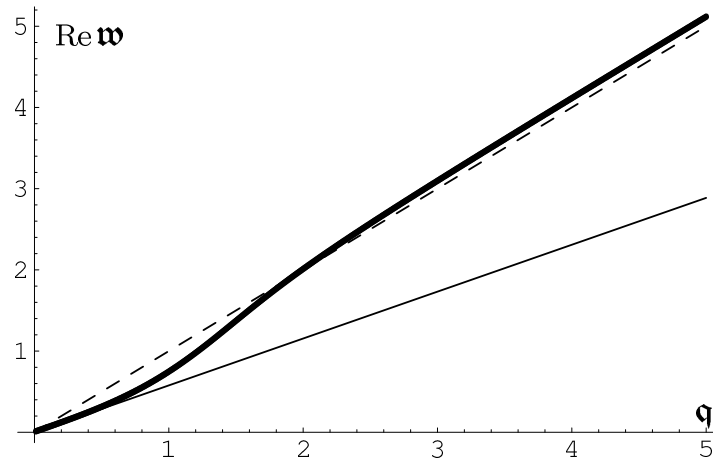**E. Shuryak and M.L., Phys.Rev.C76:021901,2007**

Introduce higher viscosity terms in

the gradient expansion of $T^{\mu\nu}$:

$$\frac{\partial_\tau (s\tau)}{s\,(\tau\, T)} = \frac{\bar{\eta}}{\pi} \left[c^2 \frac{1}{(\tau\, T)^2} + \sum_{n=2}^{\infty} \frac{\alpha_n}{(T\tau)^{2n}} \right]$$

Sound and Holography

Quasi-normal mode analysis in the AdS BH background - the sound channel
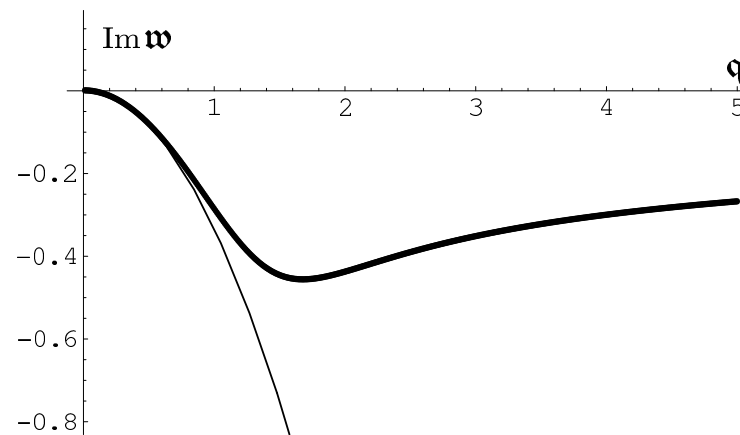
$$\Re e[\omega] = c\,q + \sum_{n=1}^{\infty} r_n\,q^{2n+1} \qquad \Im m[\omega] = -\bar{\eta}\left[c^2\,q^2 + \sum_{n=2}^{\infty} \beta_n\,q^{2n}\right]$$
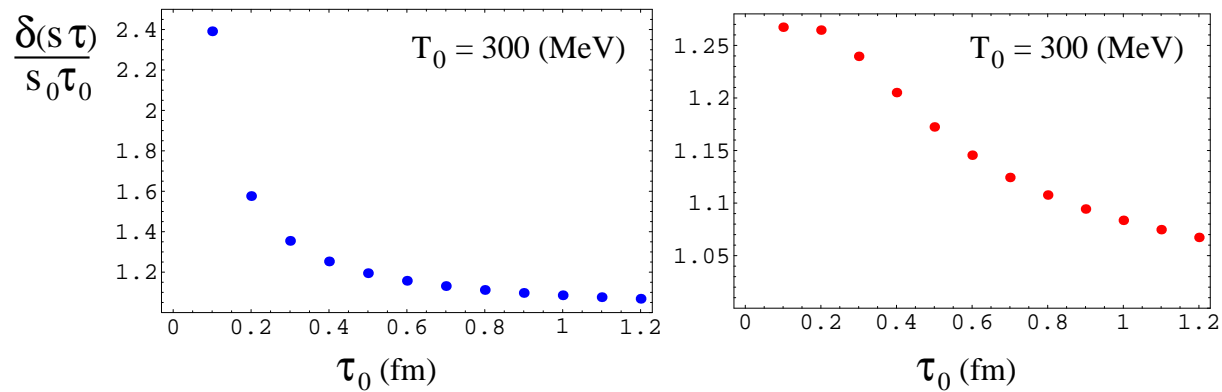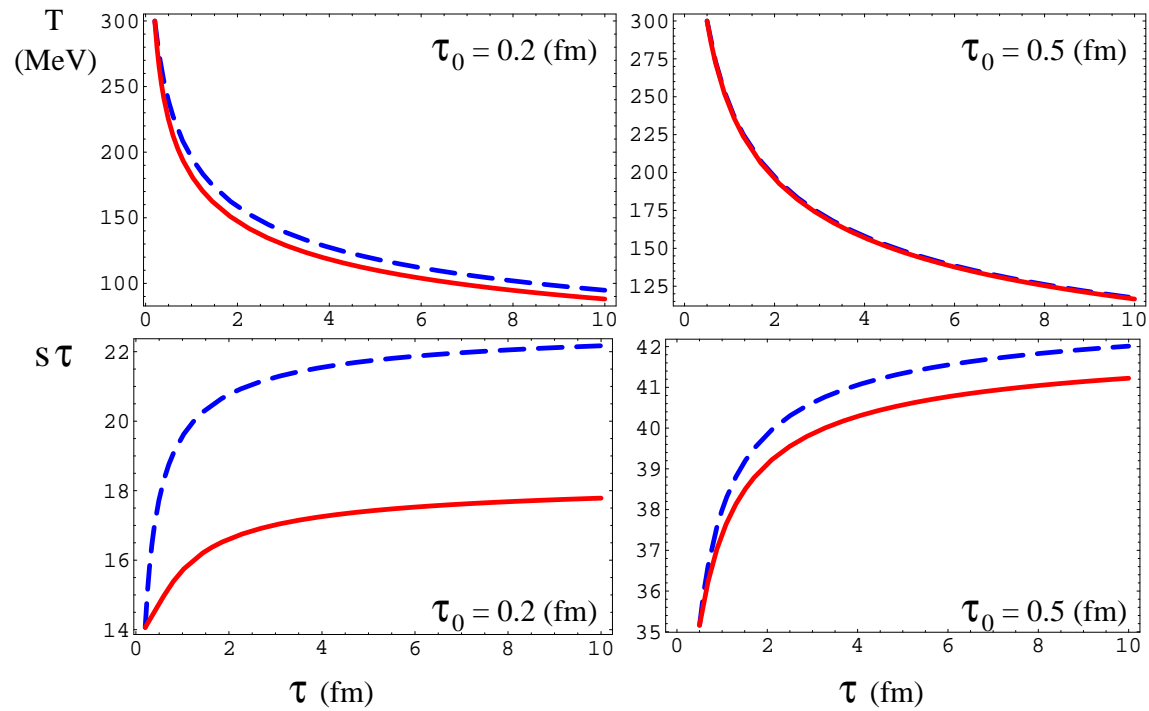
$$r_1 \rightarrow \tau_R = 2 - \ln[2]$$

R. Baier, P. Romatschke, D.T. Son, A.O. Starinets, M.A. Stephanov, arXiv:0712.2451

S. Bhattacharyya, V. E Hubeny, S. Minwalla, M. Rangamani, arXiv:0712.2456

$\beta_2 < 0$ while the second order hydro leads to $\beta_2 > 0$

How much entropy is produced by Hydro at RHIC?

Linearized Hydrodynamics to all orders

$$\Pi^{\mu\nu} = \Delta^{\mu m} \Delta^{\nu n} D_{mn,k}[\nabla] u^k$$

Tracelessness condition: $\quad \Delta^{mn} D_{mn,k}[\nabla] u^k = 0$

$$D_{mn,k} u^k = g_{mn} \left[\frac{2}{3} \eta_1 - \frac{1}{3} \eta_2 \nabla^2\right] (\nabla u) - \eta_1 [g_{mk} \nabla_n + g_{nk} \nabla_m] u^k + \eta_2 \nabla_m \nabla_n (\nabla u)$$

$$\eta_{1,2} = \eta_{1,2}[(u\nabla), \nabla^2] \rightarrow \eta_{1,2}[i\,\omega, \omega^2 - q^2] = \Re e\,\eta_{1,2} + \Im m\,\eta_{1,2}$$

$$\eta_1[\omega \rightarrow 0, q \rightarrow 0] \rightarrow \eta$$

We neglect all terms which are nonlinear in (∇u) (though some could be recovered).

Shear channel:

$$G^T_R = \frac{\eta_1 \, q^2/2}{-\,i\,\omega \;+\; \eta_1\, q^2/2} \qquad\qquad \chi^R = \Im m \; G^T_R$$

Sound channel:

$$G^L_R = \frac{2\,i\,\omega\,c^2\,q^2\,\tilde{\eta} \;-\; c^2\,q^2}{\omega^2 \;-\; c^2\,q^2 \;+\; 2\,i\,\omega\,c^2\,k^2\,\tilde{\eta}} \qquad\qquad \chi^L = \Im m \; G^L_R$$

$$\tilde{\eta} \;=\; \eta_1 \;+\; \eta_2\,(\omega^2 \;-\; 2\,q^2)/4$$

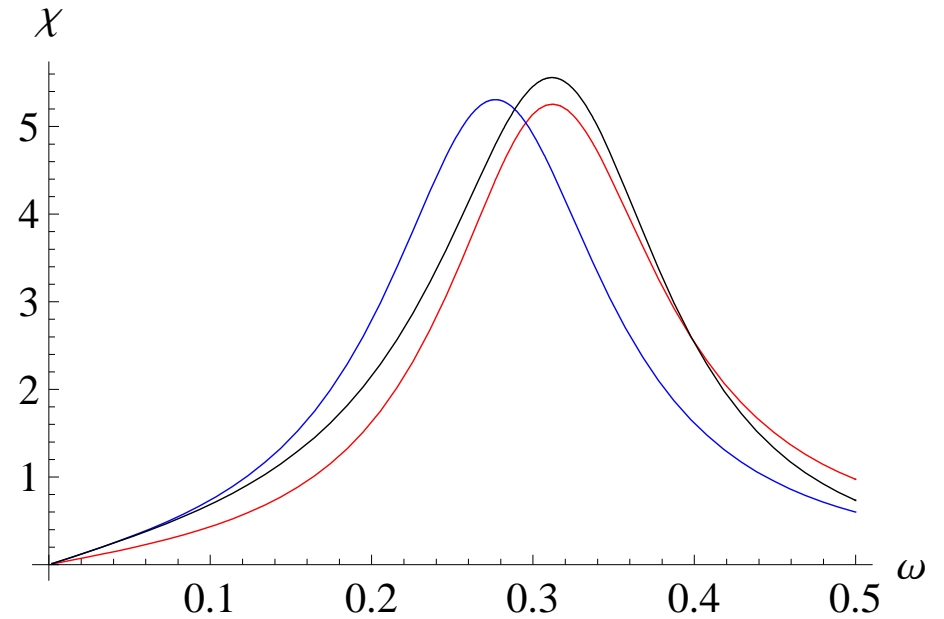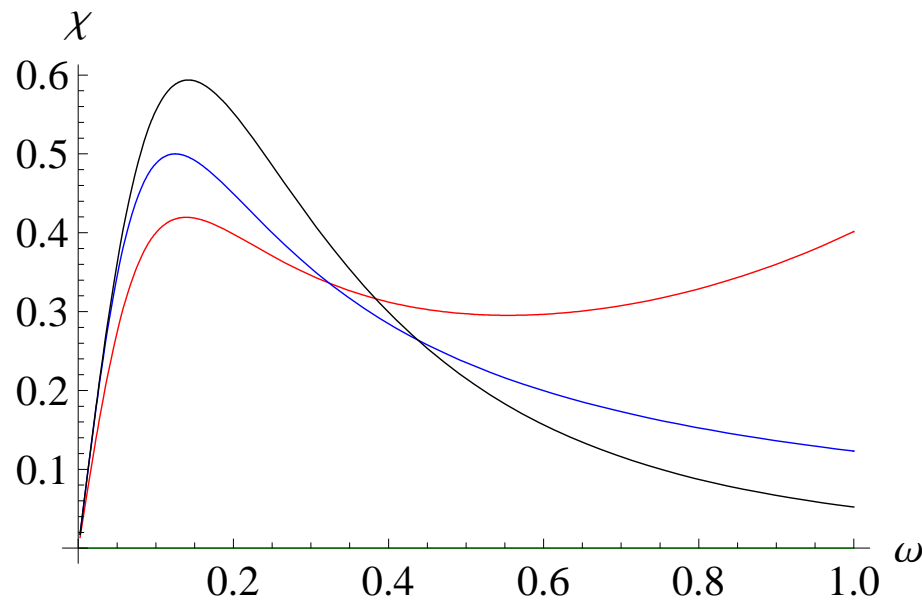In order to extract $\eta_{1,2}$ we have to invert this relations.
For that we need both imaginary and real parts of the correlators.

Poles of the correlators should reproduce the entire tower of quasi-normal modes
+ their dispersion relations.

Shear $q = 0.5$ **Sound**

red - AdS BH   P. Kovtun, A. Starinets, Phys.Rev.Lett.96:131601,2006

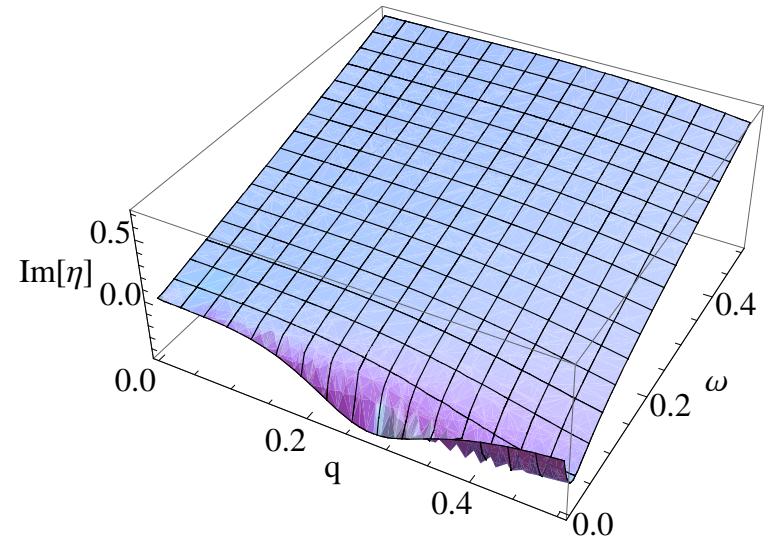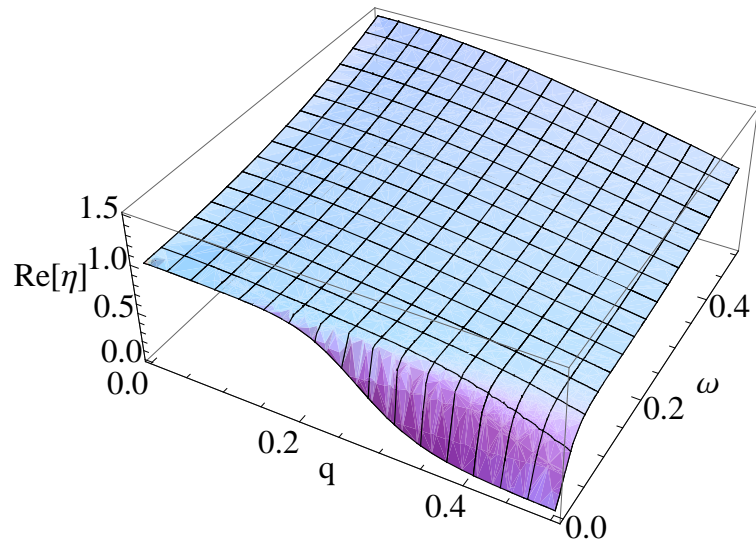blue - Navier Stokes   black - second order hydro.

Discussion point:
The spectral functions contain "non-thermal" vacuum physics, such as pair production.
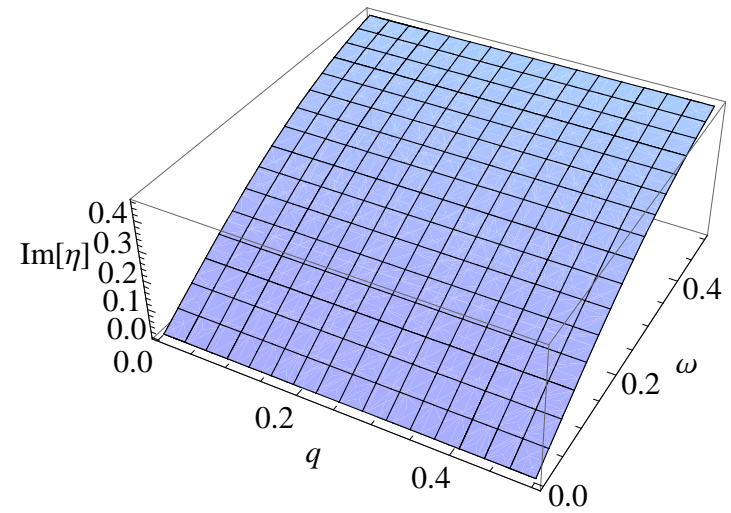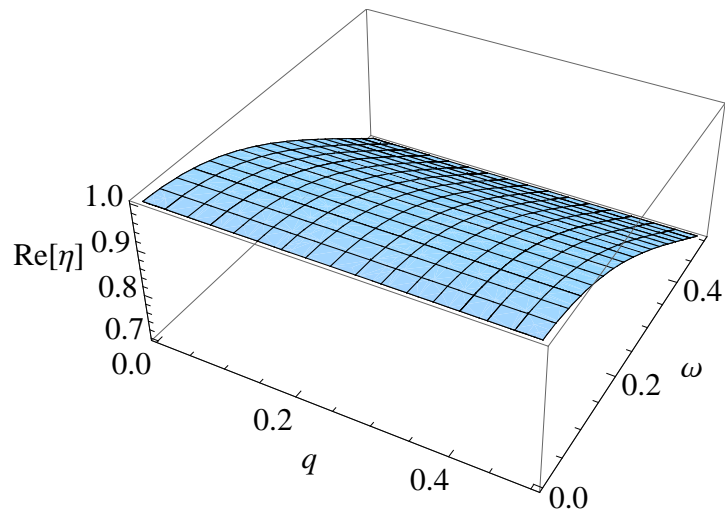Should this physics be removed when constructing hydro?
The "non-thermal" processes are real. They do occur in plasma.
Should we model them as an effective hydro?

$$\eta_1 \;=\; 1 + i\,\tau_R\,\omega + \kappa\,q^2 + \lambda\,w^2 \ldots \qquad\qquad \tau_R \;=\; 2 - \ln[2]\,, \qquad \kappa \;=\; -\,2\,, \qquad \lambda \;=\; 1 + \ln[2]$$

$$\bar{\eta} \;=\; 1 \,+\, i\,\tau_R\,\omega \,-\, \tau_R^2\,\omega^2 \ldots$$

Concluding Remarks

• **Higher order terms in the gradient expansion seem to be important at early times. Taking them into account is likely to reduce the dependence on the initial time of the evolution.**

• **The second order hydrodynamics does not agree with the all-order hydrodynamics from the AdS/CFT. This suggests that the second order hydro is potentially less trustable tool than it was previously thought.**