The BSM Orders of CPV

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Outline

The collective nature of CPV: Real vs. Imaginary

The (flavour-)invariant measures of CPV

Beyond Jarlskog: the 699 (minimal) invariants of SMEFT_6

The BSM/SMEFT sources of CPV and their sizes

Models of flavour: MFV, Alignment/Froggatt-Nielsen

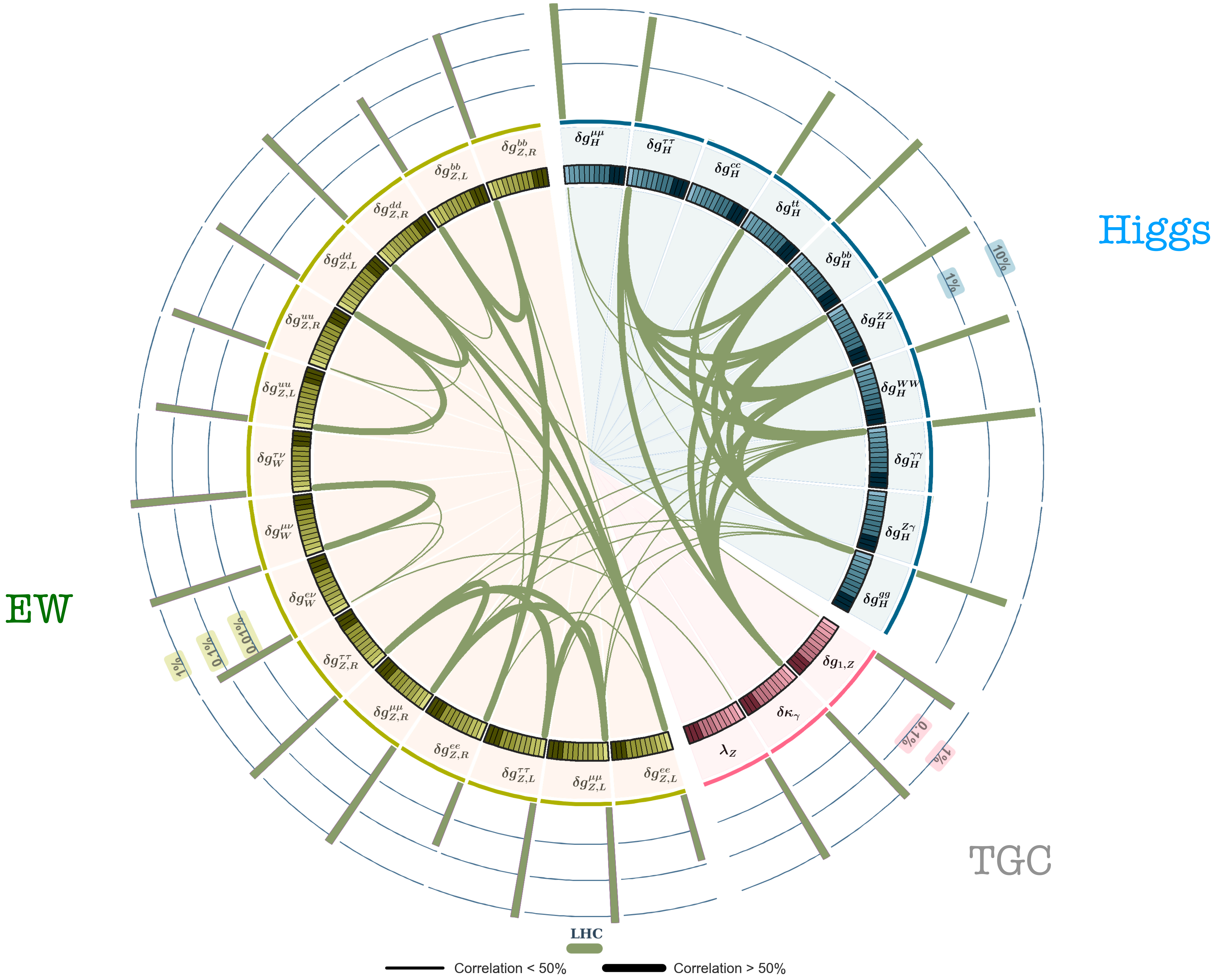
Connections with UV?

Note 1: I’ll consider only heavy/decoupling new physics
Note 2: I’ll assume that SU(2)xU(1) is linearly realised above the weak scale, i.e. SMEFT rather than HEFT. Our construction can be generalised but we haven’t gone through this exercise (yet). I’ll also assume that possible B and L violating effects are pushed to high scale irrelevant for our discussion.

SM: Once, Now, Future

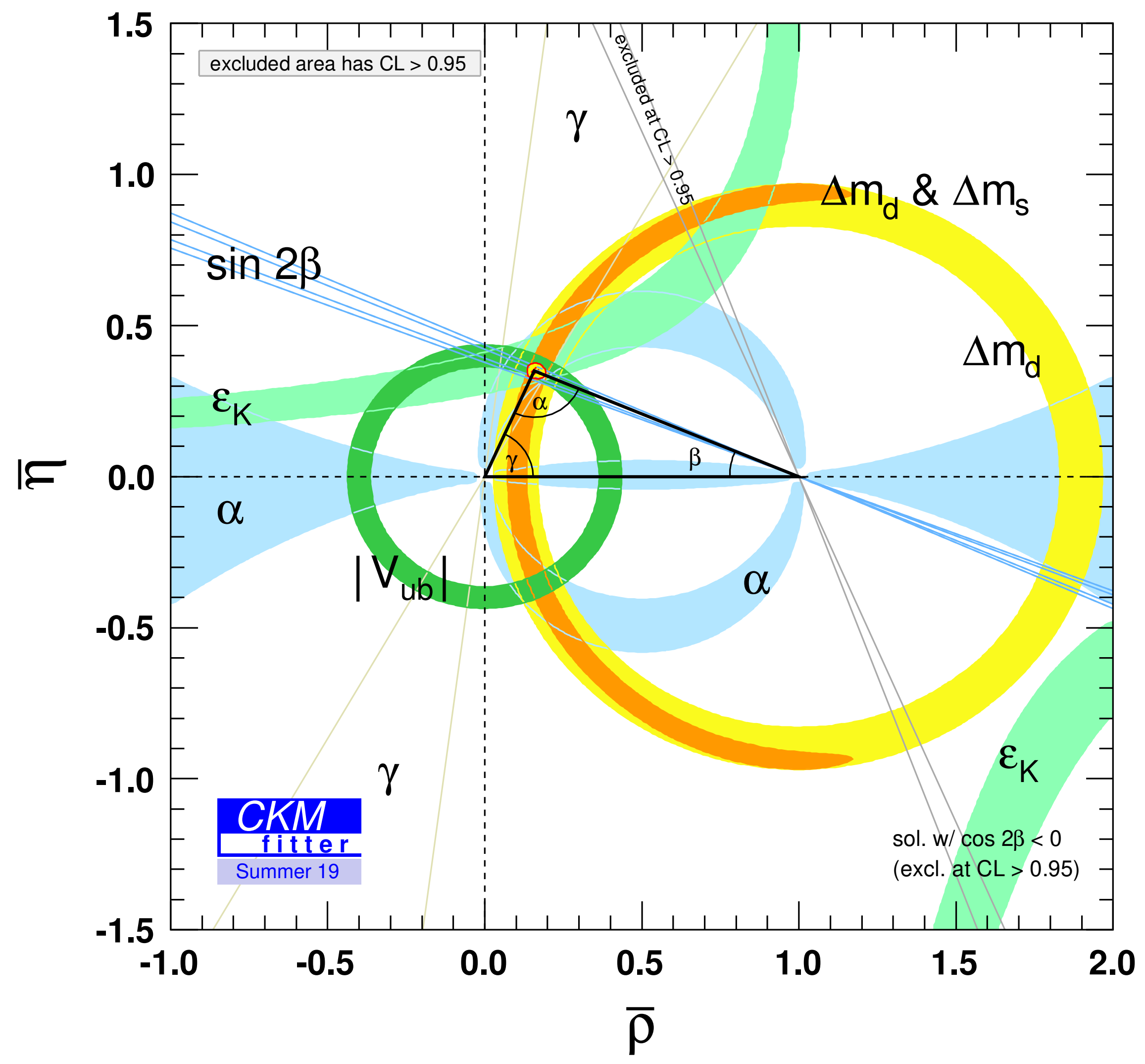
The high-pT perspective

EW known at 0.1%, TGC known at 1%, Higgs known at 10%



Many thanks to J. De Blas et al. (HEPfit) for the analysis of current data (work in progress) and to A. Paul for plotting the results

The flavour perspective



CKM picture works amazingly well
but still many anomalies

Where is New Physics?
What is its structure/symmetry?

Which Selection Rules/Symmetries?

The EFT parametrisation of New Physics

Examples of symmetries leading to different selection rules

Operator	Naive (maximal) scaling with g_*	Symmetry/Selection Rule and corresponding suppression
$O_{y_\psi} = \|H\|^2 \bar{\psi}_L H \psi_R$	g_*^3	Chiral: y_f/g_*
$O_T = (1/2)\left(H^\dagger \overleftrightarrow{D}_\mu H\right)^2$	g_*^2	Custodial: $(g'/g_*)^2$, $y_t^2/16\pi^2$
$O_{GG} = \|H\|^2 G^a_{\mu\nu} G^{a\,\mu\nu}$ $O_{BB} = \|H\|^2 B_{\mu\nu} B^{\mu\nu}$	g_*^2	Shift symmetry: $(y_t/g_*)^2$ Elementary Vectors: $(g_s/g_*)^2$ (for O_{GG}) $(g'/g_*)^2$ (for O_{BB}) Minimal Coupling: $g_*^2/16\pi^2$
$O_6 = \|H\|^6$	g_*^4	Shift symmetry: λ/g_*^2
$O_H = (1/2)(\partial^\mu \|H\|^2)^2$	g_*^2	Coset Curvature: ϵ_c
$O_B = (i/2)\left(H^\dagger \overleftrightarrow{D}^\mu H\right)\partial^\nu B_{\mu\nu}$ $O_W = (i/2)\left(H^\dagger \sigma^a \overleftrightarrow{D}^\mu H\right)\partial^\nu W^a_{\mu\nu}$	g_*	Elementary Vectors: g'/g_* (for O_B) g/g_* (for O_W)
$O_{HB} = (i/2)\left(D^\mu H^\dagger D^\nu H\right) B_{\mu\nu}$ $O_{HW} = (i/2)\left(D^\mu H^\dagger \sigma^a D^\nu H\right) W^a_{\mu\nu}$	g_*	Elementary Vectors: g'/g_* (for O_{HB}) g/g_* (for O_{HW}) Minimal Coupling: $g_*^2/16\pi^2$

Contino, Falkowski, Goertz, Grojean, Riva '16

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \sum_i \frac{c_i^{(6)}}{\Lambda^2}\mathcal{O}_i^{(6)} + \sum_j \frac{c_j^{(8)}}{\Lambda^4}\mathcal{O}_j^{(8)} + \cdots$$

Dimensional arguments impose

$$c_i^{(D)} \sim (\text{coupling})^{n_i - 2}$$

n_i=number of fields in operator $\mathcal{O}_i^{(D)}$ (independant of D)

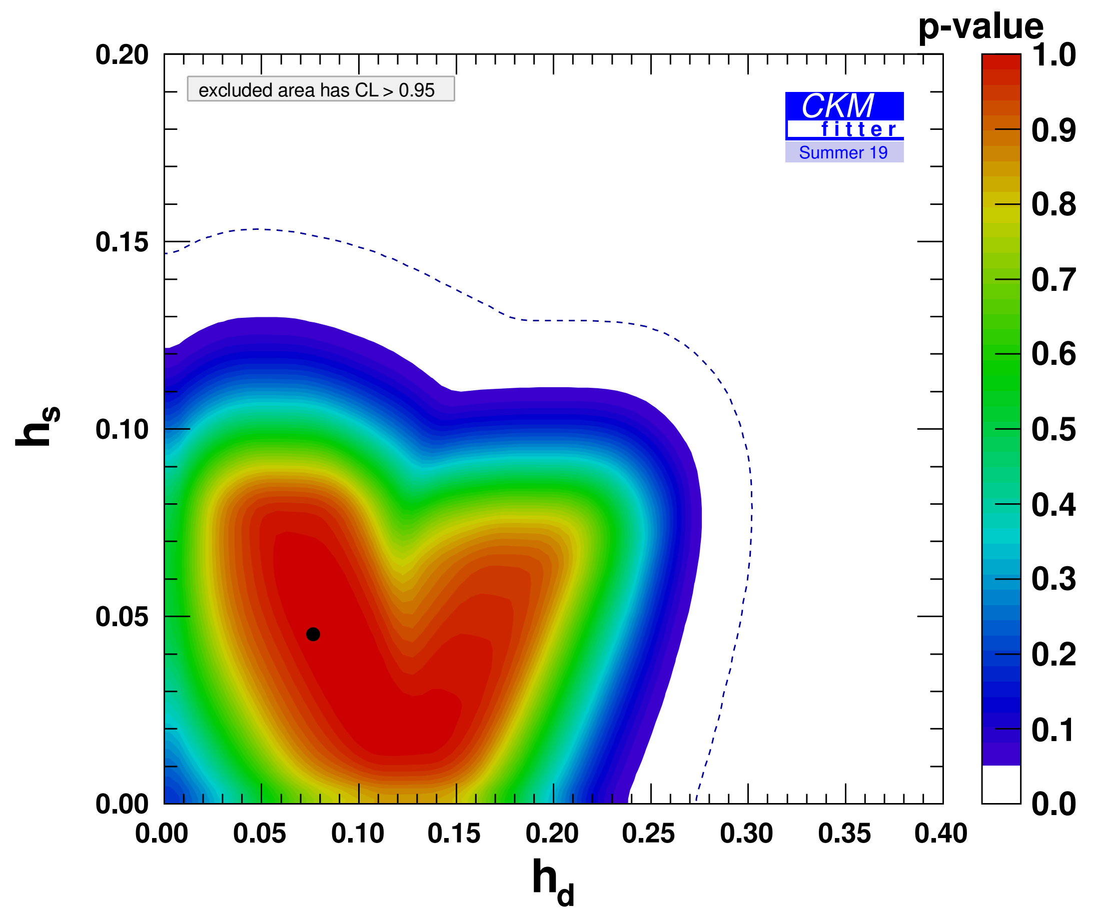
generically, coupling ~ g_* (coupling of New Physics to SM)
but there might exist **"selection rules"**
that lead to other scaling

**Does BSM respect B, L, custodial sym. , $SU(3)^5_{\text{flavour}}$, CP?
Is there any additional symmetry?**

What about CPV?

Does new physics break CP?

- Unlike B,L, CP is not an accidental symmetry of SM4
- But its violation is “screened” by the CKM selection rule (see next slides)
- As advertised by Z. Ligeti last week: BSM CPV effects can be O(1) in most loop-level FCNC processes



$$h\, e^{2i\sigma} = A_{\mathrm{NP}}(B^0 \to \overline{B}^0)/A_{\mathrm{SM}}(B^0 \to \overline{B}^0)$$

NP parameters

$$\frac{C_{ij}^2}{\Lambda^2}\left(\bar{q}_{i,L}\gamma_\mu q_{j,L}\right)^2 \;\Rightarrow\; h \simeq \frac{|C_{ij}|^2}{|V_{ti}^* V_{tj}|^2}\left(\frac{4.5\,\mathrm{TeV}}{\Lambda}\right)^2$$

Couplings	NP loop order	Sensitivity for Summer 2019 [TeV]		Phase I Sensitivity [TeV]		Phase II Sensitivity [TeV]	
		B_d mixing	B_s mixing	B_d mixing	B_s mixing	B_d mixing	B_s mixing
$\lvert C_{ij}\rvert = \lvert V_{ti}V_{tj}^*\rvert$	tree level	9	13	17	18	20	21
(CKM-like)	one loop	0.7	1.0	1.3	1.4	1.6	1.7
$\lvert C_{ij}\rvert = 1$	tree level	1×10^3	3×10^2	2×10^3	4×10^2	2×10^3	5×10^2
(no hierarchy)	one loop	80	20	2×10^2	30	2×10^2	40

Charles et al. ‘20

- On the other hand, there are already strong (indirect) constraints, e.g., EDM
- We need a map to explore CPV effect: What are the BSM sources of CPV and what could be their sizes?

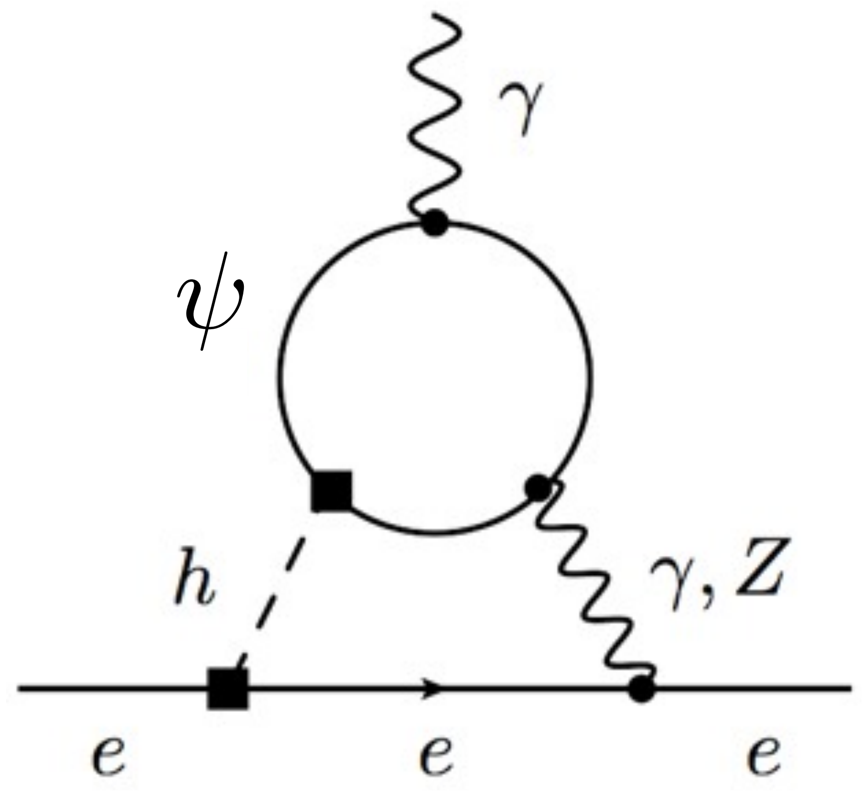
CPV is a Collective Effect

The example of electron EDM

- "Imaginary" Yukawa coupling gives rise to eEDM through Barr-Zee diagram

Brod, Haisch, Zupan '13

$$\mathcal{L} = y\, h\bar{\psi}\psi$$



- The Yukawa can be made real by chiral rotation: $\psi \to e^{i\theta\gamma^5}\psi$
- The "phase" will appear in the mass
- The CPV effect is captured by Im ($y^{\dagger}\cdot m$), which is invariant under chiral rotation

Trivial here, but can get complicated: flavour indices, links to UV parameters…

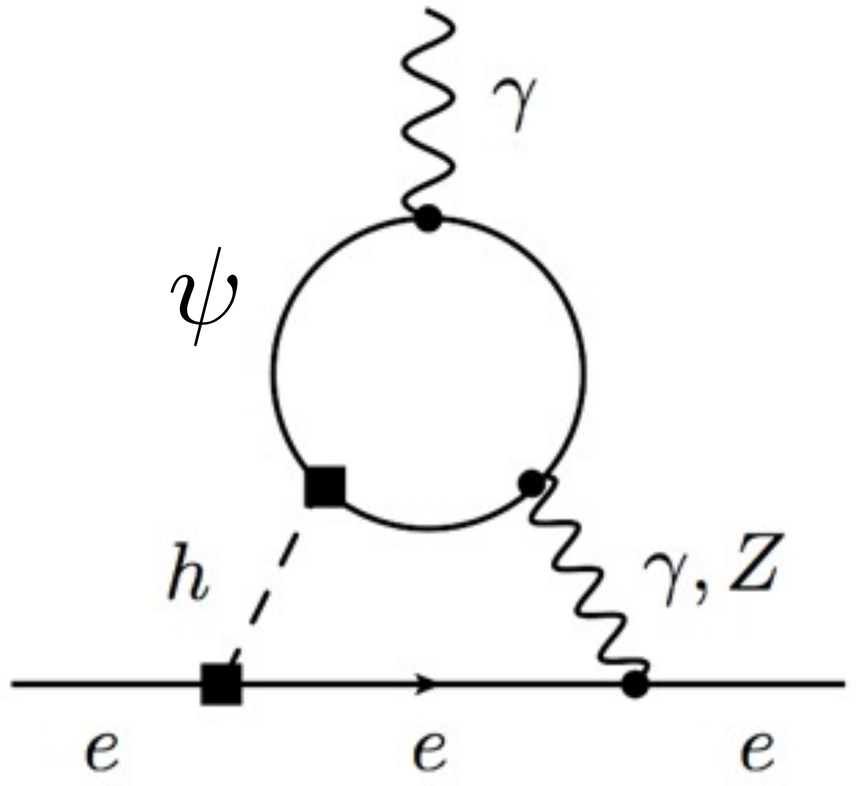
Dim.6 Yukawa's Contribution to EDMs

CP doesn't say Wilson coefficients are real

$$\mathcal{L} = \underbrace{Y_u}_{\text{3x3 complex}} \bar{Q}\tilde{H}U + \underbrace{C_{uH}}_{\text{3x3 complex}} |H|^2 \bar{Q}\tilde{H}U \quad \Rightarrow \quad \underbrace{g^{ij}_{huu}}_{Y_u^{ij} + 3v^2 C_{uH}^{ij}} h\bar{u}_i u_j$$

One can choose $U(3)_Q \times U(3)_U$ transformations to make C_{uH} or g_{huu} *real*
so CPV effects cannot simply be sourced by Im C_{uH}
Phases can be moved to mass matrices. Even in mass basis, residual U(1)'s to move phase around.

At two loops and $1/\Lambda^2$ order, Barr-Zee diagrams depends only on three phases captured by three invariants



$$\frac{d_e}{e} \propto \frac{\alpha y_e}{16\pi^3}\left(a\, I_1 + b\, I_2 + c\, I_3\right)$$

with

$$I_n = \text{Im Tr}\left(Y_u^\dagger \left(Y_u Y_u^\dagger\right)^n C_{uH}\right)$$

a, b, c functions of Y_u only

At higher loops, more phases can appear.

- How many?
- How many constraints should we impose to ensure CP is conserved?

The SM$_4$ Collective CPV

The well-known KM counting

$Y_u\ (9R+9I)$
$Y_d\ (9R+9I)$ →

	$SU(3)_Q$	$SU(3)_u$	$SU(3)_d$	$U(1)_u$	$U(1)_d$	$U(1)_B$
Y_u	3	$\bar{3}$	1	1	0	0
Y_d	3	1	$\bar{3}$	0	1	0
	$3R{+}5I$	$3R{+}5I$	$3R{+}5I$	$1I$	$1I$	~~$1I$~~ X

→ physical $9R+1I$

- The position of this physical phase is (flavour)-basis dependent, e.g.
  - Up-basis: Yu=diag, Yd= $V_{\rm CKM}$.diag
  - Down-basis: Yu=$V_{\rm CKM}^\dagger$.diag, Yd=diag
  - many other choices of flavour bases

$$V_{\rm CKM}=\begin{pmatrix}1&0&0\\0&c_{23}&s_{23}\\0&-s_{23}&c_{23}\end{pmatrix}\begin{pmatrix}c_{13}&0&s_{13}e^{-i\delta}\\0&1&0\\-s_{13}e^{i\delta}&0&c_{13}\end{pmatrix}\begin{pmatrix}c_{12}&s_{12}&0\\-s_{12}&c_{12}&0\\0&0&1\end{pmatrix}$$

Jarlskog Invariant

The SM CPV order

- The lowest order flavour invariant sensitive to CPV

$$J_4 = \mathrm{Im}\mathrm{Tr}\left([Y_u^\dagger Y_u, Y_d^\dagger Y_d]^3\right)$$

- Explicitly

$$J_4 = 6\underbrace{c_{12}s_{12}c_{13}s_{13}c_{23}s_{23}}_{\mathcal{O}\left(\lambda^6\right)}\underbrace{\left(y_c^2 - y_u^2\right)\left(y_t^2 - y_u^2\right)\left(y_t^2 - y_c^2\right)\left(y_s^2 - y_d^2\right)\left(y_b^2 - y_d^2\right)\left(y_b^2 - y_s^2\right)}_{\mathcal{O}\left(\lambda^{30}\right)}\underbrace{\sin\delta}_{\mathcal{O}\left(\lambda^0\right)}$$

Wolfenstein parametrisation $$V_{\mathrm{CKM}} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

- Even if δ~O(1), large suppression effects due to collective nature of CPV
- Important property: CP is conserved iff J_4=0 (neglecting θ_{QCD} for now)

Beyond Jarlskog: Building SM_6 invariants

Playing with new fermion bilinear interactions first

- In the Warsaw basis, Manohar et al. counted 7 Hermitian and 8 generic bilinear operators for a total of 129 phases (and 164 real parameters)

$5: \psi^2 H^3 + \text{h.c.}$	
Q_{eH}	$(H^\dagger H)(\bar{l}_p e_r H)$
Q_{uH}	$(H^\dagger H)(\bar{q}_p u_r \widetilde{H})$
Q_{dH}	$(H^\dagger H)(\bar{q}_p d_r H)$

$6: \psi^2 XH + \text{h.c.}$
$Q_{eW},\, Q_{eB}$
$Q_{uG}\ ,\, Q_{uW},\, Q_{uB}$
$Q_{dG},\, Q_{dW},\, Q_{dB}$

$7: \psi^2 H^2 D$	
$Q_{Hl}^{(1)},\, Q_{Hl}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{l}_p \gamma^\mu l_r)$, $(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{l}_p \tau^I \gamma^\mu l_r)$
Q_{He}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$
$Q_{Hq}^{(1)},\, Q_{Hq}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{q}_p \gamma^\mu q_r)$, $(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{q}_p \tau^I \gamma^\mu q_r)$
Q_{Hu}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}_p \gamma^\mu u_r)$
Q_{Hd}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}_p \gamma^\mu d_r)$
Q_{Hud}	$i(\widetilde{H}^\dagger D_\mu H)(\bar{u}_p \gamma^\mu d_r)$

$SU(3)_Q$	$SU(3)_u$	$SU(3)_d$	$SU(3)_L$	$SU(3)_e$
1	1	1	3	$\bar{3}$
3	$\bar{3}$	1	1	1
3	1	$\bar{3}$	1	1
1	1	1	$8+1$	1
1	1	1	1	$8+1$
$8+1$	1	1	1	1
1	$8+1$	1	1	1
1	1	$8+1$	1	1
1	3	$\bar{3}$	1	1

- In the limit m_ν=0, lepton numbers in each family are conserved. The WC not invariant under these U(1)'s can never show up at linear order in any amplitude: 129 → 102 phases (and 164 → 137 real parameters)

Beyond Jarlskog: Building SM_6 invariants

Examples of invariants from with bilinear operators

- For each operators, e.g. the dim.6 Yukawa operators, we can build a series of CP-odd invariants:

$$I_{u_1 \dots d_k} = \mathrm{Im}\ \mathrm{Tr}\left(Y_u^\dagger \left(Y_u Y_u^\dagger\right)^{u_1} \left(Y_d Y_d^\dagger\right)^{d_1} \dots \left(Y_u Y u^\dagger\right)^{u_k} \left(Y_d Y_d^\dagger\right)^{d_k} C_{uH}\right)$$

- Of course, they are not all independent:

e.g., for 3 families, $I_3 = \mathrm{Tr}\left(Y_u Y_u^\dagger\right) I_2 + \frac{1}{2}\left(\mathrm{Tr}\left(\left(Y_u Y_u^\dagger\right)^2\right) - \mathrm{Tr}^2\left(Y_u Y_u^\dagger\right)\right) I_1$

- If J_4=0, we can find 9 independent invariants ⇒ **minimal** basis of invariants.

"CP is conserved iff J4 and the invariants of a minimal basis are all vanishing"

- If J_4≠0, we can actually build 18 independent invariants! Not surprising, because CP-even BSM can interfere with CP-odd SM. But what was maybe unexpected is that we can build more than 9 (independent) invariants that are larger than J_4 → **maximal** basis of invariants.

Scaling of Collective CPV BSM Effects

The new invariants don't suffer from the same suppression factors

- The invariants can be evaluated in e.g. the up-flavour basis:

dim.6 up-Yukawa operator

$$I_n = \underbrace{y_u^{2n+1}}_{\mathcal{O}(\lambda^{16n+8})}\eta_u + \underbrace{y_c^{2n+1}}_{\mathcal{O}(\lambda^{8n+4})}\eta_c + \underbrace{y_t^{2n+1}}_{\mathcal{O}(\lambda^{0})}\eta_t$$

$$I_{1,1} = \underbrace{c_{13}c_{23}s_{13}s_\delta}_{\mathcal{O}(\lambda^{3})}\underbrace{\left(y_b^2 - c_{12}^2 y_d^2 - s_{12}^2 y_s^2\right) y_t}_{\mathcal{O}(\lambda^{6})}\,\rho_{ut} + \ldots$$

- You can actually build17 independent invariants larger than J_4 (the 18th is $O(J_4)$):

Minimal basis

$$\text{Rank } 1 \to \mathcal{O}(\lambda^0)$$
$$\text{Rank } 2 \to \mathcal{O}(\lambda^4)$$
$$\text{Rank } 3 \to \mathcal{O}(\lambda^8)$$

Maximal basis

$$\text{Rank } 5 \to \mathcal{O}(\lambda^{10}) \quad \text{Rank } 11 \to \mathcal{O}(\lambda^{18})$$
$$\text{Rank } 6 \to \mathcal{O}(\lambda^{12}) \quad \text{Rank } 14 \to \mathcal{O}(\lambda^{20})$$
$$\text{Rank } 9 \to \mathcal{O}(\lambda^{14}) \quad \text{Rank } 15 \to \mathcal{O}(\lambda^{22})$$
$$\text{Rank } 10 \to \mathcal{O}(\lambda^{16}) \quad \text{Rank } 17 \to \mathcal{O}(\lambda^{24})$$
$$\text{Rank } 18 \to \mathcal{O}(\lambda^{36})$$

17 (resp. 14) possible new sources of CPV larger than J_4 as long as Λ<6TeV (resp. Λ<100TeV)

Models of Flavours

MFV, first

- Other constraints from CP-even observables: totally flavour generic/anarchic dim-6 operators are severely constrained. How additional flavour structure will affect the orders of CPV computed above in the generic case?

- Let's first stick to the canonical flavour "model": Minimal Flavour Violation

$$C_{uH} = aY_u + b\left(Y_u Y_u^\dagger\right) Y_u + c\left(Y_d Y_d^\dagger\right) Y_u + \ldots$$

Generic Flavour		MFV
$\text{Rank } 1 \to \mathcal{O}\left(\lambda^0\right)$		$\text{Rank } 1 \to \mathcal{O}\left(\lambda^0\right)$
$\text{Rank } 2 \to \mathcal{O}\left(\lambda^4\right)$	➡	$\text{Rank } 2 \to \mathcal{O}\left(\lambda^8\right)$
$\text{Rank } 3 \to \mathcal{O}\left(\lambda^8\right)$		$\text{Rank } 3 \to \mathcal{O}\left(\lambda^{18}\right)$

CPV Orders in Alignment Models

Froggatt-Nielsen-type Flavour Structure

- Another popular flavour structure is alignment inherited e.g. from $U(1)_{FN}$ symmetry

- The U(1) charges of the quarks will imprint a particular scaling of the dim.6 WC:

$$Yu = \begin{pmatrix} \lambda^8 & \lambda^5 & \lambda^3 \\ \lambda^7 & \lambda^4 & \lambda^2 \\ \lambda^5 & \lambda^2 & 1 \end{pmatrix} \quad Yd = \begin{pmatrix} \lambda^7 & \lambda^6 & \lambda^6 \\ \lambda^6 & \lambda^5 & \lambda^5 \\ \lambda^4 & \lambda^3 & \lambda^3 \end{pmatrix} \quad C_{uH} = \text{generic} = \begin{pmatrix} \lambda^8 & \lambda^5 & \lambda^3 \\ \lambda^7 & \lambda^4 & \lambda^2 \\ \lambda^5 & \lambda^2 & 1 \end{pmatrix}$$

- For the dim.6 up-Yukawa operator, the scaling of the invariants and the rank structure remain unchanged. But for other operators, e.g. dim.6 down-Yukawa, the invariants get more suppressed:

		Generic		FN
$Yd^\dagger . C_{dH}$	$\propto$	$\mathcal{O}(\lambda^3)$	→	$\mathcal{O}(\lambda^6)$
$Yd^\dagger . YuYu^\dagger . C_{dH}$		$\mathcal{O}(\lambda^3)$		$\mathcal{O}(\lambda^6)$
$Yd^\dagger . YuYu^{\dagger 2} . C_{dH}$		$\mathcal{O}(\lambda^3)$		$\mathcal{O}(\lambda^6)$
$Yd^\dagger . YdYd^\dagger . C_{dH}$		$\mathcal{O}(\lambda^9)$		$\mathcal{O}(\lambda^{12})$
$Yd^\dagger . YuYu^\dagger . YdYd^\dagger . C_{dH}$		$\mathcal{O}(\lambda^9)$		$\mathcal{O}(\lambda^{12})$

```
Rank 1 -> O(λ^6)
Rank 3 -> O(λ^10)
Rank 4 -> O(λ^12)
Rank 7 -> O(λ^16)
Rank 8 -> O(λ^18)
Rank 10 -> O(λ^20)
Rank 12 -> O(λ^22)
Rank 13 -> O(λ^24)
Rank 16 -> O(λ^26)
Rank 17 -> O(λ^28)
Rank 18 -> O(λ^36)
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Still 17 larger-than-J_4 invariants

Beyond Jarlskog: 4-Fermi operators

A total of 700 (fermionic) BSM CPV minimal parameters

- In the Warsaw basis, Manohar et al. also counted the free-parameters in 4F operators: 1014 phases. As before, not all these phases can show up at leading order when the neutrino masses are taken to vanish: only 597 survive (adding to the 102 bilinear ones and J_4 for a total of 700 phases)

e.g. $C_{QuQd}\,\bar{Q}u\bar{Q}d$

$SU(3)_Q$	$SU(3)_u$	$SU(3)_d$
$1+3+6$	$\bar{3}$	$\bar{3}$

- One can build two types of 4F-invariants out of the bilinear invariants:

A-type

$$\mathrm{Im}\left(\underbrace{M^{uH}_{ij}}\underbrace{M^{dH}_{kl}}C^{QuQd}_{ijkl}\right)$$

B-type

$$\mathrm{Im}\left(\underbrace{M^{dH}_{il}}\underbrace{M^{uH\dagger}_{jk}}C^{QuQd}_{ijkl}\right)$$

matrices built out of Yu and Yd that to form bilinear invariants, e.g., $\mathrm{Im}\ \mathrm{Tr}\left(M^{uH}C_{uH}\right)$

Beyond Jarlskog: 4-Fermi operators

More invariants: minimal and maximal bases

- As for the bilinears, one can construct a minimal basis of invariants:

“CP is conserved iff J4 and the invariants of a minimal basis are all vanishing”

- The dimension of the **minimal** basis is always equal to the number of phases associated to an operator: QQQQ → 18, QuQd → 81, LLuu → 36/9 (w/wo neutrino masses) …

- But the real coefficients also contribute to CPV: the dimension of the **maximal** basis is equal to the total number of parameters associated to an operator: QQQQ → 45, QuQd → 162, LLuu → 81/27 (w/wo neutrino masses) …

Theta QCD

Can we build new invariants using Θ_{QCD}?

	$SU(3)_{Q_L}$	$U(1)_{Q_L}$	$SU(3)_{u_R}$	$U(1)_{u_R}$	$SU(3)_{d_R}$	$U(1)_{d_R}$
Q_L	$\mathbf{3}$	1	$\mathbf{1}$	0	$\mathbf{1}$	0
u_R	$\mathbf{1}$	0	$\mathbf{3}$	1	$\mathbf{1}$	0
d_R	$\mathbf{1}$	0	$\mathbf{1}$	0	$\mathbf{3}$	1
Y_u	$\mathbf{3}$	1	$\bar{\mathbf{3}}$	-1	$\mathbf{1}$	0
Y_d	$\mathbf{3}$	1	$\mathbf{1}$	0	$\bar{\mathbf{3}}$	-1
$e^{i\theta_{QCD}}$	$\mathbf{1}$	6	$\mathbf{1}$	-3	$\mathbf{1}$	-3

- Given that $\bar{\theta} = \theta - \arg\det(Y_u Y_d)$ in a flavour invariant, no new SM invariant can be constructed
- In SM6, in principle, new structure can emerge

$$\mathrm{Im}\left(e^{-i\theta_{QCD}}\epsilon^{ABC}\epsilon^{abc}Y_{u,Aa}Y_{u,Bb}C_{uH,Cc}\det Y_d\right)$$

- Probably highly suppressed in the perturbative regime of QCD ($e^{-8\pi^2/g_s^2} \sim \lambda^{37}$)
- Relevant at low scale?

Conclusions

EDM constraints don't exclude all sources of CPV

- CPV is a collective effect.
- CPV is accidentally suppressed in SM_4.
- Many new possible new sources of CPV at dim.6 level.
- Minimal basis of invariants: conditions for CP to be preserved (at $1/\Lambda^2$ order): → 699 (when $m_\nu=0$) → 48 (when $Y_d \to 0$) invariants (and J_4) have to vanish.
- Maximal basis of invariants: proper/flavour-basis independent parametrisation of the sources of CPV: many more than 699 independent invariants can be larger than J_4. Conversely, the "phases" can be appear in CP-even observables in a way that is not J_4-suppressed.