On radiative corrections in leptogenesis

Mikko Laine

(University of Bern)

Basic setup

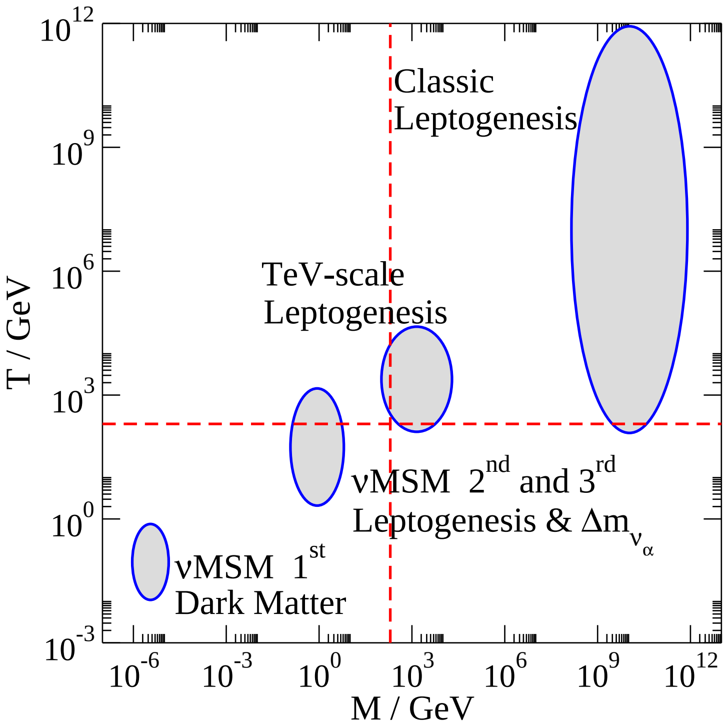
Consider an extension of SM with right-handed neutrinos:

$$\mathcal{L} = \mathcal{L}_{\mathsf{SM}} + \frac{1}{2}\bar{N}[i\partial\!\!\!/ - M]N - [h_\nu\,\bar{\ell}\,a_R\,\tilde{\phi}\,N + \mathrm{H.c.}]\ .$$

Motivation: offers for a simple description of experimentally observed neutrino masses and mixing angles.

See-saw: $m_\nu \sim \dfrac{|h_\nu|^2 v^2}{M}$ $\Rightarrow$ certain combinations of the new couplings are fixed, but the absolute scale of M is open.

Many regimes are open for cosmological exploration.



In this talk:
$T \gtrsim 160$ GeV.

This is **not** an exclusion plot, apart perhaps from Dark Matter, but a reflection of human interests.

Necessary conditions for [us explaining] baryogenesis:

- $\not{B}$
- $\not{C}$, $C\not{P}$
- non-equilibrium
- ability to compute reliably with thermal quantum field theory

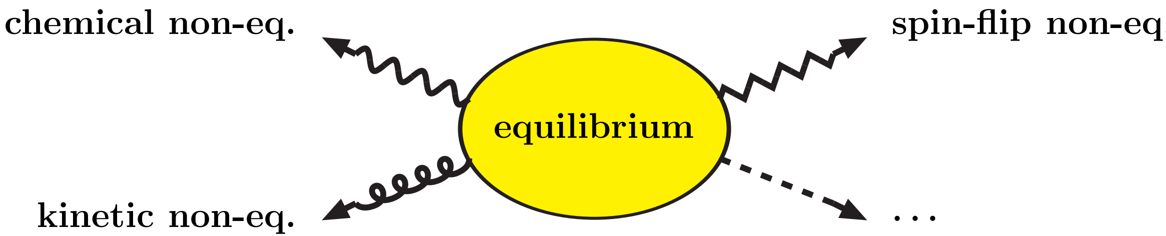
Traditionally: $C\not{P}$ and $\not{B}$ are fascinating challenges, non-equilibrium and reliable computation mundane ones.

Today: $C\not{P}$ and $\not{B}$ have been much studied and are rather well understood; non-equilibrium and reliable computation less so.

Here: some ingredients on the latter two.

Conceptual challenge: non-equilibrium

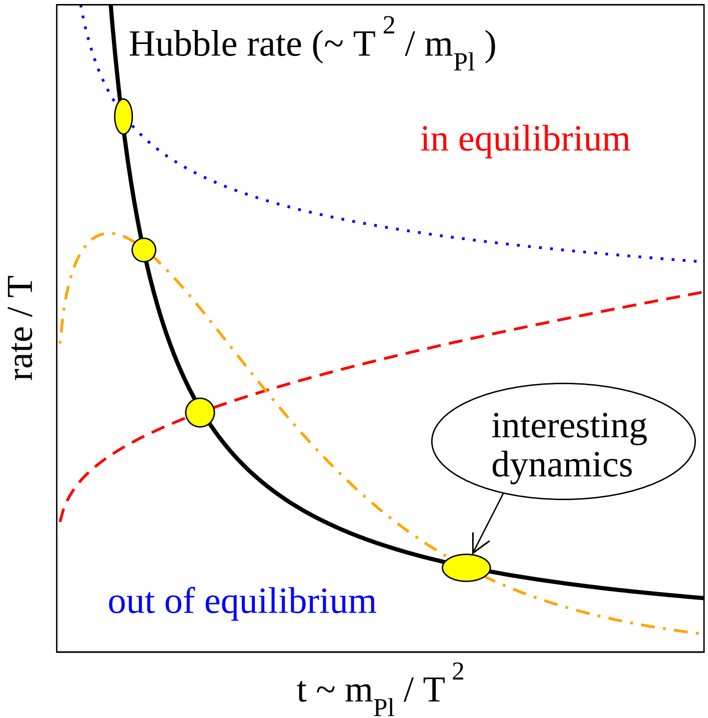
There are infinitely many ways to deviate from equilibrium.



Basic principle: with time, the system relaxes back to equilibrium (just because equilibrium is the most likely configuration).

The relaxation process is characterized by a **rate**, which is a microscopic property of the system, and depends on the deviation.

In most cases only a few deviations are “interesting”.



If an equilibration rate is fast, the excitation is in equilibrium; if it is slow, there is a supplementary conserved charge.

Most minimal scenario: one "interesting" variable.

If the deviation from equilibrium is small and we are looking at long time scales, we can expand in the deviation:

$$
\begin{aligned}
\partial_t n_L &= -\gamma_L \, n_L + \mathcal{O}(n_L^2) \\
\Rightarrow (\partial_t + 3H) n_L &= -\gamma_L \, n_L + \mathcal{O}(n_L^2) \\
\Leftrightarrow \partial_t (a^3 n_L) &= -\gamma_L \, a^3 n_L + \mathcal{O}(a^6 n_L^2) \; .
\end{aligned}
$$

$\Rightarrow$ There is only one coefficient, γ_L, which is positive.

$\Rightarrow$ There is only a decaying mode.

$\Rightarrow$ No $n_L \neq 0$ can be generated from close to equilibrium.

Second most minimal scenario: two “interesting” variables.[1]

Apart from the asymmetry n_L, consider a weakly interacting particles species N whose energy density (particles + antiparticles, like Dark Matter) is also close to falling out of equilibrium:

$$\begin{aligned}
\partial_t(a^3 n_L) &= -\gamma_L\, a^3 n_L - \gamma_{L,N}\,(a^3 n_N - a^3 n_{\text{eq}})\,, \\
\partial_t(a^3 n_N) &= -\gamma_N\,(a^3 n_N - a^3 n_{\text{eq}}) - \gamma_{N,L}\, a^3 n_L\,.
\end{aligned}$$

CP-odd: $a^3 n_L$.
CP-even: $a^3 n_N$.

[1] M. Fukugita and T. Yanagida, *Baryogenesis Without Grand Unification*, Phys. Lett. B 174 (1986) 45.

Now something can be generated out of (almost) nothing!

$n_{\text{eq}} \sim (\frac{MT}{2\pi})^{3/2} e^{-M/T}$ at $T \ll M$

$\Rightarrow n_N - n_{\text{eq}} \neq 0$

$\Rightarrow$ 1st order equation with a source term for n_L

$\Rightarrow n_L \neq 0$.

$$\begin{aligned} \partial_t(a^3 n_L) &= -\gamma_L\, a^3 n_L - \gamma_{L,N}\,(a^3 n_N - a^3 n_{\text{eq}})\ , \\ \partial_t(a^3 n_N) &= -\gamma_N\,(a^3 n_N - a^3 n_{\text{eq}}) - \gamma_{N,L}\, a^3 n_L\ . \end{aligned}$$

There are many less minimal scenarios.

- Kinetic non-equilibrium: $n_N \to f_N(k)$.
- Spin degrees of freedom: $n_N \to f_N(k, s)$.
- Flavour transitions: $n_L,\ f_N \to 3$-component vectors.
- Resonant regime: quantum coherence.
- Quadratic deviations from equilibrium.

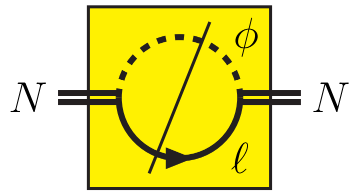
More complicated scenarios involve more coefficients. The computation of each coefficient is however subject to the same uncertainties to which we now turn.

Technical challenge: IR problem

Consider right-handed neutrino production rate γ_N

With linear response theory, γ_N can be determined **in equilibrium**.

Up to $\mathcal{O}(|h_\nu|^2)$:



The external four-momentum is on-shell: $\mathcal{K} = (\sqrt{k^2 + M^2}, \mathbf{k})$.

What are the scales of the problem?

- The properties of the SM plasma are characterized by

$$\pi T \; , \quad gT \qquad (\alpha \equiv \frac{g^2}{4\pi}) \; .$$

- To $\mathcal{O}(|h_\nu|^2)$, the scale M only appears "externally" in

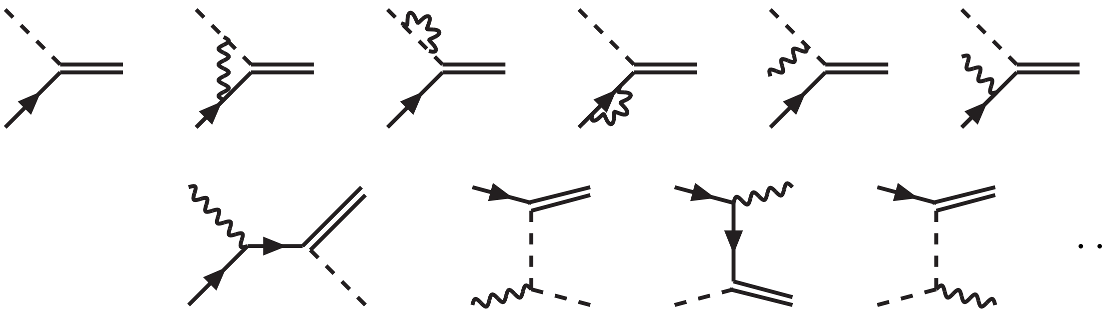
$$k_0 = \sqrt{k^2 + M^2} \; .$$

Low temperatures / large masses / late Universe

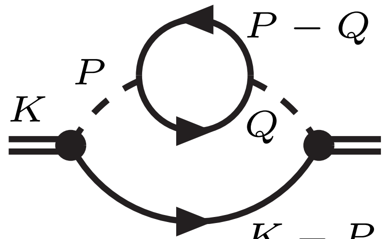
$$T \lesssim M$$

ML, *Thermal right-handed neutrino production rate in the relativistic regime,* JHEP 08 (2013) 138 [1307.4909].

Standard Model interactions up to NLO:



For illustration: corrections from the top Yukawa coupling



After cancelling numerator structures against propagators, the most non-trivial structure left over is

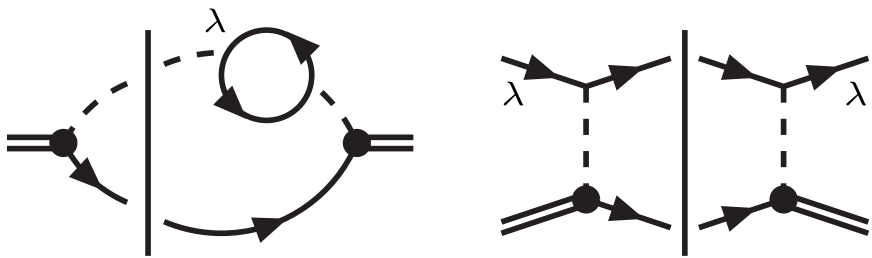
$$\widetilde{\mathcal{I}}_{\mathsf{h}} \equiv \lim_{\lambda\to 0} \sum\!\!\!\!\!\!\!\!\int_{P\{Q\}} \frac{K^2}{Q^2P^2[(Q-P)^2+\lambda^2](P-K)^2} ,$$

$$K^2 \equiv k_n^2 + k^2 .$$

The production rate is proportional to the cut:

$$\rho_{\widetilde{\mathcal{I}}_\mathsf{h}} \equiv \mathrm{Im}[\widetilde{\mathcal{I}}_\mathsf{h}]_{k_n \to -i[k_0 + i0^+]} \; ,$$

$$\mathcal{K}^2 \equiv k_0^2 - k^2 \; .$$



For $\lambda \to 0$ virtual and real processes contain soft, collinear and thermal divergences, which cancel in the sum.

Omitting one process and regulating $\lambda \to gT$ in the other leads to a wrong result and an overestimate of NLO corrections.

Cancellation of “naive” divergences

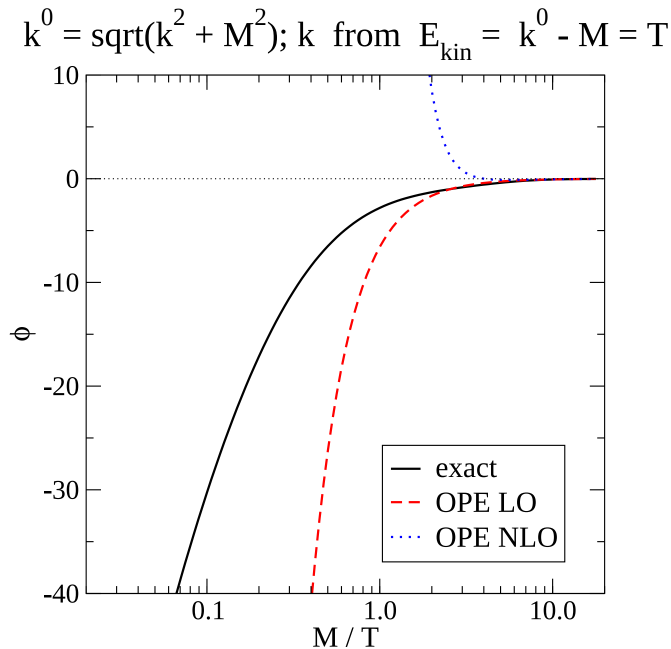
Summing together both processes, the limit $\lambda \to 0$ can be taken, and the result can be expressed as

$$\rho_{\widetilde{\mathcal{I}}_{\mathsf{h}}} = -\frac{M^2}{4(4\pi)^3}\left[\frac{1}{\epsilon} + 2\ln\frac{\bar{\mu}^2}{M^2} + 5 + \phi_T(k_0,k)\right] .$$

The function ϕ_T vanishes for $T \to 0$ but, for $T \neq 0$, depends separately on k_0 and k (i.e. breaks Lorentz invariance).

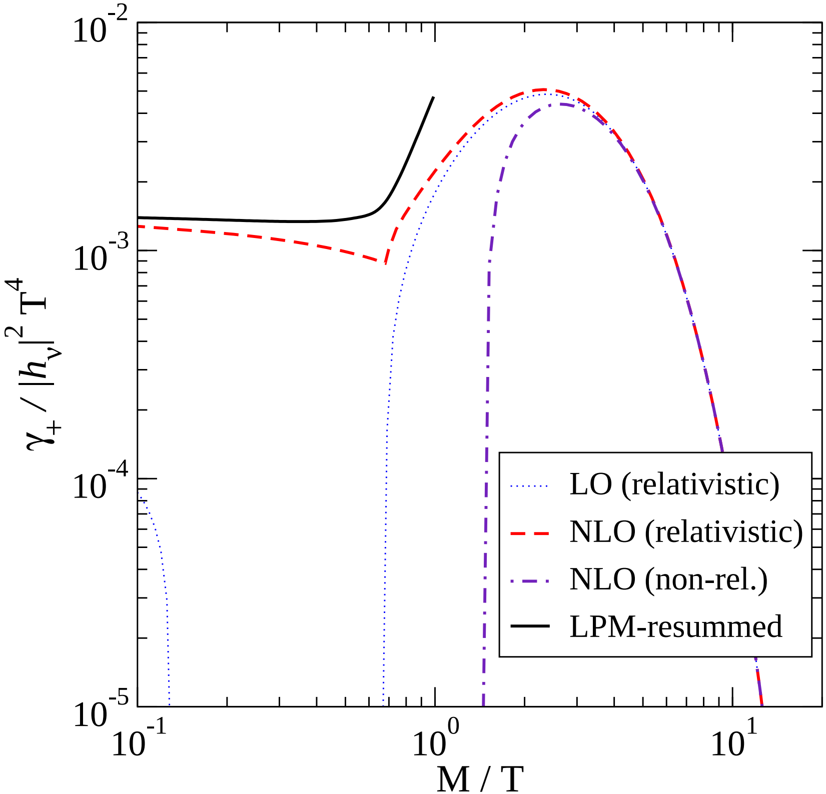
(There exists a rapidly convergent 2d integral representation for ϕ_T, but analytic expressions only as an “OPE” series in T^2/M^2.)

Breakdown of the loop expansion



$$|\phi_T(k_0,k)| \sim \frac{kT^3}{M^4} \gg \frac{1}{g^2} \quad \text{for} \quad T \gg \frac{M}{\sqrt{g}} \; .$$

The dominant divergence can be resummed into a thermal mass $m_\phi \sim gT$, but this is only a partial solution.



High temperatures / small masses / early Universe

$$\boxed{M \lesssim gT}$$

A. Anisimov, D. Besak and D. Bödeker, *Thermal production of relativistic Majorana neutrinos: Strong enhancement by multiple soft scattering*, JCAP 03 (2011) 042 [1012.3784]; D. Besak and D. Bödeker, *Thermal production of ultrarelativistic right-handed neutrinos: Complete leading-order results*, JCAP 03 (2012) 029 [1202.1288].

Techniques ("LPM") inspired by analogous QCD computations[2]

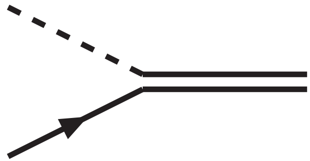
P.B. Arnold, G.D. Moore and L.G. Yaffe, *Photon emission from ultrarelativistic plasmas*, JHEP 11 (2001) 057 [hep-ph/0109064]; *Photon emission from quark gluon plasma: Complete leading order results*, JHEP 12 (2001) 009 [hep-ph/0111107].

---

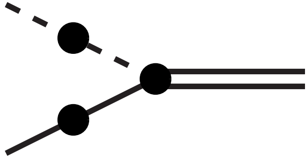
[2] See also: SLAC E-146 experiment, P.L. Anthony *et al.*, *An Accurate measurement of the Landau-Pomeranchuk-Migdal effect*, Phys. Rev. Lett. 75 (1995) 1949.

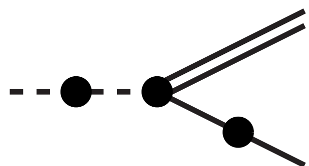
What is going on?

The naive leading-order process at high temperatures if SM particles are approximated as massless:



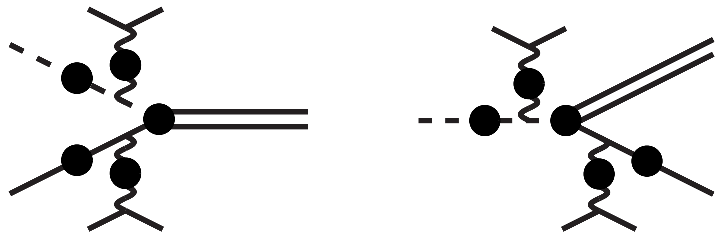
Hard Thermal Loop (HTL) resummation generates thermal masses (and other features). At very high temperatures, $m_\phi \sim gT > M$, and new channels open up.





Higher-order scatterings are not suppressed.

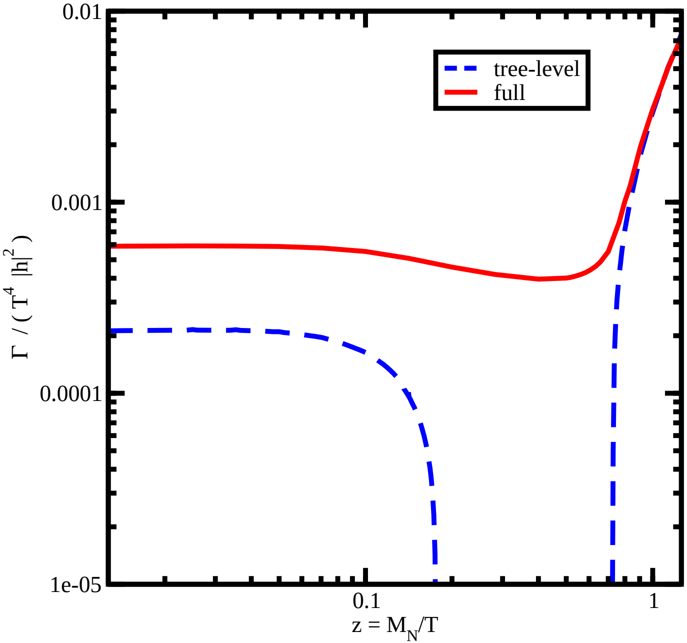
It does not cost anything to add soft gauge scatterings to these reactions: $\epsilon \sim g^2T^2/m_\text{D}^2 \sim 1$.



All such processes need to be consistently summed together.

(Resummation amounts to a solution on an inhomogeneous 2d Schrödinger equation with an imaginary light-cone potential representing the effects of these scatterings.)

This gives a substantial effect:



Anisimov et al (resummed part of the result)

In addition there are "hard" $2 \leftrightarrow 2$ scatterings which contribute at the same order.

Some care is needed for adding the hard and soft contributions together in a way which does not to introduce double counting.

(In fact a discrepancy[3] remains to be resolved.)

[3] B. Garbrecht, F. Glowna and P. Schwaller, *Scattering Rates For Leptogenesis: Damping of Lepton Flavour Coherence and Production of Singlet Neutrinos*, Nucl. Phys. B 877 (2013) 1 [1303.5498].

How does all this affect leptogenesis?

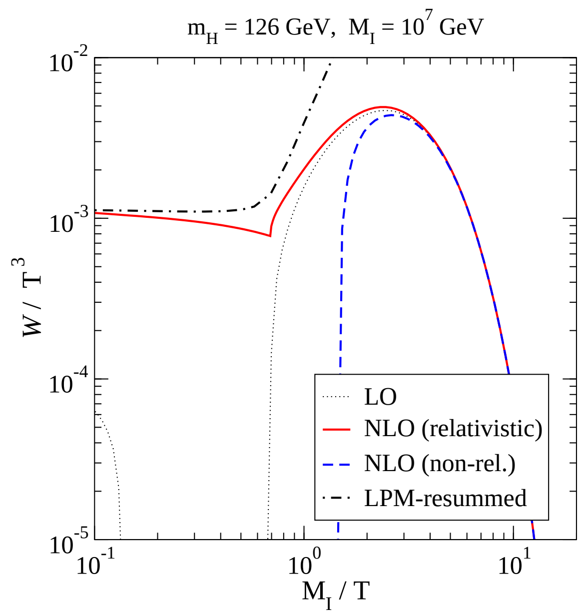
Even in the minimal framework there are several coefficients.

$$\partial_t(a^3 n_L) = -\gamma_L\, a^3 n_L - \gamma_{L,N}\,(a^3 n_N - a^3 n_{\text{eq}})\ ,$$
$$\partial_t(a^3 n_N) = -\gamma_N\,(a^3 n_N - a^3 n_{\text{eq}}) - \gamma_{N,L}\, a^3 n_L\ .$$

Apart from γ_N, the lepton number "washout rate" has been computed up to NLO: $\gamma_L \sim |h_\nu|^2\, W\, \Xi^{-1}$.

[D. Bödeker and ML, *Kubo relations and radiative corrections for lepton number washout*, 1403.2755].

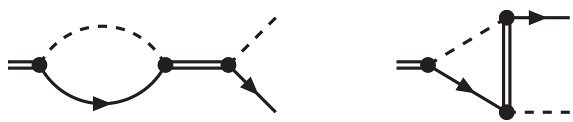
$\gamma_{N,L}$ is unimportant.



An outstanding challenge is to compute $\gamma_{L,N}$ at $T \sim M$.

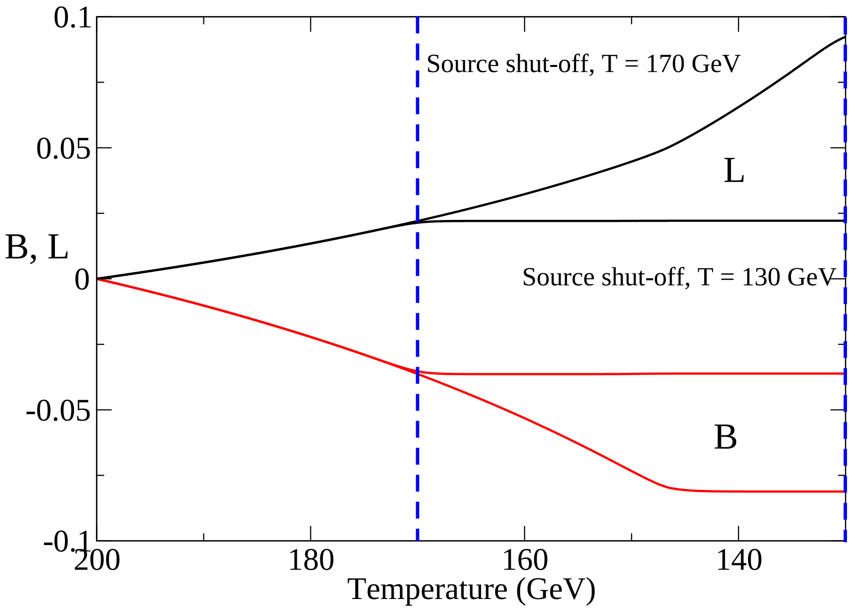
Physics: An ensemble of right-handed neutrinos ($n_N - n_{\text{eq}} \neq 0$) can decay with $\Gamma(N \to H\nu) \neq \Gamma(N \to H\bar{\nu})$ and yield $n_L \neq 0$.

Similarly to CP-violation in the kaon system, the origin might be "indirect" (related to oscillations) or "direct" (related to decays).



(My guess: it makes physical sense to go after NLO only if can do this in the relativistic and ultrarelativistic regimes.)

In the end n_L is converted to Ω_b through "sphalerons".[4]



[4] Rate is known with few percent accuracy through M. D'Onofrio, K. Rummukainen and A. Tranberg, *The Sphaleron Rate in the Minimal Standard Model*, 1404.3565.

Summary

Conceptual problem (non-equilibrium): factorization $\rightarrow$ define a finite number of coefficients that can be computed in equilibrium. (In the simplest scenario, there are just three coefficients.)

Technical problem (infrared divergences): the computation of the coefficients requires all-orders resummations of the loop expansion at temperatures larger than the particle mass.

Phenomenological consequences remain to be explored.