Experimental Control of Spatio-temporal Chaos in Parametrically Driven Surface Waves

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Talk Outline

- Motivation: Multi-frequency parametrically amplified surface waves: Why should you care?
- Review of theory/expt of two-frequency surface waves (Getting to know the neighborhood)
- Characterizing Spatio-temporally disordered states: A method to the madness
- Control of disordered states

Well -studied nonlinear systems:

Patterns: Single excited spatial mode + secondary instabilities e.g. R-B, Couette-Taylor, Faraday Instability...

Chaos: Order - Disorder in the temporal domain

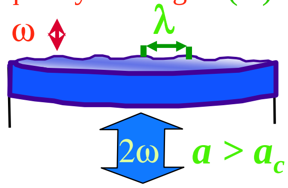
Turbulence: A large number of concurrently excited modes

Motivation:

- What nonlinear states are selected when a few distinctly different nonlinear modes are allowed to interact?
- What mechanisms govern their interactions and selection?
- What are the routes to complexity in both space and time?
- When a large number of spatially extended nonlinear states are concurrently possible in the system, how can we control the state of the system?

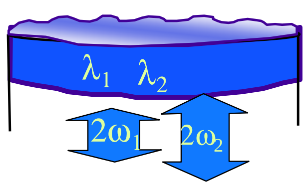
The 2-frequency Faraday Instability

Single Frequency Forcing : $a(\omega) = A\cos(2\omega t)$



$\omega \Leftrightarrow \mathbf{k}$

Two Frequency Forcing: $a(\omega_1,\omega_2) = A_1\cos(2\omega_1 t)+A_2\cos(2\omega_2 t+\phi)$



$\omega_1 \Leftrightarrow \mathbf{k}_1$

$\omega_2 \Leftrightarrow \mathbf{k}_2$

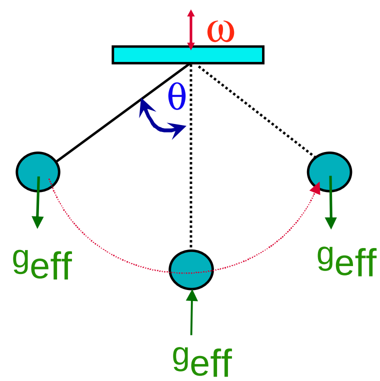
Question:

What states are selected by nonlinear interactions between k_1 and k_2 ?

Why is the fluid response subharmonic ($\omega/2$) ?

$$g_{eff} = g + a\sin(\omega t)$$

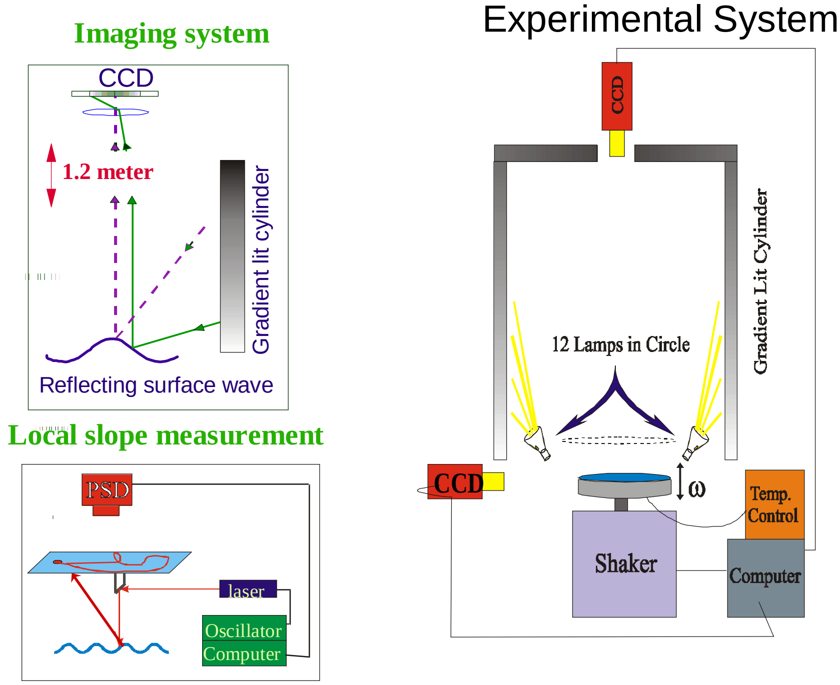
$$d^2\theta/dt^2 = g_{eff}(t)\,\theta = [g + (a\cdot\sin(\omega t)]\theta + N.L$$



The most effective forcing occurs when the pendulum is accelerated downward when its amplitude is maximal

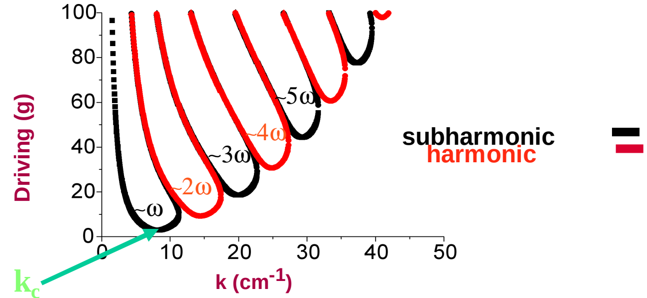
=> g_{eff} oscillates at twice the response frequency

Experimental System



Linear stability analysis: K. Kumar and L. S. Tuckerman , JFM **279**, 49 (1994).

Single Frequency Driving: $a(\omega) = A\cos(2\omega t)$



Spatially: Each tongue describes a well-defined **wave number**

Temporally: Each tongue is described by an infinite series of harmonics having a **well-defined parity when** $t \rightarrow t + \pi/\omega_0$ **:**

(odd parity) $A \rightarrow -A$ or **(even parity)** $A \rightarrow A$

Even parity ⟹ **Harmonic tongue** ⟹ $\sum_n A_n \exp[i\frac{2n}{2}\omega_0 t]$

Odd parity ⟹ **Subharmonic tongue** ⟹ $\sum_n B_n \exp[i\frac{2n+1}{2}\omega_0 t]$

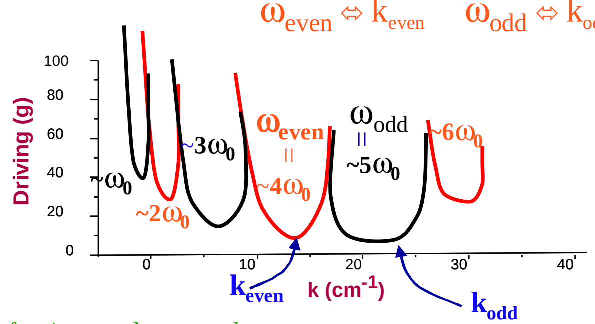
Linear stability analysis:

Besson, Edwards, Tuckerman Phys. Rev **E54**, 507 (1996)

2-frequency Driving:

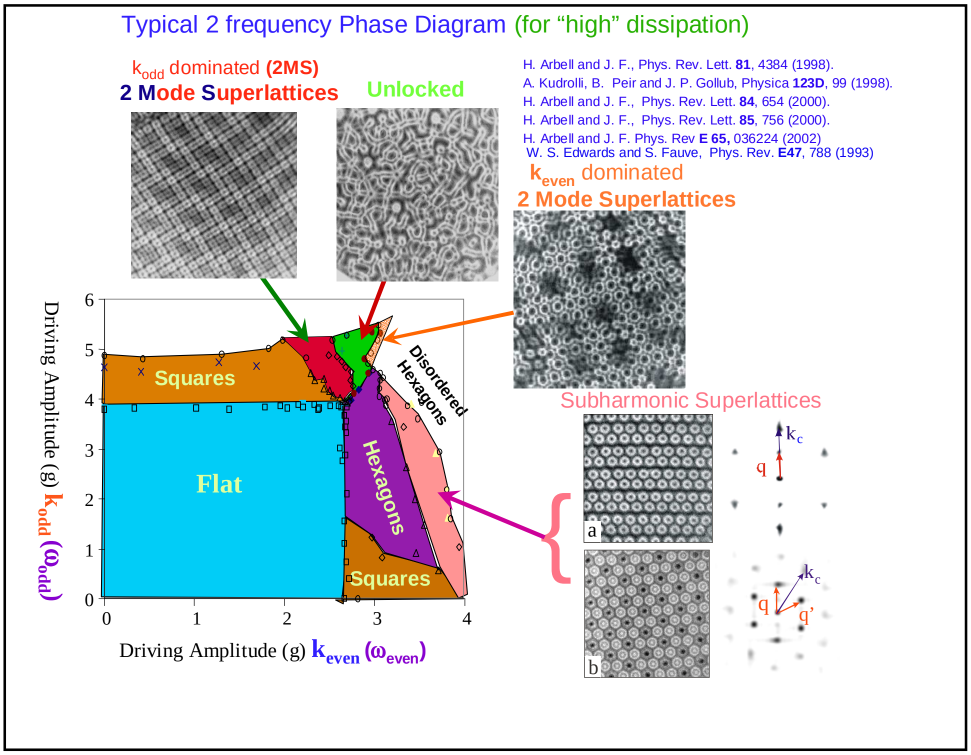
$$a(\omega_{even},\omega_{odd}) = A_{even}\cos(2\underbrace{\mathbf{n_1}\cdot\omega_0}_{\omega_{even} \Leftrightarrow k_{even}} t)+A_{odd}\cos(2\underbrace{\mathbf{n_2}\cdot\omega_0}_{\omega_{odd} \Leftrightarrow k_{odd}} t+\phi)$$

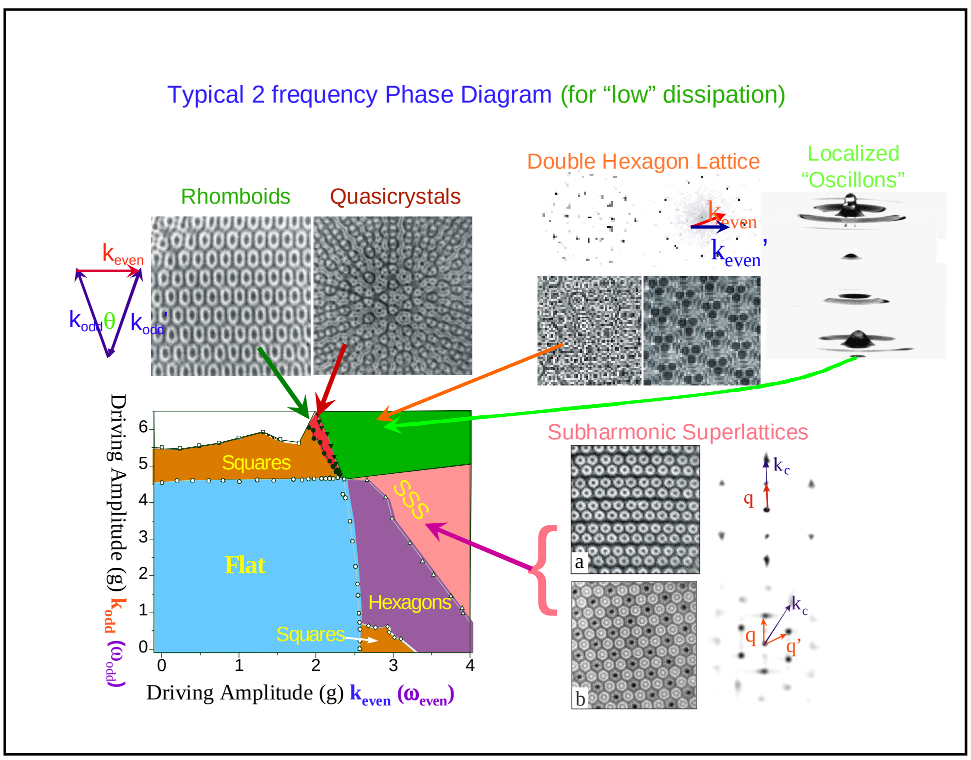
For:
$n_1 = 4$ $n_2 = 5$
$A_{even/odd} \sim A_c$
(Bicritical Point)

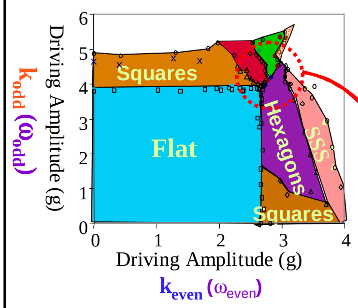


As for single frequency forcing, each tongue has:

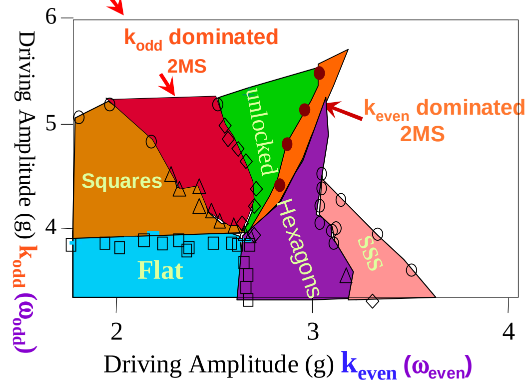
- **A well-defined critical wave number**
- A **well-defined temporal parity**
- **A dominant frequency of** $\sim \omega_i /2 = n_i\cdot\omega_0 \Leftrightarrow k_i$
- **An important difference: New stable tongues corresponding to multiples of ω_0**


Typical 2 frequency Phase Diagram (for "high" dissipation)
k_odd dominated (2MS)
2 Mode Superlattices
Unlocked
H. Arbell and J. F., Phys. Rev. Lett. 81, 4384 (1998).
A. Kudrolli, B. Peir and J. P. Gollub, Physica 123D, 99 (1998).
H. Arbell and J. F., Phys. Rev. Lett. 84, 654 (2000).
H. Arbell and J. F., Phys. Rev. Lett. 85, 756 (2000).
H. Arbell and J. F. Phys. Rev E 65, 036224 (2002)
W. S. Edwards and S. Fauve, Phys. Rev. E47, 788 (1993)
k_even dominated
2 Mode Superlattices
Disordered Hexagons
Squares
Flat
Hexagons
Squares
Subharmonic Superlattices
Driving Amplitude (g) k_odd (ω_odd)
Driving Amplitude (g) k_even (ω_even)


Typical 2 frequency Phase Diagram (for "low" dissipation)
Rhomboids
Quasicrystals
Double Hexagon Lattice
Localized "Oscillons"
k_even
k_odd
θ
k_odd'
Squares
Flat
SSS
Hexagons
Squares
Subharmonic Superlattices
Driving Amplitude (g) k_odd (ω_odd)
Driving Amplitude (g) k_even (ω_even)



From now on: we'll concentrate on the area of phase space in the vicinity of the bicritical point for "dissipative" systems

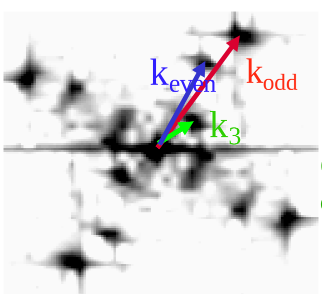


The 2MS state is constructed using ***both*** $\mathbf{k_{even}}$ and $\mathbf{k_{odd}}$.

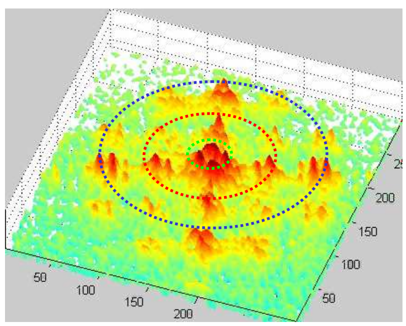
k_{even} and k_{odd} orientations are determined by:

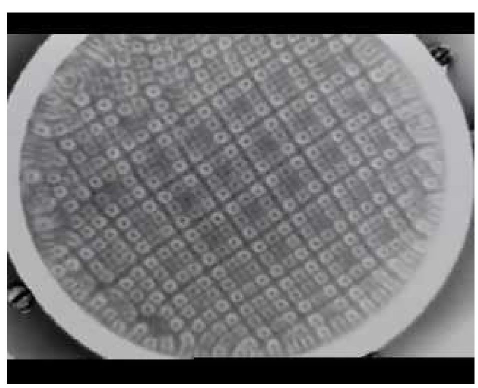
Multiwave resonant mixing in *time* and *space*

$$\begin{cases} \omega_{odd} - \omega_{even} = \omega_3 \\ \vec{k}_{odd} - \vec{k}_{even} = \vec{k}_3 \end{cases}$$

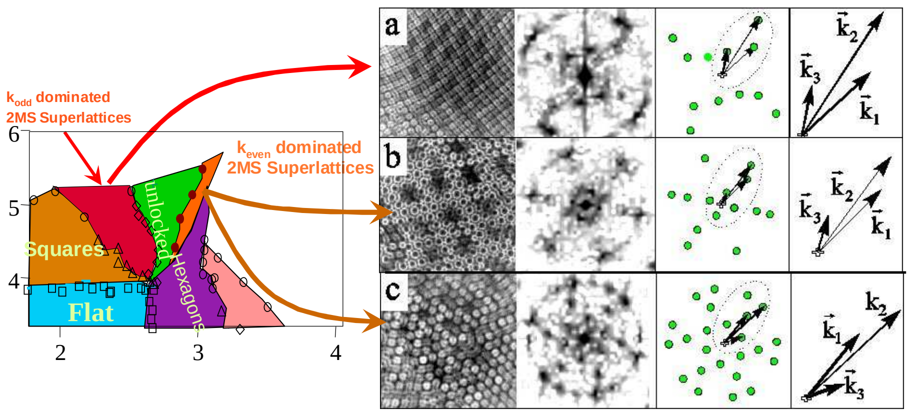


Observed for all parities and driving frequency combinations





2MS superlattices can have different underlying symmetries

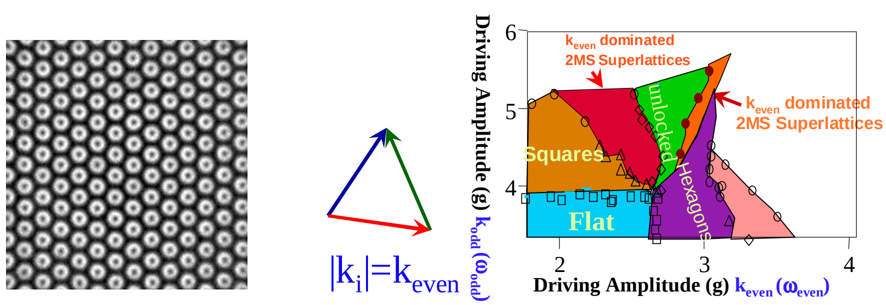


The 2MS superlattice state inherits the spatial symmetry of the dominant wave number

➔(harmonic) k_{even} ⇔ hexagonal/square symmetry;

➔ (subharmonic) k_{odd} ⇔ square symmetry

Hexagons

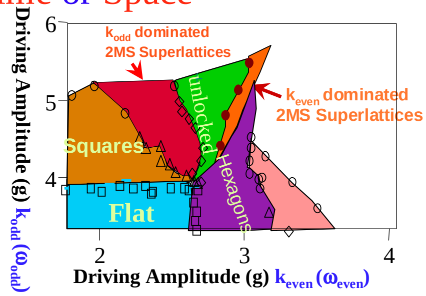


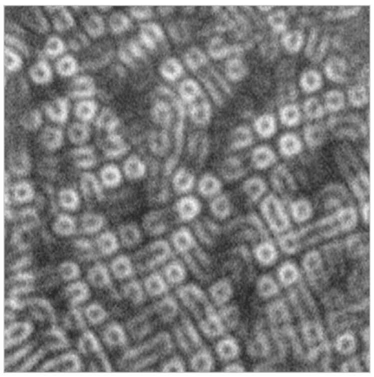
- Three wave coupling of degenerate wavevectors of length $k(\omega_{even}) = k_{even}$
- Temporally harmonic (even parity state)
- No odd-parity wavenumbers in the spatial spectrum

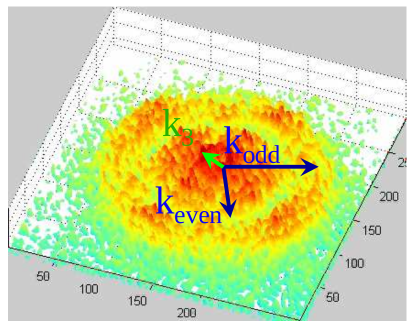
"Unlocked" States: 2 Wave Numbers

No Phase Locking in Time or Space

- Both k_{even} and k_{odd} present
- No locking or well-defined symmetry in space
- "chaotic" temporal behavior

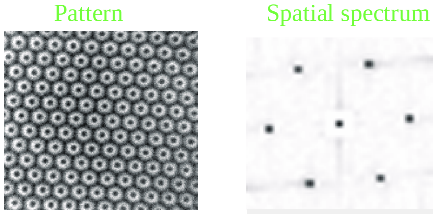






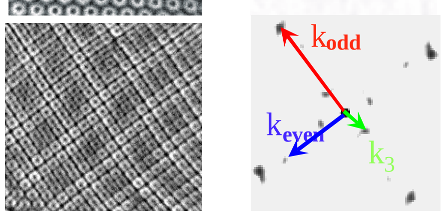
Characterizing the symmetry of a given state

Pattern | Spatial spectrum

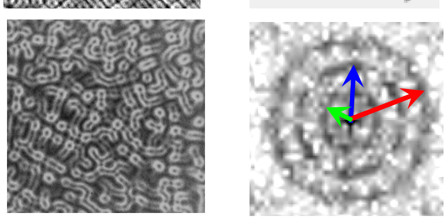


Simple Patterns
- Single wave number
- Well defined symmetry

(Hexagons 60 degree symmetry)



Superlattice states
- Multiple wave numbers
- Each wave number could have its own symmetry



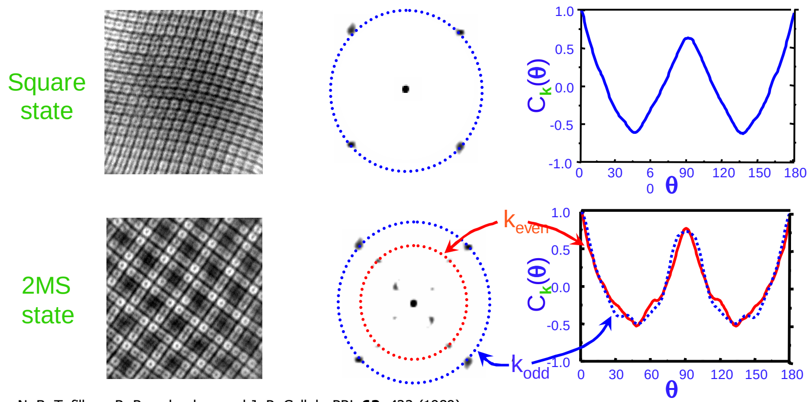
Unlocked States
- Multiple wave numbers
- No well-defined symmetry

How does one quantify the degree of symmetry of a given wave number?

Angular Correlation Functions

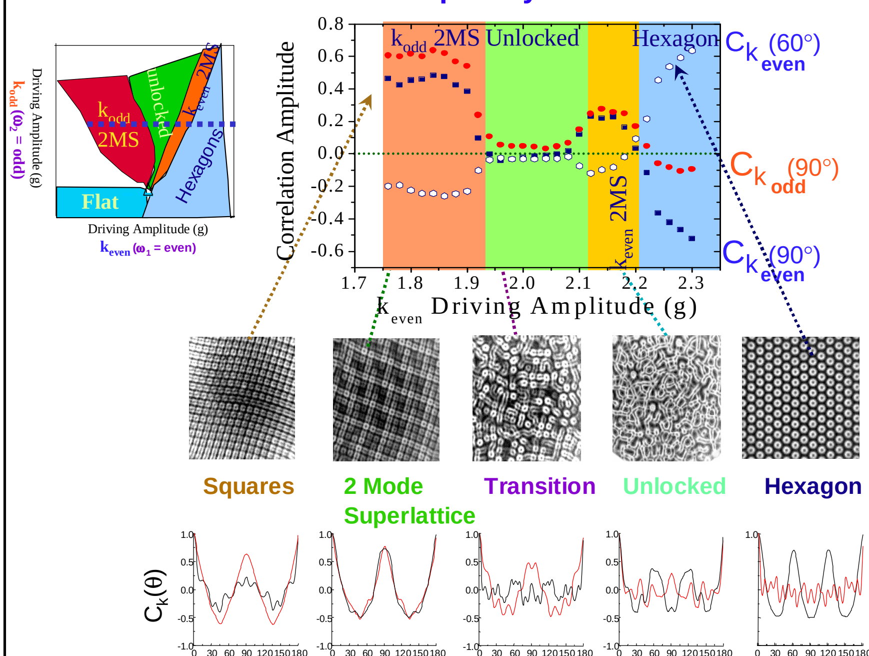
The degree of correlation of a **given wave number** can be quantified by means of the angular correlation function ⇔ $C_k(\theta)$:

$$C_k(\theta) = \langle |1/f_k| \rangle^2 \Sigma_\alpha \left[f_k(\alpha) \cdot f_k(\alpha+\theta) \right]$$

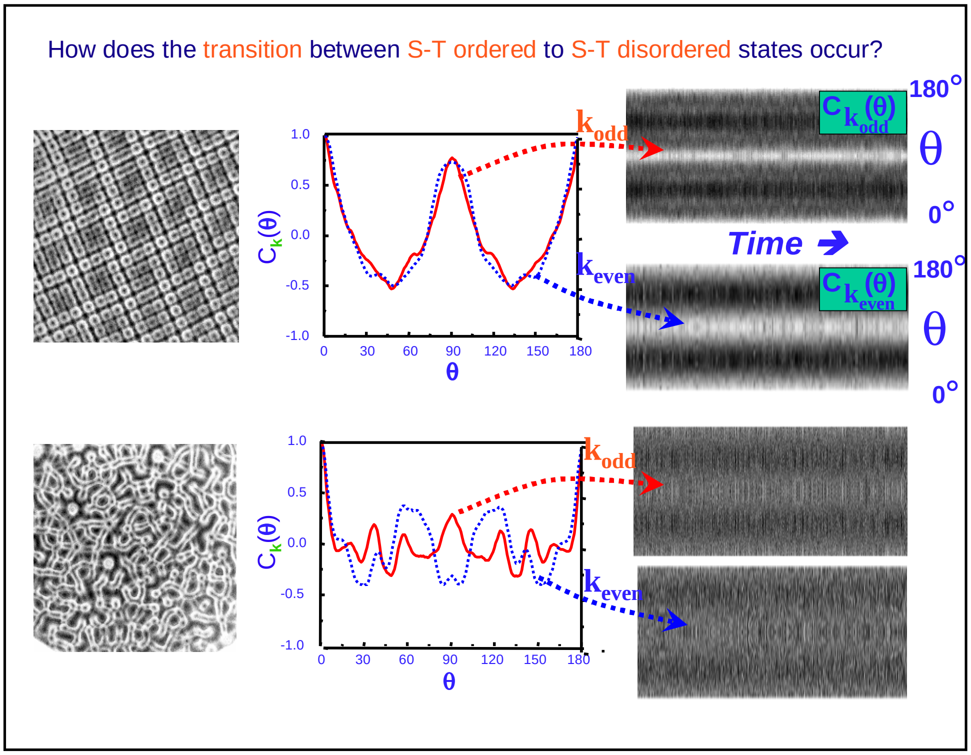


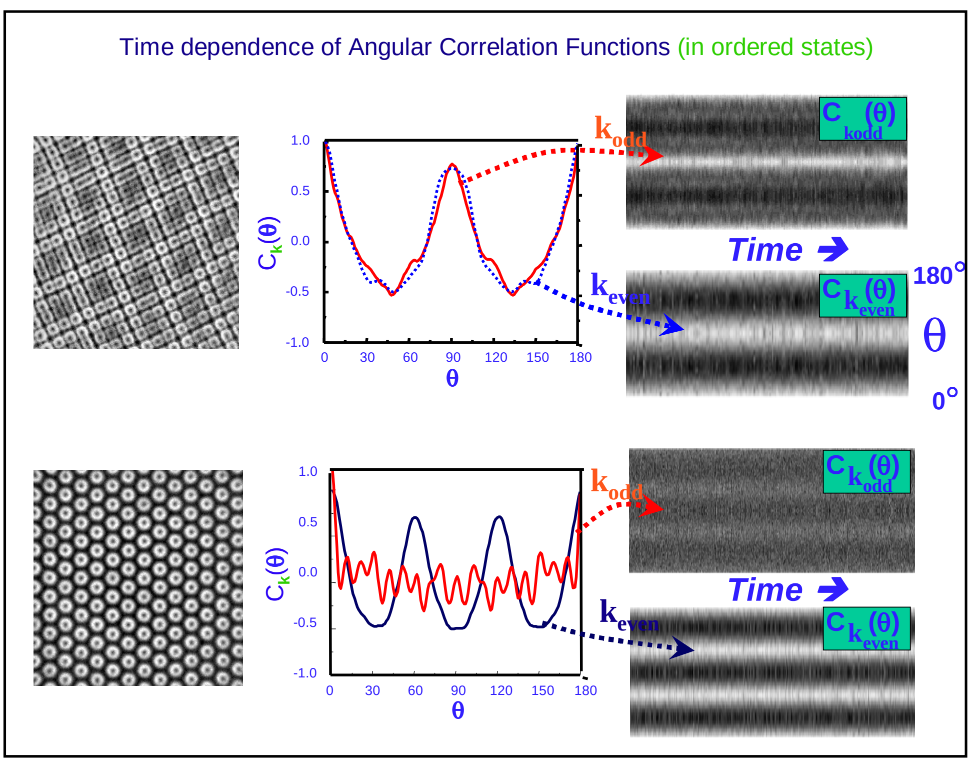
N. B. Tufillaro, R. Ramshankar, and J. P. Gollub, PRL **62**, 422 (1989).
D. Binks, M. T. Westra, and W. van de Water, PRL **79**, 5010 (1997).
H. Arbell and J. Fineberg, PRL **81**, 4384 (1998).

The transition between the spatially locked and unlocked states

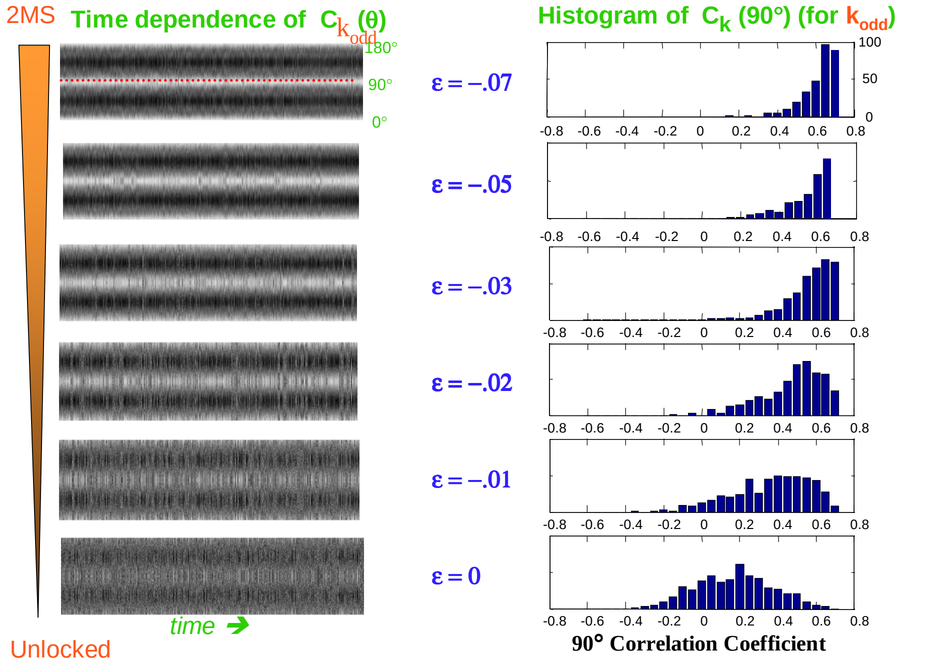


$C_k(\theta) = \langle |1/f_k| \rangle^2 \Sigma_\alpha [f_k(\alpha) \cdot f_k(\alpha+\theta)]$ Angle (degrees)


How does the transition between S-T ordered to S-T disordered states occur?
$C_k(\theta)$
θ
k_{odd}
k_{even}
$C_{k_{odd}}(\theta)$
$C_{k_{even}}(\theta)$
180°
0°
Time

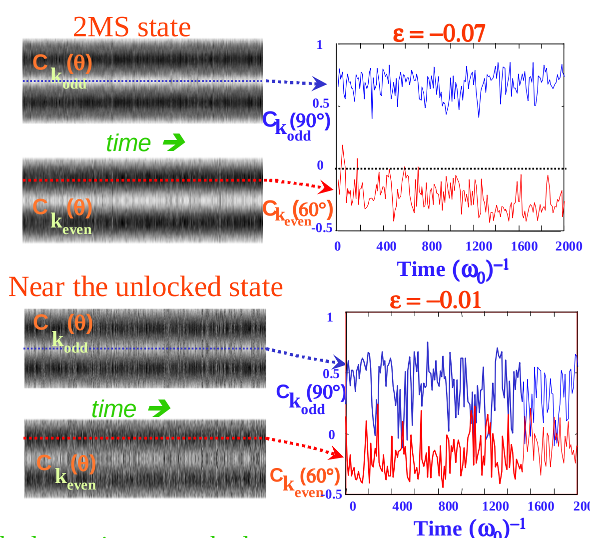

Time dependence of Angular Correlation Functions (in ordered states)
$C_k(\theta)$
θ
k_{odd}
k_{even}
$C_{k_{odd}}(\theta)$
$C_{k_{even}}(\theta)$
180°
0°
Time

How does the transition from order to disorder occur?



The loss of order occurs gradually as the unlocked phase boundary (ε=0) is approached

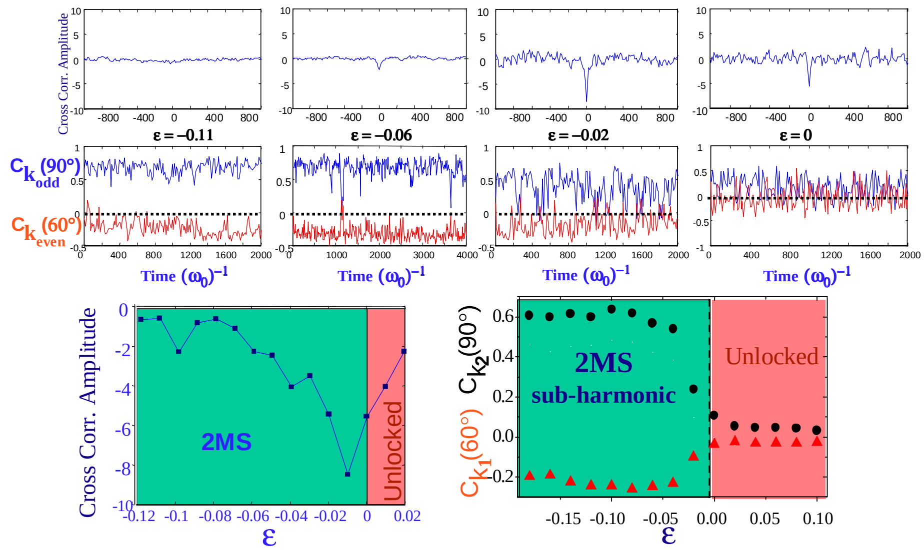
What mechanism leads to disorder in the unlocked state?



As the unlocked state is approached:

• The angular correlations of each wave number show signs of intermittency with increased fluctuations of the angular correlation as the phase boundary (ε=0) is approached

The loss of coherence of the unlocked state is due to **competition** between modes with different **symmetries**



As the unlocked state is approached:

- Fluctuations of the angular correlation increase
- The fluctuations of the symmetries of the two wave numbers are correlated
- Momentary decreased 90° correlation of k_{odd} ⇔ increased 60° correlation of k_{even}

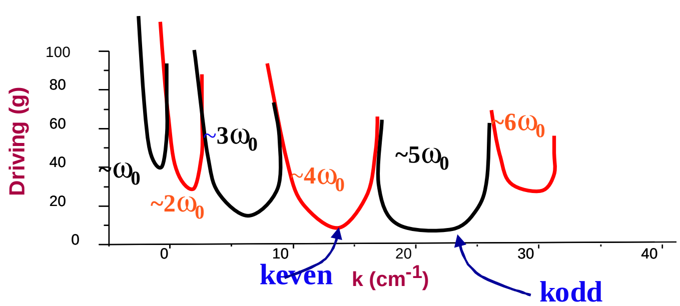
Open loop control and selection of nonlinear states by the use of a third driving frequency

We can now understand the unlocked state:

The loss of coherence is due to **competition** between degenerate modes that possess different *temporal and spatial* **symmetries**

Reminder:

- Each driving frequency generates linearly stable tongues having the same temporal parity as the driving



Open Loop Control[1] *and selection of nonlinear states*

- by the use of a third driving frequency

We can now understand the unlocked state:

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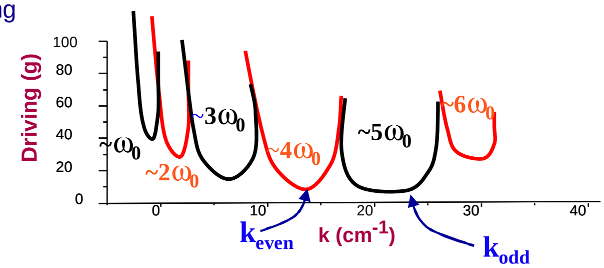
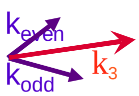


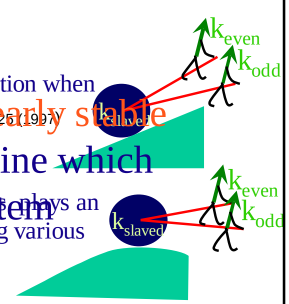
- The coupling (or "slaving") of linearly stable states to the excited ones can determine[2] which nonlinear state is selected by the system

[1] Y. Braiman and I. Goldhirsch, Phys. Rev. Lett. **66**, 2545 (1991)

[2] J. Porter and M. Silber, PRL **89**, 084501 (2002)



The influence of 3-wave interactions and symmetries on the selected nonlinear states



- Far from the bicritical point ➔ mechanism for enhanced dissipation when unstable states couple to stable (slaved) ones Zhang and Vinals, JFM 341, 225 (1997)

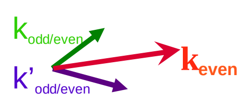
• The coupling (or "slaving") of linearly stable states to the excited ones can determine which nonlinear state is selected by the system

- Near the bicritical point ➔ coupling to stable near-critical modes plays an important role in pattern selection; either enhancing or suppressing various nonlinear states M. Silber, C. M.Topaz, and A. C. Skeldon, Physica **D143**, 205 (2000)

- Modes of the same parity can only couple to harmonic modes M. Silber and A. C. Skeldon, Phys. Rev. **E59** 5446 (1999)

Spatial resonance condition: $\vec{k}_i + \vec{k}_j = \vec{k}_{harmonic}$

Temporal resonance condition:
odd + odd = **even** parity
even + even = **even** parity



- On the harmonic side of the phase diagram: selected states belong to invariant subgroups of the broken hexagonal symmetry (each state has different spatial and temporal symmetries) D.P. Tze, A. M. Rucklidge, R.B. Hoyle, and M. Silber, Physica **146D** 367 (2000)

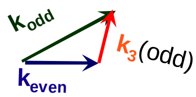
- Near the bicritical point broken approximate symmetries may determine the selection of nonlinear states. J. Porter and M. Silber, PRL **89**, 084501 (2002)

We will show that:

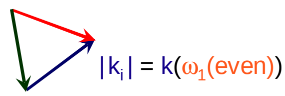
By regulating both the amplitude and parity of a third driving frequency, **Ω**, it is possible to both stabilize and control the overall nonlinear state of the system

Examples:

Unlocked + Ω_{odd} ➔ Ω couples to wavenumbers with odd parity
➔ k_{odd} (odd parity) - dominated 2MS states

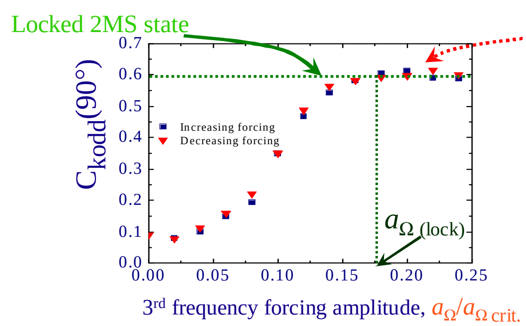


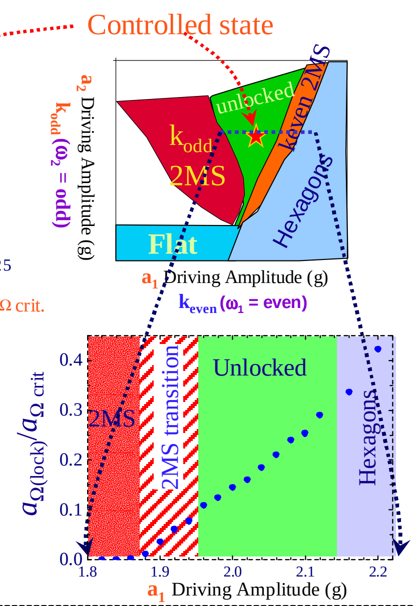
Unlocked + Ω_{even} ➔ Ω couples to wavenumbers with even parity
➔ Hexagons or k_{even} (even parity) -dominated 2MS states



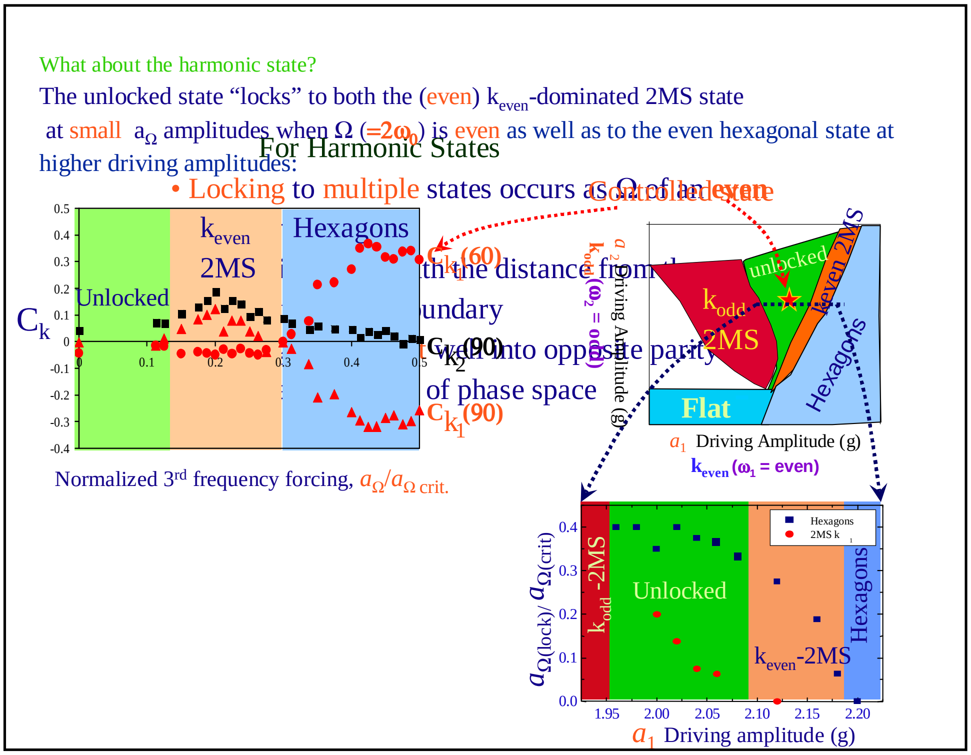
Does the control work?

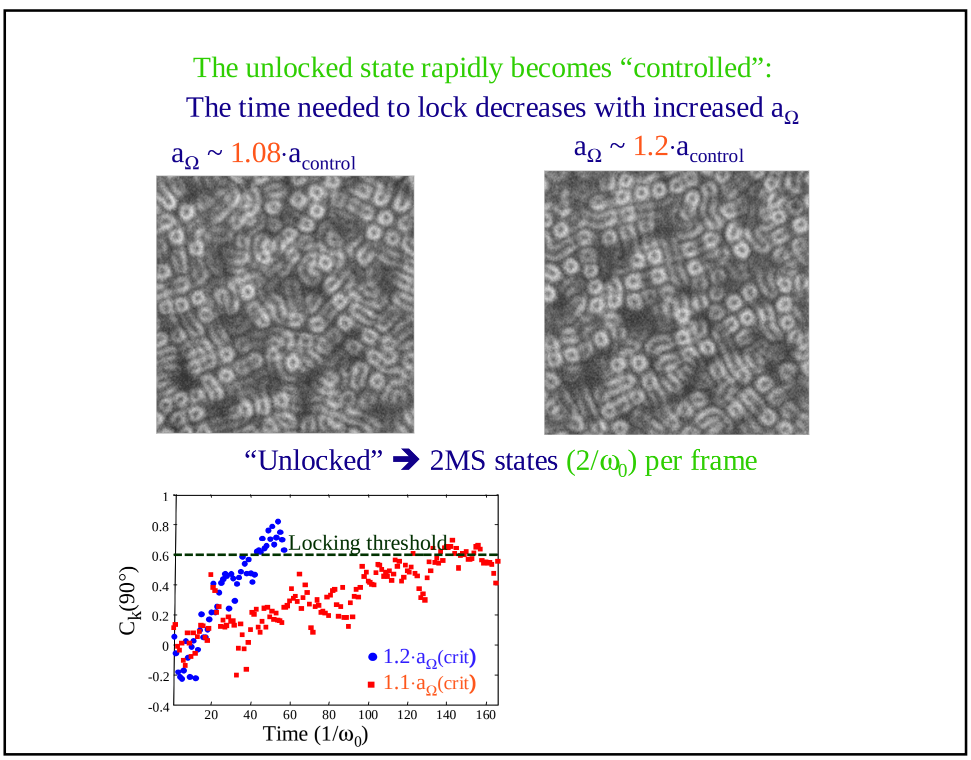
The unlocked state "locks" to the (odd) k_{odd}-dominated 2MS state at small a_Ω amplitudes when Ω (=ω_0) is odd





- Locking occurs at small a_Ω amplitudes
- $a_{\Omega(lock)}$ increases with the distance from the unlocked phase boundary
- Locking can persist well into opposite parity -dominated regions of phase space


What about the harmonic state?
The unlocked state "locks" to both the (even) k_even-dominated 2MS state
at small a_Ω amplitudes when Ω (=2ω_0) is even as well as to the even hexagonal state at higher driving amplitudes:
For Harmonic States
• Locking to multiple states occurs as Ω of an even
Controlled state
Unlocked
k_even 2MS
Hexagons
C_k
C_k1(60)
C_k2(90)
C_k1(90)
Normalized 3rd frequency forcing, a_Ω/a_Ω crit.
k_odd 2MS
unlocked
k_even 2MS
Hexagons
Flat
a_1 Driving Amplitude (g)
k_even (ω_1 = even)
k_odd (ω_2 = odd)
a_Ω(lock)/a_Ω(crit)
k_odd -2MS
Unlocked
k_even-2MS
Hexagons
a_1 Driving amplitude (g)

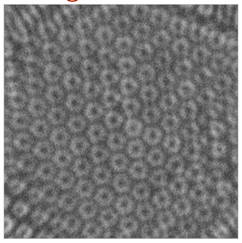

The unlocked state rapidly becomes "controlled":
The time needed to lock decreases with increased a_Ω
a_Ω ~ 1.08·a_control
a_Ω ~ 1.2·a_control
"Unlocked" ➔ 2MS states (2/ω_0) per frame
Locking threshold
C_k(90°)
1.2·a_Ω(crit)
1.1·a_Ω(crit)
Time (1/ω_0)

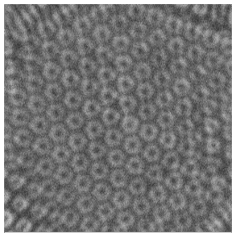
By switching between two control frequencies, rapid transitions between different states are accomplished

When switching: control is achieved in times of order of a single time step (~ $1/\omega_0$):

Hexagons ➔ 2MS

Transitions in slow motion





2MS ➔ Hexagons

Conclusions:

• Open-loop control of nonlinear states with different ST behavior can be performed by small perturbations having well-defined temporal symmetries

• We have presented the following specific examples:
  With even forcing: ST chaotic (unlocked) state ➔ either hexagons or harmonic 2MS states
  With odd forcing: ST chaotic (unlocked) state ➔ subharmonic 2MS states
  Subharmonic 2MS ⇔ Harmonic hexagons or Harmonic 2MS states

• This type of control should be generally relevant for any parametrically forced system
  e.g. nonlinear optics; forced mechanical systems; globally forced reaction diffusion

• Next…
  Use shaped controlling waveforms for more efficient control??
  Pattern engineering by incorporation of additional types of symmetries in the driving??
  …