Planet Migration with Disk Torques and Planet Scattering

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Outline

I. Giant planet migration through disk torques and planet-planet scattering events

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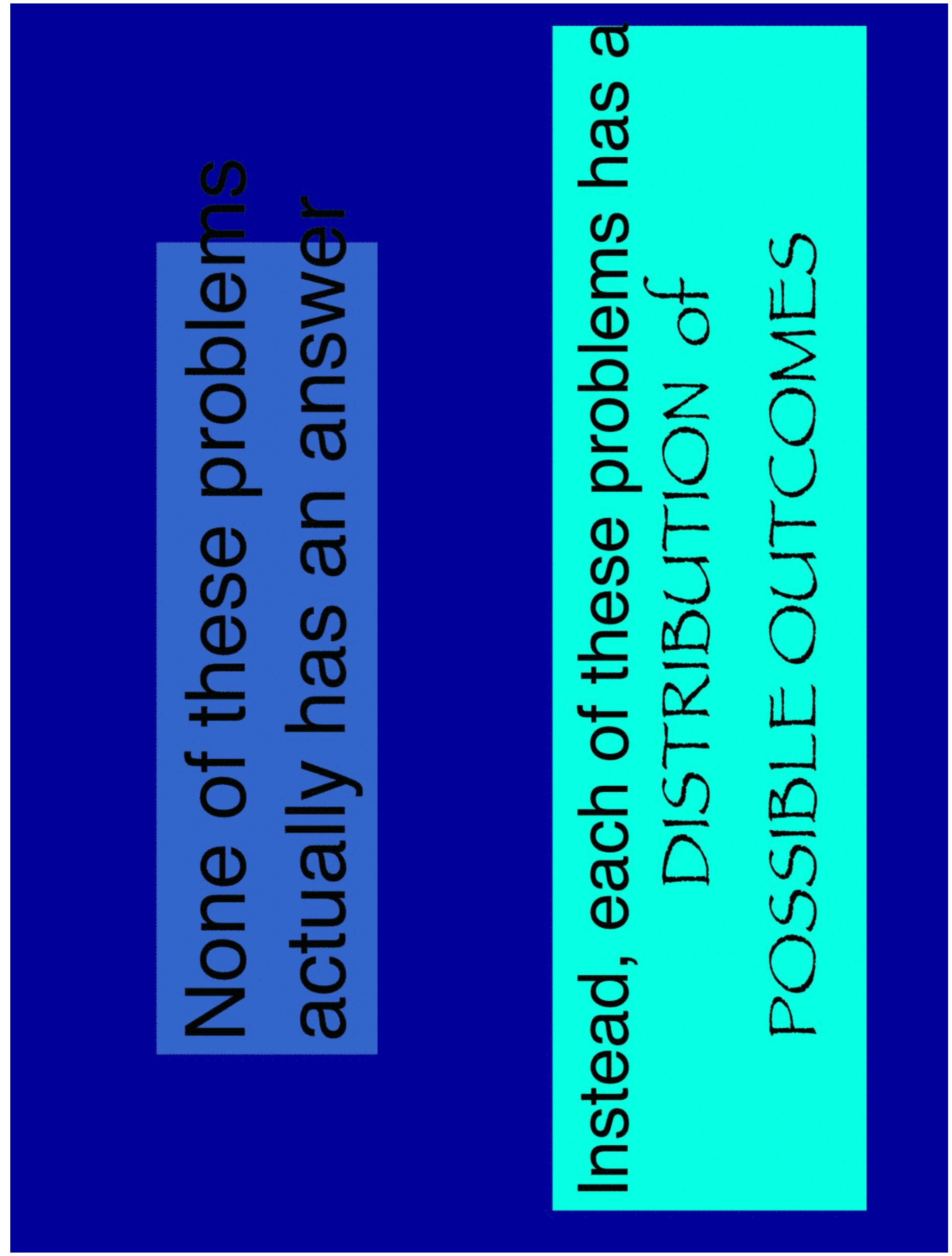
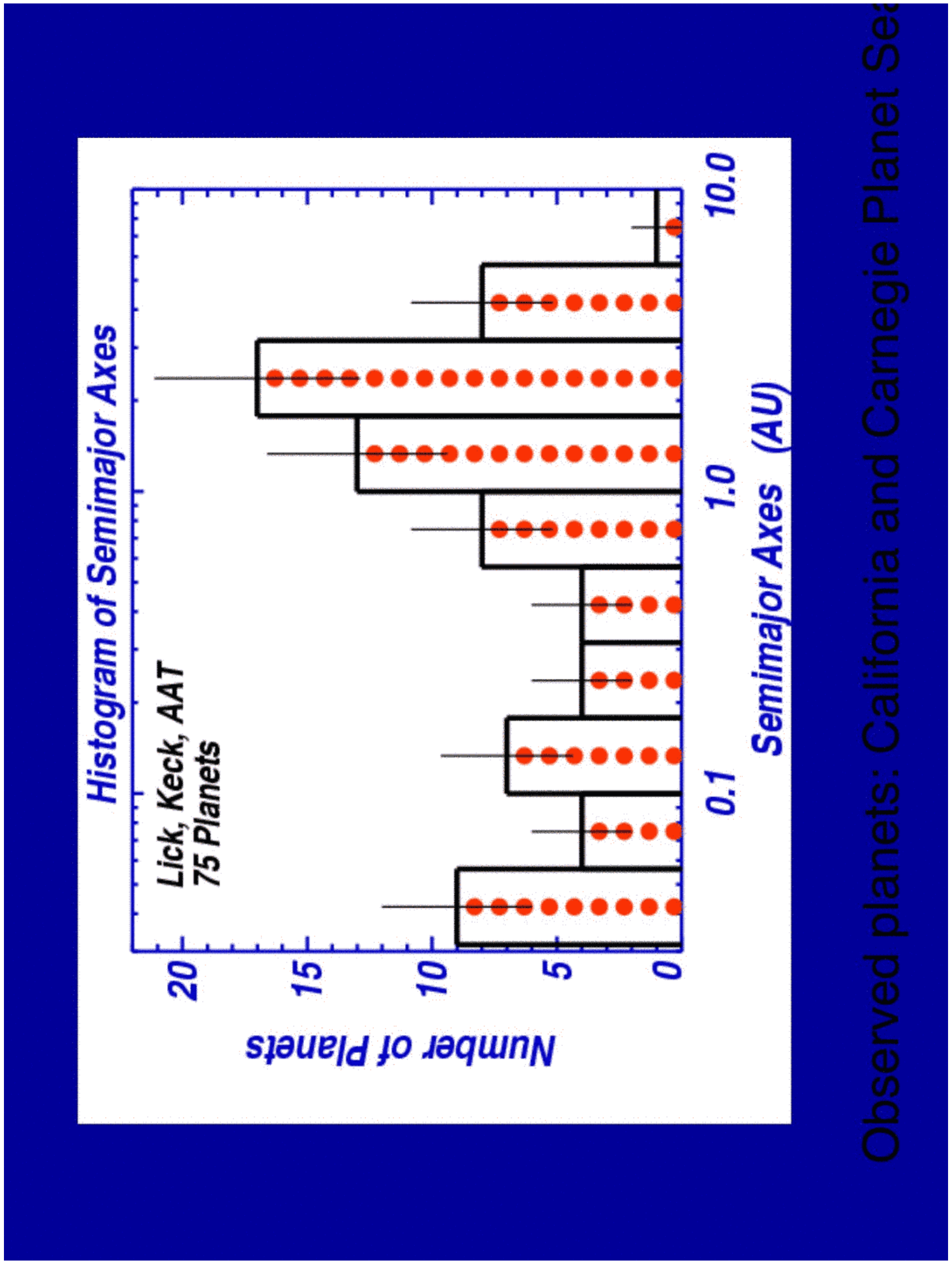
II. Type I migration with turbulent fluctuations

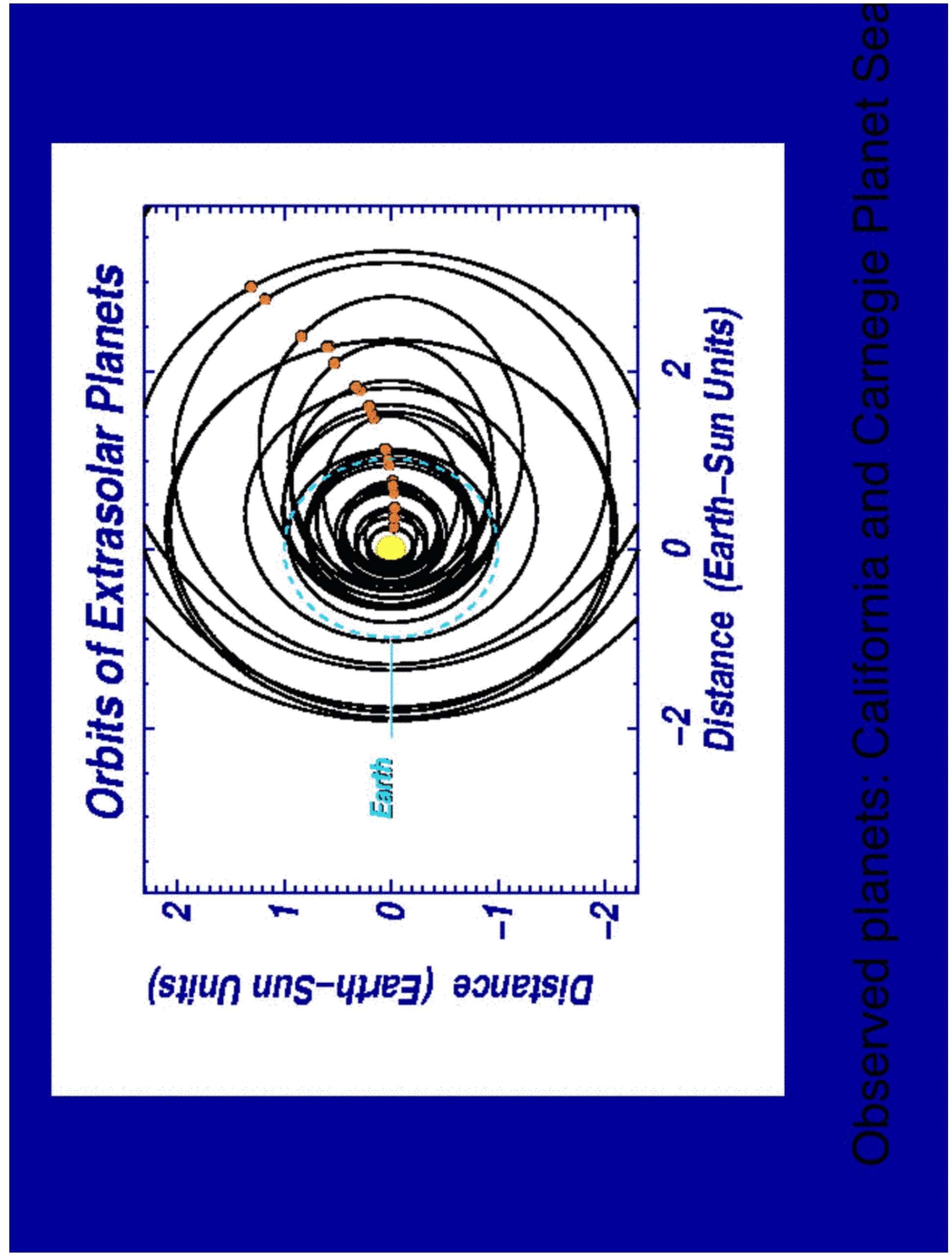
III. Solar system scattering in the solar birth aggregate

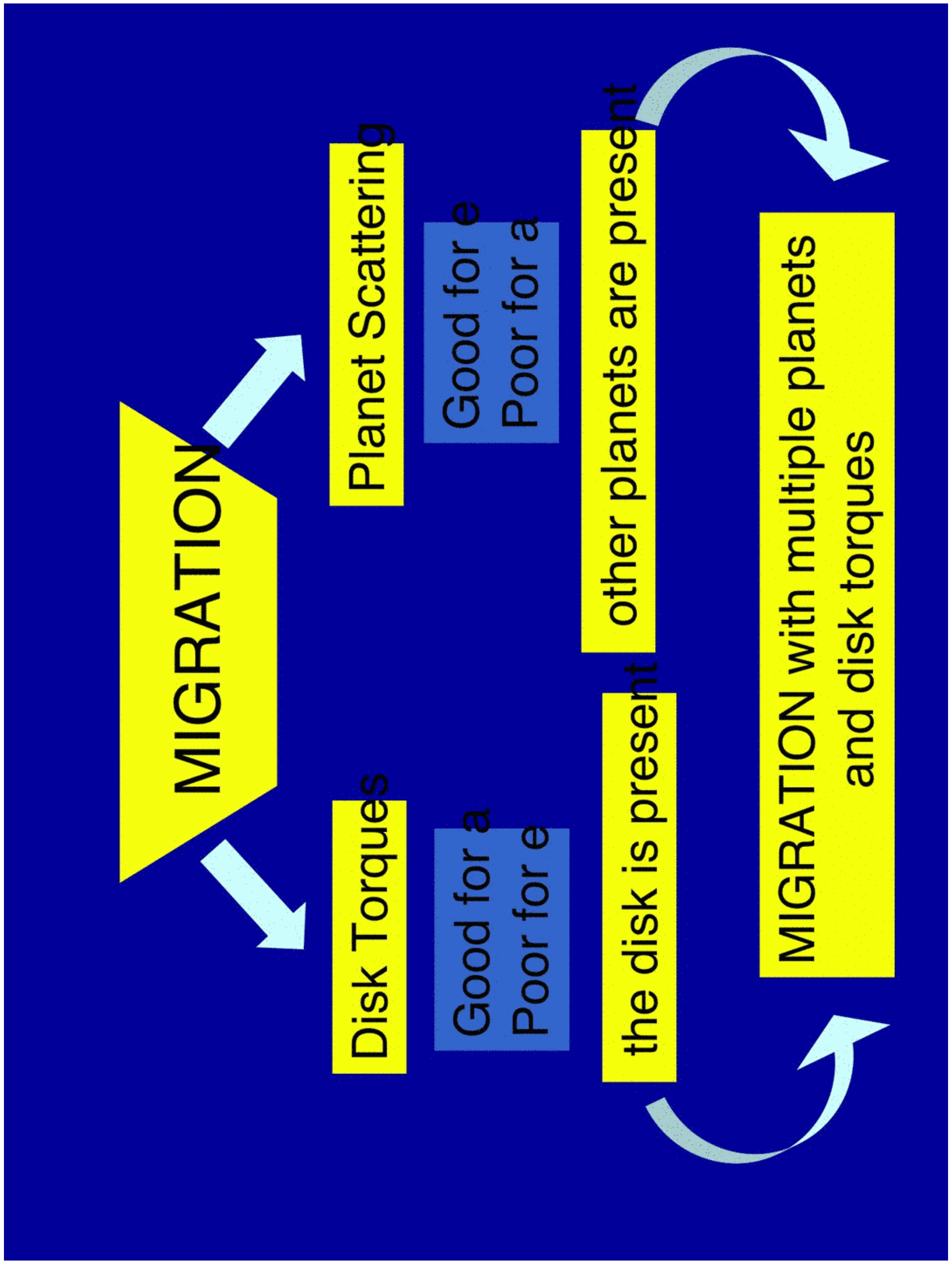
IV. Long term stability of Earth-like planets in binaries

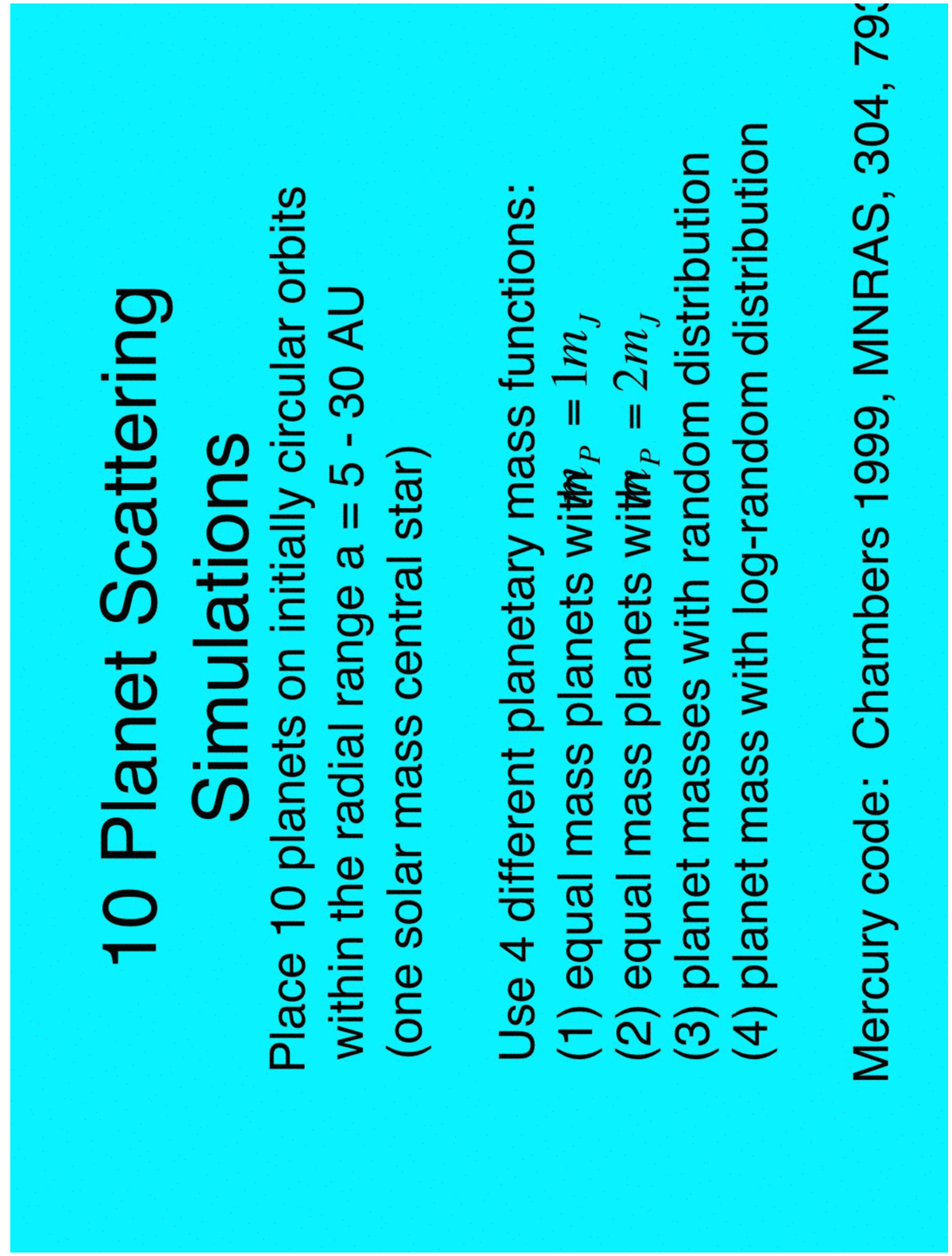

None of these problems
actually has an answer

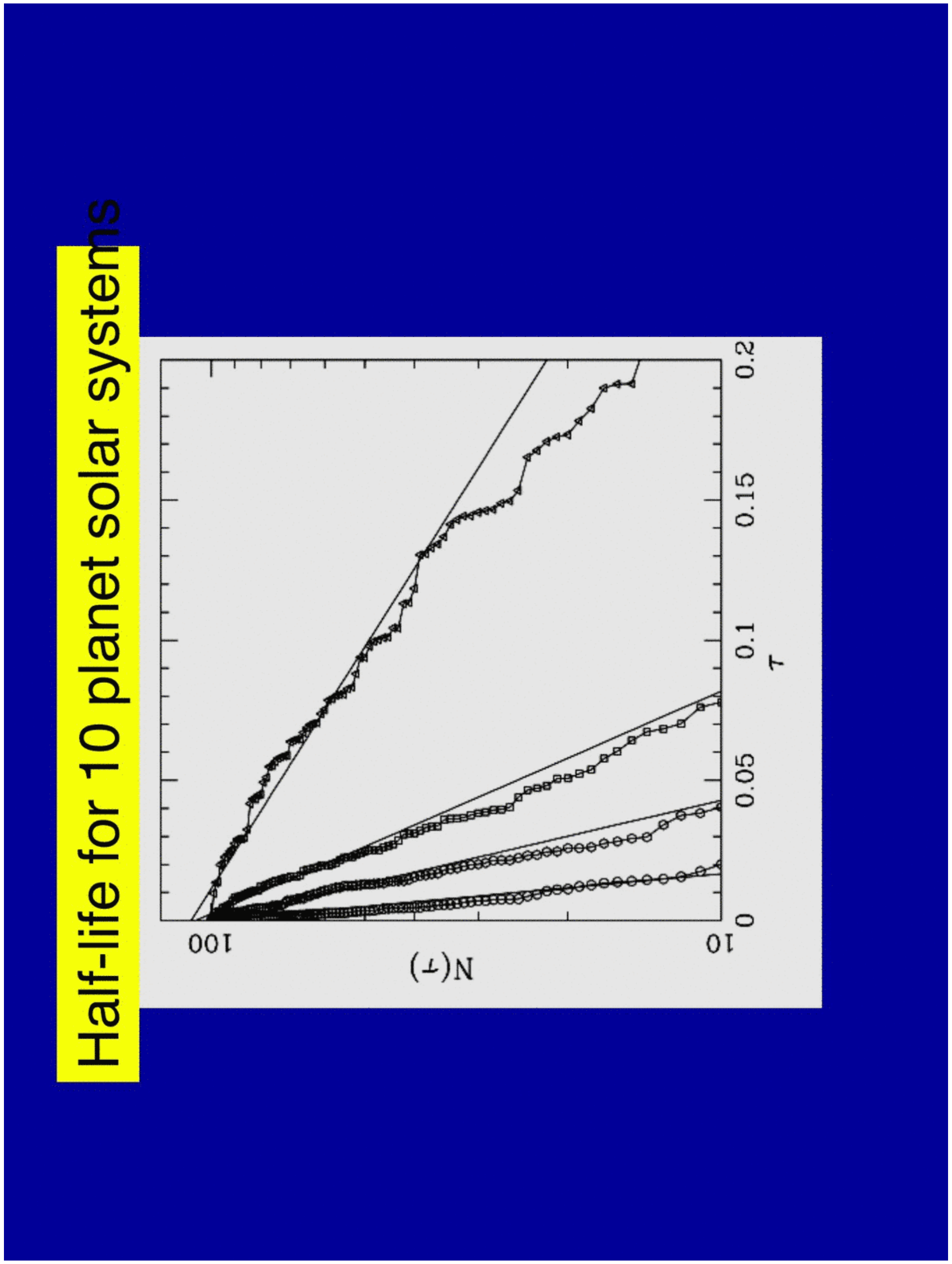

None of these problems
actually has an answer
Instead, each of these problems has a
DISTRIBUTION of
POSSIBLE OUTCOMES

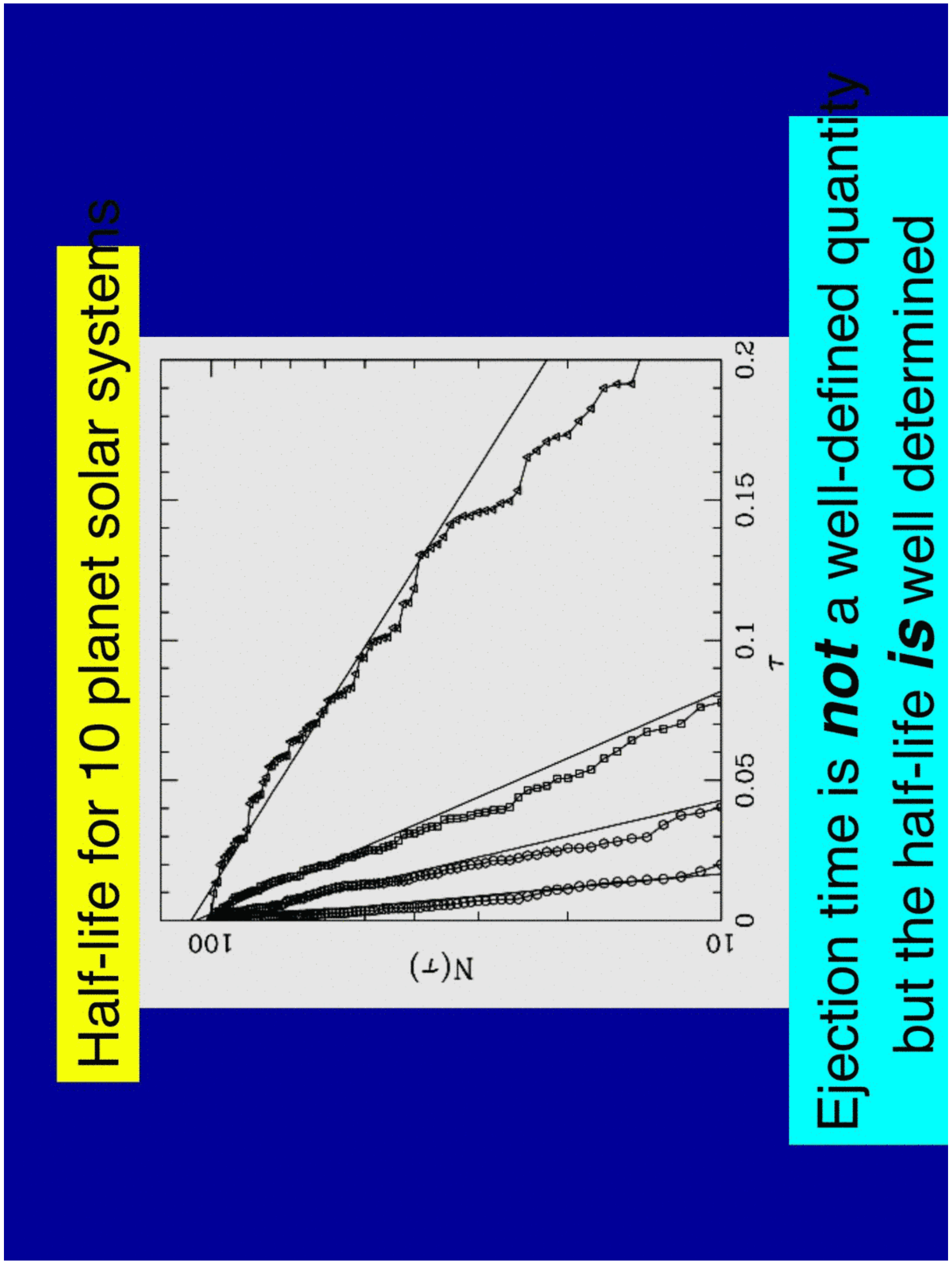

Histogram of Semimajor Axes
Lick, Keck, AAT
75 Planets
Number of Planets
0
5
10
15
20
0.1
1.0
10.0
Semimajor Axes (AU)
Observed planets: California and Carnegie Planet

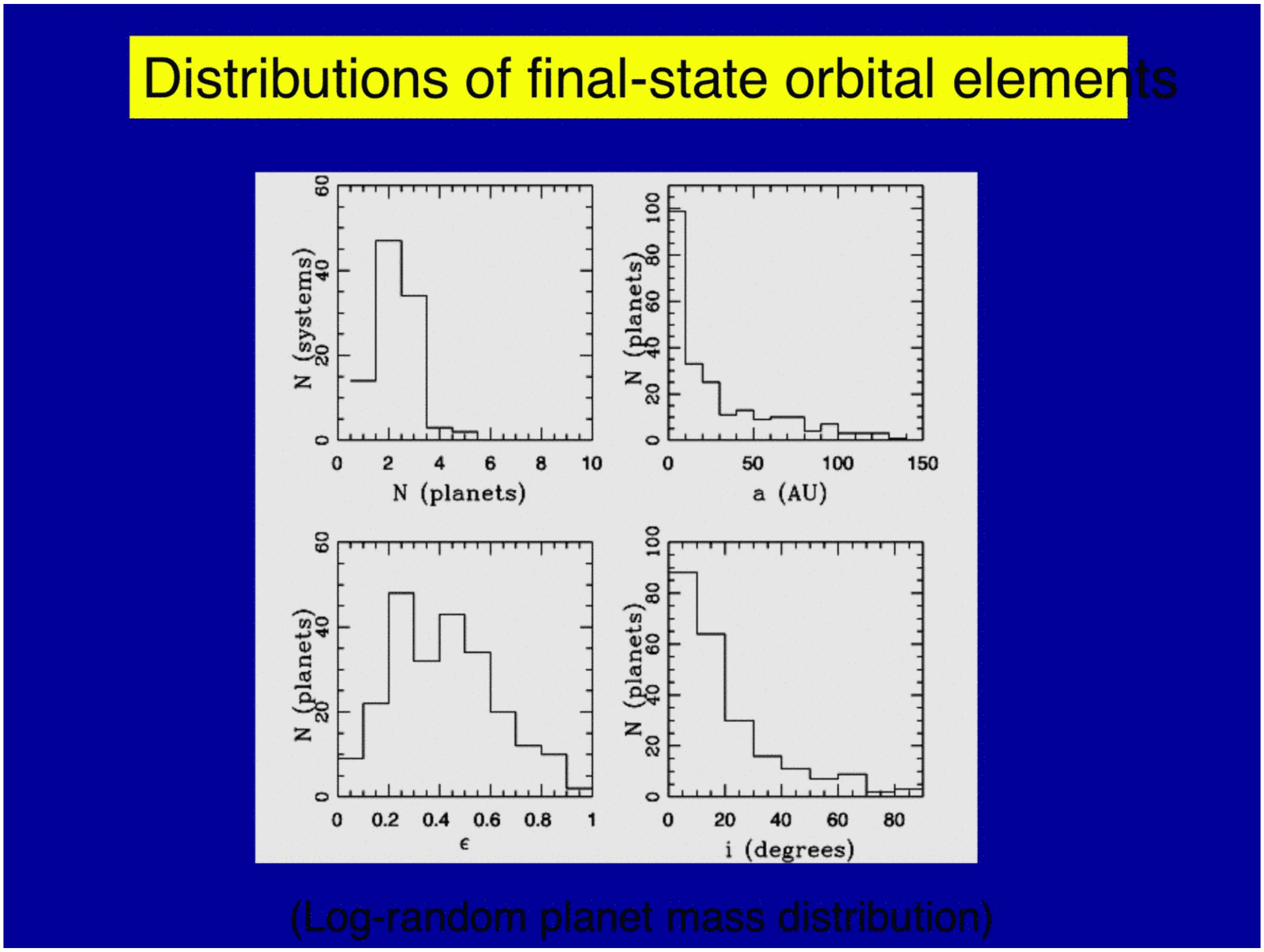

Orbits of Extrasolar Planets
Earth
Distance (Earth–Sun Units)
2
1
0
−1
−2
Distance (Earth–Sun Units)
−2
0
2
Observed planets: California and Carnegie Planet

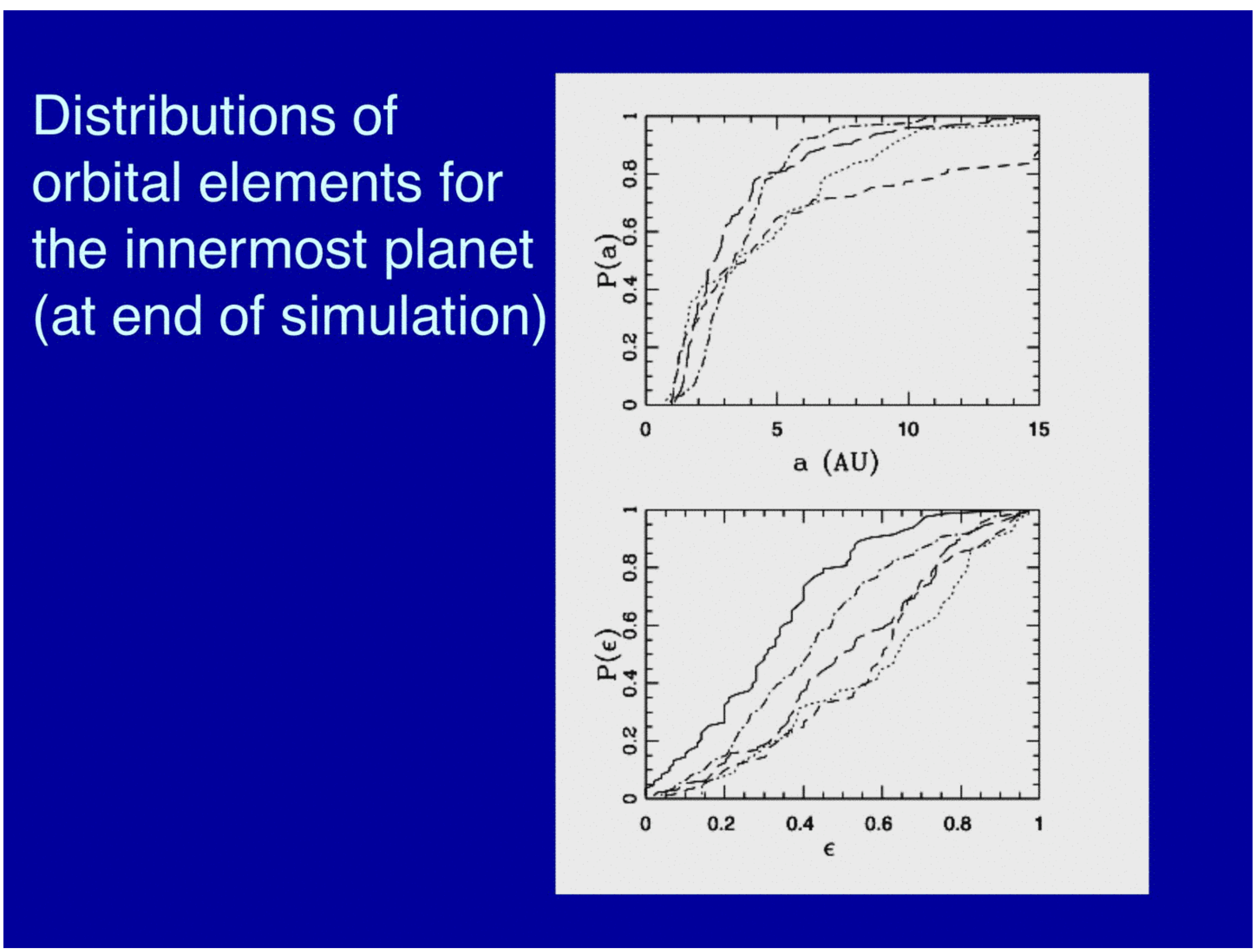

MIGRATION
Disk Torques
Good for a
Poor for e
Planet Scattering
Good for e
Poor for a
the disk is present
other planets are present
MIGRATION with multiple planets
and disk torques

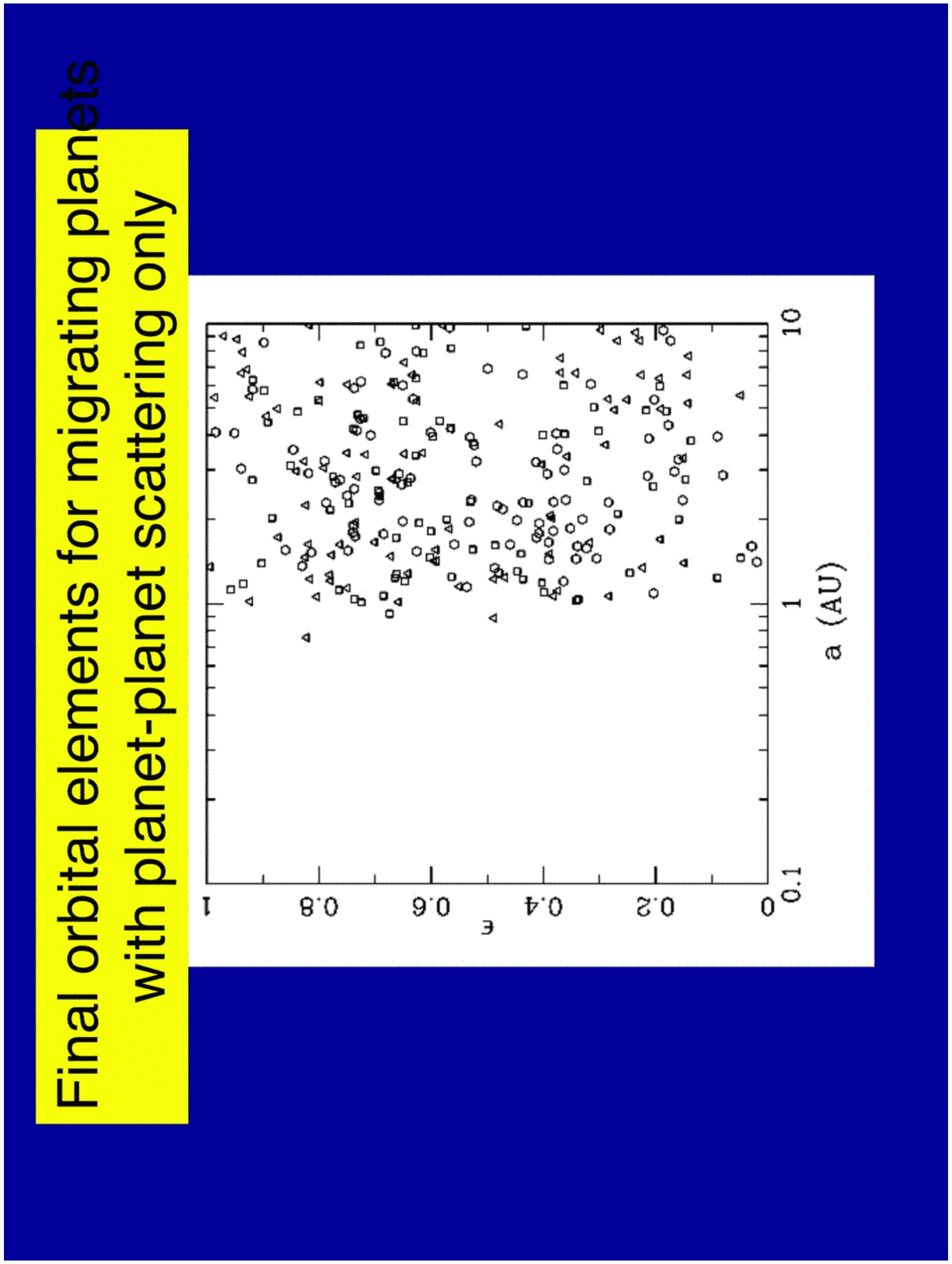

10 Planet Scattering
Simulations
Place 10 planets on initially circular orbits
within the radial range a = 5 - 30 AU
(one solar mass central star)
Use 4 different planetary mass functions:
(1) equal mass planets with $m_P = 1m_J$
(2) equal mass planets with $m_P = 2m_J$
(3) planet masses with random distribution
(4) planet mass with log-random distribution
Mercury code: Chambers 1999, MNRAS, 304, 79

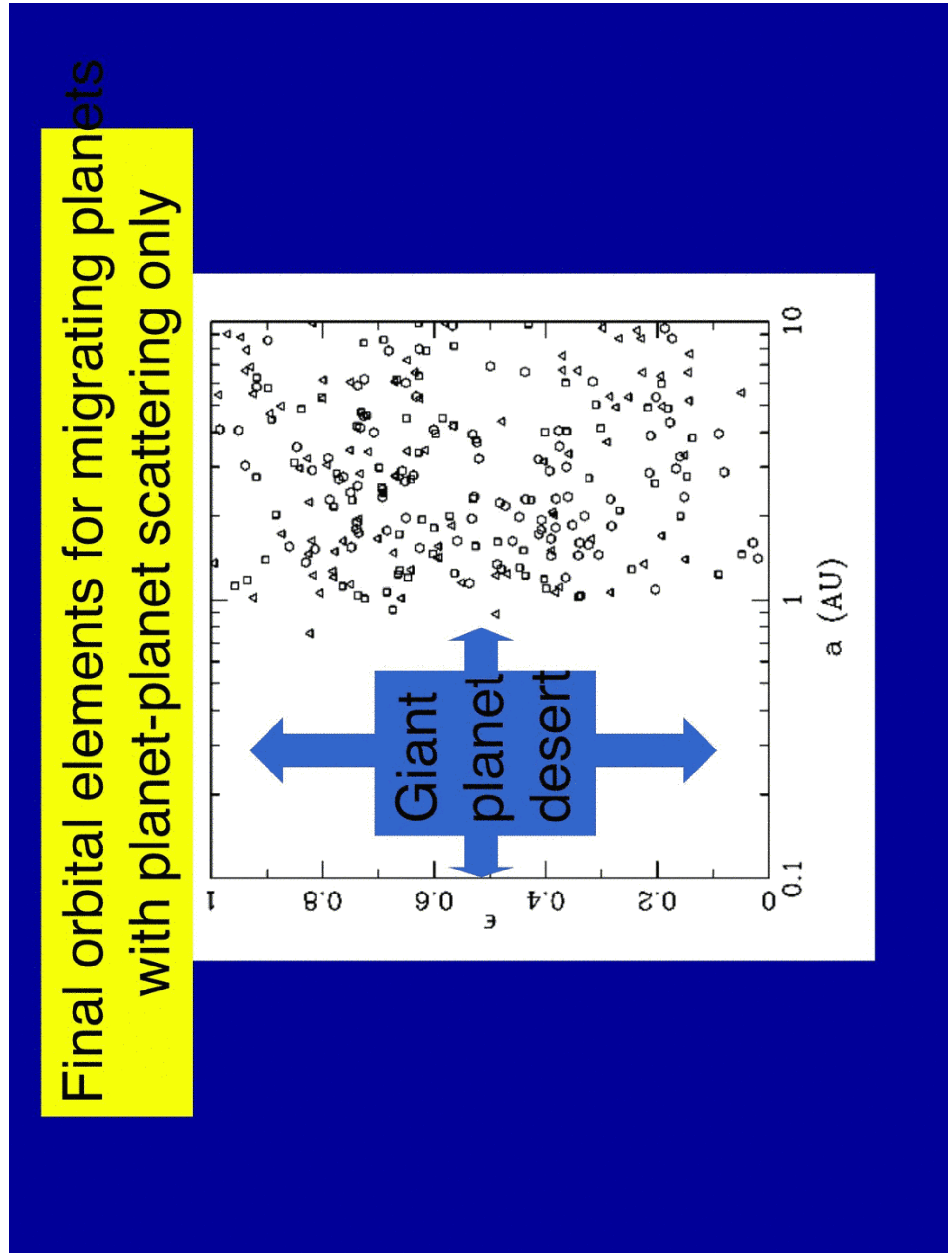

Half-life for 10 planet solar systems
N(τ)
100
10
τ
0
0.05
0.1
0.15
0.2


Half-life for 10 planet solar systems
N(τ)
100
10
τ
0
0.05
0.1
0.15
0.2
Ejection time is ***not*** a well-defined quantity
but the half-life ***is*** well determined


Distributions of final-state orbital elements
N (systems)
N (planets)
N (planets)
a (AU)
N (planets)
ε
N (planets)
i (degrees)
(Log-random planet mass distribution)


Distributions of orbital elements for the innermost planet (at end of simulation)
P(a)
a (AU)
P(ε)
ε


Final orbital elements for migrating planets
with planet-planet scattering only
e
a (AU)
0.1
1
10
0
0.2
0.4
0.6
0.8
1


Final orbital elements for migrating planets
with planet-planet scattering only
Giant
planet
desert
e
a (AU)
0.1
1
10
0
0.2
0.4
0.6
0.8
1

Maximally Efficient Scattering Disk

Starting energy: $E_0 = -\frac{GM_* m_P}{2a_0}$

Scattering changes energy $\Delta E = -\frac{GM_* \mu}{2r}$

$$a_1 = a_0(1+\mu/m_P)^{-1}, \;\; a_n = a_0(1+\mu/m_P)^{-n}$$

$$f_n = a_0/a_n = (1+\frac{M_S/m_P}{n})^n$$

$$\lim_{n\to\infty} f_n = \exp[M_S/m_P] \Rightarrow \boxed{M_{MES} = m_P \ln[a_0/a_f]}$$

Surface density distribution

$$\sigma(r) = \sum_{k=1}^{n} \frac{\mu}{2\pi r} \; \delta(r-a_k)$$ (in discrete form)

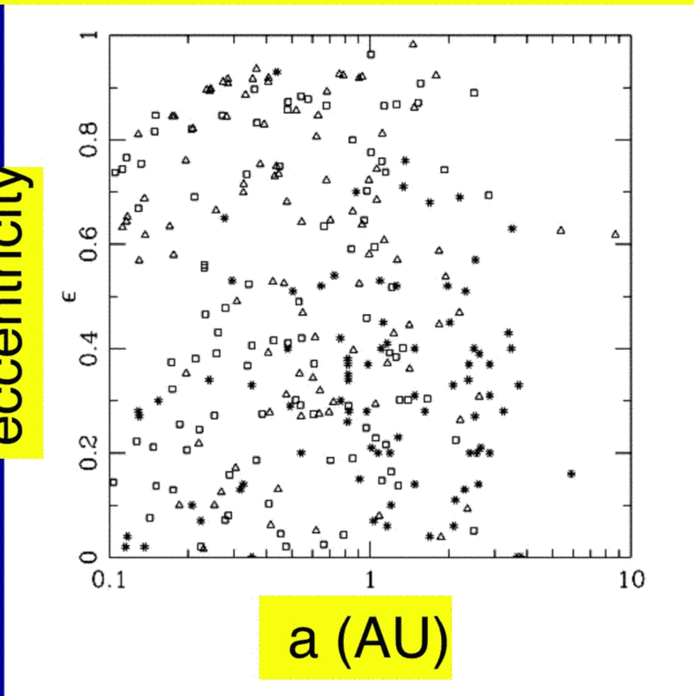
in the limit: $n \to \infty, \;\; \mu \to 0, \;\; n\mu \to const$

$$\sigma_{MES}(r) \to \frac{m_P}{2\pi r^2}$$

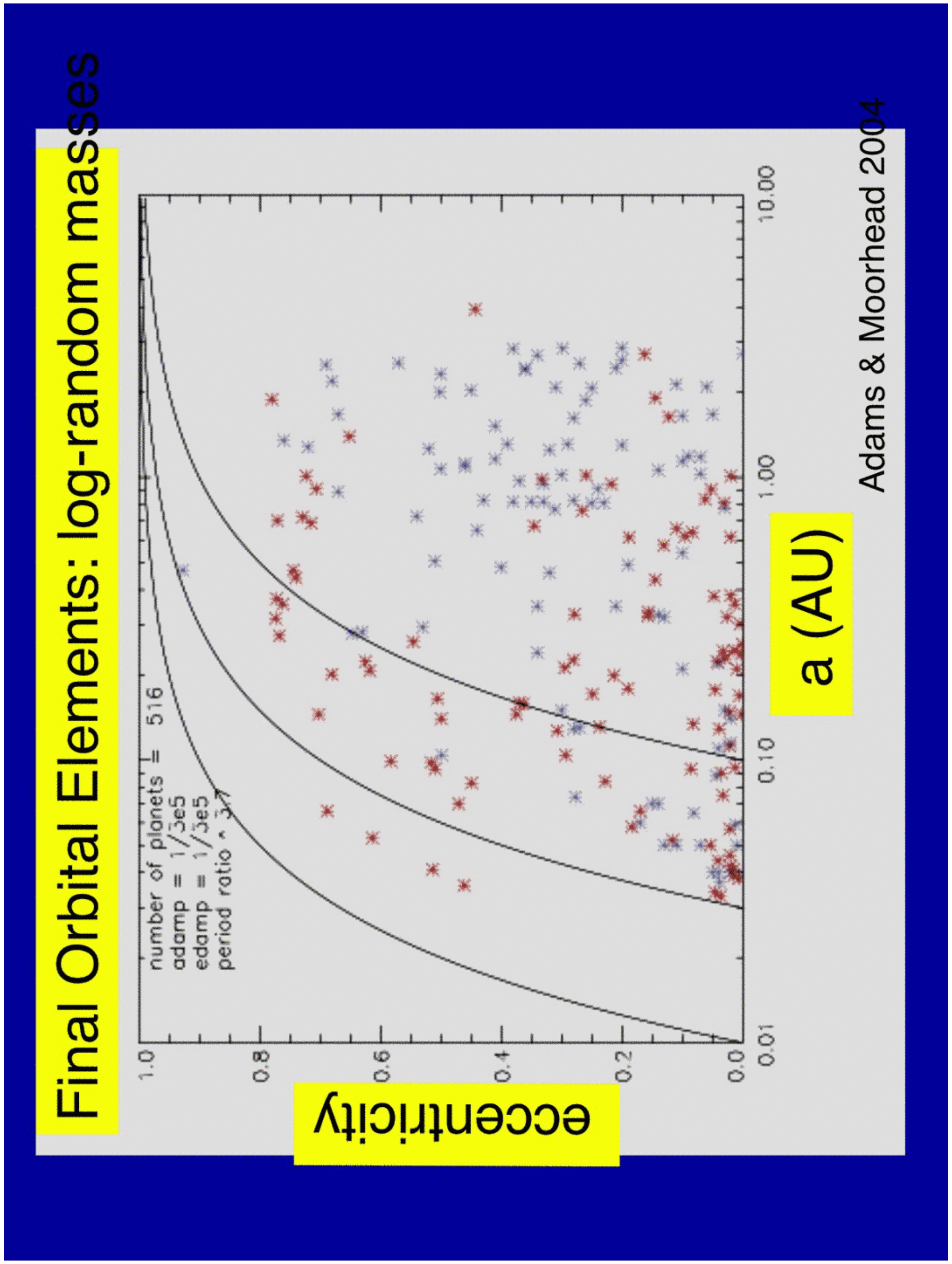
2 Planet simulations with disk torques and scattering

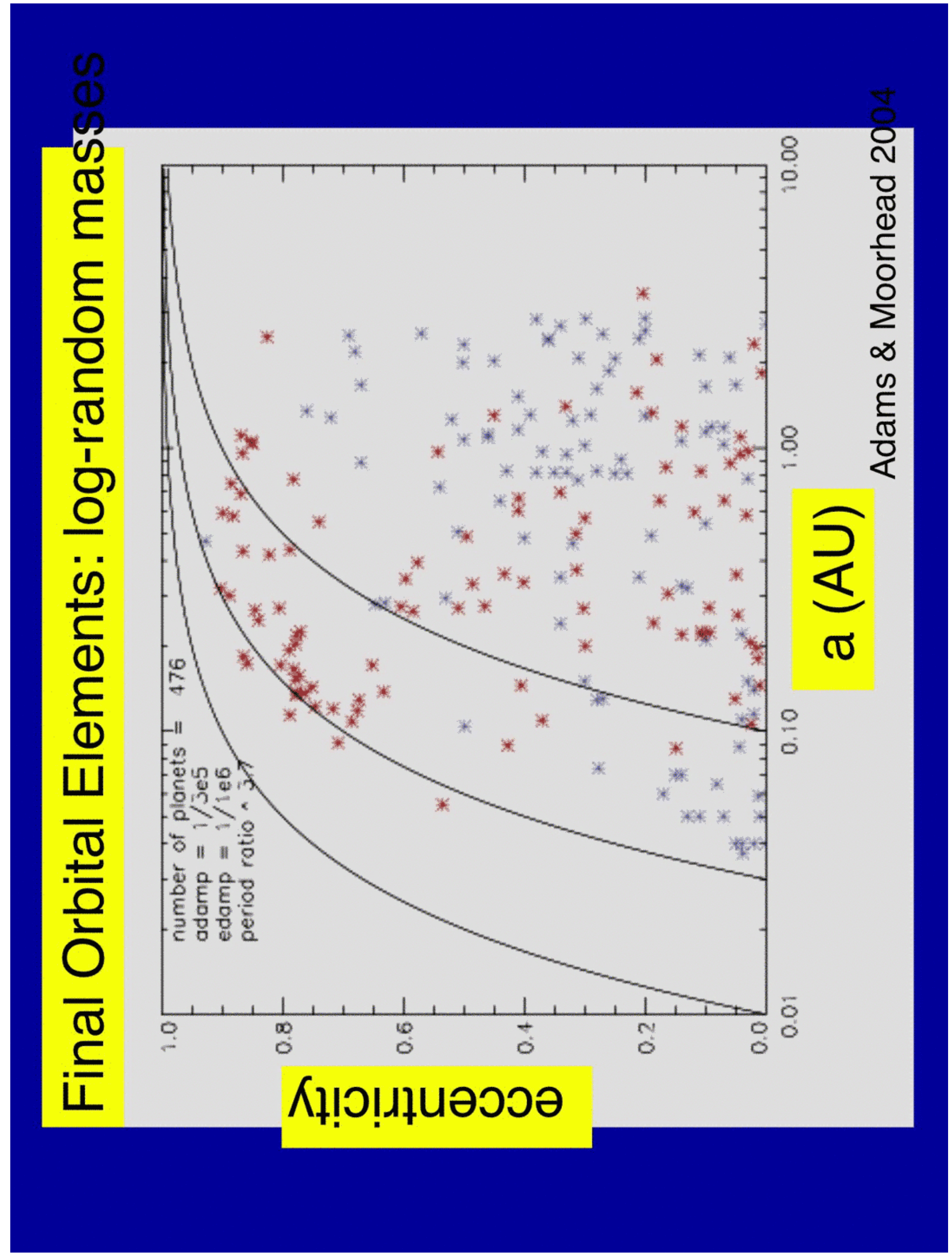
- B-S code
- Starting periods out of resonance: $P_1 = 1900d, P_2 = P_1 \times 3.736...$
- Random planet masses: $m_P = 0 - 5m_J$
  or log-random masses: $Log_{10}[m_P] = -1 \rightarrow 1$
- Viscous evolution time = 0.3 Myr
- Allow planetary collisions: $r_P = 2r_J$
- Relativistic corrections
- Tidal interactions with the central star
- Random disk lifetime = 0 - 1 Myr
- Eccentricity damping time = 1 - 3 Myr

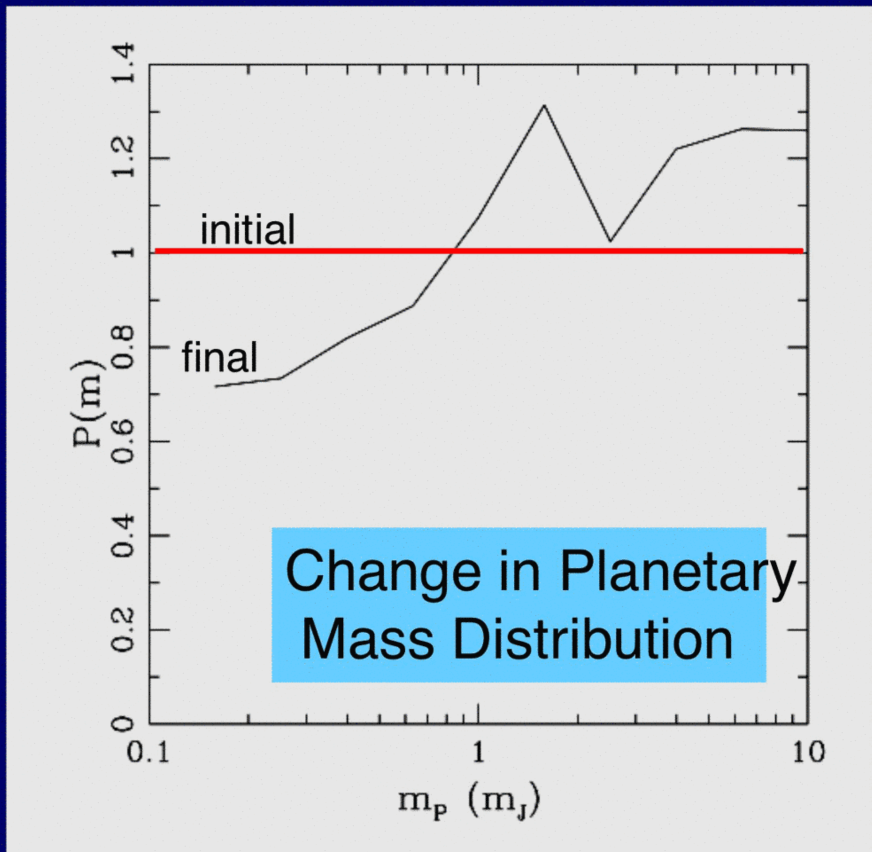
Final Orbital Elements for Migrating Planets with disk torques and planet scattering

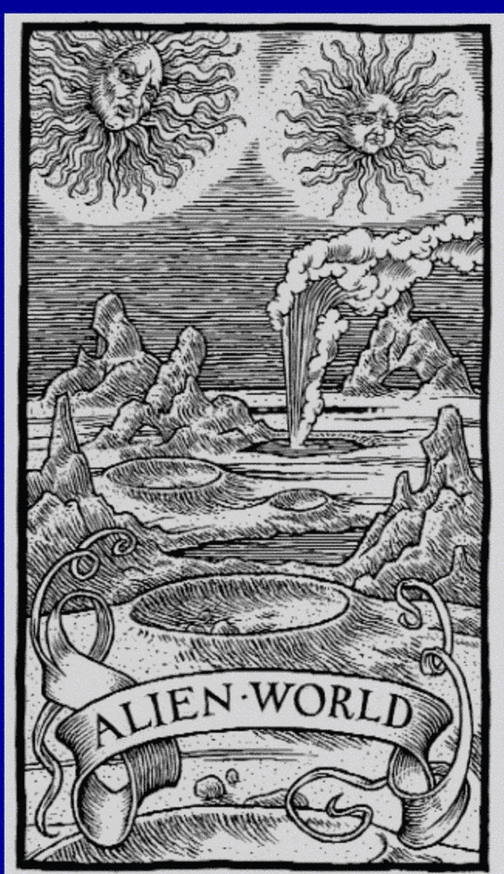


Adams and Laughlin 2003, Icarus, 163, 2


Final Orbital Elements: log-random masses
eccentricity
a (AU)
Adams & Moorhead 2004
number of planets = 516
adamp = 1/3e5
edamp = 1/3e5
1.0
0.8
0.6
0.4
0.2
0.0
0.01
0.10
1.00
10.00


Final Orbital Elements: log-random masses
eccentricity
a (AU)
Adams & Moorhead 2004
number of planets = 476
adamp = 1/3e5
edamp = 1/1e6
1.0
0.8
0.6
0.4
0.2
0.0
0.01
0.10
1.00
10.00





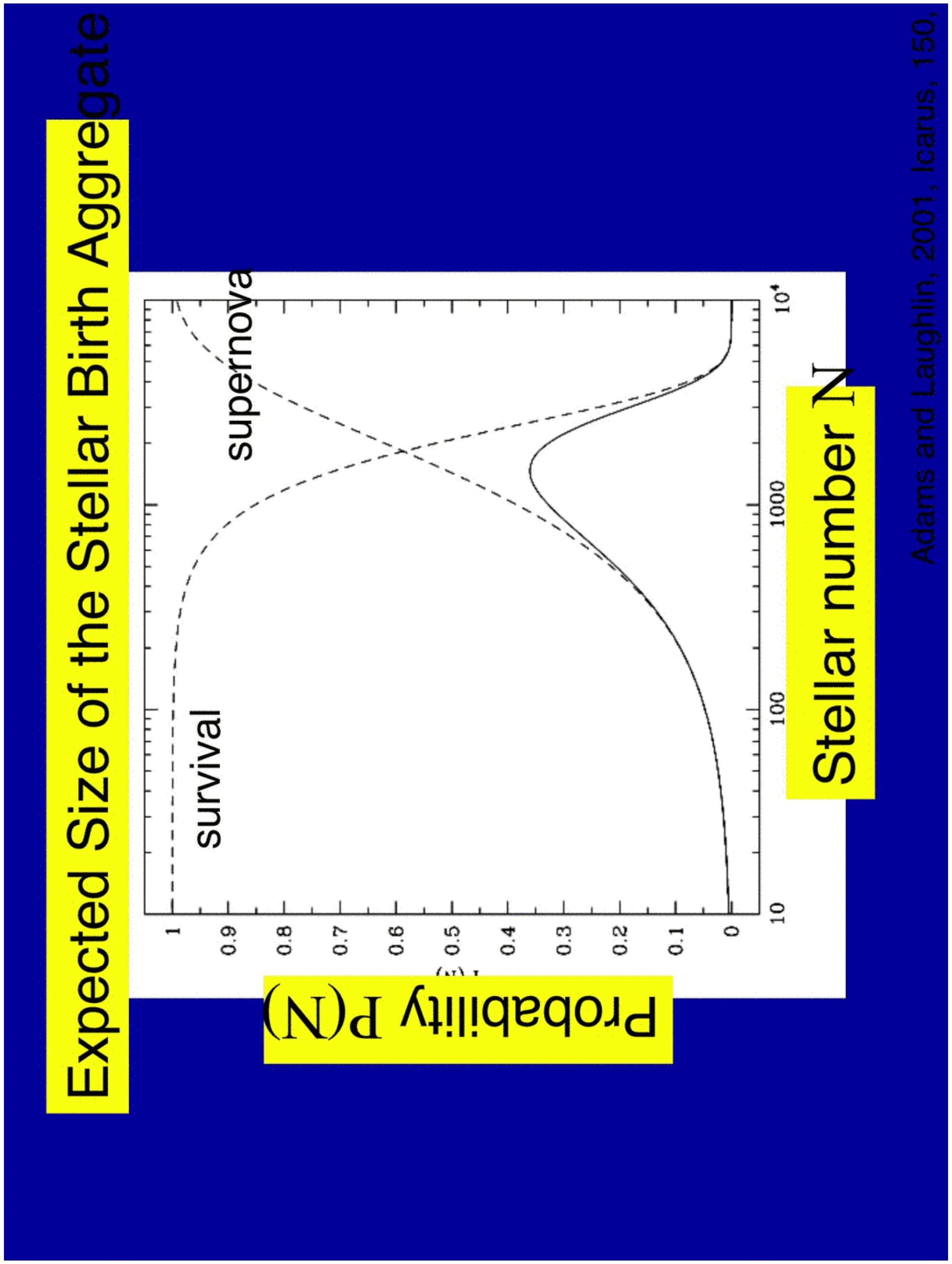
Conclusions:

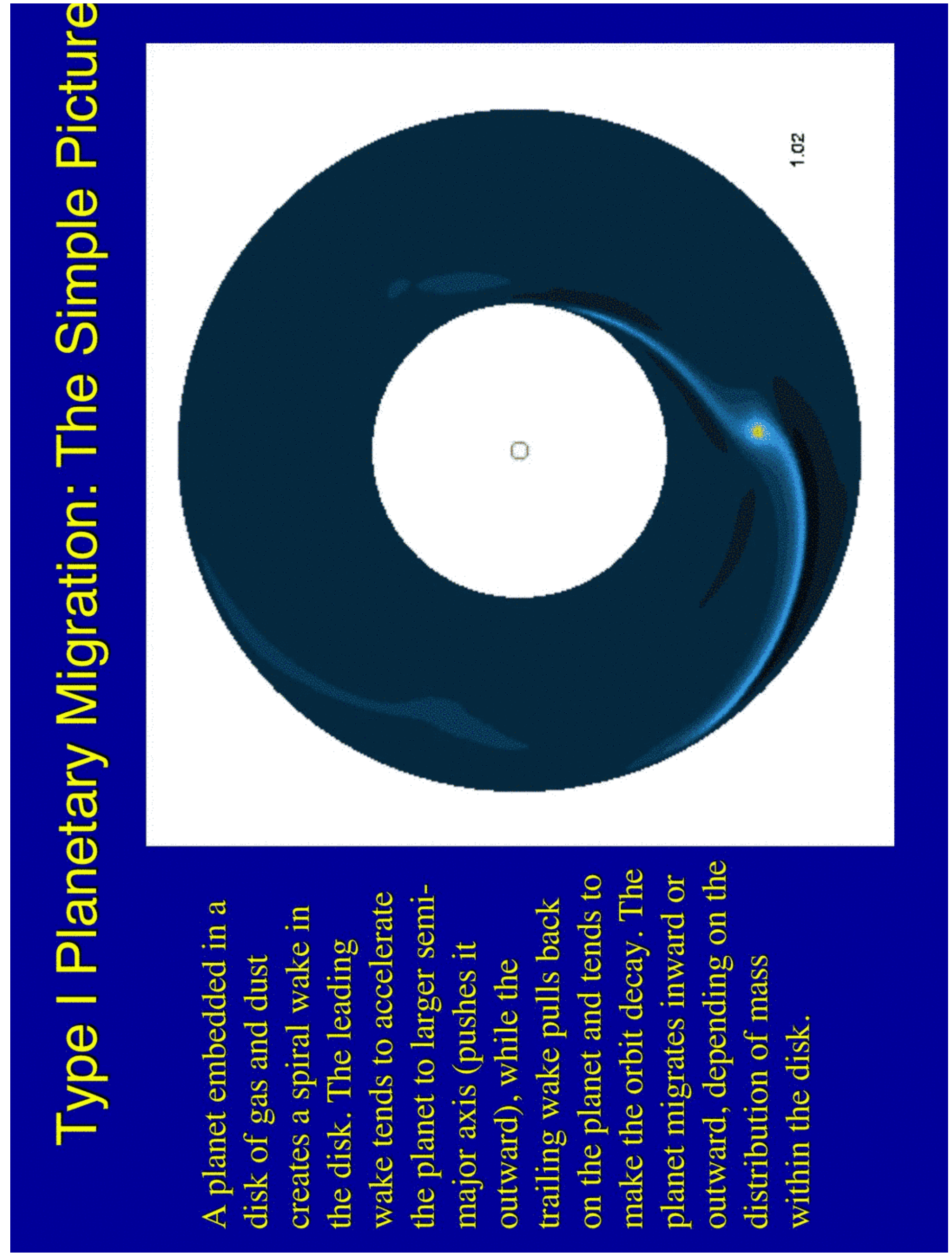
Planet scattering gives e (but not a)

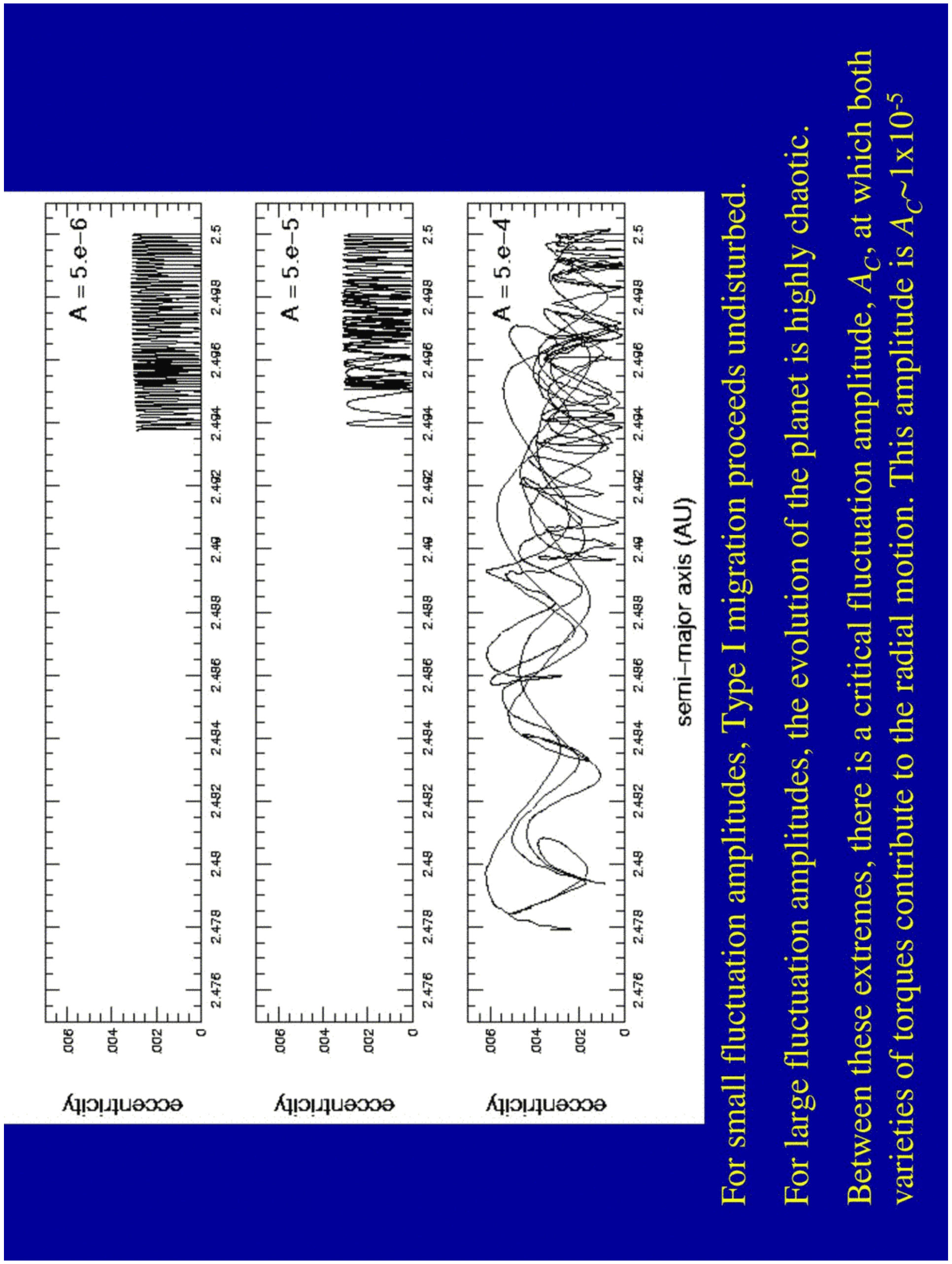
Disk torques can provide a (but not e)

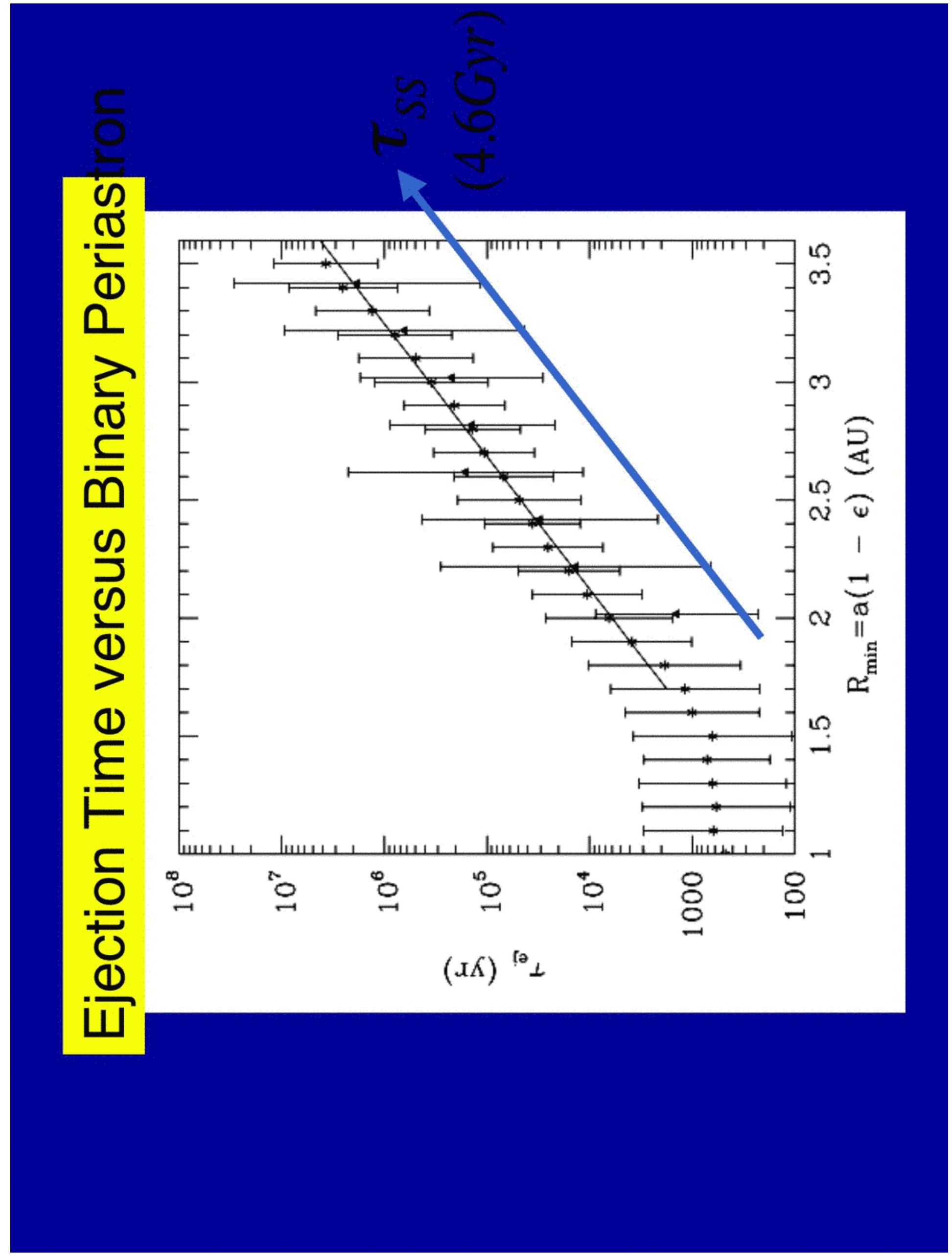
Disk torques and planet scattering can fill the entire a-e plane (as observed)

Adams & Laughlin 2003, Icarus, 163, 290


Expected Size of the Stellar Birth Aggregate
supernova
survival
Stellar number N
Probability P(N)
Adams and Laughlin, 2001, Icarus, 150,


Type I Planetary Migration: The Simple Picture
1.02
A planet embedded in a disk of gas and dust creates a spiral wake in the disk. The leading wake tends to accelerate the planet to larger semi-major axis (pushes it outward), while the trailing wake pulls back on the planet and tends to make the orbit decay. The planet migrates inward or outward, depending on the distribution of mass within the disk.


A = 5.e−6
A = 5.e−5
A = 5.e−4
eccentricity
semi−major axis (AU)
For small fluctuation amplitudes, Type I migration proceeds undisturbed.
For large fluctuation amplitudes, the evolution of the planet is highly chaotic.
Between these extremes, there is a critical fluctuation amplitude, A_C, at which both varieties of torques contribute to the radial motion. This amplitude is $A_C \sim 1 \times 10^{-5}$


Ejection Time versus Binary Periastron
τ_{SS} (4.6Gyr)
τ_{ej} (yr)
$R_{min} = a(1 - \epsilon)$ (AU)

Conclusions:

Planet scattering gives e (but not a)

Disk torques can provide a (but not e)

Disk torques and planet scattering can fill the entire a-e plane (as observed)

Adams & Laughlin 2003, Icarus, 163, 290

Solar Birth Aggregate

Supernova enrichment requires large N

$$M_* > 25 M_\odot$$

$$F_{SN} = 0.000485$$

Well ordered solar system requires small N

$$\varepsilon(Neptune) < 0.1$$

$$\Delta\Theta_j < 3.5^\circ$$

Probability of Supernovae

$$P_{SN}(N) = 1 - f_{not}^N = 1 - (1 - F_{SN})^N$$

Probability of Scattering Event

Scattering rate: $\Gamma = n\langle \sigma v \rangle$

Survival probability: $P_{survive} \propto \exp\left[-\int \Gamma dt\right]$

Known results provide n, v, t as function of N (e.g. BT87)

need to calculate the interaction cross sections $\langle \sigma \rangle$

Basic Cluster Considerations

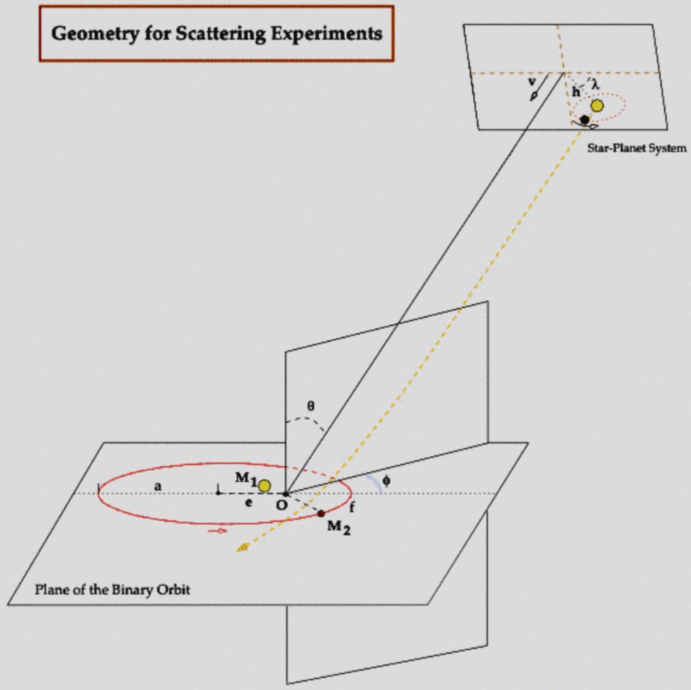
Velocity: $\langle v \rangle \approx 1 \ km/s$

Density and time: $n \approx N/4R^3$

$t_R \approx (R/v) \ N/(10 \ln N)$

$(nvt)_0 \approx N^2/40R^2 \ln N \propto N^{\mu}$

Geometry for Scattering Experiments

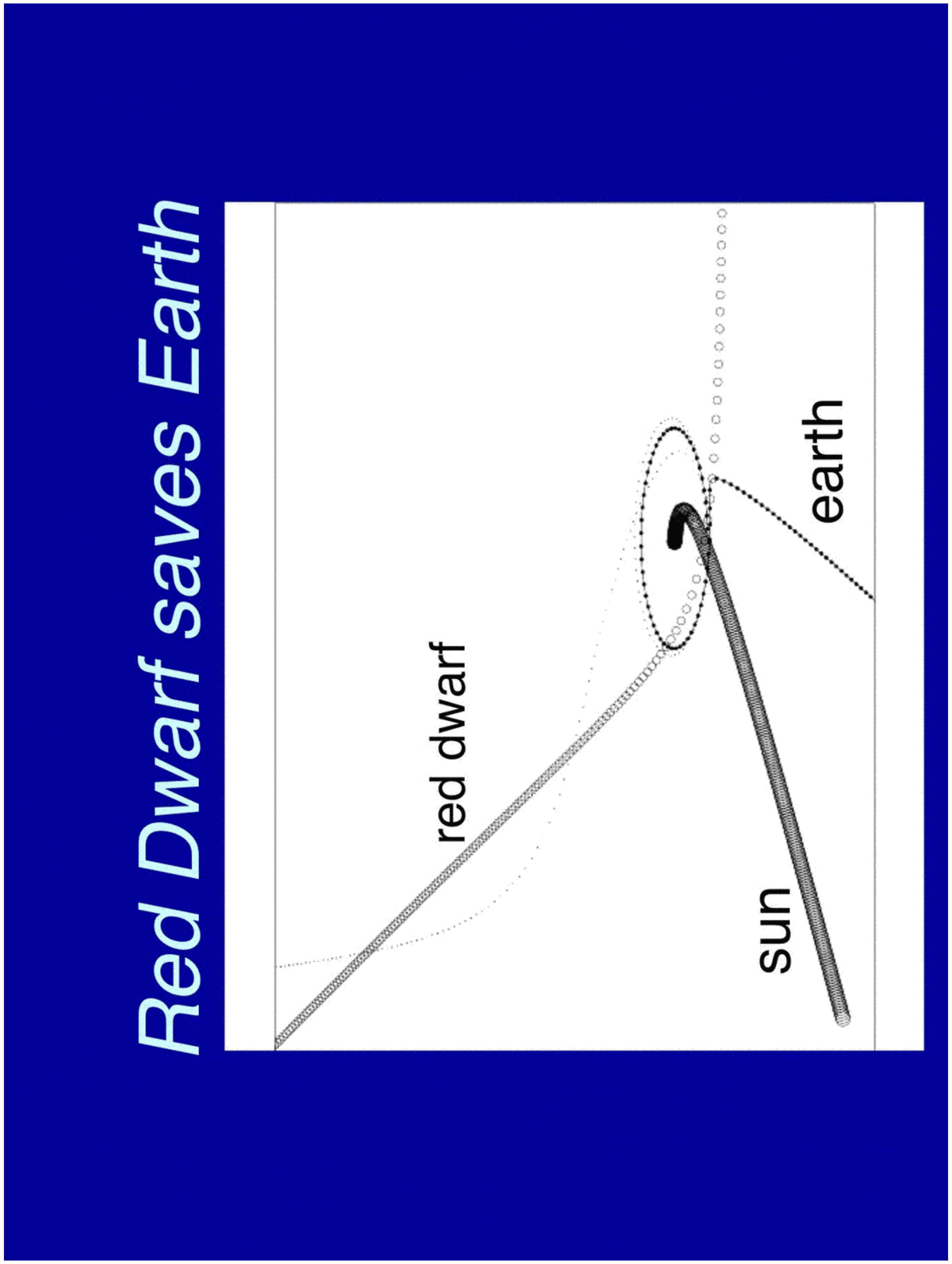


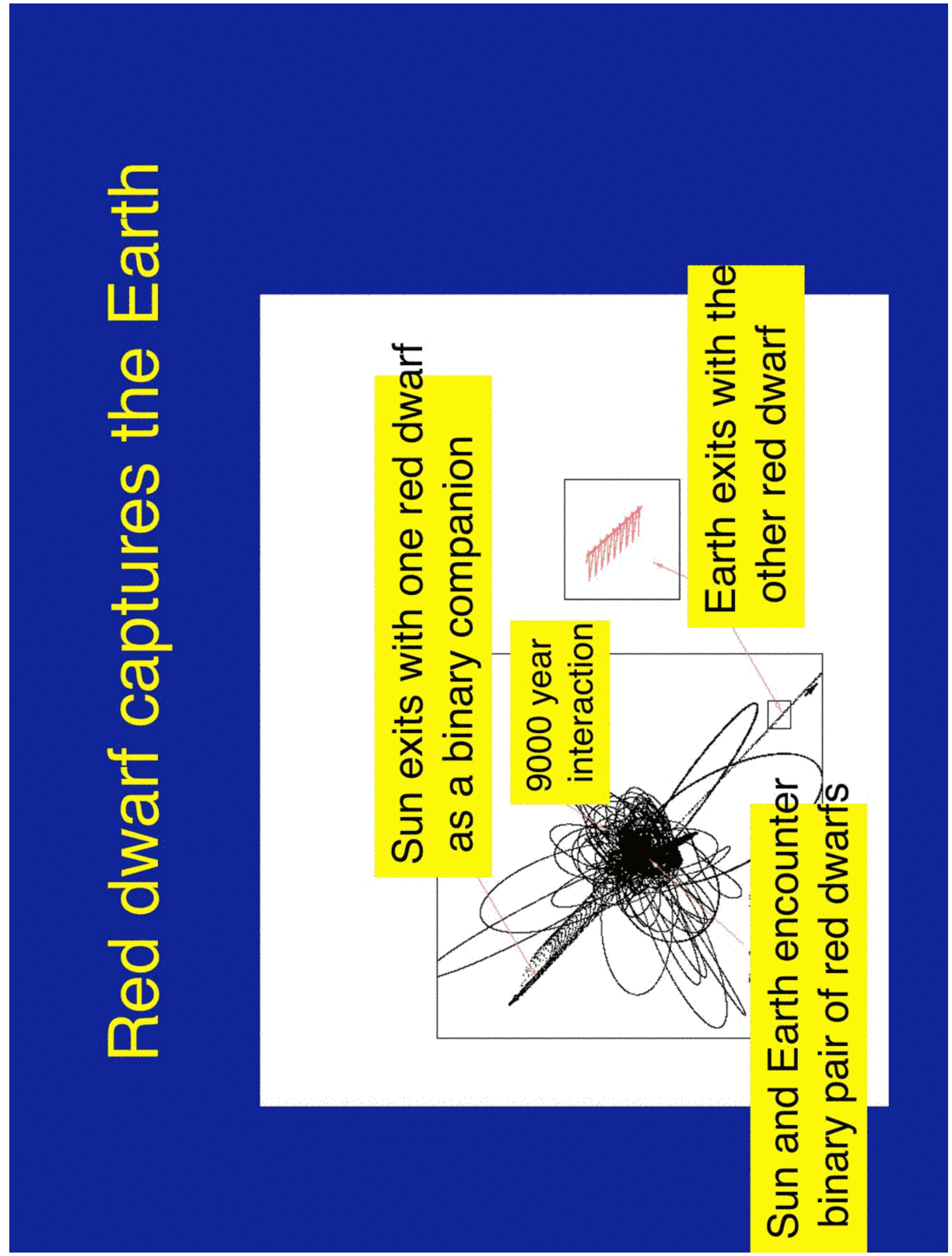
Star-Planet-Binary scattering encounters are specified by 13 parameters.

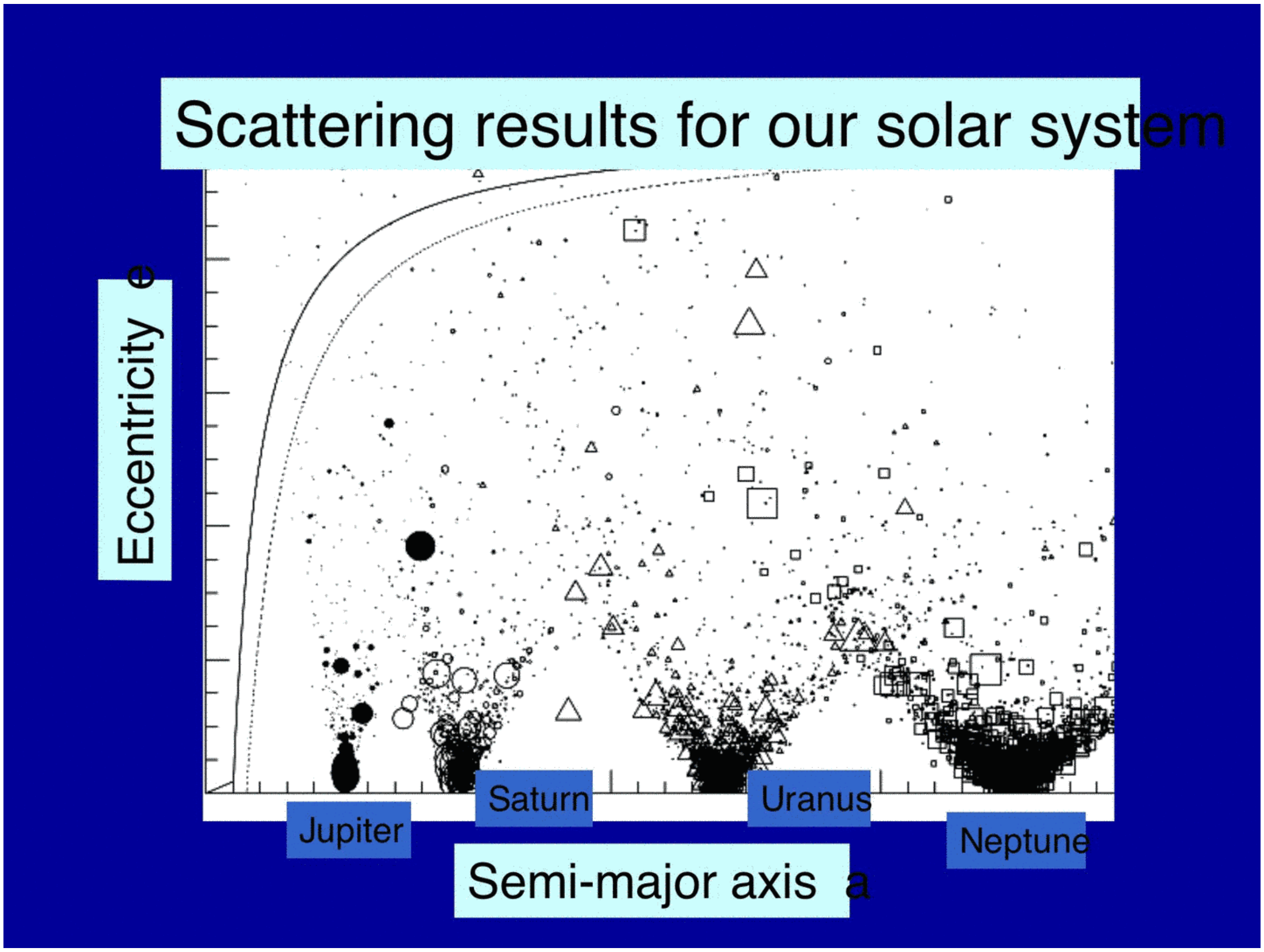
1. Parameters describing the binary orbit: m_1, m_2, e,f, a
2. Parameters describing the encounter: v, h ,ψ, θ, ϕ
3. Parameters describing a (circular) planetary orbit: r, θ_1, θ_2, θ_3

Monte Carlo Experiments

- Jupiter only, v = 1 km/s, N=40,000 realizations
- 4 giant planets, v = 1 km/s, N=50,000 realizations
- KB Objects, v = 1 km/s, N=30,000 realizations
- Earth only, v = 40 km/s, N=100,000 realizations
- 4 giant planets, v = 40 km/s, N=100,000 realizations

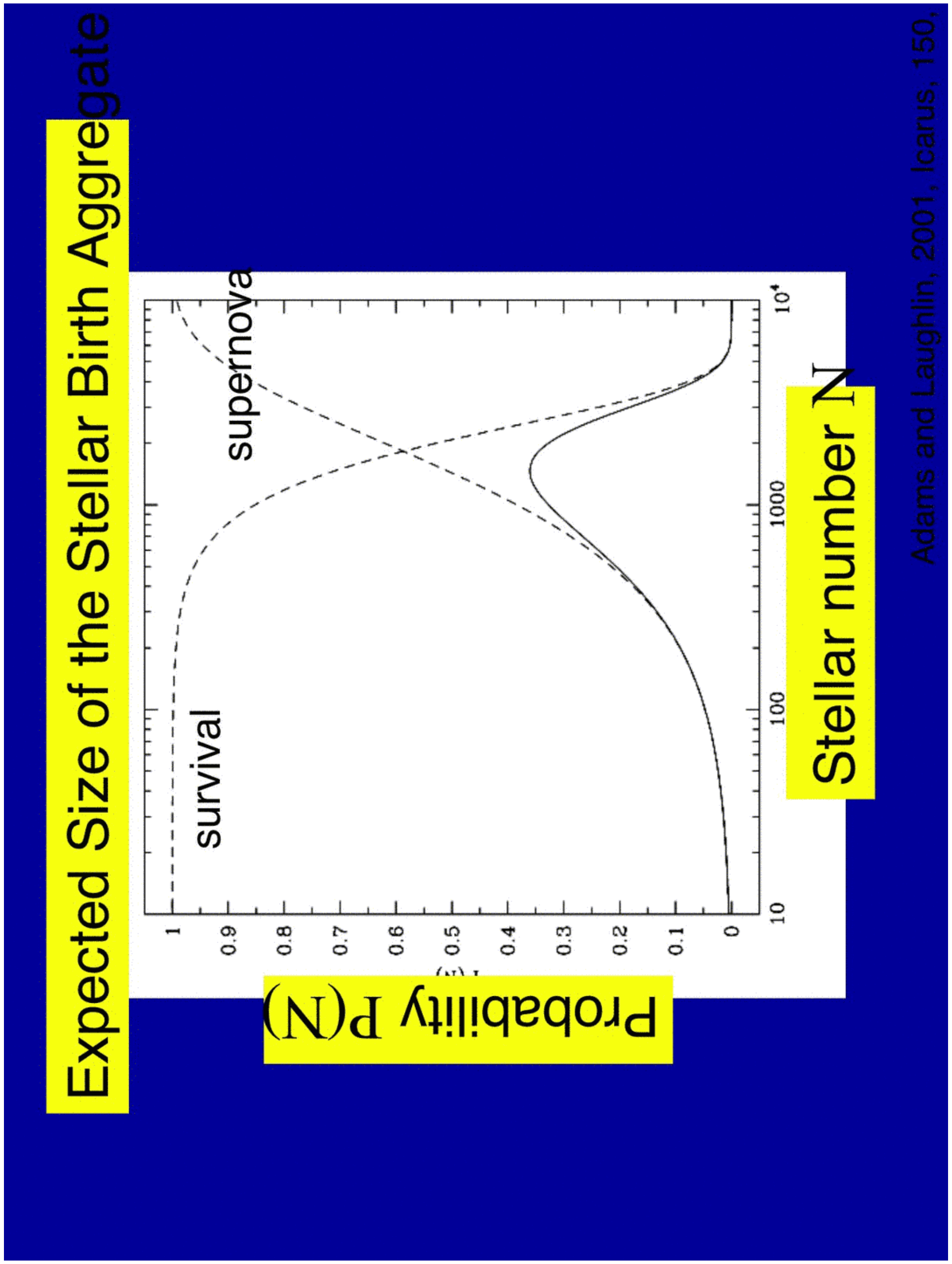

Red Dwarf saves Earth
red dwarf
earth
sun

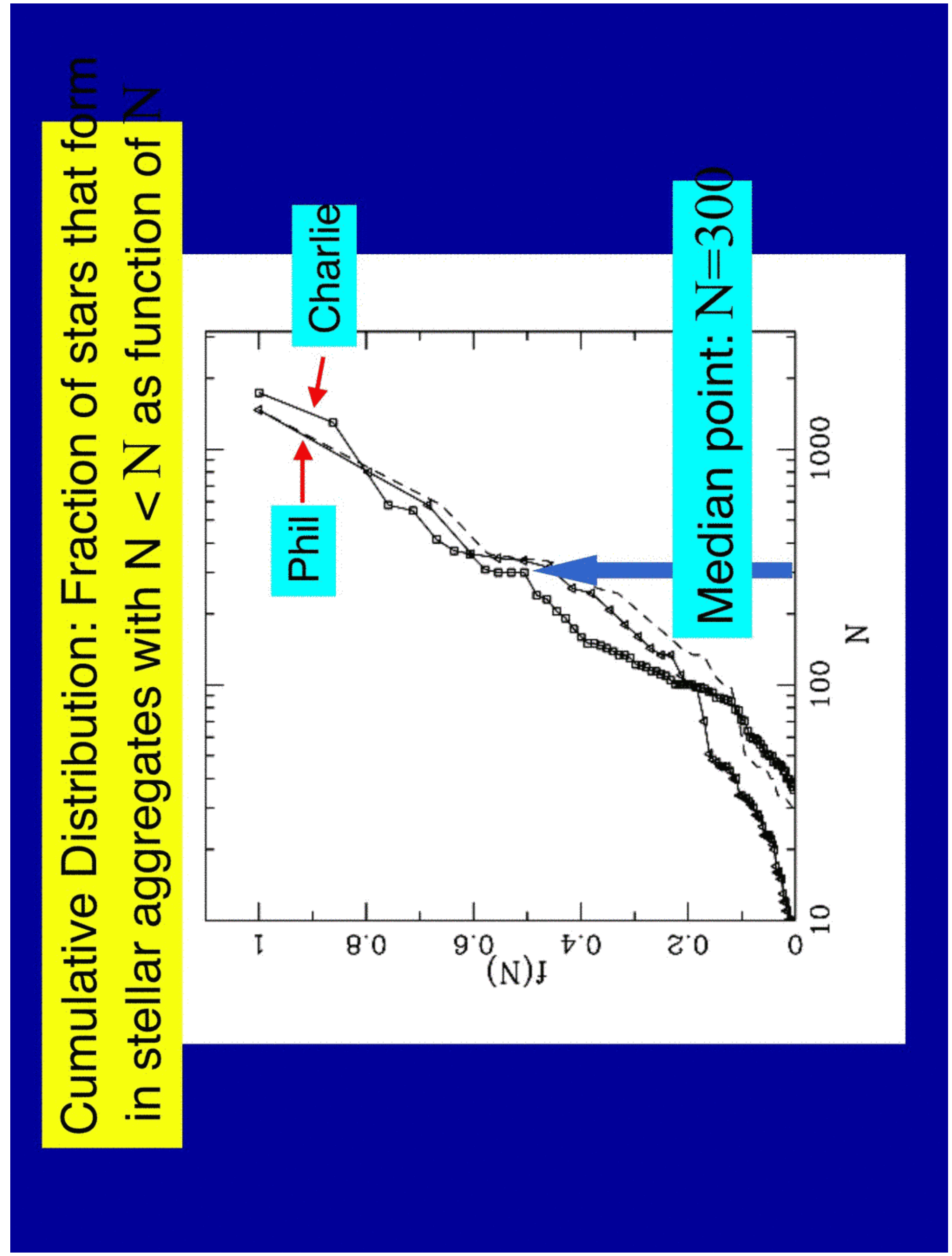

Red dwarf captures the Earth
Sun exits with one red dwarf as a binary companion
9000 year interaction
Earth exits with the other red dwarf
Sun and Earth encounter binary pair of red dwarfs



Cross Section for Solar System Disruption

$$\langle \sigma \rangle \approx (400\,AU)^2$$


Expected Size of the Stellar Birth Aggregate
Probability P(N)
Stellar number N
survival
supernova
Adams and Laughlin, 2001, Icarus, 150,
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
10
100
1000
10^4

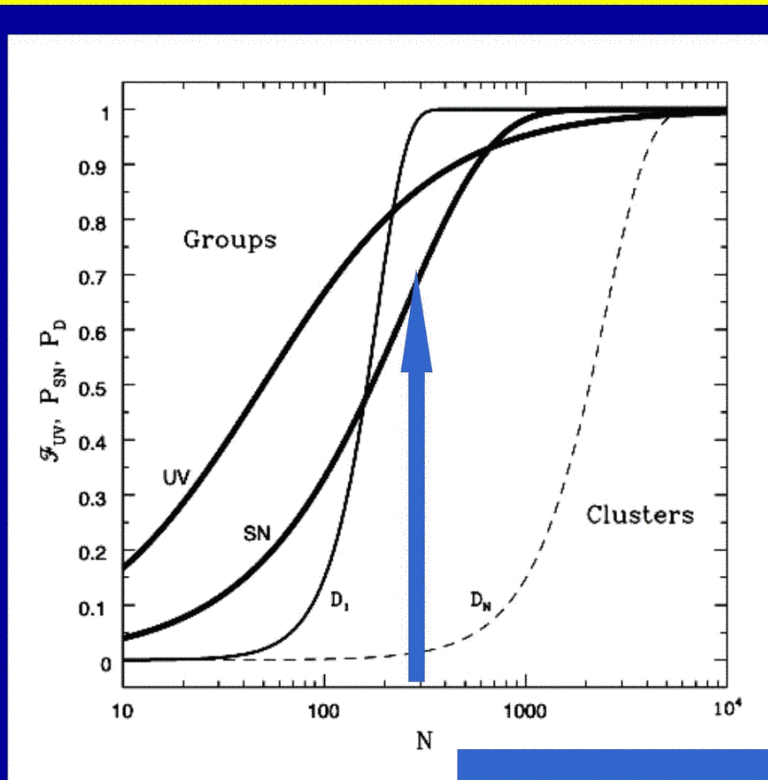

Cumulative Distribution: Fraction of stars that form in stellar aggregates with N < N as function of N
Charlie
Phil
Median point: N=300
f(N)
N
0
0.2
0.4
0.6
0.8
1
10
100
1000

Constraints on the Solar Birth Aggregate

$$\langle N \rangle \approx 2000 \pm 1100$$

$$P \approx 0.0085$$ (1 out of 120)

Probability as function of system size N

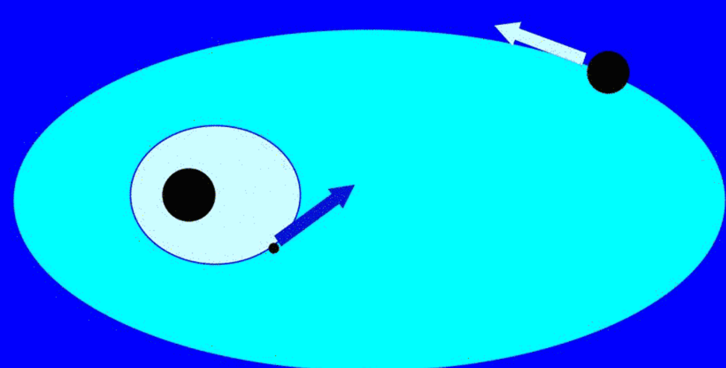


Adams and Myers 2001, ApJ, 553, 7

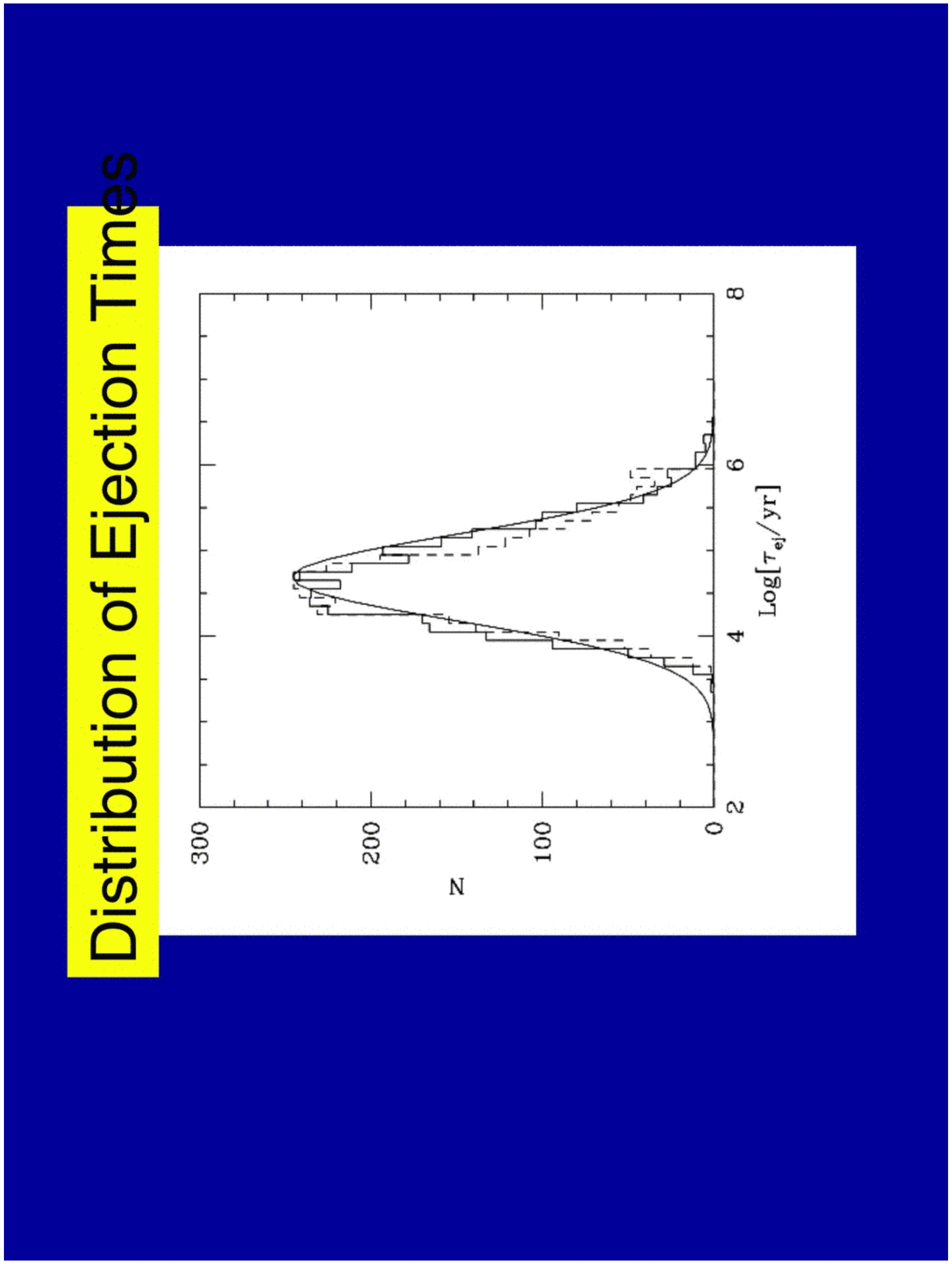
The Case for Extra-solar Terrestrial Planets

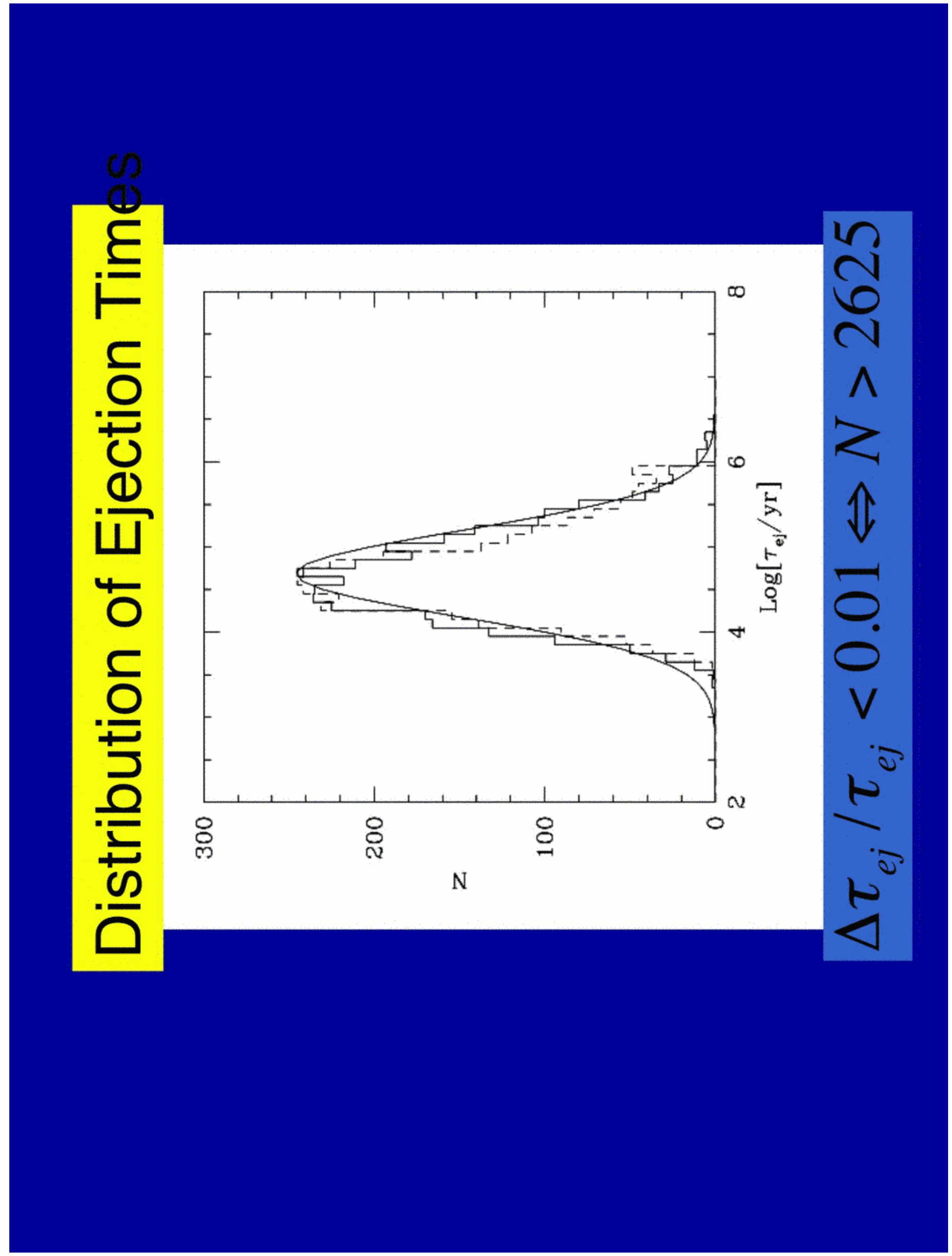
- Our solar system has terrestrial planets and no instances of astronomical creation are unique
- Our solar system has made large numbers of moons, asteroids, and other rocky bodies
- Terrestrial planets discovered in orbit around the Pulsar PSR 1257+ 12 (Wolszczan & Frail 1992)

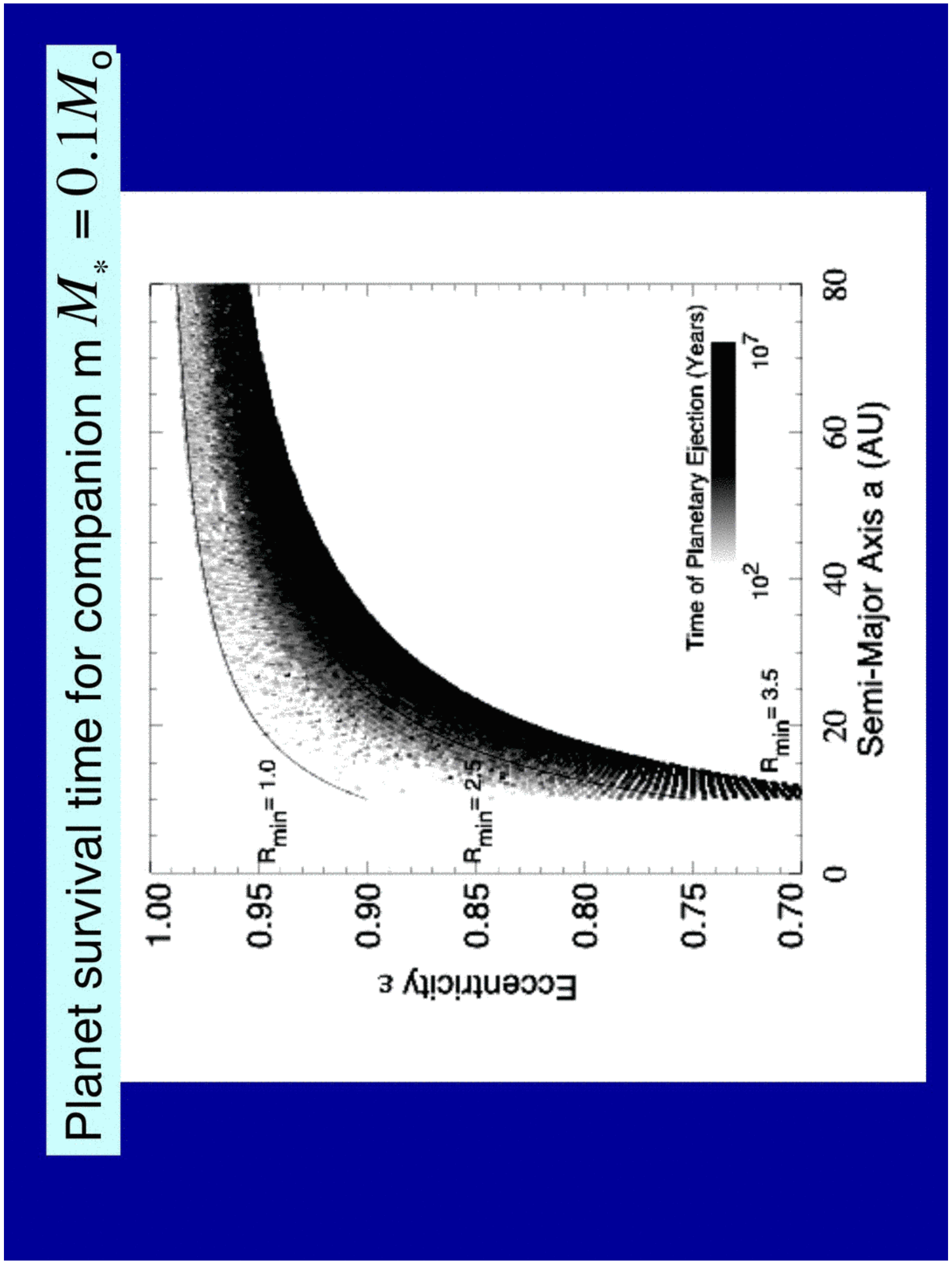
Earth-like planet in a binary system

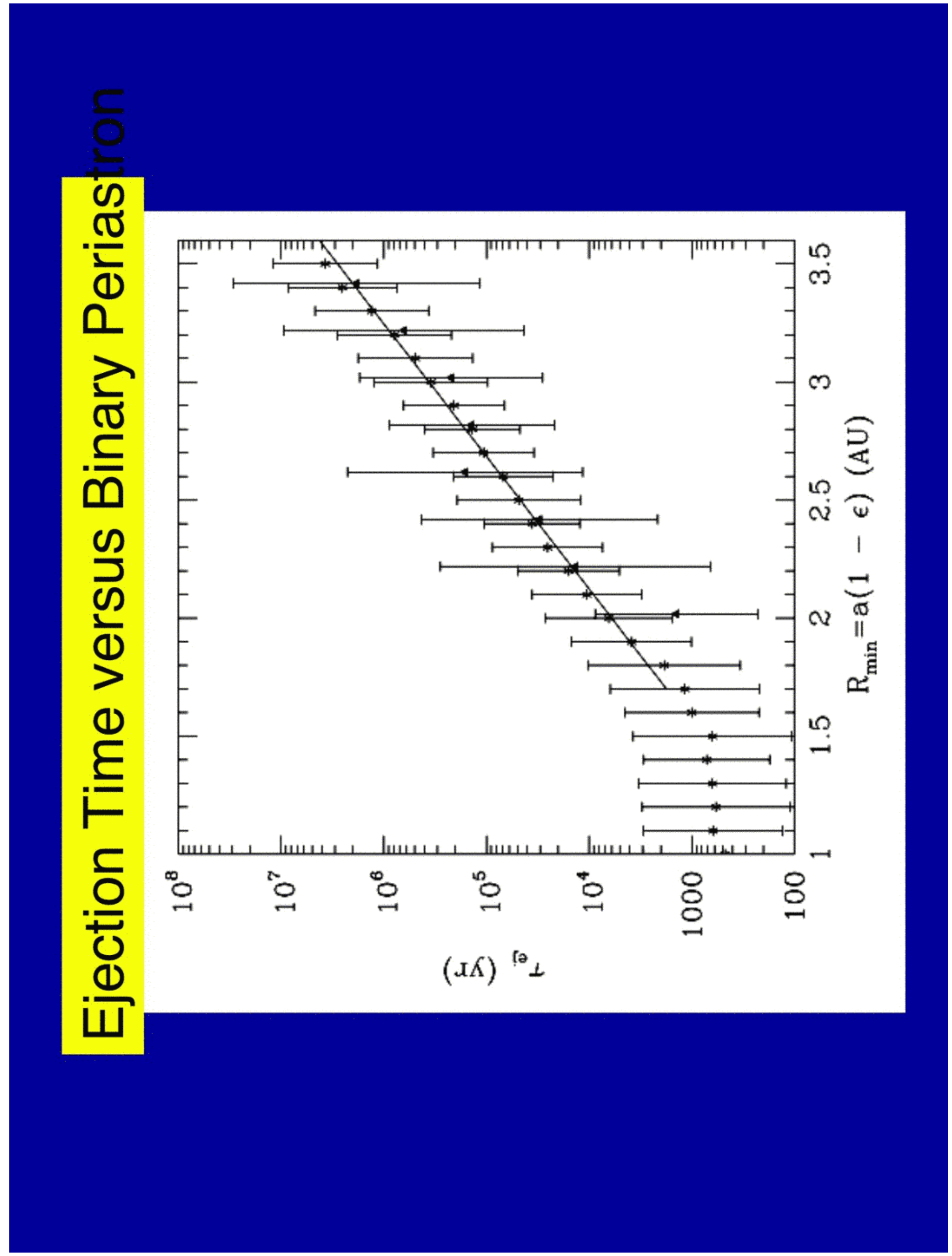


Variables: $(M_C, a_b, \varepsilon_b, i_{bE})$

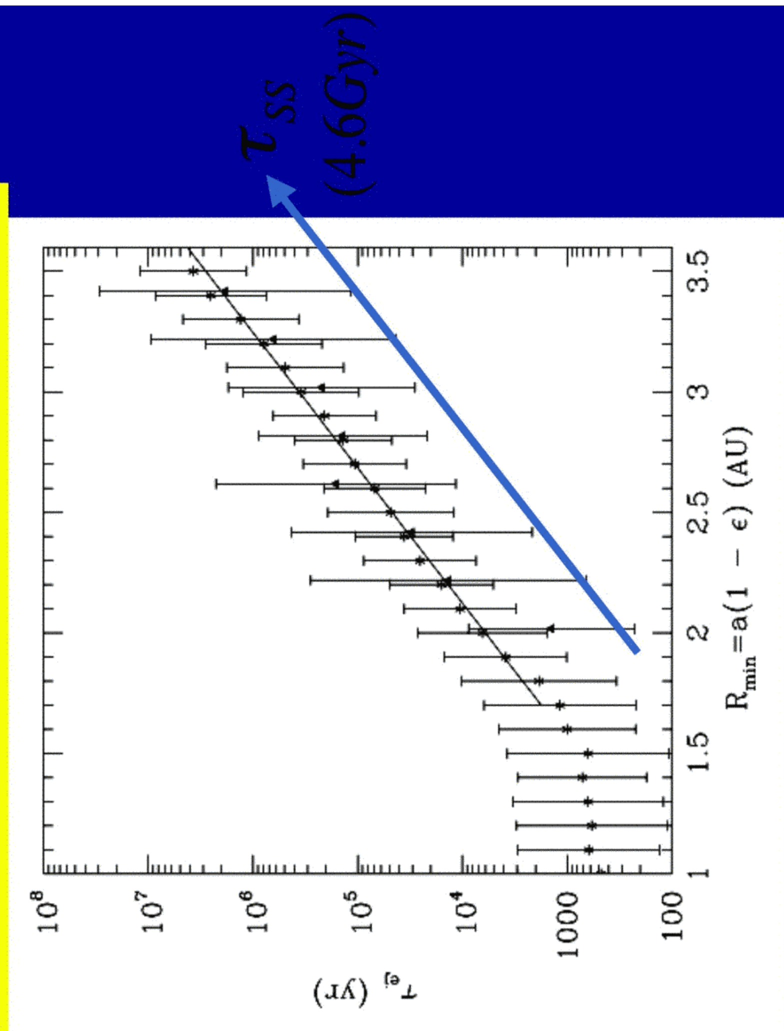

Distribution of Ejection Times
N
300
200
100
0
2
4
6
8
$\mathrm{Log}[\tau_{ej}/\mathrm{yr}]$


Distribution of Ejection Times
N
300
200
100
0
2
4
6
8
$\mathrm{Log}[\tau_{ej}/\mathrm{yr}]$
$\Delta\tau_{ej}/\tau_{ej} < 0.01 \Leftrightarrow N > 2625$


Planet survival time for companion m $M_* = 0.1 M_\odot$
Eccentricity ε
1.00
0.95
0.90
0.85
0.80
0.75
0.70
0
20
40
60
80
Semi-Major Axis a (AU)
R_{min}= 1.0
R_{min}= 2.5
R_{min}= 3.5
Time of Planetary Ejection (Years)
10^2
10^7


Ejection Time versus Binary Periastron
τ_{ej} (yr)
10^8
10^7
10^6
10^5
10^4
1000
100
1
1.5
2
2.5
3
3.5
R_{min}=a(1 − ϵ) (AU)

Ejection Time versus Binary Periastron



Conservative Estimate:

Need binary periastron p > 7 AU for the long term stability of and Earth-like planet

What fraction of the observed binary population has periastron p > 7 AU?

David et al. 2003, PASP, 115, 82

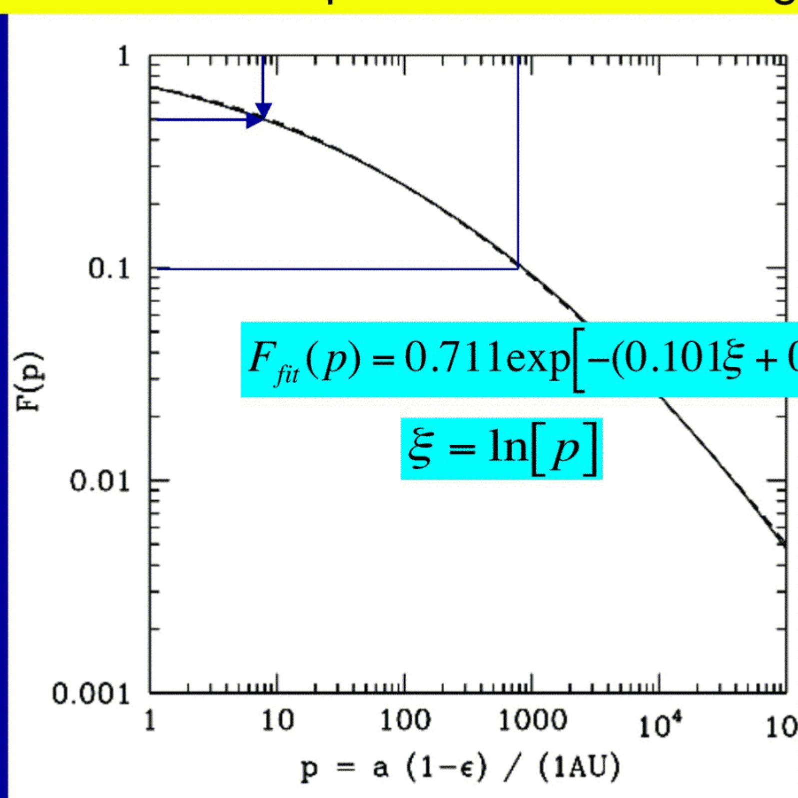
Observed Distributions

$$dP_a = f(\ln a)\, d\ln a = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x-x_0)^2}{2\sigma^2} \right]$$

$$dP_\varepsilon = 2\varepsilon\, d\varepsilon, \qquad dP_\mu = w(\mu)\, d\mu$$

e.g., Duqunenoy & Mayor 1991, A&A, 248, 485

Fraction of binaries with periastron less than given value p

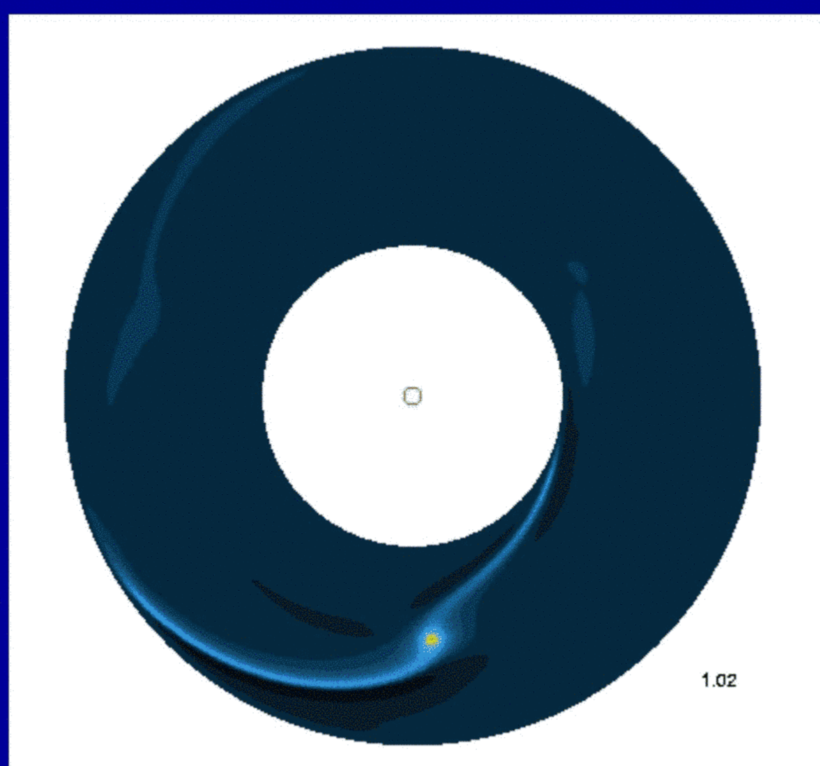


$$F_{fit}(p) = 0.711 \exp\left[-(0.101\xi + 0.0287\xi^2) \right]$$

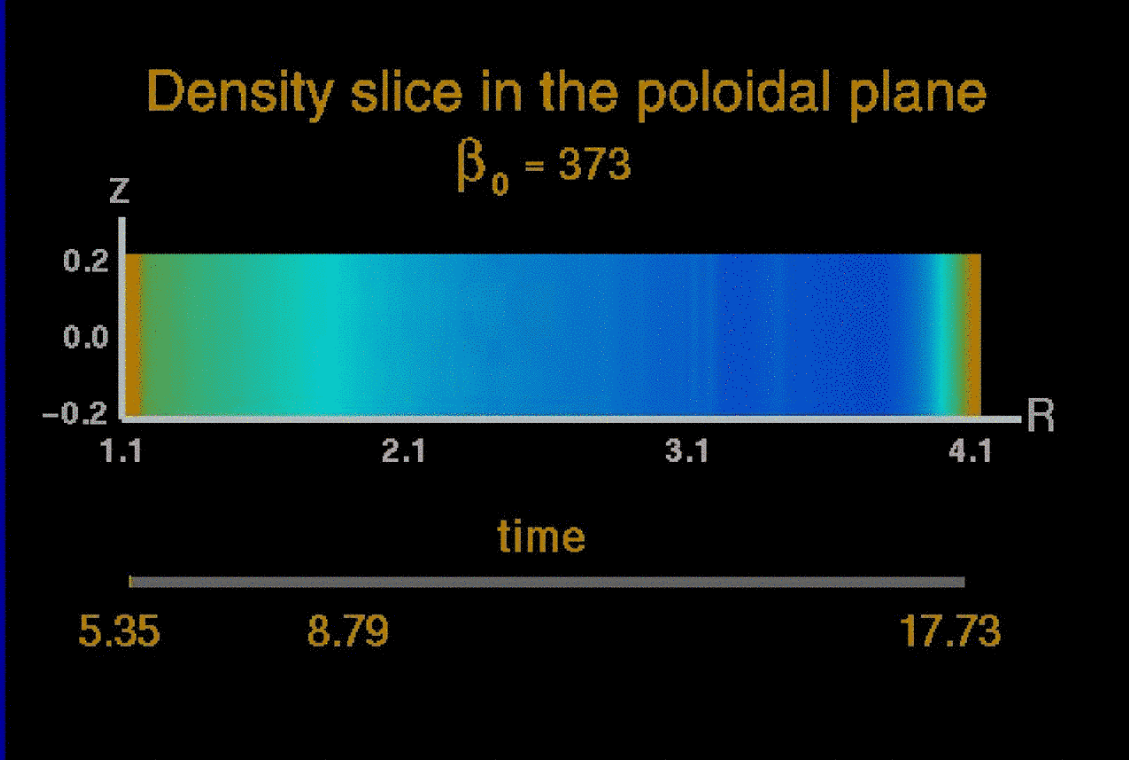
$$\xi = \ln[p]$$

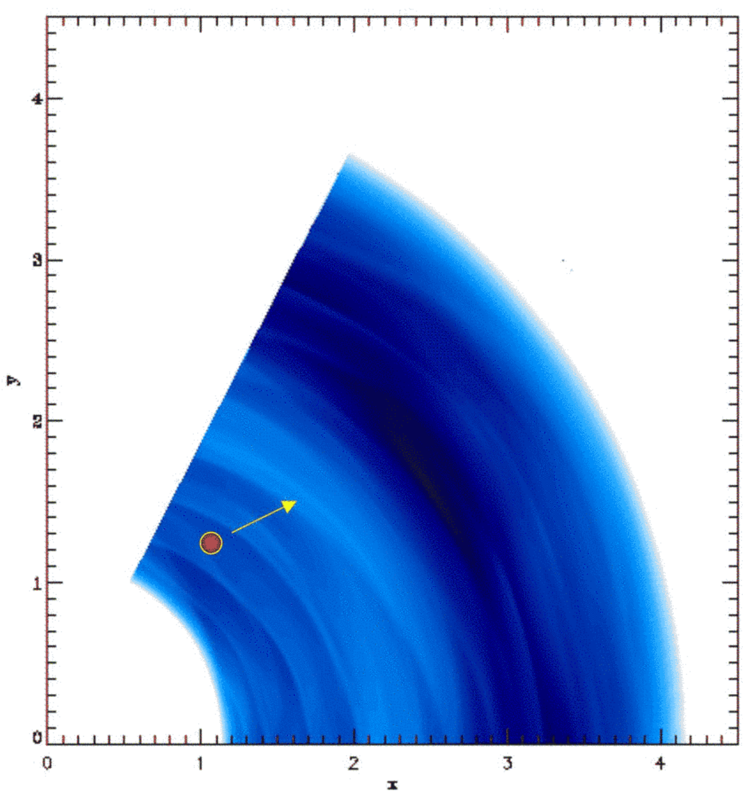
Type I Planetary Migration: The Simple Picture

A planet embedded in a disk of gas and dust creates a spiral wake in the disk. The leading wake tends to accelerate the planet to larger semi-major axis (pushes it outward), while the trailing wake pulls back on the planet and tends to make the orbit decay. The planet migrates inward or outward, depending on the distribution of mass within the disk.



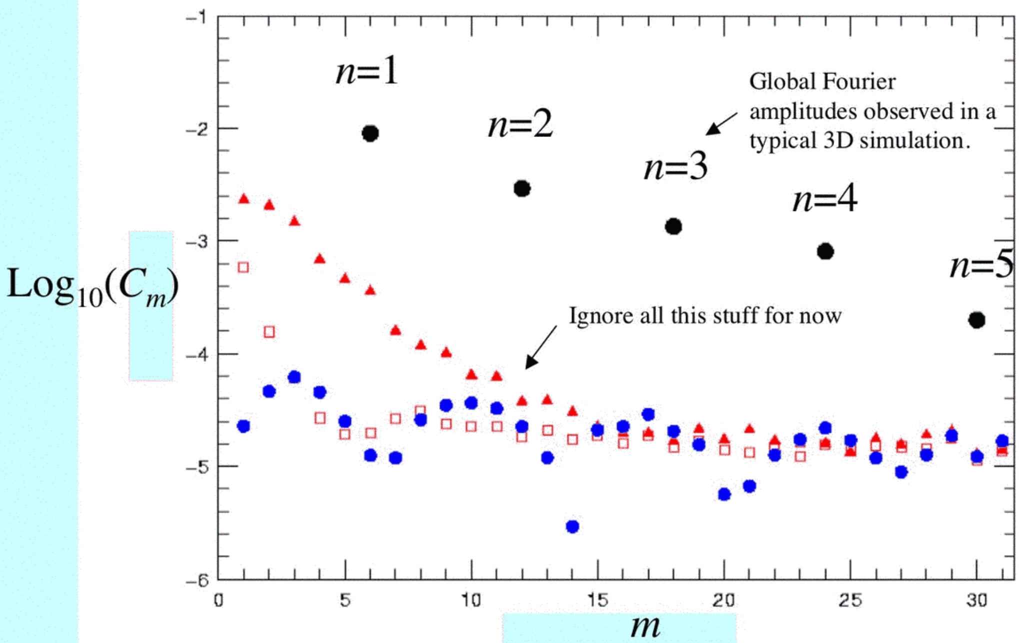
3D MHD Simulations imply self-sustained turbulence in disk





The MRI-induced turbulence leads to order-unity surface density fluctuations in the disk. These surface density perturbations provide a continuous source of stochastic gravitational torques.

A small embedded protoplanet will not affect the large-scale MRI-induced turbulence in the disk.



As the azimuthal mode number m=6n is increased, the Fourier amplitudes observed in the MHD simulations show a gradual decline. Concentration of power in low-order modes is a characteristic feature of MHD-generated turbulence.

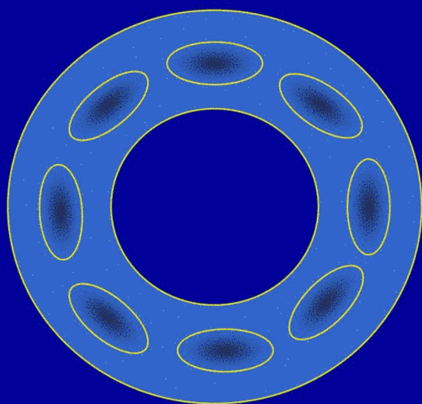
A Heuristic Model for Generating MHD Turbulence

MHD instabilities lead to surface density variations in the disk. The gravitational forces from these surface density perturbations produce torques on an embedded protoplanet. In order to study how this process works, we force a non-MHD hydrodynamical simulation by differencing a heuristic potential function:

$$\Phi = \frac{A\xi e^{-(r-r_c)^2} / \sigma^2}{r^{1/2}} \cos[m\theta - \varphi - \Omega_c \tilde{t}] \sin[\pi \frac{\tilde{t}}{\Delta t}]$$

Heuristic model (continued)

$$\Phi = \frac{A\xi e^{-(r-r_c)^2}/\sigma^2}{r^{1/2}} \cos[m\theta - \varphi - \Omega_c \tilde{t}]\sin[\pi \frac{\tilde{t}}{\Delta t}]$$

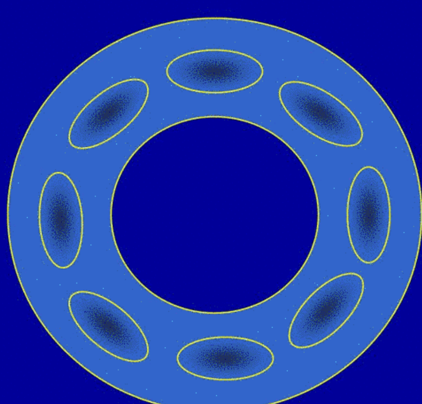


1. An individual mode first appears at time t_0, which is chosen so that the simulation contains 50 modes at any given time.

2. The pattern speed $\Omega_c = v_{KEPLER}/r_c$ in the angular term allows the potential mode to travel along with the Keplerian flow. The disk shear naturally gives the surface density response a spiral form (as seen in the MHD simulations)

Heuristic model (cont.)

$$\Phi = \frac{A\xi e^{-(r-r_c)^2}/\sigma^2}{r^{1/2}} \cos[m\theta - \varphi - \Omega_c \tilde{t}]\sin[\pi \frac{\tilde{t}}{\Delta t}]$$

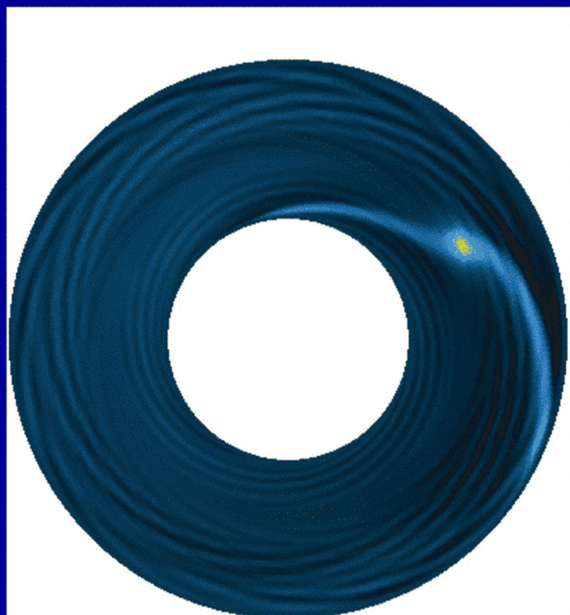


3. Individual modes come and go with a sinusoidal time dependence. An individual mode begins at time t_0 and has faded away after time Δt.

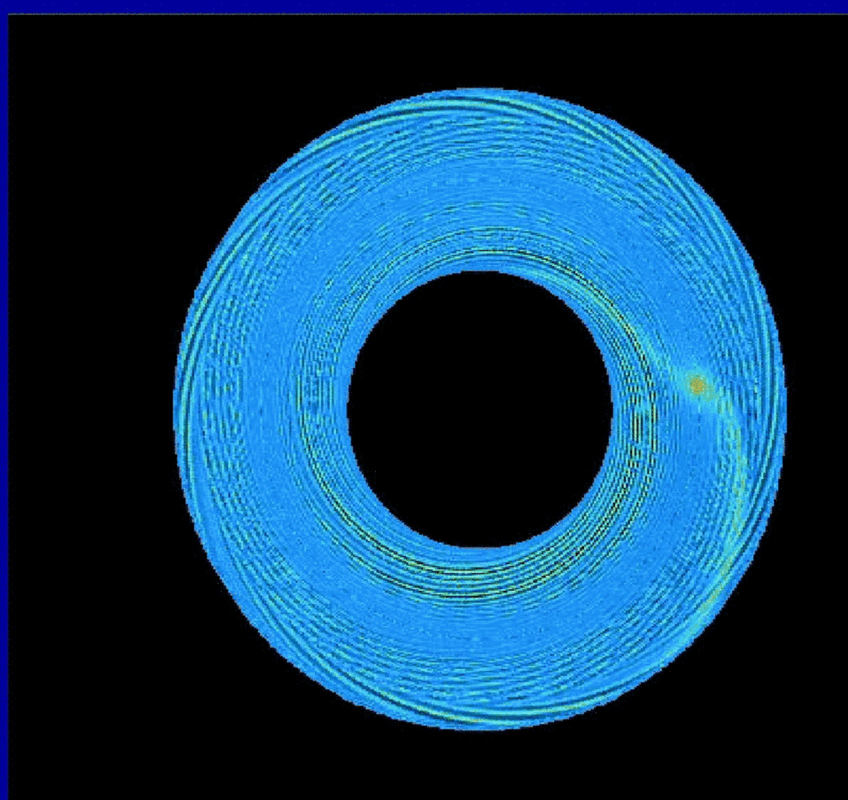
4. The duration of an individual mode is the sound crossing time in the angular direction:

$$\Delta t = t_f - t_0 = \frac{2\pi r_c}{m a_s}$$

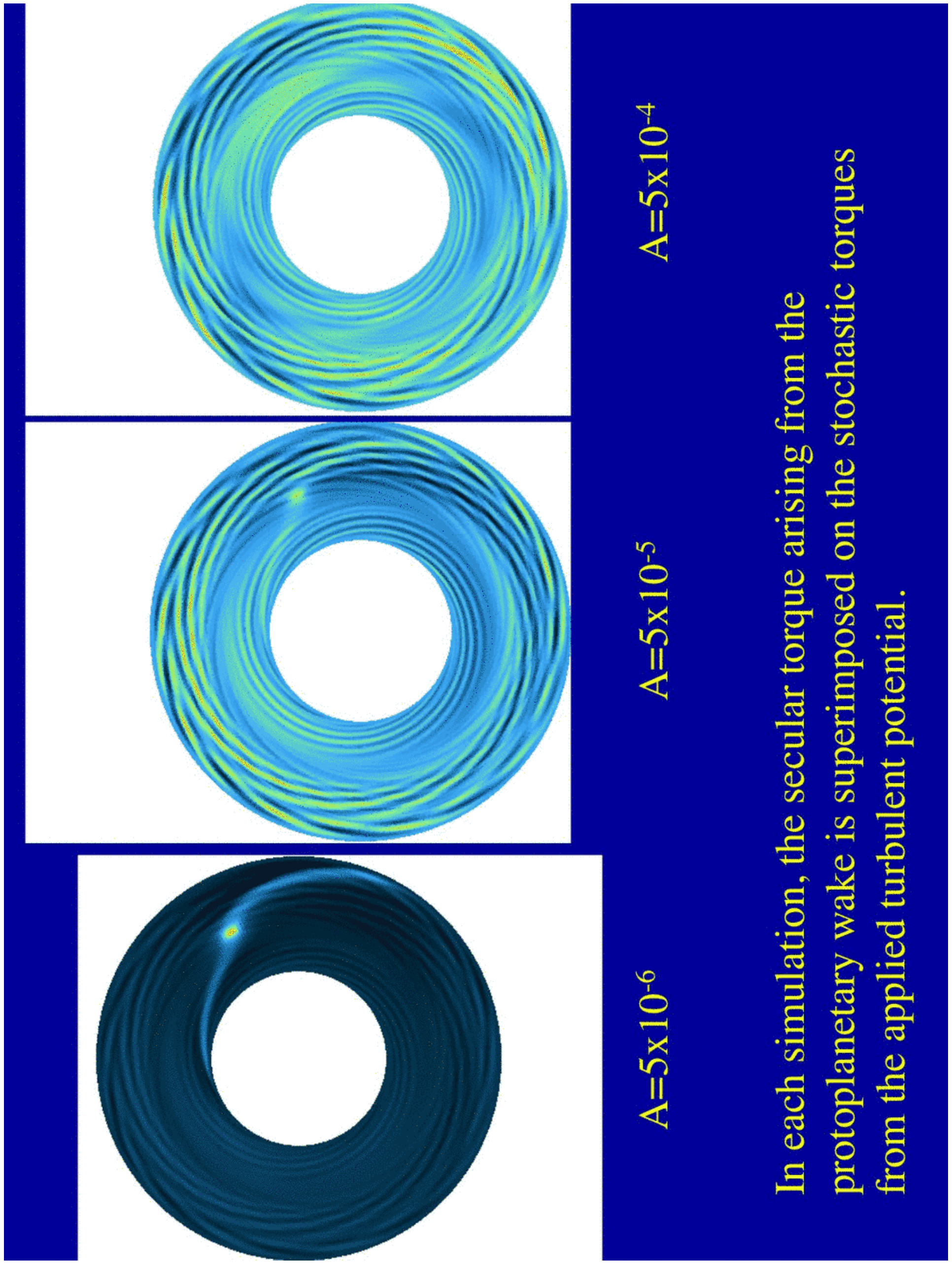
2D migration simulations with turbulent perturbations

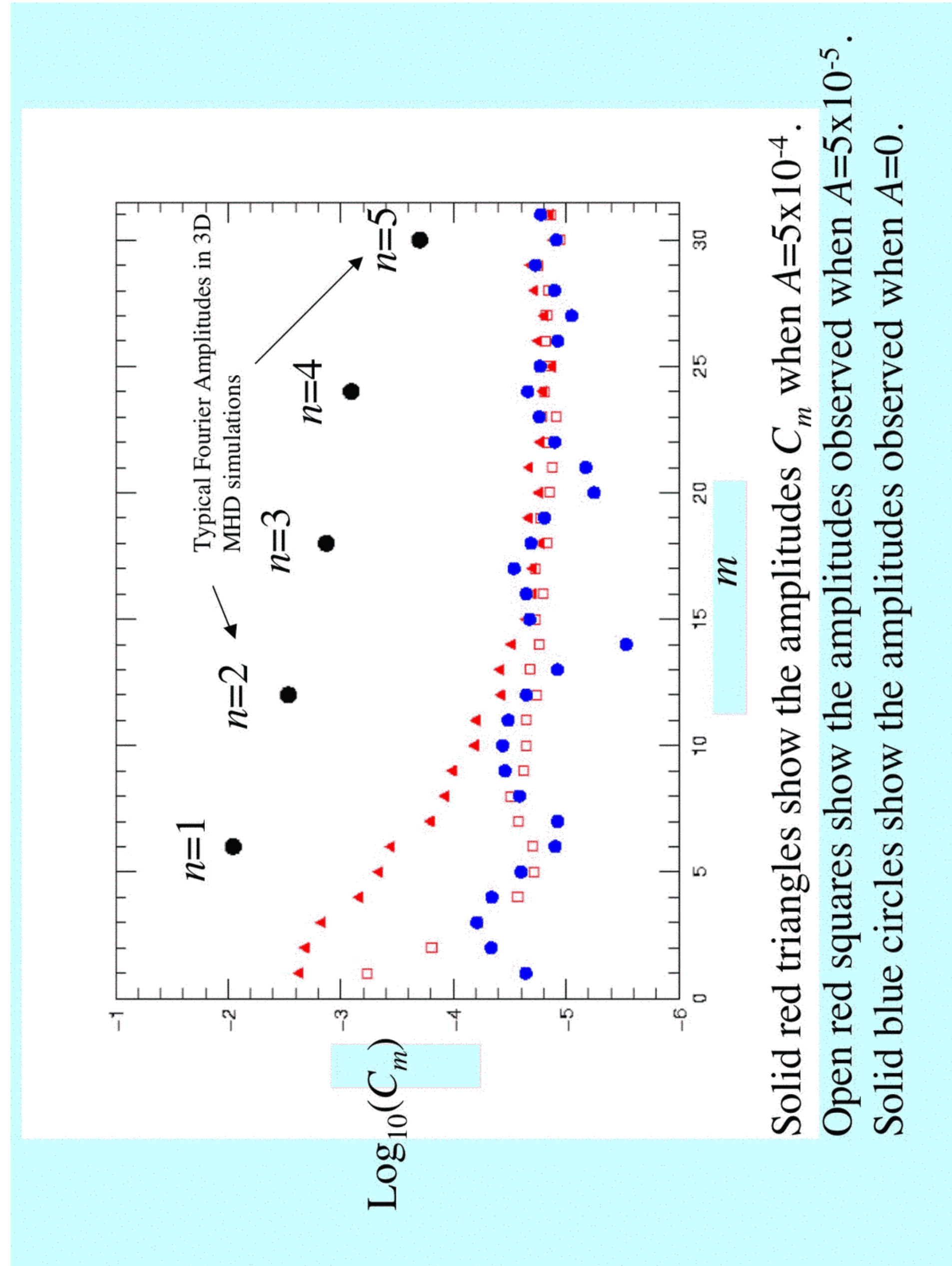


- 256 r, 512 ϕ zones
- van Leer advection
- $M_d/M_* = 0.02$
- "Locally isothermal" E.O.S.
- $G=1$, $M_*=1$, $R_{planet}=2.5$, $R_{in}=1.6$, $R_{out}=3.4$
- $A=5\times10^{-6}$, 5×10^{-5}, 5×10^{-4}
- "Sponge" boundary conditions at R_{in} and R_{out}
- 100 Planetary Orbits



~100 orbits from a simulation with A=5.0e-5


A=5x10^{-4}
A=5x10^{-5}
A=5x10^{-6}
In each simulation, the secular torque arising from the protoplanetary wake is superimposed on the stochastic torques from the applied turbulent potential.


Typical Fourier Amplitudes in 3D MHD simulations
n=1
n=2
n=3
n=4
n=5
$\mathrm{Log}_{10}(C_m)$
m
Solid red triangles show the amplitudes C_m when A=5x10^{-4}.
Open red squares show the amplitudes observed when A=5x10^{-5}.
Solid blue circles show the amplitudes observed when A=0.

The Critical Amplitude

We want to obtain a rough analytic estimate of the critical fluctuation amplitude A_c required for the expected turbulent torque to exceed the Type I migration torque.

To do this, we assume that the gravitational potential arising from the perturbed gas disk has the same functional form as the heuristic driving potential.

The amplitude of the potential acting on the planet will be reduced from that acting on the gas by a factor $\Gamma<1$.

With these assumptions, the torque exerted on the planet by a mode is:

$$\tau = -\frac{mA\xi\Gamma Mp}{r^{1/2}} e^{-(r-rc)^2/\sigma^2} \sin[m\theta - \varphi - \Omega_c \tilde{t}] \sin[\pi\tilde{t}/\Delta t]$$

We can integrate the torque over time to obtain the net change in angular momentum per mode:

$$\Delta J = \int_0^{\Delta t} \tau dt = \frac{mA\xi\Gamma Mp}{r^{1/2}} e^{-(r-r_c)^2/\sigma^2} \frac{\Delta t}{\pi} \frac{\sin(a\pi - \varphi) - \sin\varphi}{a^2 - 1}$$

where: $a \equiv (m\Omega - \Omega_c)\Delta t/\pi$

The mean torque per mode τ_1, averaged over the lifetime of the mode is thus given by:

$$\tau_1 = \frac{\Delta J}{\Delta t} = \frac{A\Gamma Mp}{r^{1/2}} F$$

where:

$$F \equiv \frac{m\xi}{\pi} e^{-(r-r_c)^2/\sigma^2} \frac{\sin(a\pi - \varphi) - \sin\varphi}{a^2 - 1}$$

At any given time in our 2D simulations, the disk contains 50 realizations of F, which tend to cancel each other out. We can compute the forcing function F_{50} by:

$$F_{50} \equiv \sum_{j=1}^{50} F_j$$

where the F_j are sampled using the same prescription as is used in the 2D simulations. The function F_{50} should average to zero, but the RMS value, $\langle F_{50} \rangle$, provides a measure of the effective strength of the torque. Numerically sampling F_{50} over many realizations shows that $\langle F_{50} \rangle \sim 0.29$. Hence, the average torque τ_t exerted on the planet is given by:

$$\tau_t = \langle F_{50} \rangle \frac{A\Gamma Mp}{r^{1/2}} \approx 0.29 \frac{A\Gamma Mp}{r^{1/2}}$$

To estimate the ratio Γ of the disk potential V to the heuristic MRI-induced potential Φ, we first use a WKB analysis (see Shu 1992, or Binney & Tremaine 1989) which relates the surface density response of the disk to an arbitrary perturbing potential in the limit that the potential is rapidly varying.

This is done by linearizing (1) Poisson's equation, (2,3) the components of the momentum equation, and (4) the continuity equation. The fluid velocities v_r and v_ϕ are eliminated, leaving an algebraic relation between the maximum value, S, of the perturbed surface density peaks at a given radius and the imposed (*i.e.* known) potential Φ:

$$S = \frac{(d\Phi/dr)^2 \sigma_d \Phi}{(m\Omega_c - m\Omega)^2 - \Omega^2 - (d\Phi/dr)^2 a_s^2}$$

We define $k = d\Phi/dr$ … The more rapid the radial variation of Φ, the stronger the surface density response.

Knowing the surface density response *S* from the imposed heuristic potential Φ, we can ask, what is the resultant potential *V* (arising from *S*) that is felt by the planet?

This relation is well known in the WKB limit:

$$V = -\frac{2\pi G S}{|k|}$$

The response factor Γ tying the applied heuristic potential to the potential felt by the planet is therefore:

$$\Gamma = \frac{V}{\Phi} \approx \frac{2\pi G \sigma_d k}{(m\Omega_c - m\Omega)^2 - \Omega^2 - k^2 a_s^2} \approx \frac{\pi \sigma_d r^2}{M_*} \frac{64}{\pi^2} \approx 0.06$$

What have we done? We've made an analytic estimate of the magnitude of the expected turbulent torque at any given time:

$$\tau_T = 0.0174 \frac{A M_P}{r^{1/2}}$$

This is the expectation value. The actual value of the torque is time-variable and can be either positive or negative.

We need to compare this torque from the surface density fluctuations to the torque expected from Type I migration.

Ward (1997) finds, for the Type I migration torque:

$$\tau_I = \beta_I \left(\frac{M_P}{M_*}\right)^2 (\pi \sigma_d r^2)(r\Omega)^2 \left(\frac{r}{H}\right)^3$$

β_I is a dimensionless amplitude estimated by Ward to be $\beta_I \sim 10^{-2}$

We can thus equate τ_I and τ_T to solve for the critical amplitude A_c above which the expected torque due to turbulence is larger than larger than the torque from standard Type I migration:

$$A_c = \frac{3.4\pi^2}{64}\beta_I\left(\frac{M_P}{M_*}\right)\left(\frac{r}{H}\right)^3 r^{1/2}(r\Omega)^2 \approx 1\times 10^{-5}$$

This value for A_c is in reasonable agreement with the numerical simulations (which were done with an Earth-mass planet).

Note that A_c is directly proportional to M_P

The two classes of torques have different qualitative effects!

The **net turbulent torque** can be either positive or negative, and it changes its value over a characteristic time t_{smp}. The response of the planet to this torque is a random walk. Over a time Nt_{smp}, the resulting change in angular momentum is:

$$(\Delta J)_T = \sqrt{N}\tau_T t_{smp}$$

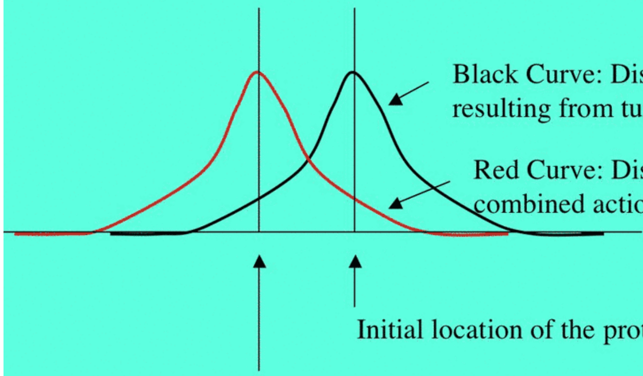
The **Type I torque** is a secular torque that continuously pushes the protoplanet in one direction. Over time Nt_{smp}, the resulting change in angular momentum is:

$$(\Delta J)_I = N\tau_I t_{smp}$$

In our simulations (both 3D MHD and 2D "HMHD"), the low-order m modes give the largest contributions to the turbulent torque. They last for roughly P_{orbit}. Hence, $t_{smp} \sim P_{orbit}$.

In order for the turbulent torques to have a decisive effect on the motion over a specified time interval T_f, the amplitude of the turbulent fluctuations must be larger than the critical amplitude by a factor:

$$A \geq \left(\frac{T_f}{P_{orbit}} \right)^{1/2} A_c$$

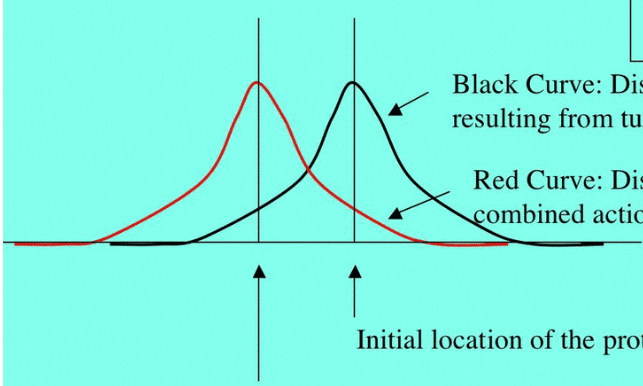


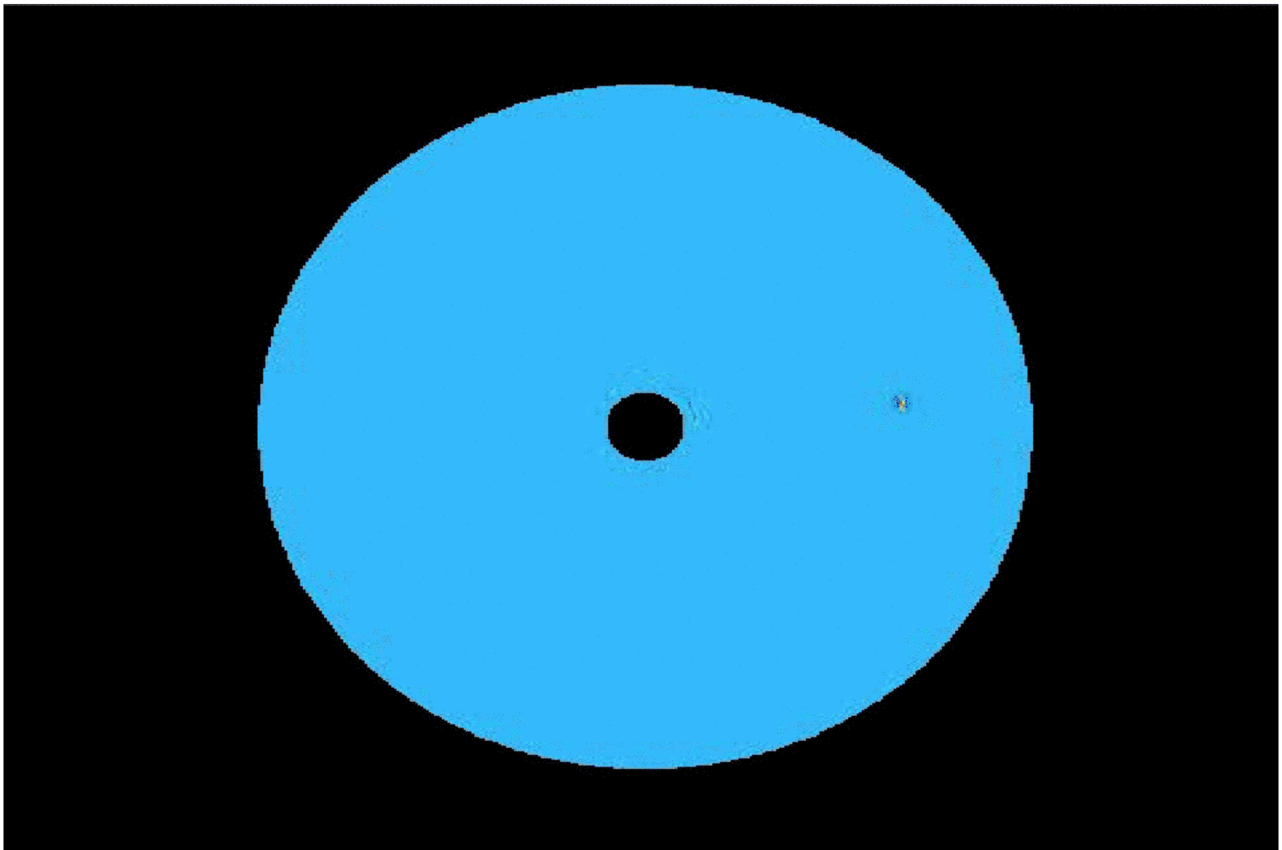
For a type I migration timescale of 10^6 years; and P_{orbit}=10 yr:

$$A \geq \left(\frac{T_f}{P_{orbit}} \right)^{1/2} A_c \longrightarrow$$

$A \sim 300 A_c = 3 \times 10^{-3}$.

This is very close to the value observed in the MHD simulations.





Future work: How does an MHD turbulent region in the inner part of the disk affect protoplanetary migration in the outer unionized region. 3D simulations should be used to study this process.

Conclusions

- Planet scattering -> e (not a)   Disk torques -> a (not e)

  Planetary migration with both disk torques and planet scattering can explain observed a-e plane

  $\langle \sigma \rangle$
- Determination of scattering cross sections

  For external enrichment scenario, Sun forms in stellar aggregate with N = 2000
- 50 percent of binary systems allow for Earth-like planet to remain stable for the age of solar system
- In Type I migration, the magnetically driven turbulent

  fluctuations that drive accretion overwhelm

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