g-2 discussion: Lattice HVP

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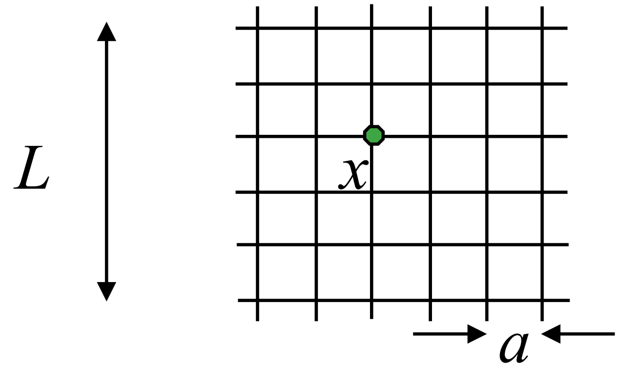
Lattice QCD Introduction

$$\mathcal{L}_{\mathrm{QCD}} = \sum_f \bar\psi_f (\not{D} + m_f)\psi_f + \frac{1}{4}\mathrm{tr} F_{\mu\nu}F^{\mu\nu}$$

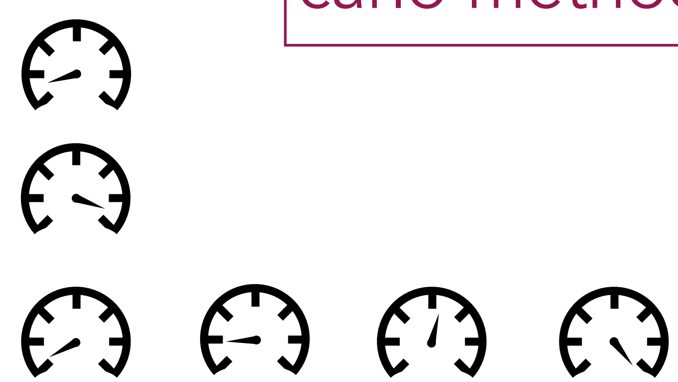


- ✦ discrete Euclidean space-time (spacing a)
  derivatives ➙ difference operators, etc…
- ✦ finite spatial volume (L)
- ✦ finite time extent (T)

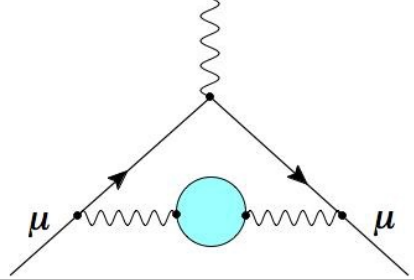
Integrals are evaluated numerically using monte carlo methods.

adjustable parameters

- ❖ lattice spacing: $a \to 0$
- ❖ finite volume, time: $L \to \infty,\ T > L$
- ❖ quark masses (m_f): $M_{H,\mathrm{lat}} = M_{H,\mathrm{exp}}$
  tune using hadron masses
  extrapolations/interpolations $m_f \to m_{f,\mathrm{phys}}$



m_{ud} m_s m_c m_b



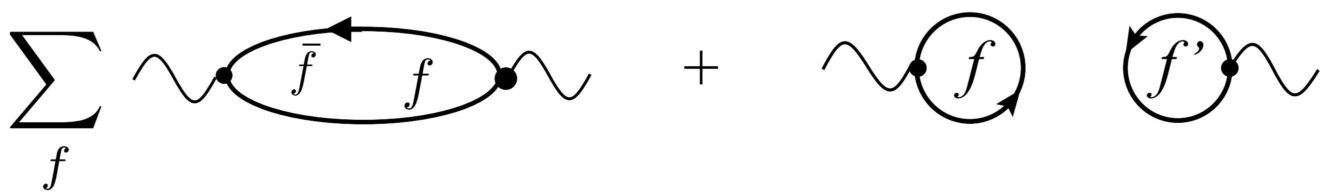
Lattice HVP: Introduction

Calculate a_μ^{HVP} in Lattice QCD:

$$a_\mu^{\mathrm{HVP,LO}} = \sum_f a_{\mu,f}^{\mathrm{HVP,LO}} + a_{\mu,\mathrm{disc}}^{\mathrm{HVP,LO}}$$

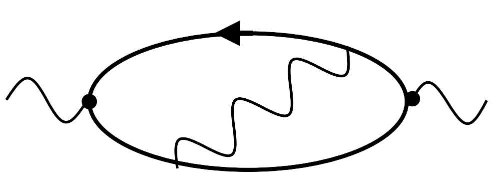
- Separate into connected for each quark flavor + disconnected contributions (gluon and sea-quark background not shown in diagrams)

  Note: almost always $m_u = m_d$



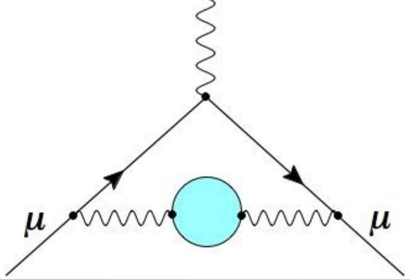
f= ud, s, c, b

- need to add QED and strong isospin breaking ($\sim m_u - m_d$) corrections:



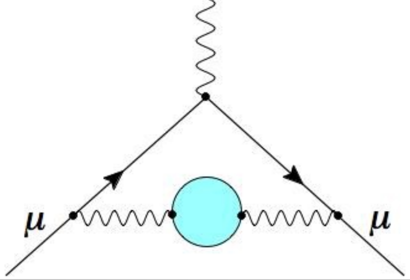
$+ \ldots$

- either perturbatively on isospin symmetric QCD background
- or by using QCD + QED ensembles with $m_u \neq m_d$

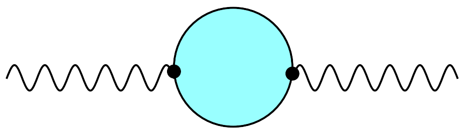


Lattice HVP: Introduction

- light-quark connected contribution:
  ~90% of total
- *s*,*c*,*b*-quark contributions
  ~8%, 2%, 0.05% of total
- disconnected contribution:
  ~2% of total
- Isospinbreaking (QED + $m_u \neq m_d$) corrections:
  ~1% of total



Lattice HVP: Introduction



$\hat{\Pi}(q^2)$

Leading order HVP correction:

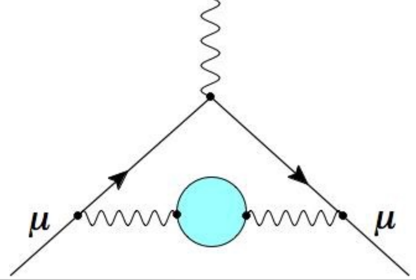
$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2)\,\hat{\Pi}(q^2)$$

- Calculate $a_\mu^{\mathrm{HVP,LO}}$ in Lattice QCD

  Compute correlation function: $C(t) = \frac{1}{3}\sum_{i,x}\langle j_i(x,t) j_i(0,0)\rangle$

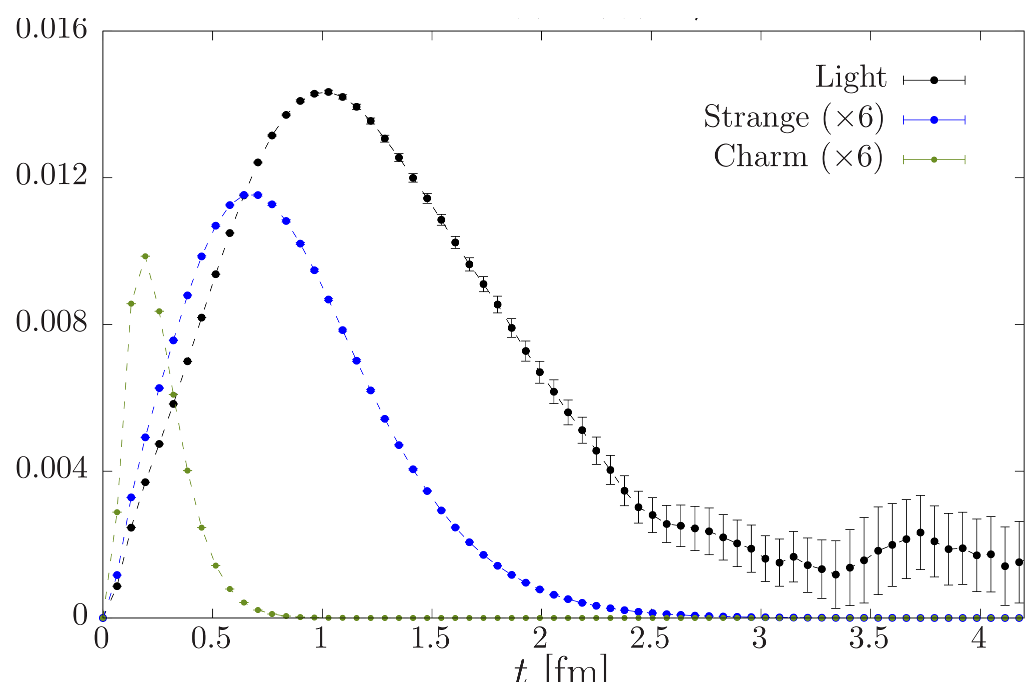
  Obtain $a_\mu^{\mathrm{HVP,LO}}$ from an integral over Euclidean time:

$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\,\tilde{w}(t)\,C(t)$$

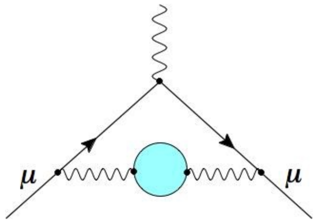


Lattice HVP: Introduction

- Target: < 0.5% total error
- Challenges:
  ✓ needs ensembles with (light sea) quark masses at their physical values
  ✓ finite volume corrections
  - continuum extrapolation
  - include QED and strong isospin breaking corrections ($m_u \neq m_d$)
  - growth of statistical errors at large Euclidean times

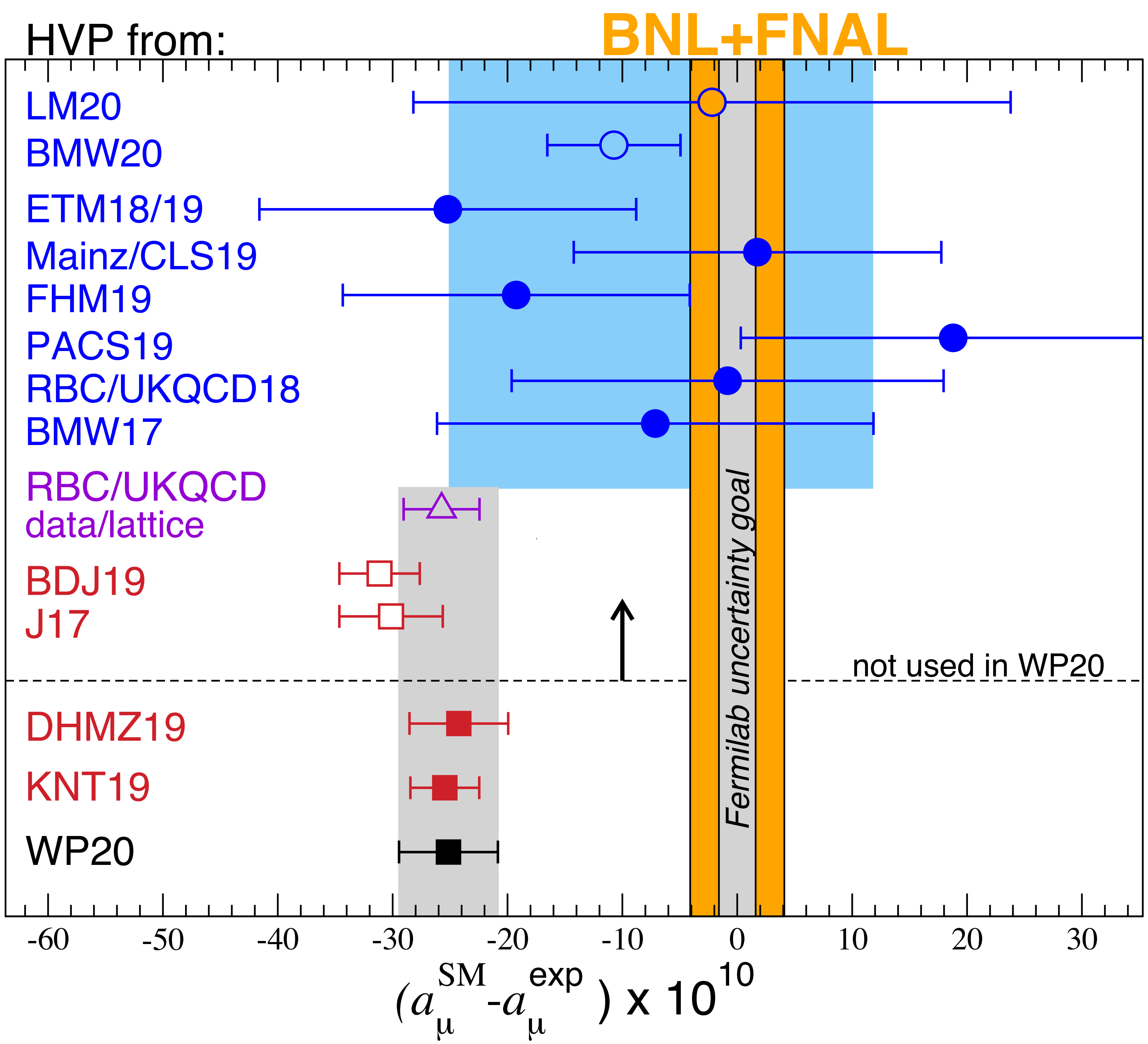


[A. Gerardin et al, PRD 2019]



HVP: Comparison

$$a_\mu^{\rm HVP} + \left[a_\mu^{\rm QED} + a_\mu^{\rm Weak} + a_\mu^{\rm HLbL}\right] \rightarrow a_\mu^{\rm SM}$$



Lattice QCD + QED

(More on consequences of BMW20 result in appendix)

hybrid: combine data & lattice

data driven

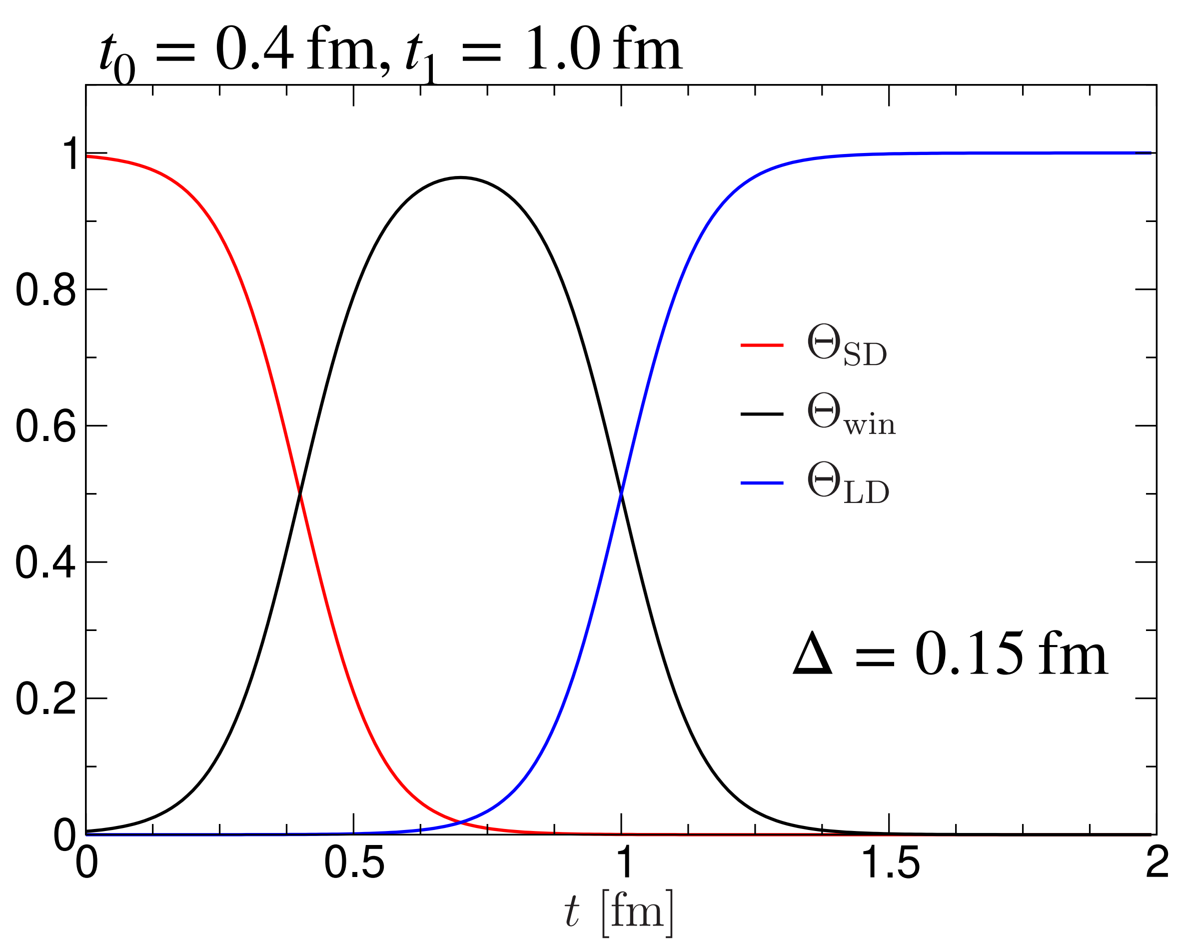
+ unitarity/analyticity constraints

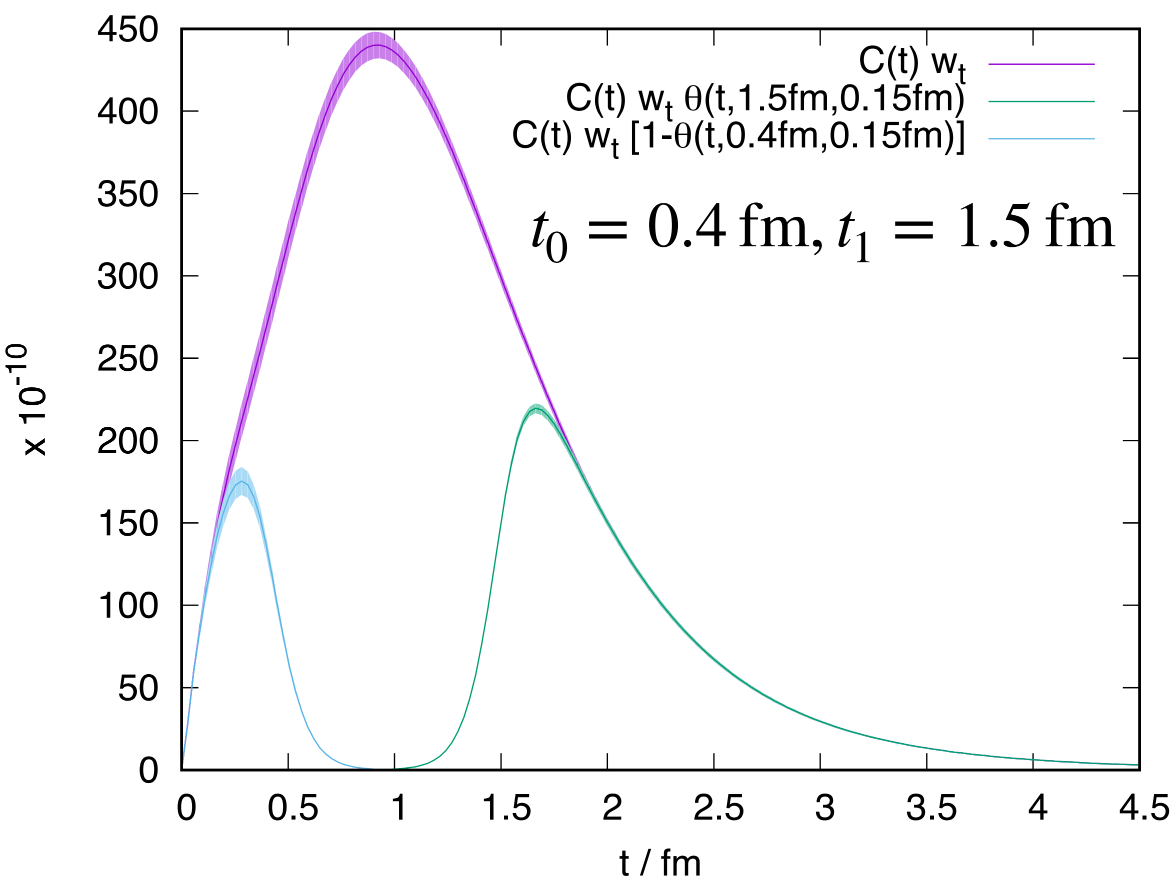
A hybrid method: windows in Euclidean time

Hybrid method: combine LQCD with R-ratio data

[T. Blum et al, arXiv:1801.07224, 2018 PRL]

- Convert R-ratio data to Euclidean correlation function (via the dispersive integral) and compare with lattice results for windows in Euclidean time
- intermediate window:
  expect reduced FV effects and discretization errors





Lattice HVP: Cross Checks

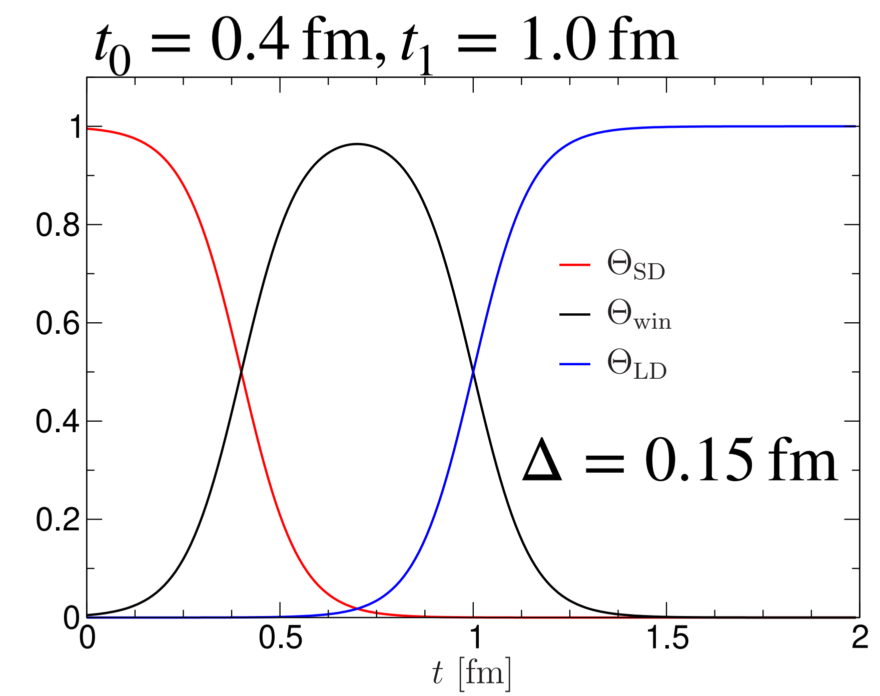
$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{w}(t)\, C(t)$$

- Use windows in Euclidean time to consider the different time regions separately.

Short Distance (SD) $t : 0 \to t_0$

Intermediate (W) $t : t_0 \to t_1$

Long Distance (LD) $t : t_1 \to \infty$



- Compute each window separately (in continuum, infinite volume limits,…) and combine

$$a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$$

Lattice HVP: Cross Checks

H. Wittig @ Lattice HVP workshop

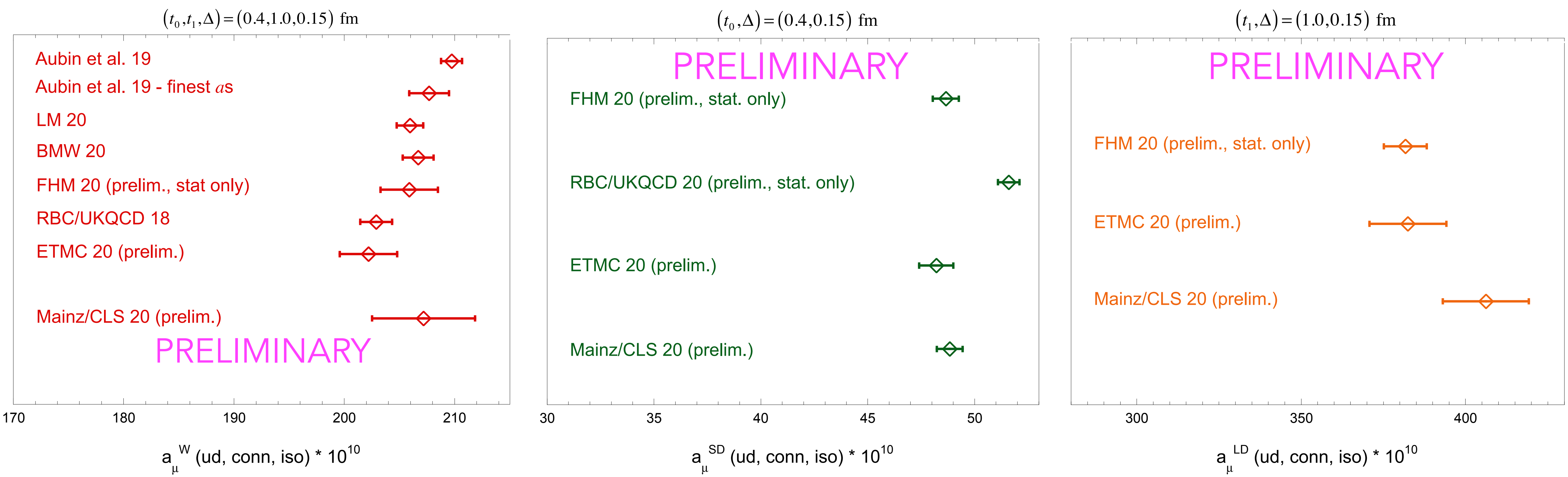
$t_0 = 0.4\,\mathrm{fm}, t_1 = 1.0\,\mathrm{fm}$

$\Delta = 0.15\,\mathrm{fm}$

$$a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$$

"Window" quantities

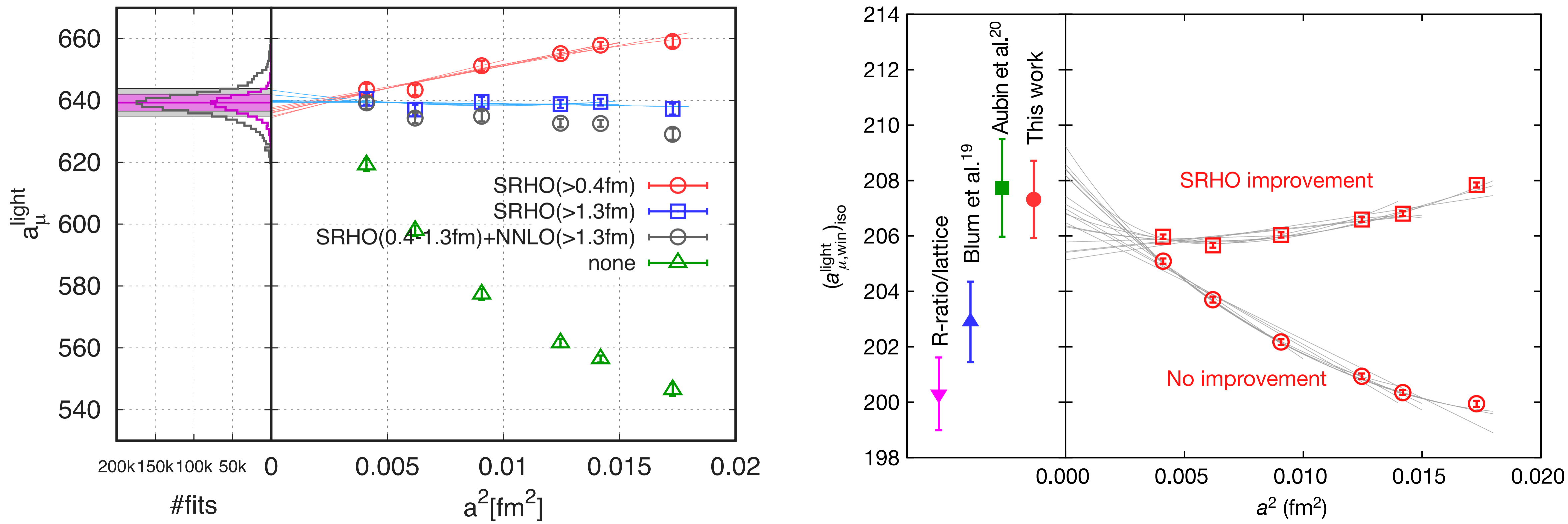
(Plots from Davide Giusti)



- Straightforward reference quantities
- Can be applied to individual contributions (light, strange, charm, disconnected,…)

Lattice HVP: results from BMW

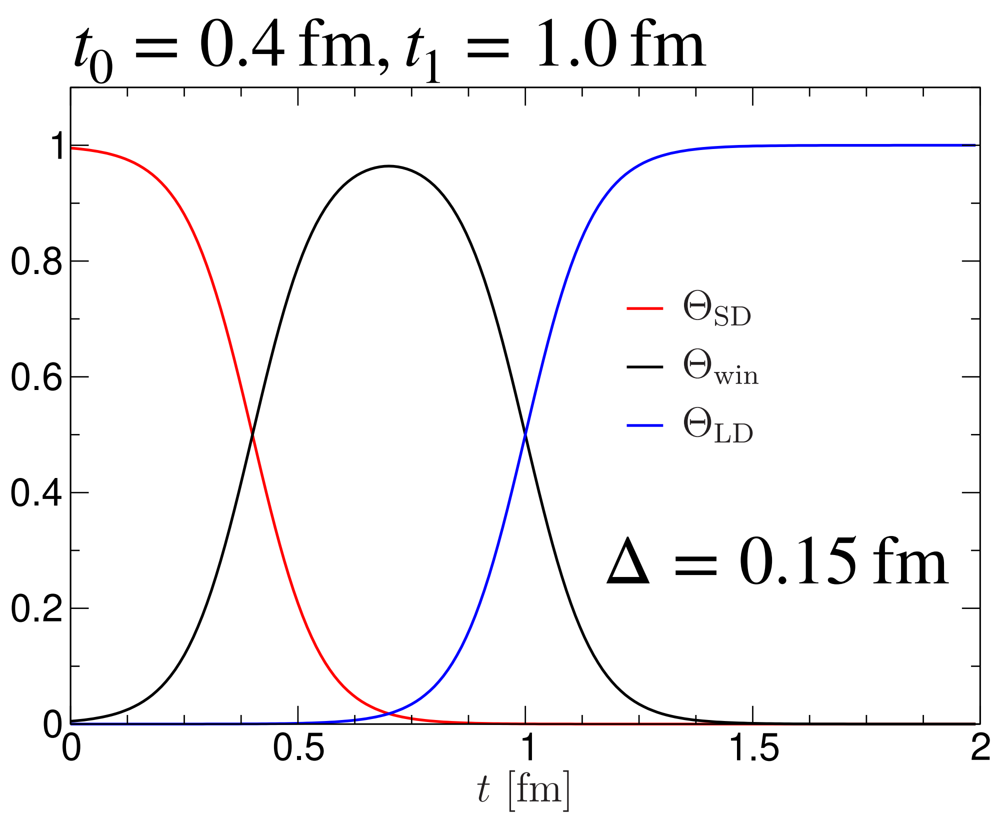
[Borsanyi et al, arXiv:2002.12347, 2021 Nature]

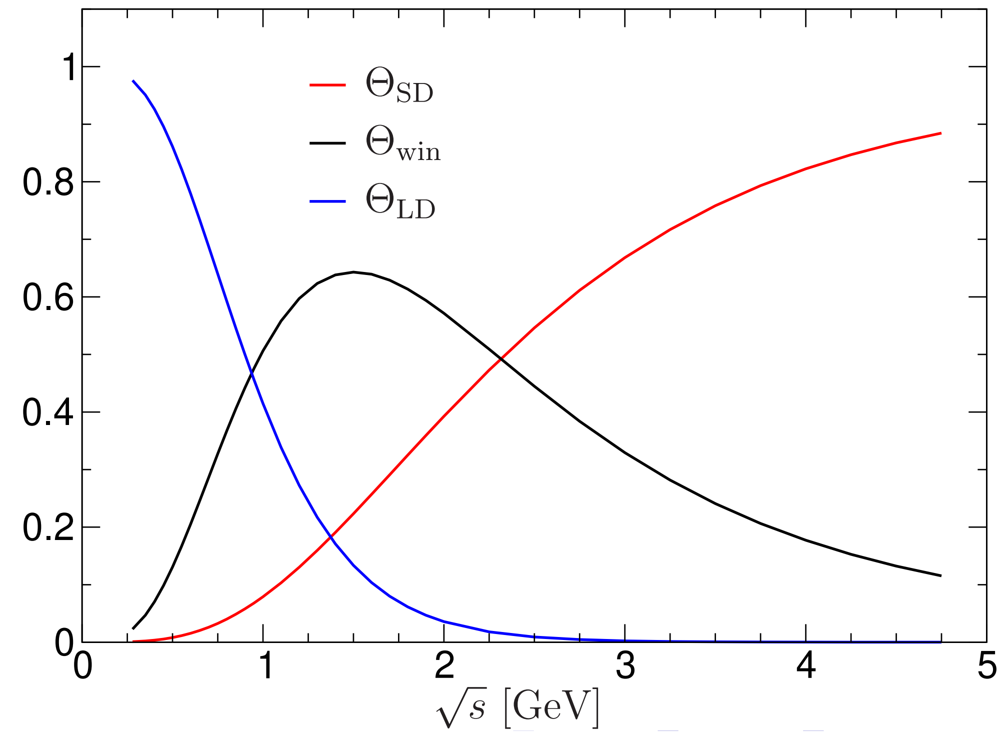


- Small statistical errors and large discretization effects (before corrections)
- Intermediate window a_μ^{W}:
  -3.7 σ tension with data-driven evaluation (KNT)
  -2.2 σ tension with RBC/UKQCD18
- Need to quantify the differences between data-driven evaluations and the BMW results for the various energy/distance scales

Windows: Euclidean time vs $\sqrt{s}$

Martin Hoferichter @ Lattice HVP workshop





$[t_0, t_1]$ intermediate window	percentage captured of $\pi\pi$ channel $\leq$ 1 GeV		
	SD	intermediate	LD
[0.4, 1.0] fm	3	28	69
[1.0, 2.0] fm	31	51	18
[1.0, 2.5] fm	31	61	9
[1.0, 3.0] fm	31	65	4

SD: $[0, t_0]$
LD: $[t_1, \infty]$
intermediate: $[t_0, t_1]$

For intermediate window:
~30% from $\sigma(\pi\pi) \lesssim 1\,\text{GeV}$