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Static potentials for finite temperature quarkonia

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in collaborations with O. Jahn, M. Laine, P. Romatschke, M. Tassler

- Continuation of Mikko Laine's talk yesterday
- I Critical assessment of static potentials used in potential models
- II Perturbative + non-perturbative aspects of new real-time potential

Motivation: the fate of quarkonia in the QCD plasma

"MEM is not reliable for reconstructing spectral functions at finite T"

Peter Petreczky, Non-Equilibrium Dynamics, KITP 08, slide 8

➡ also pursue other approaches, if only to improve default models for MEM

Here: potential models $\longleftrightarrow$ QFT ?

T=0: static potential related to effective field theory pNRQCD

expansion parameter $(E-2M)/M$; V = matching coefficient

➡ solve static Schrödinger eqn. with confining V (from the lattice)

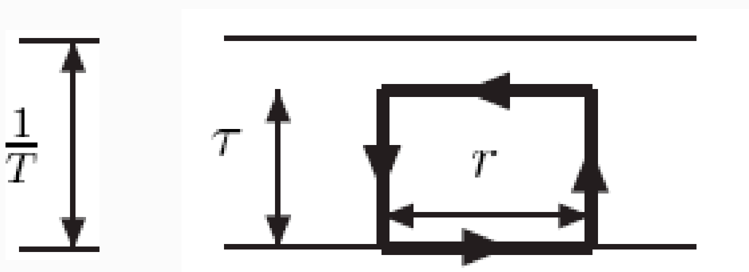
➡ very successful spectroscopy (~1%), search for hybrids, ...

finite T standard practice: argue for a 'finite T potential', proceed as above

The static potential at T=0: Wilson loop

- Euclidean correlator of gauge invariant meson operator
- integrate out quarks in the limit $M \to \infty$

$$\langle \bar{\psi}(\mathbf{x},\tau) U(\mathbf{x},\mathbf{y};\tau) \psi(\mathbf{y},\tau) \bar{\psi}(\mathbf{y},0) U^{\dagger}(\mathbf{x},\mathbf{y};0) \psi(\mathbf{x},0) \rangle \longrightarrow \mathrm{e}^{-2M\tau} W_E(|\mathbf{x}-\mathbf{y}|,\tau)$$



$U(\mathbf{x},\mathbf{y};0) \equiv U_r(0)$

$$r = |\mathbf{x}-\mathbf{y}|$$

- spectral decomposition, $T \to 0$ $\quad W_E(r,\tau \to \infty) \longrightarrow c_{01}^2 [U(\mathbf{x},\mathbf{y};0)]\, \mathrm{e}^{-V(r)\tau}$
- lattice generalization to finite T not clear, $N_\tau = \dfrac{1}{aT}$ on lattices short

Static 'potentials' at finite T, free energies: the Polyakov loop

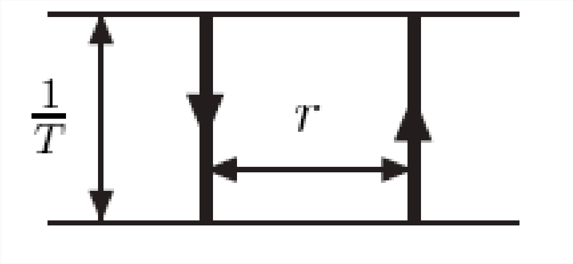
McLerran, Svetitsky PRD 81

- Static quarks propagate through periodic boundary
- finite T: sum over Boltzmann weighted excited states

➡ free energy of a static quark in a plasma: $\langle \mathrm{Tr}\, L_{\mathbf{x}} \rangle \sim \mathrm{e}^{-F_q/T}$

order parameter for confinement: $\langle L \rangle \begin{cases} = 0 \Leftrightarrow \text{confined phase,} & T < T_c \\ > 0 \Leftrightarrow \text{deconfined phase,} & T > T_c \end{cases}$

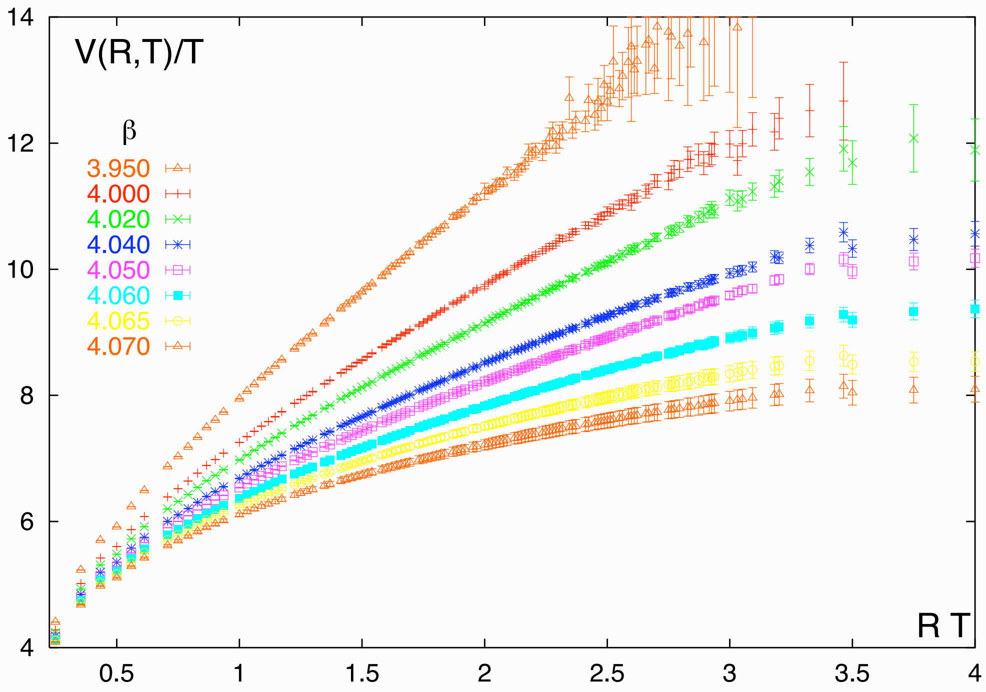
- free energy of static quark anti-quark pair:



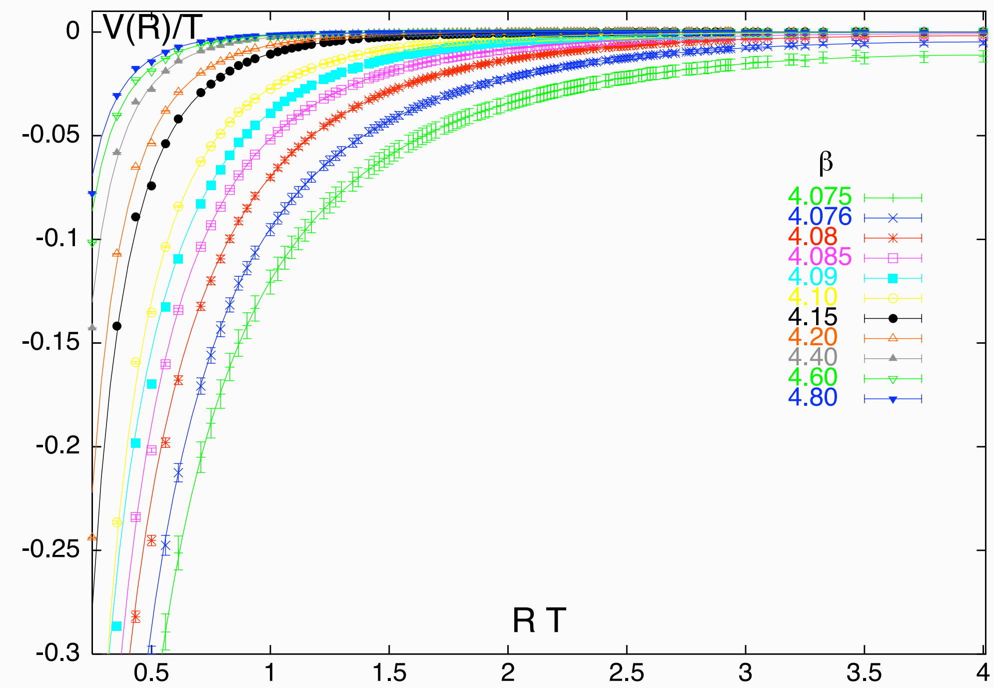
$$\mathrm{e}^{-F_{\bar{q}q}(r,T)/T} = \frac{1}{N^2} \langle \mathrm{Tr}\, L^\dagger(\mathbf{x})\, \mathrm{Tr}\, L(\mathbf{y}) \rangle, \quad r = |\mathbf{x} - \mathbf{y}|$$

The static quark free energy in the quenched limit

Bielefeld



$T < T_c$



$T > T_c$

eff. string: $$\frac{\sigma(T)}{\sigma(0)} = a\sqrt{1 - b\frac{T^2}{T_c^2}}$$

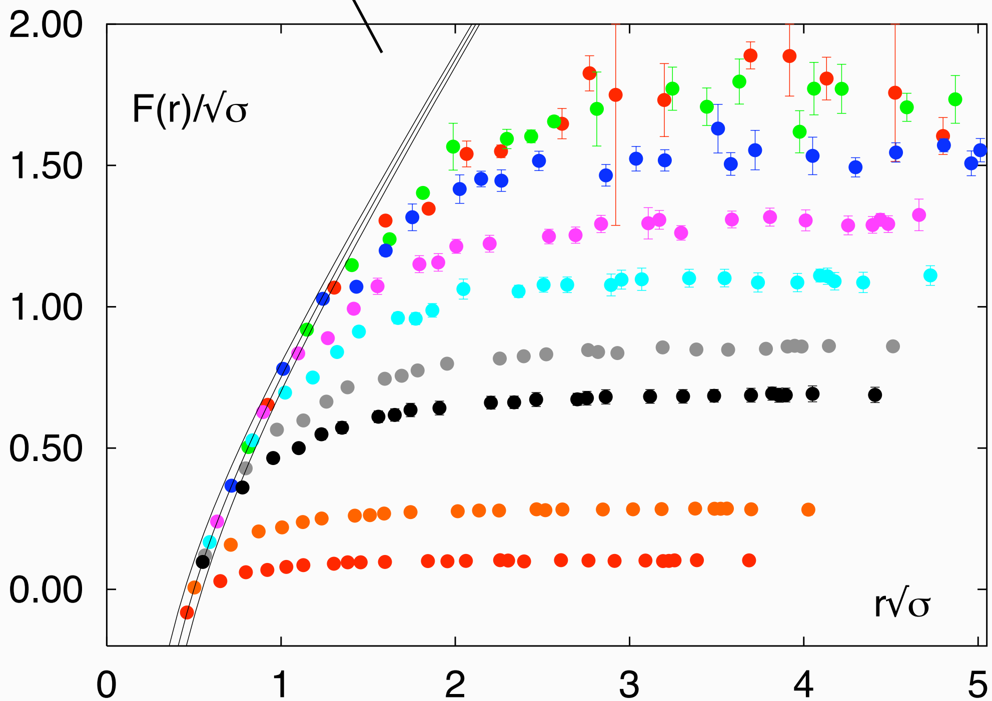
$$\frac{F_{q\bar{q}}(r,T)}{T} = -\frac{c(T)}{(rT)^d}\mathrm{e}^{-\mu(T)r}$$

The static quark free energy, dynamical

Bielefeld

with dynamical light quarks: $N_f = 3$

$V(r) = -\alpha/r + r\sigma$



$T = 0.58T_c$

$T = 1.15T_c$

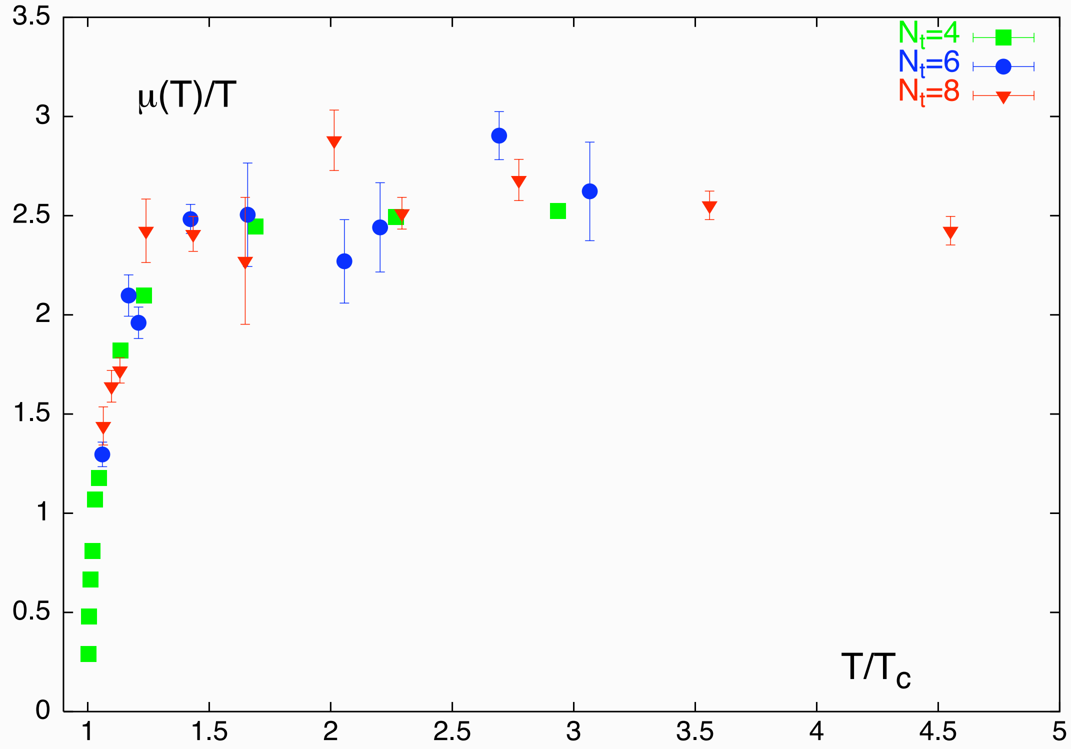
- screening of colour force at T=0 by dynamical fermions
- with increasing T screening by the plasma sets in

Screening of static quark free energy

Bielefeld

...in pure gauge theory

$$\frac{F_{q\bar{q}}(r,T)}{T} = -\frac{c(T)}{(rT)^d} \mathrm{e}^{-\mu(T) r}$$



numerically: $d \approx 3/2, \mu \approx M_{0_+^{++}}$ lightest gauge inv. screening mass

LO perturbation theory: $d = 2, \mu = 2m_D^0$ exchange of two A_0

➡ $F_{\bar{q}q}$ does not correspond to the Debye screened static potential!

Decomposition in different colour channels

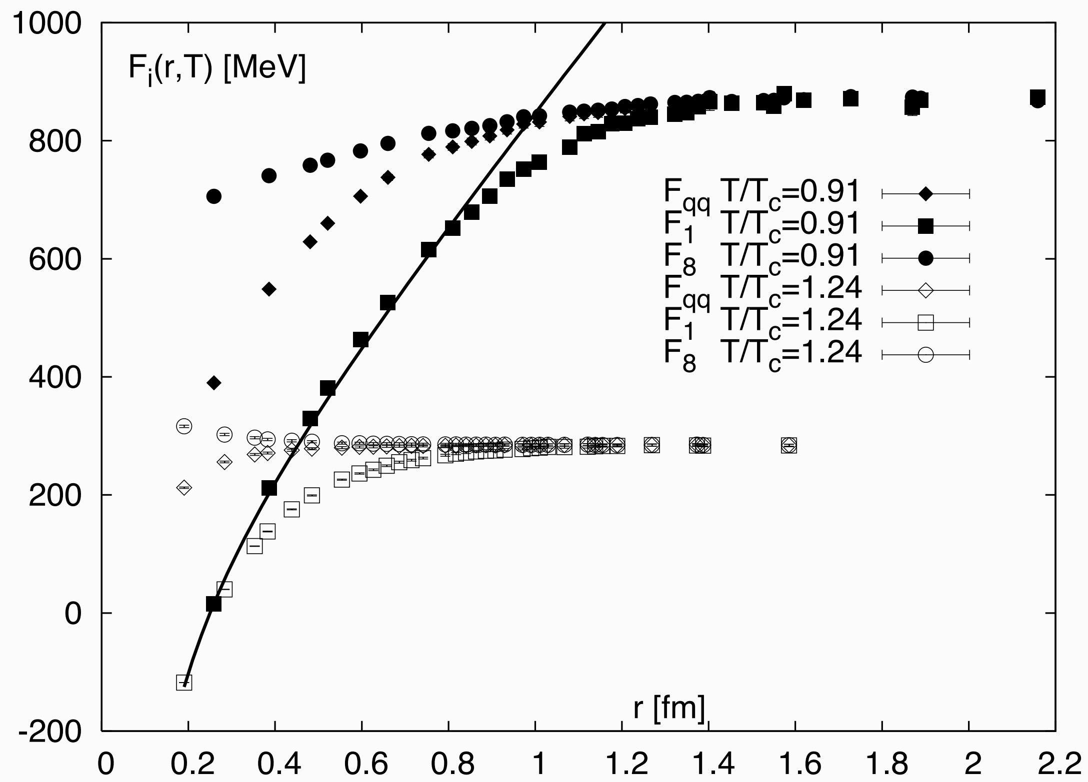
McLerran, Svetitsky PRD 81

$$
\begin{aligned}
e^{-F_{\bar{q}q}(r,T)/T} &= \frac{1}{N^2}\langle \mathrm{Tr}\, L^\dagger(\mathbf{x})\, \mathrm{Tr}\, L(\mathbf{y})\rangle = \frac{1}{N^2} e^{-F_1(r,T)/T} + \frac{N^2-1}{N^2} e^{-F_8(r,T)/T} \\
e^{-F_1(r,T)/T} &= \frac{1}{N}\langle \mathrm{Tr}\, L^\dagger(\mathbf{x}) L(\mathbf{y})\rangle, \\
e^{-F_8(r,T)/T} &= \frac{1}{N^2-1}\langle \mathrm{Tr}\, L^\dagger(\mathbf{x})\, \mathrm{Tr}\, L(\mathbf{y})\rangle - \frac{1}{N(N^2-1)}\langle \mathrm{Tr}\, L^\dagger(\mathbf{x}) L(\mathbf{y})\rangle.
\end{aligned}
$$

- correlators in ‘singlet’ and ‘octet’ channels gauge dependent
- $$F_1(r,T) \sim \frac{e^{-m_D(T) r}}{4\pi r}$$ Nadkarni PRD 86
- non-perturbative meaning?

Numerical results in Coulomb gauge

Kaczmarek, Zantow PRD 05



- \`Potentiology': solve Schödinger eq. in various potentials, check if solution allows to reconstruct lattice correlators

$$F_i, \quad U_i = F_i + TS_i \ldots$$

- different r-dependence results in different binding behaviour in Schrödinger eq.
- F_1, U_1 gauge artefact or physics?

Singlet and octet channels from gauge invariant correlators

start from meson operators: Jahn, O.P., PRD 05

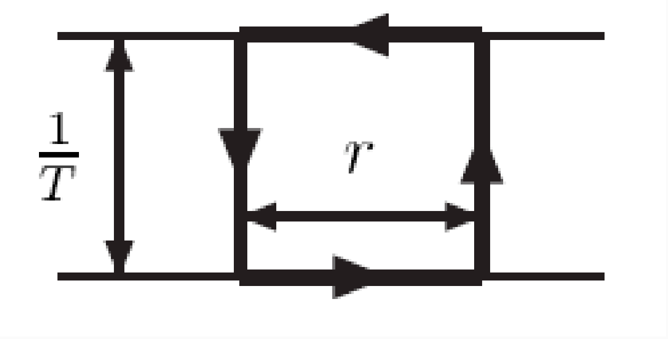
$$O(\mathbf{x},\mathbf{y}) = \bar{\psi}(\mathbf{x})U(\mathbf{x},\mathbf{y})\psi(\mathbf{y}), \quad O^a(\mathbf{x},\mathbf{y}) = \bar{\psi}(\mathbf{x})U(\mathbf{x},\mathbf{x}_0)T^a U(\mathbf{x}_0,\mathbf{y})\psi(\mathbf{y}),$$

integrate out static quarks:

$$\begin{aligned}
\langle O(\mathbf{x},\mathbf{y};0)O^\dagger(\mathbf{x},\mathbf{y};N_t)\rangle &\propto \langle \mathrm{Tr}\, L^\dagger(\mathbf{x})U(\mathbf{x},\mathbf{y};0)L(\mathbf{y})U^\dagger(\mathbf{x},\mathbf{y};N_t)\rangle \\
\langle O^a(\mathbf{x},\mathbf{y};0)O^{a\dagger}(\mathbf{x},\mathbf{y};N_t)\rangle &\propto \frac{1}{N^2-1}\langle \mathrm{Tr}\, L^\dagger(\mathbf{x})\, \mathrm{Tr}\, L(\mathbf{y})\rangle \\
&\quad -\frac{1}{N(N^2-1)}\langle \mathrm{Tr}\, L^\dagger(\mathbf{x})U(\mathbf{x},\mathbf{y};0)L(\mathbf{y})U^\dagger(\mathbf{x},\mathbf{y};N_t)\rangle
\end{aligned}$$

identical to gauge fixed correlators in axial gauge

singlet channel: periodic Wilson loop



Spectral analysis of Polyakov loop correlators

Jahn, O.P., PRD 05

$\hat{T}_0 = \mathrm{e}^{-a\hat{H}_0}$ with Kogut-Susskind Hamiltonian in temporal gauge

$$\frac{1}{N^2}\langle \mathrm{Tr}\, L^\dagger(\mathbf{x})\, \mathrm{Tr}\, L(\mathbf{y})\rangle = \frac{1}{ZN^4}\hat{\mathrm{Tr}}\,[\hat{T}_0^{N_t}\;\hat{P}^{\mathrm{F}\otimes\bar{\mathrm{F}}}] = \frac{1}{ZN^2}\sum_n \langle n_{\alpha\beta}|n_{\beta\alpha}\rangle\;\mathrm{e}^{-E_n/T}$$

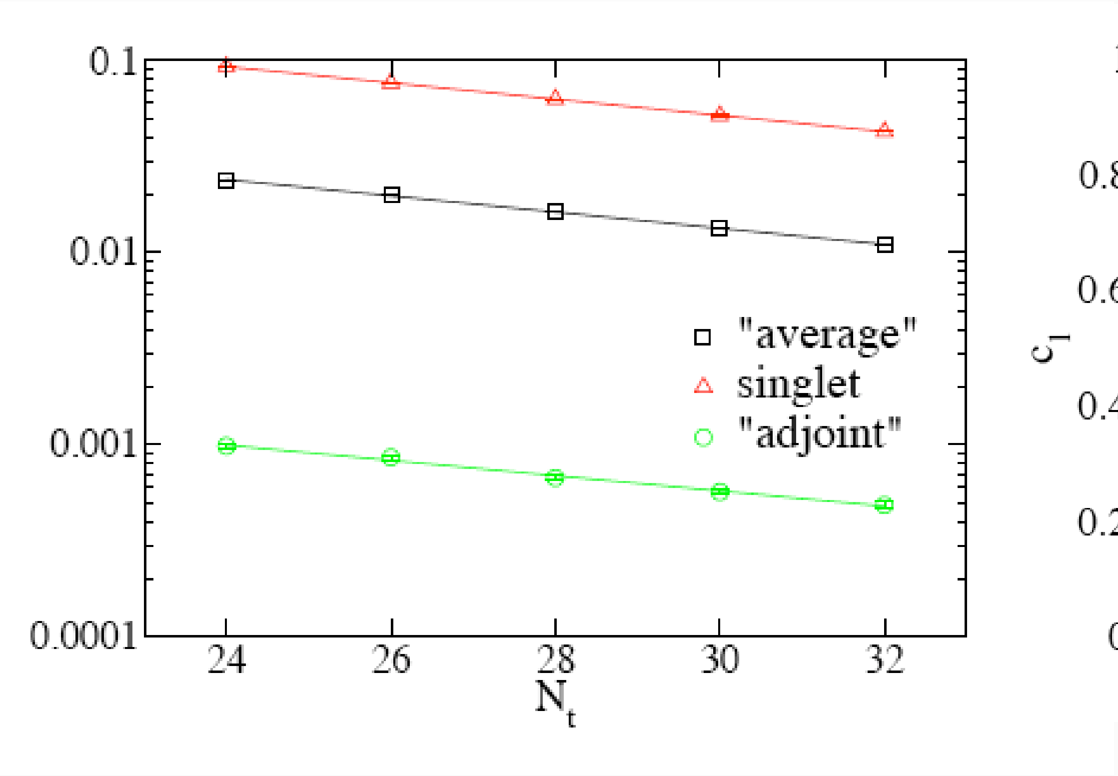
$$\langle O(\mathbf{x},\mathbf{y};0)O^\dagger(\mathbf{x},\mathbf{y};N_t)\rangle \propto \frac{1}{ZN^2}\sum_n \langle n_{\delta\gamma}|\hat{U}_{\gamma\delta}(\mathbf{x},\mathbf{y})\hat{U}^\dagger_{\alpha\beta}(\mathbf{x},\mathbf{y})|n_{\beta\alpha}\rangle\;\mathrm{e}^{-E_n/T}$$

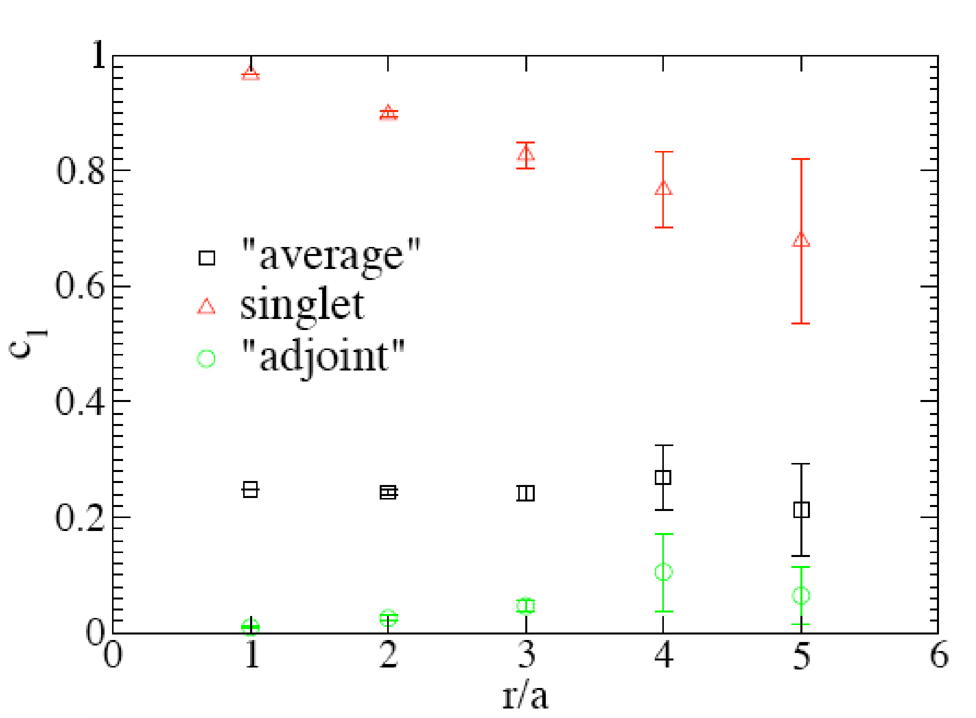
$$\langle O^a(\mathbf{x},\mathbf{y};0)O^{\dagger a}(\mathbf{x},\mathbf{y};N_t)\rangle \propto \frac{1}{ZN^2}\sum_n \langle n_{\delta\gamma}|\hat{U}^a_{\gamma\delta}(\mathbf{x},\mathbf{y})\hat{U}^{\dagger a}_{\alpha\beta}(\mathbf{x},\mathbf{y})|n_{\beta\alpha}\rangle\;\mathrm{e}^{-E_n/T}$$

- energy eigenvalues: usual (T=0) colour singlet potential in all three channels
- non-vanishing matrix elements in singlet and octet channel
- matrix elements path/gauge dependent

Numerical demonstration in 3d SU(2), T=0 limit

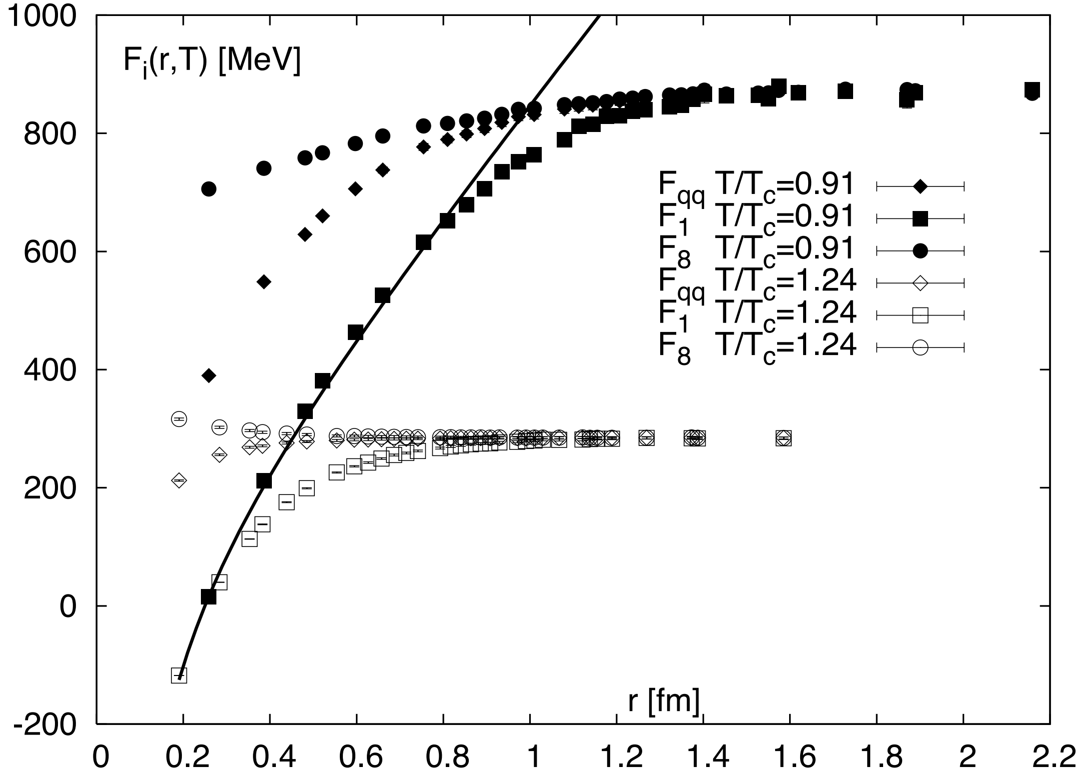
allows to isolate ground state energy and matrix element for each channel





correlators, r/a=1

matrix elements



- difference between $F_1, F_{\bar{q}q}$ is exclusively in gauge fixing function!
- binding energies from F_1, U_1 artefacts ?!

A real time static potential for finite T quarkonia

Laine, O.P., Romatschke, Tassler JHEP 07

- generalize effective theory approach to finite T
  additional scales $\pi T, gT, g^2T$, ...
- cf. Mikko Laine's talk:
  finite T correlator $C_>(t,\mathbf{r})$, related to quarkonium spectral function

$$\left\{ i\partial_t - \left[2M + V_>(t,r) - \frac{\nabla_{\mathbf{r}}^2}{M} + O\left(\frac{1}{M^2}\right) \right] \right\} C_>(t,\mathbf{r}) = 0$$

potential $\equiv$ coefficient scaling as $O(M^0)$ in t-derivative of correlator

infinitely heavy quarks: $C_>(t,\mathbf{r}) \propto W_E(it,\mathbf{r})$

$$i\partial_t W_E(it,\mathbf{r}) = V_>(t,r) W_E(it,\mathbf{r})$$

perturbative solution with HTL resummation, large time limit:

(non-relativistic: $E \ll p \quad \leftrightarrow \quad t \gg r$)

$$V_>(\infty,r) = -\frac{g^2 C_F}{4\pi}\left[m_\mathrm{D} + \frac{\exp(-m_\mathrm{D} r)}{r}\right] - \frac{i g^2 T C_F}{4\pi}\,\phi(m_\mathrm{D} r)\ ,$$

$$\text{with} \qquad \phi(x) = 2\int_0^\infty \frac{\mathrm{d}z\, z}{(z^2+1)^2}\left[1 - \frac{\sin(zx)}{zx}\right]$$

- real part familiar, Debye screening
- imaginary part due to Landau damping

Non-perturbative effects?

- Wilson loop in Minkowski time not calculable on the lattice
- scales in perturbative expansion:
  vertices g^2T, cut-off Λ, 3d confinement scale m~ g^2T
- HTL resummation accounts for effect of hard modes on soft modes,
  perturbative corrections ~ g^2T/Λ
- non-perturbative corrections from infrared modes ~ g^2T/m

➡ test for non-pert. infrared corrections via classical lattice simulations !

cf. sphaleron rate in e.-w. theory, plasma instabilities...

Classical limit in perturbation theory

$$V_>(\infty, r) = -\frac{g^2 C_F}{4\pi}\left[m_{\rm D} + \frac{\exp(-m_{\rm D} r)}{r}\right] - \frac{i g^2 T C_F}{4\pi}\,\phi(m_{\rm D} r)$$

re-instate factors of $\hbar$ in perturbative calculation: $g^2 \to g^2 \hbar, \quad \frac{1}{T} \to \frac{\hbar}{T}$

$$\lim_{\hbar \to 0} V_>(\infty, r) = -\frac{i g^2 T C_F}{4\pi}\phi(m_D r)$$

➡ only the imaginary part survives in the classical limit !

➡ test for non-perturbative effects in this sector

Imaginary part from classical lattice simulations

Laine, O.P., Tassler, JHEP 07

- Hamiltonian approach: t continuous, discretize on 3d lattice
- temporal gauge; conjugate fields ➡ $U_i(\mathbf{x},t), \quad \dot{U}_i(\mathbf{x},t) = iE_i(\mathbf{x},t)U_i(\mathbf{x},t)$
- Gauss constraint:

$$G(x) \equiv \sum_i \left[E_i(x) - U_{-i}(x)E_i(x-\hat{i})U^\dagger_{-i}(x)\right] - j^0(x) = 0$$

$$Z = \int \mathcal{D}U_i\, \mathcal{D}E_i\, \delta(G) e^{-\beta H}, \quad \beta = \frac{2N}{g^2 T a}, \quad H = \frac{1}{N}\sum_x \left[\sum_{i<j} \mathrm{Re}\,\mathrm{Tr}\,(1 - U_{ij}) + \frac{1}{2}\mathrm{Tr}\,(E_i^2)\right]$$

classical real time evolution:

$$\dot{U}_i(x) = iE_i(x)U_i(x)\ , \quad E_i = \sum_a E_i^a T^a\ , \quad \dot{E}_i^a(x) = -2\,\mathrm{Im}\,\mathrm{Tr}\,[T^a \sum_{|j|\neq i} U_{ij}(x)]$$

Quantum treatment of UV modes: HTL effective theory

Bödeker, Moore, Rummukainen PRD 00

- HTL: effective theory for soft modes by integrating out hard modes

  contains correct UV behaviour due to hard particles, take classical limit

- localize eff. action, rewriting in terms of additional on-shell particles, $W(x,v)$

$$\delta H = \frac{1}{N}\sum_x \int \frac{\mathrm{d}\Omega_v}{4\pi}\,\frac{1}{2}(am_{\rm D})^2 \mathrm{Tr}\,(W^2)$$

- discretize W's by platonic solids or spherical harmonics

- modified classical time evolution:

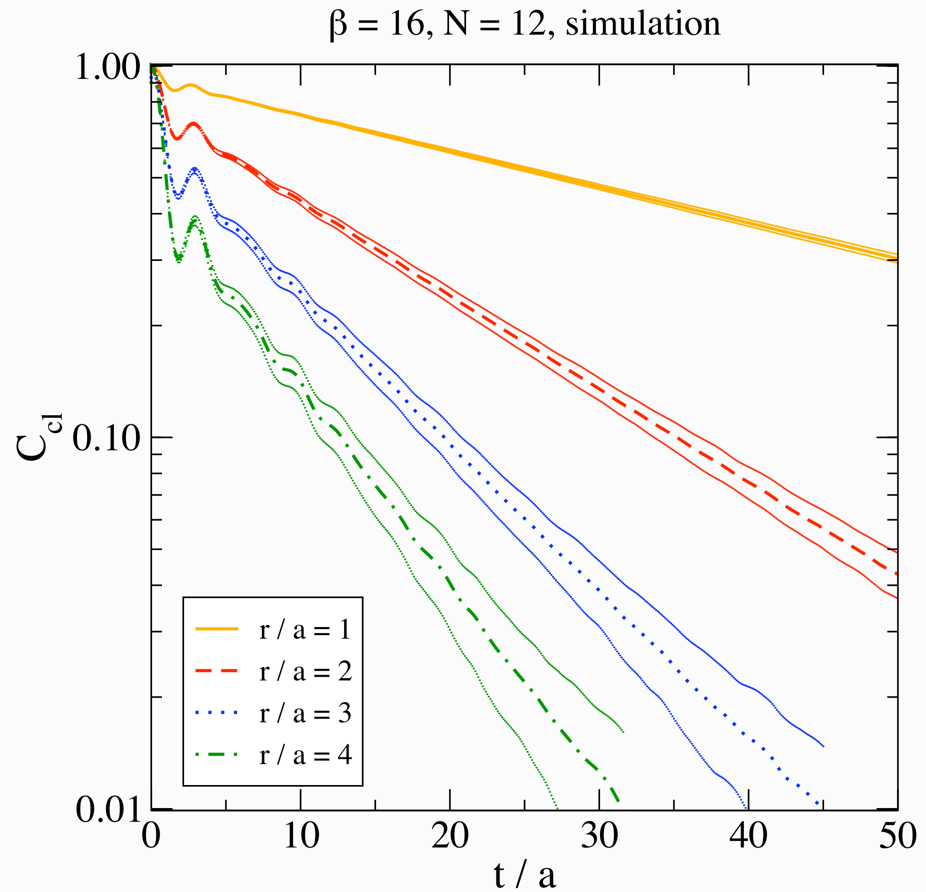
$$j^\mu(x) = (am_{\rm D})^2 \frac{1}{N_p}\sum_{n=1}^{N_p} v_n^\mu W_n(x)\;, \qquad W_n(x) \equiv W(x, v_n)$$

+ evolution eqn. for $\dot{W}_n(x) = \ldots$

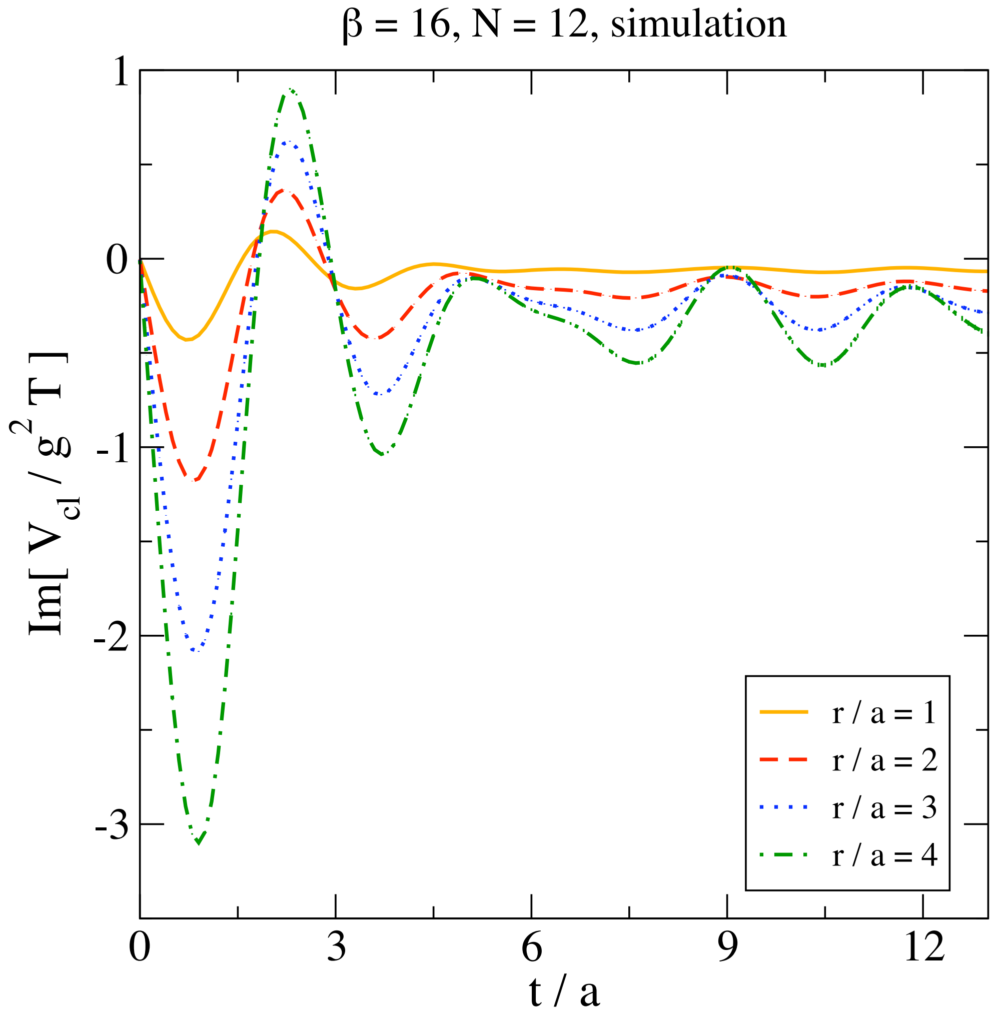
Algorithm for the simulation:

- ‘almost thermal’ gauge fields by Monte Carlo from 3d eff. theory
- generate electric field from Gaussian distribution
- project onto the sub-space satisfying Gauss’ law
- evolve the fields using the equation of motion until thermalized
- switch on measurements:

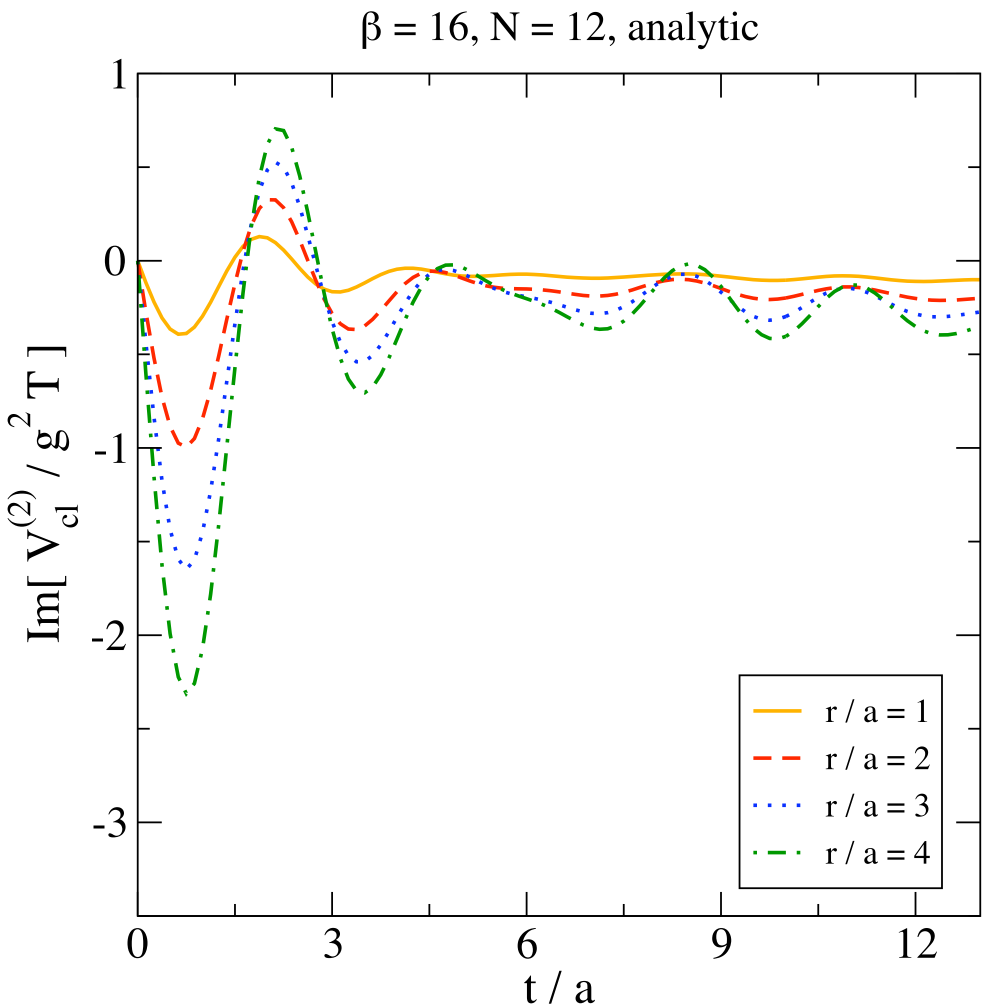
$$C_{\rm cl}(t,r) = \frac{1}{N}\mathrm{Tr}\,\left\langle U_r^\dagger(t) U_r(0)\right\rangle$$



$$V_{\rm cl}(t,r) = \frac{i\partial_t C_{\rm cl}(t,r)}{C_{\rm cl}(t,r)}$$



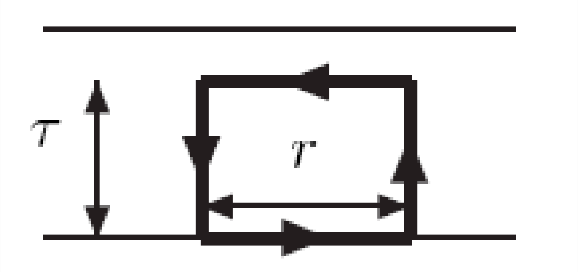
purely classical simulation



classical lattice perturbation theory

non-perturbative strengthening of damping

What is needed for the real part?



$$W_E(\tau = it, r)$$

within potential approach, do not need full time dependence,

'only' $$\lim_{t \to \infty} W_E(\tau = it, r)$$

Conclusions

- many potential models for finite T quarkonia field-theoretically questionable
- reformulation of potential approach in terms of effective QFT

  → new real-time static potential

- solution possible in HTL-resummed perturbation theory

  → Debye screening and Landau damping (Im V)

- Im V calculable non-perturbatively in classical lattice simulations; comparison: HTL pert. theory captures qualitative physics
- Still needed: non-pert. check for Re V