Beyond Oligarchy:
The Final Chapter of Planet Formation

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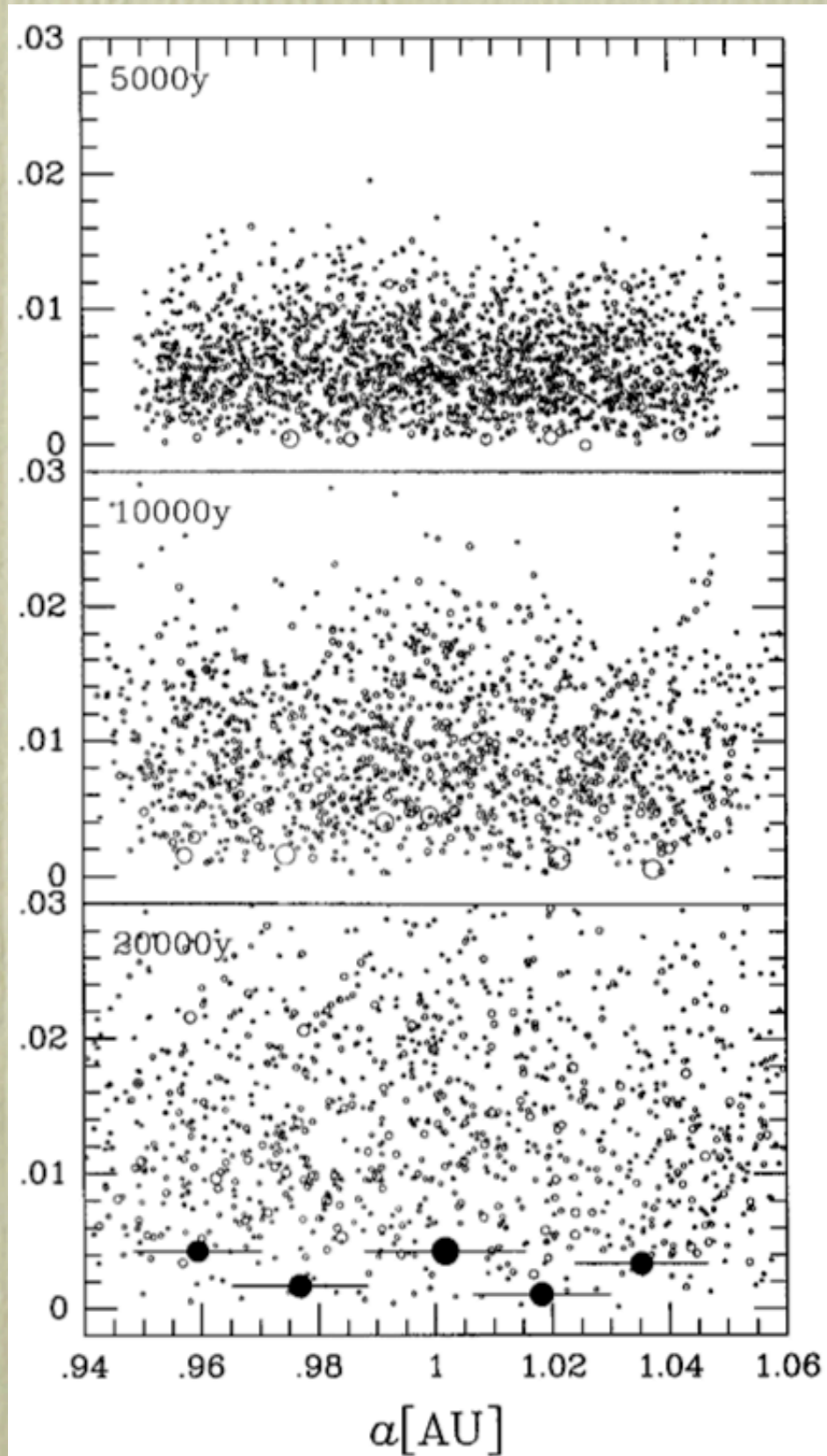Re'em Sari

Observations

30 Hill
Radii

50 Hill
Radii

0.72 AU

1 AU

1.5 AU

15 Hill
Radii

19 AU

30 AU
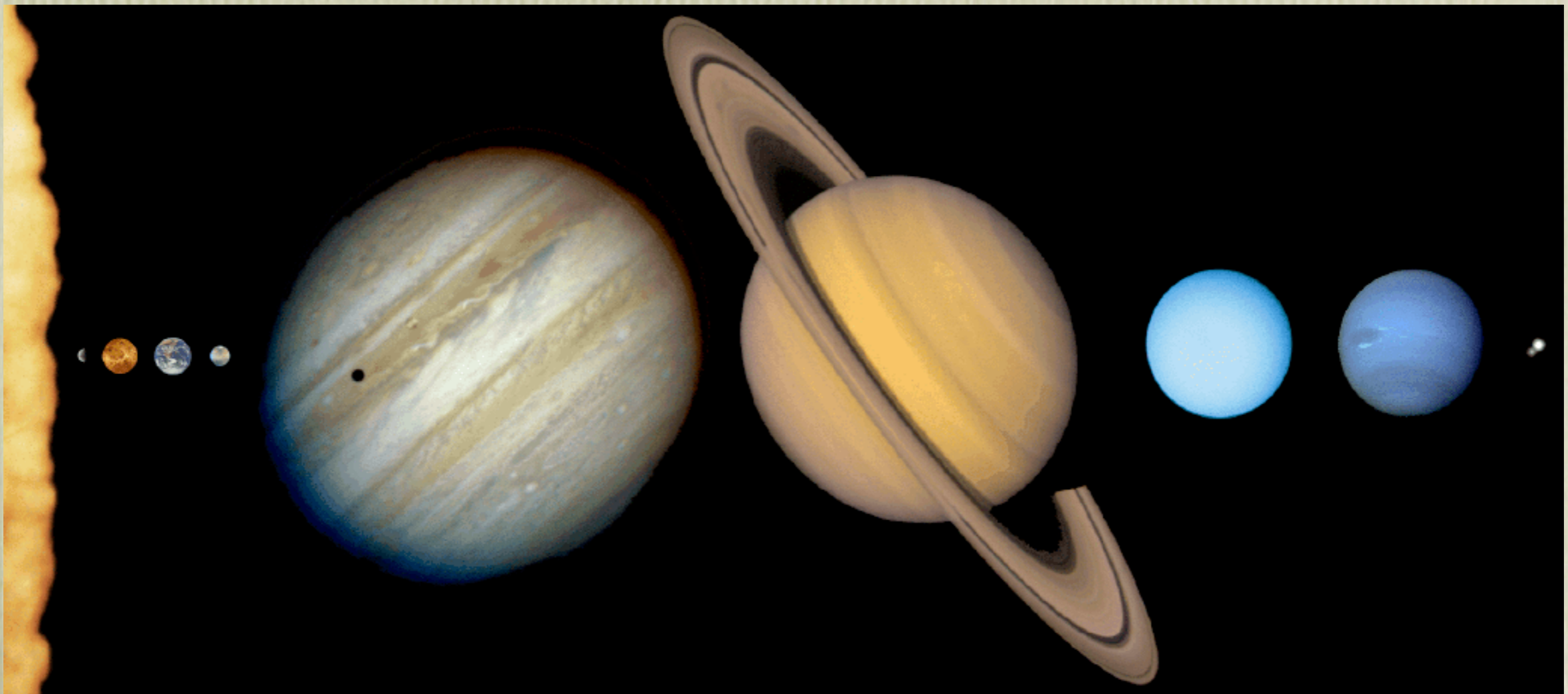
Questions

- When does oligarchy end, and what happens next?

- What sets the spacing between the planets?

- Why are their orbits circular and coplanar?

- How long did it take them to form?
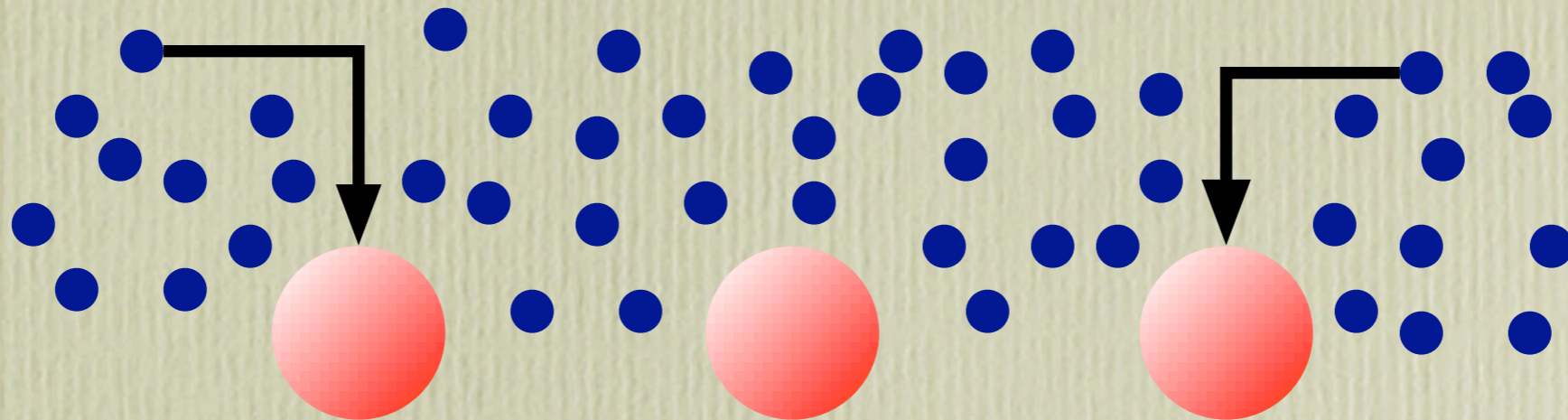
Oligarchy

e

Big bodies:

- have comparable masses

- are spaced by a few Hill radii

(From Kokubo & Ida '98)
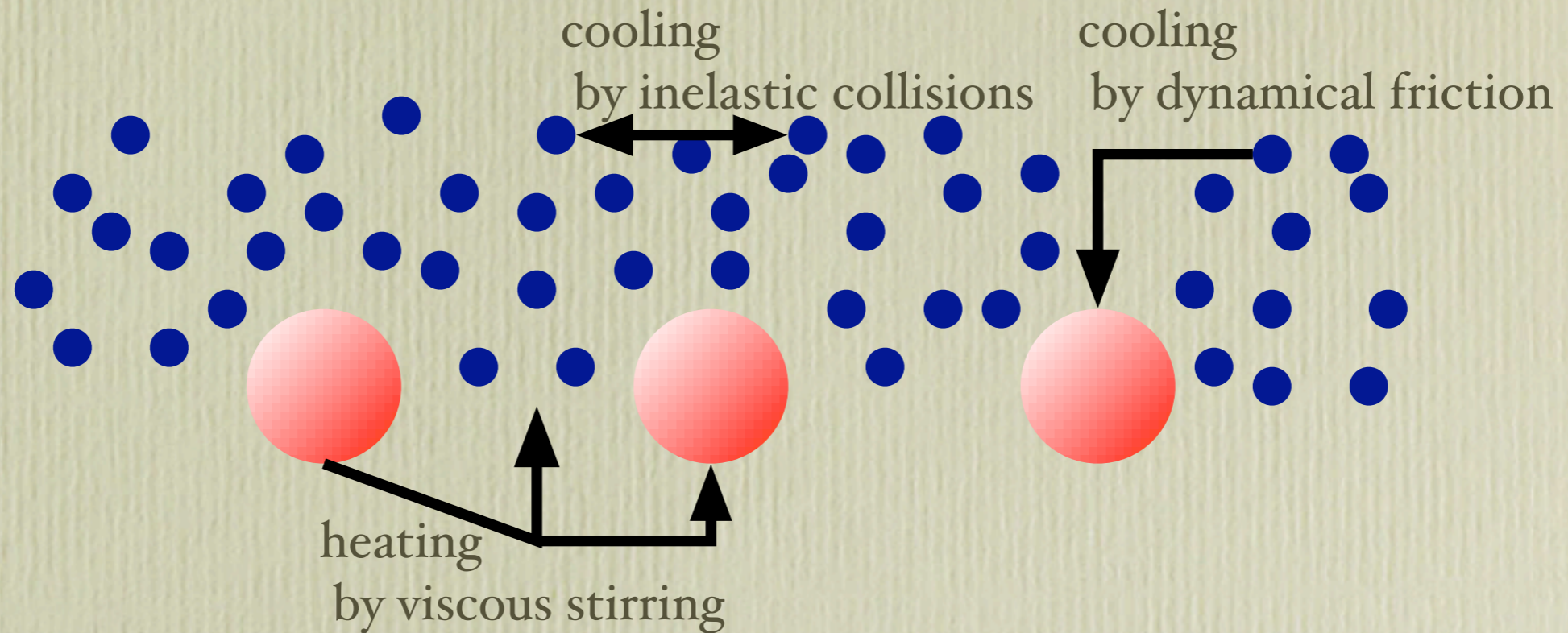
Accretion During Oligarchy

Accretion Rate:

$$\frac{1}{M}\frac{dM}{dt} \sim \underbrace{\left(\frac{v_{\mathrm{esc}}}{u}\right)^2}_{\substack{\text{gravitational} \\ \text{focusing} \\ \text{factor}}} \underbrace{\left(\frac{\sigma\Omega}{\rho R}\right)}_{\substack{\text{geometric} \\ \text{accretion} \\ \text{rate}}} \Rightarrow \begin{cases} 100 & \text{Myr for Earth} \\ 1 & \text{Gyr for Jupiter's core} \\ 1000 & \text{Gyr for Neptune} \end{cases}$$

Heating and Cooling During Oligarchy

cooling
by inelastic collisions
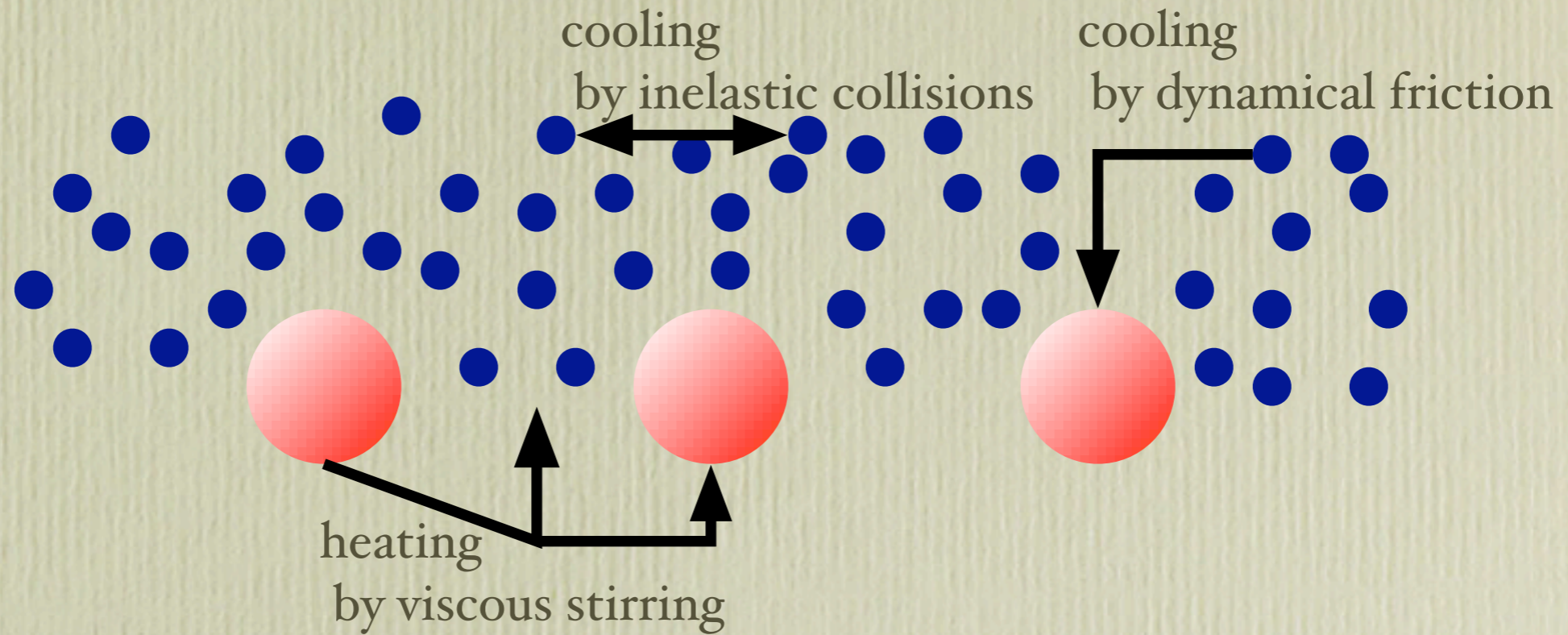
cooling
by dynamical friction

heating
by viscous stirring

Heating = Cooling

On small bodies: $\dfrac{\Sigma\Omega}{\rho R}\left(\dfrac{v_{\mathrm{esc}}}{u}\right)^4 \sim \dfrac{\sigma\Omega}{\rho s}$ $\qquad \Rightarrow u \sim v_{\mathrm{esc}}\left(\dfrac{\Sigma}{\sigma}\dfrac{s}{R}\right)^{1/4}$

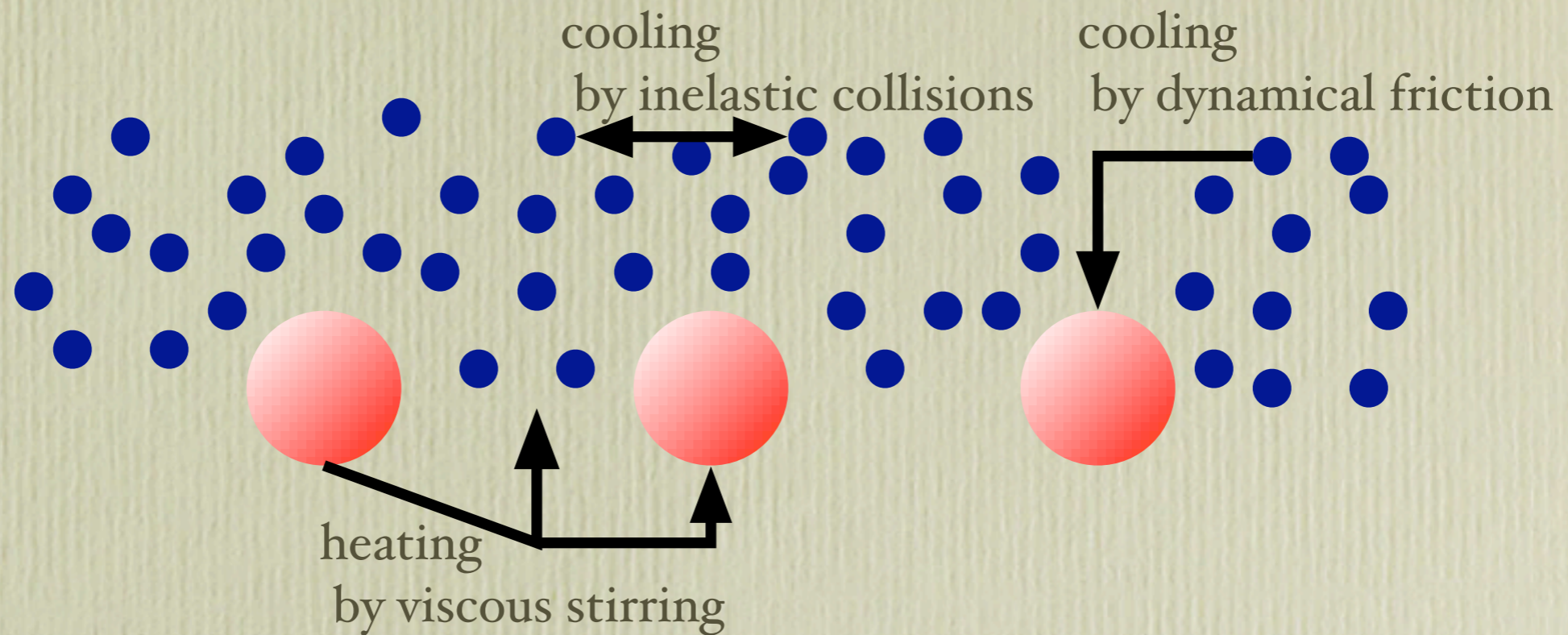Very small s $\Rightarrow$ very small u $\Rightarrow$ very fast accretion

Heating and Cooling During Oligarchy

cooling
by inelastic collisions

cooling
by dynamical friction

heating
by viscous stirring

Heating = Cooling

On big bodies: $\dfrac{\Sigma\Omega}{\rho R}\left(\dfrac{v_{\text{esc}}}{v}\right)^4 \sim \dfrac{\sigma\Omega}{\rho R}\left(\dfrac{v_{\text{esc}}}{u}\right)^4 \Rightarrow v \sim u\left(\dfrac{\Sigma}{\sigma}\right)^{1/4}$

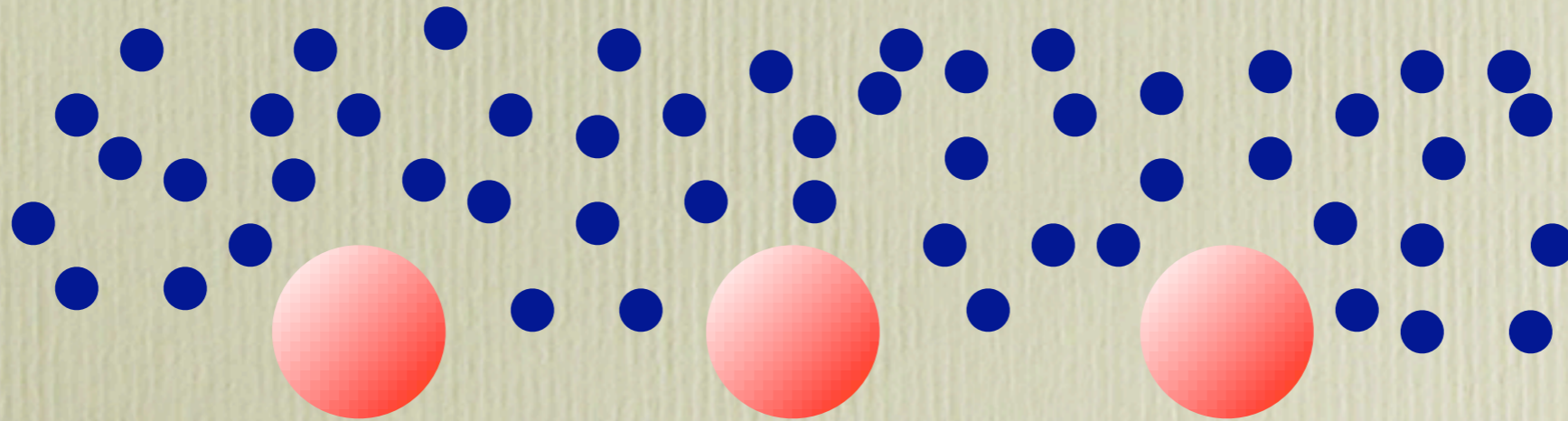# Heating and Cooling
During Oligarchy

cooling
by inelastic collisions

cooling
by dynamical friction

heating
by viscous stirring

Heating = Cooling

On big bodies: $\dfrac{\Sigma\Omega}{\rho R}\left(\dfrac{v_{\mathrm{esc}}}{v}\right)^4$ $\underset{>}{X}$ $\dfrac{\sigma\Omega}{\rho R}\left(\dfrac{v_{\mathrm{esc}}}{X_V}\right)^4$ $\Rightarrow$ v runs away
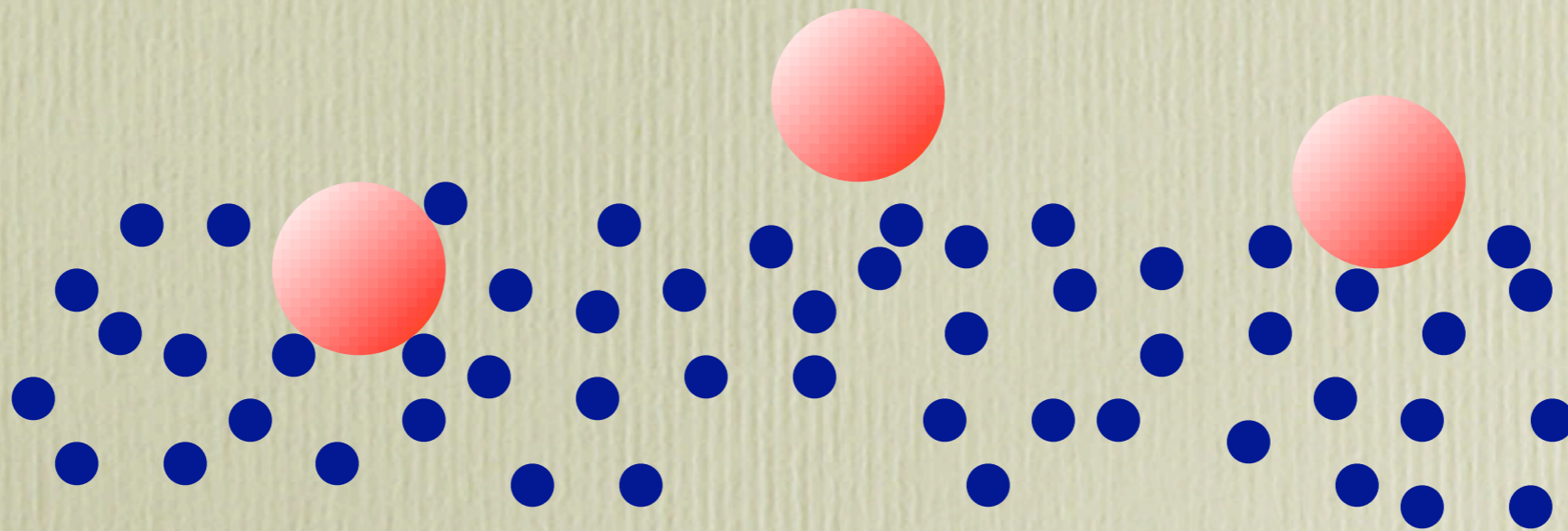
Beyond Oligarchy: Unrestrained Heating

When $\Sigma > \sigma$, heating of big bodies exceeds cooling
$\Rightarrow$ **heating runs away**
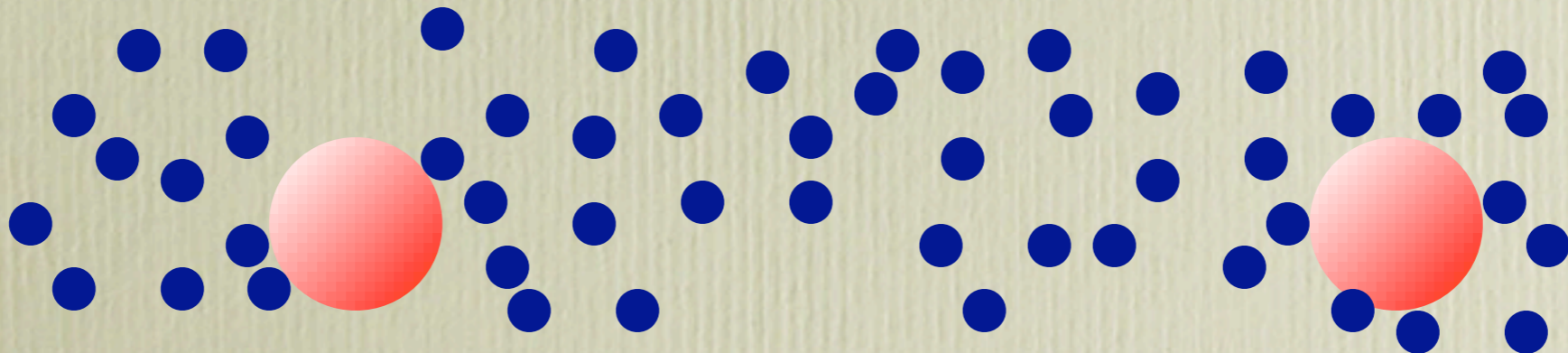
Beyond Oligarchy: Unrestrained Heating

When $\Sigma > \sigma$, heating of big bodies exceeds cooling
$\Rightarrow$ **heating runs away**

For Uranus and Neptune
$$v_{\text{escape}} > v_{\text{orbital}}, \text{ so}$$

- Some big bodies ejected to Oort cloud or beyond in time $\frac{1}{10}\Omega^{-1}\left(\frac{M_\odot}{M_{\text{Neptune}}}\right)^2 \sim 1 \text{ Gyr}$

- Remaining big bodies cooled by small bodies

- Continue until no large-scale chaos $\Rightarrow$ no heating

- Remaining small bodies probably not accreted

For Venus, Earth, and Mars
$$v_{\text{escape}} < v_{\text{orbital}} \text{ , so}$$

- Ejection is impossible

- Remaining big bodies collide and coalesce in time $\left(\dfrac{\sigma\Omega}{\rho R}\right)^{-1} \sim 100$ Myr

- Continue until no large-scale chaos. Residual small bodies can cool planets.
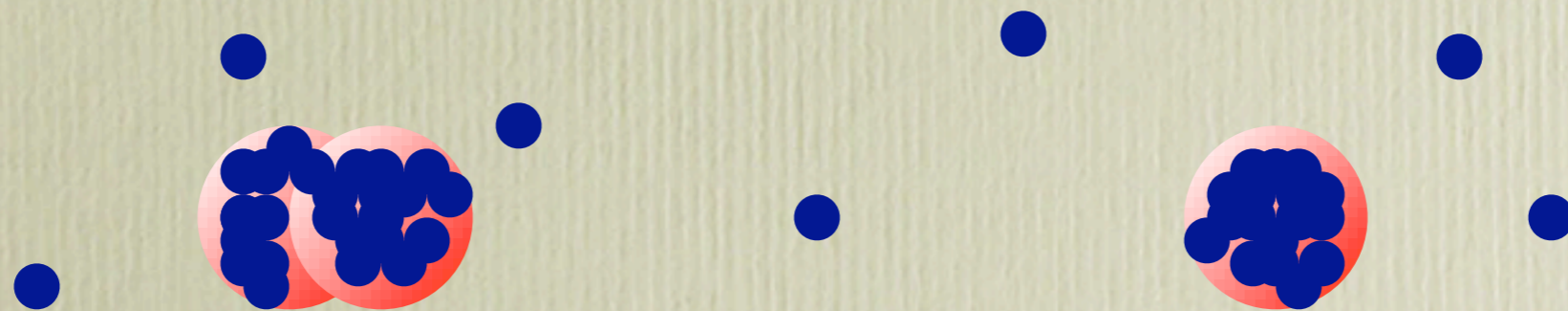
Answers

When does oligarchy end, and what happens next?

When $\Sigma = \sigma$, heating runs away

Answers

What sets the spacing between planets?

Stability $\Rightarrow$ no large-scale chaos

Answers
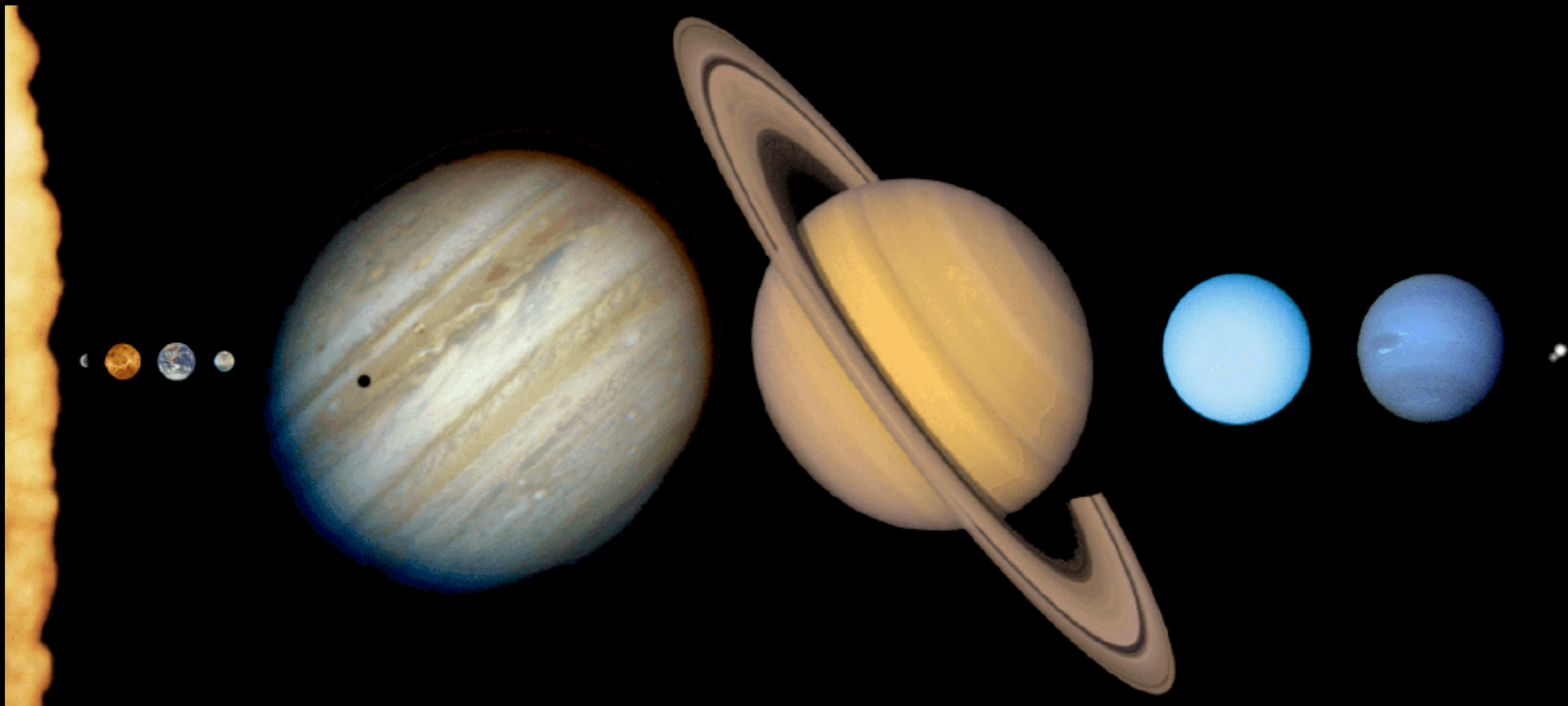
Why are their orbits circular and coplanar?

Small bodies damp velocities

Answers

How long did it take them to form?

- In inner Solar System: $\sim$ 100 Myr

- In outer Solar System: $\ll$ 100 Myr, but to eject neighbours: 1 Gyr

The End