



# ***The Onset of Fibrillation following a Heart Attack***

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# *Basic Problem*

## Some Statistics:

- Cardiovascular disease is the **number 1 killer** in America (about  $10^6$  deaths per year).
- At least 250,000 people per year die from a heart attack before reaching a hospital.



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## Some Statistics:

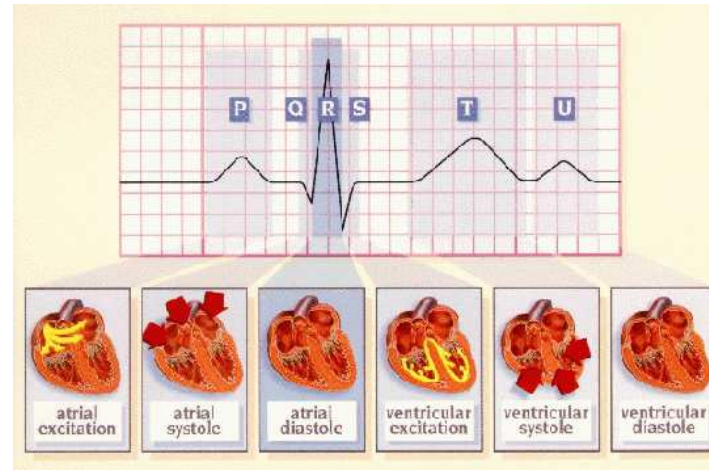
- Cardiovascular disease is the **number 1 killer** in America (about  $10^6$  deaths per year).
- At least 250,000 people per year die from a heart attack before reaching a hospital.

## Fundamental Questions:

- What are the mechanisms underlying the initiation of fibrillation following a heart attack?
- Why do only some heart attacks result in fibrillation?
- What can be done to prevent a heart attack from initiating fibrillation?



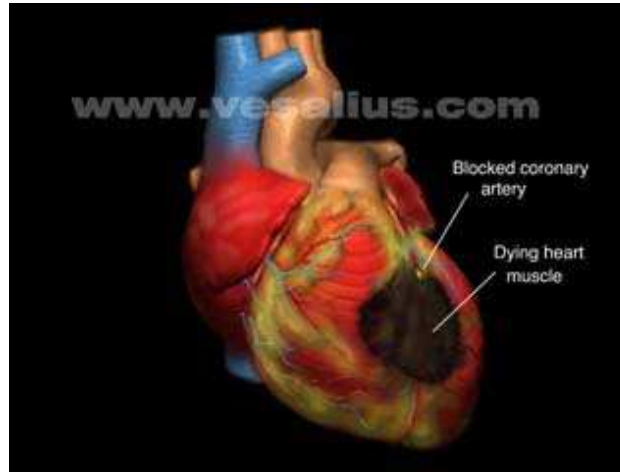
# Conduction system of the heart



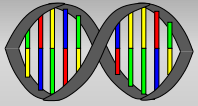
- Electrical signal originates in the SA node.
- The signal propagates across the atria, through the AV node, and throughout the ventricles.
- The muscle cells contract in unison, and then relax awaiting the next signal.



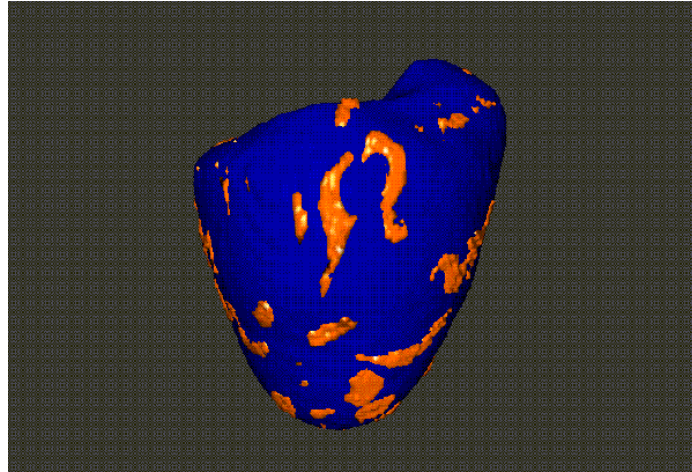
# *What is a heart attack?*



- A coronary occlusion leads to loss of blood flow to some region of the heart.
- Ischemia develops.
- Fibrillation may or may not occur.



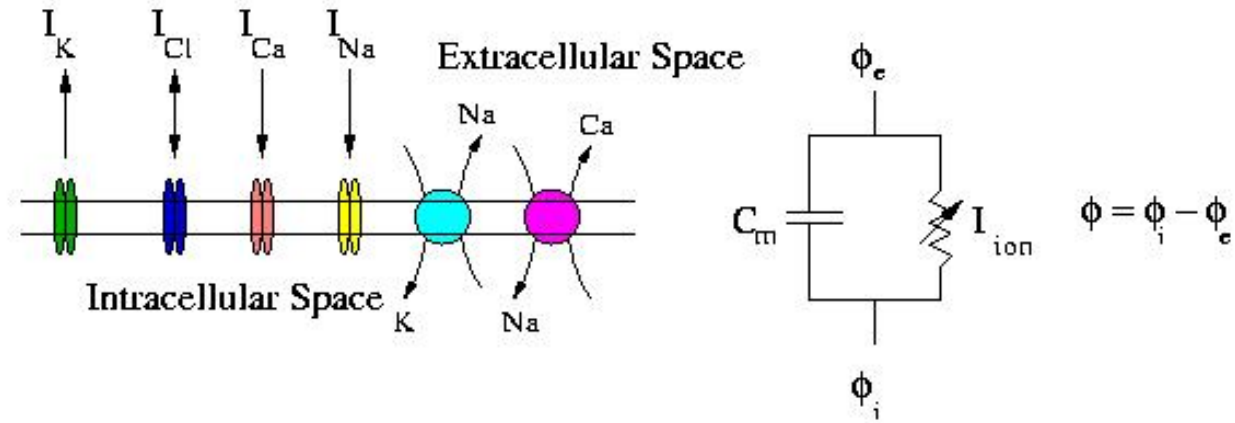
# *Surface View of Fibrillation*



Movie

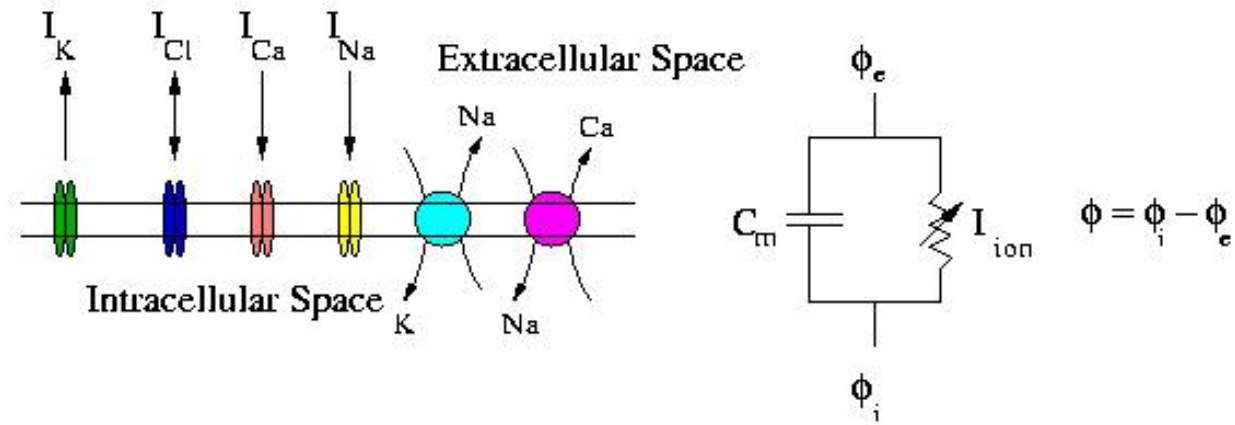


# Modeling Cardiac Electrical Activity





# Modeling Cardiac Electrical Activity



Transmembrane potential  $\phi$  is regulated by transmembrane ionic currents and capacitive currents:

$$C_m \frac{d\phi}{dt} + I_{ion}(\phi, w) = I_{in} \quad \text{where} \quad \frac{dw}{dt} = g(\phi, w), \quad w \in R^n$$



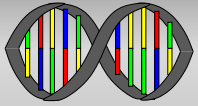


Examples include:

- Beeler-Reuter model (complicated bad model)
- Luo-Rudy model(s) (more complicated, bad models)
- Winslow-Jafri, Noble, etc. (even more complicated, bad models)
- Two Variable Models (FHN, Morris-Lecar, Puschino, Mitchell-Schaeffer-Karma, etc. - highly simplified bad models)

$$I_{ion} = g_{Na} m_{\infty}(\phi) h(\phi - \phi_{Na}) + g_K(\phi - \phi_K)$$

$$\tau_h(\phi) \frac{dh}{dt} = h_{\infty}(\phi) - h$$



# ***Suggested Mechanisms for Initiation of Reentrant Activity***

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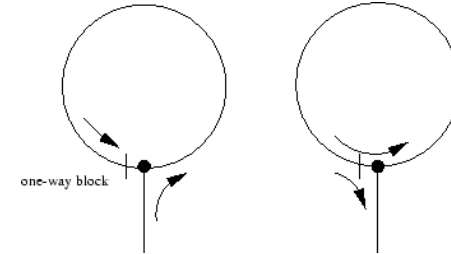
Fundamental question: How is a dynamical system moved from one state (the normal heartbeat) to another (reentry)?



# *Suggested Mechanisms for Initiation of Reentrant Activity*

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- Anatomical - One way block on a closed 1D loop

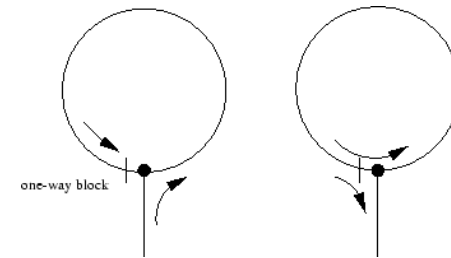




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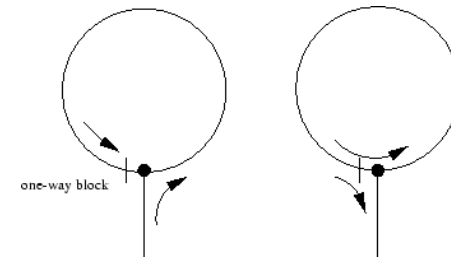
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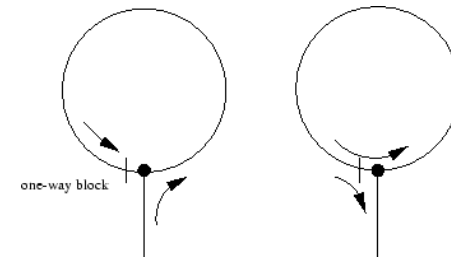
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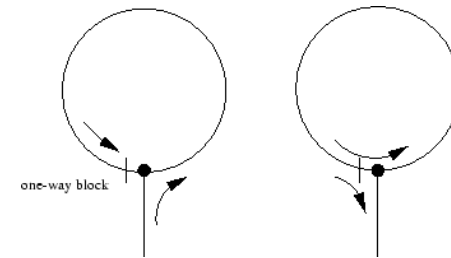
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- Dispersion (i.e. spatial inhomogeneity) of Refractoriness (Moe, et al.).

Unfortunately, none of these has much explanatory power for fibrillation onset following heart attacks.



# *Hypothesis:*

Fibrillation following a heart attack is the result of

- A **border zone arrhythmia** (autonomous pacing)
- that evokes a **breakup instability**.

Important questions:

- What is a **border zone arrhythmia**?
- What is a **breakup instability**?
- Is a border zone arrhythmia capable of evoking a breakup instability?

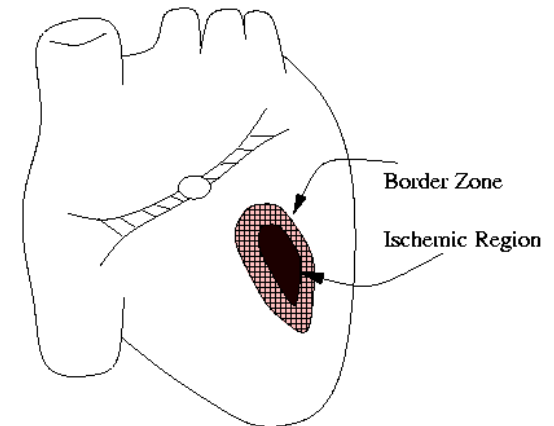




# Border Zone Arrhythmias

Following coronary occlusion:

- Extracellular Potassium increases,
- The resting potential of affected cells increases,
- A current, called current of injury, is created,
- which may stimulate action potentials.

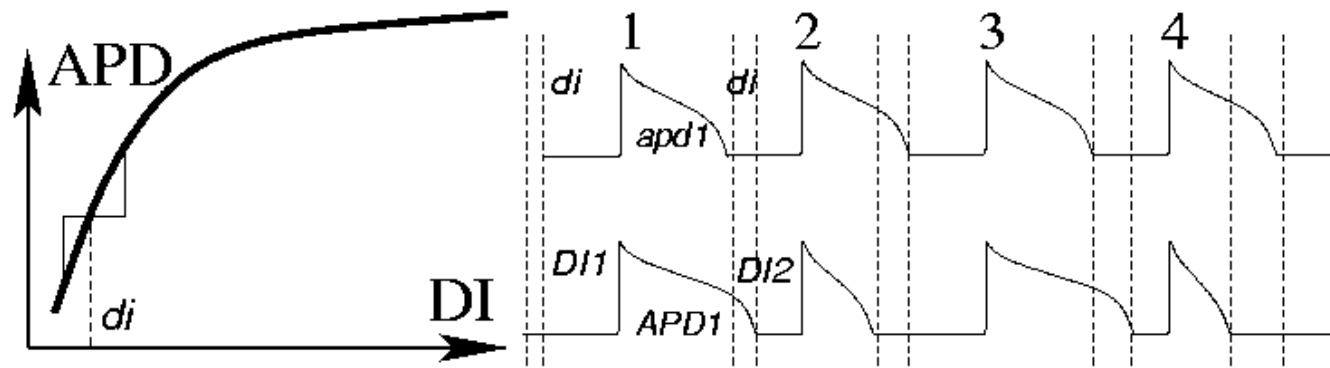


Simple Examples:

- Uncoupled Cells Coupled Cells
- 1D Cable



# APD Alternans



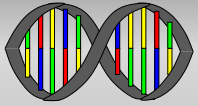
## Action Potential Duration Restitution Curve

$$APD_n + DI_n = BCL.$$

where  $APD_n = A(DI_{n-1})$  is the restitution curve. It follows that

$$DI_n = BCL - A(DI_{n-1}),$$

[APD Map Animated](#)



# *The APD Instability*

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Stable Pulse on a Ring

Unstable Pulse on a Ring

Collapse of Unstable Pulse

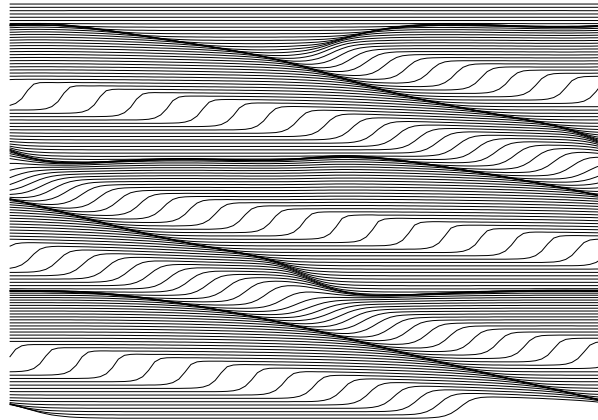


# *The APD Instability*

Stable Pulse on a Ring

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Collapse of Unstable Pulse



APD Instability in 2 Dimensions



## ***Key Observation:***

The breakup instability does **not** require a spiral core to drive it.



Observations:

- The breakup always occurs at some distance from the center. Why?
- The frequency of the spiral is larger than can be achieved by pacing. Why?



## ***Key Observation:***

The breakup instability does **not** require a spiral core to drive it.



Key Question:

- Can a periodic source or a constant stimulus evoke the breakup instability?



## ***Key Question:***

Can a periodic source or a constant stimulus evoke the breakup instability?

- Answer #1: **NO!**
  - Reason #1: Periodic stimuli cannot excite tissue fast enough to evoke the breakup instability.
  - Reason #2: The APD for a stimulated action potential is too long.
  - Illustration: 1D Cable w/ Monophasic Forcing



## ***Key Question:***

Can a periodic source or a constant stimulus evoke the breakup instability?

- Answer #2: **YES!**
  - Reason: If action potentials are shortened via hyperpolarization, then the breakup instability can be evoked.
  - Illustration: 1D Cable w/ Biphasic Forcing

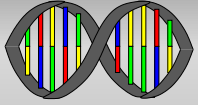
Question: Is "Biphasic Forcing" possible?





## ***Important Facts:***

- During ischemia, ATP levels in the cell decrease.
- $K_{ATP}$  channels (ATP activated potassium channels) have increased activity when ATP is depleted.
- Increased  $K_{ATP}$  activity shortens the APD of affected cells.
- Shortened action potentials in the border zone provide a source of "biphasic forcing".
- With shortened action potentials, a border zone arrhythmia can elicit the breakup instability.

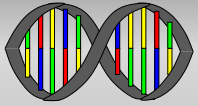


# *Consequences*

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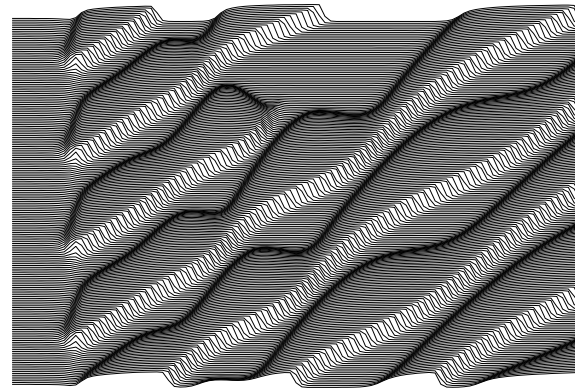
Results from Simulations:

1D Border Zone Arrhythmia



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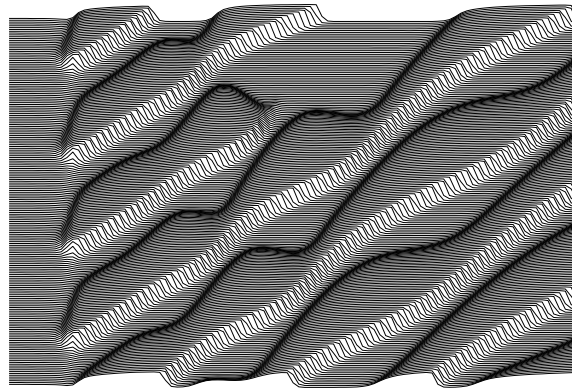
### 1D Border Zone Arrhythmia





## Results from Simulations:

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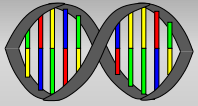


### 2D Border Zone Arrhythmia



## ***Conclusion:***

- A possible mechanism for the initiation of fibrillation following a heart attack is:
  - A **border zone region** resulting from locally high extracellular potassium causes autonomous pacing,
  - A **decrease in ATP** activates  $K_{ATP}$  channels locally which shortens action potentials,
  - Rapid pacing with shortened action potentials exposes the **breakup instability**,
  - resulting in VT and/or fibrillation onset.
- The mathematical observations:
  - This is a generic behavior (a bifurcation), requiring no external stimulus or special physical assumptions.



# *Many Remaining Questions*

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There are numerous unresolved questions:

- Does this bifurcation occur in more complicated ionic models?



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(It took only ten minutes of fooling around with parameters to produce the last 1D figure.)



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- Does this bifurcation occur in more complicated ionic models?
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- What are the controlling parameters and parameter ranges? (It took only ten minutes of fooling around with parameters to produce the last 1D figure.)
- Does this have anything to do with reperfusion arrhythmias?



# *Acknowledgments*

## Collaborators

- Sasha Panfilov, University of Utrecht
- Brad Peercy, Rice University
- Eric Cytrynbaum, UC Davis, University of British Columbia

## Notes

- Funding provided by a grant from the NSF.
- This talk can be viewed at  
<http://www.math.utah.edu/keener/lectures/onset>
- No Microsoft products were used or harmed during the production of this talk.