Nonabelian Plasma Instabilities

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Nonequilibrium Dynamics in Particle Physics and Cosmology

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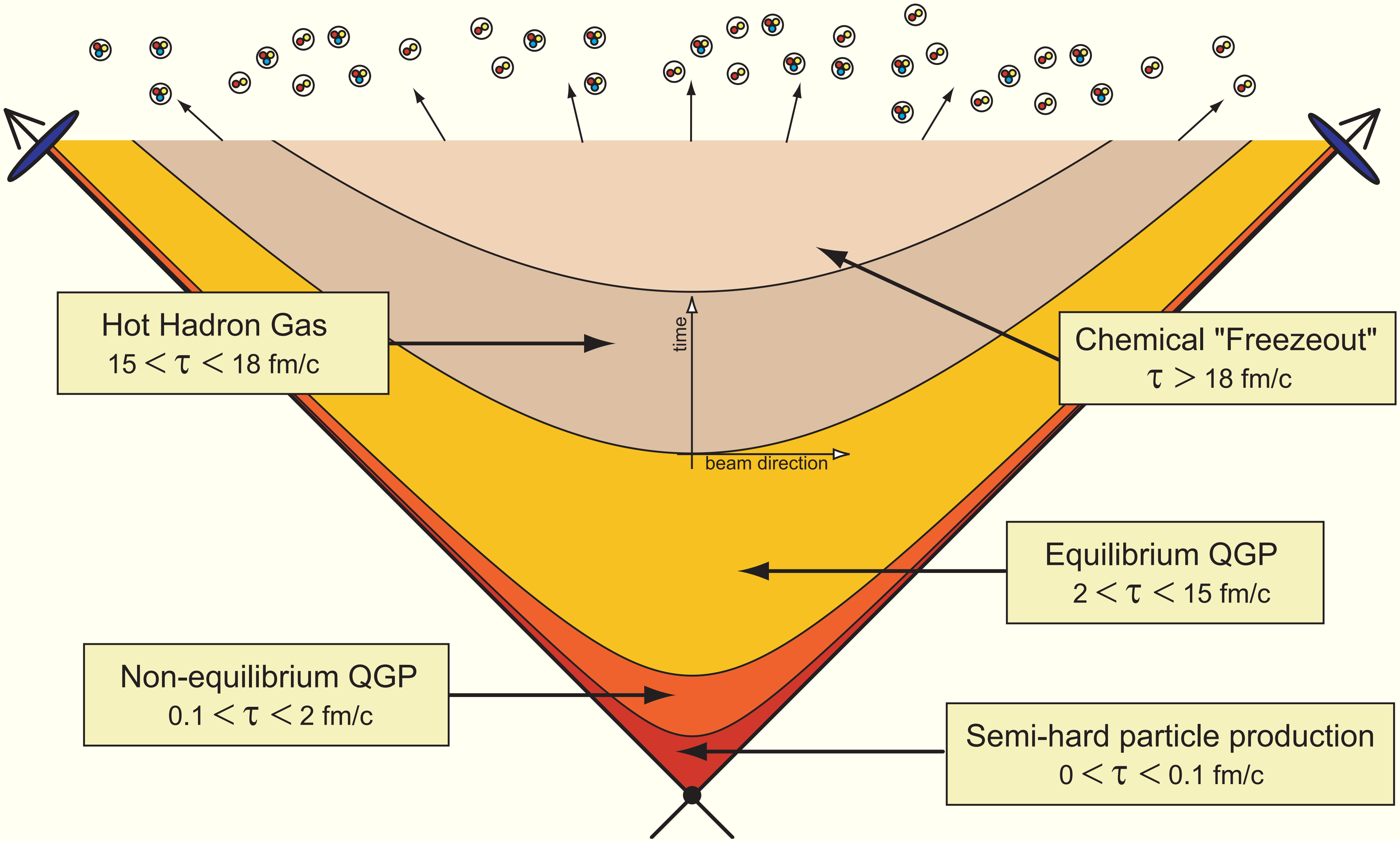
Heavy-ion collision timescales and “epochs” @ LHC

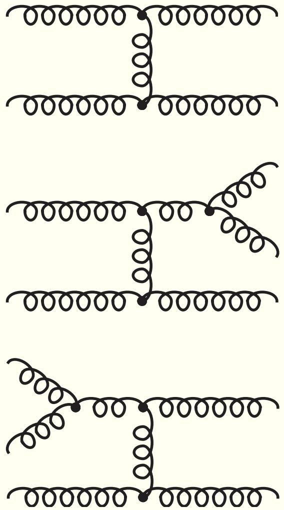


*1 fm/c $\simeq 3 \times 10^{-23}$ seconds

Motivation – Isotropization/Thermalization

- Need to understand mechanisms and time scales necessary for the isotropization and equilibration of a QGP at weak coupling.

- Consider pure glue. Processes include:



  - $2 \leftrightarrow 2$ elastic scattering (super slow)
  - Inelastic processes, e.g. $2 \to 3$ and $3 \to 2$ processes
  - Effect of *soft background fields* : expansion is in $g\,A$ not g; CGC $A \sim 1/g$

- Equilibrium: background fields screen the interaction (Debye)

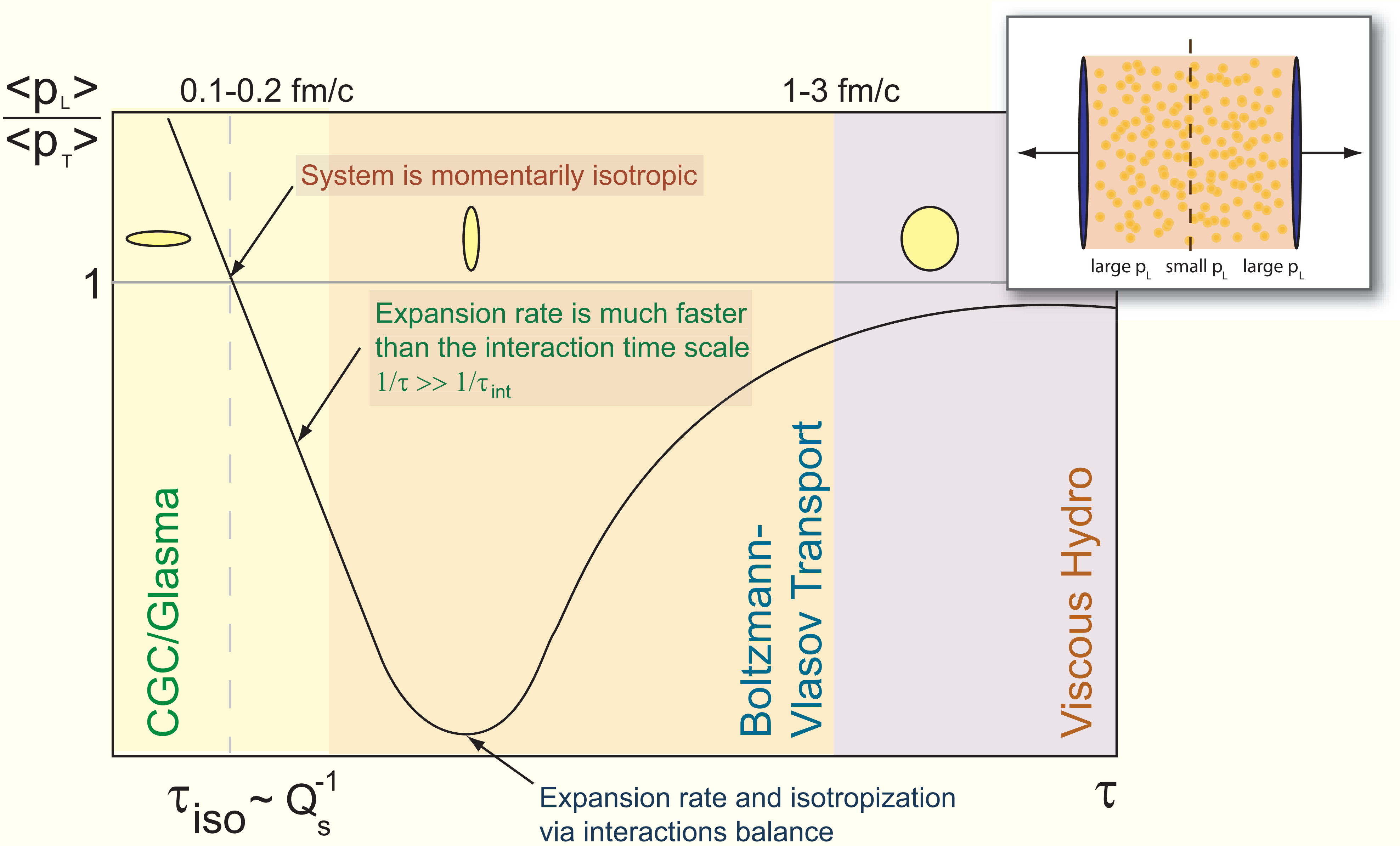
- Non-equilibrium: background fields can have non-trivial dynamics and can have a large effect on the particles' motion

Improving upon Bottom-Up Thermalization

- Previous leading order perturbative results included 2 $\leftrightarrow$ 2, 2 $\rightarrow$ 3, and 3 $\rightarrow$ 2 processes [R. Baier, A. Mueller, D. Son, and D. Schiff, hep-ph/0009237]
- "Bottom-up" thermalization : soft modes isotropize and equilibrate first, then the hard modes $\rightarrow$ $\tau_{\rm therm} \sim \alpha_s^{-13/5} Q_s^{-1}$
- At RHIC $Q_s \sim$ 1.5 - 2 GeV and $\alpha_s \sim$ 0.3 $\rightarrow$ $\tau_{\rm therm} \sim$ 2 - 3 fm/c
- Bottom-up calculation ignored effect of local anisotropy in momentum space on soft physics (field dynamics)

In anisotropic systems plasma instabilities are present which will accelerate isotropization and thermalization.

Momentum Space Anisotropy Time Dependence



- Analytic Results from Linearized Transport Theory -

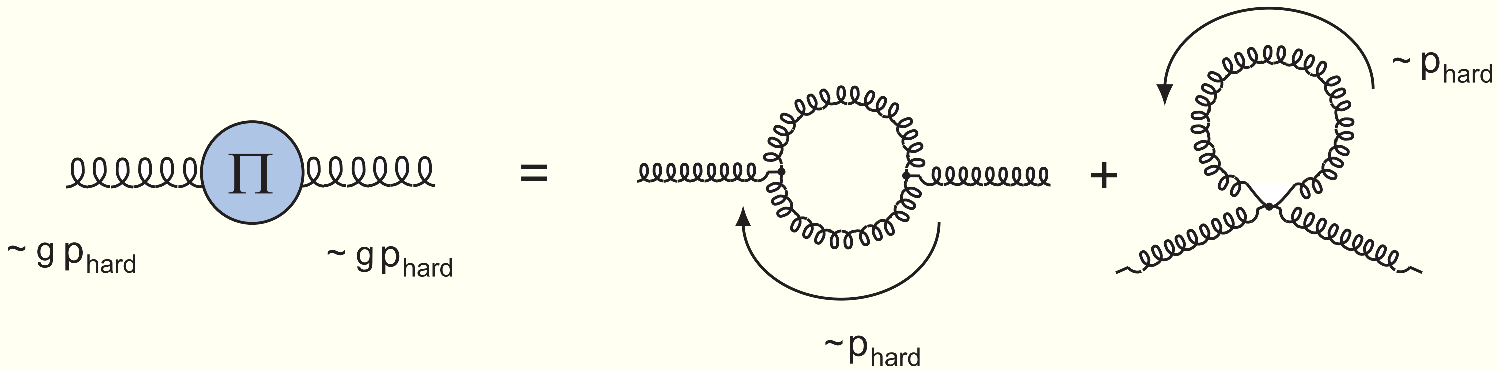
Gluon Polarization – The Chromo-Weibel Instability

The high-energy medium gluon polarization tensor can be obtained by linearizing collisionless transport theory : $f(p,x) \to f(\mathbf{p}) + \delta f(p,x)$

$$[v \cdot D_x, \delta f(p,x)] + g v_\mu F^{\mu\nu} \partial_\nu^{(p)} f(\mathbf{p}) = 0$$

$$D_\mu F^{\mu\nu} = J^\nu = g \int_p v^\nu \delta f(p,x)$$

or diagrammatically using "hard-loop" perturbation theory



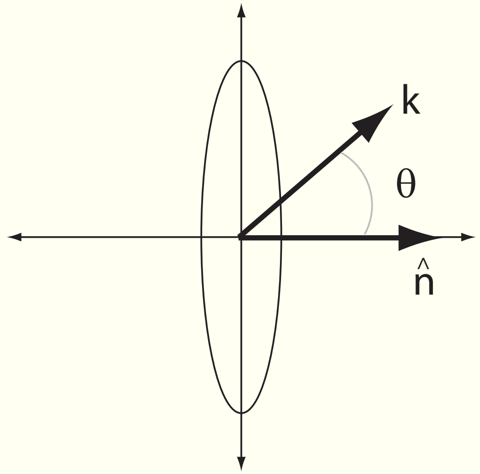
$$\Pi^{ij}_{ab}(\omega,\mathbf{k}) = -g^2 \delta_{ab} \int_{\mathbf{p}} v^i \frac{\partial f(\mathbf{p})}{\partial p^l} \left(\delta^{jl} - \frac{v^j k^l}{\omega - \mathbf{v}\cdot\mathbf{k} + i\epsilon} \right)$$

S. Mrówczyński (1994); P. Romatschke and MS, hep-ph/0304092

The nature of the anisotropy

For simplicity assume that the anisotropic distribution function can be obtained from an arbitrary isotropic distribution function by a change of its argument.

$$f(p^2) \rightarrow f(p^2 + \xi(\mathbf{p}\cdot\mathbf{n})^2)$$



The polarization tensor can then be written as

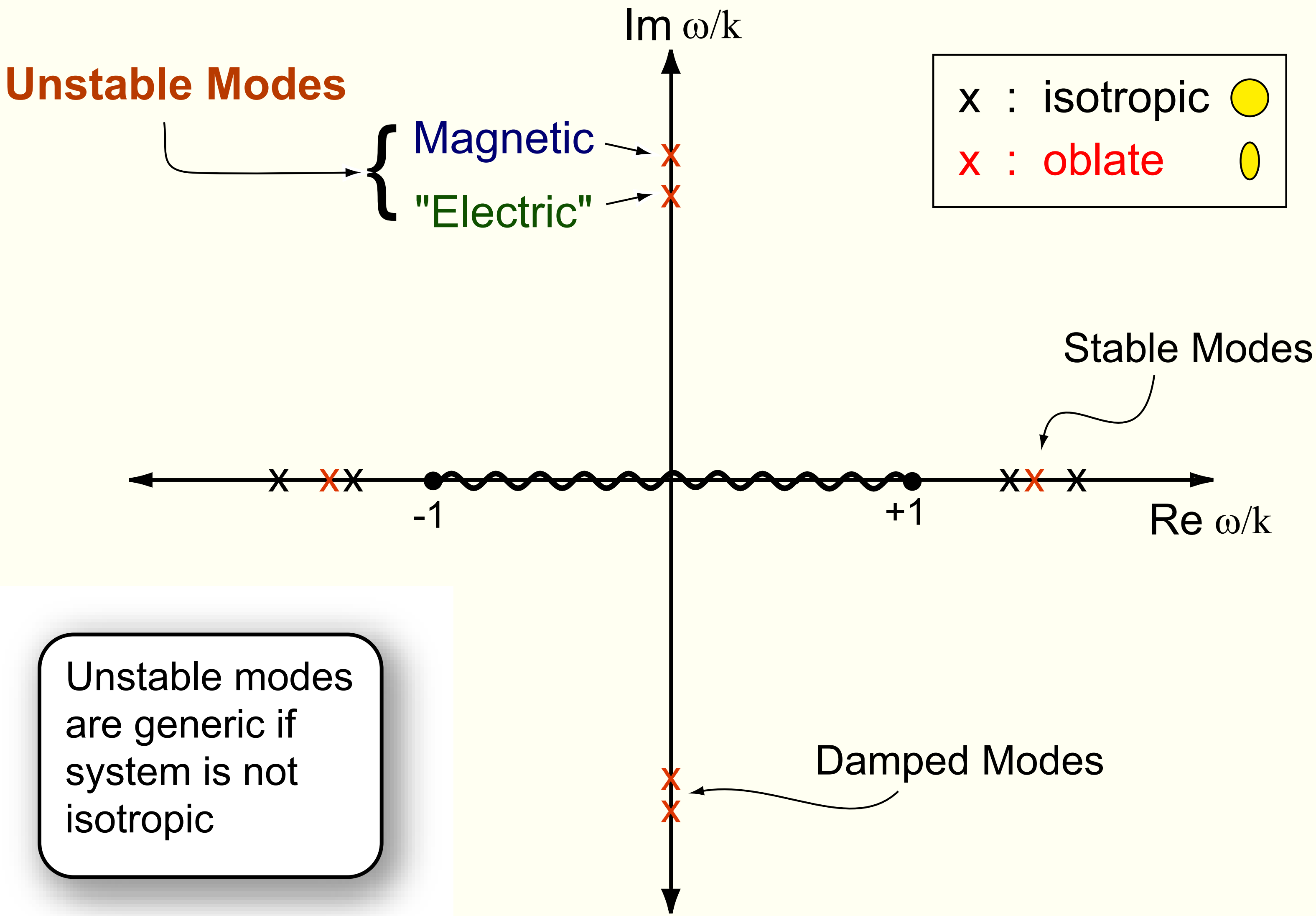
$$\Pi^{ij}_{ab}(\omega,k) = m_D^2\,\delta_{ab}\int\frac{d\Omega}{4\pi}v^i\frac{v^l+\xi(\mathbf{v}\cdot\mathbf{n})n^l}{(1+\xi(\mathbf{v}\cdot\mathbf{n})^2)^2}\left(\delta^{jl}-\frac{v^j k^l}{\omega-\mathbf{v}\cdot\mathbf{k}+i\epsilon}\right)$$

where m_D is the *isotropic* Debye mass

$$m_D^2 = -\frac{g^2}{2\pi^2}\int_0^\infty dp\,p^2\frac{df(p^2)}{dp} \sim g^2 p_{\rm hard}^2$$

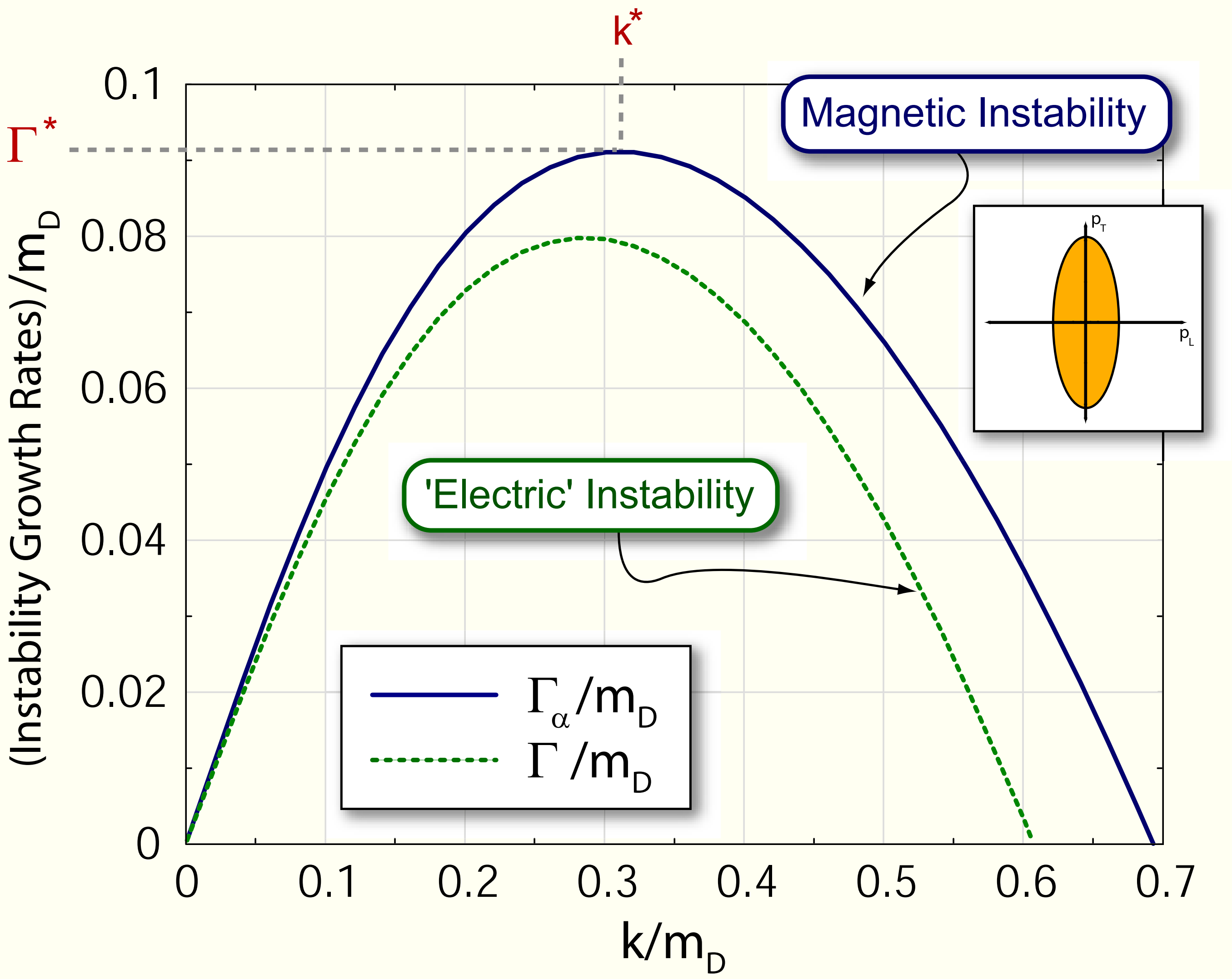
P. Romatschke and MS, hep-ph/0304092.

Anisotropic Gluonic Collective Modes ($\xi > 0$)



Unstable modes are generic if system is not isotropic

Unstable Mode Spectrum – Oblate Distribution



Instability growth rates as a function of momentum for $\langle p_T^2 \rangle / \langle p_L^2 \rangle \simeq 10$ and $\theta_{\rm glue} = \pi/8$ with respect to the beamline.

Using $\alpha_s = 0.3$ and $Q_s \sim 1.5 - 2$ GeV

$$m_D = g Q_s \rightarrow 3 - 4 \text{ GeV}$$

$$m_D \sim \sqrt{\frac{g^2 N_c n_g}{Q_s}} \sim \sqrt{\frac{12\left(\frac{100}{\text{fm}^3}\right)}{1.5 - 2 \text{ GeV}}} \rightarrow 2 - 3 \text{ GeV}$$

$$\Gamma \sim 0.5 - 2.4 \text{ GeV}$$

e-Folding time 0.1 - 0.4 fm/c

P. Romatschke and MS, hep-ph/0304092

Time scales

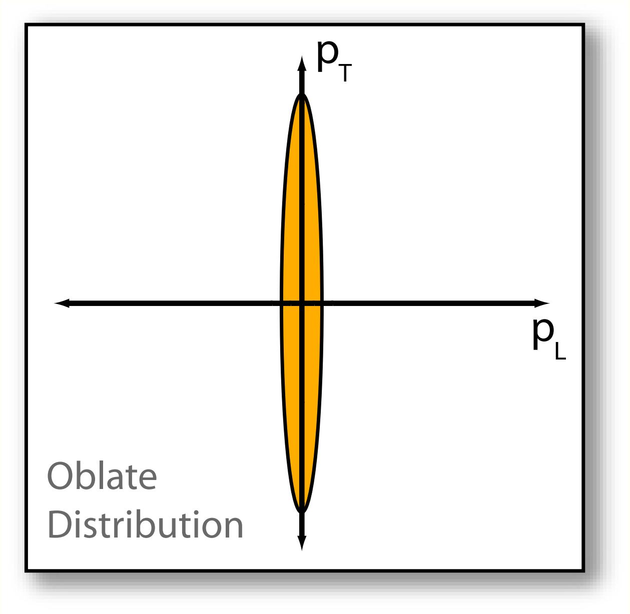
- This picture strictly only holds at leading order in $\alpha_s = g^2/4\pi$.
- Instability time scale: $t_{\text{instability}} \sim m_{D,\text{iso}}^{-1} \sim (\sqrt{\alpha}_s Q_s)^{-1}$
- Collisional time scale: $t_{\text{hard collisions}} \sim (\alpha_s^2 Q_s)^{-1}$

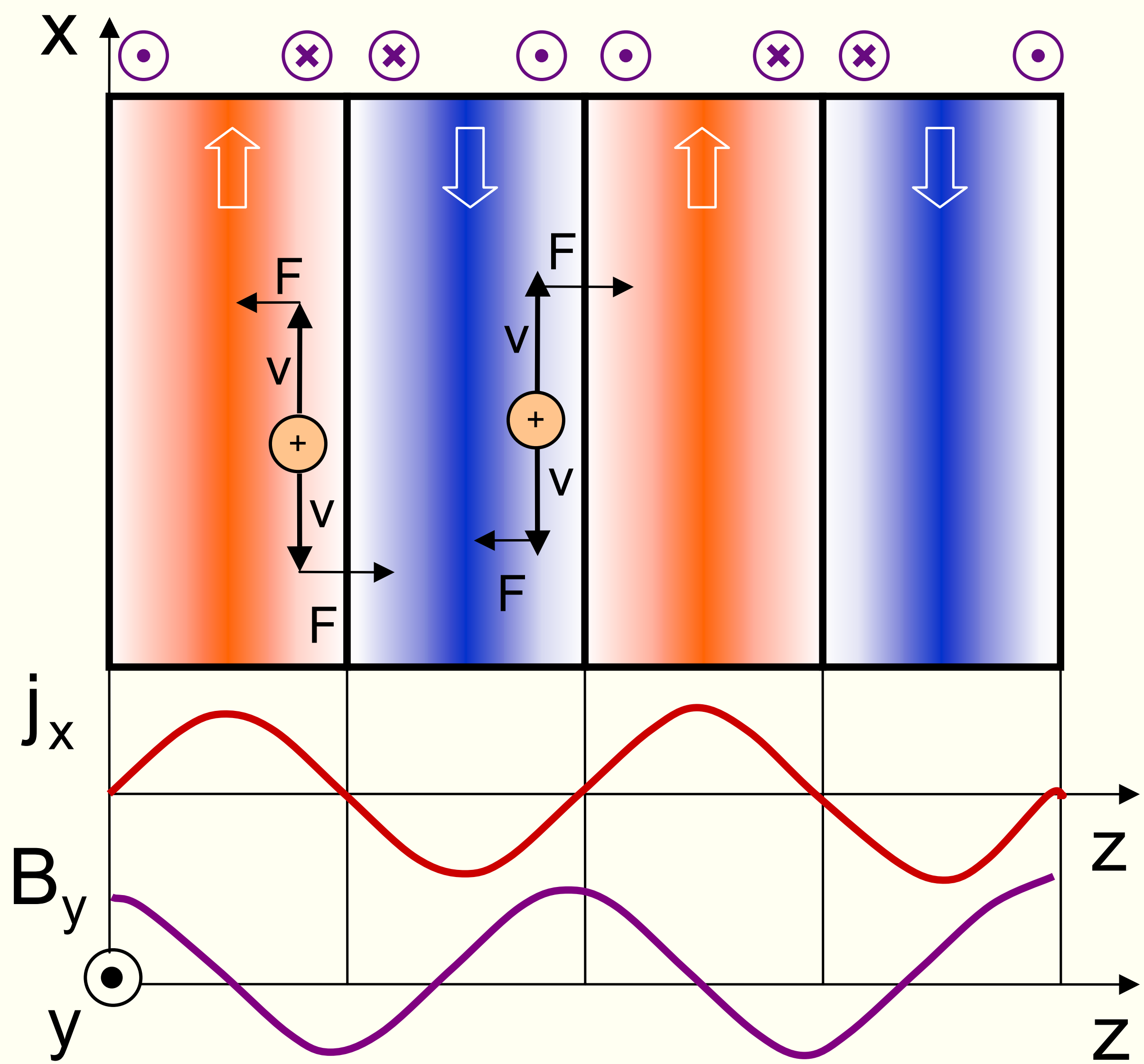
α_s	$t_{\text{collisions}}/t_{\text{instability}}$
0.01	1000
0.1	30
0.3	6

Can include collisions in the Boltzmann-Vlasov equation and it has been shown that for $\xi \gtrsim 1$ instabilities persist even for $\alpha_s = 0.3$.

[B. Schenke, MS, C. Greiner, and M. Thoma, hep-ph/0603029]

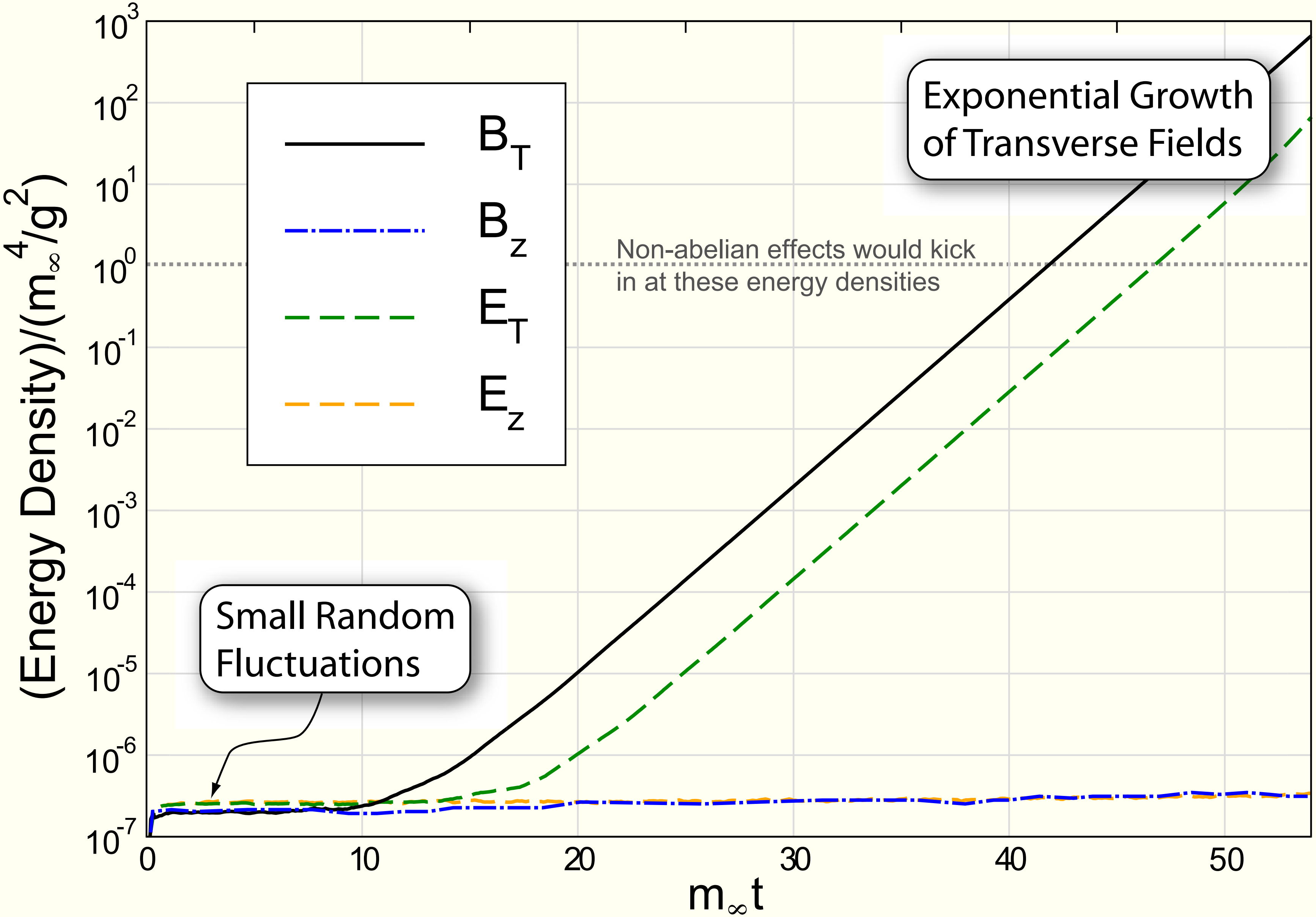
Current Filamentation in Abelian (QED) Plasmas





E. Weibel, PRL 2, 83 (1959)

Anisotropic Abelian Plasma – Weibel Instability



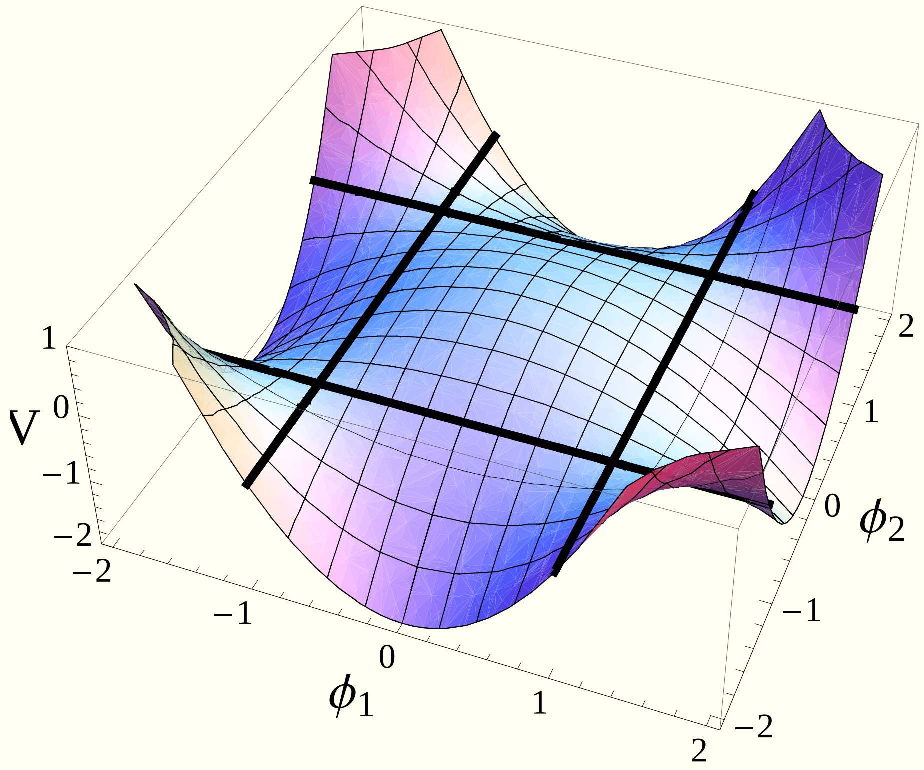
Anisotropic QCD Hard-loop Effective Action

Require gauge invariance

$\downarrow$

Effective action for soft fields

$$S_{\rm soft} = S_{\rm QCD} + S_{\rm HL}$$



$$S_{\rm HL} \quad = \quad \frac{g^2}{2}\int_{x,\mathbf{p}}\left[f(\mathbf{p})\,F_{\mu\nu}(x)\frac{p^\nu p^\rho}{(p\cdot D)^2}F_\rho{}^\mu(x) + i\frac{C_F}{2}\tilde{f}(\mathbf{p})\bar{\Psi}(x)\frac{p\cdot\gamma}{p\cdot D}\Psi(x)\right]$$

St. Mrówczyński, A. Rebhan, and MS, 2004

- Real-Time Lattice Simulations -

3+1 Real-Time Lattice Simulation (Pure Glue)

Numerically solve the equations of motion resulting from the hard-loop effective action on a space + velocity lattice in temporal gauge.

$$j^\mu[A] = -g^2 \int_{\mathbf{p}} \frac{1}{2|\mathbf{p}|}\, p^\mu \, \frac{\partial f(\mathbf{p})}{\partial p^\beta} W^\beta(x;\mathbf{v})$$

with

$$[p \cdot D(A)] W_\beta(x;\mathbf{v}) = F_{\beta\gamma}(A) p^\gamma$$

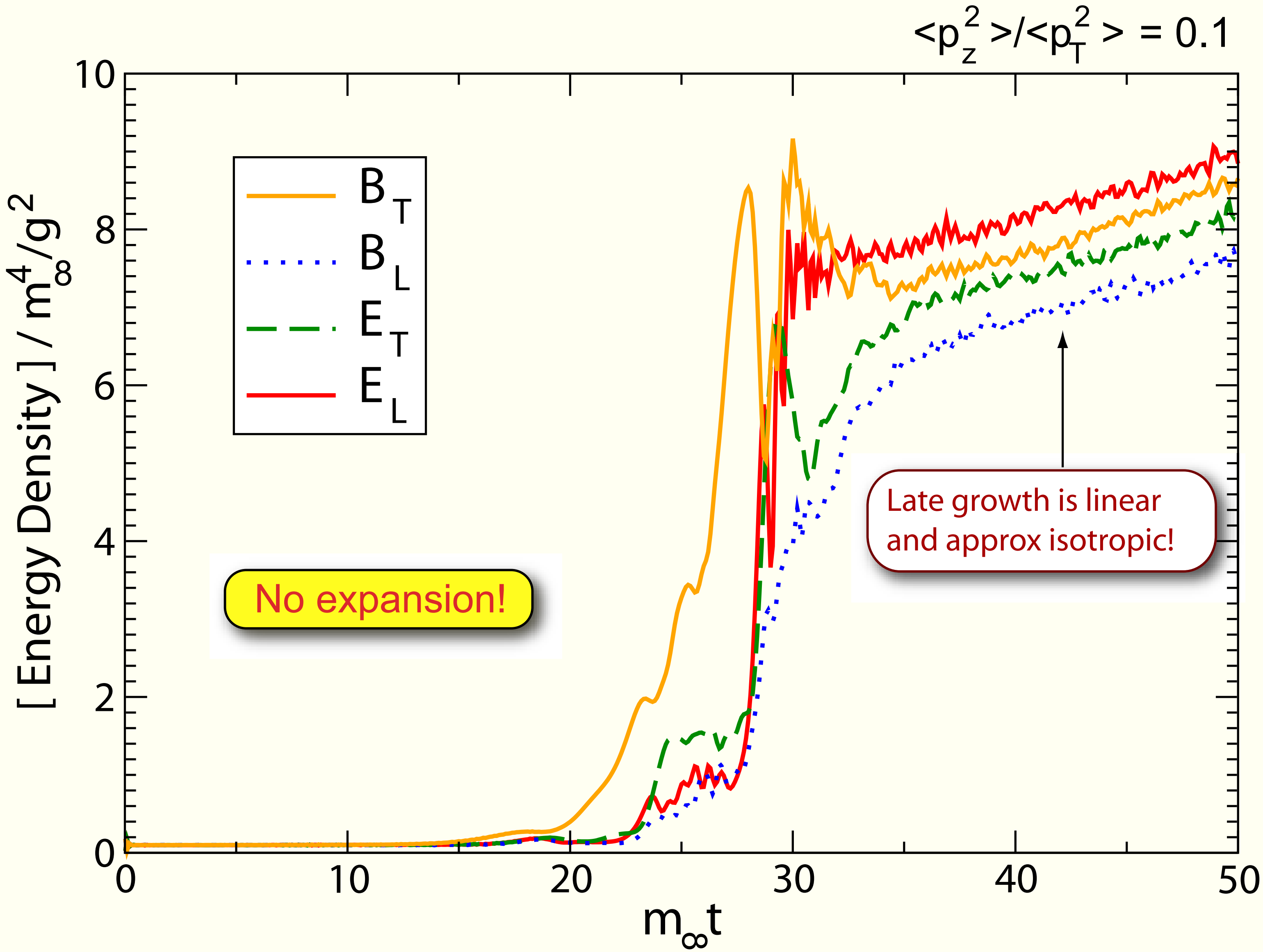
This has to be solved with the Yang-Mills equation

$$D_\mu(A) F^{\mu\nu} = j^\nu$$

where $\nu = 0$ is the Gauss law constraint.

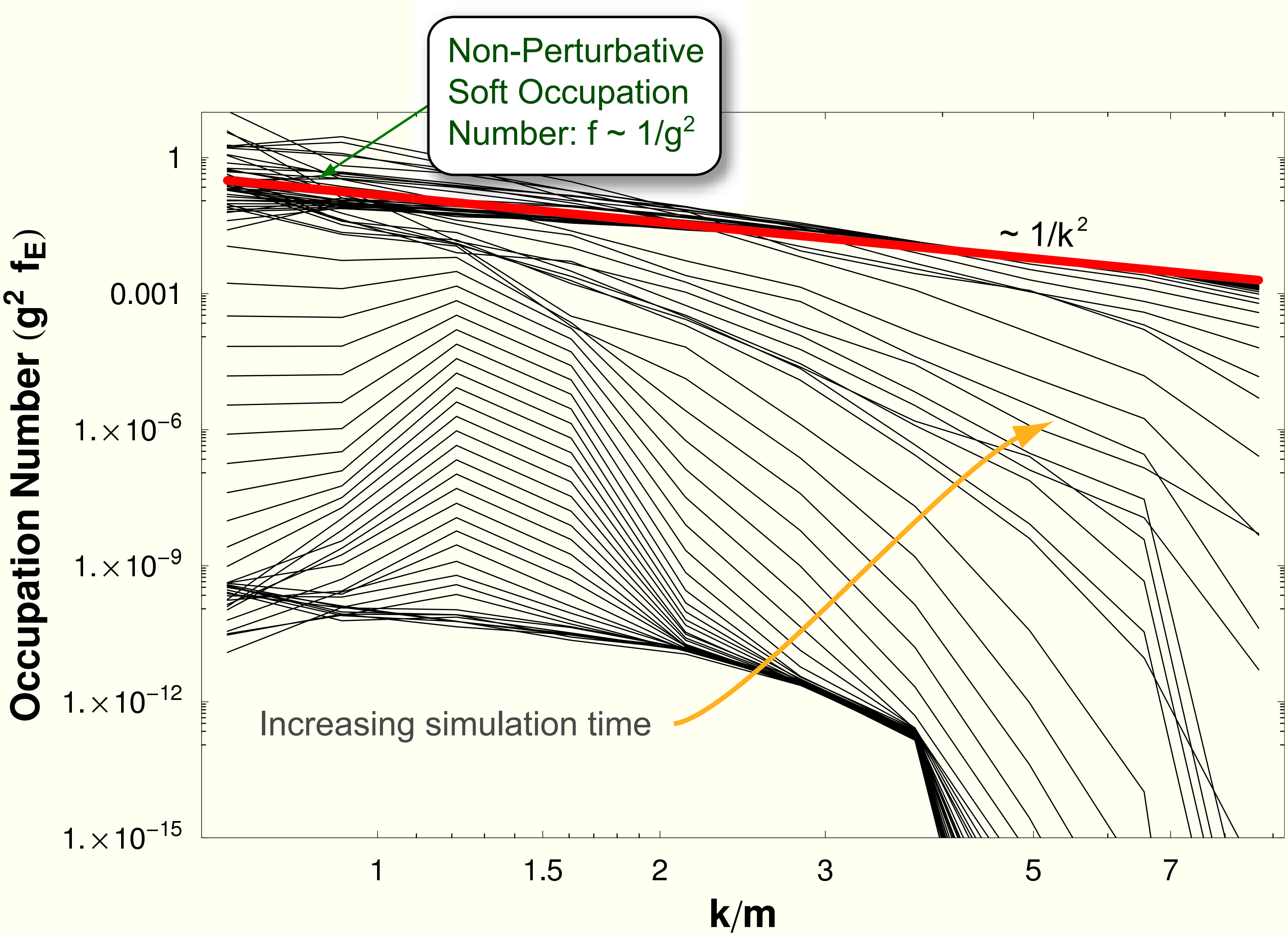
Rebhan, Romatschke, and MS, 2005; Arnold, Moore, and Yaffe, 2005

3D SU(2) Hard-Loop Results



A. Rebhan, P. Romatschke, and MS, hep-ph/0505261; P. Arnold, G. Moore, and L. Yaffe, hep-ph/0505212; D. Bodeker and K. Rummukainen, arXiv:0705.0180v1

Kolmogorov cascade $\longrightarrow$ Turbulent Fields?



- Come to Kari Rummukainen’s discussion for more details and systematics -

3D Colored-Particle-in-Cell Simulations (CPIC)

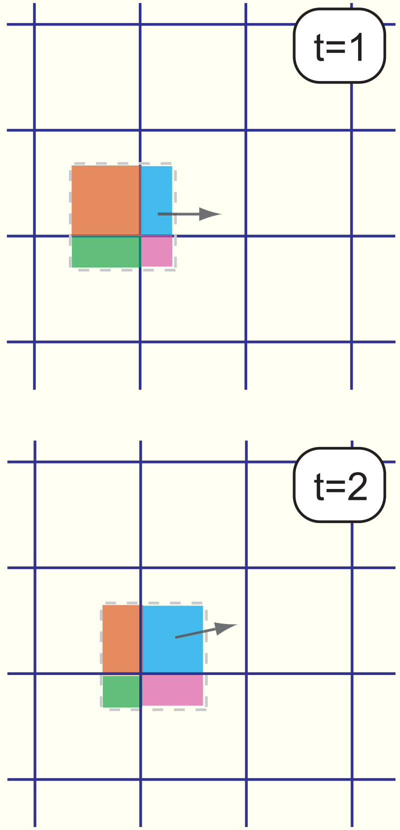
Hard-loop approximation strictly only applies when we ignore the back-reaction of the particles on their self-generated fields. How can we go beyond hard-loops?

Include back-reaction by solving collision-less transport equation without linearization

$$p^\mu[\partial_\mu - g q^a F^a_{\mu\nu}\partial^\nu_p - g f_{abc} A^b_\mu q^c \partial_{q^a}] f(t, \boldsymbol{x}, \boldsymbol{p}, q) = 0$$

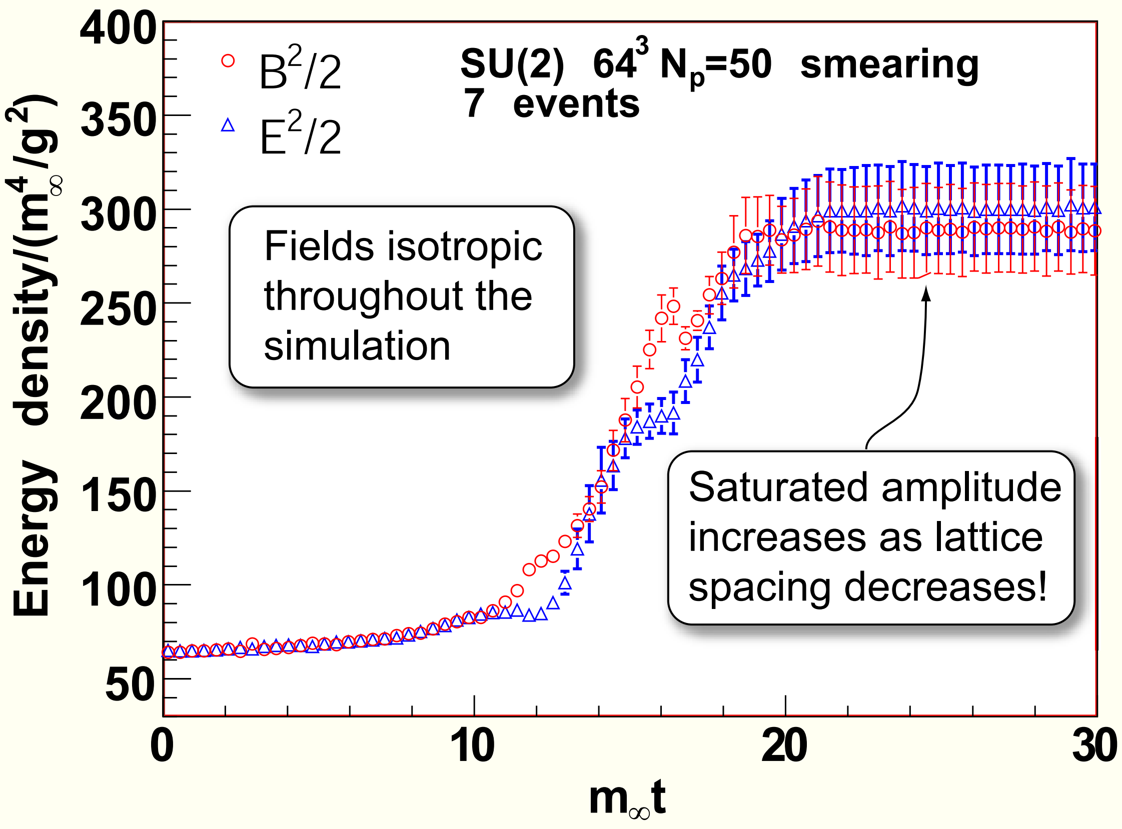
Coupled to the Yang-Mills equation for the soft gluon fields

$$D_\mu F^{\mu\nu} = J^\nu = g \int \frac{d^3p}{(2\pi)^3} dq\, q\, v^\nu f(t, \boldsymbol{x}, \boldsymbol{p}, q)$$

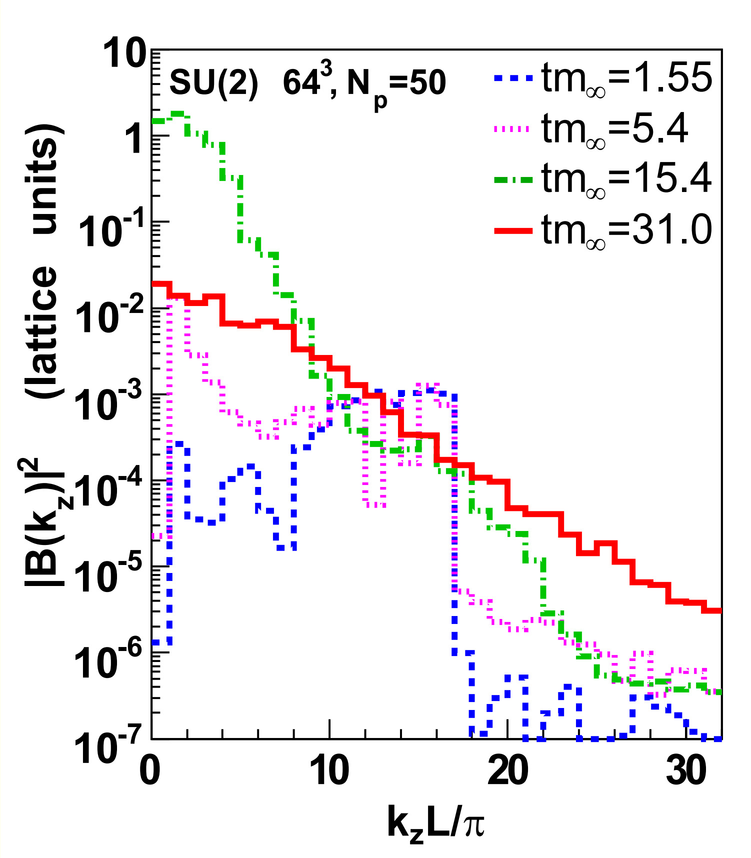


A. Dumitru, Y. Nara, and MS, hep-ph/0604149

CPIC Results – Ultraviolet Avalanche



$L = 5$ fm, $p_{\mathrm{hard}} = 16$ GeV, $g^2\, n_g = 10/\mathrm{fm}^3$, $m_\infty = 0.12$ GeV.

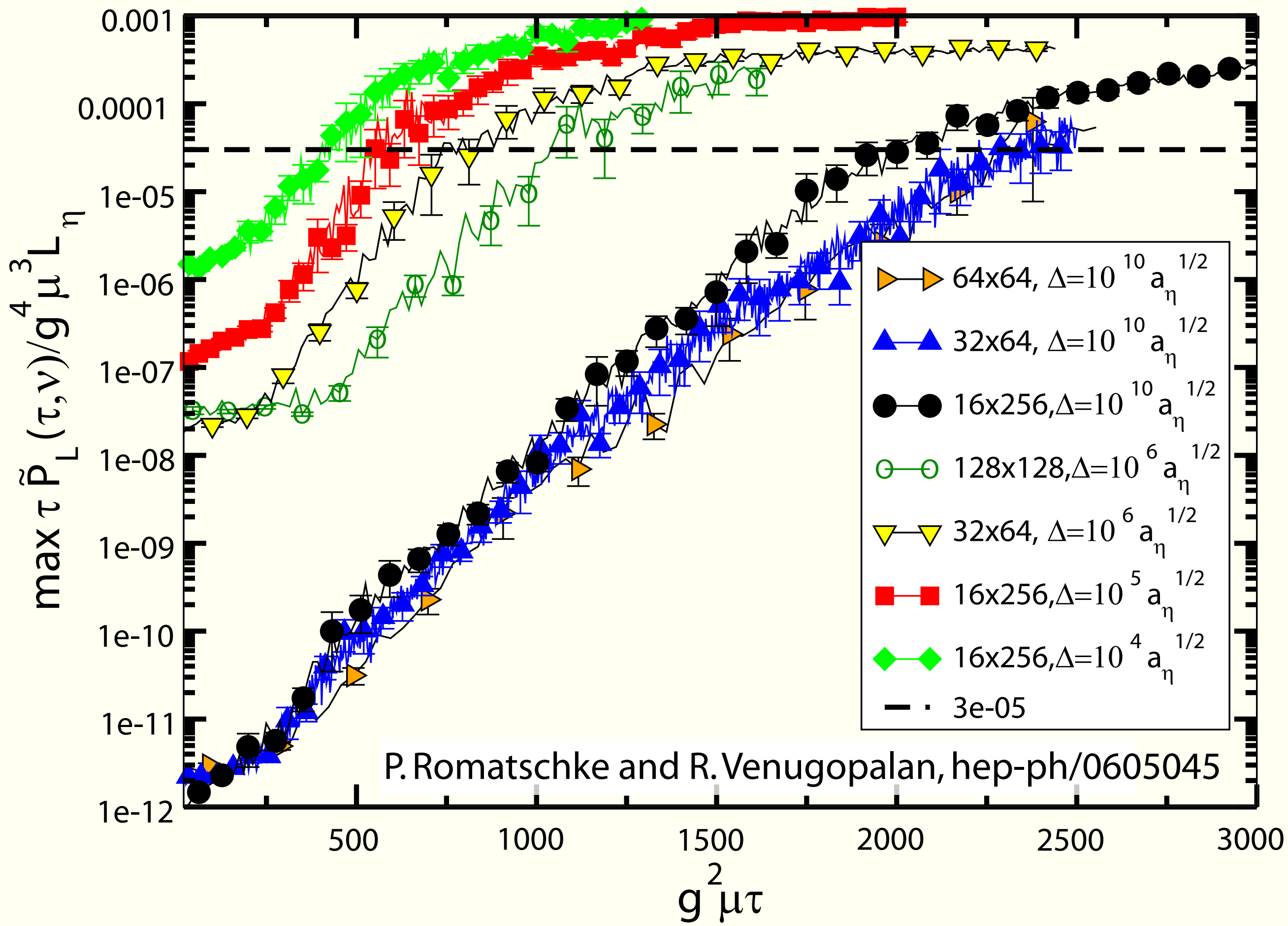


Coulomb gauge-fixed color-magnetic field spectrum at four different times.

A. Dumitru, Y. Nara, and MS, hep-ph/0604149

Instabilities in classical YM – The unstable glasma

Instabilities also seen in expanding classical Yang-Mills solutions which include rapidity fluctuations.



Growth $\sim e^{\sqrt{Q_s\tau}}$ agrees with HL calculation!

[P. Arnold, J. Lenaghan, and G. Moore, hep-ph/0307325]

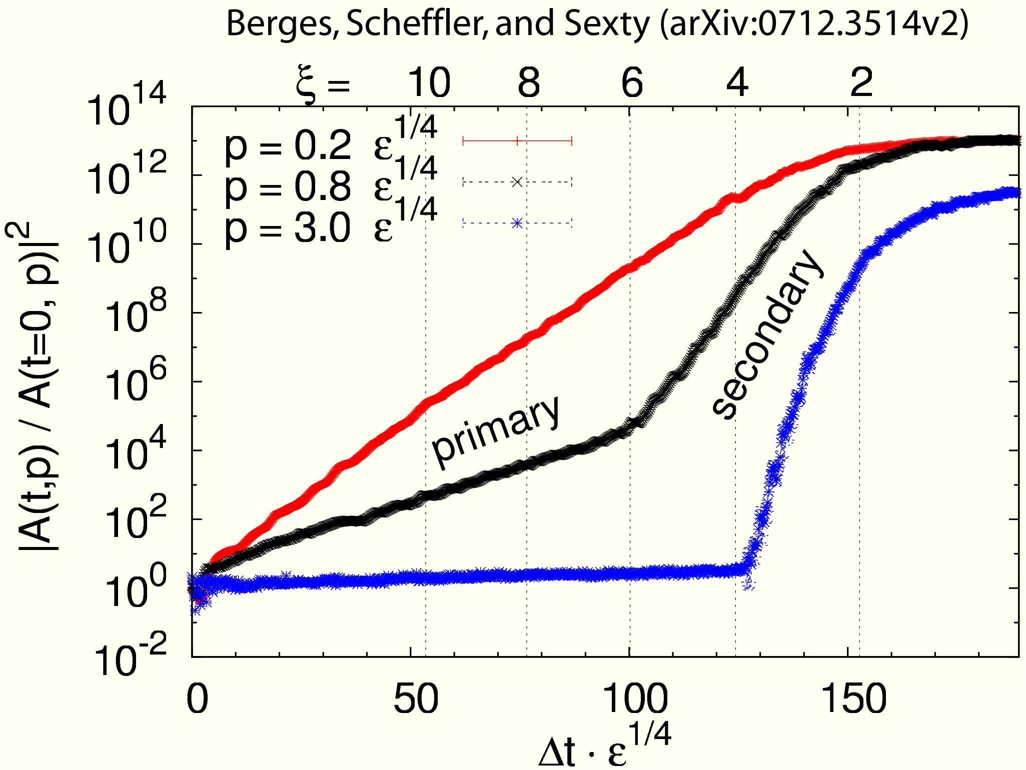
[P. Romatschke and A. Rebhan, hep-ph/0605064]

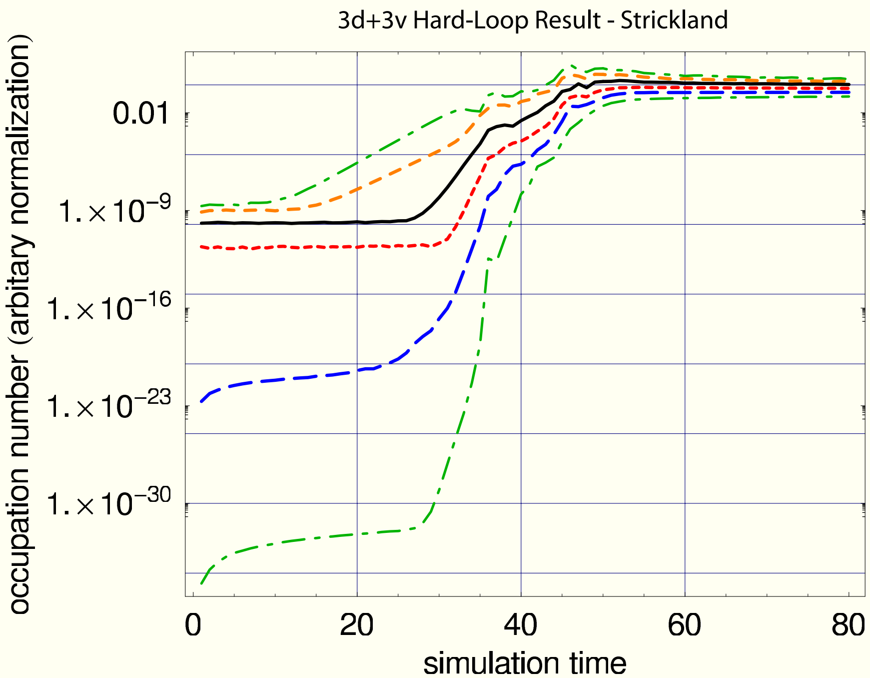
Initial spectrum of rapidity fluctuations from CGC camp

[K. Fukushima, F. Gelis, and L. McLerran, hep-ph/0610416]

Instabilities in classical YM – Non-expanding

Recently there have also been measurements of the instability growth rate, induced spectrum, etc within classical SU(2) Yang-Mills by Berges, Scheffler, and Sexty (arXiv:0712.3514v2).





- Including expansion + field-particle coupling -

Instabilities induced by expansion - Free Streaming Bkg

Assuming a color neutral background distribution function $f_0(\mathbf{p},\mathbf{x},t)$ which satisfies

$$v \cdot \partial\, f_0(\mathbf{p},\mathbf{x},t) = 0, \qquad v^\mu = p^\mu/p^0,$$

the gauge covariant Boltzmann-Vlasov equations for colored perturbations δf_a of an approximately collisionless plasma have the form

$$v \cdot D\, \delta f_a(\mathbf{p},\mathbf{x},t) = g v_\mu F_a^{\mu\nu} \partial_\nu^{(p)} f_0(\mathbf{p},\mathbf{x},t),$$

which have to be solved self-consistently with the non-Abelian Maxwell equations

$$D_\mu F_a^{\mu\nu} = j_a^\nu = g \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu}{2p^0} \delta f_a(\mathbf{p},\mathbf{x},t).$$

P. Romatschke and A. Rebhan, hep-ph/0605064; A. Rebhan, MS, and M. Attems, forthcoming.

Instabilities induced by expansion - Free Streaming Bkg

Go to comoving coordinates $\tilde{x}^\alpha = (\tau, x^1, x^2, \eta)$ with metric $ds^2 = d\tau^2 - d\mathbf{x}_\perp^2 - \tau^2 d\eta^2$ and $\tilde{V}^\alpha = \left(\cosh(y-\eta),\, \cos\phi,\, \sin\phi,\, \frac{1}{\tau}\sinh(y-\eta)\right)$

$$\frac{1}{\tau}\tilde{D}_\alpha(\tau \tilde{F}^{\alpha\beta}) = \tilde{j}^\beta$$

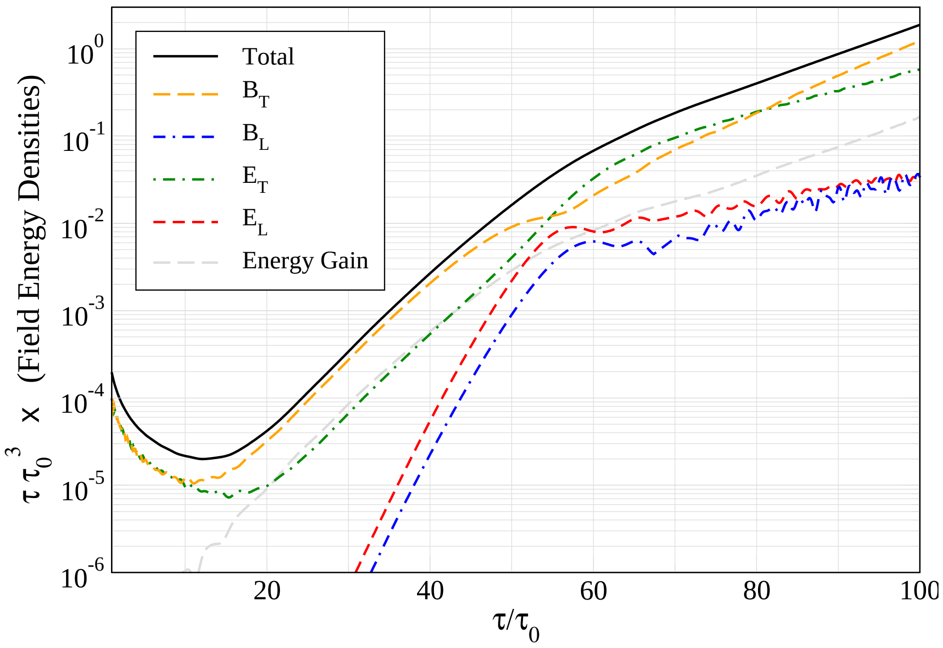
$$\tilde{V}\cdot\tilde{D}\,\mathcal{W} = \left(\tilde{V}^{\underline{i}}\tilde{F}_{\underline{i}\tau} + \frac{\tau^2}{\tau_{\rm iso}^2}\tilde{V}^\eta\tilde{F}_{\eta\tau}\right)\tilde{V}^\tau + \tilde{V}^{\underline{i}}\tilde{V}^\eta\tilde{F}_{\underline{i}\eta}\left(1-\frac{\tau^2}{\tau_{\rm iso}^2}\right).$$

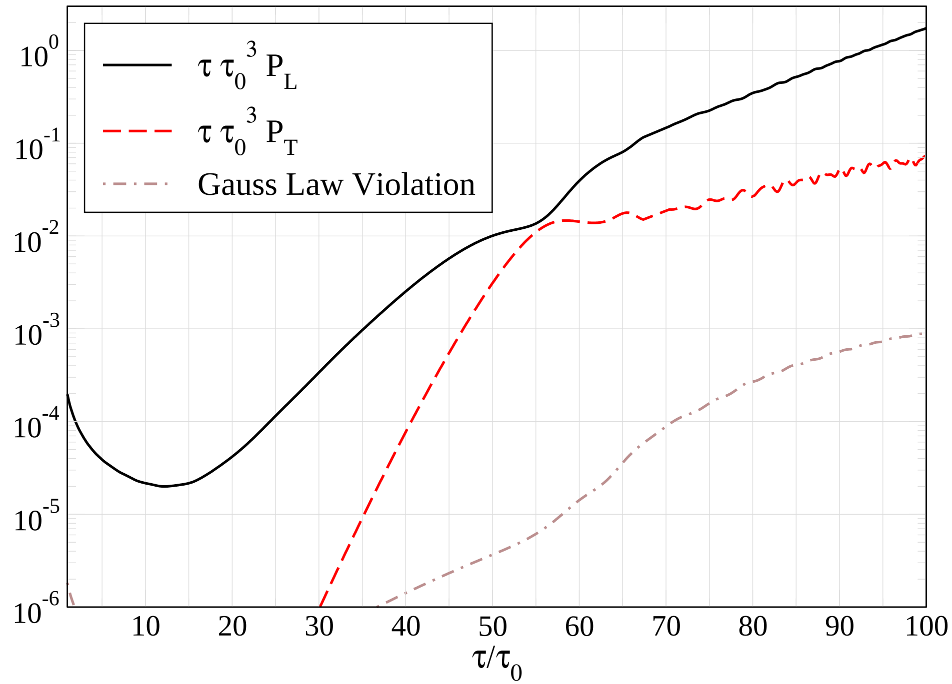
$$\tilde{j}^\alpha = -m_D^2\,\frac{1}{2}\int_0^{2\pi}\frac{d\phi}{2\pi}\int_{-\infty}^{\infty}dy\,\tilde{V}^\alpha\left(1+\frac{\tau^2}{\tau_{\rm iso}^2}\sinh^2(y-\eta)\right)^{-2}\mathcal{W}(\tilde{x};\phi,y)$$

P. Romatschke and A. Rebhan, hep-ph/0605064; A. Rebhan, MS, and M. Attems, forthcoming.

1s×3v Numerical Results - Energies and Pressures

Perform simulation assuming that induced fields only depend on rapidity and are constant in the transverse directions → 1s × 3v simulations. Captures the physics of the most unstable modes and provides a reference point for future 3s × 3v simulations.



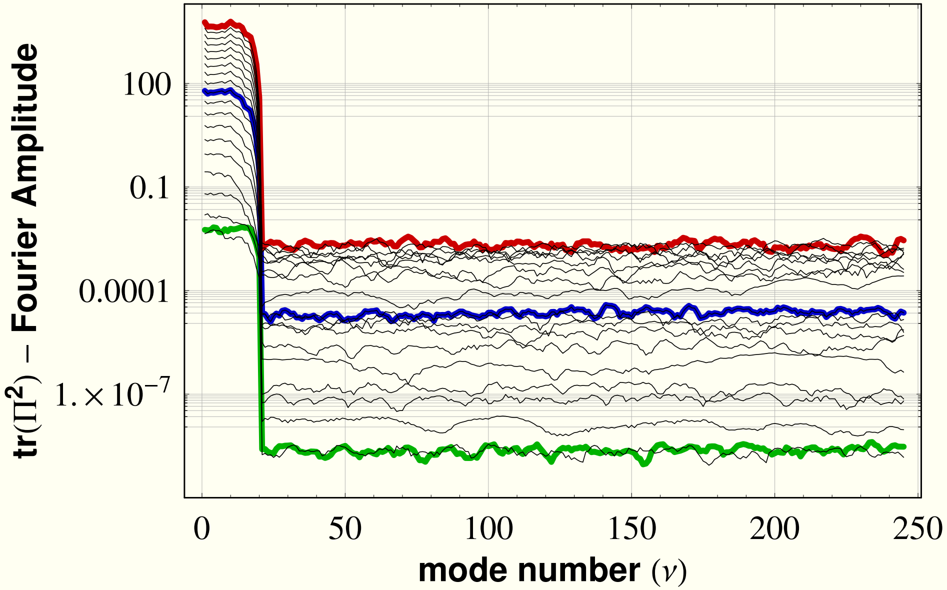


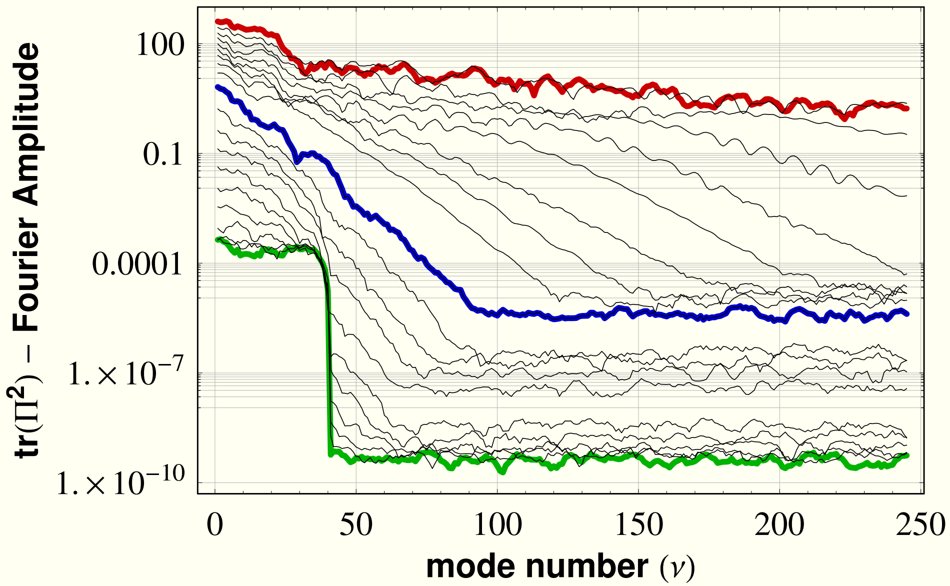
A. Rebhan, MS, and M. Attems, forthcoming.

1s$\times$3v Numerical Results - Spectra

Spectra from (left) an abelian run and (right) a non-abelian run showing Fourier decomposition of modes at different times.

Non-abelian run shows “quasi-thermal” spectra at intermediate times; qualitatively different than abelian case.





A. Rebhan, MS, and M. Attems, forthcoming.

Conclusions and Outlook

- Anisotropic plasmas are qualitatively different than isotropic ones.
- Hard-Loop : Fields show isotropic linear growth and UV cascade.
- CPIC : Rapid isotropic field growth followed by UV "avalanche".
- Classical YM : rapidity fluctuations $\rightarrow$ the "glasma" is unstable to becoming a QGP! Instabilities also seen in a static box.
- The same instability exists in weakly-coupled supersymmetric gauge theories; just need to rescale the Debye mass. QUESTION: Do these instabilities also exist in the strong coupling limit????
- Need to pin down the possible phenomenological effect of plasma instabilities: Systematic calculations of p_T-p_L anisotropy observables such as jet effects and E&M signatures.
- CPICv2 : code now includes stochastic collisions between particles. Can now be used to measure transport properties, jet energy deposition, medium response to energy deposition, etc.