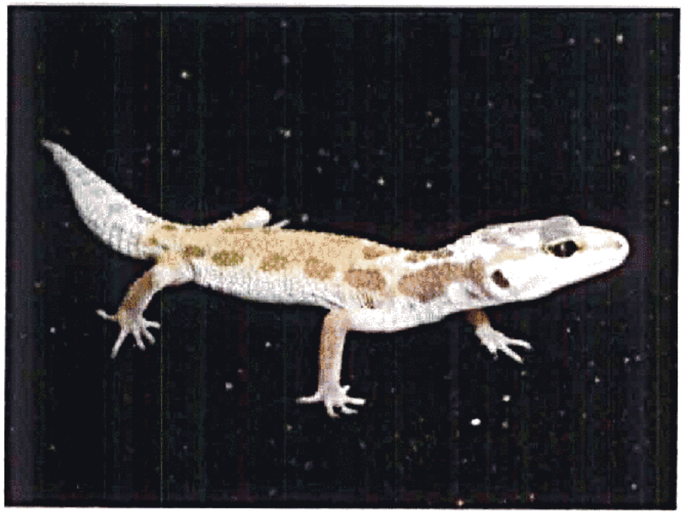
Contact Mechanics, Adhesion and Rubber Friction

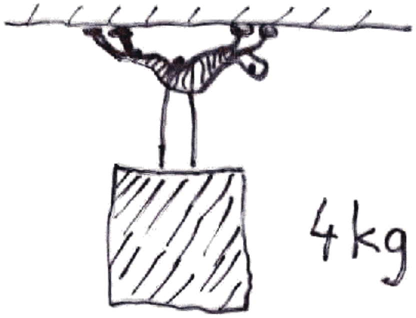
Bo Persson
IFF FZ-Juelich, Germany

* CONTACT MECHANICS
* ADHESION
* RUBBER FRICTION

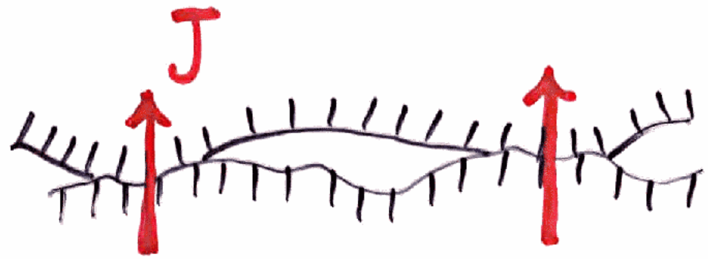
Sliding Friction: Physics Principles and Applications, Second Edition, Springer (2000)

Surface Science Reports 33, 83 (1999)

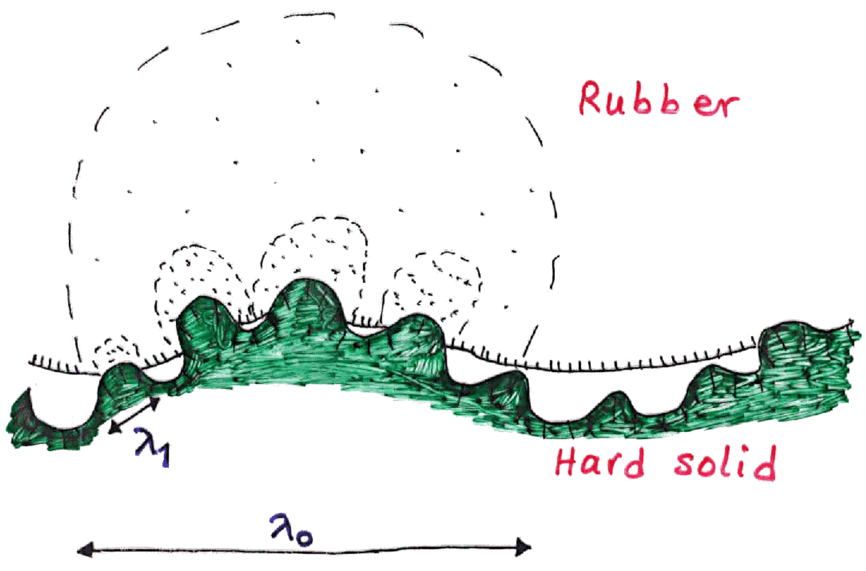




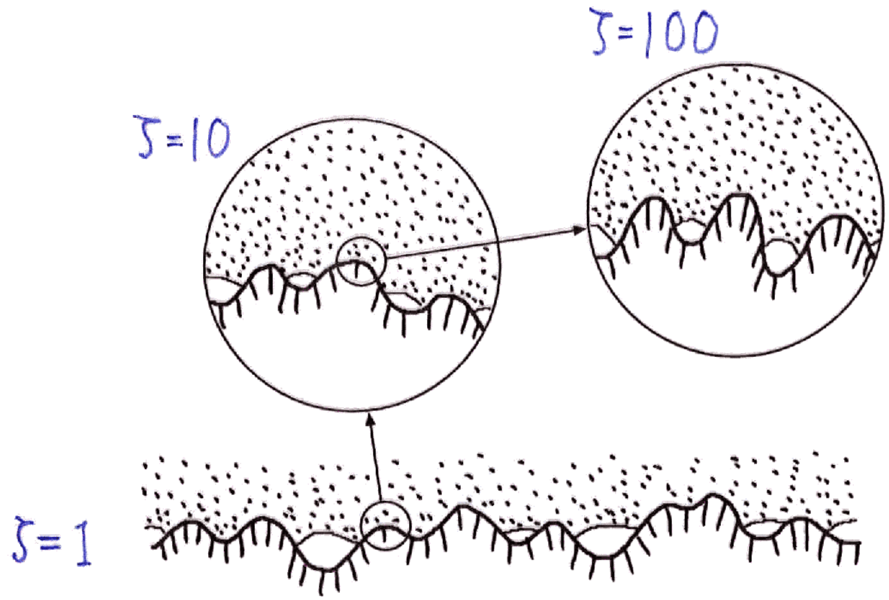
- Contact resistivity
- Heat transfer



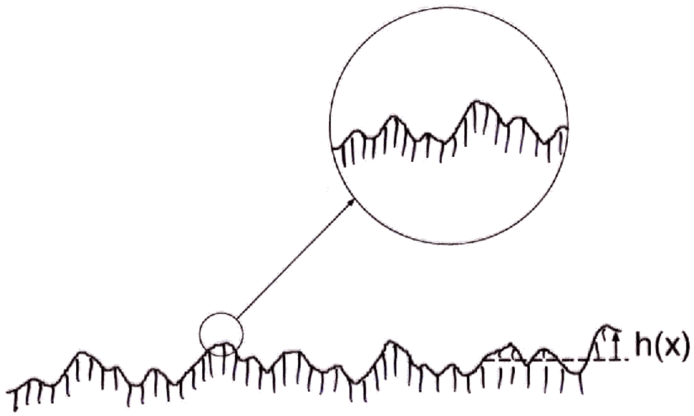
- Sealing



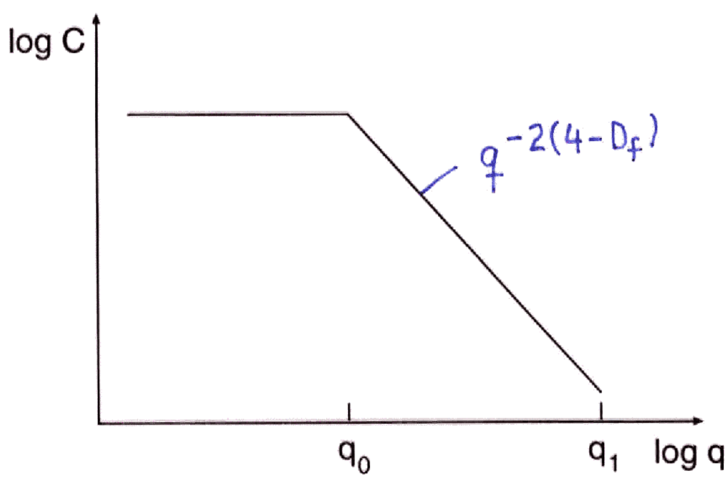
Many length scales:

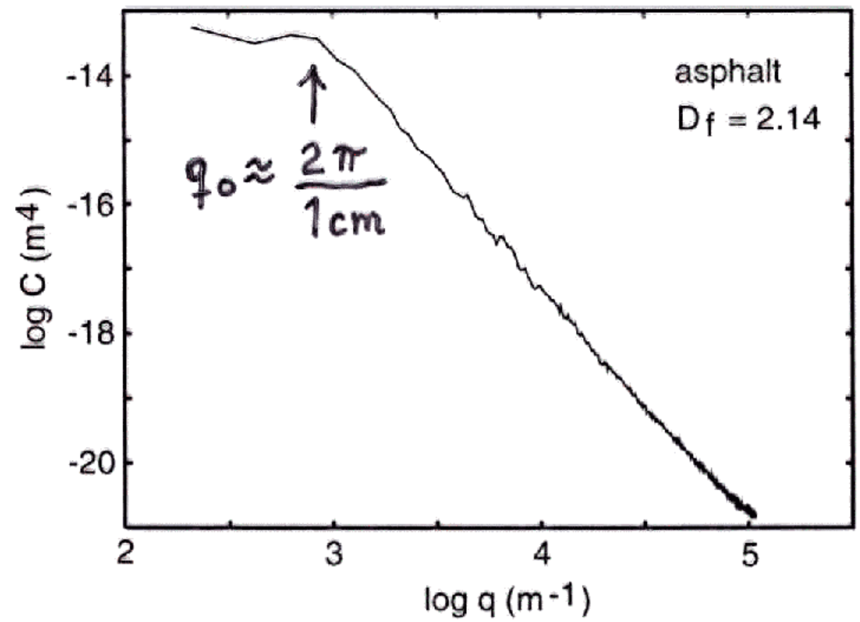


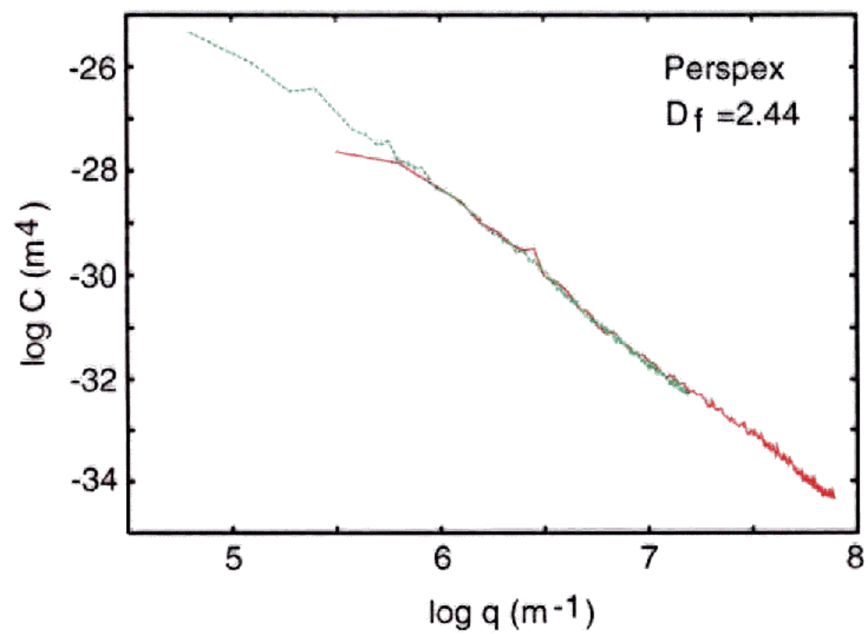
Self Affine Fractal Surface

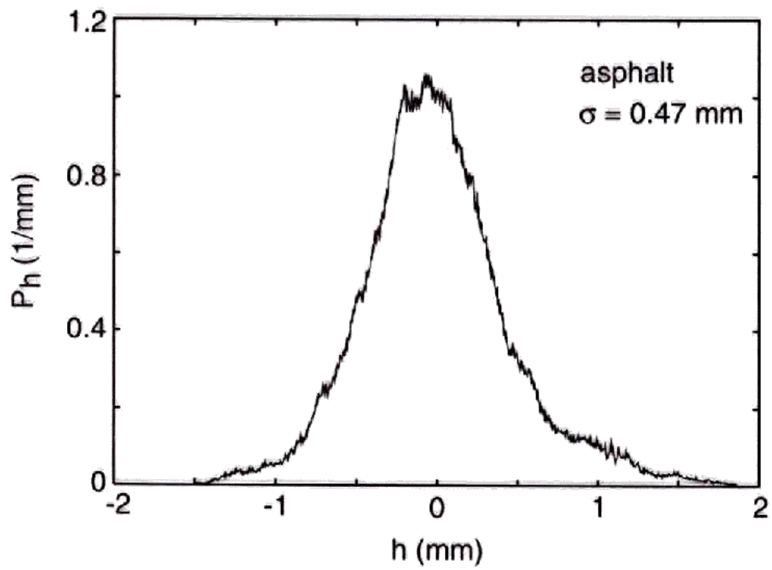


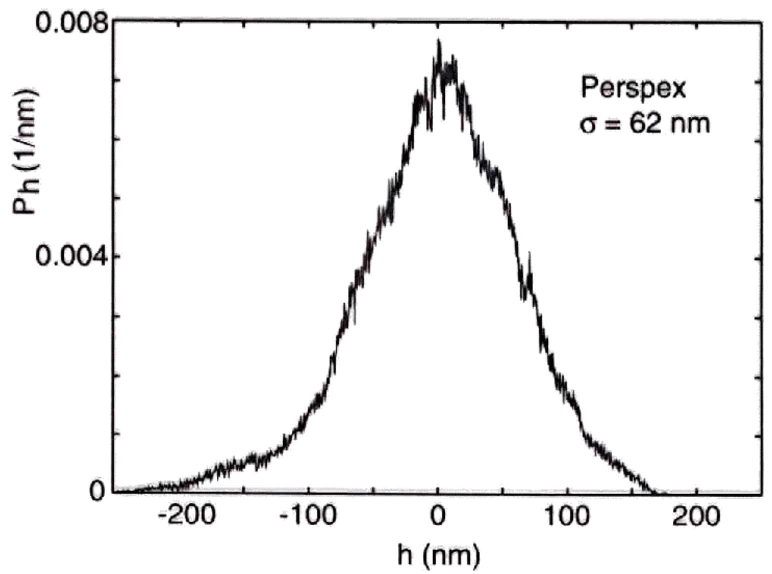
$$C(q) = \int d^2x \langle h(\mathbf{x})h(\mathbf{0})\rangle e^{i\mathbf{q}\cdot\mathbf{x}}, \qquad \langle h(\mathbf{x})\rangle = 0$$



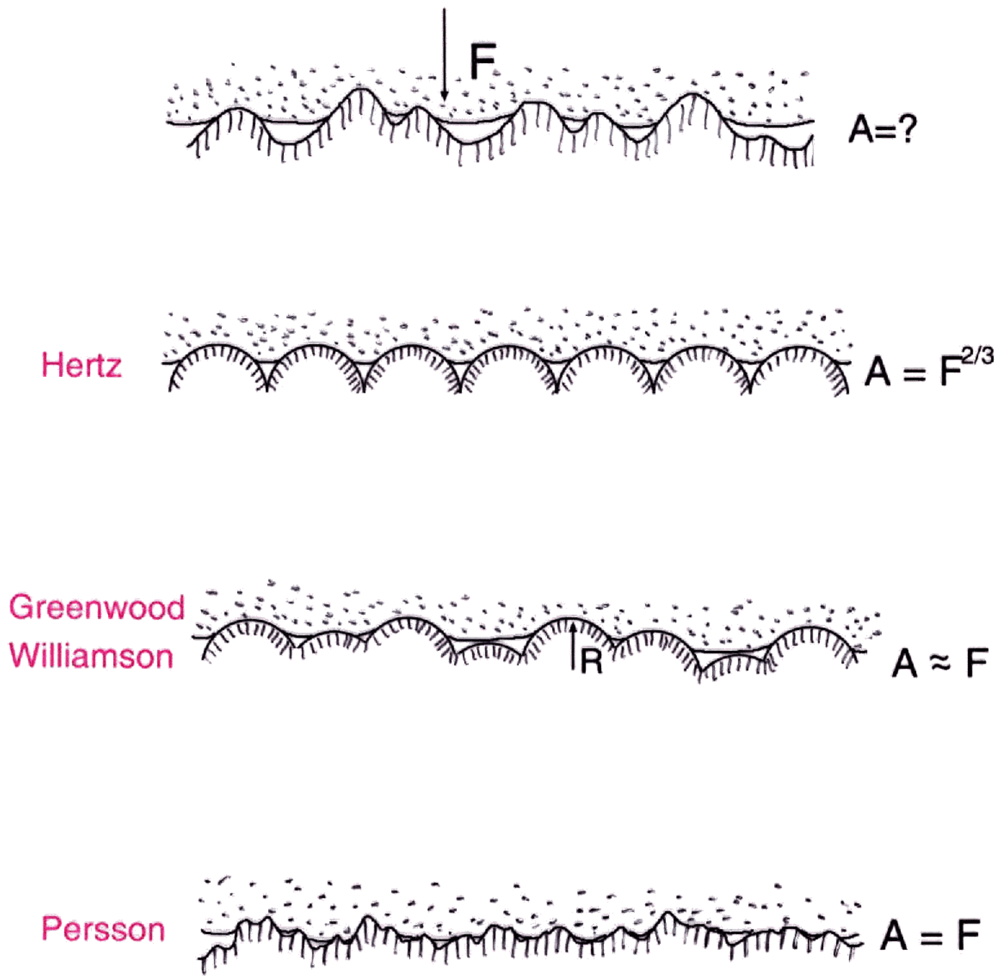








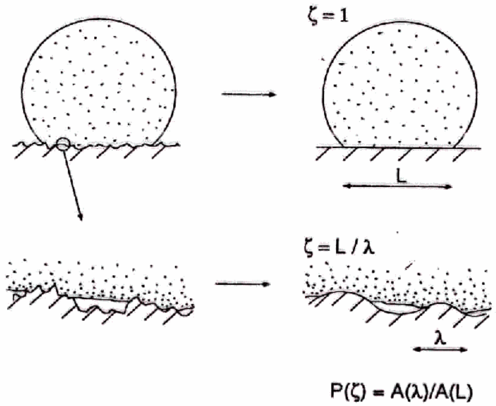
Contact Theories

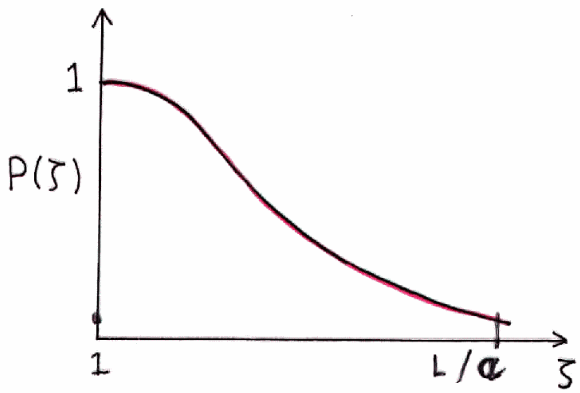


Phys. Rev. Lett. 87, 1161 (2001)

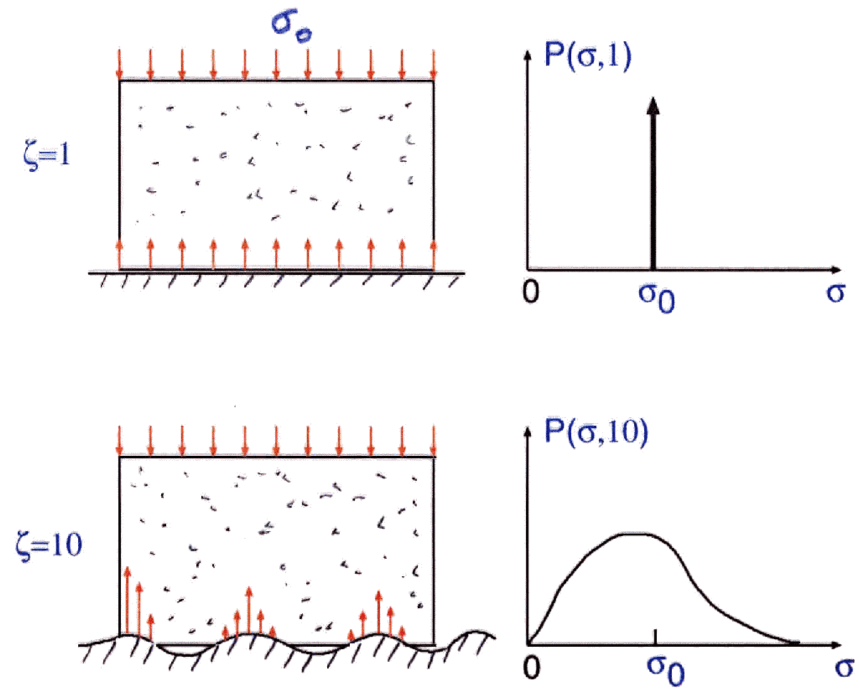
J. Chem. Phys. 115, 3840 (2001)

New Contact Theory





Stress probability distribution function P(σ,ζ)



$$P(\zeta) = \int_0^\infty d\sigma \, P(\sigma, \zeta)$$

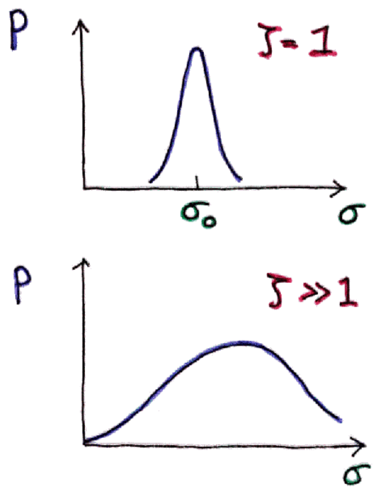
Stress probability distribution

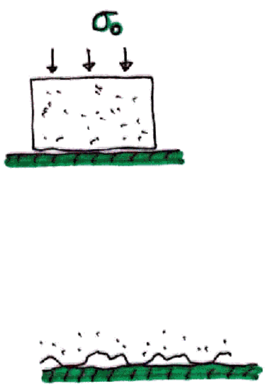
$P(\sigma, \zeta)$

$$\frac{\partial P}{\partial \zeta} = G'(\zeta)\sigma_0^2 \frac{\partial^2 P}{\partial \sigma^2}$$

$$G(\zeta) = \frac{\pi}{4}\left[\frac{E}{(1-\nu^2)\sigma_0}\right]^2 \int_{q_L}^{\zeta q_L} dq\, q^3 C(q).$$

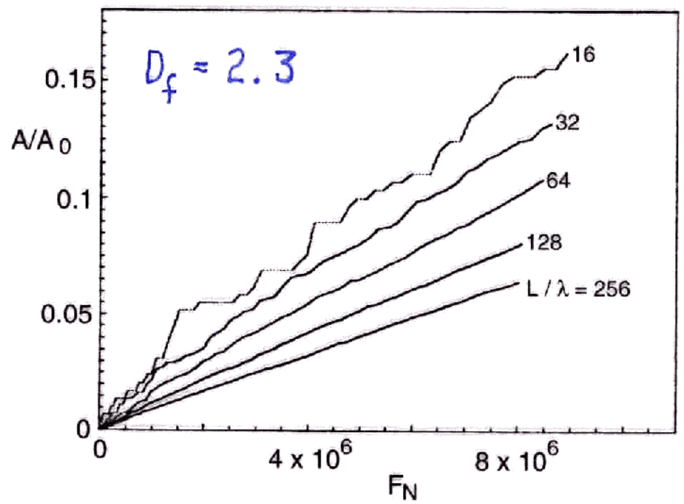
$$C(q) = \frac{1}{(2\pi)^2}\int d^2x \,\langle h(\mathbf{x})h(\mathbf{0})\rangle e^{-i\mathbf{q}\cdot\mathbf{x}},$$





$P(0, \zeta) = 0$

Brunetto et al (2001)



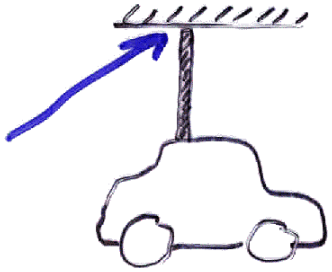
$A(\lambda) \sim (\lambda/L)^{1-H}$

$H = 3 - D_f \rightarrow 1 - H = D_f - 2 = 0.3$

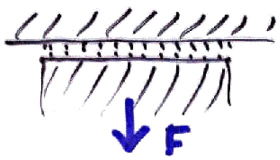
$A(\lambda) \sim (L/\lambda)^{-0.3}$

"Experimental" slope 1; 1.24, 1.57, 1.93, 2.32

Theory: 1, 1.23, 1.52, 1.87, 2.30

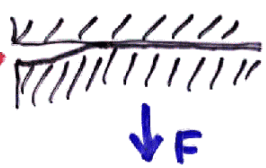


$1\ cm^2$ with van der Waals bonding



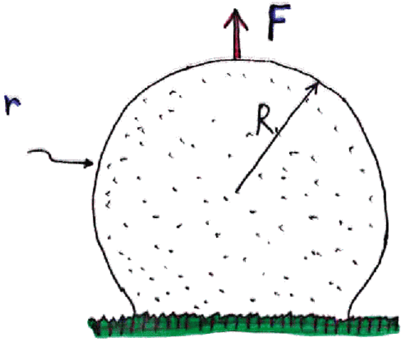
Never uniform bond breaking!

crack



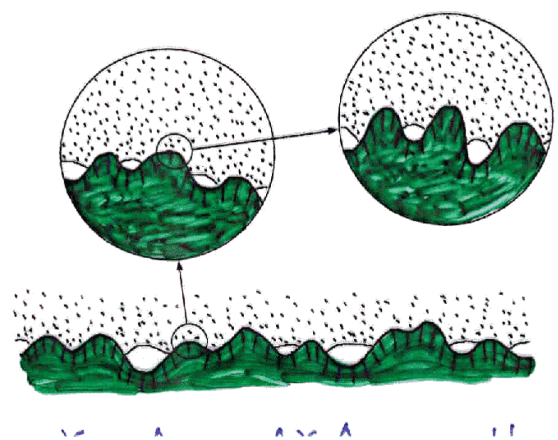
The effect of surface roughness on the adhesion of elastic solids

rubber ball

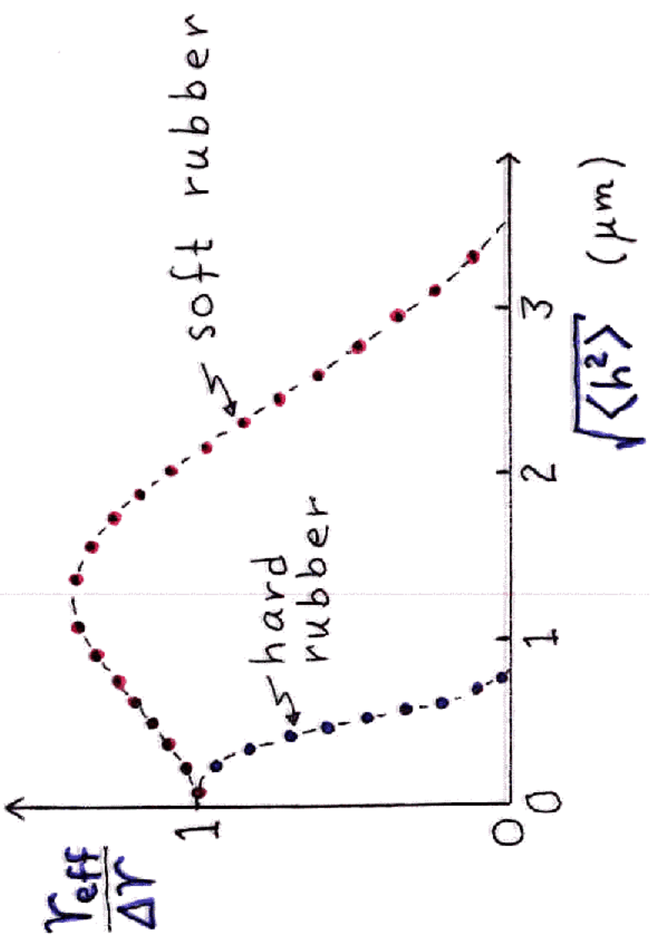


JKR theory

$$F_{\text{pull-off}} = \frac{3\pi}{2} R \gamma_{\text{eff}}$$



Experiment: Fuller, Tabor, Briggs, Briscoe, Roberts



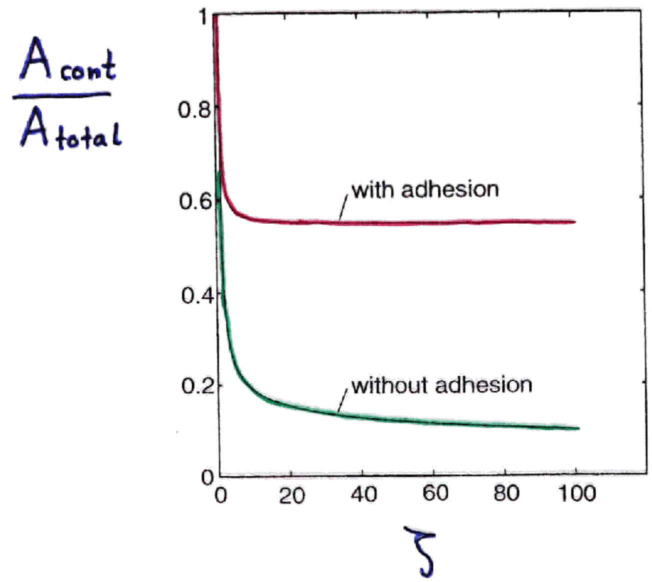
Stress probability distribution

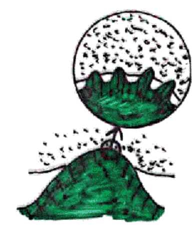
$P(\sigma, \zeta)$

$$\frac{\partial P}{\partial \zeta} = f(\zeta) \frac{\partial^2 P}{\partial \sigma^2} \qquad f(\zeta) \sim E^2 C(q_L \zeta)$$

$P(0, \zeta) = 0$ no adhesion

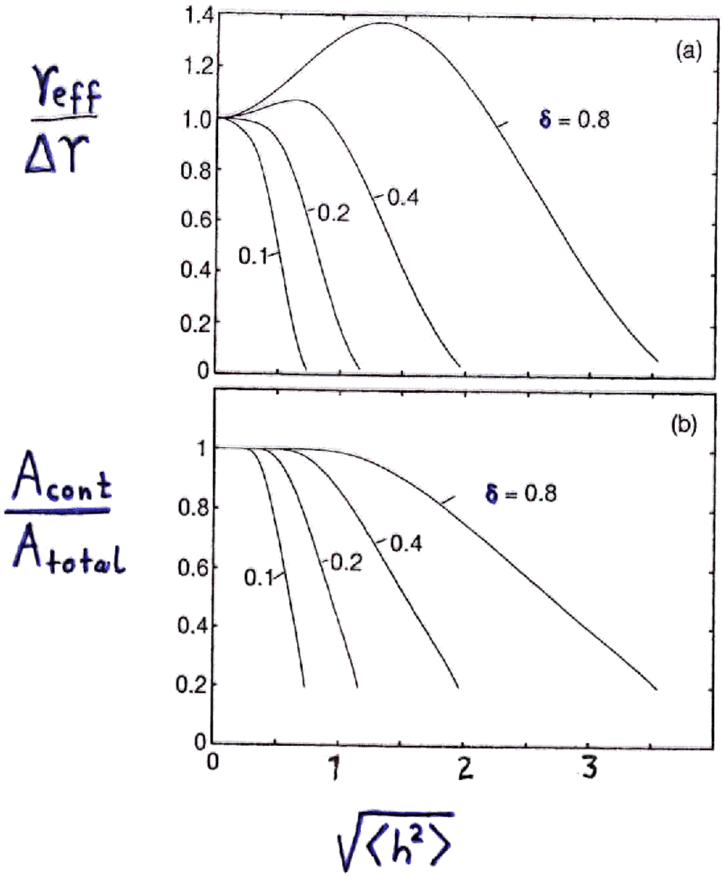
$P(-\sigma_a(\zeta), \zeta) = 0$ with adhesion



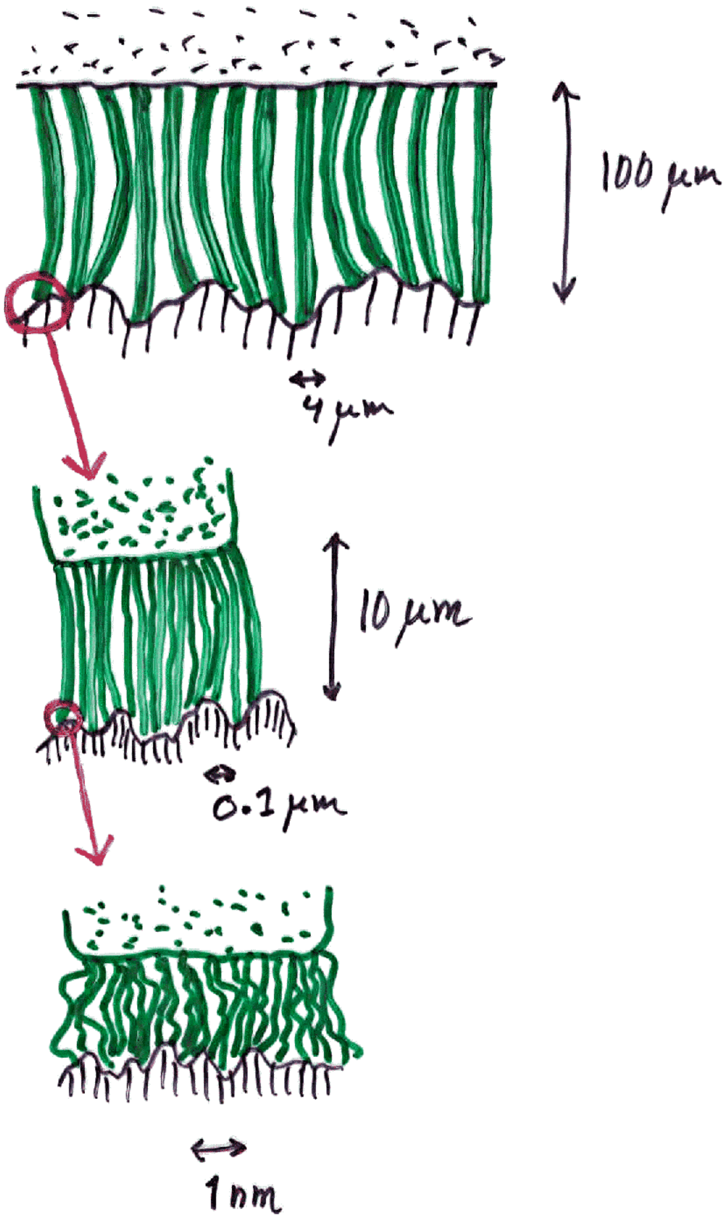


Eur. Phys. J. E.

Theory result

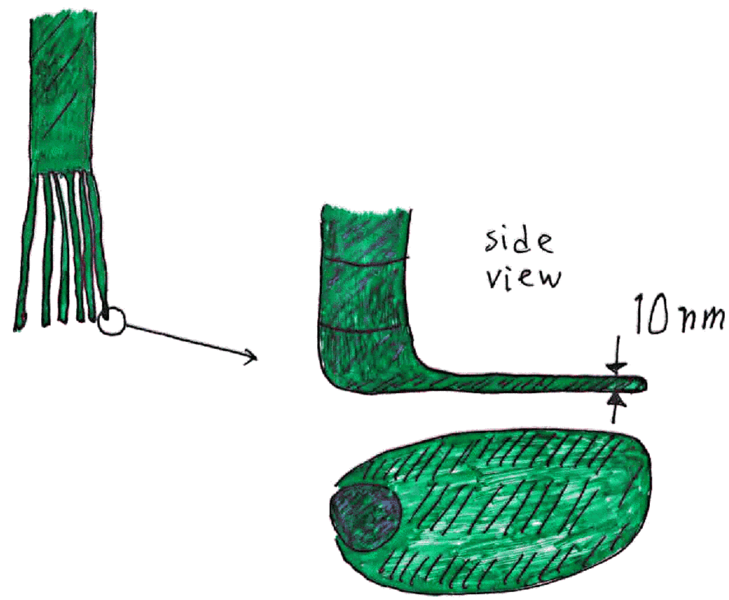


$$\delta \sim \frac{\Delta\gamma}{E}$$



3

• Adhesion - insect, Lizard



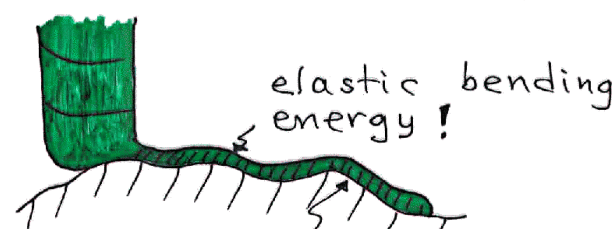
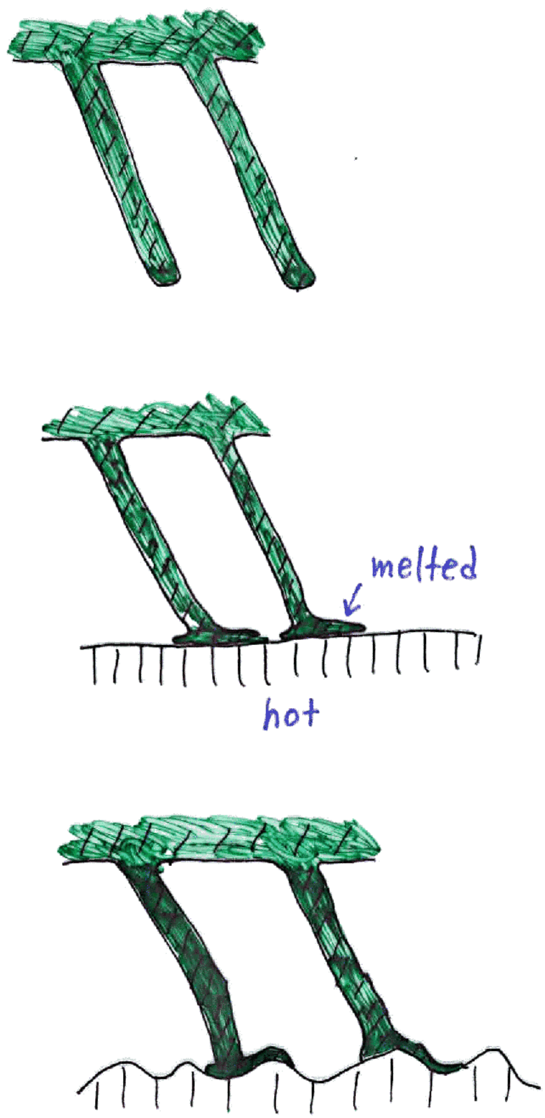


Plate-adhesion to


melted
hot

$$P(\sigma,\zeta) = \langle \delta(\sigma - \sigma_1(x,\zeta)) \rangle$$

$$P(\sigma,\zeta+\Delta\zeta) = \langle \delta(\sigma - \sigma_1 - \Delta\sigma) \rangle$$

$$= \int d\sigma' \langle \delta(\sigma' - \Delta\sigma)\, \delta(\sigma - \sigma_1 - \sigma') \rangle$$

$$= \int d\sigma' \langle \delta(\sigma' - \Delta\sigma) \rangle\, P(\sigma - \sigma', \zeta)$$

$$\langle \delta(\sigma' - \Delta\sigma) \rangle = \frac{1}{2\pi} \int dw \langle e^{iw(\sigma' - \Delta\sigma)} \rangle$$

$$\approx \frac{1}{2\pi} \int dw\; e^{iw\sigma'} \left(1 - \frac{w^2}{2} \langle \Delta\sigma^2 \rangle\right)$$

$$P(\sigma,\zeta^*) + \frac{\partial P}{\partial \zeta}(\sigma,\zeta)\,\Delta\zeta$$

$$= \int d\sigma'\; P(\sigma - \sigma', \zeta)$$

$$\times \left[\delta(\sigma') + \frac{1}{2} \frac{\partial^2}{\partial \sigma'^2} \delta(\sigma') \langle \Delta\sigma^2 \rangle \right]$$

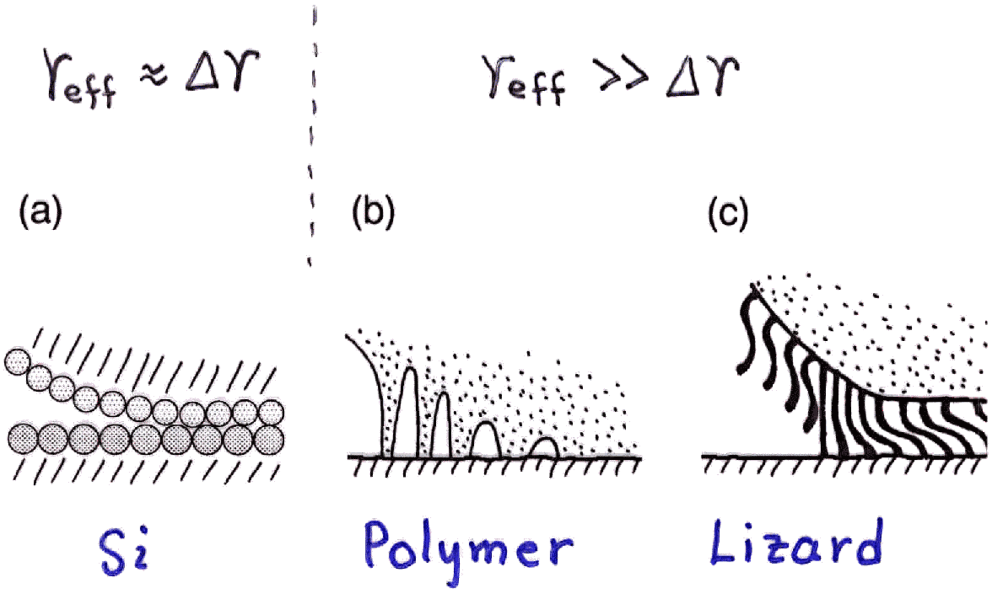
$$\frac{\partial P}{\partial \zeta} = f(\zeta) \frac{\partial^2 P}{\partial \sigma^2}, \quad f = \frac{1}{2} \frac{\langle \Delta\sigma^2 \rangle}{\Delta\zeta}$$

$$P(\sigma,\zeta) = \langle \delta(\sigma - \sigma_1(x,\zeta)) \rangle$$

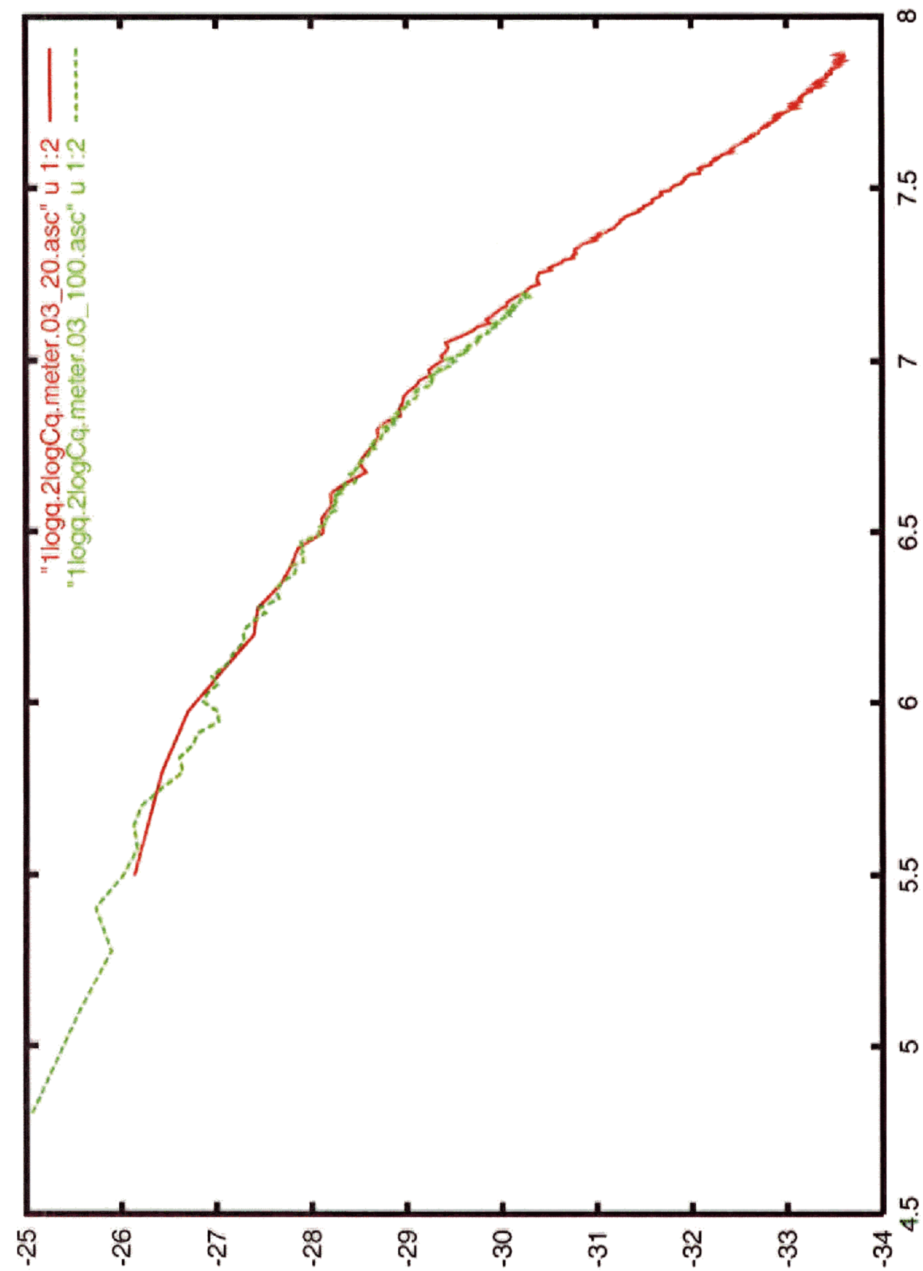
$$= \frac{1}{A_0} \int d^2x\; \delta(\sigma - \sigma_1(x,\zeta))$$

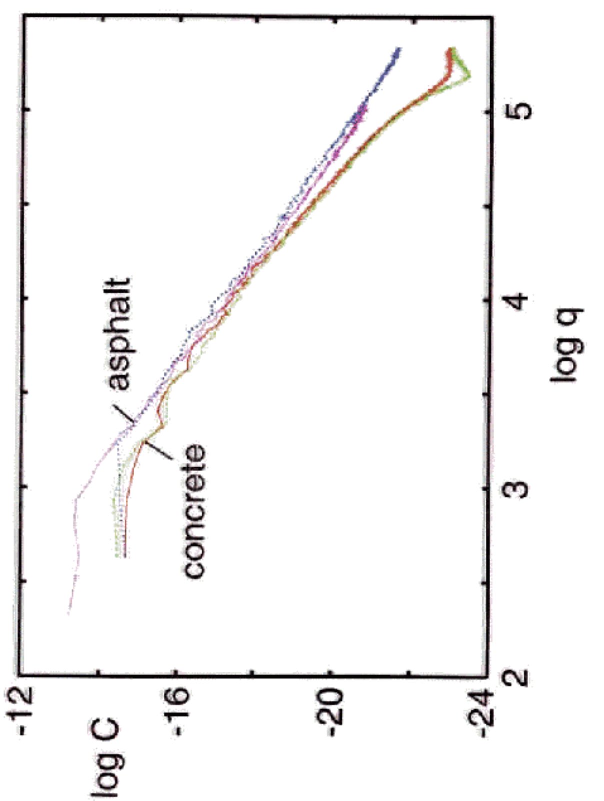
$\rightarrow$

$$\int_{-\sigma_a}^{\infty} d\sigma\; P(\sigma,\zeta) = \frac{A(\zeta)}{A_0}$$

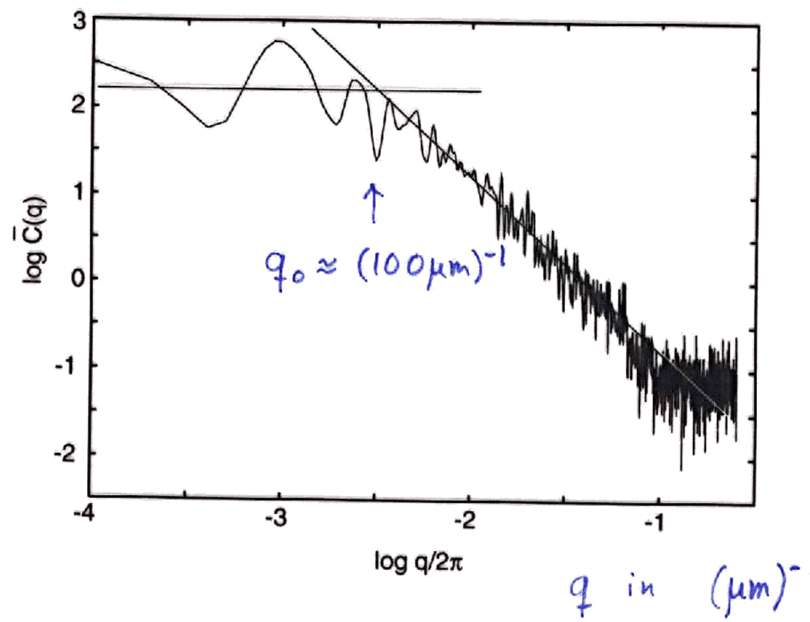


Strong adhesion results from "long bonds" rather than "strong bonds"

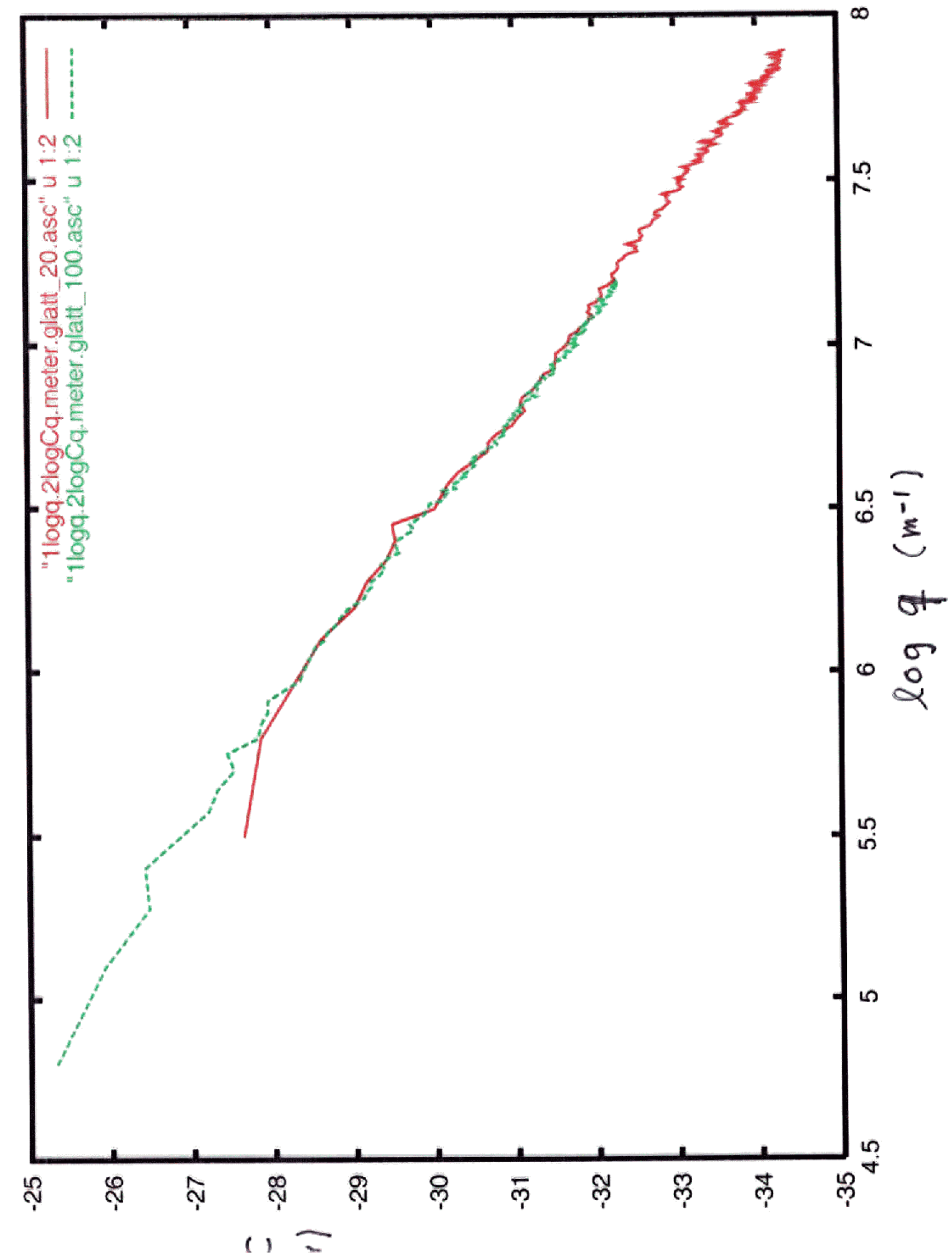


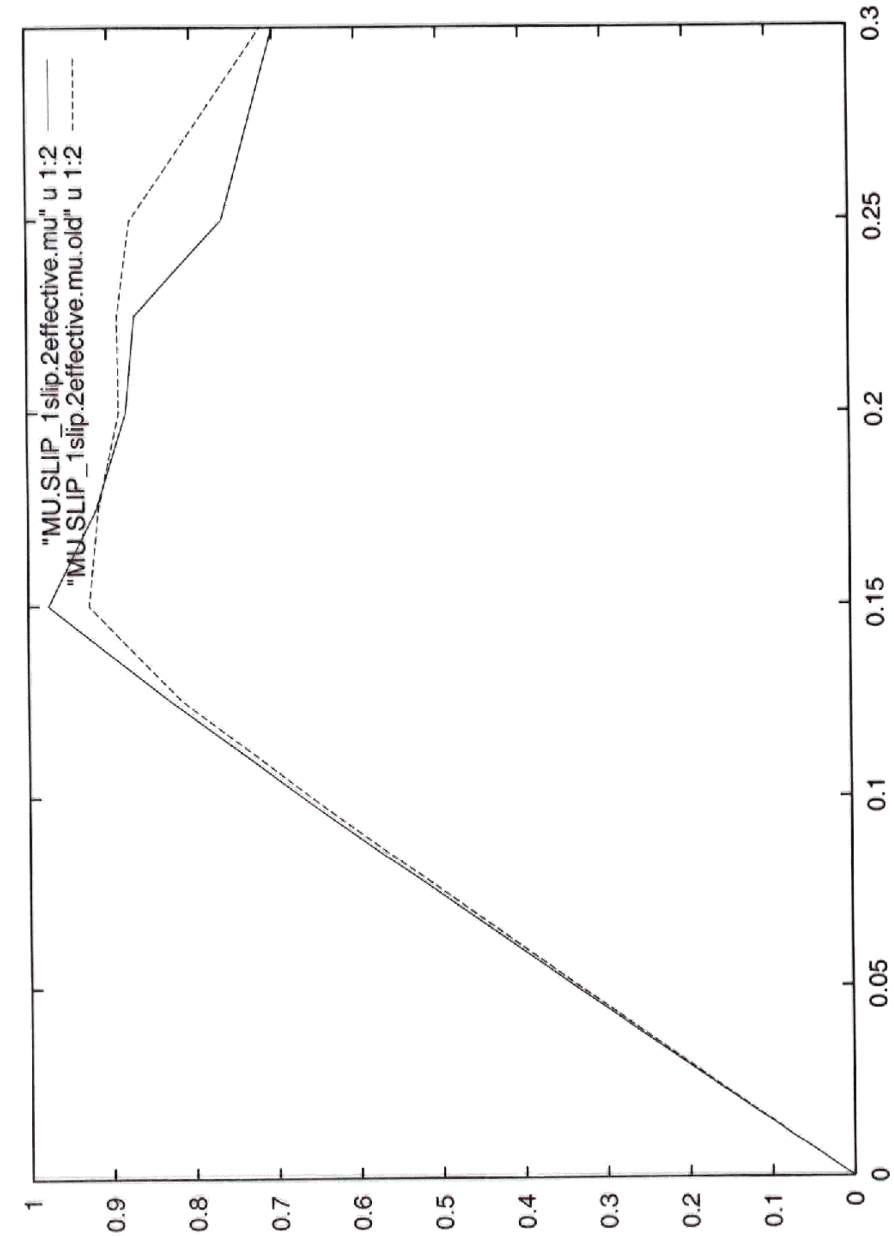


Rail- track



$D_f \approx 2.5$


"1logq.2logCq.meter.glatt_20.asc" u 1:2
"1logq.2logCq.meter.glatt_100.asc" u 1:2
log q (m^-1)


"MU.SLIP_1slip.2effective.mu" u 1:2
"MU.SLIP_1slip.2effective.mu.old" u 1:2

$$P(\sigma,\zeta) = \langle \delta[\sigma - \sigma_1(\mathbf{x})] \rangle$$

$$\sigma_1(\mathbf{x}) = \sum_{q<\zeta q_L} \sigma(\mathbf{q})\, e^{i\mathbf{q}\cdot\mathbf{x}}$$

$$q_L = \frac{2\pi}{L}$$

$$\frac{\partial P}{\partial \zeta} = G'(\zeta)\,\sigma_0^2\, \frac{\partial^2 P}{\partial \sigma^2}$$

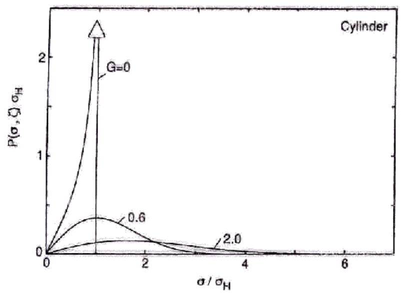
$$G(\zeta) = \frac{\pi}{4}\left[\frac{E}{(1-\nu^2)\sigma_0}\right]^2 \int_{q_L}^{\zeta q_L} dq\, q^3\, C(q)$$

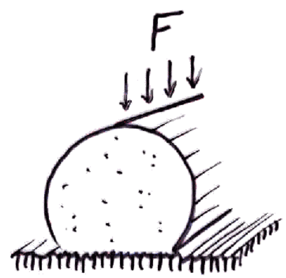
$$C(q) = \frac{1}{(2\pi)^2}\int d^2x\, \langle h(\mathbf{x})\, h(\mathbf{0})\rangle\, e^{-i\mathbf{q}\cdot\mathbf{x}}$$

$$P(\sigma,1) = P_0(\sigma)$$

$$P(0,\zeta) = P(\infty,\zeta) = 0$$

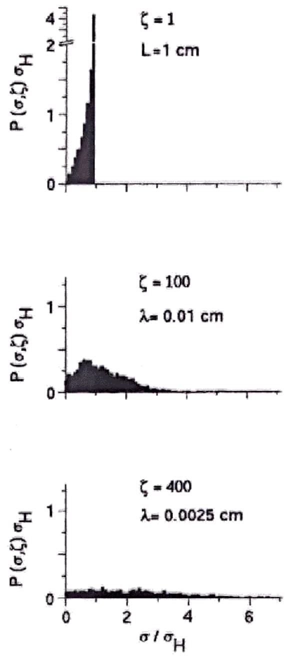
$$A(\lambda) = \frac{4(1-\nu^2)}{q_0 h_0}\left(\frac{1-H^2}{\pi H}\right)^{1/2} \frac{F_N}{E}\left(\frac{\lambda}{\lambda_0}\right)^{1-H}$$





$$\sigma(x) = \sigma_H \left(1 - \left[\frac{x}{a_H}\right]^2\right)^{\frac{1}{2}}$$

$$\langle \delta(\sigma - \sigma(x)) \rangle$$

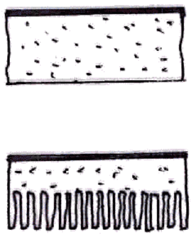


$$A_0 = L_x L_y$$

$$A = 0.62\, A_0$$

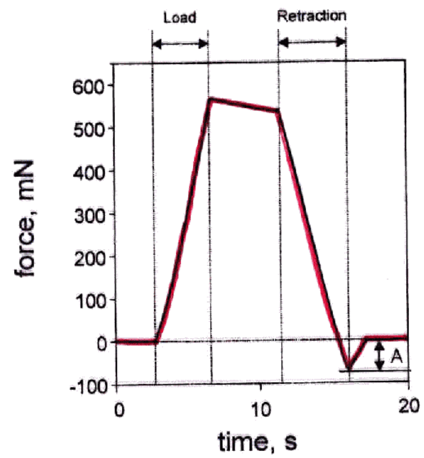
$$A = 0.33\, A_0$$

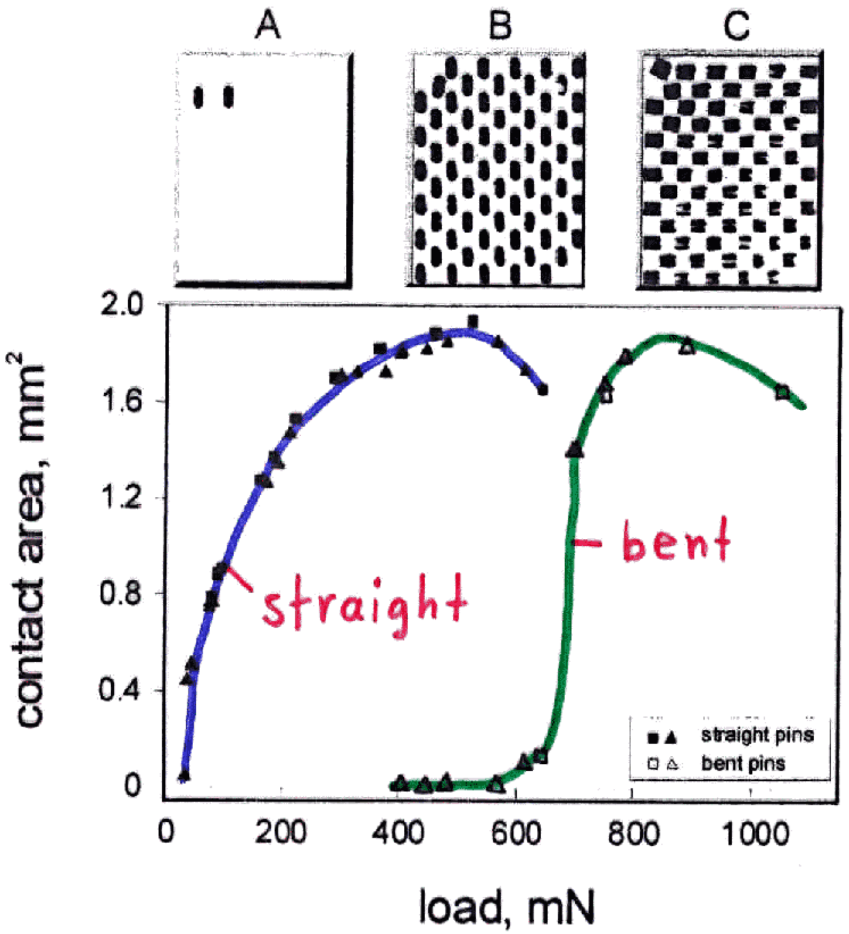
F Bucher et al

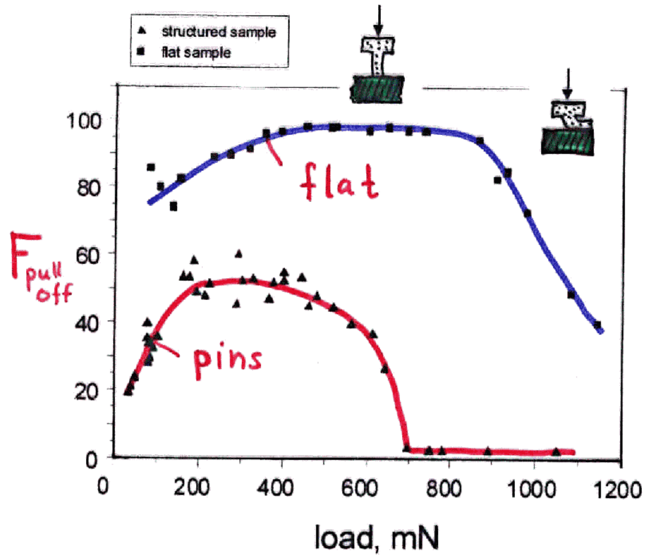


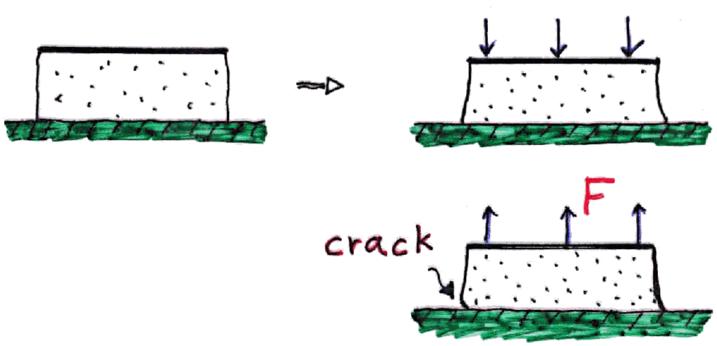
Polyvinylsiloxane

Peressadko, Gorb



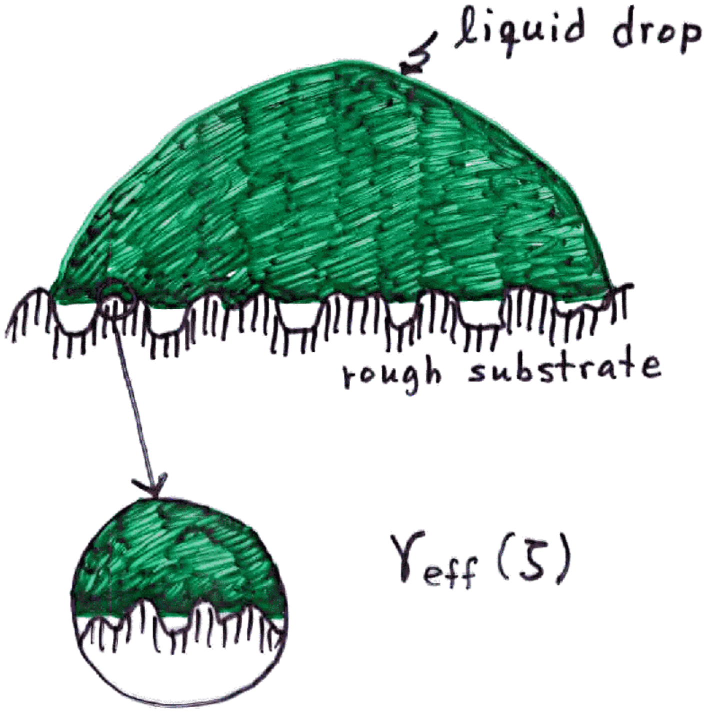

A
B
C
contact area, mm^2
2.0
1.6
1.2
0.8
0.4
0
0
200
400
600
800
1000
load, mN
straight
bent
straight pins
bent pins

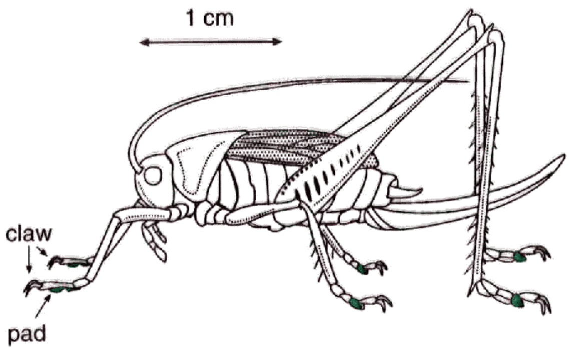

structured sample
flat sample
100
80
60
40
20
0
0
200
400
600
800
1000
1200
load, mN
$F_{pull\ off}$
flat
pins


F
crack

Future activity:

- Viscoelastic solids
- Wetting on rough substrate





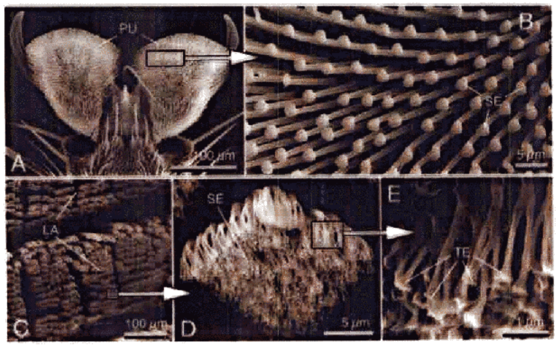


Fig. 1. Biological attachment surfaces, scanning electron microscopy. A-B. Pads (pulvilli) of the leg of the fly *Episyrphus balteatus* are divided into setae with flat tips. C-E. Pads of the leg of the gecko *Gecko gekko* consist of lamellae, which are subdivided into setae branching into even finer terminal elements. LA, lamellae; PU, pulvilli; SE, setae; TE, terminal elements.

1

Unsolved problems

- Adhesion with viscoelastic solid

rubber

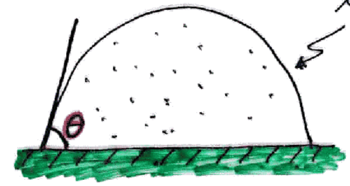
short time (hard solid)

↓ time

long time (soft solid)

2

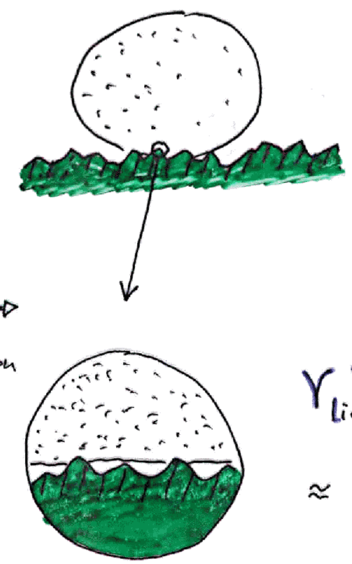
- Droplet on rough substrate



$$\gamma_{lv} \cos\theta = \gamma_{sv} - \gamma_{ls}$$

lotus effect

$$\gamma_{ls} \to \gamma_{ls}^{eff}$$



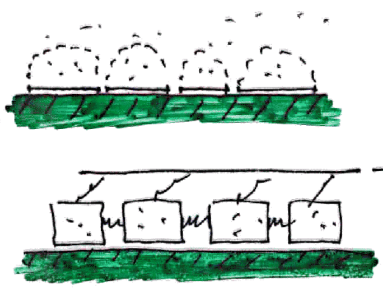
Roughness on many length scales ⟶ Renormalization group procedure!

$$\gamma^{\text{effective}}_{\text{liquid-solid}} \approx \gamma_{sv} + \gamma_{lv}$$

$$\gamma_{lv} \cos\theta \approx \gamma_{sv} - [\gamma_{sv} + \gamma_{lv}]$$

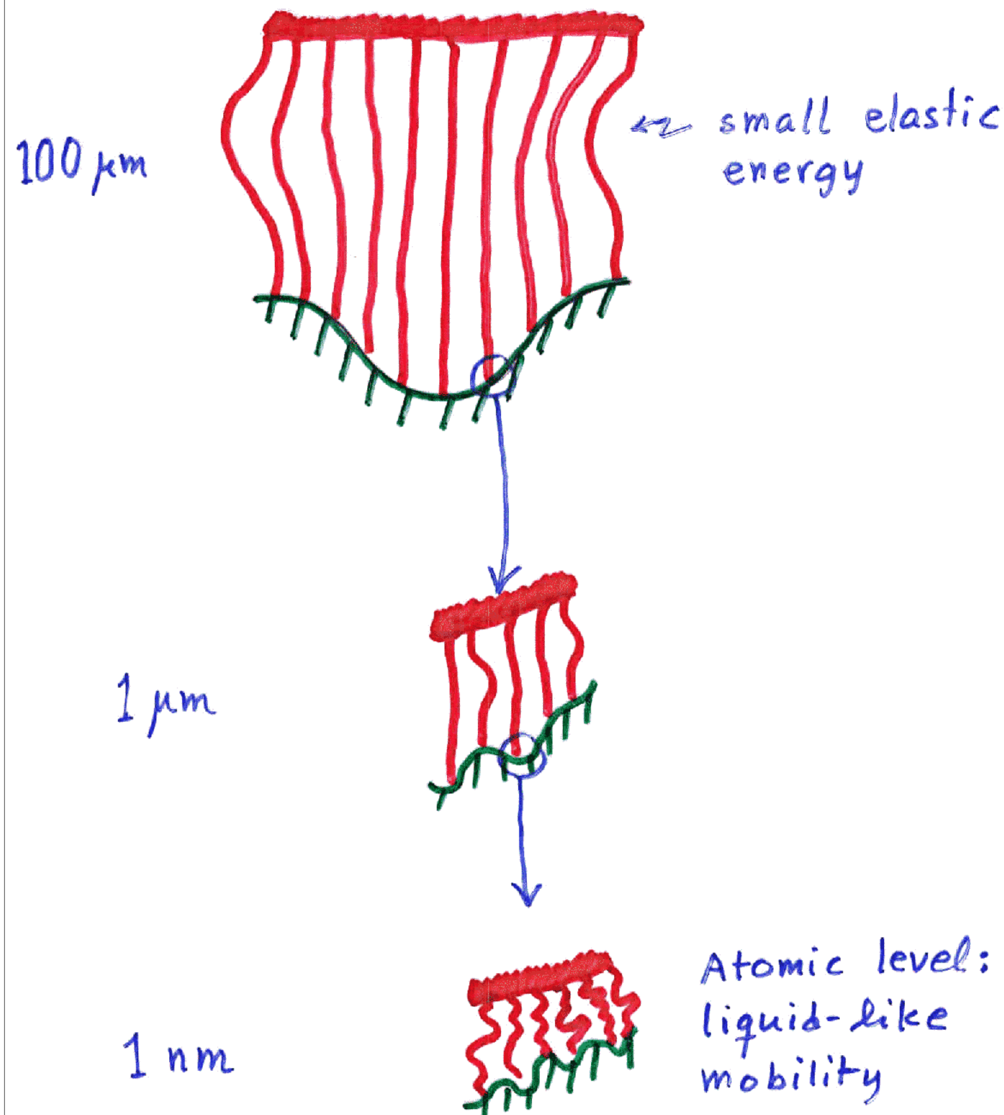
4

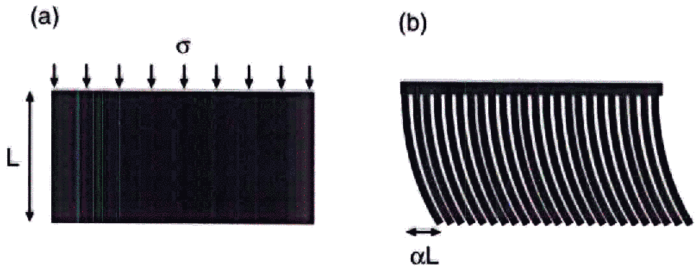
- Rubber friction on smooth substrate



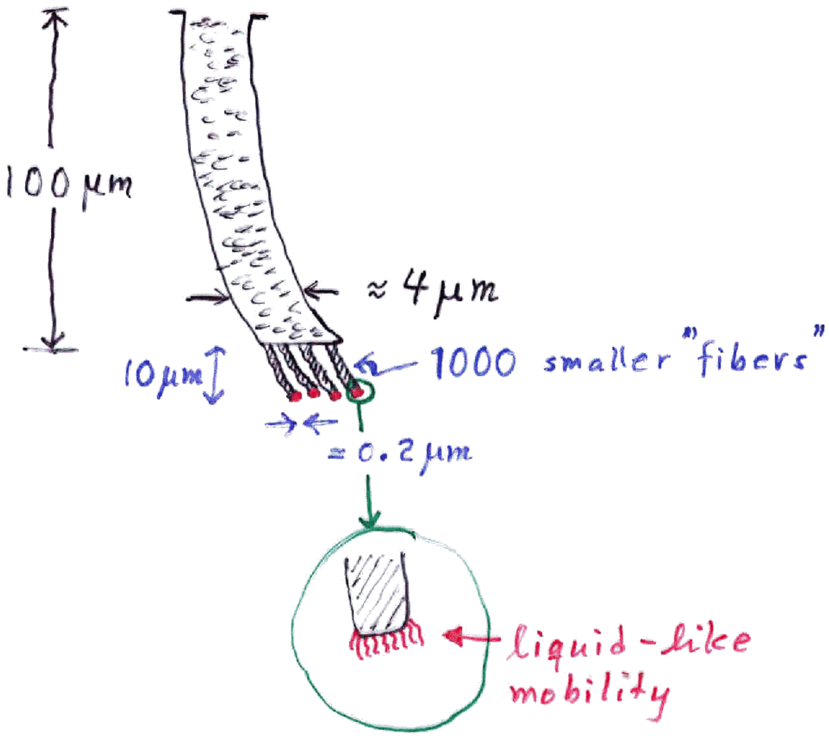
random stick-slip
or
wave-propagation

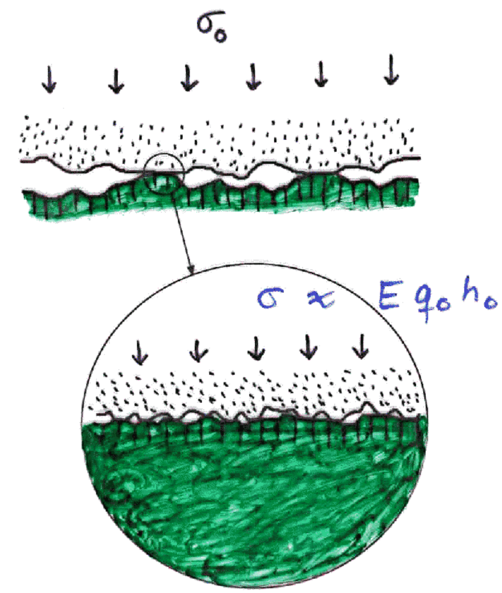
Schallamach



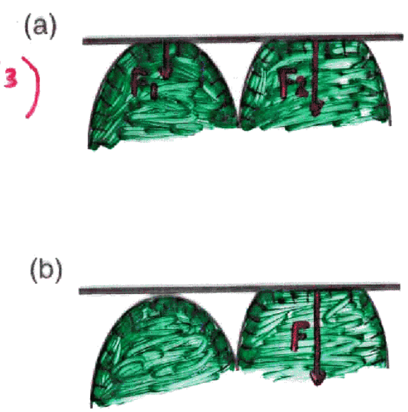


E $\qquad\qquad$ $E^* \approx E\left(\frac{R}{L}\right)^2$





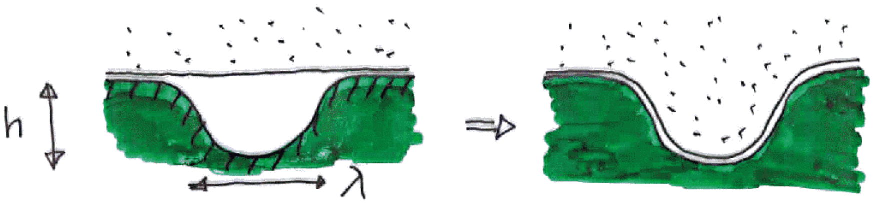
$A \sim a\left(F_1^{2/3} + F_2^{2/3}\right)$



$A' \sim aF^{2/3} = a\left(F_1 + F_2\right)^{2/3} < A$

7

Role of rubber-substrate adhesion



$$E_{el} \approx \sigma \lambda^2 h \approx E \frac{h}{\lambda} \lambda^2 h$$

$$= E \lambda h^2$$

$$E_{ad} \approx \Delta\gamma \lambda^2$$

$$E_{ad} \approx E_{el} \longrightarrow \frac{h}{\lambda} \approx \left(\frac{\Delta\gamma}{E\lambda}\right)^{\frac{1}{2}}$$

$$\left.\begin{array}{l} \Delta\gamma \approx 3\ \mathrm{meV}/\mathrm{\AA}^2 \\ E \approx 10^6\ \mathrm{N/m^2} \\ h/\lambda \approx 1 \end{array}\right\} \longrightarrow \lambda \approx 10^3\ \mathrm{\AA}$$

Adhesion