Which EFT for the LHC?

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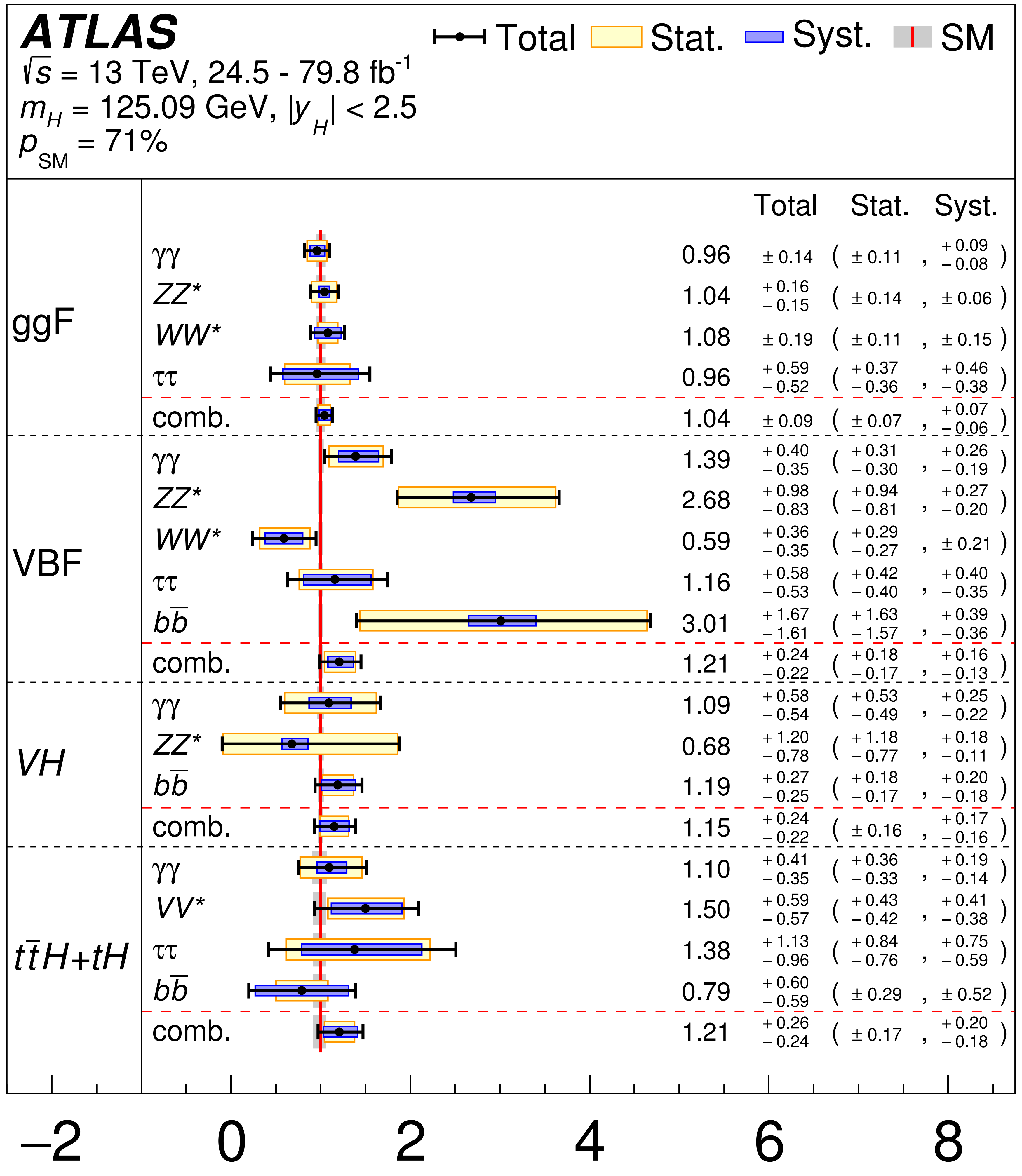
Based in part on
"Is SMEFT enough?" [2008.08597] w/ **Tim Cohen, Xiaochuan Lu,** and **Dave Sutherland**
+ work in progress w/ same + **Ian Banta**

Inspired by
R. Alonso, E. Jenkins, A. Manohar [1511.00724, 1605.03602]

KITP "New Physics from Precision at High Energies" 04/06/21

Measurements → Meaning

[ATLAS 1909.02845]



Precision Higgs measurements a key program of LHC3/HL-LHC.

Anticipated 5-10% precision provides unprecedented tests. Future colliders to ~0.5%

Interpreting either agreement or disagreement with SM invites an EFT framework.*

Strong motivation to develop and understand Higgs EFTs!

This talk: which EFT for the LHC?

*unless new physics is light.

Higgs EFTs

SM ***SU(2)$_L$xU(1)$_Y$***

$$(D_\mu H)^\dagger (D^\mu H) - m^2|H|^2 - \lambda|H|^4$$

HEFT* ***U(1)$_{em}$***

[Feruglio '93, Bagger et al. '93, …]

$$\frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}\left[vF(h/v)\right]^2(\partial\vec{n})^2 - V(h) + \ldots$$

SMEFT ***SU(2)$_L$xU(1)$_Y$***

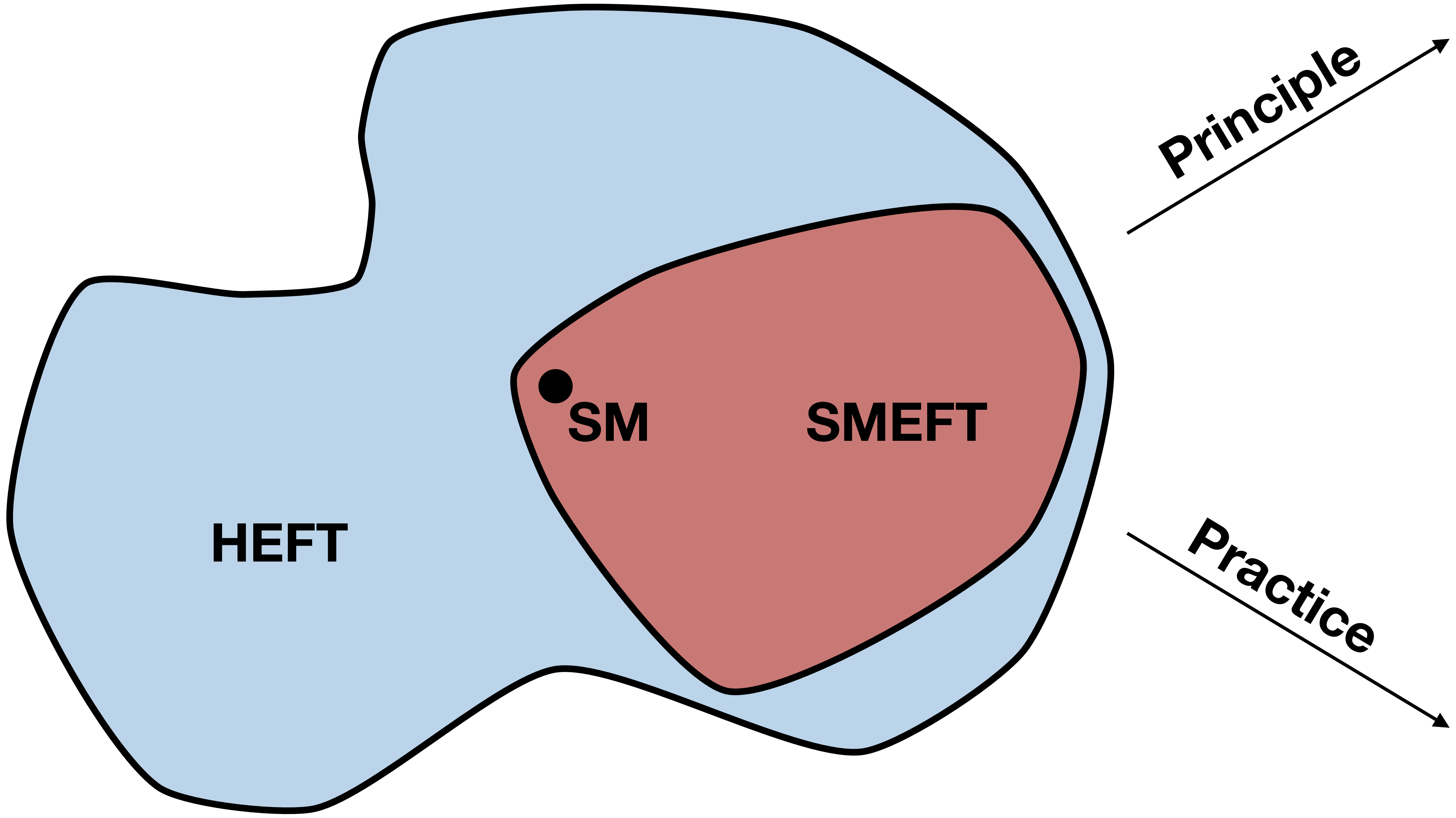
[Weinberg '79, Buchmuller, Wyler '86, …]

$$(D_\mu H)^\dagger (D^\mu H) - m^2|H|^2 - \lambda|H|^4 + \frac{c_H}{2\Lambda^2}(\partial_\mu|H|^2)^2 + \frac{c_6}{\Lambda^2}|H|^6 + \ldots$$

*Alternately, "Higgs-Electroweak Chiral Lagrangian", …

Comprehensive review: [Brivio, Trott 1703.10924]

Why bother

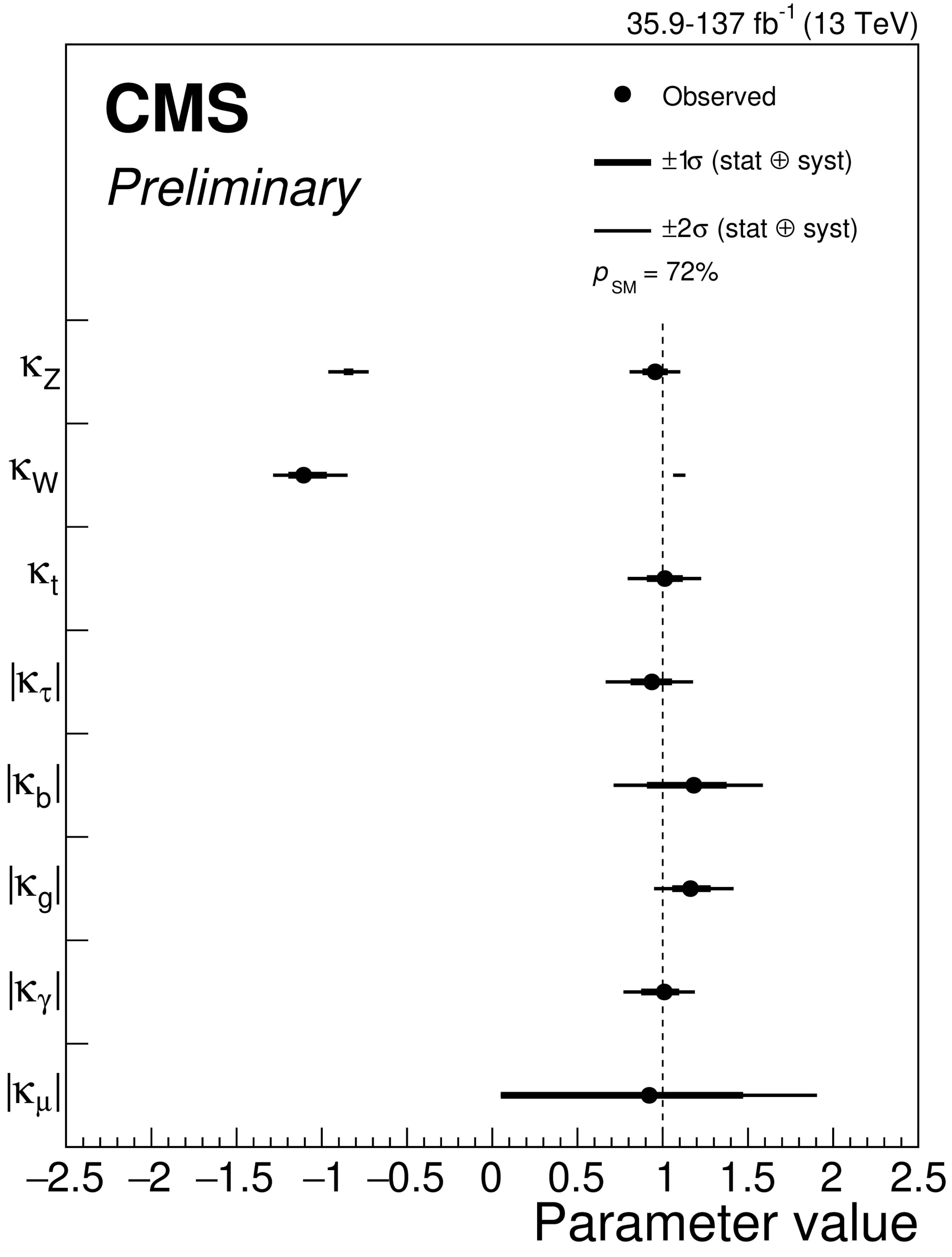


One of these is the theory of nature*, might as well study the heck out of it

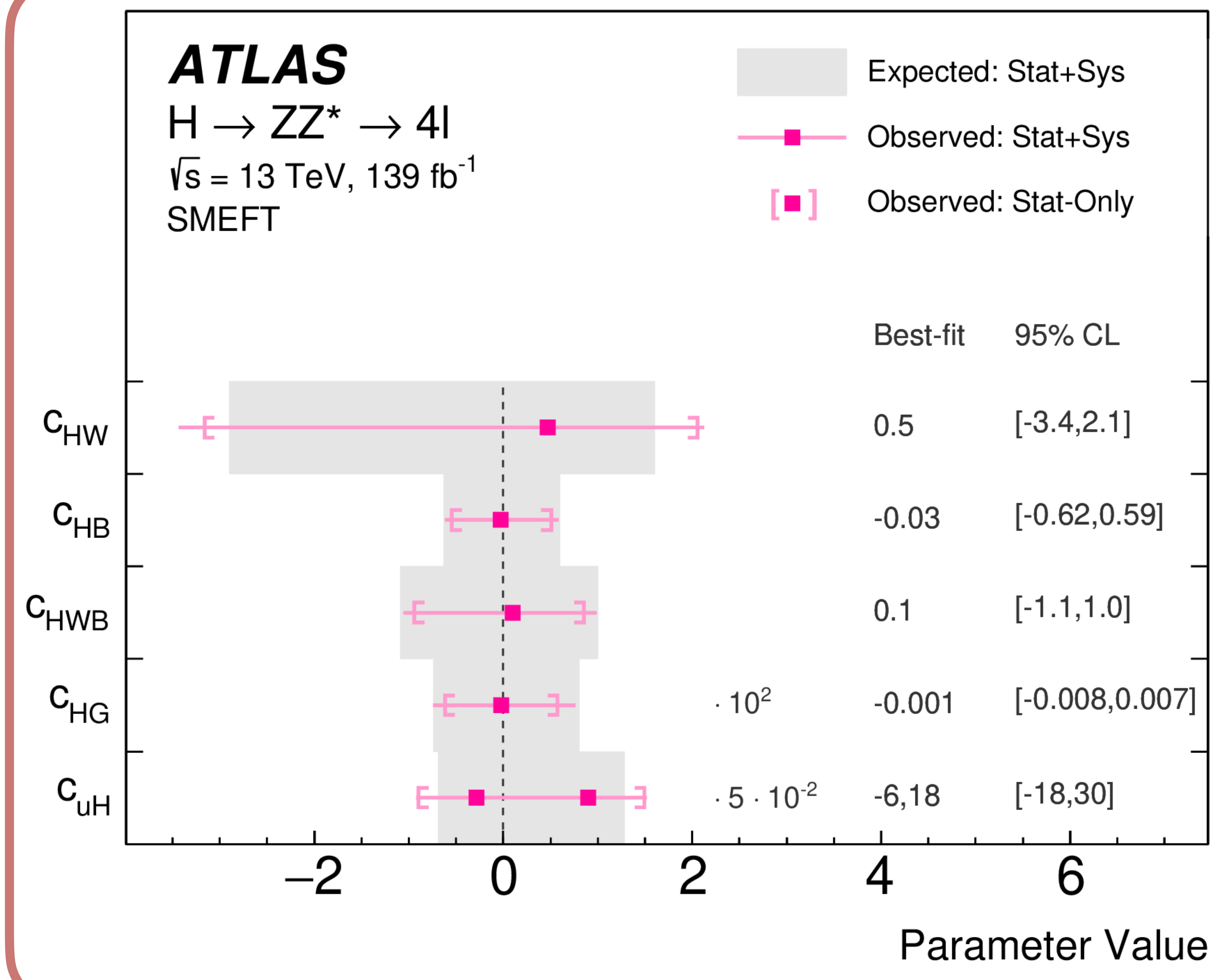
Provide meaningful framework for experimental tests of Higgs properties

*unless new physics is light.

Data!



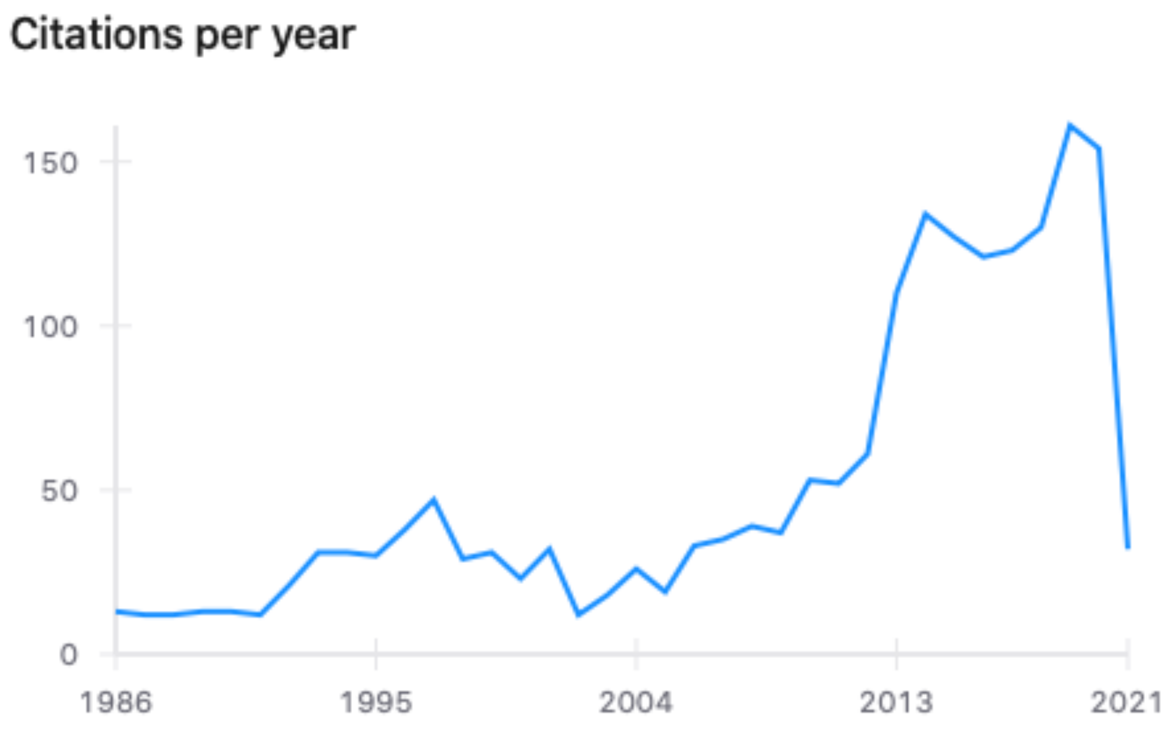
35.9-137 fb-1 (13 TeV)
CMS
Preliminary
Observed
±1σ (stat ⊕ syst)
±2σ (stat ⊕ syst)
p SM = 72%
κZ
κW
κt
|κτ|
|κb|
|κg|
|κγ|
|κμ|
–2.5 –2 –1.5 –1 –0.5 0 0.5 1 1.5 2 2.5
Parameter value



ATLAS
H → ZZ* → 4l
√s = 13 TeV, 139 fb-1
SMEFT
Expected: Stat+Sys
Observed: Stat+Sys
Observed: Stat-Only
Best-fit
95% CL
cHW
0.5
[-3.4,2.1]
cHB
-0.03
[-0.62,0.59]
cHWB
0.1
[-1.1,1.0]
cHG
· 10²
-0.001
[-0.008,0.007]
cuH
· 5 · 10⁻²
-6,18
[-18,30]
−2 0 2 4 6
Parameter Value

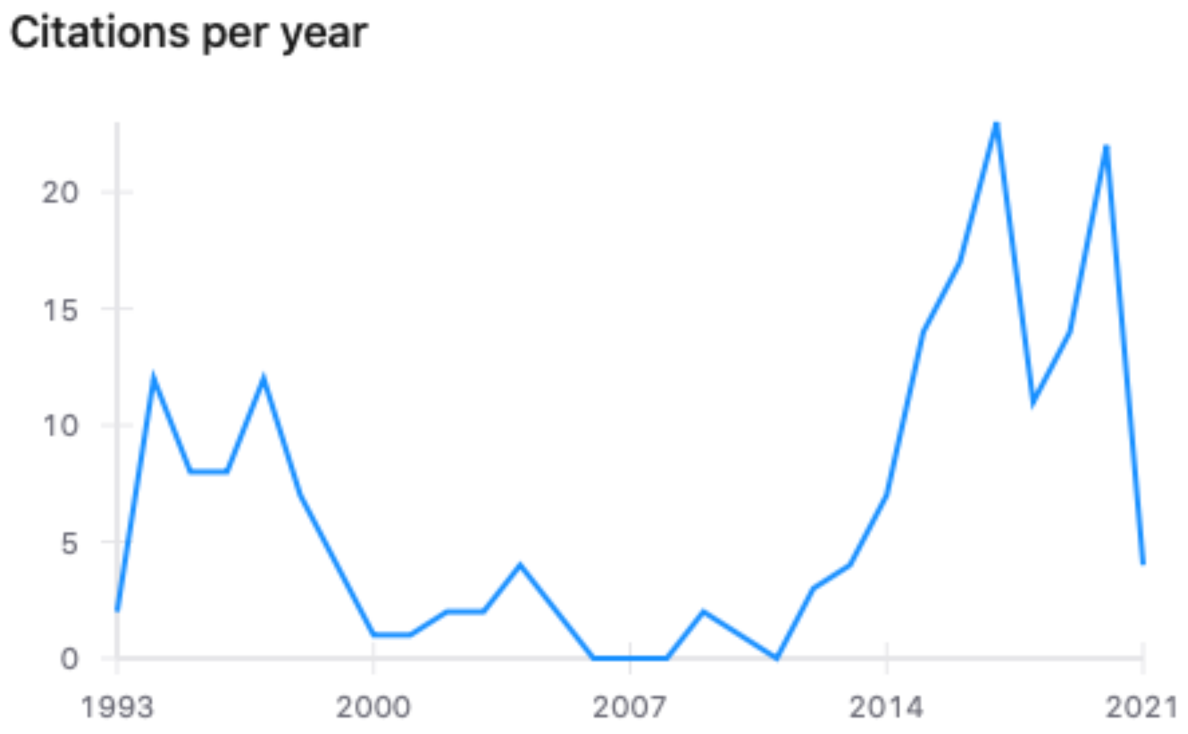
Which EFT?

Vastly more progress in SMEFT since c. 2012 (precision, fits, projections, theorems,…)



Effective Lagrangian Analysis of New Interactions and Flavor Conservation

W. Buchmuller (CERN), D. Wyler (Zurich, ETH)
Aug, 1985



The Chiral approach to the electroweak interactions

F. Feruglio (Padua U. and INFN, Padua)
Sep, 1992

Seems justified: SU(2)xU(1) an apparently good symmetry, no O(1) deviations or custodial symmetry violation

(When) Is HEFT necessary?

See also: [Burgess, Matias, Pospelov ’99; Grinstein & Trott ’07; Alonso, Gavela, Merlo, Rigolin, Yepes ’12; Espriu, Mescia, Yencho ’13; Buchalla, Cata, Krause ’13; Brivio et al. ’13; Falkowski & Rattazzi ‘ 19]

For this talk: focus exclusively on scalar sector in the global limit, assume custodial symmetry, restrict to 2-derivative order.

The Standard Model EFT

SMEFT: EFT where 4 scalar d.o.f. are arranged into an SU(2) doublet (equivalently, O(4) fundamental; assuming custodial symmetry):

$$\vec{\phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix}, \qquad \vec{\phi} \to O\vec{\phi}, \qquad H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

where $O \in O(4) \supset SU(2) \times U(1)$

"Electroweak symmetry is linearly realized."

$$\mathcal{L}_{\mathrm{SM}} = \frac{1}{2}(\partial\vec{\phi} \cdot \partial\vec{\phi}) - \frac{1}{4}\lambda(\vec{\phi} \cdot \vec{\phi} - v^2)^2$$

$$\mathcal{L}_{\mathrm{SMEFT}} = \frac{1}{2}A(\vec{\phi} \cdot \vec{\phi})(\partial\vec{\phi} \cdot \partial\vec{\phi}) + \frac{1}{2}B\left(\vec{\phi} \cdot \vec{\phi}\right)(\vec{\phi} \cdot \partial\vec{\phi})^2 - V\left(\vec{\phi} \cdot \vec{\phi}\right) + \mathcal{O}\left(\partial^4\right)$$

Reminder: only worrying about scalars up to 2 derivatives…

The Higgs EFT

Alternately, HEFT:
construct EFT out of
singlet h and Goldstones π_i

No presumed relation between h, π

$$h \qquad \vec{n} = \begin{pmatrix} n_1 = \pi_1/v \\ n_2 = \pi_2/v \\ n_3 = \pi_3/v \\ n_4 = \sqrt{1 - n_1^2 - n_2^2 - n_3^2} \end{pmatrix}$$

$$h \to h\,, \qquad \vec{n} \to O\vec{n}\,, \qquad O \in O(4)$$

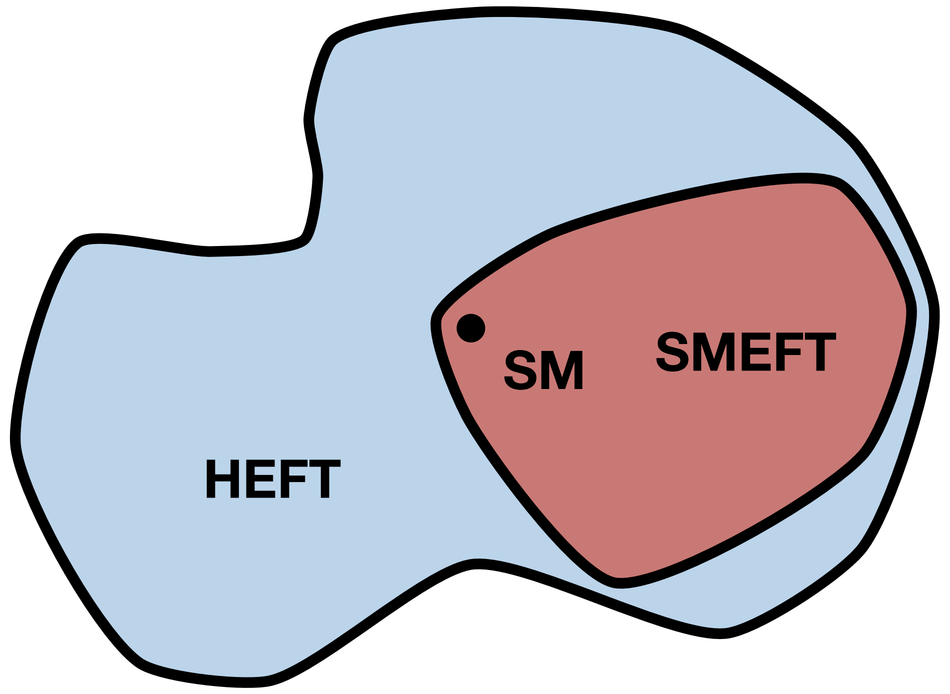
“Electroweak symmetry is nonlinearly realized.”

$$\mathcal{L}_{\rm SM} = \frac{1}{2}\left(\partial h\right)^2 + \frac{1}{2}(v+h)^2\left(\partial\vec{n}\right)^2 - \frac{1}{4}\lambda(h^2+2vh)^2$$

$$\mathcal{L}_{\rm HEFT} = \frac{1}{2}[K\left(h\right)]^2(\partial h)^2 + \frac{1}{2}[vF\left(h\right)]^2(\partial\vec{n})^2 - V\left(h\right) + \mathcal{O}\left(\partial^4\right)$$

(*K(h)* redundant, conventional to redefine h to set *K(h)* = 1; retaining *K(h)* clearer for matching)

SM ⊂ SMEFT ⊂ HEFT



[R. Alonso, E. Jenkins, A. Manohar 1511.00724 & 1605.03602]

$$\vec{\phi} = (v+h)\,\vec{n}\,(\pi)\,; \quad \vec{\phi}\cdot\vec{\phi} = (v+h)^2$$

SMEFT can always be written as HEFT:

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}A(\vec{\phi}\cdot\vec{\phi})(\partial\vec{\phi}\cdot\partial\vec{\phi}) + \frac{1}{2}B\left(\vec{\phi}\cdot\vec{\phi}\right)(\vec{\phi}\cdot\partial\vec{\phi})^2 - V\left(\vec{\phi}\cdot\vec{\phi}\right)\\ &= \frac{1}{2}\left[A + (v+h)^2\,B\right](\partial h)^2 + \frac{1}{2}\,(v+h)^2\,A(\partial\vec{n})^2 - V\end{aligned}$$

Correlations at every order between h, v

HEFT cannot always be written as SMEFT:

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}[K(h)]^2(\partial h)^2 + \frac{1}{2}[vF(h)]^2(\partial\vec{n})^2 - V(h)\\ &= \frac{1}{2}\frac{v^2F}{\vec{\phi}\cdot\vec{\phi}}(\partial\vec{\phi})^2 + \frac{1}{2}(\vec{\phi}\cdot\partial\vec{\phi})^2\frac{1}{\vec{\phi}\cdot\vec{\phi}}\left(K^2 - \frac{v^2F^2}{\vec{\phi}\cdot\vec{\phi}}\right) - \tilde{V}(\vec{\phi}\cdot\vec{\phi})\end{aligned}$$

Generically non-analytic at the origin

HEFT or SMEFT?

When can a theory be written as HEFT but not SMEFT?

Maybe you can always just tell by eye…

$$\mathcal{L} = \frac{1}{2}\left(1 + \frac{h}{2v}\right)^2 (\partial h)^2 + \frac{1}{2}(v+h)^2\left(\frac{3}{4} + \frac{h}{4v}\right)^2 (\partial \vec{n})^2 - V$$

Definitely HEFT, right?

HEFT or SMEFT?

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Definitely HEFT, right?

But now let's perform the field redefinition

$$h \to \tilde{h} \equiv h + \frac{1}{4v}h^2$$

HEFT or SMEFT?

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Definitely HEFT, right?

But now let's perform the field redefinition

$$h \to \tilde{h} \equiv h + \frac{1}{4v}h^2$$

$$\mathcal{L} = \frac{1}{2}(\partial\tilde{h})^2 + \frac{1}{2}(\tilde{v}+\tilde{h})^2(\partial\vec{n})^2 + \cdots = |\partial\tilde{H}|^2 + \dots$$

Actually the SM

Field redefinitions readily obscure the distinction at the level of the Lagrangian.

A Geometric Perspective

***Instead*: classify EFTs based on geometry.**

Two-derivative terms define a metric on the scalar field manifold

$$\mathcal{L} = \frac{1}{2} g_{ij}(\phi) \partial\phi^i \partial\phi^j - V(\phi)$$

Field space corresponds to a (possibly curved) manifold with functions (e.g. V) defined on it; the field parameterization corresponds to charts on the manifold. Use geometric invariants to classify EFTs.

Long history (primarily) applied to nonlinear sigma models, e.g.
[Honerkamp '72; Tataru '75; Alvarez-Gaume, Freedman, Mukhi '81, …]

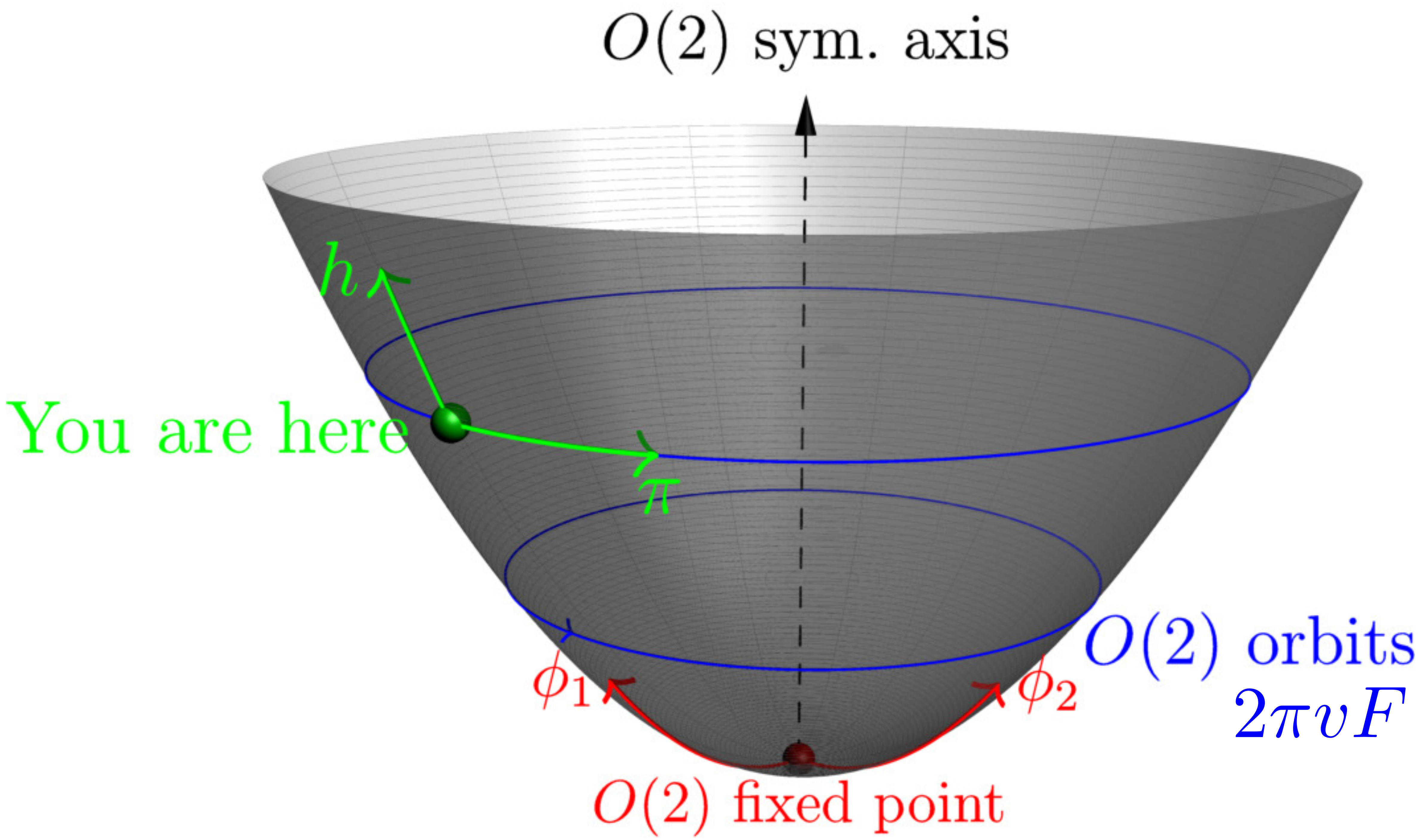
Application to HEFT: [Alonso, Jenkins, Manohar 1511.00724 & 1605.03602]
(Applied to SMEFT: [Helset, Martin, Trott 2001.01453])

SM: flat manifold **HEFT:** curved manifold

SMEFT: curved manifold w/ O(4) invariant point

A Geometric Perspective

(Think O(4), but O(2) is easier to illustrate)



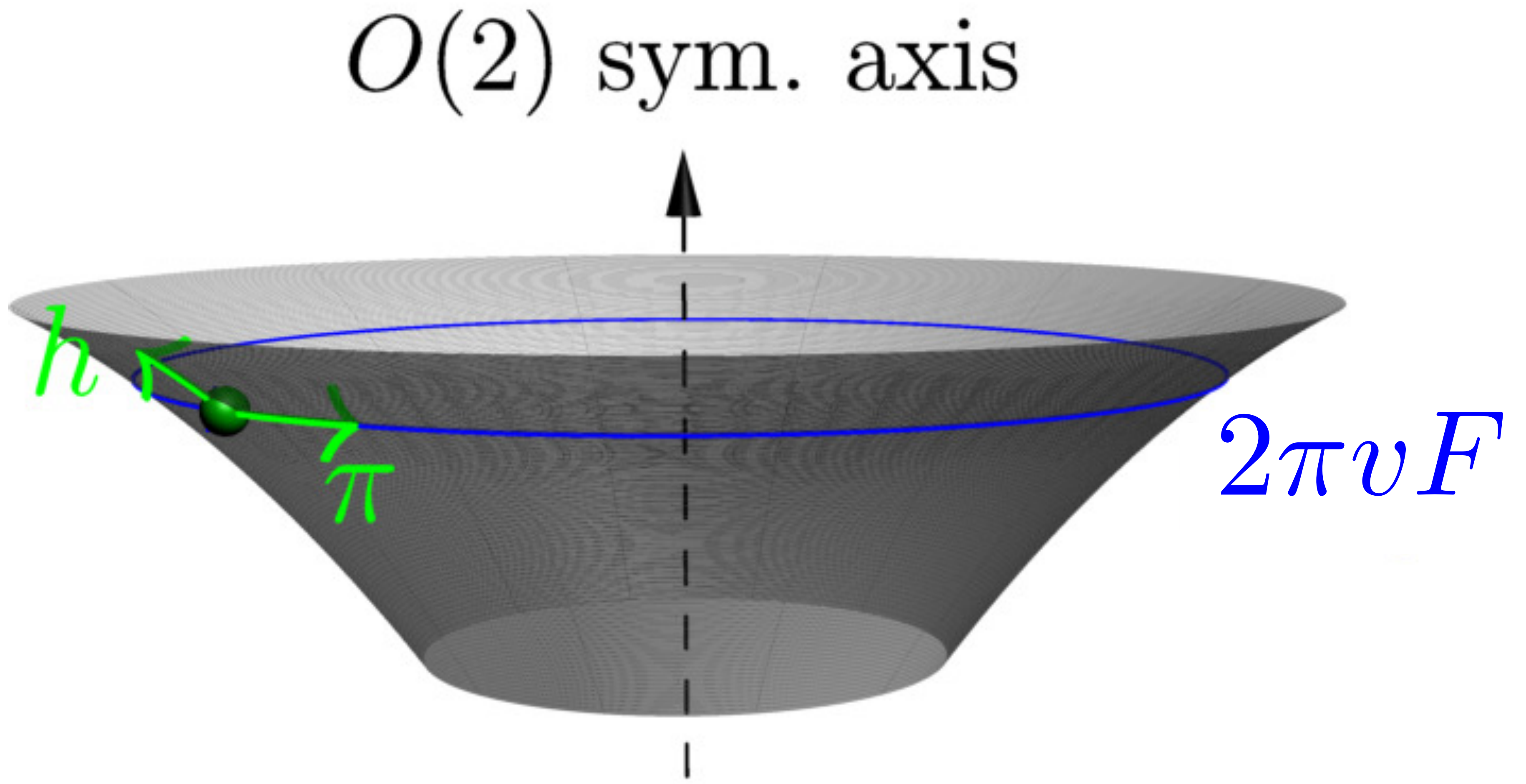
$$\mathcal{L}_{\mathrm{HEFT}} = \frac{1}{2}(\partial h)^2 + \frac{1}{2}[vF(h)]^2(\partial\vec{n})^2 - V(h) + \mathcal{O}\left(\partial^4\right)$$

SMEFT if O(4) fixed point on manifold → F(h) = 0 somewhere (say, *h* = -*v*)

HEFT not SMEFT: Case I

[Alonso, Jenkins, Manohar 1605.03602]

When there's a hole s.t. *h* = -*v* is not on the manifold
(no O(4) fixed point about which to expand in SMEFT coordinates)



$$\mathcal{L}_{\text{HEFT}} = \frac{1}{2}(\partial h)^2 + \frac{1}{2}[vF(h)]^2(\partial\vec{n})^2 - V(h) + \mathcal{O}(\partial^4)$$

Corresponds to $F(h) \neq 0$ **everywhere**

HEFT not SMEFT: Case I

How does this arise? *When UV physics also breaks the symmetry.*

A toy example: 2AHM, i.e. two Higgses charged under a U(1) gauge symmetry

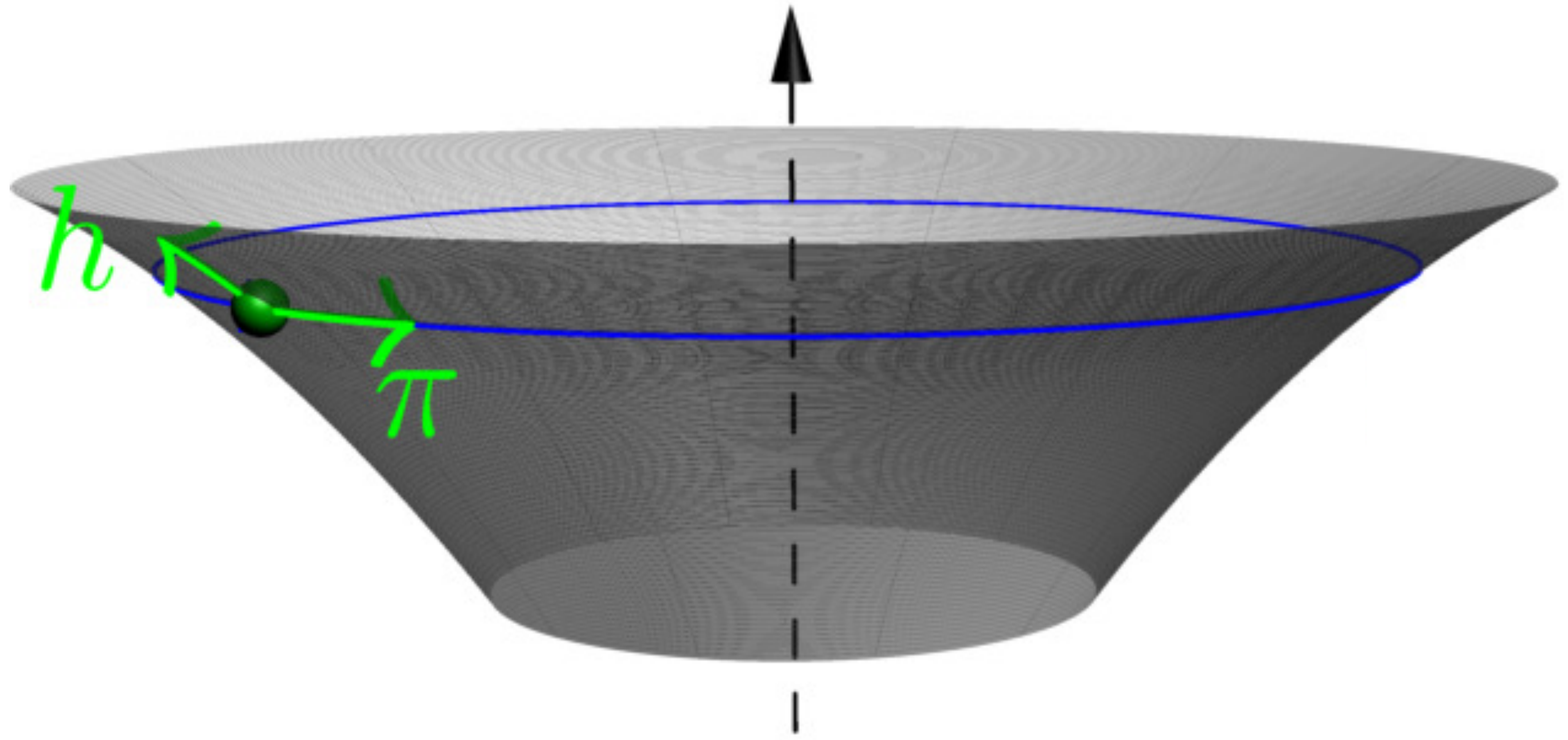
Acquire vevs s.t. $v^2 \equiv 4v_1^2 + v_2^2$

Field	Q
H_1	$+2$
H_2	$+1$

Spectrum: light Higgs *h*, goldstone *π*, heavy fields *H, Π*

Integrate out *H, Π* to obtain EFT of *h, π*

$$K(h) = 1, \qquad F(h) = \frac{1}{v}\sqrt{4(v_1 + c_\alpha h)^2 + (v_2 + s_\alpha h)^2}$$



Generically F(h) ≠ 0 everywhere for nonzero v_1, v_2

HEFT not SMEFT: Case II

When there's a cone or cusp at *h*=-*v*

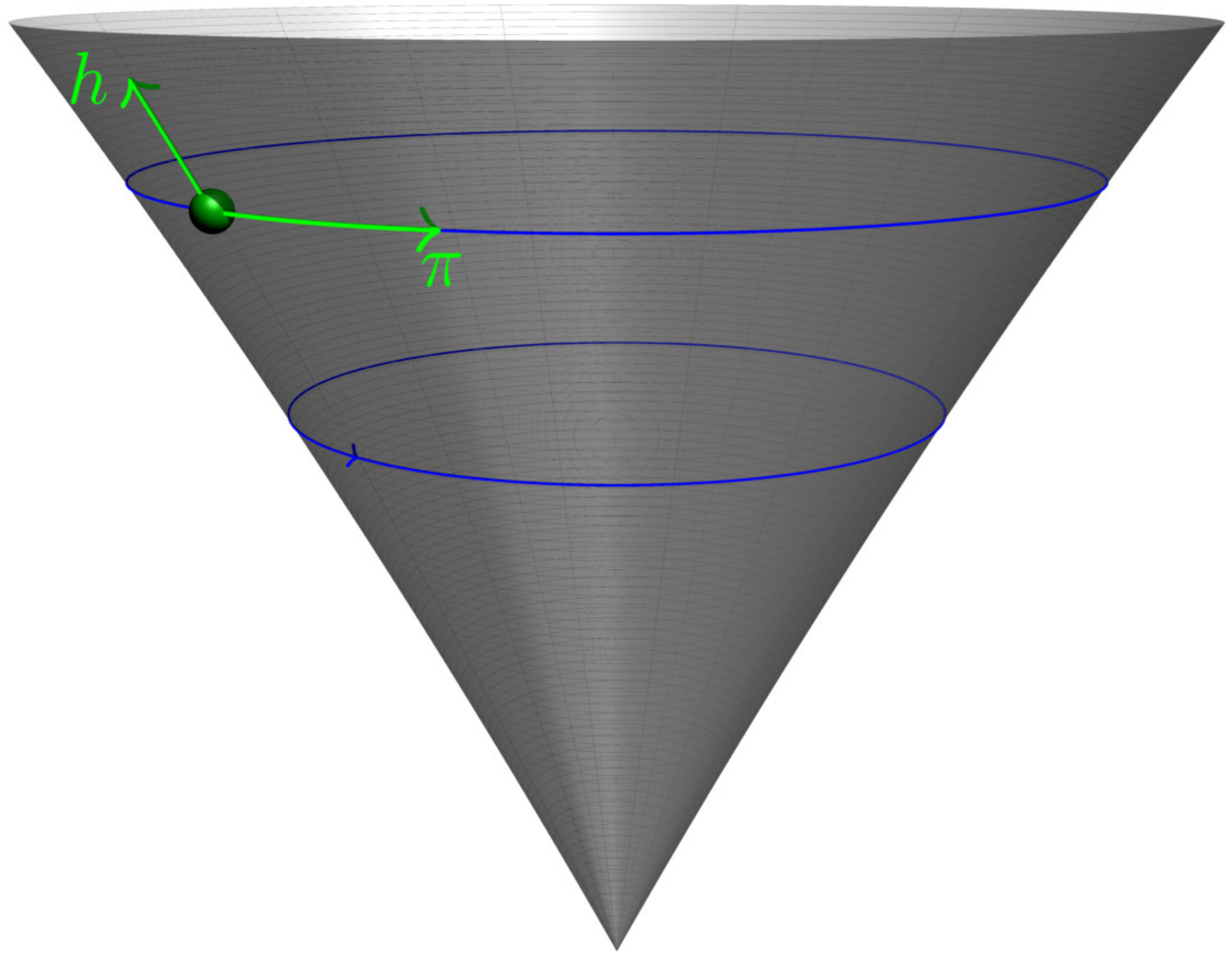
Can often tell by inspecting F(h), V(h) for non-analyticities, but this does not always work.

$$\mathcal{L}_{\mathrm{HEFT}} = \frac{1}{2}(\partial h)^2 + \frac{1}{2}[vF(h)]^2(\partial\vec{n})^2 - V(h)$$

But can diagnose singularities as in GR:

If $(\nabla^2)^n R$ and $(\nabla^2)^{n+1} V$

are finite at *h*=-*v*, then can write HEFT as SMEFT (gives the requisite infinite set of conditions!)



Otherwise, there is a cone/cusp and HEFT is required.

HEFT not SMEFT: Case II

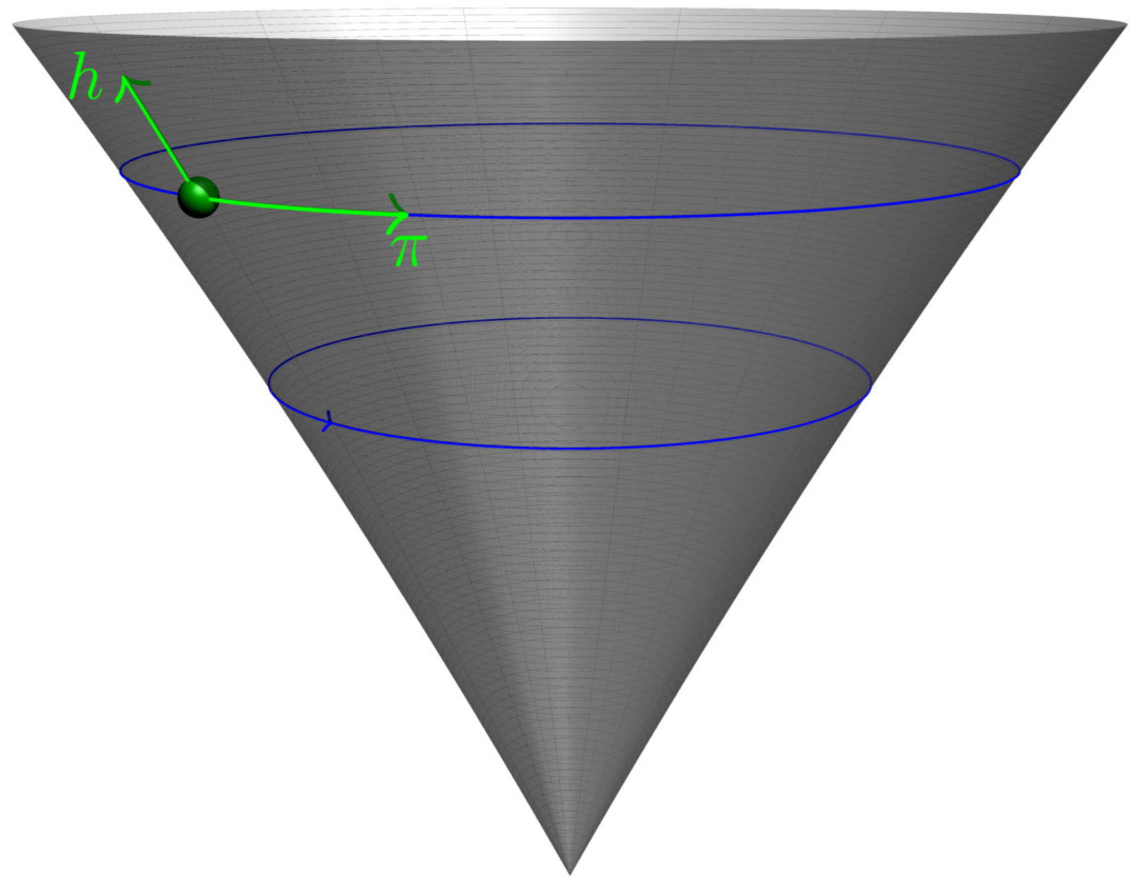
How does this arise? *When a field becomes massless.*

An example: integrating out anything that acquires
all of its mass from EWSB, e.g. *M=0* limit of

$$\mathcal{L} \supset \bar{\psi}_1(i\,\partial\!\!\!/ - M)\psi_1 + \bar{\psi}_2(i\,\partial\!\!\!/ - M)\psi_2 - y\bar{\psi}_1 H\psi_2 + \text{h.c.}$$

F(h=-v) = 0, so okay according to Case I

Compute Ricci scalar: $$R(h=-v) \propto \frac{|y|^4}{16\pi^2}\frac{1}{M^2}$$



When *M ≠ 0*, curvature finite and SMEFT is consistent

For *M=0*, curvature blows up.
K, *F*, and *V* all non-analytic at *h=-v* due to *log(v+h)*

HEFT as SMEFT IFF

$$\mathcal{L}_{\text{HEFT}} = \frac{1}{2}(\partial h)^2 + \frac{1}{2}[vF(h)]^2(\partial\vec{n})^2 - V(h) + \mathcal{O}\left(\partial^4\right)$$

1. ***F(h*) = 0*** at some h=h* (candidate O(4) f.p.)

2. ***Metric is analytic*** at h=h*: F(h), K(h) admit convergent Taylor expansions here, and curvature invariants $\sim\nabla^n R$ are finite for n≥0.

3. ***Potential is analytic*** at h=h*: V(h) admits convergent Taylor expansion here, and invariants $\sim\nabla^n V$ are finite for n≥0.

Satisfying these conditions ensures the theory admits a SMEFT expansion around the O(4) fixed point. However, a further consideration: *that expansion should converge at our vacuum (h=0).*

SMEFT Convergence

Even when SMEFT exists, the SMEFT expansion may not converge at our vacuum.

Clear example: for SMEFT with $\Lambda < v$, $\mathcal{L} \supset \sum_{n=1}^{\infty} c_n \frac{|H|^{4+2n}}{\Lambda^{2n}}$ diverges, w/out optimal truncation

To make this more concrete…

Consider a singlet scalar with nonzero bare mass,

$$\mathcal{L}_{\mathrm{UV}} = |\partial H|^2 + \mu_h^2 |H|^2 - \frac{1}{2}\lambda_h |H|^4 + \frac{1}{2} S \left(-\partial^2 - m^2 - \kappa |H|^2\right) S$$

Integrating out the scalar gives 0- & 2-derivative effective lagrangian for H:

$$\delta\mathcal{L}_{\mathrm{Eff}}^{(0)} = \frac{1}{(4\pi)^2}\frac{1}{4}\left(m^2 + \kappa |H|^2\right)^2 \left(\ln \frac{\mu^2}{m^2 + \kappa |H|^2} + \frac{3}{2}\right) \qquad \delta\mathcal{L}_{\mathrm{Eff}}^{(2)} = \frac{1}{(4\pi)^2}\frac{1}{4}\frac{1}{6}\frac{\kappa^2}{m^2 + \kappa |H|^2}\left(\partial |H|^2\right)^2$$

SMEFT Convergence

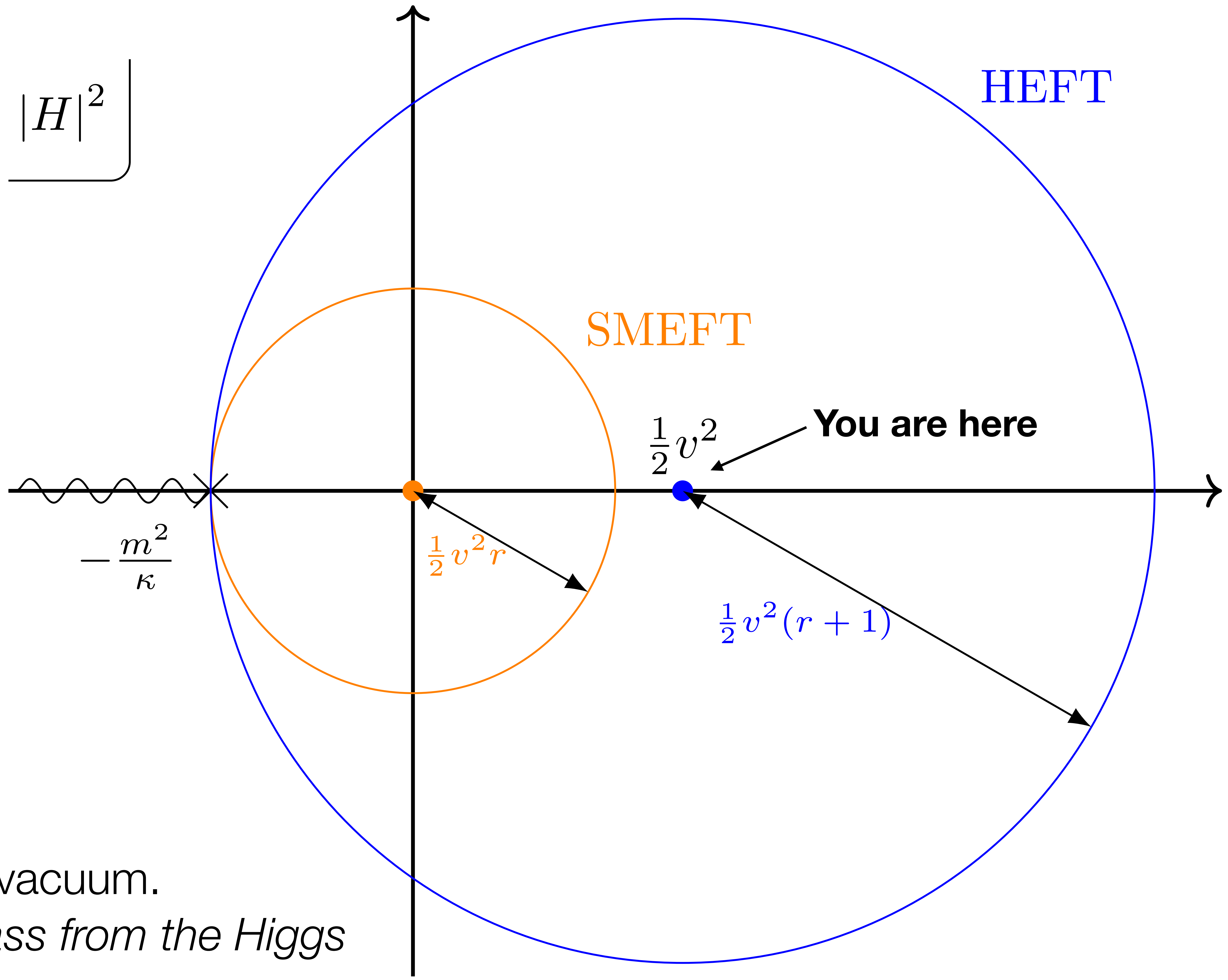
Consider analytic structure of the effective Lagrangian in the complex $|H|^2$ plane

$$r \equiv \frac{\text{bare mass}^2}{\text{mass}^2 \text{ from Higgs}} = \frac{m^2}{\frac{1}{2}\kappa v^2}$$

Branch cut at $|H|^2 = -\frac{m^2}{\kappa} \Rightarrow$

SMEFT radius of convergence is $v^2 r/2$

HEFT radius of convergence is $v^2(r+1)/2$



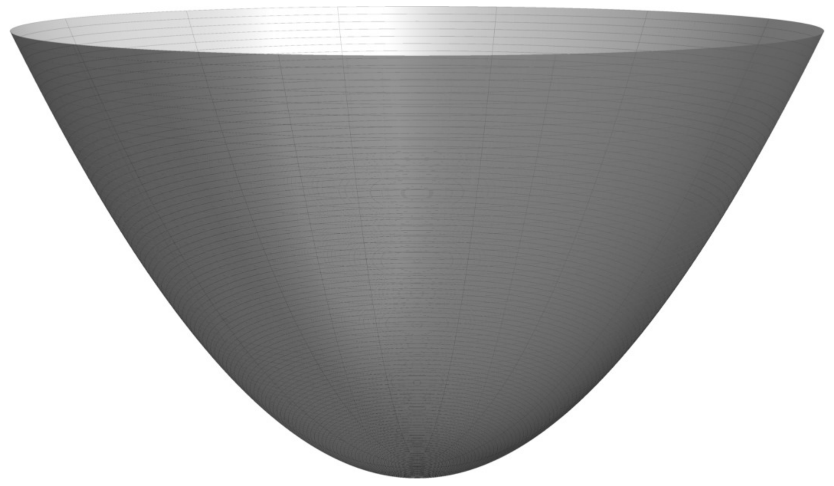
$r < 1$: SMEFT expansion does not converge at our vacuum.
HEFT required by states w/ more than half of their mass from the Higgs

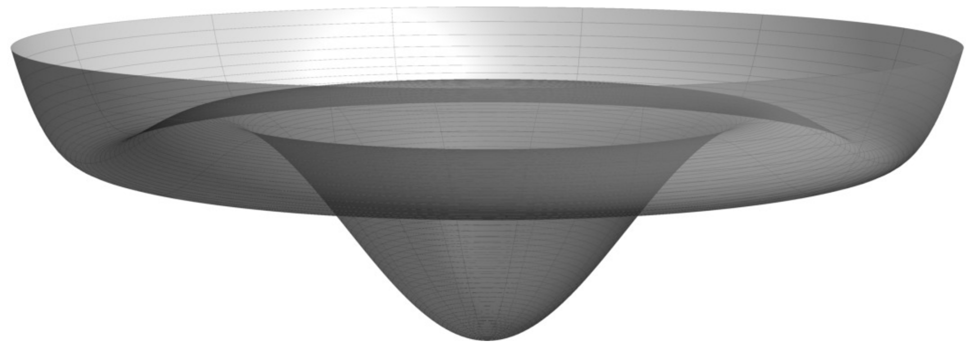
SMEFT Convergence

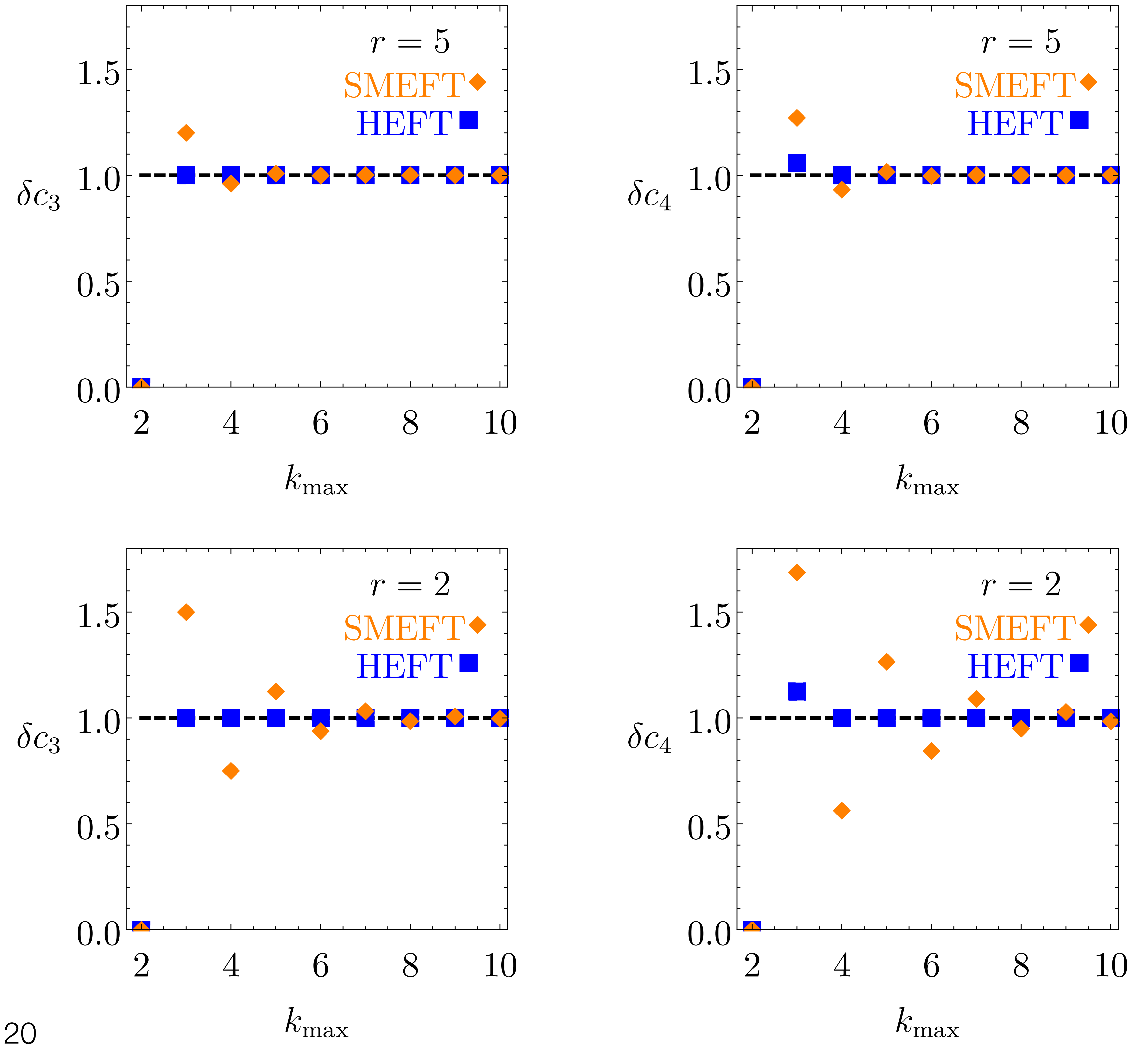
Even for $r \gtrsim 1$, HEFT can capture true corrections to SM using fewer terms in the relevant expansion than SMEFT.

[Englert et al. 1403.7191; Brehmer et al. 1510.03443] Improve agreement between truncation of SMEFT and true corrections by defining matching scale as physical mass of new particles in the broken phase ("v-improved matching").

Practically amounts to matching in HEFT, converting to SMEFT coordinates.







HEFTons*

*Working name, suggestions welcome, “Higglets” and “Baryons” unfortunately already taken.

HEFT required whenever a new particle acquires more than half of its mass from the Higgs.

Many such HEFTons viable, consistent with all existing data (see also [Bonnefoy et al. 2011.10025])

Conclusions

- Universal geometric criteria for HEFT vs. SMEFT:

- *Many* ways to get $U(1)_{em}$ Higgs EFT starting from SU(2)xU(1) symmetry in the UV, consistent w/ data. Perhaps premature to focus heavily on SMEFT interpretations.

- HEFT can be the preferred EFT for data even when both HEFT & SMEFT expansions valid.

- Open questions: How do we get experimental access to the (possible) O(4) fixed point? What is the connection between the geometric picture and statements about the scale of unitarity breakdown in SMEFT vs. HEFT? When does all this matter?

→ Dave Sutherland's talk

Which EFT for the LHC?

Part II: The Unitarity Violations of HEFT, Geometrically

Based on work with I. Banta, T. Cohen, N. Craig and X. Lu

Dave Sutherland

INFN, Sezione di Trieste

KITP Precision21, April 6th 2021

H2020 MSCA COFUND
G.A. 754496

The plan

We've seen how UV completions shape the target space manifold in the SM scalar sector, and how amenable these shapes are to a SMEFT description.

I aim to show how amplitudes measure the shape of the manifold locally about our vacuum. Thereby, we connect our geometric understanding of HEFT back to recent results about unavoidable TeV scale ($\sim 4\pi v$) unitarity cutoffs in HEFT theories, and the masses of the UV states that must unitarize them.

At the end, I'll speculate what this means for 'Which EFT for the LHC?'.

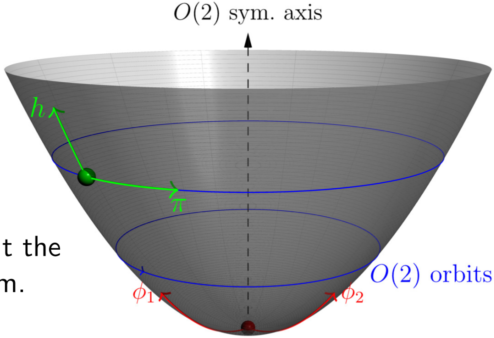
Manifold redux

Draw a 2d manifold (h and one π coordinate).
$O(4) \to O(2) \approx$ EW sym.

Dynamics encoded by a metric and potential.

HEFT is an expansion about our vacuum.

SMEFT is an expansion about the electroweak preserving vacuum.

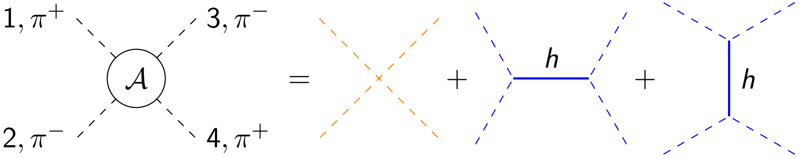


A HEFT is poorly described by SMEFT when sufficient violence is done to the manifold between us and the EW preserving vacuum.

This talk: Amplitudes probe such effects by measuring the variation of curvatures about our vacuum. Unitarity bounds result.

A review of unitarity violation in $W^+W^- \to W^+W^-$ (1)

$$\mathcal{L} = \frac{1}{2}(\partial h)^2 + \frac{1}{2}(vF(h))^2(\partial\vec{n})^2 - V(h) \supset +\frac{1}{2v^2}\left[\partial_\mu(\pi^+\pi^-)\right]^2 + 2\overline{F'}\, h\partial\pi^+\partial\pi^-$$



$$\mathcal{A} = -\frac{1}{v^2}(s+t) + \overline{F'}^2\left[\frac{s^2}{s-m_h^2} + \frac{t^2}{t-m_h^2}\right]$$

$$= \left(\overline{F'}^2 - \frac{1}{v^2}\right)(s+t) + \overline{F'}^2\left[2m_h^2 + \frac{m_h^4}{s-m_h^2} + \frac{m_h^4}{t-m_h^2}\right]$$

Note:[1]

- we use vertices with different numbers of Higgses;
- the $\mathrm{O}(p^2)$ part has no kinematic pole.

[1] A $\overline{\text{bar}}$ denotes a quantity evaluated at the our vacuum $h = \pi_i = 0$. $\overline{F} = 1$.

A review of unitarity violation in $W^+W^- \to W^+W^-$ (2)

Put the amplitude in a correctly normalized s-wave state[2]

$$|\hat{M}| = \frac{\left|\int \mathrm{d}\Pi_i \mathrm{d}\Pi_f \mathcal{A}\right|}{\left(\int \mathrm{d}\Pi_i\right)^{\frac{1}{2}} \left(\int \mathrm{d}\Pi_f\right)^{\frac{1}{2}}} \overset{E \gg m_W}{=} \frac{1}{8\pi} \frac{E^2}{2} \left| \overline{F'}^2 - \frac{1}{v^2} \right| + \mathrm{O}(E^0)$$

Unitarity circle arguments say $|\hat{M}| \lesssim 1$, so there is a unitarity bound on the CoM energy

$$E \lesssim \sqrt{16\pi} \left| \overline{F'}^2 - \frac{1}{v^2} \right|^{-\frac{1}{2}}$$

$E \to \infty$ if the hWW coupling is SM like: $\overline{F'} = \frac{1}{v}$.

Four legs good, more legs better: I will argue that, for any weakly coupled UV completion, better unitarity bounds can be obtained from higher point amplitudes.

[2] E is the center of mass energy.

The four point amplitude in terms of R and V

Derivative key: ',' partial, ';' covariant (Nagai, Tanabashi, Tsumura, and Uchida 2019)

$$\mathcal{A} = -\left(\frac{1}{v^2} - \overline{F'}^2\right)(s+t) + 2m_h^2\overline{F'}^2 + m_h^4\overline{F'}^2\left[\frac{1}{s-m_h^2} + \frac{1}{t-m_h^2}\right]$$

$$\overline{R}_{+--+} = -\overline{g}_{+-,+-} + \overline{\Gamma}^h_{+-}\overline{\Gamma}_{-+h} = \frac{1}{v^2} - \overline{F'}^2$$

$$\overline{V}_{;(h+-)} = -\overline{V}_{,hh}\overline{\Gamma}^h_{+-} = -m_h^2\overline{F'}$$

$$\overline{V}_{;(++--)} = 2\overline{V}_{,hh}\overline{\Gamma}^h_{+-}\overline{\Gamma}^h_{+-} = 2m_h^2\overline{F'}^2$$

$$\mathcal{A} = -\overline{R}_{+--+}(s+t) + \overline{V}_{;(++--)} + \overline{V}_{;(h+-)}\overline{g}^{hh}\overline{V}_{;(h+-)}\left[\frac{1}{s-m_h^2} + \frac{1}{t-m_h^2}\right]$$

The components of R are **sectional curvatures**

$$\overline{R}_{+--+} \equiv \overline{\mathcal{K}_\pi} \implies E < \sqrt{16\pi}|\overline{\mathcal{K}_\pi}|^{-\frac{1}{2}}$$

(Alonso, Jenkins, and Manohar 2016)

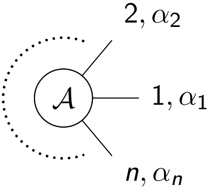
n point amplitude in terms of R and V

For a general scalar EFT

$$\mathcal{L} = \frac{1}{2} g_{\alpha\beta}(\vec{\phi}) \partial_\mu \phi^\alpha \partial^\mu \phi^\beta - V(\vec{\phi}) + \mathrm{O}\big(\partial^4\big) \,,$$

the tree-level amplitudes are

$$\left(\prod_{i=1}^{n} \overline{g}^{1/2}_{\alpha_i \alpha_i} \right) \mathcal{A} =$$



$$= \overline{V}_{;(\alpha_1 \ldots \alpha_n)} + \sum_{1 \le i < j \le n} s_{ij} \left(\tfrac{n-3}{n-1} \right) \left[\overline{R}_{\alpha_i(\alpha_1 \alpha_2 | \alpha_j ; | \alpha_3 \ldots \hat{\alpha}_i \ldots \hat{\alpha}_j \ldots \alpha_n)} + \mathrm{O}\big(\overline{R}^2\big) \right]$$

$$+ \text{ factorizable pieces} \,.$$

(Assuming $\overline{g}_{\alpha\beta}$ diagonal. Show by going to an *inertial frame*, a.k.a. normal coordinates. E.g. (Alvarez-Gaume, Freedman, and Mukhi 1981))

$\mathcal{A}\left(\pi_i \pi_j h^{n-2}\right)$

Following (Falkowski and Rattazzi 2019)

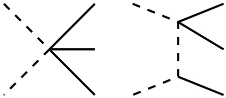
With the application of geometric/kinematic identities:

$$\begin{aligned}\mathcal{A}\left(\pi_i \pi_j h^{n-2}\right) =& \overline{V}_{;(\pi_i \pi_j h \ldots h)} + \overline{R}_{\pi_i h h \pi_j; h \ldots h}\left(s_{12} - \frac{2m_h^2}{n-1}\right) \\ &+ \mathrm{O}\left(\overline{R}^2\right) + \text{factorizable pieces} \\ =& \overline{V}_{;(\pi_i \pi_j h \ldots h)} - \delta_{ij}\, \overline{\partial_h^{n-4}\mathcal{K}_h}\left(s_{12} - \frac{2m_h^2}{n-1}\right) \\ &+ \mathrm{O}\left(\overline{R}^2\right) + \text{factorizable pieces}\,.\end{aligned}$$

The parts of the $n > 4$ amplitudes that grow with energy are *derivatives* of sectional curvatures.

$$\mathcal{A}\left(\pi_i \pi_j \to h^{n-2}\right) = -E^2\,\delta_{ij}\,\overline{\partial_h^{n-4}\mathcal{K}_h} + \mathrm{O}(E^0)$$

Unpack this result



Take the $\mathrm{O}(E^2)$ part of $\mathcal{A}\left(\pi_1\pi_1 \to h^3\right)$

$$\mathcal{A}\left(\pi_1\pi_1 \to h^3\right) = -E^2\overline{\partial_h \mathcal{K}_h} = E^2\overline{\partial_h\left(\frac{F''}{F}\right)} = E^2\left(\overline{F}''' - \overline{F}''\overline{F}'\right)$$

Sub in particular UV examples

$$\textbf{SM}:\ vF = (v+h) \qquad \Longrightarrow\ \mathcal{A} = 0$$

$$\textbf{unSMEFTy}:\ vF = (v+h) + \frac{\epsilon}{v^2}h^3 \qquad \Longrightarrow\ \mathcal{A} = \frac{6\epsilon}{v^3}E^2$$

$$\textbf{SMEFTy}:\ vF = (v+h) + \frac{\epsilon}{v^2}(v+h)^3 \qquad \Longrightarrow\ \mathcal{A} = 0 + \mathrm{O}(\epsilon^2)$$

Parametrically faster growth in cases where the deviations are non-SMEFT like.

SM kinetic term $\Longrightarrow$ 4 point amplitude unitary

SMEFT term $\Longrightarrow$ higher point amplitude unitary(ish)

Correlations present in many other amplitudes

(Abu-Ajamieh, Chang, Chen, and Luty 2020)

$$\mathcal{L} = \mathcal{L}_{\rm SM} - \delta_3 \frac{m_h^2}{2v} h^3 - \delta_4 \frac{m_h^2}{8v^2} h^4 - \sum_{n=5}^{\infty} \frac{c_n}{n!} \frac{m_h^2}{v^{n-2}} h^n + \cdots$$

$$+ \delta_{Z1} \frac{m_Z^2}{v} h Z^\mu Z_\mu + \delta_{W1} \frac{2m_W^2}{v} h W^{\mu +} W_\mu^- + \delta_{Z2} \frac{m_Z^2}{2v^2} h^2 Z^\mu Z_\mu + \delta_{W2} \frac{m_W^2}{v} h^2 W^{\mu +} W_\mu^-$$

$$+ \sum_{n=3}^{\infty} \left[\frac{c_{Zn}}{n!} \frac{m_Z^2}{v^n} h^n Z^\mu Z_\mu + \frac{c_{Wn}}{n!} \frac{2m_W^2}{v^n} h^n W^{\mu +} W_\mu^- \right] + \cdots$$

$$- \delta_{t1} \frac{m_t}{v} h \bar{t} t - \sum_{n=2}^{\infty} \frac{c_{tn}}{n!} \frac{m_t}{v^n} h^n \bar{t} t + \cdots$$

Process	$\times \frac{E^4}{1152\pi^3 v^4}$
$hZ^2 \to hZ^2$	$[4\delta_{V1} - 2\delta_{V2} + \frac{1}{2} c_{V3}]$
$h^2 Z \to Z^3$	$-\frac{\sqrt{3}}{2}[4\delta_{V1} - 2\delta_{V2} + \frac{1}{2} c_{V3}]$
$h^2 W^+ \to Z^2 W^+$	$-\frac{1}{2}[4\delta_{V1} - 2\delta_{V2} + \frac{1}{2} c_{V3}]$
$h^2 Z \to Z W^+ W^-$	$-\frac{1}{\sqrt{2}}[4\delta_{V1} - 2\delta_{V2} + \frac{1}{2} c_{V3}]$
$h^2 W^+ \to W^+ W^- W^+$	$-[4\delta_{V1} - 2\delta_{V2} + \frac{1}{2} c_{V3}]$
$hZW^+ \to hZW^+$	$[36\delta_{V1} - 13\delta_{V2} + 2c_{Vc}]$
$hW^+W^+ \to hW^+W^+$	$[36\delta_{V1} - 13\delta_{V2} + 2c_{V3}]$
$hW^+W^- \to hW^+W^-$	$-[28\delta_{V1} - 9\delta_{V2} + c_{V3}]$
$hZ^2 \to hW^+W^-$	$-\sqrt{2}[32\delta_{V1} - 11\delta_{V2} + \frac{3}{2} c_{V3}]$

Process	$\times \frac{(\frac{1}{2}c_{t2} - \delta_{t1}) m_t E^2}{32\pi^2 v^3}$
$\bar{t}_R t_R \to Z h^2$	$i\sqrt{N_c}$
$h^2 \to Z \bar{t}_L t_L$	$i\sqrt{\frac{N_c}{3}}$
$Zh \to h \bar{t}_L t_L$	$i\sqrt{\frac{2N_c}{3}}$
$t_R Z \to t_L h^2$	$\frac{i}{\sqrt{6}}$
$t_R h \to t_L Z h$	$\frac{i}{\sqrt{3}}$
$\bar{t}_R t_R \to Z^2 h$	$-\sqrt{N_c}$
$Z^2 \to \bar{t}_L t_L h$	$-\sqrt{\frac{N_c}{3}}$
$Zh \to \bar{t}_L t_L Z$	$-\sqrt{\frac{2N_c}{3}}$
$t_R h \to t_L Z^2$	$-\frac{1}{\sqrt{6}}$
$t_R Z \to t_L Z h$	$-\frac{1}{\sqrt{3}}$

Unitarity bound for $\mathcal{A}(\pi_i\pi_j \to h^n)$

For $2 \to n$, the s-wave state $|\hat{M}|^2 \sim \frac{1}{8\pi}\left(\frac{1}{(n-2)!}\right)^2 \left(\frac{E}{4\pi}\right)^{2(n-2)} |\mathcal{A}|^2$, see e.g. (Abu-Ajamieh, Chang, Chen, and Luty 2020)[3]

Unitarity bound

$$E < 4\pi \times \left|\frac{\overline{\partial_h^{n-2}\mathcal{K}_h}}{n!}\right|^{-\frac{1}{n}} \times b_n \times (n!)^{\frac{1}{n}}$$

$$= \begin{cases} 8^{\frac{1}{4}}\sqrt{16\pi} \times \left|\overline{\mathcal{K}_h}\right|^{-\frac{1}{2}} & n = 2 \\ 4\pi v_* \times (n!)^{\frac{1}{n}} & n = \text{'a few'} \end{cases}$$

v_* is the scale of '∂_h' $\approx$ the radius of convergence of $\mathcal{K}_h$.

$4\pi v_*$ is a general bound for HEFT. If poorly described by SMEFT, $v_* \sim v$.

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[3] b_n is an $\mathrm{O}(1)$ fudge factor

$$\left(\frac{1}{b_n}\right)^{2n} = \frac{(4\pi)^2}{8(n-1)}\left(1 - \frac{2m_h^2}{(n+1)E^2}\right)^2 \times \frac{\text{Vol. } n \text{ body Higgs PS}}{\text{Vol. } n \text{ body massless PS}}.$$

Example: loop-level scalar singlet EFT

(Cohen, Craig, Lu, and Sutherland 2021)

$$\mathcal{L}_{\text{UV}} = |\partial H|^2 + \mu_H^2 |H|^2 - \lambda_H |H|^4 + \frac{1}{2} S \left(-\partial^2 - m^2 - \kappa |H|^2\right) S\,.$$

Assume $m^2, \kappa > 0$. Integrate out S to get

$$\mathcal{L}_{\text{EFT}} = \frac{1}{2}\left(1 + \delta \frac{\kappa (v+h)^2}{2 m^2 + \kappa (v+h)^2}\right)(\partial h)^2 + \frac{1}{2}(v+h)^2 (\partial \vec{n})^2 - V(h)$$

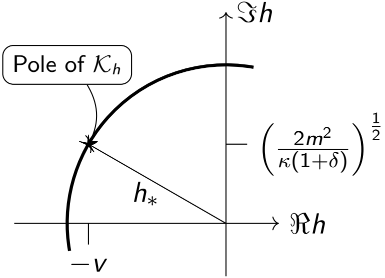
where $\delta = \frac{\kappa}{96\pi^2}$ controls the coupling strength.

Sectional curvatures

$$\mathcal{K}_h = \delta \frac{\kappa}{2} \frac{m^2}{\left(m^2 + \frac{1}{2}\kappa (1+\delta)(v+h)^2\right)^2}\,,$$

$$\mathcal{K}_\pi = \delta \frac{\kappa}{2} \frac{1}{\left(m^2 + \frac{1}{2}\kappa (1+\delta)(v+h)^2\right)}\,.$$

Example: loop-level scalar singlet scales



Sectional curvatures

$$\mathcal{K}_h = \delta \frac{\kappa}{2} \frac{m^2}{\left(m^2 + \frac{1}{2}\kappa\,(1+\delta)\,(v+h)^2\right)^2}\,,$$

$$\mathcal{K}_\pi = \delta \frac{\kappa}{2} \frac{1}{\left(m^2 + \frac{1}{2}\kappa\,(1+\delta)\,(v+h)^2\right)}\,.$$

Radius of convergence, $v_* \approx h_* = \sqrt{v^2 + \dfrac{2m^2}{\kappa(1+\delta)}}$.

Mass of scalar at vacuum, $m_S^2 = m^2 + \dfrac{1}{2}\kappa v^2 \approx \dfrac{1}{2}\kappa v_*^2$.

If $\kappa \sim (4\pi)^2$ and $m^2 \sim (4\pi v)^2$, $\overline{\mathcal{K}_h}^{-1/2} \sim v_*$.

If S gets majority of its mass from EWSB, $v_* \sim v$.

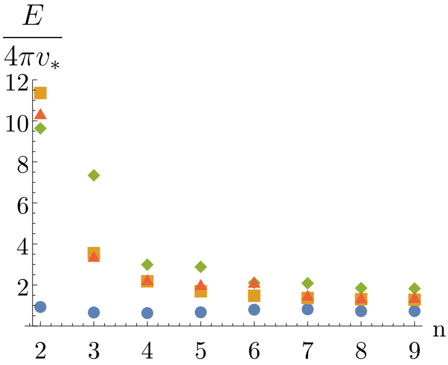
Example: loop-level scalar singlet unitarity cutoff

Unitarity bound

$$E < 4\pi \times \left| \frac{\overline{\partial_h^{n-2}\mathcal{K}_h}}{n!} \right|^{-\frac{1}{n}} \times b_n \times (n!)^{\frac{1}{n}}$$

Strongly coupled: $\overline{\mathcal{K}_h}^{-1/2} \sim v_*$

not SMEFT: $v_* \sim v$



	$\frac{m^2}{v^2}$	$\frac{\kappa}{2}$	$\frac{v_*}{v}$	SMEFT
●	$(4\pi)^2$	$(4\pi)^2$	2.1	?
■	1	$(4\pi)^2$	1.2	✗
◆	$(4\pi)^2$	1	14.	✓
▲	1	1	2.	?

Technical summary

Unitarity bound

$$E < 4\pi \times \left|\frac{\overline{\partial_h^{n-2}\mathcal{K}_h}}{n!}\right|^{-\frac{1}{n}} \times b_n \times (n!)^{\frac{1}{n}}$$

$$= \begin{cases} 8^{\frac{1}{4}}\sqrt{16\pi} \times \left|\overline{\mathcal{K}_h}\right|^{-\frac{1}{2}} & n = 2 \\ 4\pi v_* \times (n!)^{\frac{1}{n}} & n = \text{`a few'} \end{cases}$$

Scalar amplitudes are geometric! We identified parts of HEFT amplitudes $\propto$ derivatives of sectional curvatures at our vacuum.

This allows us to probe locally: how curved our manifold is, and how rapidly this is changing. We derived unitarity bounds sensitive to these two scales.

Manifolds poorly described by SMEFT can't be flat over a large region, leading inexorably to TeV scale unitarity cutoffs.

Which EFT for the LHC?

For any given deviation from SM interactions that we may measure at the LHC, we can always find both a SMEFT operator, and a HEFT operator, to fit it.

The problem is the correlations baked into SMEFT truncated to a given mass dimension order.

We have shown that there are plausible UV theories that can deviate from this pattern of effects in principle.
In practice, could we ever measure a BSM effect that is not fit by a dimension 6 SMEFT operator?

SMEFT vs. HEFT, practically

In SMEFT, electroweak symmetry and decoupling is manifest.[4]

In HEFT, what you see is what you get (e.g. fewer problems with input parameter schemes).

SMEFT is clearly the theorists' choice (and has some theoretical assumptions baked in), whereas HEFT clearly maps onto experiment.

[4] Although it seems that in HEFT, in a specific basis (normal coordinates), decoupling should be manifest as well.

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