Some Geometric Stuff.

Key collaborators on these developments:

A. Helset

T. Corbett

A. Martin

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DET FRIE FORSKNINGSRÅD
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Consequences of the Higgs field becoming a number

The Higgs field takes on a vev, recall what happens:

$$\mathcal{L}_{\rm SM} = -\frac{1}{4} G^A_{\mu\nu} G^{A\mu\nu} - \frac{1}{4} W^I_{\mu\nu} W^{I\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + (D_\mu H^\dagger)(D^\mu H) + \sum_{\psi=q,u,d,l,e} \overline{\psi}\, i \not{D}\, \psi$$
$$- \lambda \left(H^\dagger H - \frac{1}{2} v^2 \right)^2 - \left[H^{\dagger j} \overline{d}\, Y_d\, q_j + \widetilde{H}^{\dagger j} \overline{u}\, Y_u\, q_j + H^{\dagger j} \overline{e}\, Y_e\, l_j + \text{h.c.} \right],$$

$$D \leq 4$$

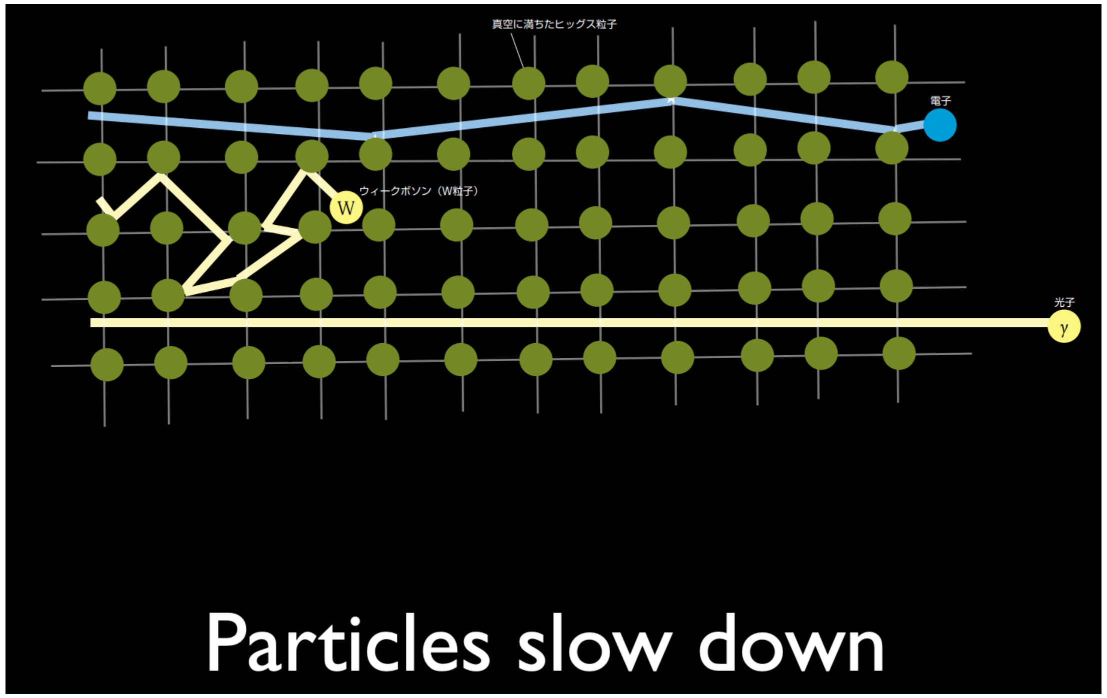


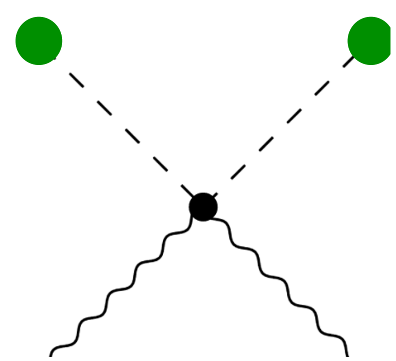
Image credit: Hitoshi's Higgs2020 talk

Gives masses, mass eigenstate fields, useful combinations of fields and couplings

Consequences of the Higgs field becoming a number

The Higgs field takes on a vev, recall what happens:

$$(D_\mu H^\dagger)(D^\mu H)$$



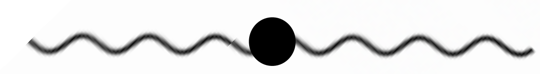
4-point

$$U_{BC} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{i}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & c_{\overline{\theta}} & s_{\overline{\theta}} \\ 0 & 0 & -s_{\overline{\theta}} & c_{\overline{\theta}} \end{bmatrix}$$

$$\mathcal{W}_B^\nu = U_{BC} \mathcal{A}^{C,\nu}$$

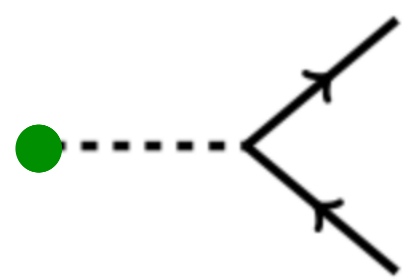
$$\mathcal{W}_B = \{W_1, W_2, W_3, B\}$$

$$\mathcal{A}_C = \{W^+, W^-, Z, A\}$$

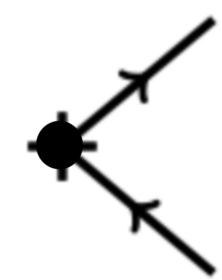


2-point (mass)

$$\left[H^{\dagger j} \overline{d} Y_d q_j + \widetilde{H}^{\dagger j} \overline{u} Y_u q_j + H^{\dagger j} \overline{e} Y_e l_j + \text{h.c.}\right]$$

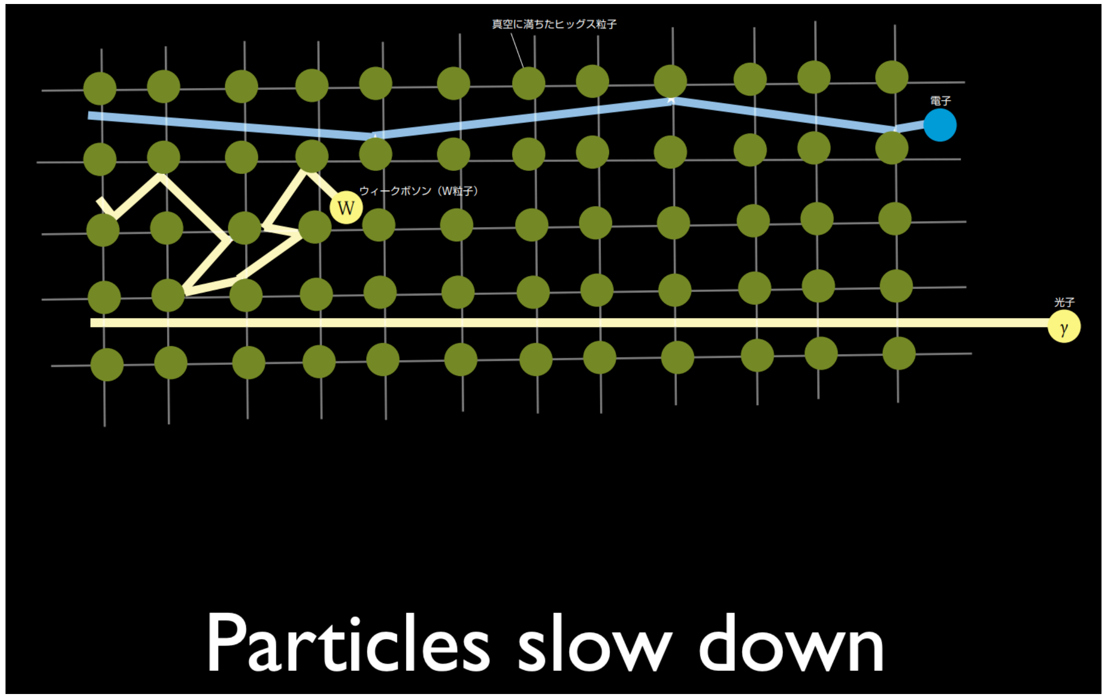


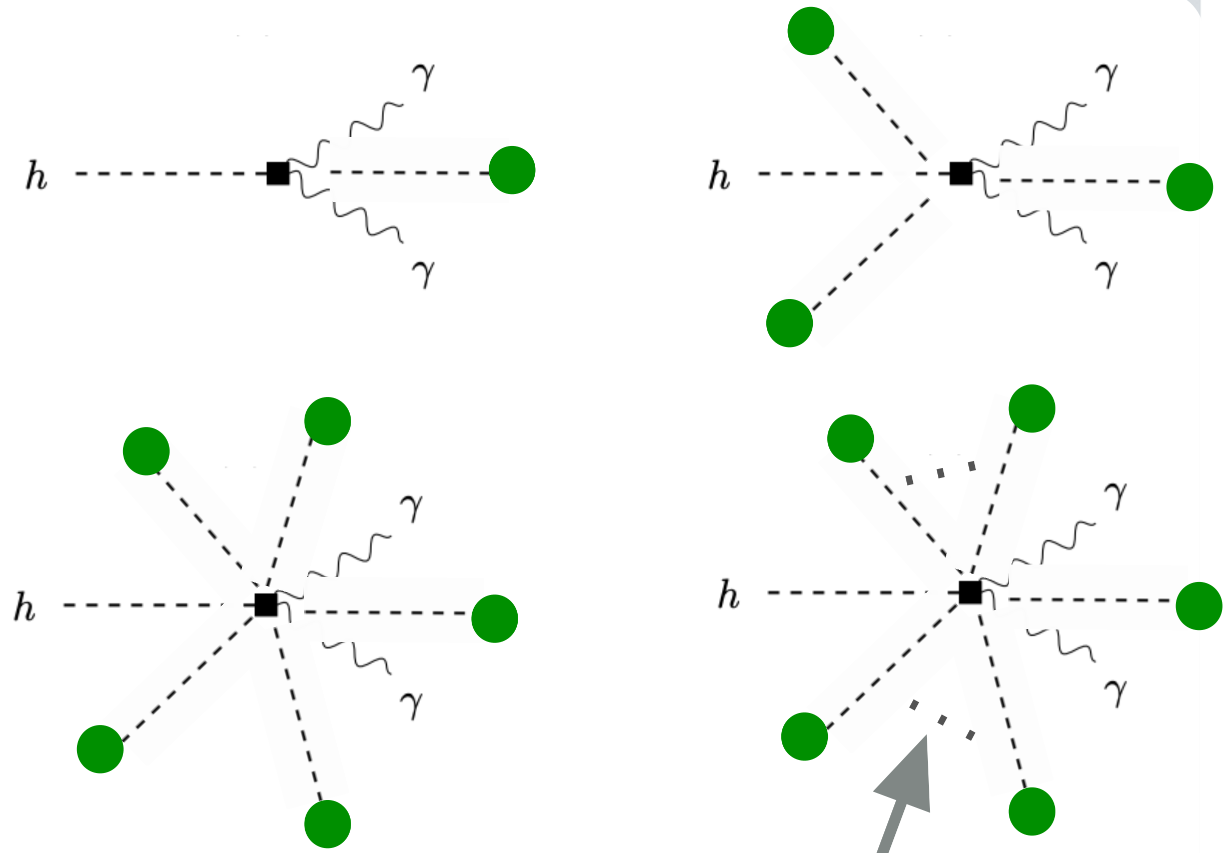
3-point



2-point (mass)

What is the Geometric SMEFT?





Powers of $\frac{H^\dagger H}{\Lambda^2}$
and symmetry generators

$$D \leq 4$$

$$D > 4$$

Gives masses, mass eigenstate fields.

Gives geometries that define the mass eigenstate fields and interactions in the EFT

Curved SMEFT spaces: scalar fields

- Curved SMEFT field space manifest in background field formulation

In general terms: G. A. Vilkovisky, Nucl. Phys. B234 (1984) 125.

Metric on Higgs field space, SM a **FLAT** field space

$$\mathcal{L}_{\rm scalar,kin} = \frac{1}{2} h_{IJ}(\phi) \left(D_\mu \phi\right)^I \left(D^\mu \phi\right)^J ,$$

Where $H = \frac{1}{\sqrt{2}} \begin{bmatrix} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{bmatrix}$

$$\sqrt{h}^{IJ} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 - \frac{1}{4}\tilde{C}_{HD} & 0 \\ 0 & 0 & 0 & 1 + \tilde{C}_{H\Box} - \frac{1}{4}\tilde{C}_{HD} \end{bmatrix}$$

here $\tilde{C}_i = \frac{\langle H^\dagger H \rangle}{\Lambda^2} C_i$

Small perturbations so positive semi-definite Matrix and unique square root

(sqrt) Metric in SMEFT, a *curved* field space

$$R^I_{JKL} \neq 0$$

1002.2730 Burgess, Lee, Trott
1511.00724 Alonso, Jenkins, Manohar
1605.03602 Alonso, Jenkins, Manohar

Curved SMEFT space: gauge fields

- Similarly in the gauge coupling space a curved field space

Metric on gauge field space, SM a **FLAT** field space

$$\mathcal{L}_{\text{gauge,kin}} = -\frac{1}{4} g_{AB}(\phi) \mathcal{W}^A_{\mu\nu} \mathcal{W}^{B,\mu\nu},$$

Where $\mathcal{W}^A = (W^1, W^2, W^3, B)$

$$\sqrt{g}^{AB} = \begin{bmatrix} 1+\tilde{C}_{HW} & 0 & 0 & 0 \\ 0 & 1+\tilde{C}_{HW} & 0 & 0 \\ 0 & 0 & 1+\tilde{C}_{HW} & -\frac{\tilde{C}_{HWB}}{2} \\ 0 & 0 & -\frac{\tilde{C}_{HWB}}{2} & 1+\tilde{C}_{HB} \end{bmatrix}$$

here $\tilde{C}_i = \frac{\langle H^\dagger H \rangle}{\Lambda^2} C_i$

1803.08001 Helset, Paraskevas, Trott
1909.08470 Corbett, Helset, Trott

(sqrt) Metric in SMEFT, a *curved* field space

All orders SM Lagrangian parameters

- Low n-point interactions of fields are parameterised in terms of couplings,

2001.01453 Helset, Martin, Trott

$$\bar{g}_2 = g_2 \sqrt{g}^{11} = g_2 \sqrt{g}^{22},$$

$$\bar{g}_Z = \frac{g_2}{c^2_{\theta_Z}} \left(c_{\bar{\theta}} \sqrt{g}^{33} - s_{\bar{\theta}} \sqrt{g}^{34} \right) = \frac{g_1}{s^2_{\theta_Z}} \left(s_{\bar{\theta}} \sqrt{g}^{44} - c_{\bar{\theta}} \sqrt{g}^{34} \right),$$

$$\bar{e} = g_2 \left(s_{\bar{\theta}} \sqrt{g}^{33} + c_{\bar{\theta}} \sqrt{g}^{34} \right) = g_1 \left(c_{\bar{\theta}} \sqrt{g}^{44} + s_{\bar{\theta}} \sqrt{g}^{34} \right),$$

- Masses

$$\bar{m}^2_W = \frac{\bar{g}^2_2}{4} \sqrt{h_{11}}^2 \bar{v}^2_T, \qquad \bar{m}^2_Z = \frac{\bar{g}^2_Z}{4} \sqrt{h_{33}}^2 \bar{v}^2_T \qquad \bar{m}^2_A = 0.$$

- Mixing angles:

$$s^2_{\theta_Z} = \frac{g_1(\sqrt{g}^{44} s_{\bar{\theta}} - \sqrt{g}^{34} c_{\bar{\theta}})}{g_2(\sqrt{g}^{33} c_{\bar{\theta}} - \sqrt{g}^{34} s_{\bar{\theta}}) + g_1(\sqrt{g}^{44} s_{\bar{\theta}} - \sqrt{g}^{34} c_{\bar{\theta}})},$$

$$s^2_{\bar{\theta}} = \frac{(g_1 \sqrt{g}^{44} - g_2 \sqrt{g}^{34})^2}{g_1^2[(\sqrt{g}^{34})^2 + (\sqrt{g}^{44})^2] + g_2^2[(\sqrt{g}^{33})^2 + (\sqrt{g}^{34})^2] - 2 g_1 g_2 \sqrt{g}^{34}(\sqrt{g}^{33} + \sqrt{g}^{44})}.$$

(Interesting way to think of the Weinberg angle)

All orders expressions are known now

- All orders scalar metric -leading to gauge boson masses in SMEFT

$$h_{IJ} = \left[1 + \phi^2 C_{H\Box}^{(6)} + \sum_{n=0}^{\infty} \left(\frac{\phi^2}{2}\right)^{n+2} \left(C_{HD}^{(8+2n)} - C_{H,D2}^{(8+2n)}\right)\right] \delta_{IJ} + \frac{\Gamma_{A,J}^{I} \phi_K \Gamma_{A,L}^{K} \phi^L}{2} \left(\frac{C_{HD}^{(6)}}{2} + \sum_{n=0}^{\infty} \left(\frac{\phi^2}{2}\right)^{n+1} C_{H,D2}^{(8+2n)}\right).$$

- All orders gauge metric - gives mass eigenstate couplings in SMEFT

$$\begin{aligned} g_{AB}(\phi_I) = &\left[1 - 4 \sum_{n=0}^{\infty} \left(C_{HW}^{(6+2n)}(1-\delta_{A4}) + C_{HB}^{(6+2n)}\delta_{A4}\right)\left(\frac{\phi^2}{2}\right)^{n+1}\right]\delta_{AB} \\ &- \sum_{n=0}^{\infty} C_{HW,2}^{(8+2n)} \left(\frac{\phi^2}{2}\right)^{n} \left(\phi_I \Gamma_{A,J}^{I} \phi^J\right)\left(\phi_L \Gamma_{B,K}^{L} \phi^K\right)(1-\delta_{A4})(1-\delta_{B4}) \\ &+ \left[\sum_{n=0}^{\infty} C_{HWB}^{(6+2n)} \left(\frac{\phi^2}{2}\right)^{n}\right]\left[(\phi_I \Gamma_{A,J}^{I} \phi^J)(1-\delta_{A4})\delta_{B4} + (A \leftrightarrow B)\right], \end{aligned}$$

- <u>Number of operator forms saturate in geosmeft.</u>
  This is due to reducing possible generator insertions on the Higgs manifold

$$T_{ij}^a T_{k\ell}^a = \frac{1}{2}\left(\delta_{i\ell}\delta_{jk} - \frac{1}{N}\delta_{ij}\delta_{k\ell}\right)$$

SM weak-mass eigenstate relations

- Weak eigenstates

Mass eigenstate

$$\hat{\mathcal{W}}^{A,\nu} = \delta^{AB}\, U_{BC} \hat{\mathcal{A}}^{C,\nu},$$
$$\hat{\alpha}^{A} = \delta^{AB}\, U_{BC} \hat{\beta}^{C},$$
$$\hat{\phi}^{J} = \delta^{JK}\, V_{KL} \hat{\Phi}^{L},$$

Rotations

Flat field space's.
Due to $D \leq 4$

$$U_{BC} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{i}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & c_{\overline{\theta}} & s_{\overline{\theta}} \\ 0 & 0 & -s_{\overline{\theta}} & c_{\overline{\theta}} \end{bmatrix} \qquad V_{JK} = \begin{bmatrix} \frac{-i}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\phi^{J} = \{\phi_1, \phi_2, \phi_3, \phi_4\}, \Phi^{K} = \{\Phi^-, \Phi^+, \chi, h\}$$

$$\alpha^{A} = \{g_2\, g_2, g_2, g_1\}, \qquad \mathcal{W}^{A} = \{W_1, W_2, W_3, B\},$$

$$\beta^{C} = \left\{ \frac{g_2\,(1-i)}{\sqrt{2}}, \frac{g_2\,(1+i)}{\sqrt{2}}, \sqrt{g_1^2 + g_2^2}(c_{\overline{\theta}}^2 - s_{\overline{\theta}}^2), \frac{2\, g_1\, g_2}{\sqrt{g_1^2 + g_2^2}} \right\}, \qquad \mathcal{A}^{C} = (\mathcal{W}^+, \mathcal{W}^-, \mathcal{Z}, \mathcal{A})\,.$$

What else could you write?

SMEFT weak-mass eigenstate relations

- Weak eigenstates

Mass eigenstate

$$\hat{\mathcal{W}}^{A,\nu} = \sqrt{g}^{AB} U_{BC} \hat{\mathcal{A}}^{C,\nu},$$
$$\hat{\alpha}^{A} = \sqrt{g}^{AB} U_{BC} \hat{\beta}^{C},$$
$$\hat{\phi}^{J} = \sqrt{h}^{JK} V_{KL} \hat{\Phi}^{L},$$

1909.08470 Corbett, Helset, Trott
(True in any operator basis.)

Generator transform

$$\gamma^{I}_{C,J} = \frac{1}{2}\tilde{\gamma}^{I}_{A,J} \sqrt{g}^{AB} U_{BC}.$$

SMEFT field space metrics
(Now known to all orders)

Rotations

$$U_{BC} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{i}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & c_{\overline{\theta}} & s_{\overline{\theta}} \\ 0 & 0 & -s_{\overline{\theta}} & c_{\overline{\theta}} \end{bmatrix} \qquad V_{JK} = \begin{bmatrix} \frac{-i}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\phi^J = \{\phi_1, \phi_2, \phi_3, \phi_4\}, \Phi^K = \{\Phi^-, \Phi^+, \chi, h\}$$

$$\alpha^A = \{g_2\, g_2, g_2, g_1\}, \qquad \mathcal{W}^A = \{W_1, W_2, W_3, B\},$$

$$\beta^C = \left\{ \frac{g_2\,(1-i)}{\sqrt{2}}, \frac{g_2\,(1+i)}{\sqrt{2}}, \sqrt{g_1^2 + g_2^2}(c^2_{\overline{\theta}} - s^2_{\overline{\theta}}), \frac{2\, g_1\, g_2}{\sqrt{g_1^2+g_2^2}} \right\}, \qquad \mathcal{A}^C = (\mathcal{W}^+, \mathcal{W}^-, \mathcal{Z}, \mathcal{A}).$$

What else could you write? Nothing that generalises to all orders.

Dim 6 SMEFT EW Lagrangian terms

- EW sector parameters redefined in the SMEFT (already in SMEFTsim)

$$\begin{bmatrix} \mathcal{W}^3_\mu \\ \mathcal{B}_\mu \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} v_T^2 C_{HWB} \\ -\frac{1}{2} v_T^2 C_{HWB} & 1 \end{bmatrix} \begin{bmatrix} \cos\overline{\theta} & \sin\overline{\theta} \\ -\sin\overline{\theta} & \cos\overline{\theta} \end{bmatrix} \begin{bmatrix} \mathcal{Z}_\mu \\ \mathcal{A}_\mu \end{bmatrix},$$

Mass redefinitions

$$M_W^2 = \frac{\overline{g}_2{}^2 v_T^2}{4},$$
$$M_Z^2 = \frac{v_T^2}{4}(\overline{g}_1{}^2 + \overline{g}_2{}^2) + \frac{1}{8} v_T^4 C_{HD} (\overline{g}_1{}^2 + \overline{g}_2{}^2) + \frac{1}{2} v_T^4 \overline{g}_1 \overline{g}_2 C_{HWB}.$$

Mixing angle redefinitions

$$\sin\overline{\theta} = \frac{\overline{g}_1}{\sqrt{\overline{g}_1{}^2 + \overline{g}_2{}^2}} \left[1 + \frac{v_T^2}{2} \frac{\overline{g}_2}{\overline{g}_1} \frac{\overline{g}_2{}^2 - \overline{g}_1{}^2}{\overline{g}_2{}^2 + \overline{g}_1{}^2} C_{HWB}\right]$$
$$\cos\overline{\theta} = \frac{\overline{g}_2}{\sqrt{\overline{g}_1{}^2 + \overline{g}_2{}^2}} \left[1 - \frac{v_T^2}{2} \frac{\overline{g}_1}{\overline{g}_2} \frac{\overline{g}_2{}^2 - \overline{g}_1{}^2}{\overline{g}_2{}^2 + \overline{g}_1{}^2} C_{HWB}\right]$$

Interactions to remaining SM fields via:

$$D_\mu = \partial_\mu + i \frac{\overline{g}_2}{\sqrt{2}} \left[\mathcal{W}^+_\mu T^+ + \mathcal{W}^-_\mu T^-\right] + i \overline{g}_Z \left[T_3 - \overline{s}^2 Q\right] \mathcal{Z}_\mu + i \overline{e} Q \mathcal{A}_\mu,$$

$$\overline{e} = \frac{\overline{g}_1 \overline{g}_2}{\sqrt{\overline{g}_2{}^2 + \overline{g}_1{}^2}} \left[1 - \frac{\overline{g}_1 \overline{g}_2}{\overline{g}_2{}^2 + \overline{g}_1{}^2} v_T^2 C_{HWB}\right]$$

$$\overline{g}_Z = \sqrt{\overline{g}_2{}^2 + \overline{g}_1{}^2} + \frac{\overline{g}_1 \overline{g}_2}{\sqrt{\overline{g}_2{}^2 + \overline{g}_1{}^2}} v_T^2 C_{HWB}$$

$$\overline{s}^2 = \sin^2\overline{\theta} = \frac{\overline{g}_1{}^2}{\overline{g}_2{}^2 + \overline{g}_1{}^2} + \frac{\overline{g}_1 \overline{g}_2 (\overline{g}_2{}^2 - \overline{g}_1{}^2)}{(\overline{g}_1{}^2 + \overline{g}_2{}^2)^2} v_T^2 C_{HWB}.$$

1312.2014 Alonso, Jenkins, Manohar, Trott

LO Automation of this approach

- Need to keep all operators and carefully compute S matrix elements avoiding uncontrolled approximations (and human error)
- Automation of LO (geo)SMEFT already in the SMEFTsim package

https://arxiv.org/abs/1709.06492
https://arxiv.org/abs/2012.11343

Generalisation for composite ops

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \mathcal{L}^{(7)} + ..., \qquad \mathcal{L}^{(d)} = \sum_{i=1}^{n_d} \frac{C_i^{(d)}}{\Lambda^{d-4}} Q_i^{(d)} \quad \text{for } d > 4,$$

$$v/M < 1$$

$$\mathcal{L}_{\rm SMEFT} = \sum_i f_i(\alpha \cdots) G_i(I, A \cdots),$$

Derivative expansion

Composite operator form
With minimal scalar field
coordinate dependence

Vev expansion

Scalar field coordinate dependence
And insertions of symmetry generators

$D^\mu \phi$

Mixes expansions, but grouped with derivative forms.

Generalisation for composite ops

- Such connections can be defined from the Lagrangian expansion constructively

$$h_{IJ}(\phi) = \frac{g^{\mu\nu}}{d} \frac{\delta^2 \mathcal{L}_{\rm SMEFT}}{\delta(D_\mu\phi)^I \, \delta(D_\nu\phi)^J}\Bigg|_{\mathcal{L}(\alpha,\beta\cdots)\to 0}.$$

non-trivial Lorentz-index-carrying Lagrangian terms and spin connections $\{\mathcal{W}^A_{\mu\nu}, (D^\mu\Phi)^K, \bar\psi\sigma^\mu\psi, \bar\psi\psi\cdots\}$

- Limited number of such connections for up to three point functions

$V(\phi)$ $\quad h_{IJ}(\phi)(D_\mu\phi)^I(D_\mu\phi)^J, \quad g_{AB}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\mu\nu}, \quad k^A_{IJ}(\phi)(D_\mu\phi)^I(D_\nu\phi)^J\,\mathcal{W}^{\mu\nu}_A,$
$f_{ABC}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\nu\rho}\mathcal{W}^{C,\mu}_\rho,$

With fermions $\quad Y(\phi)\bar\psi_1\psi_2, \quad L_{I,A}(\phi)\bar\psi_1\gamma^\mu\tau_A\psi_2(D_\mu\phi)^I, \quad d_A(\phi)\bar\psi_1\sigma^{\mu\nu}\psi_2\mathcal{W}^A_{\mu\nu},$

Gluon fields $\quad k_{\mathcal{AB}}(\phi)G^{\mathcal{A}}_{\mu\nu}G^{\mathcal{B},\mu\nu}, \quad k_{\mathcal{ABC}}(\phi)G^{\mathcal{A}}_{\nu\mu}G^{\mathcal{B},\rho\nu}G^{\mathcal{C},\mu\rho}, \quad c(\phi)\bar\psi_1\sigma^{\mu\nu}T_{\mathcal{A}}\psi_2G^{\mathcal{A}}_{\mu\nu}.$

Generalisation for composite ops

- Such connections can be defined from the Lagrangian expansion constructively

$$h_{IJ}(\phi) = \frac{g^{\mu\nu}}{d} \frac{\delta^2 \mathcal{L}_{\rm SMEFT}}{\delta(D_\mu\phi)^I \, \delta(D_\nu\phi)^J}\Bigg|_{\mathcal{L}(\alpha,\beta\cdots)\to 0}.$$

non-trivial Lorentz-index-carrying Lagrangian terms and spin connections $\{\mathcal{W}^A_{\mu\nu}, (D^\mu\Phi)^K, \bar{\psi}\sigma^\mu\psi, \bar{\psi}\psi\cdots\}$

- Limited number of such connections for up to three point functions

This is a non trivial fact proven in 2001.01453 Helset, Martin, Trott

There is a theory choice here - its REMOVE DERIVATIVE OPS, USE EOM.

Same reasoning built into, and led to the "Warsaw basis".
Also why we were able to renormalise the Warsaw basis completely in 2013.

EFT Industry standard in flavour physics, chiral pert theory etc.

An instant pay off of this approach

- Growth in operator forms in connections *Always* saturate to fixed number, this is just the simplest organization exploiting this

	Mass Dimension				
Field space connection	6	8	10	12	14
$h_{IJ}(\phi)(D_\mu\phi)^I(D^\mu\phi)^J$	2	2	2	2	2
$g_{AB}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\mu\nu}$	3	4	4	4	4
$k_{IJA}(\phi)(D^\mu\phi)^I(D^\nu\phi)^J\mathcal{W}^A_{\mu\nu}$	0	3	4	4	4
$f_{ABC}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\nu\rho}\mathcal{W}^{C,\mu}_\rho$	1	2	2	2	2
$Y^u_{pr}(\phi)\bar{Q}u+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$Y^d_{pr}(\phi)\bar{Q}d+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$Y^e_{pr}(\phi)\bar{L}e+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$d^{e,pr}_A(\phi)\bar{L}\sigma_{\mu\nu}e\mathcal{W}^{\mu\nu}_A+$ h.c.	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$d^{u,pr}_A(\phi)\bar{Q}\sigma_{\mu\nu}u\mathcal{W}^{\mu\nu}_A+$ h.c.	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$d^{d,pr}_A(\phi)\bar{Q}\sigma_{\mu\nu}d\mathcal{W}^{\mu\nu}_A+$ h.c.	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$L^{\psi_R}_{pr,A}(\phi)(D^\mu\phi)^J(\bar{\psi}_{p,R}\gamma_\mu\sigma_A\psi_{r,R})$	N_f^2	N_f^2	N_f^2	N_f^2	N_f^2
$L^{\psi_L}_{pr,A}(\phi)(D^\mu\phi)^J(\bar{\psi}_{p,L}\gamma_\mu\sigma_A\psi_{r,L})$	$2\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$

2001.01453 Helset, Martin, Trott

- Once we have things to dim eight it is sufficient in many observables

Mases
Couplings and mixing angles
TGC, Higgs to ZZ,WW
QGC,TGC + Higgs

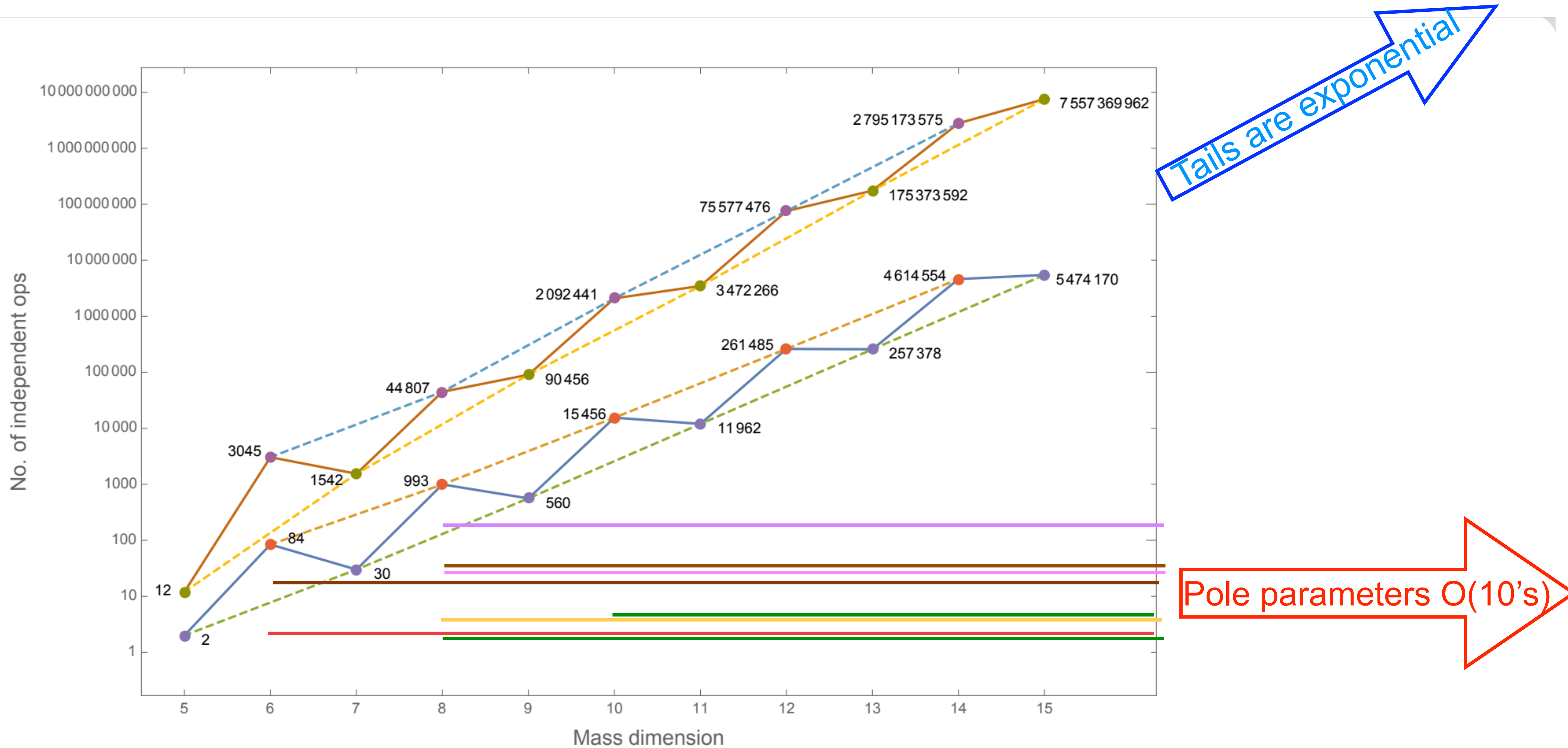
Yukawas

Dipoles

W,Z couplings to fermions +higgs

- Basis choice changes entries in these geometric structures, but geometric organization exist in any basis. The trend of saturation of effects at dimension eight is a general feature.

Whats under control?



- General growth in operator forms from Hilbert series

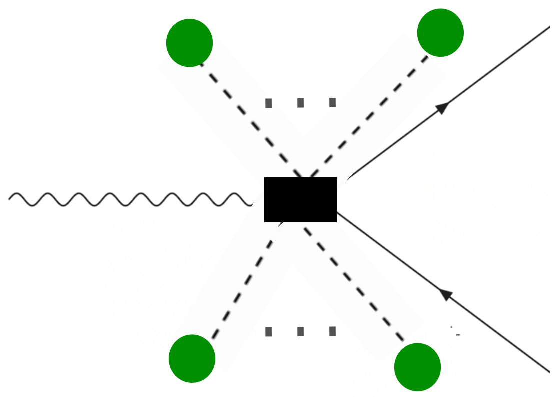
  https://arxiv.org/abs/1503.07537 https://arxiv.org/pdf/1512.03433.pdf

  https://arxiv.org/abs/1510.00372 https://arxiv.org/abs/1706.08520

GeoSMEFT All orders result ex.

2001.01453 Helset, Martin, Trott

- What does this allow one to do?



Consider a $W^{\pm}, Z$ coupling to a fermion bilinear.

The all orders coupling in the SMEFT is a sum of two field space connections.

$\bar{\psi} i \not{D} \psi$:with a consistent change weak to mass eigenstates in SMEFT

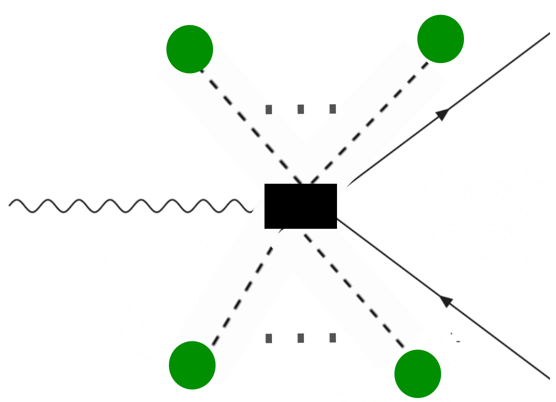
Added to this is the scalar, fermion connection (with a background field expectation)

$$L^{\psi_R}_{pr,A}(\phi)(D^\mu \phi)^J (\bar{\psi}_{p,R} \gamma_\mu \sigma_A \psi_{r,R})$$
$$L^{\psi_L}_{pr,A}(\phi)(D^\mu \phi)^J (\bar{\psi}_{p,L} \gamma_\mu \sigma_A \psi_{r,L})$$

GeoSMEFT All orders result ex.

2001.01453 Helset, Martin, Trott

- What does this allow one to do?



$\mathcal{A}^C = (\mathcal{W}^+, \mathcal{W}^-, \mathcal{Z}, \mathcal{A})$

Consider a $W^{\pm}, Z$ coupling to a fermion bilinear.

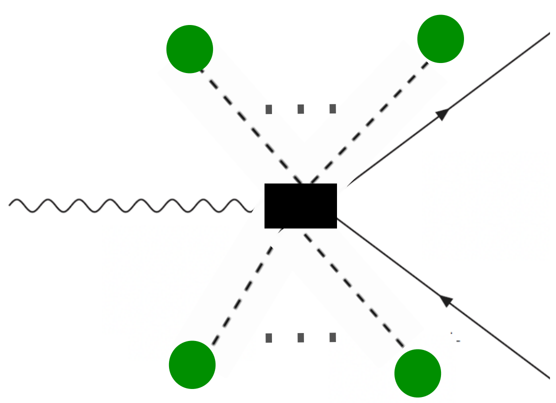
$$-\mathcal{A}^{A,\mu}(\bar{\psi}_p \gamma_\mu \bar{\tau}_A \psi_r)\delta_{pr} + \mathcal{A}^{C,\mu}(\bar{\psi}_p \gamma_\mu \sigma_A \psi_r)\langle L^{\psi,pr}_{I,A}\rangle(-\gamma^I_{C,4})\bar{v}_T,$$

Compact all $\bar{v}_T/\Lambda$ orders answer!

GeoSMEFT All orders result ex.

2001.01453 Helset, Martin, Trott

- What does this allow one to do?



Consider a $W^{\pm}, Z$ coupling to a fermion bilinear.

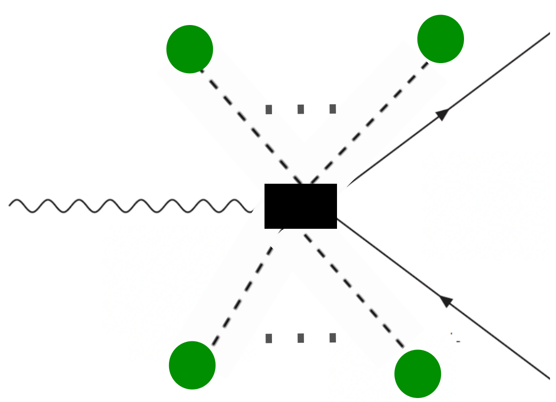
$$-\mathcal{A}^{A,\mu}(\bar{\psi}_p \gamma_\mu \bar{\tau}_A \psi_r)\delta_{pr} + \mathcal{A}^{C,\mu}(\bar{\psi}_p \gamma_\mu \sigma_A \psi_r)\langle L^{\psi,pr}_{I,A}\rangle(-\gamma^{I}_{C,4})\bar{v}_T,$$

The coupling of the canonically normalised mass eigenstate fields is then

$$\begin{aligned}
\langle \mathcal{Z}|\bar{\psi}_p \psi_r\rangle &= \frac{\bar{g}_Z}{2}\,\bar{\psi}_p\,\not{\epsilon}_{\mathcal{Z}}\,\left[(2 s^2_{\theta_Z} Q_\psi - \sigma_3)\delta_{pr} + \sigma_3 \bar{v}_T \langle L^{\psi,pr}_{3,3}\rangle + \bar{v}_T \langle L^{\psi,pr}_{3,4}\rangle\right]\,\psi_r, \\
\langle \mathcal{A}|\bar{\psi}_p \psi_r\rangle &= -\bar{e}\,\bar{\psi}_p\,\not{\epsilon}_{\mathcal{A}}\,Q_\psi\,\delta_{pr}\,\psi_r, \\
\langle \mathcal{W}_{\pm}|\bar{\psi}_p \psi_r\rangle &= -\frac{\bar{g}_2}{\sqrt{2}}\bar{\psi}_p(\not{\epsilon}_{\mathcal{W}^{\pm}})\,T^{\pm}\left[\delta_{pr} - \bar{v}_T \langle L^{\psi,pr}_{1,1}\rangle \pm i \bar{v}_T \langle L^{\psi,pr}_{1,2}\rangle\right]\,\psi_r.
\end{aligned}$$

GeoSMEFT All orders result ex.

- Can build up observable quantities, such as a decay width.



Consider a $W^{\pm}, Z$ coupling to a fermion bilinear.

$$-\mathcal{A}^{A,\mu}(\bar{\psi}_p \gamma_\mu \bar{\tau}_A \psi_r)\delta_{pr} + \mathcal{A}^{C,\mu}(\bar{\psi}_p \gamma_\mu \sigma_A \psi_r)\langle L_{I,A}^{\psi,pr}\rangle(-\boldsymbol{\gamma}_{C,4}^{I})\bar{v}_T,$$

- Two body decay widths:

$$\bar{\Gamma}_{Z\to\bar{\psi}\psi} = \sum_\psi \frac{N_c^\psi}{24\pi}\sqrt{\bar{m}_Z^2}|g_{\rm eff}^{Z,\psi}|^2\left(1-\frac{4\bar{M}_\psi^2}{\bar{m}_Z^2}\right)^{3/2}$$

$$\bar{\Gamma}_{W\to\bar{\psi}\psi} = \sum_\psi \frac{N_c^\psi}{24\pi}\sqrt{\bar{m}_W^2}|g_{\rm eff}^{W,\psi}|^2\left(1-\frac{4\bar{M}_\psi^2}{\bar{m}_W^2}\right)^{3/2}$$

$$g_{\rm eff}^{Z,\psi} = \frac{\bar{g}_Z}{2}\left[(2s_{\theta_Z}^2\,Q_\psi - \sigma_3)\delta_{pr} + \bar{v}_T\langle L_{3,4}^{\psi,pr}\rangle + \sigma_3\bar{v}_T\langle L_{3,3}^{\psi,pr}\rangle\right]$$

$$g_{\rm eff}^{W,q_L} = -\frac{\bar{g}_2}{\sqrt{2}}\left[V_{\rm CKM}^{pr} - \bar{v}_T\langle L_{1,1}^{q_L,pr}\rangle \pm i\bar{v}_T\langle L_{1,2}^{q_L,pr}\rangle\right],$$

$$g_{\rm eff}^{W,\ell_L} = -\frac{\bar{g}_2}{\sqrt{2}}\left[U_{\rm PMNS}^{pr,\dagger} - \bar{v}_T\langle L_{1,1}^{\ell_L,pr}\rangle \pm i\bar{v}_T\langle L_{1,2}^{\ell_L,pr}\rangle\right],$$

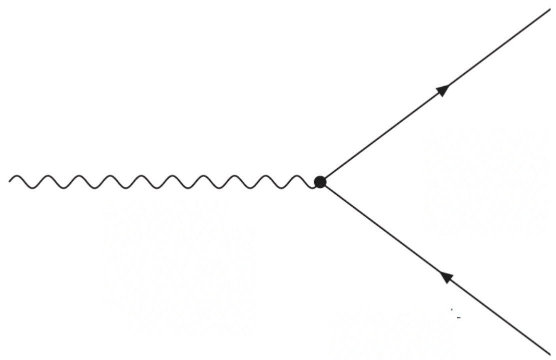
- Can do LEP to dim 8 in about 3 weeks of work if you learn this stuff.

Pause.

Current geosmeft limitations

GeoSMEFT Pushing to higher n points

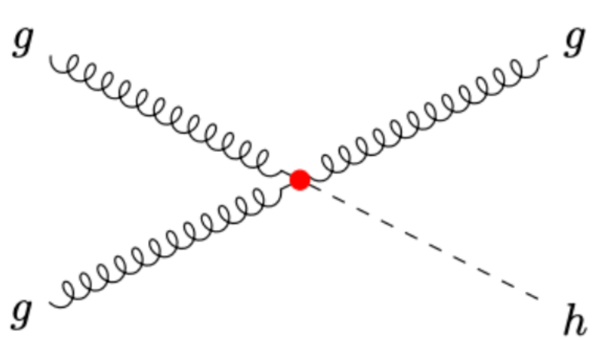
- Can build up observable quantities, such as a decay width.

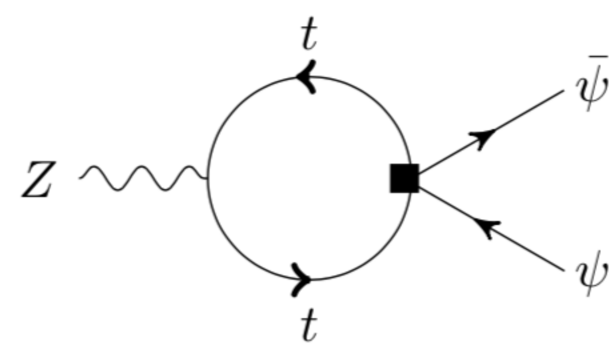


Consider a $W^{\pm}, Z$ coupling to a fermion bilinear.

$$-\mathcal{A}^{A,\mu}(\bar{\psi}_p\gamma_\mu\bar{\tau}_A\psi_r)\delta_{pr} + \mathcal{A}^{C,\mu}(\bar{\psi}_p\gamma_\mu\sigma_A\psi_r)\langle L^{\psi,pr}_{I,A}\rangle(-\gamma^{I}_{C,4})\bar{v}_T,$$

- Not all physics is derivable from two and three point functions





GeoSMEFT Pushing to higher n points

- Limited number of such connections for up to three point functions

  This is a non trivial fact proven for: $F = \{H, \psi, \mathcal{W}^{\mu\nu}\}$ via the following:

$$D^2 F \;\Rightarrow\; \boxed{\text{EOM}} \text{ and higher-points,}$$

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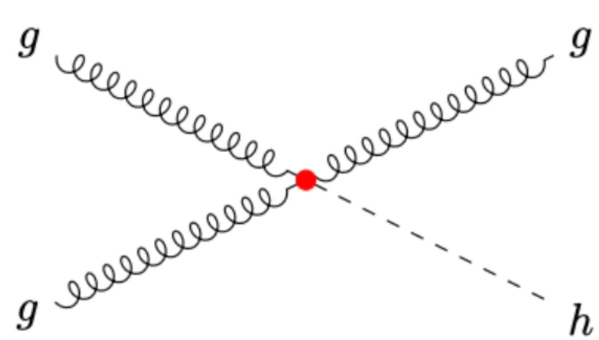
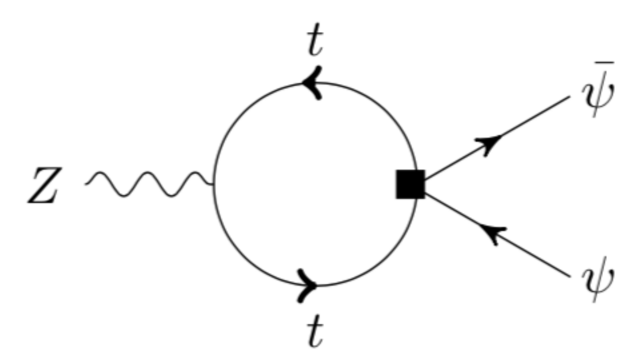
$$f(H)(D_\mu F_1)(D_\nu F_2)D_{\{\mu\nu\}}F_3 \;\Rightarrow\; \boxed{\text{EOM}} \text{ and higher-points.}$$

$$f(\phi)\, F_1\, (D_\mu F_2)\, (D_\mu F_3) \Rightarrow (D_\mu f(\phi))\, (D_\mu F_1)\, F_2\, F_3 + \frac{1}{2}(D^2 f(\phi))\, F_1\, F_2\, F_3 + \boxed{\text{EOM}},$$





- How to incorporate such higher n-point effects is the key challenge.
- Pert corrections advancing fast- higher n points also moving.

GeoSMEFT Pushing to higher n points

- Note these integration by parts steps were used

$$
\begin{aligned}
& f(H)(D_\mu F_1)(D_\nu F_2)D_{\{\mu\nu\}}F_3 \\
&= - f(H)\left[(D^2 F_1)(D_\nu F_2) + (D_\mu F_1)(D_\mu D_\nu F_2) + (D_\mu D_\nu F_1)(D_\mu F_2) + (D_\nu F_1)(D^2 F_2)\right](D_\nu F_3) \\
&\quad - (D_\mu f(H))\left[(D_\mu F_1)(D_\nu F_2) + (D_\nu F_1)(D_\mu F_2)\right](D_\nu F_3)
\end{aligned}
$$

$$
f(\phi)\, F_1\, (D_\mu F_2)\, (D_\mu F_3) \Rightarrow (D_\mu f(\phi))\, (D_\mu F_1)\, F_2\, F_3 + \frac{1}{2}(D^2 f(\phi))\, F_1\, F_2\, F_3 + \boxed{\text{EOM}},
$$

These steps were critical to reducing the number of connections for two and three point functions. This just fails for four points and higher.

One knows that there are an infinite set of higher derivative terms lurking In higher n points, dependent on $\{D_\mu \phi^I, D_{\{\mu,\nu\}}\phi^I, D_{\{\mu,\nu,\rho\}}\phi^I, \cdots\},$

This is a problem for measurements away from SM resonances.

Generators for SMEFT vs SM

Generators on scalar SMEFT space

- To think in a unified gauge space manifold need generators (reformulate SM generators)

$$(D^\mu \phi)^I = (\partial^\mu \delta^I_J - \tfrac{1}{2}\mathcal{W}^{A,\mu}\tilde{\gamma}^I_{A,J})\phi^J$$

$$\tilde{\epsilon}^A{}_{BC} = g_2\, \epsilon^A{}_{BC}, \quad \text{with } \tilde{\epsilon}^1{}_{23} = +g_2,$$

$$\tilde{\gamma}^I_{A,J} = \begin{cases} g_2\, \gamma^I_{A,J}, & \text{for } A = 1,2,3 \\ g_1 \gamma^I_{A,J}, & \text{for } A = 4. \end{cases}$$

$$\gamma^I_{1,J} = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad \gamma^I_{2,J} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$\gamma^I_{3,J} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \gamma^I_{4,J} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}.$$

(this last one also "i")

1803.08001 Helset, Paraskevas, Trott

- Some interesting math here, we also define $\Gamma^I_{A,K} = \gamma^I_{A,J}\, \gamma^J_{4,K}$

$$\Gamma^I_{1,J} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, \quad \Gamma^I_{2,J} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad \Gamma^I_{3,J} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \Gamma^I_{4,J} = -\mathbb{I}_{4\times 4}.$$

Field space connections to all orders

2001.01453 Helset, Martin, Trott

- Field space connection for W,Z coupling to fermion pairs $(D^\mu \phi)^I \, \bar{\psi} \Gamma_\mu \psi$

$$
\begin{aligned}
\mathcal{Q}^{1,(6+2n)}_{\substack{H\psi \\ pr}} &= (H^\dagger H)^n H^\dagger \, i\overleftrightarrow{D}^\mu H \bar{\psi}_p \gamma_\mu \psi_r, \\
\mathcal{Q}^{3,(6+2n)}_{\substack{H\psi \\ pr}} &= (H^\dagger H)^n \, H^\dagger \, i\overleftrightarrow{D}^\mu_a H \bar{\psi}_p \gamma_\mu \sigma_a \psi_r, \\
\mathcal{Q}^{2,(8+2n)}_{\substack{H\psi \\ pr}} &= (H^\dagger H)^n (H^\dagger \sigma_a H) \, H^\dagger \, i\overleftrightarrow{D}^\mu H \bar{\psi}_p \gamma_\mu \sigma_a \psi_r, \\
\mathcal{Q}^{\epsilon,(8+2n)}_{\substack{H\psi \\ pr}} &= \epsilon^a_{bc} \, (H^\dagger H)^n \, (H^\dagger \sigma_c H) \, H^\dagger \, i\overleftrightarrow{D}^\mu_b H \bar{\psi}_p \gamma_\mu \sigma_a \psi_r.
\end{aligned}
$$

Not that many op forms. Closed form field space connection.

$$
\begin{aligned}
L^{\psi,pr}_{J,A} &= -(\phi\,\gamma_4)_J \delta_{A4} \sum_{n=0}^{\infty} C^{1,(6+2n)}_{\substack{H\psi \\ pr}} \left(\frac{\phi^2}{2}\right)^n - (\phi\,\gamma_A)_J (1-\delta_{A4}) \sum_{n=0}^{\infty} C^{3,(6+2n)}_{\substack{H\psi_L \\ pr}} \left(\frac{\phi^2}{2}\right)^n \\
&+ \frac{1}{2} (\phi\,\gamma_4)_J (1-\delta_{A4}) \left(\phi_K \Gamma^K_{A,L} \phi^L\right) \sum_{n=0}^{\infty} C^{2,(8+2n)}_{\substack{H\psi_L \\ pr}} \left(\frac{\phi^2}{2}\right)^n \\
&+ \frac{\epsilon^A_{BC}}{2} (\phi\,\gamma_B)_J \left(\phi_K \Gamma^K_{C,L} \phi^L\right) \sum_{n=0}^{\infty} C^{\epsilon,(8+2n)}_{\substack{H\psi_L \\ pr}} \left(\frac{\phi^2}{2}\right)^n .
\end{aligned}
$$

Notice the clean form due to generator structure and real fields.

Field space connections to all orders

2001.01453 Helset, Martin, Trott

- Off shell operators contributing to three points $(D_\mu\phi)^I \sigma_A (D_\nu\phi)^J \mathcal{W}^A_{\mu\nu}$

$$
\begin{aligned}
Q^{(8+2n)}_{HDHB} &= i\,(H^\dagger H)^{n+1}(D_\mu H)^\dagger (D_\nu H) B^{\mu\nu}, \\
Q^{(8+2n)}_{HDHW} &= i\delta_{ab}(H^\dagger H)^{n+1}(D_\mu H)^\dagger \sigma^a (D_\nu H) W_b^{\mu\nu}, \\
Q^{(8+2n)}_{HDHW,2} &= i\epsilon_{abc}(H^\dagger H)^{n}(H^\dagger \sigma^a H)(D_\mu H)^\dagger \sigma^b (D_\nu H) W_c^{\mu\nu}, \\
Q^{(10+2n)}_{HDHW,3} &= i\delta_{ab}\delta_{cd}(H^\dagger H)^{n}(H^\dagger \sigma^a H)(H^\dagger \sigma^c H)(D_\mu H)^\dagger \sigma^b (D_\nu H) W_d^{\mu\nu}.
\end{aligned}
$$

This connection saturates last in op dimension. This is due to EOM reduction. No entries at dim 6 in Warsaw basis.

$$
\begin{aligned}
k^A_{IJ}(\phi) = &-\frac{1}{2}\gamma^I_{4,J}\delta_{A4}\sum_{n=0}^{\infty} C^{(8+2n)}_{HDHB}\left(\frac{\phi^2}{2}\right)^{n+1} - \frac{1}{2}\gamma^I_{A,J}(1-\delta_{A4})\sum_{n=0}^{\infty} C^{(8+2n)}_{HDHW}\left(\frac{\phi^2}{2}\right)^{n+1} \\
&- \frac{1}{8}(1-\delta_{A4})\left[\phi_K \Gamma^K_{A,L}\phi^L\right]\left[\phi_M \Gamma^K_{B,L}\phi^N\right]\gamma^I_{B,J}\sum_{n=0}^{\infty} C^{(10+2n)}_{HDHW,3}\left(\frac{\phi^2}{2}\right)^{n} \\
&+ \frac{1}{4}\epsilon_{ABC}\left[\phi_K \Gamma^K_{B,L}\phi^L\right]\gamma^I_{C,J}\sum_{n=0}^{\infty} C^{(8+2n)}_{HDHW,2}\left(\frac{\phi^2}{2}\right)^{n}.
\end{aligned}
$$

Input parameter/scheme dependence

Strong input scheme dependence in SMEFT

$\{\hat{\alpha}_{ew}, \hat{M}_Z, \hat{G}_F, \hat{M}_h\}$ Scheme

$$\delta m_Z^2 = \frac{\hat{M}_Z^2}{2}\tilde{C}_{HD} + \frac{2^{3/4}\sqrt{\pi\hat{\alpha}}\,\hat{M}_Z}{\hat{G}_F^{1/2}}\tilde{C}_{HWB},$$

$$\delta s_{\hat{\theta}}^2 = 0.17\tilde{C}_{HD} + 0.79\tilde{C}_{HWB} + 0.76\tilde{C}_{Hl}^{(3)} - 0.34\tilde{C}'_{ll},$$

$$\frac{\delta\Gamma_Z}{\Gamma_Z^{SM}} = -0.82\tilde{C}_{HWB} - 0.67\tilde{C}_{HD} - 0.19\tilde{C}_{Hl}^{(1)} - 2.06\tilde{C}_{Hl}^{(3)} - 0.19\tilde{C}_{He} + 0.47\tilde{C}_{Hq}^{(1)} + 1.61\tilde{C}_{Hq}^{(3)} + 0.26\tilde{C}_{Hu} - 0.19\tilde{C}_{Hd} + 1.35\tilde{C}'_{ll}$$

$$\frac{\delta\alpha}{2\,\hat{\alpha}} = 0,$$

$$\frac{\delta M_W^2}{\hat{M}_W^2} = -\frac{s_{2\hat{\theta}}}{4\,c_{2\hat{\theta}}}\left(\frac{c_{\hat{\theta}}}{s_{\hat{\theta}}}\tilde{C}_{HD} + \frac{s_{\hat{\theta}}}{c_{\hat{\theta}}}2\sqrt{2}\,\delta G_F + 4\tilde{C}_{HWB}\right),$$

$$\frac{\delta\Gamma_W}{\Gamma_W^{SM}} = -3.97\tilde{C}_{HWB} - 1.80\tilde{C}_{HD} - 3.52\tilde{C}_{Hl}^{(3)} + 1.33\tilde{C}_{Hq}^{(3)} + 2.10\tilde{C}'_{ll}\,.$$

$\{\hat{M}_W, \hat{M}_Z, \hat{G}_F, \hat{M}_h\}$ Scheme

$$\delta m_Z^2 = \frac{\hat{M}_Z^2}{2}\tilde{C}_{HD} + 2\,\hat{M}_Z\,\hat{M}_W\sqrt{1 - \frac{\hat{M}_W^2}{\hat{M}_Z^2}}\,\tilde{C}_{HWB},$$

$$\delta s_{\hat{\theta}}^2 = -0.39\tilde{C}_{HD} - 0.42\tilde{C}_{HWB},$$

$$\frac{\delta\Gamma_Z}{\Gamma_Z^{SM}} = 0.46\tilde{C}_{HWB} - 0.07\tilde{C}_{HD} - 0.18\tilde{C}_{Hl}^{(1)} - 1.37\tilde{C}_{Hl}^{(3)} - 0.18\tilde{C}_{He} + 0.47\tilde{C}_{Hq}^{(1)} + 1.61\tilde{C}_{Hq}^{(3)} + 0.24\tilde{C}_{Hu} - 0.18\tilde{C}_{Hd} + \tilde{C}'_{ll},$$

$$\frac{\delta\alpha}{2\,\hat{\alpha}} = -\frac{\delta G_F}{\sqrt{2}} + \frac{\delta m_Z^2}{\hat{M}_Z^2}\frac{\hat{M}_W^2}{2\,(\hat{M}_W^2 - \hat{M}_Z^2)} - \tilde{C}_{HWB}\frac{\hat{M}_W}{\hat{M}_Z}\sqrt{1 - \frac{\hat{M}_W^2}{\hat{M}_Z^2}},$$

$$\frac{\delta M_W^2}{\hat{M}_W^2} = 0,$$

$$\frac{\delta\Gamma_W}{\Gamma_W^{SM}} = \frac{4}{3}\left(\tilde{C}_{Hq}^{(3)} - \tilde{C}_{Hl}^{(3)}\right) + \tilde{C}'_{ll}.$$

- At leading order (tree level) already strong input parameter dependence, different than case in SM!

Strong input scheme dependence in SMEFT

$\{\hat\alpha_{ew}, \hat M_Z, \hat G_F, \hat M_h\}$ Scheme

$$\delta m_Z^2 = \frac{\hat M_Z^2}{2}\tilde C_{HD} + \frac{2^{3/4}\sqrt{\pi\hat\alpha}\,\hat M_Z}{\hat G_F^{1/2}}\tilde C_{HWB},$$

$$\delta s_\theta^2 = 0.17\tilde C_{HD} + 0.79\tilde C_{HWB} + 0.76\tilde C_{Hl}^{(3)} - 0.34\tilde C'_{ll},$$

$$\frac{\delta\Gamma_Z}{\Gamma_Z^{SM}} = -0.82\tilde C_{HWB} - 0.67\tilde C_{HD} - 0.19\tilde C_{Hl}^{(1)} - 2.06\tilde C_{Hl}^{(3)} - 0.19\tilde C_{He}$$

$$+\,0.47\tilde C_{Hq}^{(1)} + 1.61\tilde C_{Hq}^{(3)} + 0.26\tilde C_{Hu} - 0.19\tilde C_{Hd} + 1.35\tilde C'_{ll}$$

$$\frac{\delta\alpha}{2\,\hat\alpha} = 0,$$

$$\frac{\delta M_W^2}{\hat M_W^2} = -\frac{s_{2\hat\theta}}{4\,c_{2\hat\theta}}\left(\frac{c_{\hat\theta}}{s_{\hat\theta}}\tilde C_{HD} + \frac{s_{\hat\theta}}{c_{\hat\theta}}2\sqrt{2}\,\delta G_F + 4\tilde C_{HWB}\right),$$

$$\frac{\delta\Gamma_W}{\Gamma_W^{SM}} = -3.97\tilde C_{HWB} - 1.80\tilde C_{HD} - 3.52\tilde C_{Hl}^{(3)} + 1.33\tilde C_{Hq}^{(3)} + 2.10\tilde C'_{ll}\,.$$

$\{\hat M_W, \hat M_Z, \hat G_F, \hat M_h\}$ Scheme

$$\delta m_Z^2 = \frac{\hat M_Z^2}{2}\tilde C_{HD} + 2\,\hat M_Z\,\hat M_W\sqrt{1-\frac{\hat M_W^2}{\hat M_Z^2}}\,\tilde C_{HWB},$$

$$\delta s_\theta^2 = -0.39\tilde C_{HD} - 0.42\tilde C_{HWB},$$

$$\frac{\delta\Gamma_Z}{\Gamma_Z^{SM}} = 0.46\tilde C_{HWB} - 0.07\tilde C_{HD} - 0.18\tilde C_{Hl}^{(1)} - 1.37\tilde C_{Hl}^{(3)} - 0.18\tilde C_{He}$$

$$+\,0.47\tilde C_{Hq}^{(1)} + 1.61\tilde C_{Hq}^{(3)} + 0.24\tilde C_{Hu} - 0.18\tilde C_{Hd} + \tilde C'_{ll},$$

$$\frac{\delta\alpha}{2\,\hat\alpha} = -\frac{\delta G_F}{\sqrt{2}} + \frac{\delta m_Z^2}{\hat M_Z^2}\,\frac{\hat M_W^2}{2\,(\hat M_W^2 - \hat M_Z^2)} - \tilde C_{HWB}\frac{\hat M_W}{\hat M_Z}\sqrt{1-\frac{\hat M_W^2}{\hat M_Z^2}},$$

$$\frac{\delta M_W^2}{\hat M_W^2} = 0,$$

$$\frac{\delta\Gamma_W}{\Gamma_W^{SM}} = \frac{4}{3}\left(\tilde C_{Hq}^{(3)} - \tilde C_{Hl}^{(3)}\right) + \tilde C'_{ll}.$$

- Completely expected from decoupling theorem. ***UV physics preserving SM symmetries being absorbed into measured low scale parameters now.***

Need input parameters defined at all orders

$\{\hat{M}_W, \hat{M}_Z, \hat{G}_F, \hat{M}_h\}$ Scheme

Sorted in

Hays, Helset, Martin Trott: 2007.00565

Input parameter dependence
Increased order by order
due to Lagrangian parameters
being redefined geometrically

D $\{\hat{m}_W, \hat{m}_Z, \hat{G}_F\}$ input-parameter scheme at all orders in $(\bar{v}_T^2/\Lambda^2)^n$

In this scheme we can again use Eqn. (E.2) to define a shift to $\bar{g}_Z$. We also use

$$\bar{g}_2 = g_2 \sqrt{g}^{11} = \frac{2\hat{m}_W}{\sqrt{h_{11}}\bar{v}_T}. \tag{D.1}$$

and

$$g_1 = g_2 \frac{(s_{\bar{\theta}}\sqrt{g}^{33} + c_{\bar{\theta}}\sqrt{g}^{34})}{(c_{\bar{\theta}}\sqrt{g}^{44} + s_{\bar{\theta}}\sqrt{g}^{34})} \tag{D.2}$$

to solve for $s^2_{\bar{\theta}}$ via

$$s^2_{\bar{\theta}} = \frac{1}{\left[\left(\sqrt{g}^{44}\right)^2 + \left(\sqrt{g}^{34}\right)^2\right]^2} \left\{ -\left(\frac{g_2\sqrt{g}_-}{\bar{g}_Z}\right)^2 \left[\left(\sqrt{g}^{44}\right)^2 - \left(\sqrt{g}^{34}\right)^2\right] + \left(\sqrt{g}^{44}\right)^2 \left[\left(\sqrt{g}^{44}\right)^2 + \left(\sqrt{g}^{34}\right)^2\right] \right.$$
$$\left. - 2\left(\frac{g_2\sqrt{g}_-}{\bar{g}_Z}\right) \sqrt{(\sqrt{g}^{44})^2(\sqrt{g}^{34})^2 \left[\left(\sqrt{g}^{44}\right)^2 + \left(\sqrt{g}^{34}\right)^2 - \left(\frac{g_2\sqrt{g}_-}{\bar{g}_Z}\right)^2\right]} \right\}. \tag{D.3}$$

The remaining Lagrangian parameters can then be defined via

$$\bar{e} = \frac{\bar{g}_2}{\sqrt{g}^{11}} \left(s_{\bar{\theta}}\sqrt{g}^{33} + c_{\bar{\theta}}\sqrt{g}^{34}\right), \tag{D.4}$$

and

$$s^2_{\bar{\theta}_Z} = \frac{\bar{e}}{\bar{g}_Z} \frac{(s_{\bar{\theta}}\sqrt{g}^{44} - c_{\bar{\theta}}\sqrt{g}^{34})}{(c_{\bar{\theta}}\sqrt{g}^{44} + s_{\bar{\theta}}\sqrt{g}^{34})}. \tag{D.5}$$

In both schemes, $\bar{g}_Z$ and $s^2_{\bar{\theta}_Z}$ have the same definition in terms or other "barred" Lagrangian parameters.