Neutrino Clouds

Instabilities and speeded-up flavor equilibration in neutrino clouds

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Domain of Application:

Neutrino clouds with # densities of about (7 MeV)^3

(but with non-thermal distributions)

just under "neutrino-surface" in SN.

Phenomena:

1. Rapid exchange of ν flavors

2. Consequent hardening of ν_e spectrum

……softening of $\nu_{\mu,\tau}$ spectra.

Mechanics:

Instabilities in (mean-field) non-linear evolution equations.

Bonus:

Beyond the mean-field……..

comparison of stable and unstable

Time scales:

Very Fast::

$$\Gamma_F = G_F \, n_e \sim [10^{-2} \text{ cm}]^{-1}$$

RFS 2004,2008

Medium fast:

$$\Gamma_{\text{med}} = \sqrt{\Gamma_F \, \Gamma_{\text{osc}}} \sim [10^{2} \text{ cm}]^{-1}$$

Raffelt et al 2006
2007
? Fuller talk & refs
RFS 2004
2005

Oscillation:

$$\Gamma_{\text{osc}} = \frac{\delta \, m^2}{p_\nu} \sim [10^{6} \text{ cm}]^{-1}$$

ν – ν Interactions:

Density operators:

$$\rho_{i,j}(\mathbf{p}) = a_i(\mathbf{p})^\dagger \, a_j(\mathbf{p})$$

$$\bar{\rho}_{i,j}(\mathbf{p}) = \bar{a}_j(\mathbf{p})^\dagger \, \bar{a}_i(\mathbf{p})$$

Forward Hamiltonian:

Angle dependence is key

$$H_{\nu\nu}(\rho) = \frac{\sqrt{2}G_F}{V} \sum_{\mathbf{p},\mathbf{q}} \sum_{\{i,j\}=e,x} [1 - \cos(\theta_{\mathbf{p},\mathbf{q}})] \times \Big[\Big(\rho_{i,j}(\mathbf{p}) - \bar{\rho}_{i,j}(\mathbf{p}) \Big) \Big(\rho_{j,i}(\mathbf{q}) - \bar{\rho}_{j,i}(\mathbf{q}) \Big)$$
$$+ \Big(\rho_{i,i}(\mathbf{p}) - \bar{\rho}_{i,i}(\mathbf{p}) \Big) \Big(\rho_{j,j}(\mathbf{q}) - \bar{\rho}_{j,j}(\mathbf{q}) \Big) \Big]$$

This term doesn't contribute anything.

Sum is over states, p, q that are occupied by ν's of some flavor.

Commutation rules:

$$[\rho_{i,j}(\mathbf{p}), \rho_{k,l}(\mathbf{p}')] = [\delta_{i,l}\rho_{k,j}(\mathbf{p}) - \delta_{j,k}\rho_{i,l}(\mathbf{p})]\delta_{\mathbf{p},\mathbf{p}'}$$

$$[\bar{\rho}_{i,j}(\mathbf{p}), \bar{\rho}_{k,l}(\mathbf{p}')] = [-\delta_{i,l}\bar{\rho}_{k,j}(\mathbf{p}) + \delta_{j,k}\bar{\rho}_{i,l}(\mathbf{p})]\delta_{\mathbf{p},\mathbf{p}'}$$

Equations of motion:

$$i\frac{d}{dt}\rho_{i,j}(\mathbf{p}) = \frac{-\sqrt{2}G_F}{V}\sum_{\mathbf{q}}\sum_k \Big[\rho_{i,k}(\mathbf{p})[\rho_{k,j}(\mathbf{q}) - \bar{\rho}_{k,j}(\mathbf{q})]$$
$$-\rho_{j,k}(\mathbf{p})[\rho_{i,k}(\mathbf{q}) - \bar{\rho}_{i,k}(\mathbf{q})]\Big][1-\cos(\theta_{\mathbf{p},\mathbf{q}})]$$
$$+|\mathbf{p}|^{-1}[\Lambda, \rho(\mathbf{p})]_{i,j} \ ,$$

oscillation term

Mean field approximation:

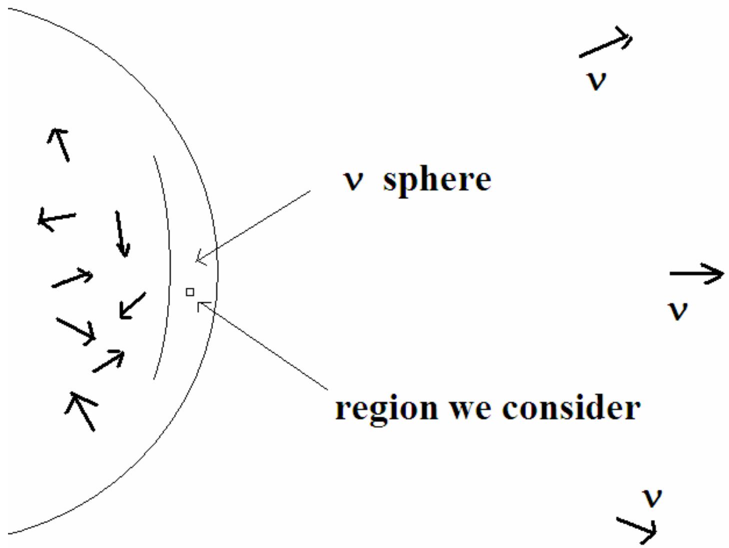
$$\langle \rho_{i,j}(p)\rho_{k,l}(p')\rangle = \langle \rho_{i,j}(p)\rangle\langle \rho_{k,l}(p')\rangle$$

Pastor and Raffelt (2002)

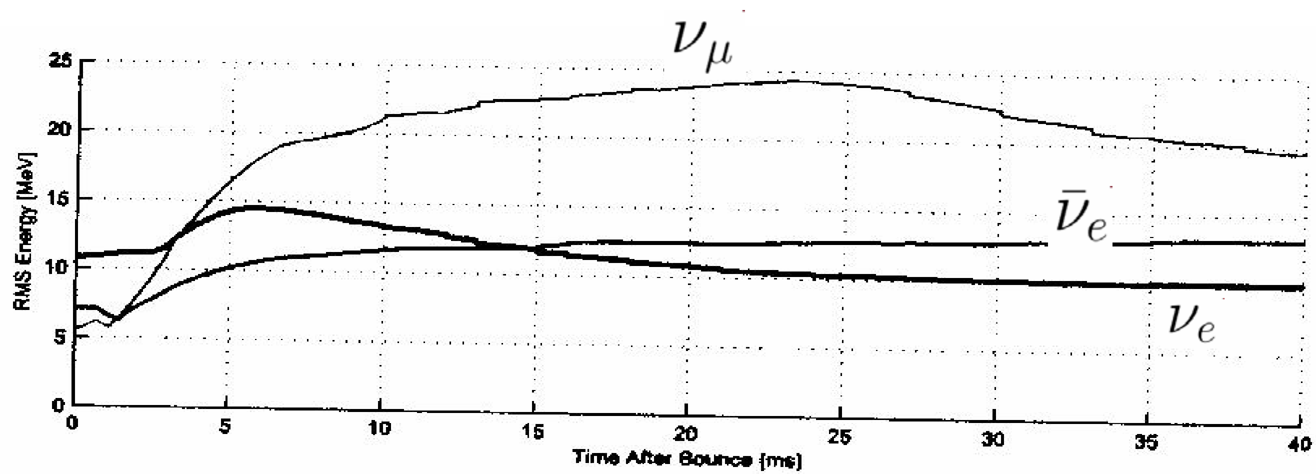
Take flavor diagonal initial conditions:

$$\rho_{e,e} \neq 0 \quad , \quad \rho_{x,x} \neq 0 \quad , \quad \rho_{x,e} = \rho_{e,x} = 0$$

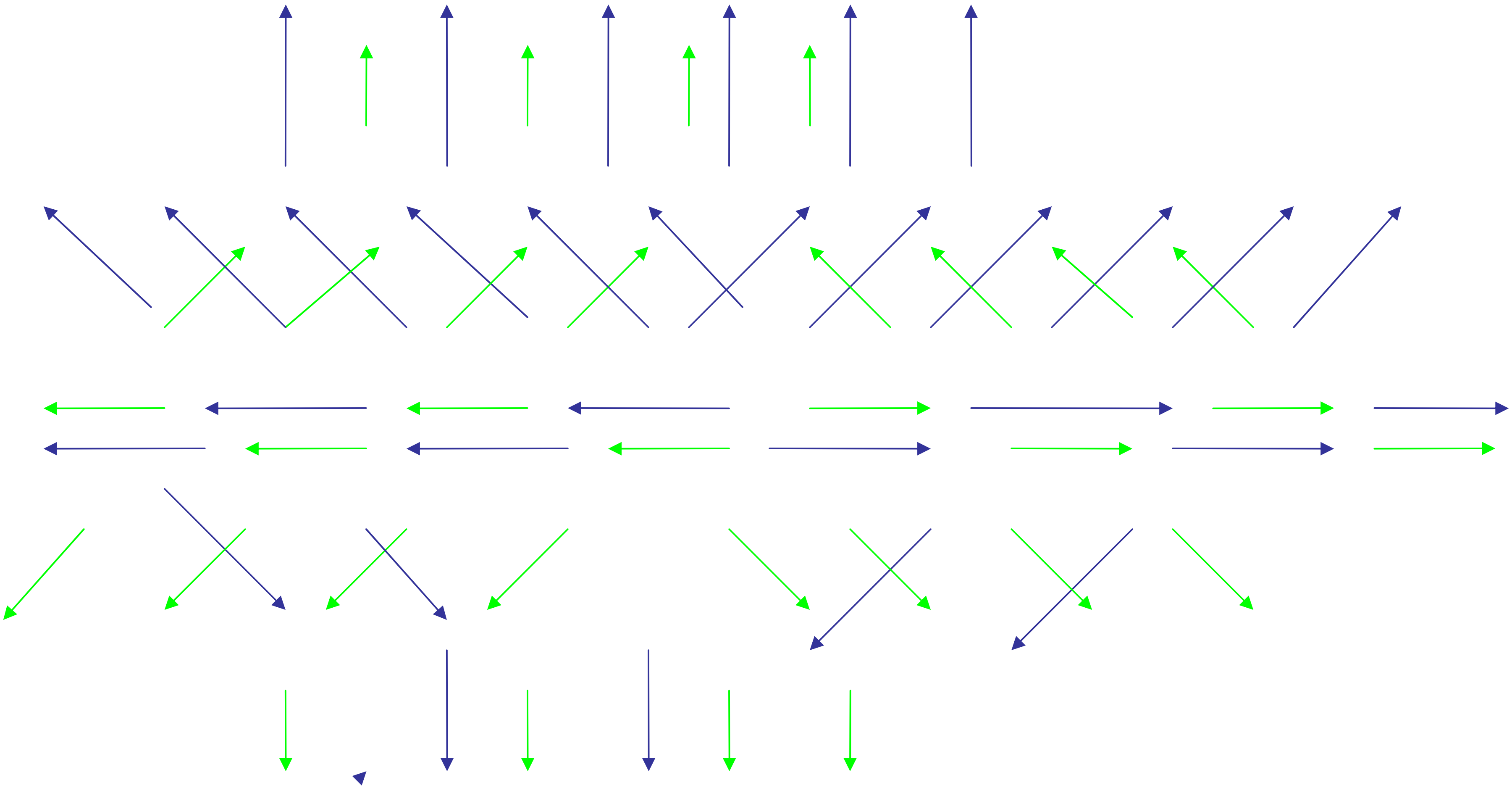
and solve



ν
ν sphere
ν
region we consider
ν



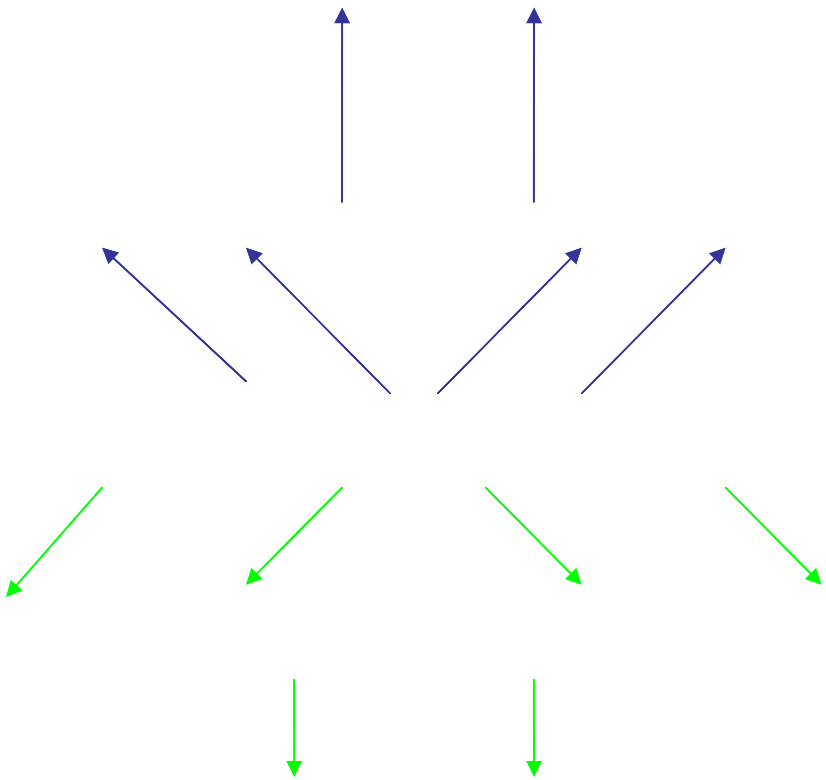
ν_μ
$\bar{\nu}_e$
ν_e
RMS Energy [MeV]
Time After Bounce [ms]
0
5
10
15
20
25
30
35
40



Momentum distribution ν_μ , ν_e near ν -sphere

We can delete ν_μ , ν_e when paired in angle.

So, in effect,



Two beams------N up , N down

For the up-moving states define,

$$\sigma_i^{(3)} = \rho_{e,e}(p_i) - \rho_{x,x}(p_i) \quad ; \quad \sigma_i^{(+)} = \rho_{e,x}(p_i) \quad ; \quad \sigma_i^{(-)} = \rho_{x,e}(p_i)$$

For the down-moving states similarly,

$$\tau_i^{(3)} \ , \ \tau_i^{(+)} \ , \ \tau_i^{(-)}$$

Collective coordinates

$$S^{(3)} = \sum_i \sigma_i^{(3)} \ , \ S^{(+)} = \sum_i \sigma_i^{(+)} \ , \ T^{(3)} = \sum_i \tau_i^{(3)} \ , \ T^{(+)} = \sum_i \tau_i^{(+)}$$

Hamiltonian

$$H = G[\,2\,S^{(+)}T^{(-)} + 2\,S^{(-)}T^{(+)} + \zeta\,S^{(3)}\,T^{(3)}\,]$$

ζ =1 neutrinos

ζ =0 photons

E of M

$$i\frac{d}{dt}S^{(+)} = G[\,T^{(+)}\,S^{(3)} - \zeta\,S^{(+)}\,T^{(3)}\,]$$
$$i\frac{d}{dt}T^{(+)} = G[\,S^{(+)}\,T^{(3)} - \zeta\,T^{(+)}\,S^{(3)}\,]$$

$$i\frac{d}{dt}S^{(3)} = G[\,S^{(+)}\,T^{(-)} - \zeta\,S^{(-)}\,T^{(+)}\,]$$
$$i\frac{d}{dt}T^{(3)} = G[\,S^{(-)}\,T^{(+)} - \zeta\,S^{(+)}\,T^{(-)}\,]$$

Initials:

$$S^{(3)}(t=0) = N \ , \ T^{(3)}(t=0) = -N$$

$$S^{(\pm)}(t=0) = 0 \ , \ T^{(\pm)}(t=0) = 0$$

Photon-photon scat:

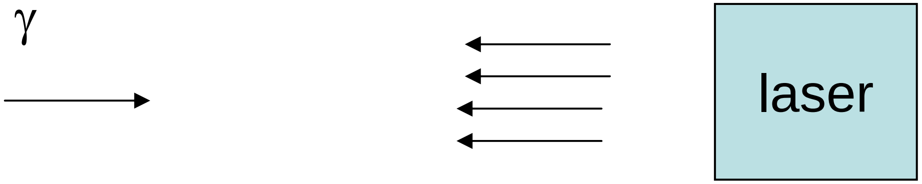
$$L_I = \int d^3x \frac{2\alpha^2}{45m^4}[(\mathbf{E^2} - \mathbf{B^2})^\mathbf{2} + \mathbf{7}(\mathbf{E}\cdot\mathbf{B})^\mathbf{2}]$$

(polarizations now take the place of flavors and Heisenberg-Euler replaces Z-exchange.)

G. L. Kotkin and V. G. Serbo, Phys. Lett. **B413**,122 (1997)

Laser: 2.35 eV, $E/E_{\rm crit} \approx 1.5 \times 10^{-6}$

100 MeV γ



Both beams linearly polarized.

Angle between polarizations not = $n\pi/2$

Mean distance for scattering of the photon –from cross-section and laser beam density -- 10^9 cm.

Question: What is distance for polarization exchange?

Answer: 3 cm. (Kotkin and Serbo)

Colliding photon clouds

Now with one cloud unpolarized and the other polarized:
The polarized cloud loses polarization in distance 3 log[N] cm.

RFS Phys.Rev.Lett. 93 (2004) 133601

Two beams.

For the up-moving states define,

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For the down-moving states similarly,

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Collective coordinates

$$S^{(3)} = \sum_i \sigma_i^{(3)} \quad , \quad S^{(+)} = \sum_i \sigma_i^{(+)} \quad , \quad T^{(3)} = \sum_i \tau_i^{(3)} \quad , \quad T^{(+)} = \sum_i \tau_i^{(+)}$$

Hamiltonian

$$H = G[\,2\,S^{(+)}T^{(-)} + 2\,S^{(-)}T^{(+)} \,+\, \zeta\, S^{(3)}\, T^{(3)}\,]$$

ζ =1 neutrinos

ζ =0 photons

E of M

$$i\frac{d}{dt}\,S^{(+)} = G[\,T^{(+)}\,S^{(3)} - \zeta\,S^{(+)}\,T^{(3)}\,]$$
$$i\frac{d}{dt}\,T^{(+)} = G[\,S^{(+)}\,T^{(3)} - \zeta\,T^{(+)}\,S^{(3)}\,]$$

$$i\frac{d}{dt}\,S^{(3)} = G[\,S^{(+)}\,T^{(-)} - \zeta\,S^{(-)}\,T^{(+)}\,]$$
$$i\frac{d}{dt}\,T^{(3)} = G[\,S^{(-)}\,T^{(+)} - \zeta\,S^{(+)}\,T^{(-)}\,]$$

Initials:

$$S^{(3)}(t=0) \;=N \quad , \quad T^{(3)}(t=0) \;= -N$$

$$S^{(\pm)}(t=0) \;=0 \quad , \quad T^{(\pm)}(t=0) \;=0$$

With these initial conditions (and in mean field):

Nothing happens.

But when ζ=0 there is an instability for rapid growth of a perturbation.

Eigenvalues of linearized problem:

$$\lambda = \pm G\, N \sqrt{\zeta^2 - 1}$$

Growth $e^{i\lambda t}$

So - in the photon problem $\zeta=0$ we would get mixing time scale, $t_{\text{mix}} \sim (NG)^{-1} = \Gamma_F^{-1}$

if $\langle S^{(+)}(0) \rangle \neq 0$ no matter how tiny

In ν problem ζ=1 --- no fast mixing here, but with some ν osc. terms (inverted hierarchy)

get:

$$\text{Im}\,\lambda = \sqrt{\Gamma_F\, \Gamma_{\text{osc}}}$$

"medium fast"

Beyond the mean field:

With no oscillation term or no Initial tilt------- the MF equations say that nothing happens.

But we can estimate in PT a flavor mixing time

$$t_{\text{mix}} \sim G^{-1} N^{-1/2}$$

Or, we can just solve the system

Case A: ζ=1 --- stable (neutrinos)

$$t_{\text{mix}} \sim G^{-1} N^{-1/2}$$

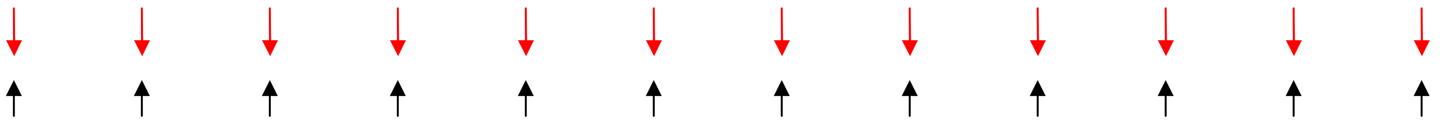
(again)

Friedland & Lunardini (2003)

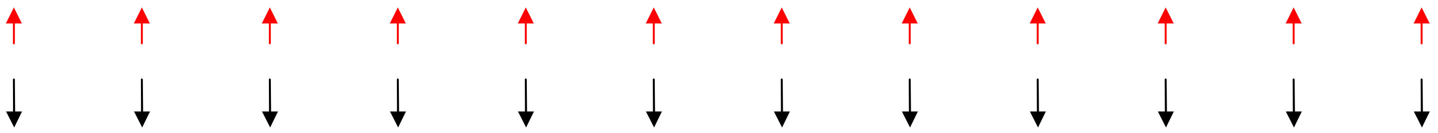
Case B: ζ=0 ---- unstable (photons)

$$t_{\text{mix}} \sim (NG)^{-1} \log N \to (G n_\nu)^{-1} \log N$$

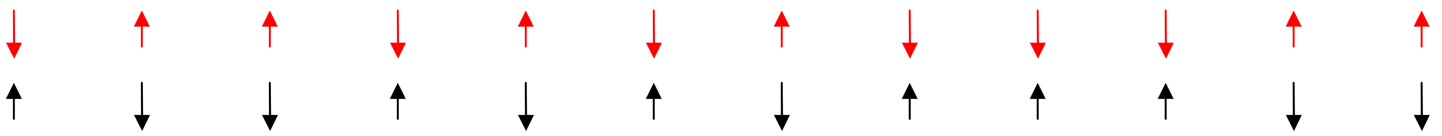
Spin system: How long for this:



to go into this?



or this?

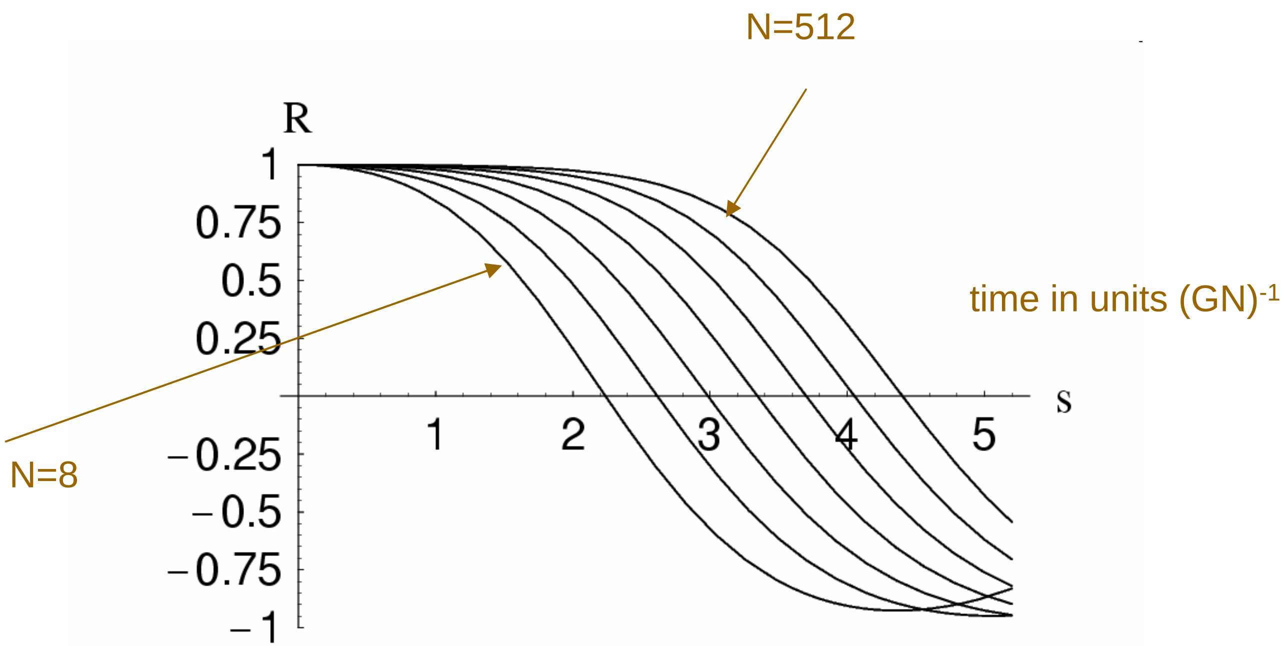


under the influence of

$$H_1 = g \sum_{i,j} [\sigma_i^+ \sigma_j^- + \sigma_i^+ \sigma_j^-]$$

or

$$H_2 = g \sum_{i,j} [\sigma_i^+ + \sigma_i^+][\sigma_j^- + \sigma_j^-]$$


N=512
R
1
0.75
0.5
0.25
time in units (GN)-1
s
1
2
3
4
5
-0.25
N=8
-0.5
-0.75
-1

We defined MF approximation by taking operators, O:

$$R^{(+)} \; , \; S^{(+)} \; , \; R^{(3)} - S^{(3)}$$

taking commutators , [H,O] , to get E of M ,

and then <>'s to get MF eqns.

Now, instead, take the operators

$$W = R^{(3)} - S^{(3)} \; , \; X = iR^{(+)}S^{(-)} \; , \; U = iR^{(+)}S^{(-)} = X^*(\text{in MF})$$

$$Y = S^{(+)}S^{(-)} \; , \; Z = R^{(-)}R^{(+)}$$

commute with H and take <>'s , getting closed set of 6 eqns.

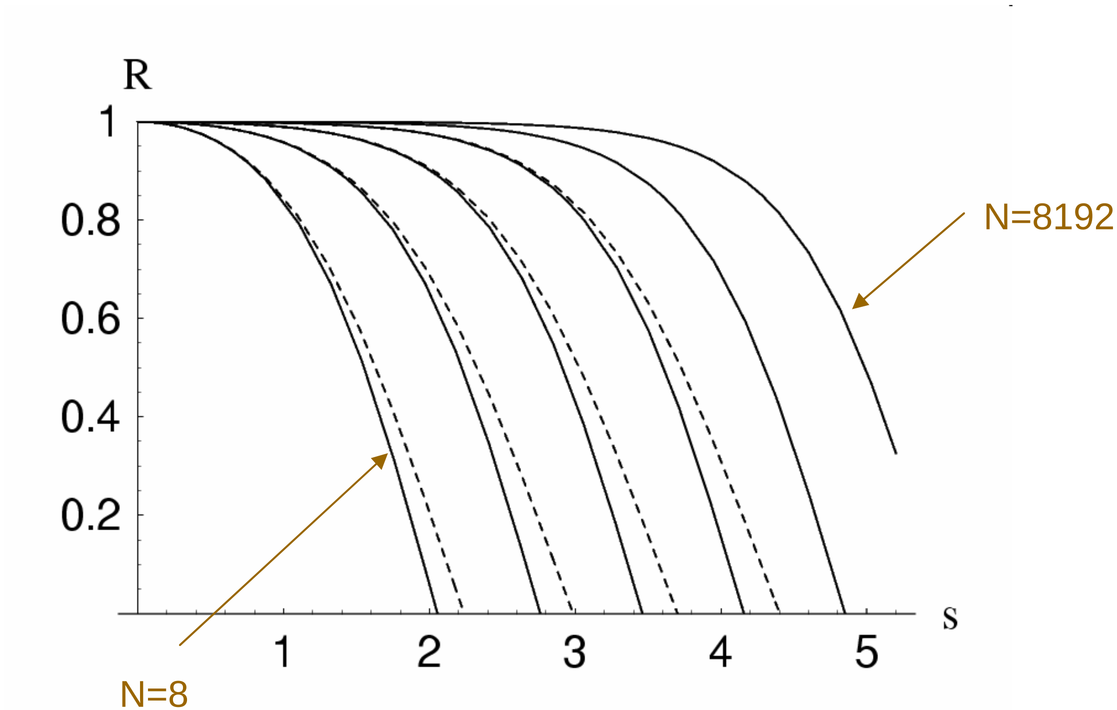
Scaling time and density:

$$s = NG\,t \qquad\qquad w = \frac{S^{(3)}}{N}$$

$$\frac{d^2}{ds^2}\,w = 2w\,(w^2 - 1) + \frac{2\,w^2}{N}$$

$$w(0) = 0 \quad , \quad \frac{d}{ds}w|_{s=0} = 0$$

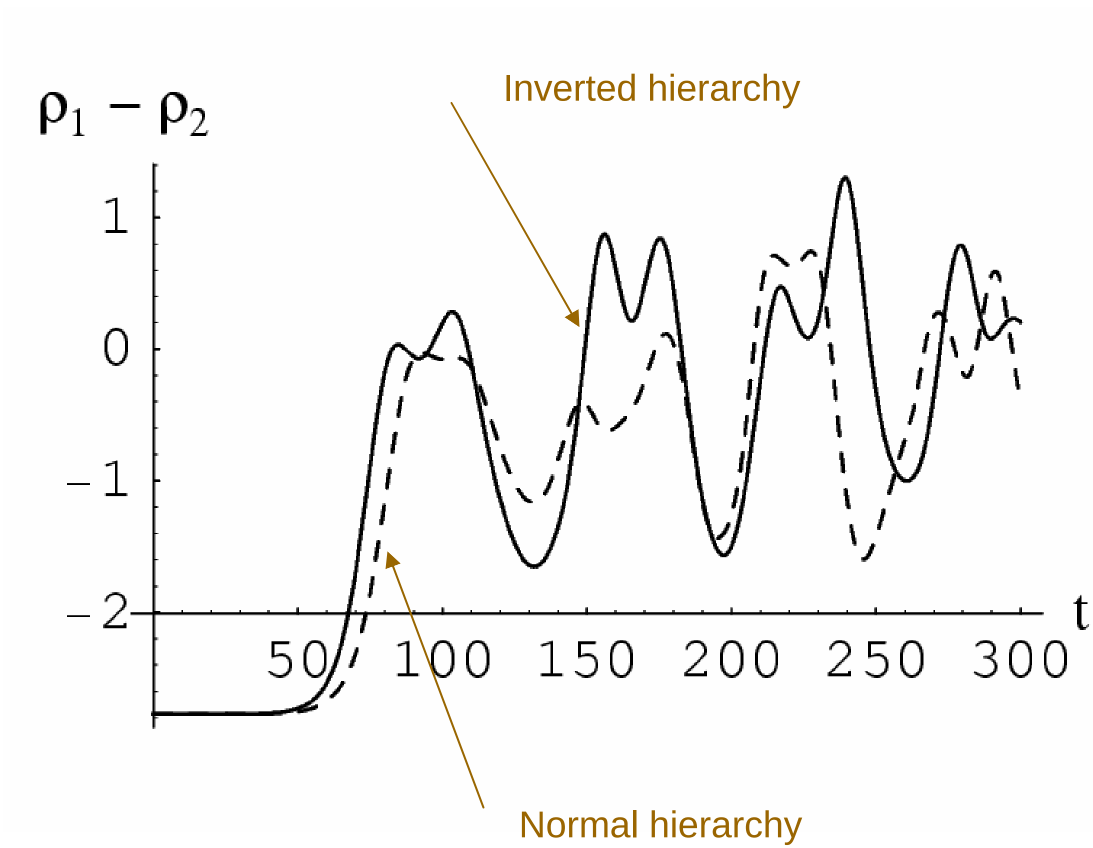
Steps= factors of 4



Solid curves --- solutions to $$\frac{d^2}{ds^2} w = 2w\,(w^2-1) + \frac{2\,w^2}{N}$$

Dashed curves --- solutions

Fourteen beams



Results of solving 56 coupled nonlinear equations over 1000’s of oscillation times.

However : We have found that complete mixing ensues whenever the 28x28 matrix for the linear response has a complex eigenvalue.

Are there other systems beside SN where this stuff could matter?

Maybe in cosmo. models with sterile neutrino dark matter.

The game is to have neutrinos with mass = 10 KeV ?, but to make the sin θ in ordinary mixing to light ν so small that X ray background Is no problem. Dynamical accelerants depending on ν chemical potentials may make it possible.

or

If $\nu + \nu \longrightarrow$ S+S coupling exists

Then wholesale conversion could occur just after (conventional) ν decoupling.