



Friction and fracture: stick-slip motion

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Frictional shear cracks

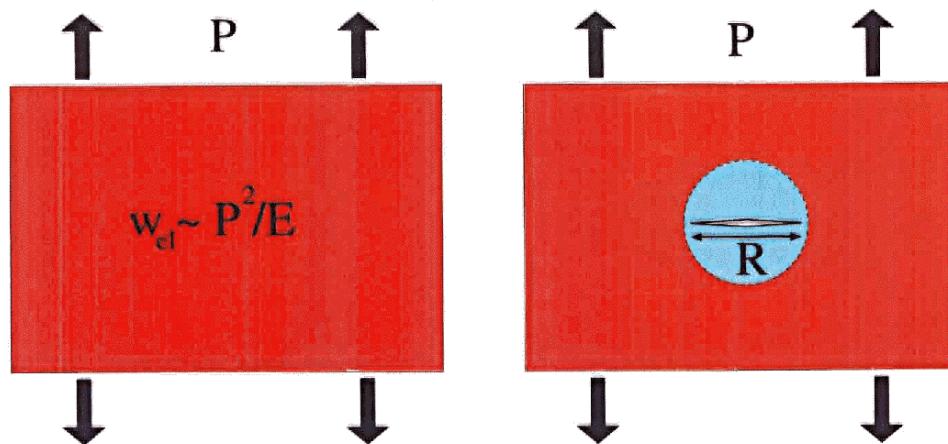
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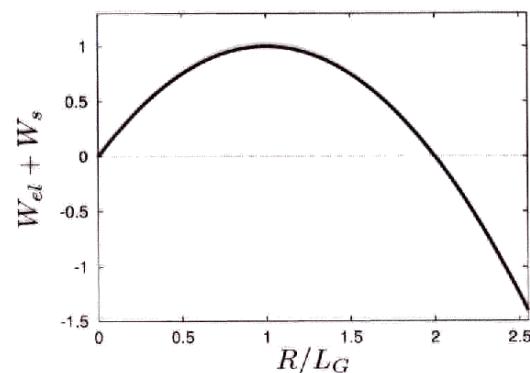
1. Introduction: two essentially independent lines of research
 - Fracture Mechanics
 - Friction
2. Continuum Model for Frictional shear cracks: Synthesis of ideas
 - Two states of the interface
 - Steady state motion of the solid body
 - Stick-Slip motion below the critical velocity



Theory of Cracks

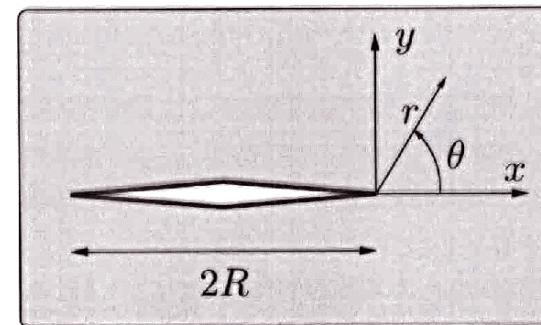
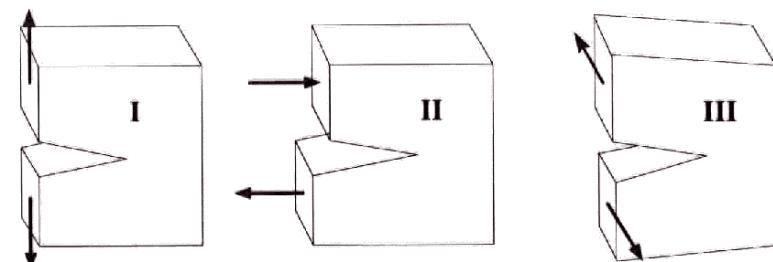


- Elastic relaxation in the area $\sim R^2$: $W_{el} \sim -\frac{P^2 R^2}{E}$
- Increase of surface energy: $W_s \sim \alpha R$



Griffith length: $L_G \sim \frac{E\alpha}{P^2}$

Near tip behavior



$$\sigma_{ij} = \frac{K}{r^{1/2}} f_{ij}(\theta)$$

with a *universal* function $f_{ij}(\theta)$ for each loading mode.

stress intensity factor: $K \sim PR^{1/2}$ contains the full information about the crack.

Griffith equilibrium: $K^2/E \sim \alpha \Rightarrow$ selection of R

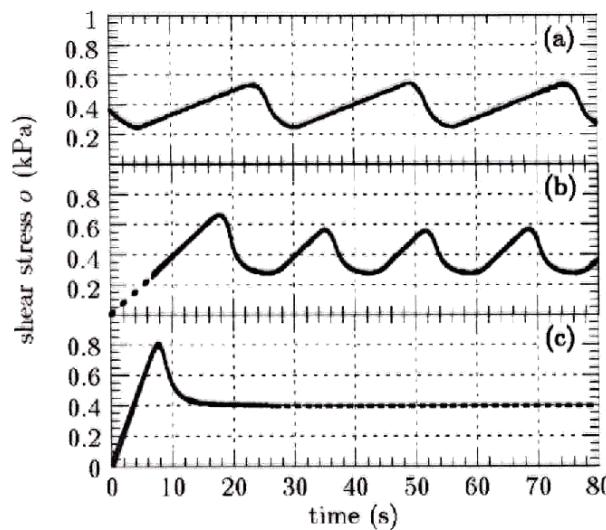
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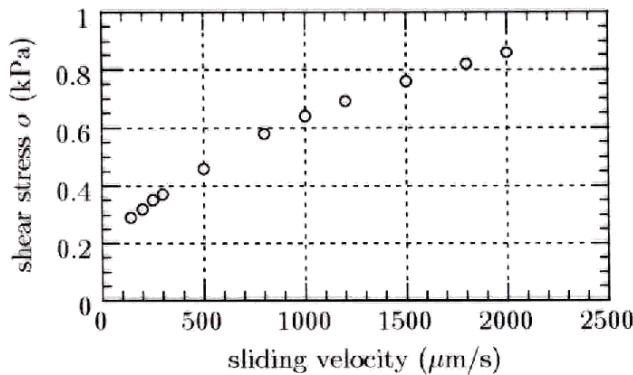
Experimental results

(T.Baumberger, C.Caroli, O.Ronsin, PRL **88**, 075509, (2002))



Two dynamical behaviors:

- $V < V_c$
stick slip (a,b)
- $V > V_c$
uniform slip (c)



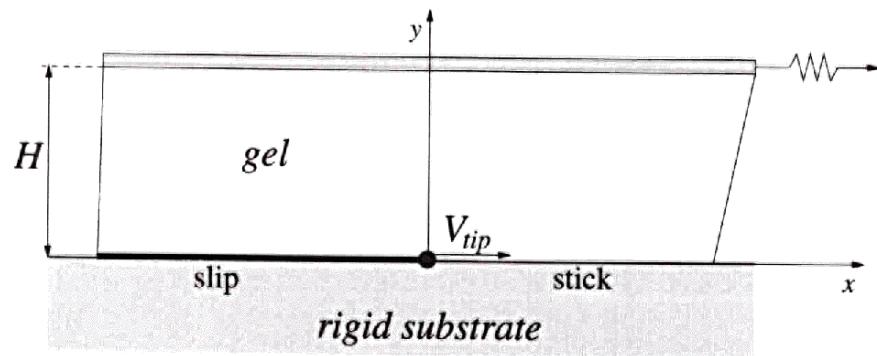
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Theoretical model

Relaxation of the shear stress with the slipping region advance:



Two different states of the gel-glass surface,
fracture surface energy $\gamma > 0$ independent of V_{tip}
Linear elasticity, displacement field $\mathbf{u} = (u_x, u_y)$.

$$x \rightarrow -\infty : \quad u_{xy} = 0, \quad \sigma_{xy} = 0$$

$$x \rightarrow +\infty : \quad u_{xy} \rightarrow u_{xy}^\infty, \quad \sigma_{xy} \rightarrow \sigma_{xy}^\infty = 2\mu u_{xy}^\infty$$

plane strain ($u_z = 0$)

$y = 0, x > 0$: $u_y = 0$ and $u_x = 0$ (stick)

$y = 0, x < 0$: $u_y = 0$ and given σ_{xy} as a function of u_x (slip)

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Griffith threshold, uniform slipping

Fracture surface energy γ

Energy balance \Rightarrow crack propagating with $V_{tip} > 0$ if

$$\Delta \equiv \frac{(\sigma_{xy}^\infty)^2 H}{2\mu\gamma} > 1$$

Finite drag force is necessary for slipping

Stable steady sliding $\Rightarrow \Delta \geq 1$, uniform $\sigma_{xy} \geq \sqrt{2\gamma\mu/H}$

Suppose linear viscous friction at $x < 0$: $\sigma_{xy} = \alpha \dot{u}_x$,

$$\sigma_{xy} = \alpha V$$

Uniform slipping is stable against healing (sticking) if the drag speed $V > V_c$,

$$V_c = S \sqrt{\frac{2\gamma}{\mu H}}, \quad S = \frac{\mu}{\alpha}$$

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Exact solution

Viscous friction: $\sigma_{xy} = \alpha \dot{u}_x$ at $x < 0$, $y = 0$

Uniform boundary condition \Rightarrow
uniform solution with $\lambda = 1/2 + \varepsilon$:

$$u_x = A \operatorname{Re} [y(x+iy)^{\lambda-1} - i(3-4\nu)(x+iy)^\lambda / \lambda]$$

$$u_y = A \operatorname{Re} [iy(x+iy)^{\lambda-1}]$$

$$\varepsilon \approx \frac{1}{2\pi} \frac{3-4\nu}{1-\nu} \frac{V_{tip}}{S}$$

$$\text{Energy balance} \implies \varepsilon = \frac{\ln \Delta}{\ln(H/a)}$$

$$V_{tip} = 2\pi \frac{1-\nu}{3-4\nu} \frac{\ln \Delta}{\ln(H/a)} S, \quad \varepsilon \ll 1$$

\rightsquigarrow Approximate solution if $\Delta \ll 1$

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Approximate solution

Assume $\sigma_{xy} = 0$ at $x < 0$, $y = 0$

Quasistatic approximation \Rightarrow singularity with $\lambda = 1/2$:

$$\begin{aligned} u_x &= A \operatorname{Re} [y(x+iy)^{\lambda-1} - i(3-4\nu)(x+iy)^\lambda/\lambda] \\ u_y &= A \operatorname{Re} [iy(x+iy)^{\lambda-1}] \end{aligned}$$

(valid if $\sqrt{x^2 + y^2} \lesssim H$)

Energy flow density $j_i = \sigma_{ik}\dot{u}_k + \frac{1}{2}\sigma_{jk}u_{jk}V_{tip}^i$

Elastic energy flux into the crack tip

$$J_0 = 2\pi\mu(3-4\nu)(1-\nu)V_{tip}A^2$$

equals the surface energy change

$$J_0 = \gamma V_{tip}$$

$$\Rightarrow A = A(\gamma)$$

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The global energy conservation law is

$$J_0 + J_d = \mu(u_{xy}^\infty)^2 HV_{tip}$$

where J_d is the energy flux through the sliding surface (energy release due to the friction).

$$J_d = \int_{-\infty}^0 dx \sigma_{xy} \dot{u}_x ,$$

the main logarithmic contribution is

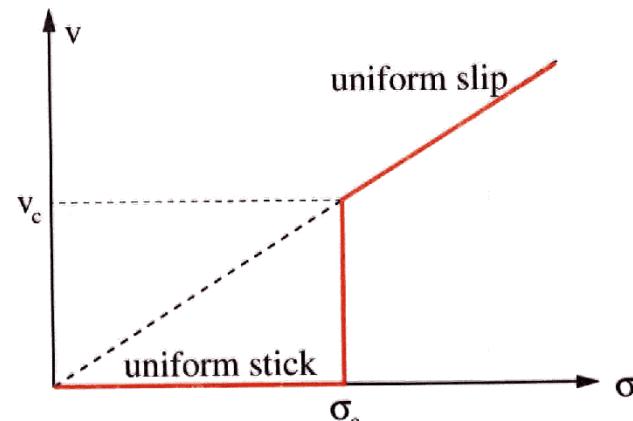
$$J_d = \alpha(3-4\nu)^2 A^2 \ln\left(\frac{H}{a}\right)$$

At scales smaller than a
linear theory (elasticity, viscous friction) is not valid.

$$V_{tip} = 2\pi \frac{1-\nu}{3-4\nu} \frac{\Delta-1}{\ln(H/a)} S , \quad \Delta-1 \ll 1$$

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Phase diagram: Velocity vs. shear stress**Given stress**

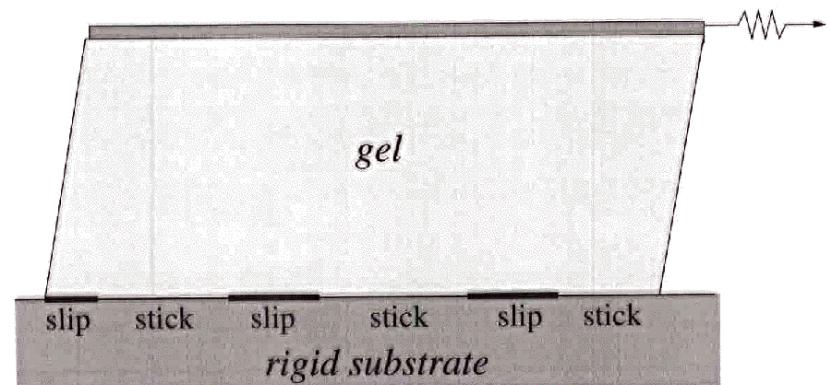
- Uniform stick for $\sigma < \sigma_c$
- Uniform slip for $\sigma > \sigma_c$

Given velocity

- Uniform slip for $v > v_c$
- Stick-slip motion for $v < v_c$
- Average shear stress: $\sigma = \sigma_c$

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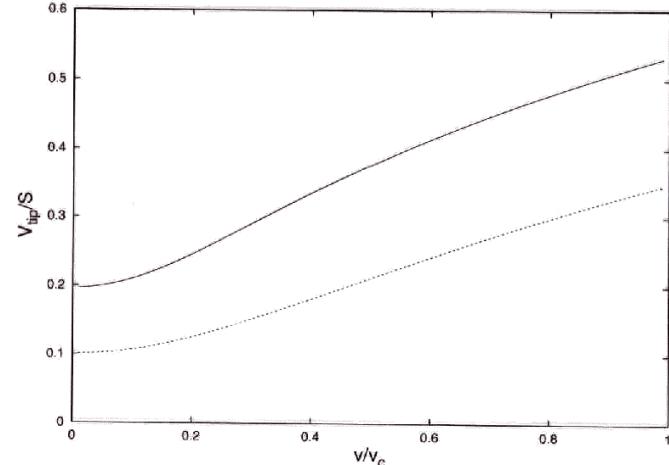
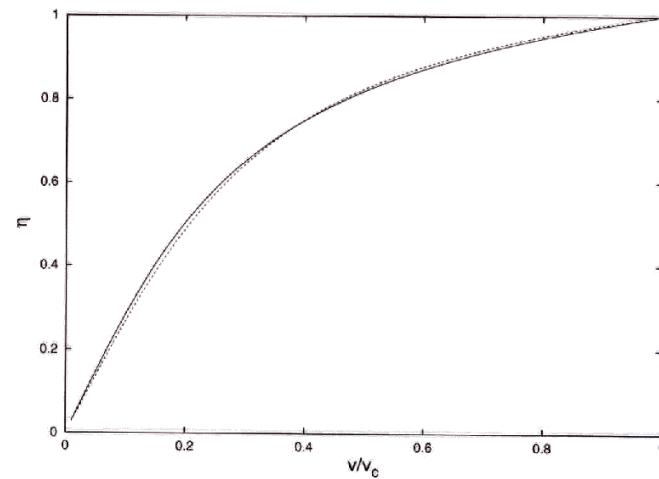
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Periodic stick slip motionPulling velocity v Crack velocity V_{tip} Stress σ Periodicity Λ Fraction of slip phase η

- Only two equations of motion to determine four parameters for given pulling velocity v
⇒ two degrees of freedom

Results

$$\sigma = \sigma_c, \quad \min\{\Lambda_{stick}, \Lambda_{slip}\} \approx H.$$



The beauty of fracture...

