Karlsruher Institut für Technologie

Particle Physics Phenomenology
after the Higgs Discovery

Mixed QCD-electroweak corrections to the production of Z and W bosons and their impact on the W mass measurements at the LHC

Arnd Behring and Kirill Melnikov

based on work done in collaboration with F. Buccioni (Oxford), F. Caola (Oxford), M. Delto (KIT), M. Jaquier (KIT), R. Röntsch (CERN)

hep-ph: 2103.02671, 2009.10386, 2005.10221, 1909.08428

KITP Santa Barbara Program “New Physics from Precision at High Energy”

Precision electroweak tests and the W mass

Since the Standard Model of particle physics is a renormalizable quantum theory, any physical quantity can be fully predicted once a few parameters of the theory have been (experimentally) established.

$$m_Z, G_F, \alpha_s(m_Z), \alpha_{\rm em}(m_Z), m_H, m_t, m_b, .., V_{\rm CKM}$$

Any physical quantity that can be measured with high precision and is not one of the input parameters, can be used to test the intrinsic consistency of the Standard Model at the loop level.

The mass of the W-boson is one of such quantities. When written in terms of the input parameters it reads

$$m_W^2 = m_Z^2 \left(1 - \frac{\pi\alpha(1+\Delta r(m_t, m_H, m_Z, \alpha, ..)}{\sqrt{2} G_F m_Z^2} \right)$$

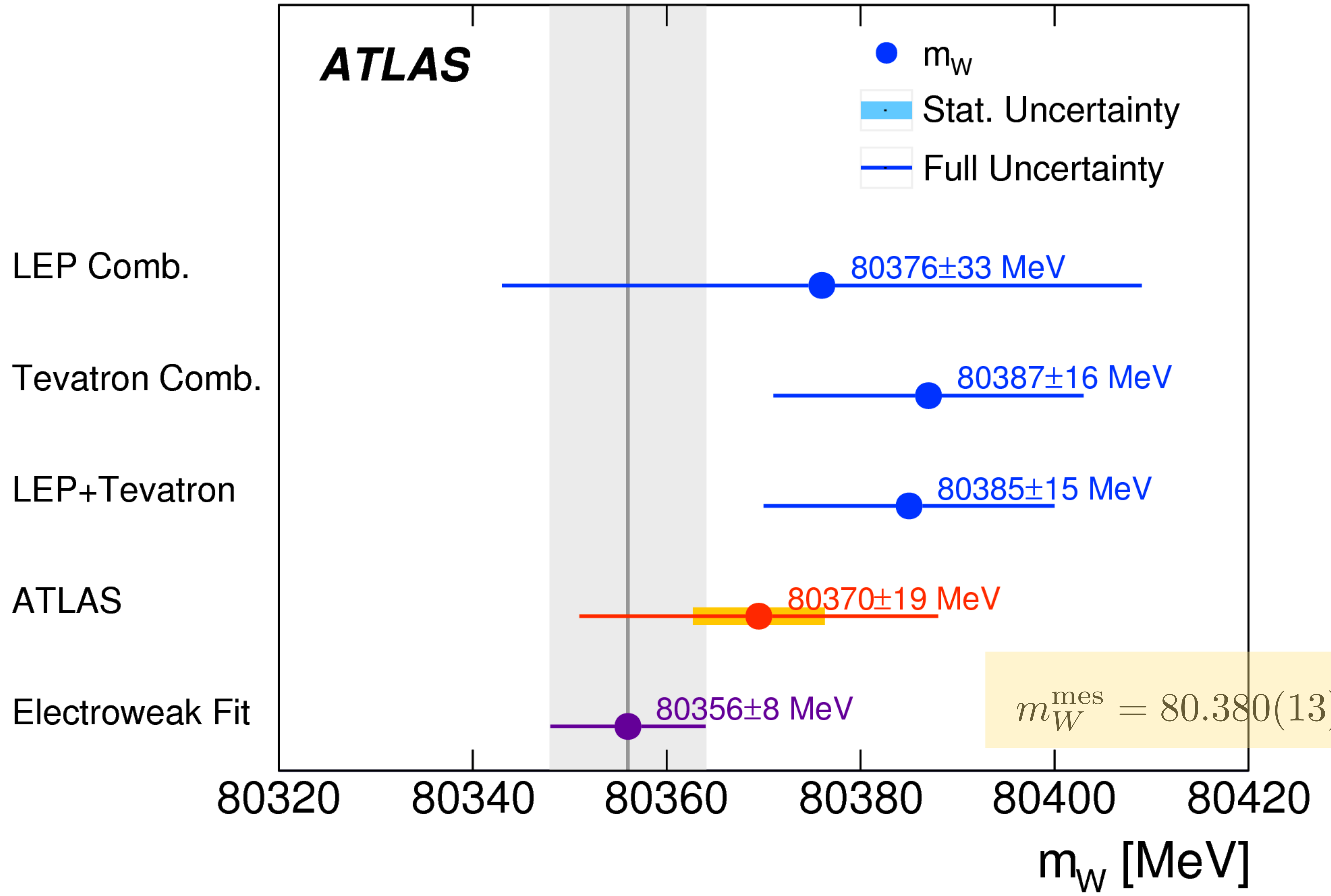
Djouadi, Verzegnassi, Kniehl, Gambino, van der Bij, Fleischer, Tarasov, Jegerlehner, Degrassi, Vicini, Freitas, Hollik, Weiglein, Awramik, Czakon, Onishchenko, Veretin, Chetyrkin, Kühn, Steinhauser, etc.

parameter	measurement	full EWK fit without m_H	full EWK fit with m_H
M_H [GeV]	125.09 ± 0.15	91 ± 19	125.09 ± 0.15
M_W [GeV]	80.380 ± 0.013	80.374 ± 0.01	80.360 ± 0.006
Γ_W [GeV]	2.085 ± 0.042	2.092 ± 0.001	2.091 ± 0.001
m_t [GeV]	172.9 ± 0.5	172.9 ± 0.5	173.1 ± 0.5
$\sin^2\theta^l_{\rm eff}$	0.2314 ± 0.00023	0.2314 ± 0.00009	0.23152 ± 0.00006
M_Z [GeV]	91.188 ± 0.002	91.188 ± 0.002	91.188 ± 0.002
$\sigma^0_{\rm had}$ [nb]	41.54 ± 0.037	41.482 ± 0.015	41.483 ± 0.015
Γ_Z [GeV]	2.495 ± 0.002	2.495 ± 0.001	2.495 ± 0.001
A_c	0.67 ± 0.027	0.6683 ± 0.0003	0.6679 ± 0.0002
A_b	0.923 ± 0.02	0.9347 ± 0.00006	0.93462 ± 0.00004
A_l (SLD)	0.1513 ± 0.00207	0.14797 ± 0.00073	0.14707 ± 0.00044
A_l (LEP)	0.1465 ± 0.0033	0.14797 ± 0.00073	0.14707 ± 0.00044
$A^l_{\rm FB}$	0.0171 ± 0.001	0.01642 ± 0.00016	0.01622 ± 0.0001
$A^c_{\rm FB}$	0.0707 ± 0.0035	0.0742 ± 0.0004	0.0737 ± 0.0002
$A^b_{\rm FB}$	0.0992 ± 0.0016	0.1037 ± 0.0005	0.1031 ± 0.0003
R^0_l	20.767 ± 0.025	20.747 ± 0.018	20.744 ± 0.018
R^0_c	0.1721 ± 0.003	0.17226 ± 0.00008	0.17225 ± 0.00008
R^0_b	0.21629 ± 0.00066	0.2158 ± 0.00011	0.21581 ± 0.00011
$\Delta\alpha^{(5)}_{\rm had}$ [10^{-5}]	2760 ± 9	0.02761 ± 9	2757 ± 9
$\alpha_s(M_Z)$	0.1181 ± 0.0011	0.1198 ± 0.003	0.1197 ± 0.003

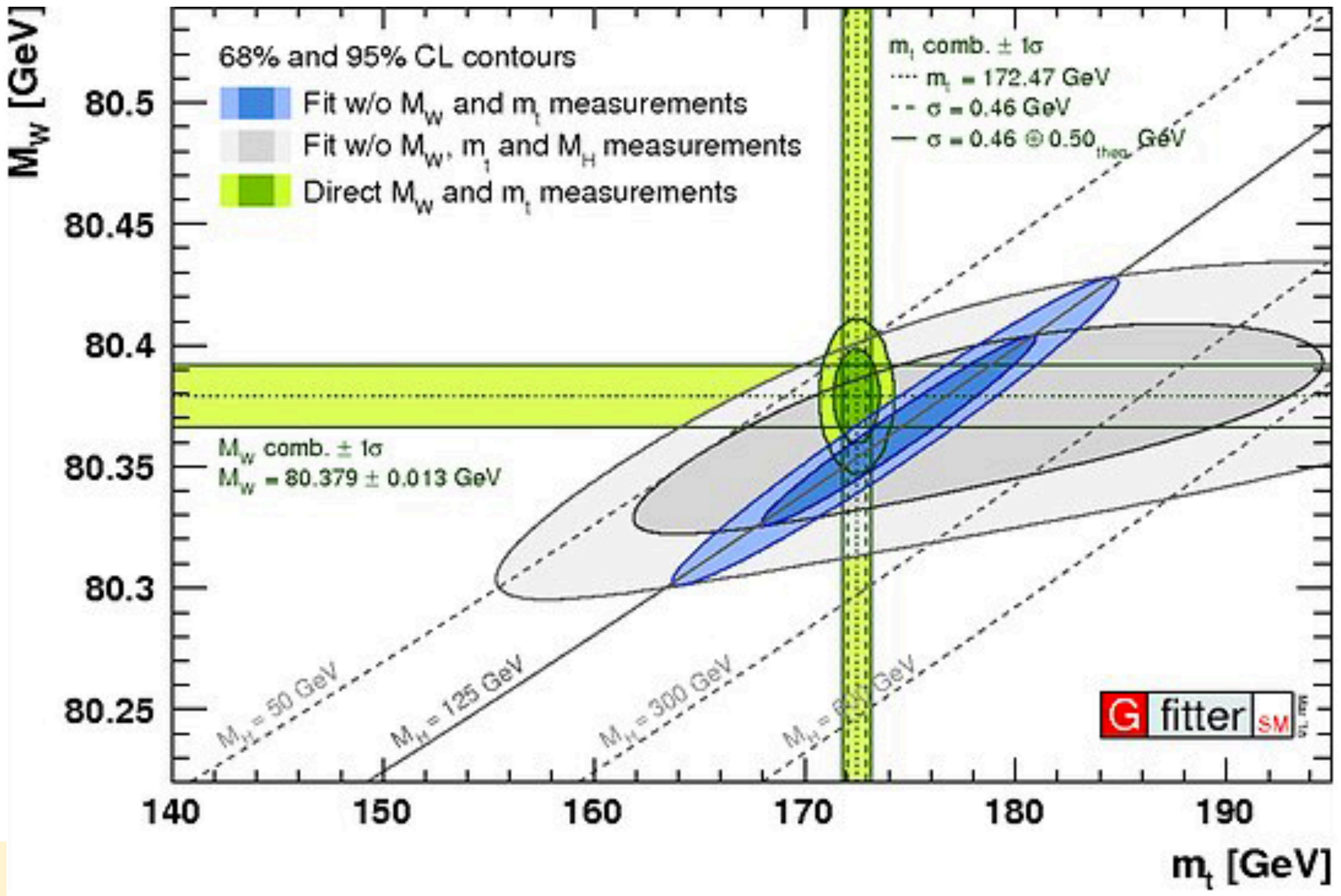
Gfittter: Haller et al. (2018); table taken from Erler and Schott

W mass measurements: an overall picture

Measurements of the W mass have a long history. They started from the threshold scan at LEP2 and continued with studies at the Tevatron Run I and Run II. There is also an LHC measurement by the ATLAS collaboration.



$$m_W^{\mathrm{mes}} = 80.380(13)\ \mathrm{GeV}$$



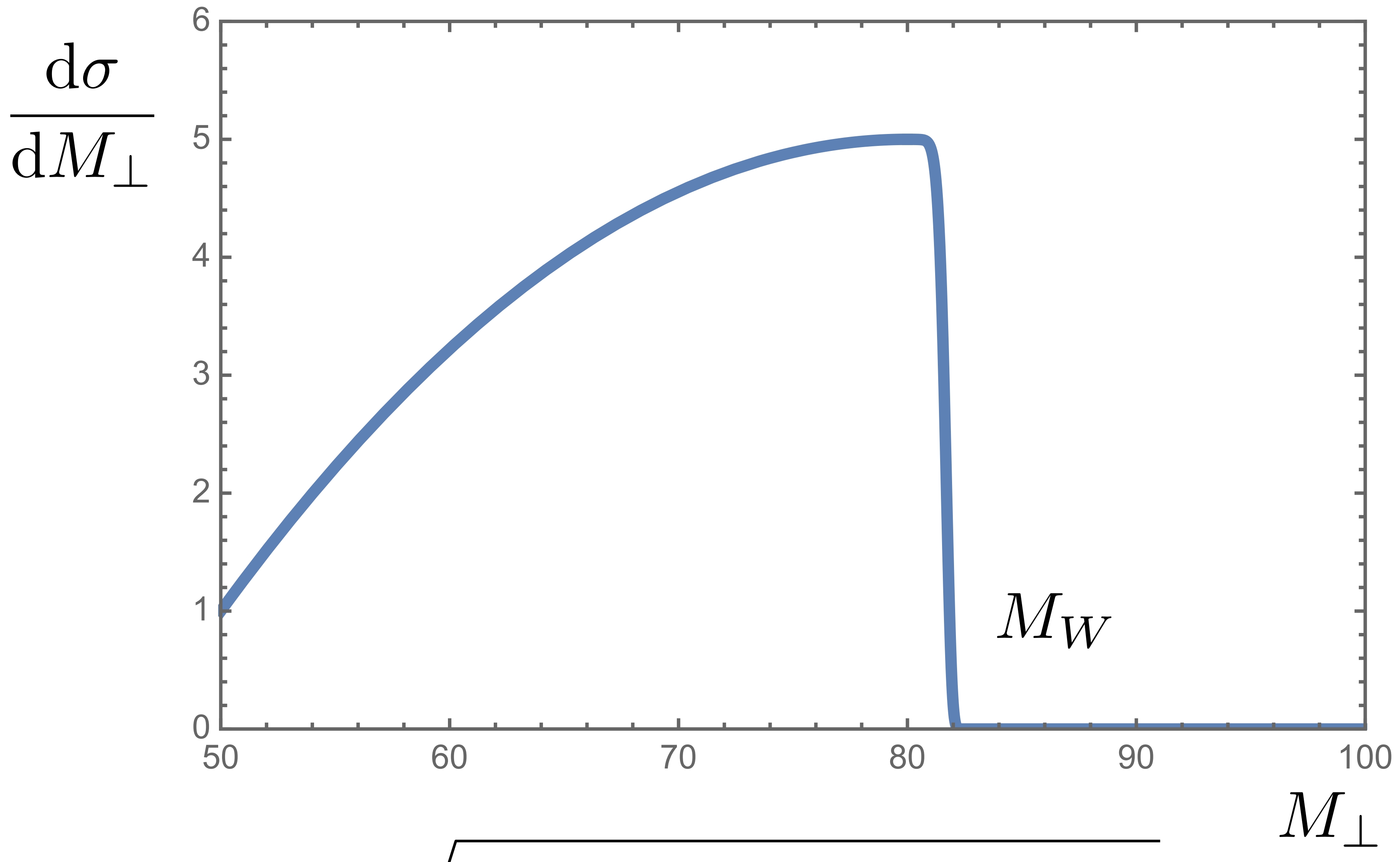
Results of direct measurements are slightly (1.5 sigma?) higher than the result of electroweak fit.

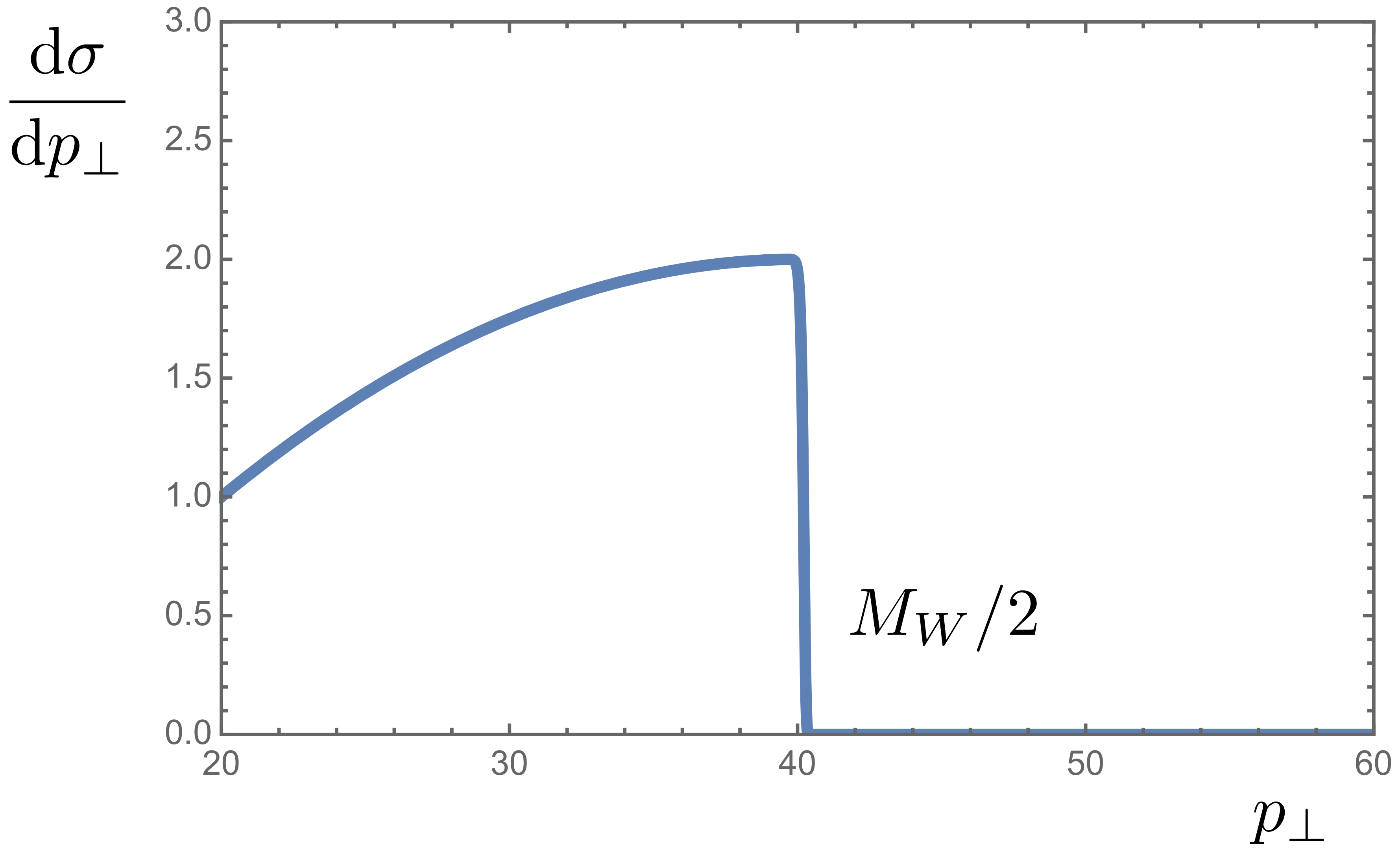
Hadron colliders already play and will continue to play the leading role in measuring the W mass. Precision achieved in the EW fit sets target precision for future measurements.

W mass measurements at a hadron collider

The W mass is extracted from observables sensitive to its value.   The kinematic edge at the mass value is a perfect, super-sensitive observable.

For the purpose of the W mass measurement, two variables with an edge have been used at hadron colliders. They are the transverse mass and the transverse momentum of the charged lepton. In the simplest case (LO modelling and ideal detectors) these distributions have edges at m_W and $m_W/2$, respectively.





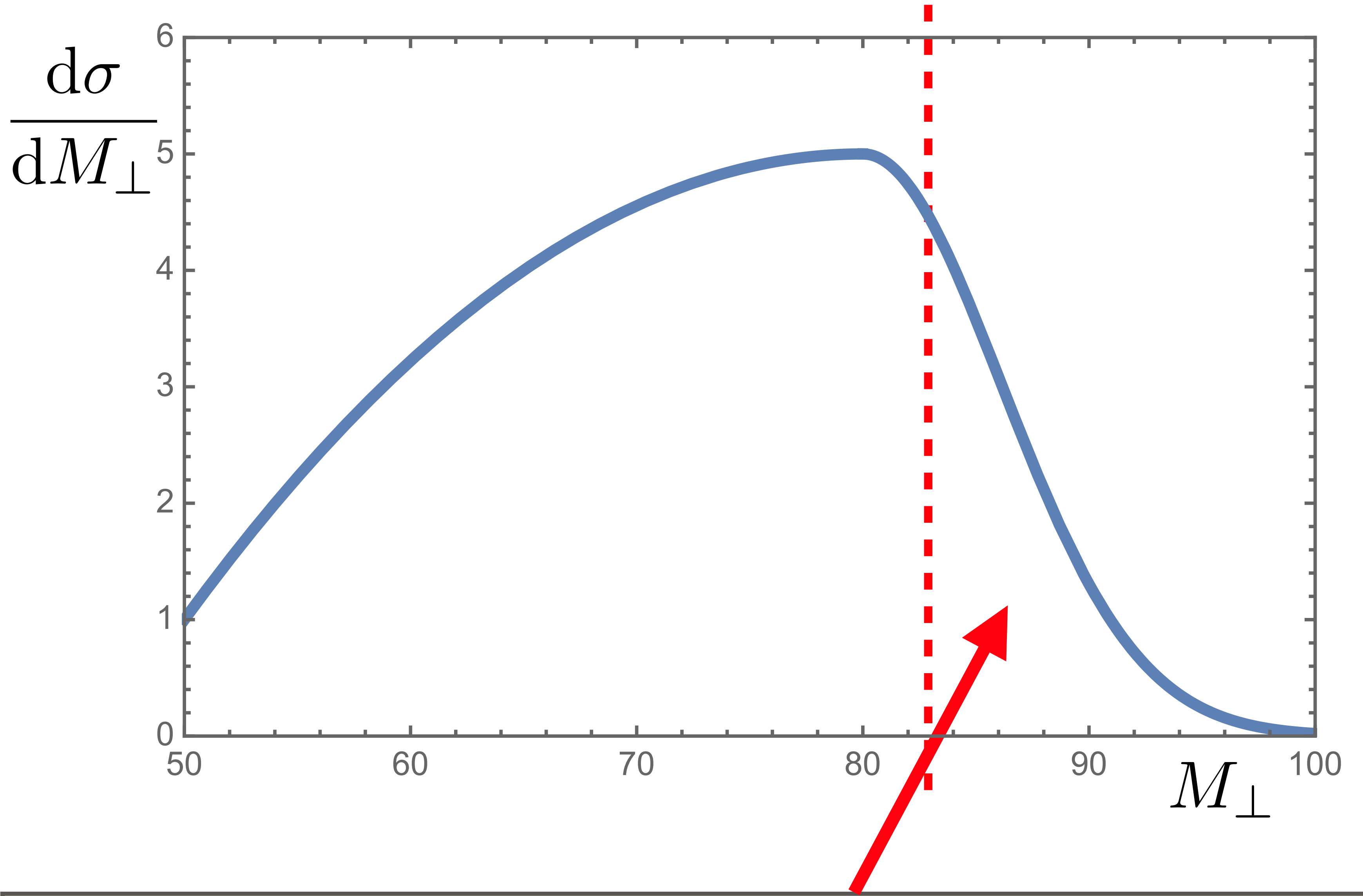
$$M_\perp = \sqrt{2p_{\perp,e^+}p_{\perp,\nu}(1-\cos\phi_{e^+\nu})}$$

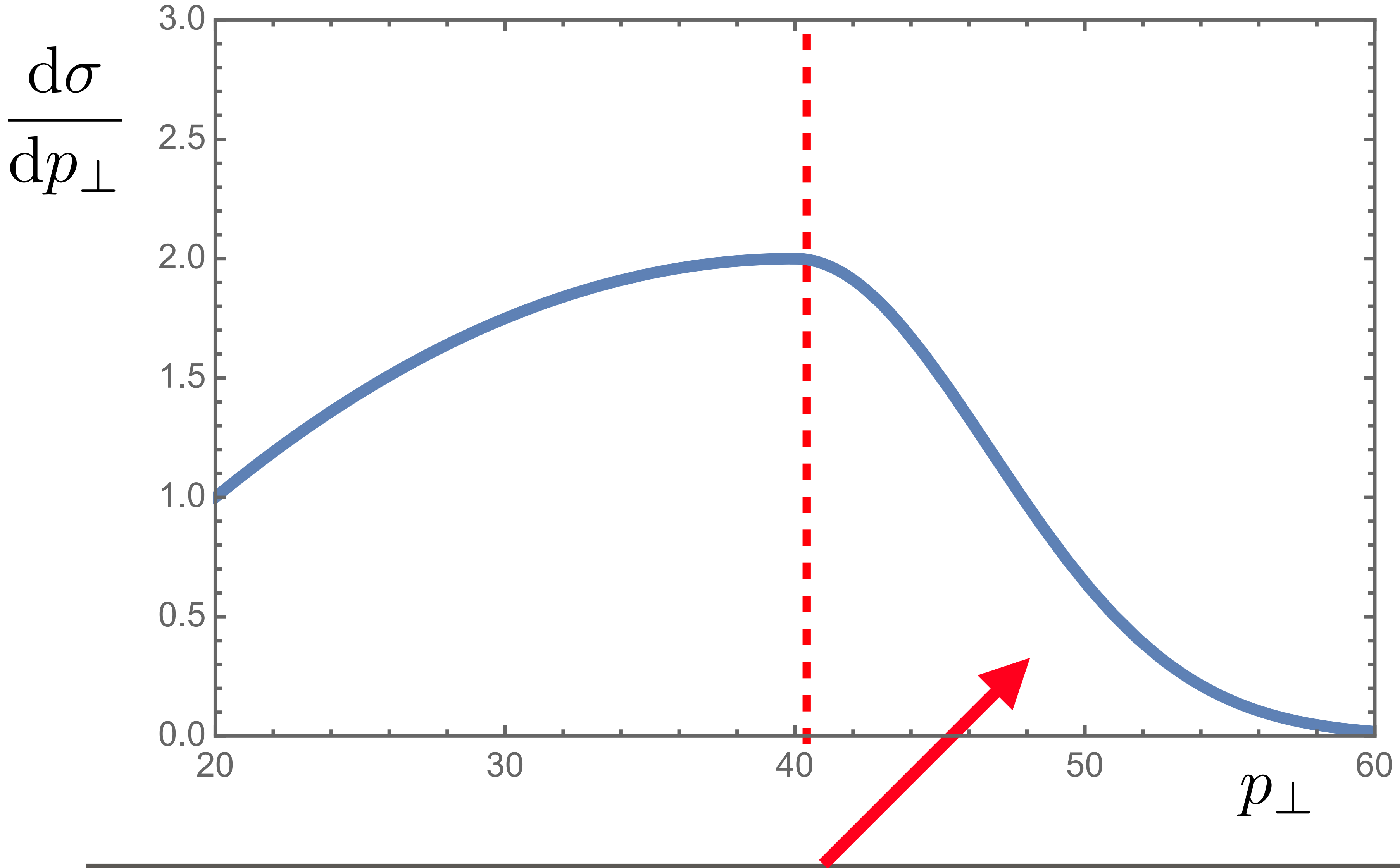
The two observables have different sensitivity to experimental uncertainties and the quality of theoretical modelling.

W mass measurements at a hadron collider

The W mass is extracted from observables sensitive to its value.   The kinematic edge at the mass value is a perfect, super-sensitive observable.

For the purpose of the W mass measurement, two variables with an edge have been used at hadron colliders. They are the transverse mass and the transverse momentum of the charged lepton. In the simplest case (LO modelling and ideal detectors) these distributions have edges at m_W and $m_W/2$, respectively.





BEYOND THE EDGE: DETECTOR EFFECTS, MOSTLY

QCD AND QED INITIAL STATE RADIATION, MOSTLY

The two observables have different sensitivity to experimental uncertainties and the quality of theoretical modelling.

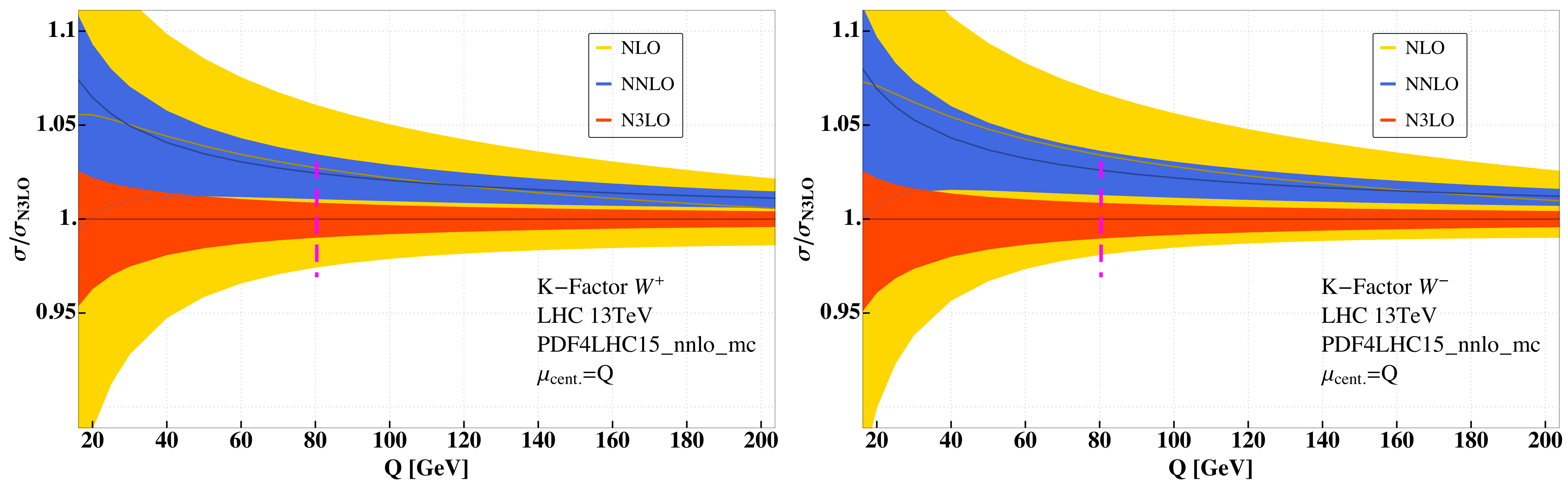
Theory for the W mass measurements at hadron colliders

We need a reliable way to describe the W transverse mass distribution and/or the charged lepton transverse momentum distribution with realistic selection cuts.

In principle, this can be done using standard theoretical tools including fixed order computations (NLO, NNLO, N3LO QCD, NLO electroweak), resummations, showers, etc. The precision with which even the simplest observables can be predicted within this framework is a percent or worse.

$$\mathrm{d}\sigma = \sum_{ij} \int \mathrm{d}x_1 \mathrm{d}x_2 \; f_i(x_1) f_2(x_2) \; \mathrm{d}\sigma_{ij}(x_1, x_2)$$

$$\mathrm{d}\sigma_{ij} = \sigma_{ij}^{(0,0)} + \alpha_s \sigma_{ij}^{(1,0)} + \alpha_{ew} \sigma_{ij}^{(0,1)} + \alpha_s^2 \sigma_{ij}^{(2,0)} + \alpha_s^3 \sigma_{ij}^{(3,0)} + \alpha_s \alpha_{ew} \sigma_{ij}^{(1,1)} + \ldots$$



Duhr, Dulat, Mistlberger

Theory for the W mass measurements at hadron colliders

Since the goal is to extract the W mass with a precision of about 10 MeV, we need to control observables from which the W mass is extracted to a tune of 10^{-2} percent. This is beyond what can be achieved using any existing theoretical tool that is based on first principles.

This has two consequences:

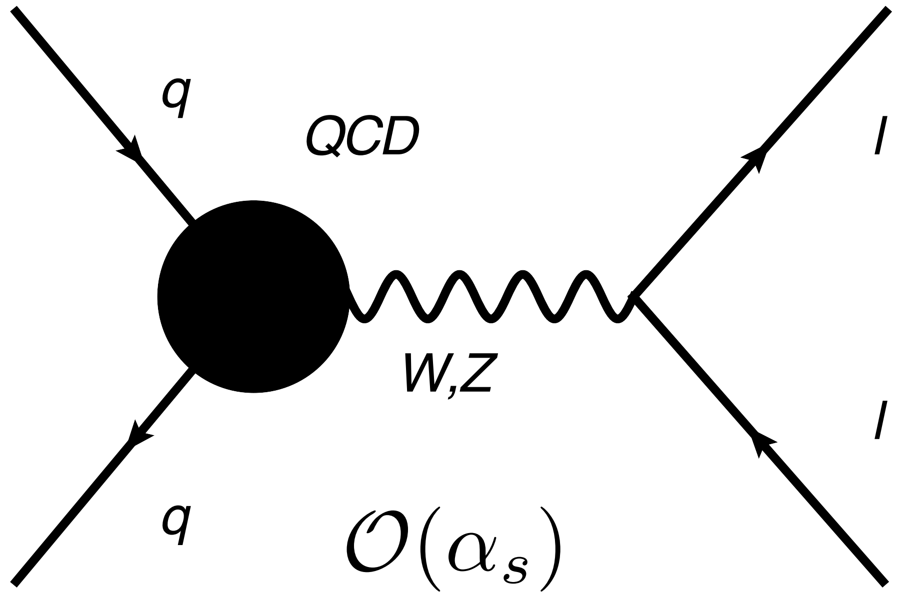
1) one must give up on using advanced theory to model relevant distributions. Instead, one measures the Z distributions, parametrizes them in a QCD-motivated way and transfers them to the W case arguing that QCD does not distinguish between Z and W production.

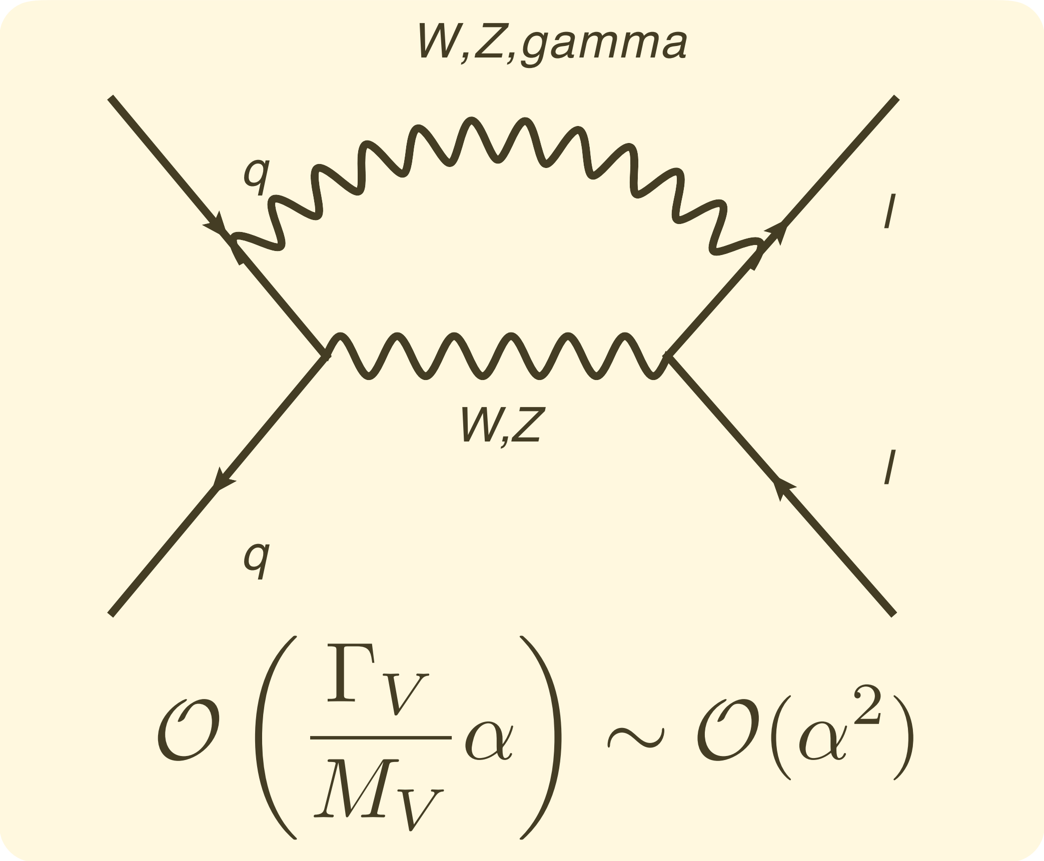
2) given the target precision of 10^{-2} percent, small effects that distinguish between Z and W distributions may matter. Electroweak corrections to Z and W production are obvious examples of the (potentially) relevant effects.

Electroweak effects, with and without QCD

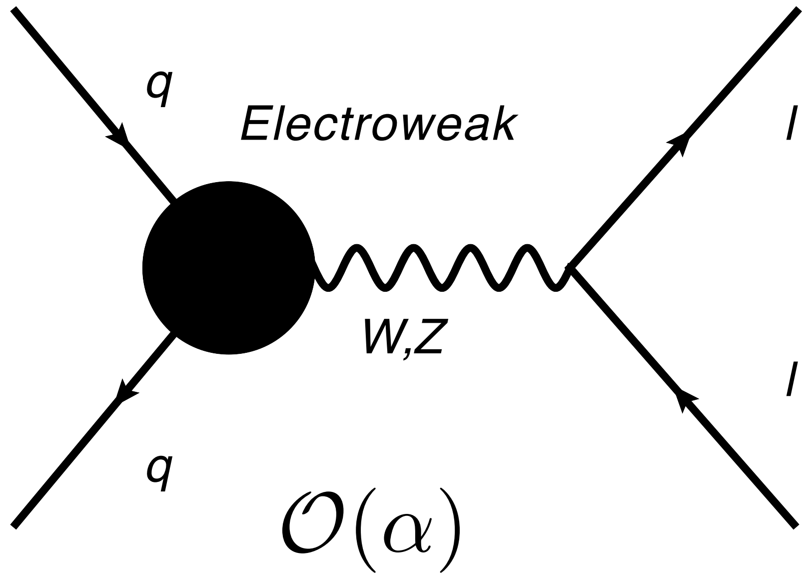
Computation of electroweak corrections to Z and W production brings in additional challenges.This happens because, in contrast to the QCD corrections, both initial and final states carry electroweak charges and, therefore, can interact with each other.

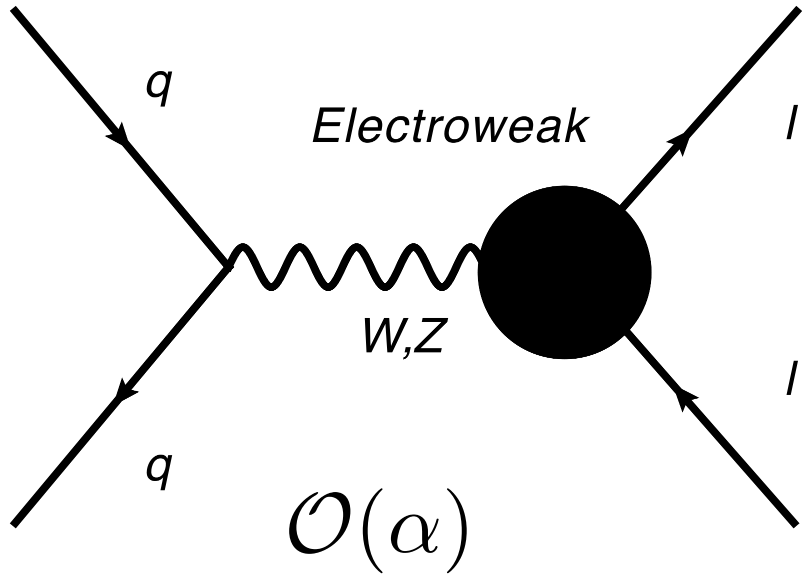
However, compared to the resonance (mass-shell) production, thee “non-factorizable” contributions have an additional suppression by $\Gamma_V / M_V \sim \alpha$. For this reason, they can be neglected as long as the NNLO electroweak factorizable corrections are not required.





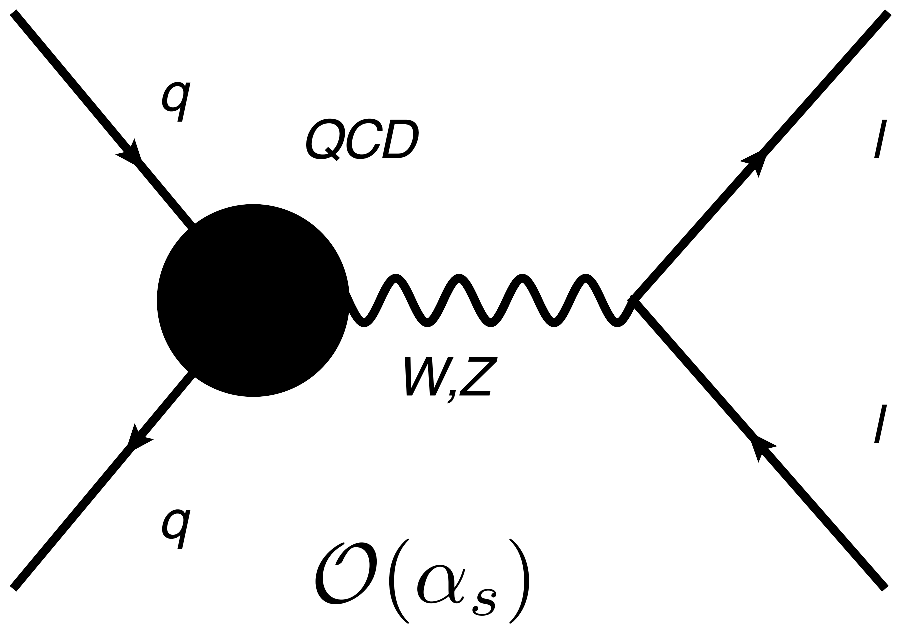
Huss, Dittmaier, Schwinn

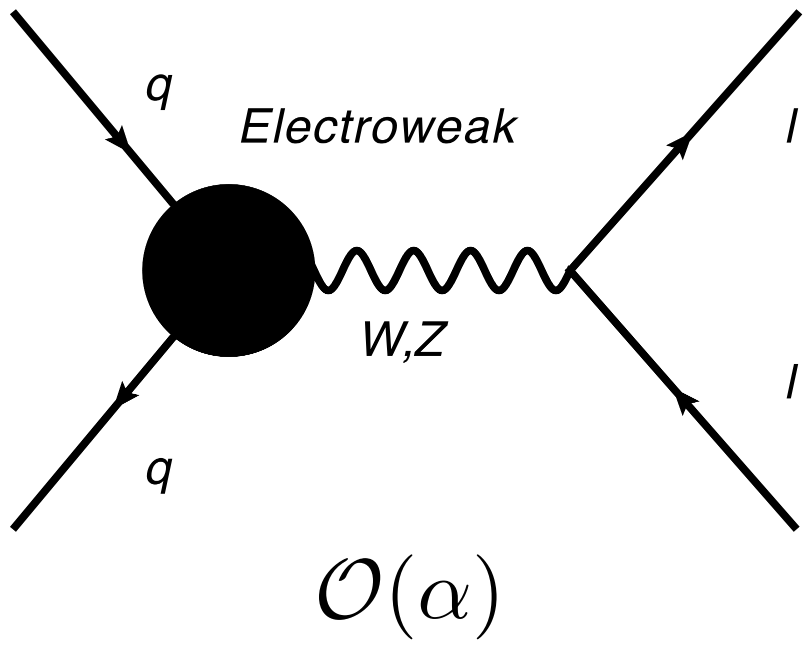


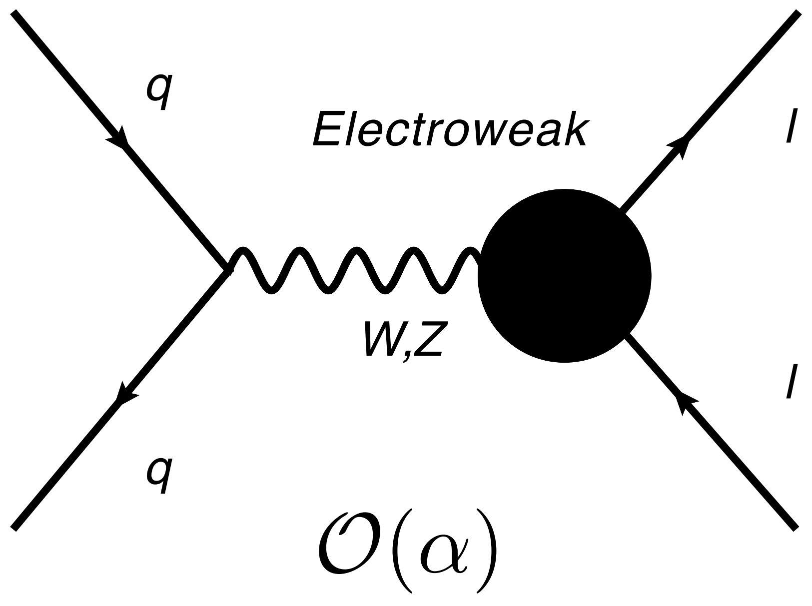


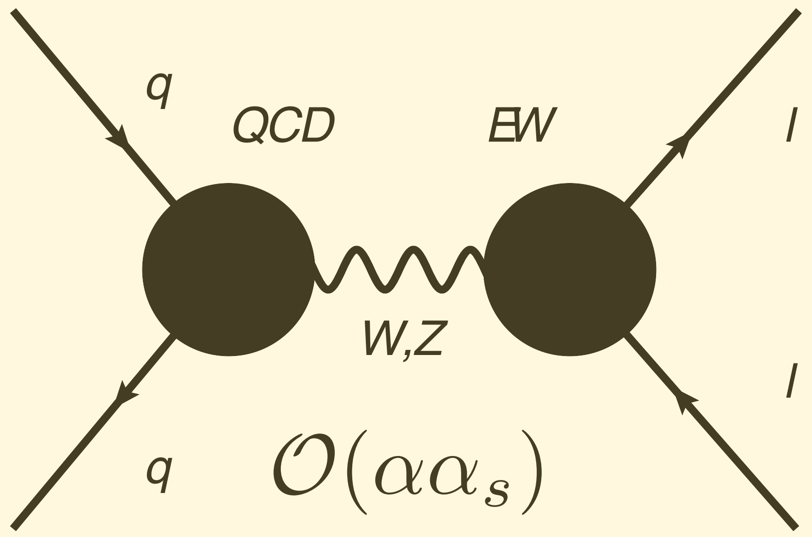
Electroweak effects, with and without QCD

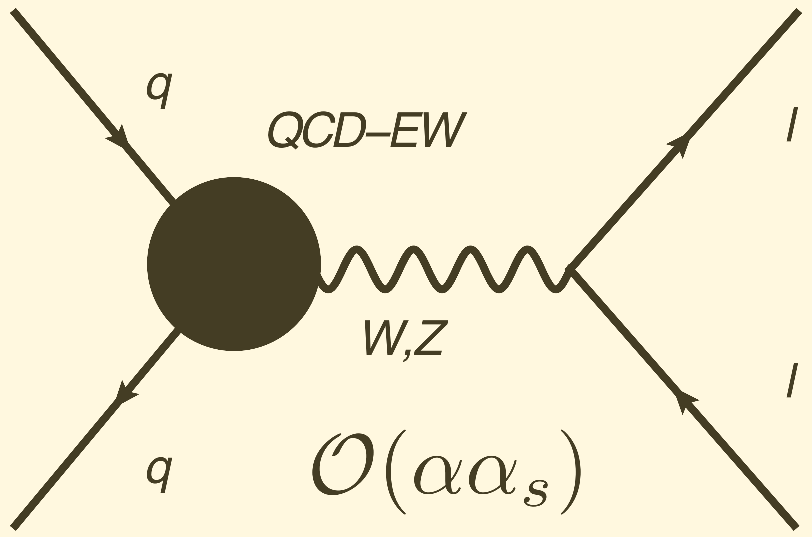
Given the target precision of 0.1 per mille, also the so-called mixed QCD-electroweak factorizable corrections may be needed.





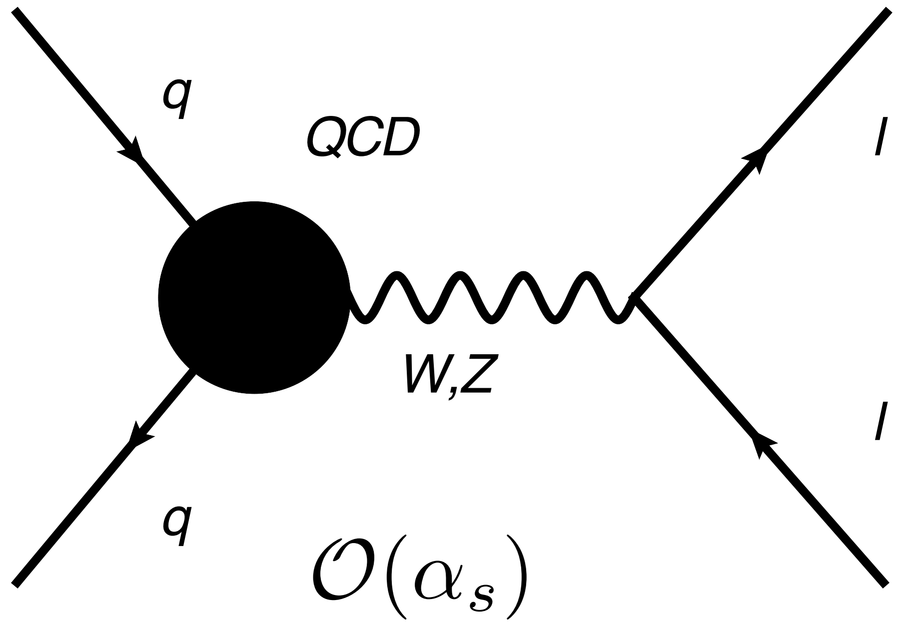


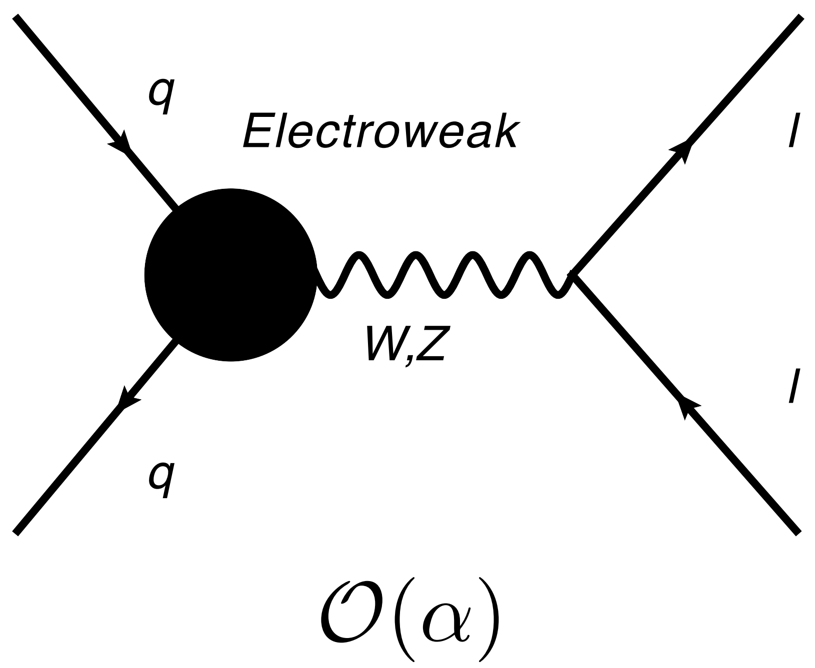


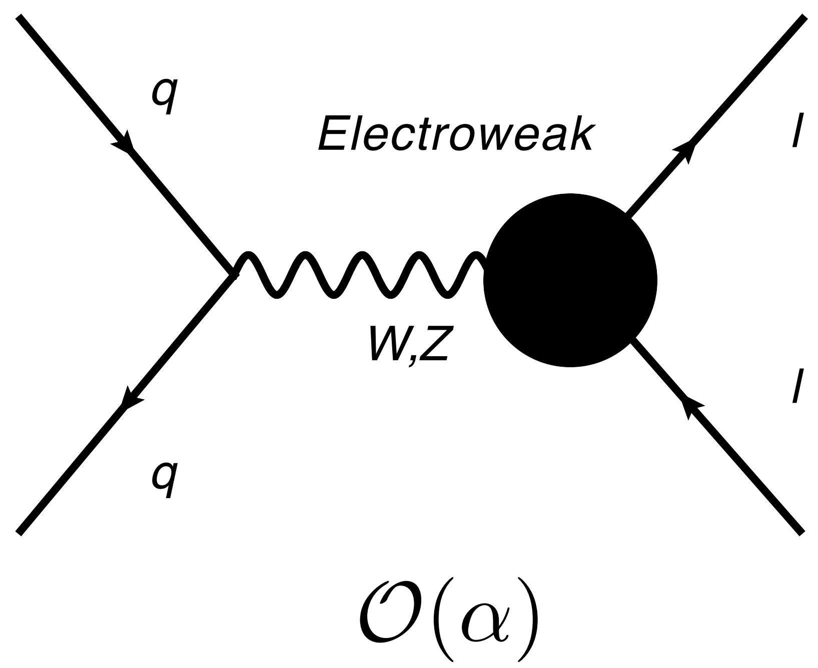


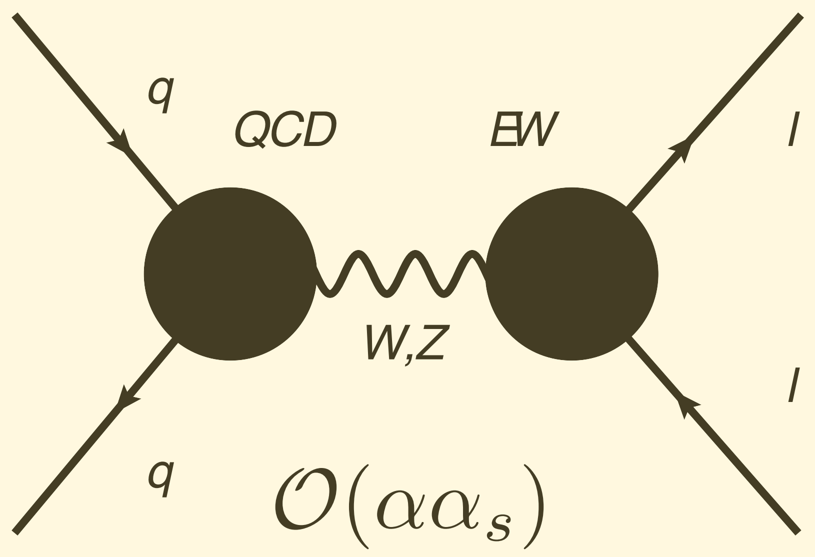
Electroweak effects, with and without QCD

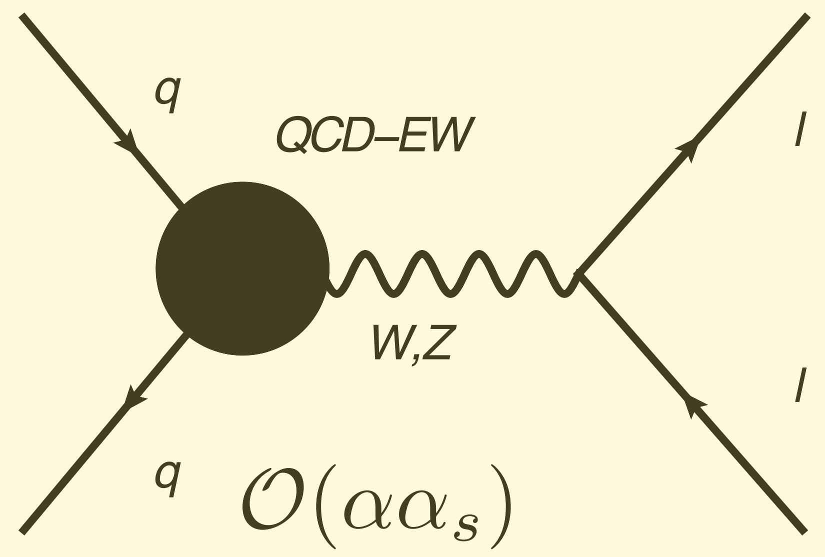
Given the target precision of 0.1 per mille, also the so-called mixed QCD-electroweak factorizable corrections may be needed.











Recent work by: De Florian, Der, Fabre; Cieri, De Florian, Der, Mazzitelli; Bonciani, Buccioni, Rana, Vicini,;Buonocore, Grazzini, Kallweit, Savoini, Tramontano

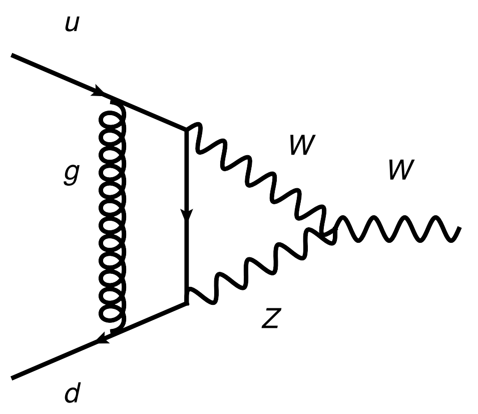
Huss, Dittmaier, Schwinn NLO$\otimes$NLO

This talk

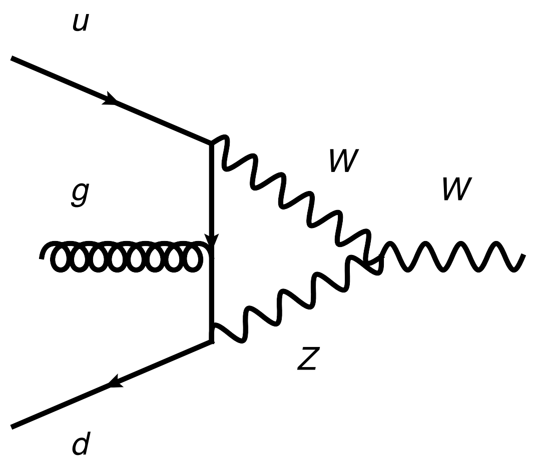
Mixed QCD-electroweak effects in the production

Fully-differential description of mixed QCD-electroweak effects in the production of (on-shell) electroweak bosons is a complicated problem.

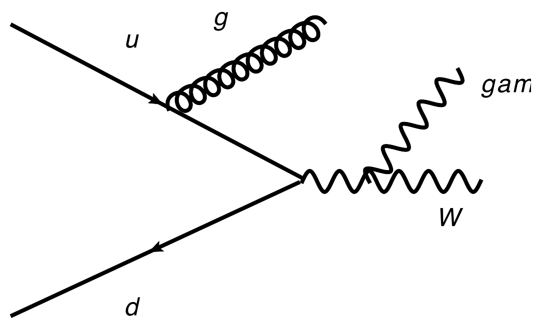
We need the two-loop form factor for mixed QCD-EW corrections.   Massive internal/external lines.



One-loop amplitudes for mixed QCD-EW corrections are required. Massive internal/external lines.



Real emission contributions with a photon and a QCD parton have to be dealt with. These contributions develop infra-red and collinear singularities and cannot be evaluated directly.

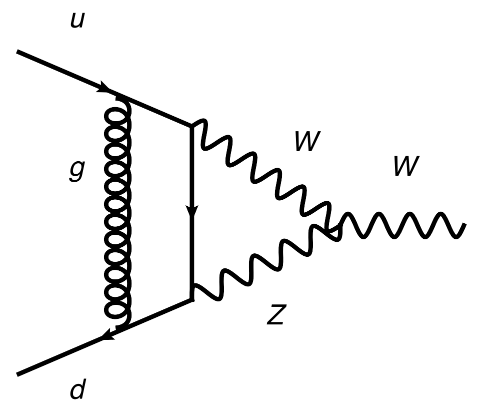


Thanks to the recent advances in the field of perturbative QCD, it becomes relatively straightforward to deal with these problems.

Mixed QCD-electroweak effects in the production

Fully-differential description of mixed QCD-electroweak effects in the production of (on-shell) electroweak bosons is a complicated problem.

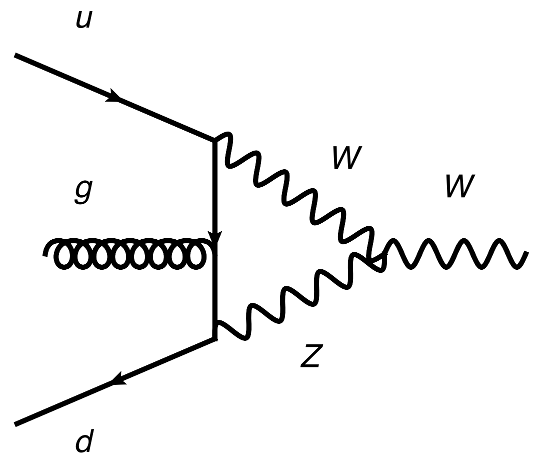
Need the two-loop form factor for mixed QCD-EW corrections. Massive internal/external lines.



Reduction to master integrals, differential equations, canonical bases, GPLs.

Reduze2 (von Manteuffel, Studerus), Henn et. al.

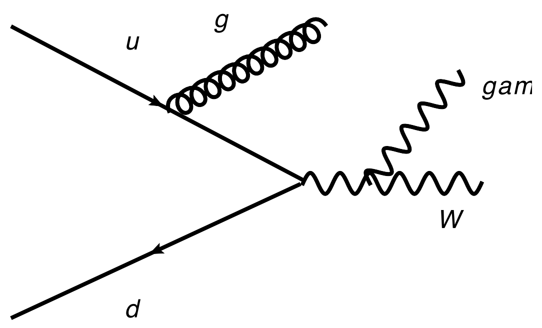
One-loop amplitudes for mixed QCD-EW corrections are required. Massive internal/external lines.



OpenLoops for numerical computation of one-loop amplitudes.

Cascioli, Maierhöfer, Pozzorini, Buccioni, Zoller et al.

Real emission contributions with a photon and a gluon have to be dealt with. These contributions develop infra-red and collinear singularities and cannot be evaluated directly.



An adaptation of NNLO QCD subtractions. We employ the so-called nested soft-collinear subtraction scheme. For the Z boson, no adaptation is required; just the color factors need to be adjusted.

Caola, Melnikov, Röntsch et. al.

Mixed QCD-electroweak effects in the production

The formula that describes fully-differential mixed QCD-electroweak corrections to vector boson production is relatively compact. Here is an example of the W production in an association with a gluon and a photon.

Parts of the results are explicit. Other parts are written in terms of operators that act on four-dimensional, physical matrix elements and four-dimensional phase spaces extracting universal soft and collinear limits in terms of splitting functions, eikonal functions as well as reduced-multiplicity phase spaces and matrix elements.

It is convenient to write the cross section in a way that separates contributions with different multiplicities and/or kinematic features of final/initial state particles.

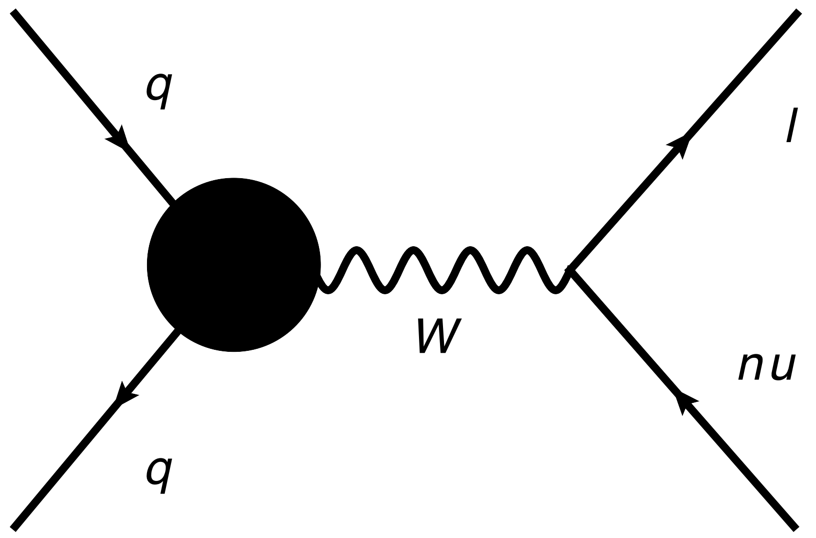
$$\mathrm{d}\sigma^{\mathrm{QCD}\otimes\mathrm{EW}}_{u\bar{d}\to W(g\gamma)} = \mathrm{d}\sigma^{\mathrm{elastic}}_{u\bar{d}\to W(g\gamma)} + \mathrm{d}\sigma^{\mathrm{boost}}_{u\bar{d}\to W(g\gamma)} + \mathrm{d}\sigma^{\mathcal{O}_{\mathrm{NLO}}}_{u\bar{d}\to W(g\gamma)} + \mathrm{d}\sigma^{\mathrm{regulated}}_{u\bar{d}\to W(g\gamma)}$$

Mixed QCD-electroweak effects in the production

The elastic piece arises from the double-unresolved real emissions of photons and gluons (both soft and/or soft/collinear). and from the finite remainders of virtual corrections.

$$\boxed{\mathrm{d}\sigma^{\mathrm{QCD}\otimes\mathrm{EW}}_{u\bar{d}\to W(g\gamma)} = \mathrm{d}\sigma^{\mathrm{elastic}}_{u\bar{d}\to W(g\gamma)} + \mathrm{d}\sigma^{\mathrm{boost}}_{u\bar{d}\to W(g\gamma)} + \mathrm{d}\sigma^{\mathcal{O}_{\mathrm{NLO}}}_{u\bar{d}\to W(g\gamma)} + \mathrm{d}\sigma^{\mathrm{regulated}}_{u\bar{d}\to W(g\gamma)}}$$

$$\begin{aligned} 2s \cdot \mathrm{d}\sigma^{\mathrm{elastic}}_{u\bar{d}\to W(g\gamma)} = &\left(\frac{\alpha_s(\mu)}{2\pi}\,\frac{\alpha_{EW}}{2\pi}\right) C_F \left[\frac{8\pi^4}{45}\left(Q_u^2+Q_d^2\right) + \left(\frac{4\pi^2}{3} - \frac{\pi^4}{3}\right) Q_W^2\right] \langle F_{\mathrm{LM}}(1_u, 2_{\bar{d}})\rangle \\ &+ \left(\frac{\alpha_{EW}}{2\pi}\right)\left[\frac{\pi^2}{3}\left(Q_u^2+Q_d^2\right) + \left(2 - \frac{\pi^2}{2}\right) Q_W^2\right] \left\langle F^{\mathrm{fin,QCD}}_{\mathrm{LV}}(1_u, 2_{\bar{d}})\right\rangle \\ &+ \left(\frac{\alpha_s(\mu)}{2\pi}\right)\left[C_F\,\frac{2\pi^2}{3}\right] \left\langle F^{\mathrm{fin,EW}}_{\mathrm{LV}}(1_u, 2_{\bar{d}})\right\rangle + \left\langle F^{\mathrm{fin,QCD}\otimes\mathrm{EW}}_{\mathrm{LVV+LV^2}}(1_u, 2_{\bar{d}})\right\rangle . \end{aligned}$$



$$\langle F_{\mathrm{LM}}(1_u, 2_{\bar{d}})\rangle = \mathcal{N} \int \mathrm{d}\Phi_W \, |\mathcal{M}(1_u, 2_{\bar{d}}; W)|^2.$$

$$\langle F_{\mathrm{LV,LVV,etc.}}(1_u, 2_{\bar{d}})\rangle = \mathcal{N} \int \mathrm{d}\Phi_W \, |\mathcal{M}^{\mathrm{loops}}(1_u, 2_{\bar{d}}; W)|^2.$$

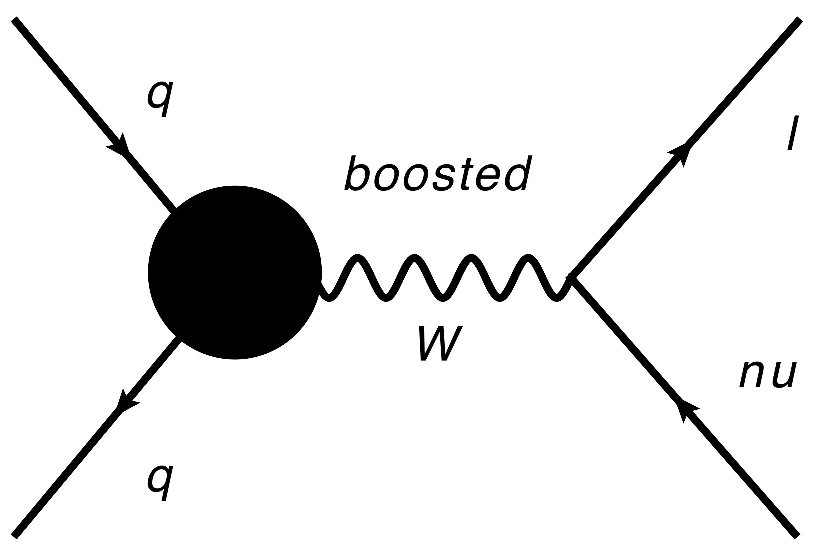
Mixed QCD-electroweak effects in the production

The boosted contribution arises from final states where either a gluon and a photon are collinear to incoming partons or one of them is collinear and the other is soft.

$$\mathrm{d}\sigma^{\mathrm{QCD}\otimes\mathrm{EW}}_{u\bar{d}\to W(g\gamma)} = \mathrm{d}\sigma^{\mathrm{elastic}}_{u\bar{d}\to W(g\gamma)} + \mathrm{d}\sigma^{\mathrm{boost}}_{u\bar{d}\to W(g\gamma)} + \mathrm{d}\sigma^{\mathcal{O}_{\mathrm{NLO}}}_{u\bar{d}\to W(g\gamma)} + \mathrm{d}\sigma^{\mathrm{regulated}}_{u\bar{d}\to W(g\gamma)}$$

$$\begin{aligned}
2s\cdot \mathrm{d}\sigma^{\mathrm{boost}}_{u\bar{d}\to W(g\gamma)} &= \left(\frac{\alpha_{EW}}{2\pi}\,\frac{\alpha_s(\mu)}{2\pi}\right)\Bigg\{C_F(Q_u^2+Q_{\bar{d}}^2)\int\limits_0^1 \mathrm{d}z_1\mathrm{d}z_2\widetilde{\mathcal{P}}^{\mathrm{NLO}}_{qq}(z_1,E_c)\frac{F_{\mathrm{LM}}(z_1\cdot 1_u, z_2\cdot 2_{\bar{d}})}{z_1 z_2}\widetilde{\mathcal{P}}^{\mathrm{NLO}}_{qq}(z_2,E_c)\\
&\quad + C_F\int\limits_0^1 \mathrm{d}z\,\sum_{i=1}^{2}P^{\mathrm{NNLO}}_{qq}(Q_i,Q_{j\neq i},z)\left\langle F^{(i)}_{\mathrm{LM}}(1_u,2_{\bar{d}}\,|\,z)\right\rangle\Bigg\}\\
&\quad + \int\limits_0^1 \mathrm{d}z\,\mathcal{P}^{\mathrm{NLO}}_{qq}(z)\sum_{i=1}^{2}\left[\left(\frac{\alpha_{EW}}{2\pi}\right)Q_i^2\left\langle F^{(i),\mathrm{fin,QCD}}_{\mathrm{LV}}(1_u,2_{\bar{d}}\,|\,z)\right\rangle + C_F\left(\frac{\alpha_s(\mu)}{2\pi}\right)\left\langle F^{(i),\mathrm{fin,EW}}_{\mathrm{LV}}(1_u,2_{\bar{d}}\,|\,z)\right\rangle\right].
\end{aligned}$$

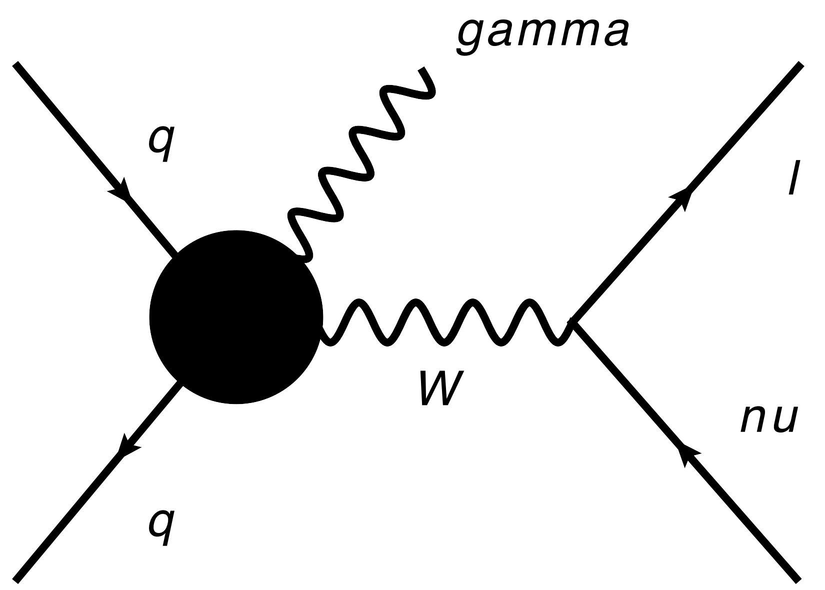
$$\begin{aligned}
P^{\mathrm{NNLO,d}}_{qq} &= \left(8-\frac{2\pi^2}{3}\right)D_1(z) - \zeta_2(1-z) + \left(2\zeta_2(1+z)-\frac{8}{1-z}\right)\ln(1-z) - 2z\ln(z)\\
&\quad + \frac{2z(1+z^2)\left(\ln^2(z)-2\ln(z)\ln(1-z)\right)}{(1-z)^2} + \frac{1+z^2}{1-z}\left(\zeta_2\ln(z)+2\left[2\ln(1-z)-\ln(z)\right]\mathrm{Li}_2(1-z)\right)\\
&\quad + 2(1-z)\mathrm{Li}_2(1-z).
\end{aligned}$$



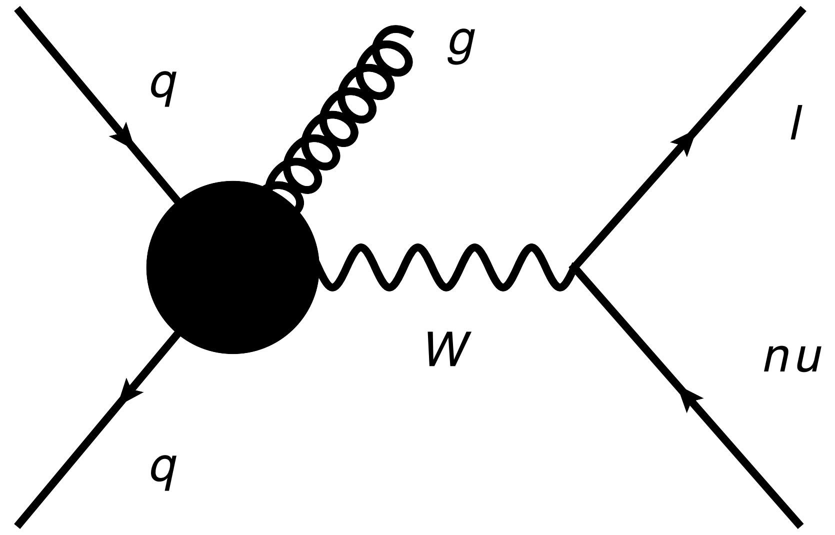
Mixed QCD-electroweak effects in the production

The $\mathcal{O}_{\rm NLO}$ terms describe contributions of $W\,\gamma$ and $W\,g$ final states. They arise from virtual corrections to these final state and from remnants of $W\,\gamma\,g$ state in case that. either a gluon or a photon become unresolved.

$$\boxed{{\rm d}\sigma^{\rm QCD\otimes EW}_{u\bar d\to W(g\gamma)} = {\rm d}\sigma^{\rm elastic}_{u\bar d\to W(g\gamma)} + {\rm d}\sigma^{\rm boost}_{u\bar d\to W(g\gamma)} + {\rm d}\sigma^{\mathcal{O}_{\rm NLO}}_{u\bar d\to W(g\gamma)} + {\rm d}\sigma^{\rm regulated}_{u\bar d\to W(g\gamma)}}$$



$$\begin{aligned}
2s\cdot{\rm d}\sigma^{\mathcal{O}_{\rm NLO}}_{u\bar d\to W(g\gamma)} &= \left\langle \mathcal{O}^{\gamma}_{\rm NLO}\left[F^{\rm fin,QCD}_{\rm LV}(1_u,2_{\bar d};4_\gamma)\right]\right\rangle + \left\langle \mathcal{O}^{g}_{\rm NLO}\left[F^{\rm fin,EW}_{\rm LV}(1_u,2_{\bar d};4_g)\right]\right\rangle \\
&+ \left(\frac{\alpha_s(\mu)}{2\pi}\right) C_F\left[\frac{2\pi^2}{3}+6L_c\right]\langle \mathcal{O}^{\gamma}_{\rm NLO}\left[F_{\rm LM}(1_u,2_{\bar d};4_\gamma)\right]\rangle + \left(\frac{\alpha_{EW}}{2\pi}\right)\Bigg\{ (Q_u^2+Q_d^2)\left[\frac{\pi^2}{3}+3L_c\right] \\
&+ Q_W^2\left[2L_c^2-5L_c-\frac{1}{2}\ln^2\left(\frac{1-\beta}{1+\beta}\right)-\frac{1}{\beta}\ln\left(\frac{1-\beta}{1+\beta}\right)\right] + Q_W\sum_{i=1}^{2} Q_i\Bigg[4L_c^2-\frac{3}{2}\ln\left(\frac{2p_i\cdot p_W}{M_W^2}\right) \\
&- 4L_c\ln\left(\frac{2p_i\cdot p_W}{M_W^2}\right)+\ln^2\left(\frac{2p_i\cdot p_W}{M_W^2}\right)+2{\rm Li}_2\left(1-\frac{1-\beta}{\kappa_{iW}}\right)+2{\rm Li}_2\left(1-\frac{1+\beta}{\kappa_{iW}}\right)\Bigg]\Bigg\}\langle \mathcal{O}^{g}_{\rm NLO}\left[F_{\rm LM}(1_u,2_{\bar d};4_g)\right]\rangle \\
&+ \int_0^1{\rm d}z\sum_{\substack{i,j=1\\ i\neq j}}^{2}\Bigg\{\left(\frac{\alpha_s(\mu)}{2\pi}\right)C_F\left\langle \mathcal{O}^{\gamma}_{\rm NLO}\left[\left(\widetilde{\mathcal{P}}^{\rm NLO}_{qq}(z,E_c)+\eta_{\gamma j}\ln(\eta_{\gamma i})\bar P^{\rm AP,0}_{qq,R}(z)\right)F^{(i)}_{\rm LM}(1_u,2_{\bar d};4_\gamma\mid z)\right]\right\rangle \\
&+ \left(\frac{\alpha_{EW}}{2\pi}\right)Q_i^2\left\langle \mathcal{O}^{g}_{\rm NLO}\left[\left(\widetilde{\mathcal{P}}^{\rm NLO}_{qq}(z,E_c)+\eta_{gj}\ln(\eta_{gi})\bar P^{\rm AP,0}_{qq,R}(z)\right)F^{(i)}_{\rm LM}(1_u,2_{\bar d};4_g\mid z)\right]\right\rangle\Bigg\}.
\end{aligned}$$



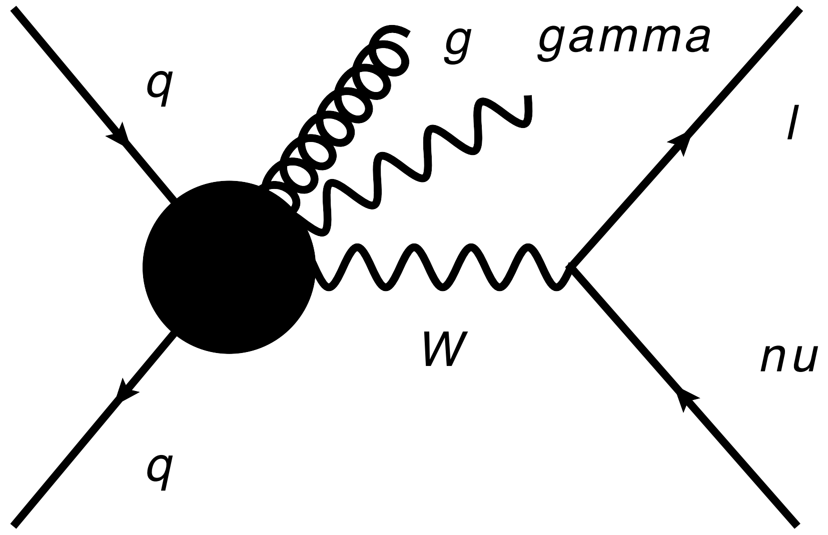
$$\mathcal{O}^{f}_{\rm NLO} = (I-S_f)(I-C_{1f}-C_{2f})$$

Mixed QCD-electroweak effects in the production

The regulated term is the only contribution that involves the fully-resolved component $W\,\gamma\,g$. It is computed numerically starting from four-dimensional matrix elements for $0 \to q\,\bar{q}'\,W\,\gamma\,g$. Subtraction terms involve known eikonal contributions and splitting functions. Also partition functions are required.

$$\boxed{\mathrm{d}\sigma^{\mathrm{QCD}\otimes\mathrm{EW}}_{u\bar{d}\to W(g\gamma)} = \mathrm{d}\sigma^{\mathrm{elastic}}_{u\bar{d}\to W(g\gamma)} + \mathrm{d}\sigma^{\mathrm{boost}}_{u\bar{d}\to W(g\gamma)} + \mathrm{d}\sigma^{\mathcal{O}_{\mathrm{NLO}}}_{u\bar{d}\to W(g\gamma)} + \mathrm{d}\sigma^{\mathrm{regulated}}_{u\bar{d}\to W(g\gamma)}}$$

$$2s\cdot\mathrm{d}\sigma^{\mathrm{regulated}}_{u\bar{d}\to W(g\gamma)} = \langle (I-S_g)(I-S_\gamma)\Xi_1^{q\bar{q}} F_{\mathrm{LM}}(1_u,2_{\bar{d}};4_g,5_\gamma)\rangle$$



$$\Xi^{q\bar{q}} = \Xi_1^{q\bar{q}} + \Xi_2^{q\bar{q}} + \Xi_3^{q\bar{q}} + \Xi_4^{q\bar{q}}$$

$$\begin{aligned}
\Xi_1^{q\bar{q}} &= (I-C_{\gamma g,1})(I-C_{g1})\omega^{\gamma1,g1}\theta_A + (I-C_{\gamma g,1})(I-C_{\gamma1})\omega^{\gamma1,g1}\theta_B + (I-C_{\gamma g,2})(I-C_{g2})\omega^{\gamma2,g2}\theta_A \\
&\quad + (I-C_{\gamma g,2})(I-C_{\gamma2})\omega^{\gamma2,g2}\theta_B + (I-C_{g2})(I-C_{\gamma1})\omega^{\gamma1,g2} + (I-C_{g1})(I-C_{\gamma2})\omega^{\gamma2,g1}, \\
\Xi_2^{q\bar{q}} &= C_{\gamma g,1}(I-C_{g1})\omega^{\gamma1,g1}\theta_A + C_{\gamma g,1}(I-C_{\gamma1})\omega^{\gamma1,g1}\theta_B + C_{\gamma g,2}(I-C_{g2})\omega^{\gamma2,g2}\theta_A + C_{\gamma g,2}(I-C_{\gamma2})\omega^{\gamma2,g2}\theta_B, \\
\Xi_3^{q\bar{q}} &= -C_{g2}C_{\gamma1}\omega^{\gamma1,g2} - C_{\gamma2}C_{g1}\omega^{\gamma2,g1}, \\
\Xi_4^{q\bar{q}} &= C_{g1}\left[\omega^{\gamma1,g1}\theta_A + \omega^{\gamma2,g1}\right] + C_{\gamma1}\left[\omega^{\gamma1,g1}\theta_B + \omega^{\gamma1,g2}\right] + C_{g2}\left[\omega^{\gamma2,g2}\theta_A + \omega^{\gamma1,g2}\right] + C_{\gamma2}\left[\omega^{\gamma2,g2}\theta_B + \omega^{\gamma2,g1}\right]
\end{aligned}$$

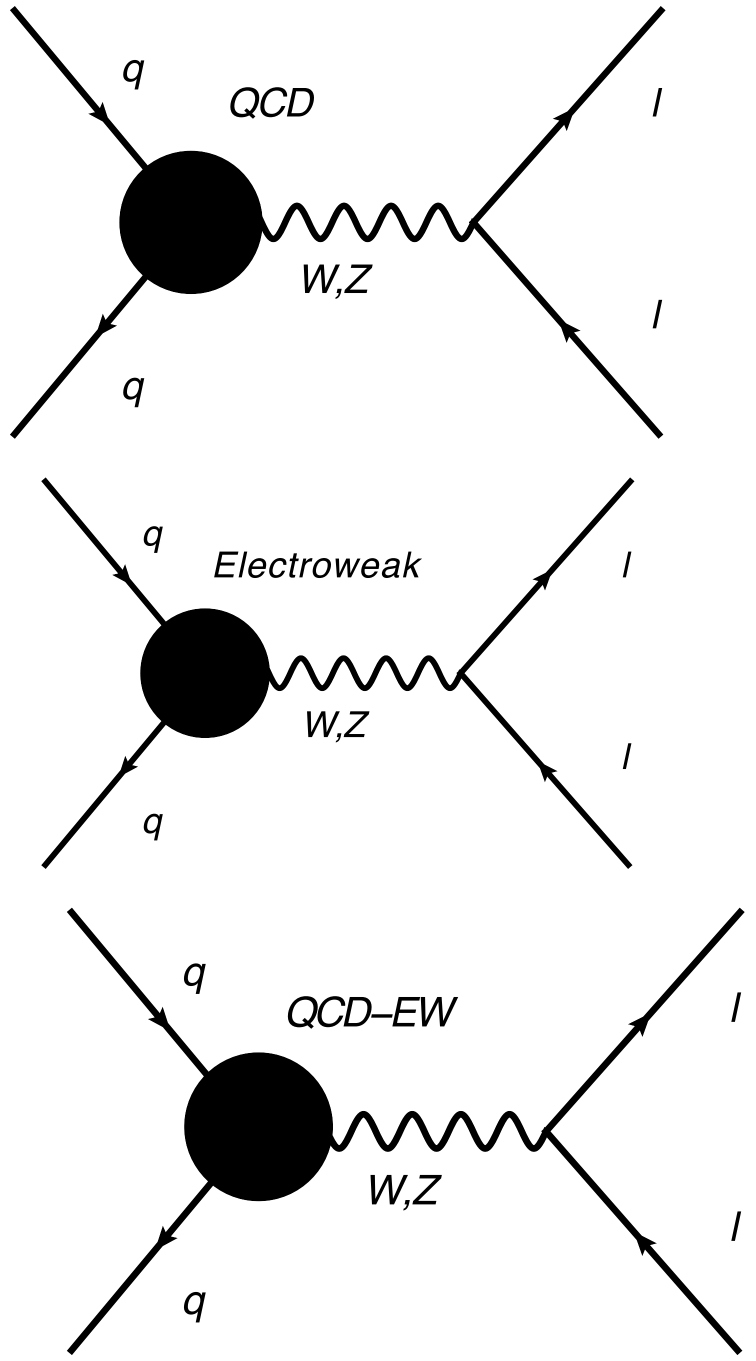
Mixed QCD-electroweak effects in the production

Fiducial cross sections for the W⁺ boson production at the 13 TeV LHC. We use NNPDF3.1lixQED parton distribution functions and employ on-shell renormalization scheme with G_F as the input parameter. Corrections to W decays are not included.

$$p_\perp^l,\ p_\perp^\nu > 15\ \mathrm{GeV} \qquad |y_e| < 2.4$$

$$\sigma_{pp\to W^+} = \sigma_{\rm LO} + \Delta\sigma_{{\rm NLO},\alpha_s} + \Delta\sigma_{{\rm NLO},\alpha}, + \Delta\sigma_{{\rm NNLO},\alpha\alpha_s} + \ldots.$$

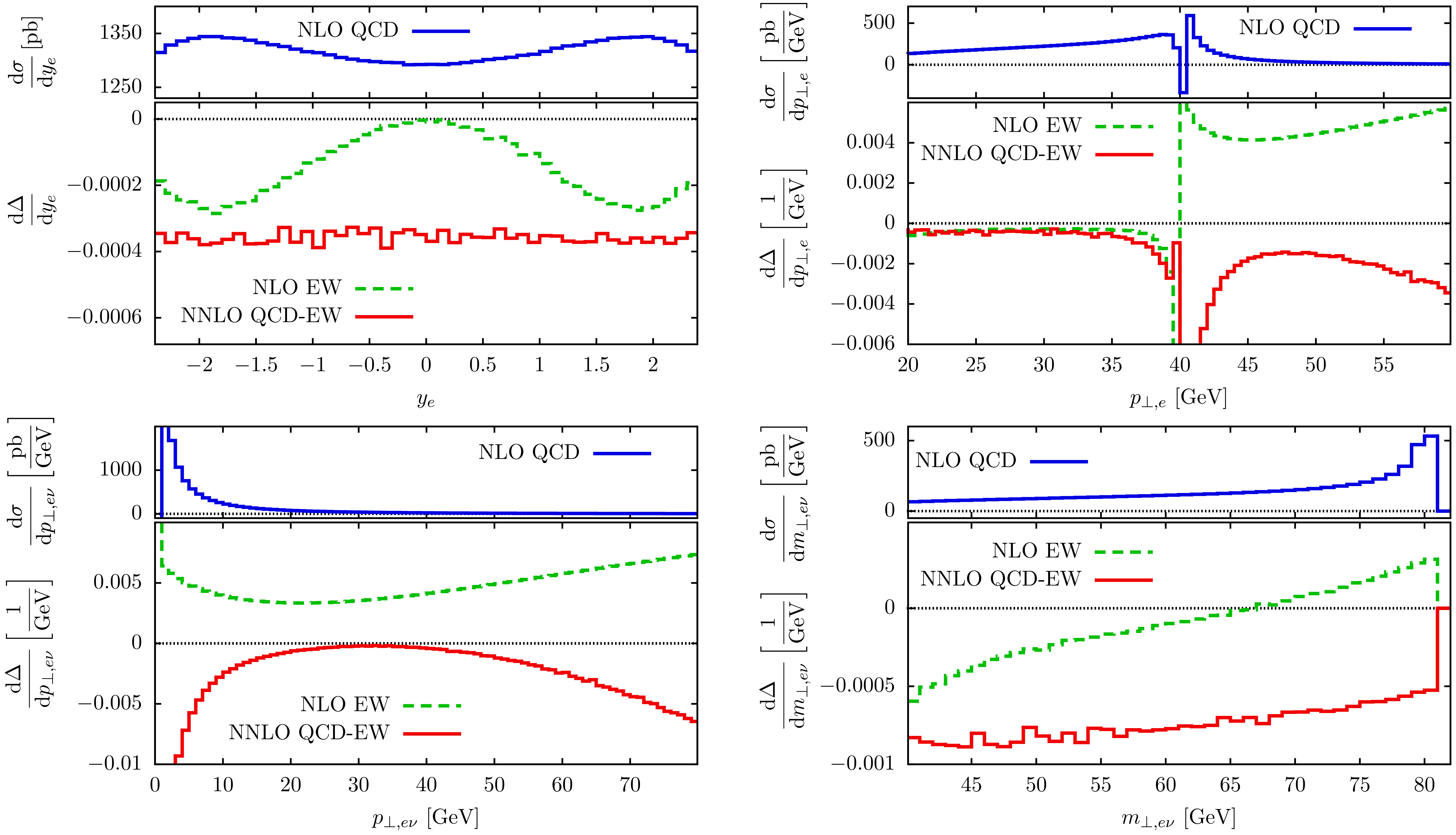
σ[pb]	channel	$\mu = M_W$	$\mu = M_W/2$	$\mu = M_W/4$
$\sigma_{\rm LO}$		6007.6	5195.0	4325.9
$\Delta\sigma_{{\rm NLO},\alpha_s}$	all ch.	508.8	1137.0	1782.2
	$q\bar{q}'$	1455.2	1126.7	839.2
	qg/gq	-946.4	10.3	943.0
$\Delta\sigma_{{\rm NLO},\alpha}$	all ch.	2.1	-1.0	-2.6
	$q\bar{q}'$	-2.2	-5.2	-6.7
	$q\gamma/\gamma q$	4.2	4.2	4.04
$\Delta\sigma_{{\rm NNLO},\alpha_s\alpha}$	all ch.	-2.4	-2.3	-2.8
	$q\bar{q}'/qq'$	-1.0	-1.2	-1.0
	qg/gq	-1.4	-1.2	-2.1
	$q\gamma/\gamma q$	0.06	0.03	-0.04
	$g\gamma/\gamma g$	-0.12	0.04	0.30

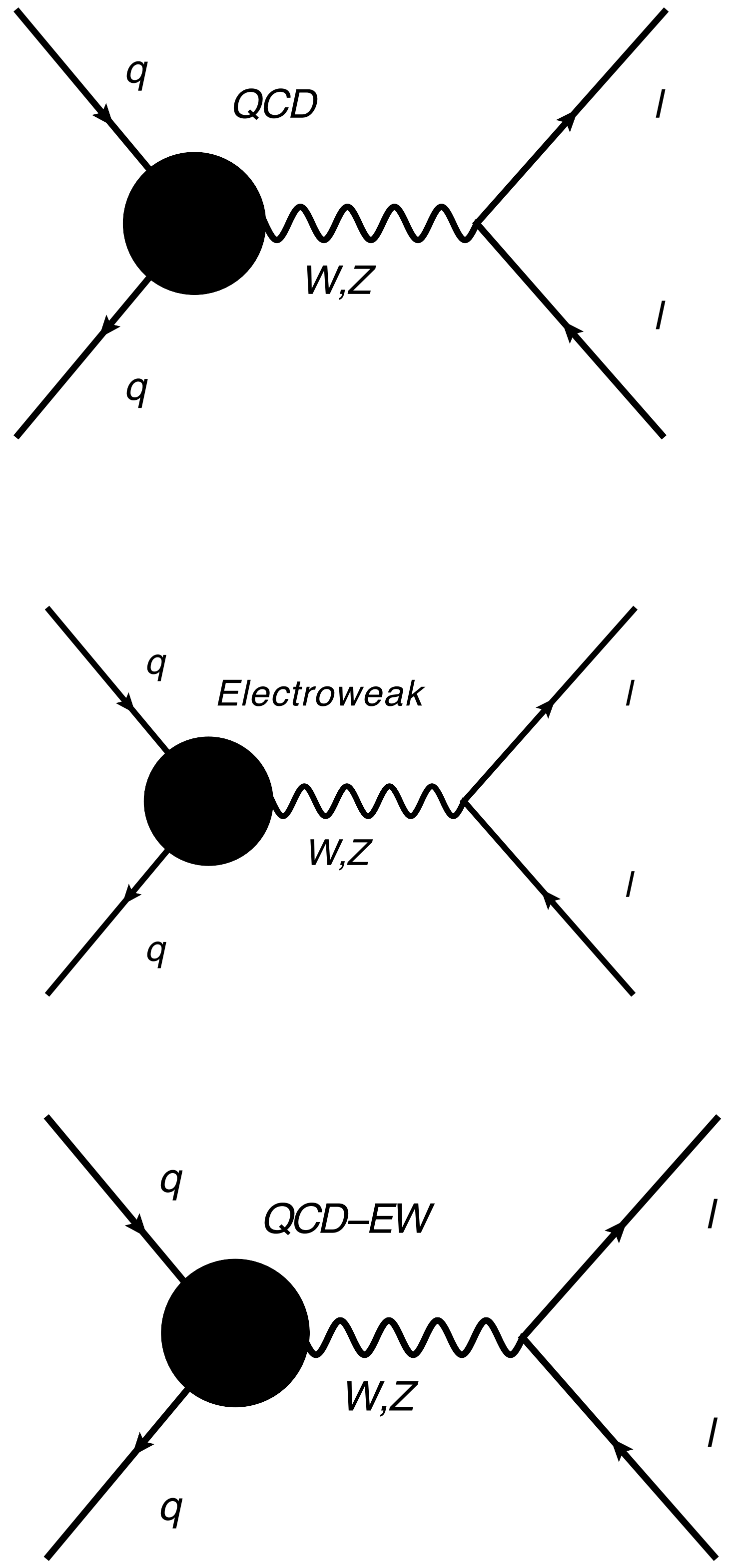


Mixed QCD-EW corrections are about 0.5 per mille; not obviously irrelevant for the extraction of the W mass at the LHC!
Mixed QCD-EW corrections are comparable to EW corrections (the consequence of G_F input scheme).

Mixed QCD-electroweak effects in the production

Similar (marginally) large effects are seen in kinematic distributions (lower panes are normalized to NLO QCD dsitributions).





Mixed QCD-EW corrections to Z and W boson production and their impact on the W mass measurements at the LHC

Arnd Behring and Kirill Melnikov

Institute for Theoretical Particle Physics (TTP)
Karlsruhe Institute for Technology

based on arxiv:2005.10221 [hep-ph], arxiv:2009.10386 [hep-ph] and arxiv:2103.02671 [hep-ph]

in collaboration with Federico Buccioni, Fabrizio Caola, Maximilian Delto, Matthieu Jaquier and Raoul Röntsch

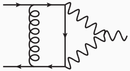
April 1st, 2021 – KITP program “New Physics from Precision at High Energies”

Mixed QCD-EW corrections to on-shell W and Z production

Mixed QCD-EW corrections to $pp \to W/Z$ have been discussed for many years

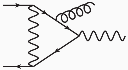
Calculation became possible due to progress on several bottlenecks

- Double Virtual: Complicated integrals with internal and external masses



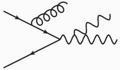
→ Progress on differential equations, iterated integrals etc.

- Real Virtual: Sufficiently stable numerics close to singular limits



→ OpenLoops can provide this in an automated way

- Double Real: IR singularities require NNLO subtraction scheme



→ Profit from progress on NNLO QCD subtraction schemes

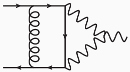
→ We derive estimates for shifts of W mass due to mixed QCD-EW corrections

Mixed QCD-EW corrections to on-shell W and Z production

Mixed QCD-EW corrections to $pp \to W/Z$ have been discussed for many years

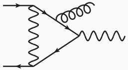
Calculation became possible due to progress on several bottlenecks

- Double Virtual: Complicated integrals with internal and external masses



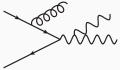
$\rightarrow$ Progress on differential equations, iterated integrals etc.

- Real Virtual: Sufficiently stable numerics close to singular limits



$\rightarrow$ OpenLoops can provide this in an automated way

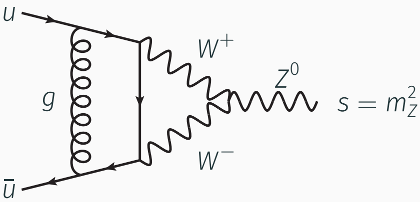
- Double Real: IR singularities require NNLO subtraction scheme

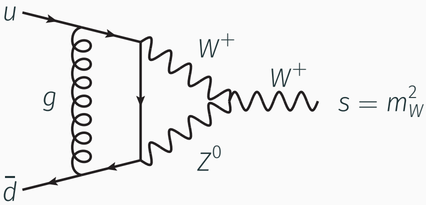


$\rightarrow$ Profit from progress on NNLO QCD subtraction schemes

$\rightarrow$ We derive estimates for shifts of W mass due to mixed QCD-EW corrections

Two-loop amplitudes



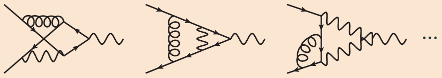


What needs to be calculated? $\rightarrow$ Only on-shell form factors
(Narrow-width approximation simplifies the problem)

- Z: Mixed QCD-EW corrections are known [Kotikov, Kühn, Veretin '07]
- W: Mixed QCD-EW corrections were not yet publicly available
  $\rightarrow$ We calculated the missing integrals and completed the form factor

Calculation of the W form factor

44 Feynman diagrams



This is a non-trivial, but tractable calculation.

Feynman rules, γ algebra, IBP reductions, ...

35 master integrals

$$I \sim \int \frac{[\mathrm{d}^d k_1][\mathrm{d}^d k_2]}{[k_2^2 - m_W^2]\dots[(k_2 - p_{12})^2 - m_Z^2]}$$

10 MI with internal W and Z

Calculated using differential equations

$$\partial_z I(z,\varepsilon) = A(z,\varepsilon) I(z,\varepsilon) \quad \text{with} \quad z = \frac{m_W^2}{m_Z^2}$$

with the equal mass case ($z = 1$) as boundary conditions

25 MI known in the literature
[Aglietti, Bonciani '03] [Aglietti, Bonciani '04]
[Bonciani, Di Vita, Mastrolia, Schubert '16]

Results can be expressed in terms of well-understood iterated integrals (GPLs)

$$G_{a,\vec{b}}(y) = \int_0^y \frac{G_{\vec{b}}(t)}{t-a} \mathrm{d}t\,, \qquad G_a(y) = \int_0^y \frac{1}{t-a} \mathrm{d}t\,, \qquad G_0(y) = \ln(y)\,, \qquad z = \frac{y}{(1+y)^2}$$

Mixed QCD-EW corrections to the W form factor

The result for the form factor can be brought into a compact form.

Infrared poles are predicted by a “Catani-like” formula:

$$\left\langle F_{\mathrm{LVV+LV^2}}^{\mathrm{QCD}\otimes\mathrm{EW}} \right\rangle = \left(\frac{\alpha_s(\mu)}{2\pi} \frac{\alpha_{\mathrm{EW}}}{2\pi} \right) \left[I_{12,\mathrm{QCD}} \cdot I_{12,\mathrm{EW}} + \frac{e^{\varepsilon\gamma_E}}{\Gamma(1-\varepsilon)} \frac{H^W_{\mathrm{QCD}\otimes\mathrm{EW}}}{\varepsilon} \right] \langle F_{LM} \rangle$$
$$+ \left(\frac{\alpha_s(\mu)}{2\pi} \right) I_{12,\mathrm{QCD}} \left\langle F_{\mathrm{LV}}^{\mathrm{fin,EW}} \right\rangle + \left(\frac{\alpha_{\mathrm{EW}}}{2\pi} \right) I_{12,\mathrm{EW}} \left\langle F_{\mathrm{LV}}^{\mathrm{fin,QCD}} \right\rangle$$
$$+ \left\langle F_{\mathrm{LVV+LV^2}}^{\mathrm{fin,QCD}\otimes\mathrm{EW}} \right\rangle .$$

Mixed QCD-EW corrections to the W form factor

The result for the form factor can be brought into a compact form.

Infrared poles are predicted by a "Catani-like" formula:

$$\left\langle F^{\mathrm{QCD}\otimes\mathrm{EW}}_{\mathrm{LVV+LV^2}}\right\rangle = \left(\frac{\alpha_s(\mu)}{2\pi}\frac{\alpha_{\mathrm{EW}}}{2\pi}\right)\left[I_{12,\mathrm{QCD}}\cdot I_{12,\mathrm{EW}} + \frac{e^{\varepsilon\gamma_E}}{\Gamma(1-\varepsilon)}\frac{H^W_{\mathrm{QCD}\otimes\mathrm{EW}}}{\varepsilon}\right]\langle F_{LM}\rangle$$
$$+\left(\frac{\alpha_s(\mu)}{2\pi}\right) I_{12,\mathrm{QCD}}\left\langle F^{\mathrm{fin,EW}}_{\mathrm{LV}}\right\rangle + \left(\frac{\alpha_{\mathrm{EW}}}{2\pi}\right) I_{12,\mathrm{EW}}\left\langle F^{\mathrm{fin,QCD}}_{\mathrm{LV}}\right\rangle$$
$$+\left\langle F^{\mathrm{fin,QCD}\otimes\mathrm{EW}}_{\mathrm{LVV+LV^2}}\right\rangle .$$

Building blocks:

$$I_{12,\mathrm{QCD}} = \left[\frac{e^{\varepsilon\gamma_E}}{\Gamma(1-\varepsilon)}\right]\left(\frac{\mu^2}{M_W^2}\right)^{\varepsilon}\left[-2C_F\cos(\pi\varepsilon)\left(\frac{1}{\varepsilon^2}+\frac{3}{2\varepsilon}\right)\right]$$
$$I_{12,\mathrm{EW}} = \left[\frac{e^{\varepsilon\gamma_E}}{\Gamma(1-\varepsilon)}\right]\left(\frac{\mu^2}{M_W^2}\right)^{\varepsilon}\left[-Q_uQ_d\cos(\pi\varepsilon)\left(\frac{2}{\varepsilon^2}+\frac{3}{\varepsilon}\right)+(Q_d-Q_u)Q_W\left(\frac{1}{\varepsilon^2}+\frac{5}{2\varepsilon}\right)\right]$$
$$H^W_{\mathrm{QCD}\otimes\mathrm{EW}} = C_F\left[Q_u^2+Q_d^2\right]\left(\frac{\pi^2}{2}-6\zeta_3-\frac{3}{8}\right)$$

Mixed QCD-EW corrections to the W form factor

The result for the form factor can be brought into a compact form.

Infrared poles are predicted by a “Catani-like” formula:

$$\left\langle F^{\text{QCD}\otimes\text{EW}}_{\text{LVV+LV}^2} \right\rangle = \left(\frac{\alpha_s(\mu)}{2\pi} \frac{\alpha_{\text{EW}}}{2\pi} \right) \left[I_{12,\text{QCD}} \cdot I_{12,\text{EW}} + \frac{e^{\varepsilon\gamma_E}}{\Gamma(1-\varepsilon)} \frac{H^W_{\text{QCD}\otimes\text{EW}}}{\varepsilon} \right] \langle F_{LM} \rangle$$
$$+ \left(\frac{\alpha_s(\mu)}{2\pi} \right) I_{12,\text{QCD}} \left\langle F^{\text{fin,EW}}_{\text{LV}} \right\rangle + \left(\frac{\alpha_{\text{EW}}}{2\pi} \right) I_{12,\text{EW}} \left\langle F^{\text{fin,QCD}}_{\text{LV}} \right\rangle$$
$$+ \left\langle F^{\text{fin,QCD}\otimes\text{EW}}_{\text{LVV+LV}^2} \right\rangle .$$

- Pole structure *almost* factorises into NLO QCD $\times$ NLO EW
- Finite remainder $\left\langle F^{\text{fin,QCD}\otimes\text{EW}}_{\text{LVV+LV}^2} \right\rangle$ also consists of a factorising (NLO QCD $\times$ NLO EW) and a non-factorising part

Mixed QCD-EW corrections to the W form factor

$$
\begin{aligned}
&\Re\widetilde{\mathcal{M}}_{\text{mix}} = \\
&\quad (Q_u^2+Q_d^2)C_F\Bigg[\frac{1}{\epsilon}\left(-\frac{3}{16}+\frac{1}{4}\pi^2-3\zeta_3\right)+\left(\frac{3}{8}-\frac{1}{2}\pi^2+6\zeta_3\right)\ln\left(\frac{M_W^2}{\mu^2}\right)+\frac{1}{4}\frac{(27z+13)(1-z)^2}{z^3}H_1(z) \\
&\quad +\frac{(1-z)^2(1+z)}{z^3}\left(\frac{3}{4}H_1(z)\pi^2-\frac{9}{2}H_{1,0,0}(z)-\frac{9}{2}H_{1,0,1}(z)\right)-\frac{1}{4}\frac{(5z+3)(1-z)(1+z)}{z^3}H_{-1,0}(z) \\
&\quad +\frac{(1-z)(1+z)^2}{z^3}\Big(-\frac{3}{2}H_{-1,-1,0}(z)+\frac{3}{2}H_{-1,0,0}(z)+3H_{-1,0,1}(z)+2H_{-1,-1,-1,0}(z)-2H_{-1,-1,0,0}(z) \\
&\quad -6H_{-1,-1,0,1}(z)-2H_{-1,0,-1,0}(z)+H_{-1,0,0,1}(z)+H_{0,-1,0,0}(z)+4H_{0,-1,0,1}(z)+\Big(-\frac{1}{4}H_{-1}(z)+\frac{1}{6}H_{-1,-1}(z) \\
&\quad -\frac{1}{6}H_{0,-1}(z)\Big)\pi^2-3H_{-1}(z)\zeta_3\Big)+\frac{1}{32}\frac{7z^2-72z+64}{z^2}+\frac{1}{24}\frac{50z^2-5z-16}{z^2}\pi^2-\frac{3}{2}\frac{8z^2-z-2}{z^2}\zeta_3-\frac{11}{180}\pi^4 \\
&\quad +\frac{(1-z)}{z^2}\Big(\frac{1}{2}(9z+11)H_{0,1}(z)-\frac{1}{2}(3z+4)H_{0,0,1}(z)+\frac{1}{4}(23z+16)H_{0,0}(z)+(3z+2)\Big(\frac{1}{2}H_{0,-1,0}(z) \\
&\quad -\frac{17}{8}H_0(z)\Big)\Big)+\frac{(z^2+3z+1)(1-z)}{z^3}\left(\frac{1}{3}H_{0,1}(z)\pi^2-2H_{0,1,0,0}(z)-2H_{0,1,0,1}(z)\right)\Bigg]+C_F\Bigg[\frac{z+2}{1-z}\Big(-\frac{1}{6}H_{0,0}(z)\pi^2 \\
&\quad +4H_0(z)\zeta_3\Big)+\frac{1}{8}\frac{(5z-2)(2z^2+12z+11)}{(1-z)z^2}H_{0,1}(z)+\frac{1}{8}\frac{43z^2+7z-16}{(1-z)z^2}H_{0,0}(z)-\frac{1}{48}\frac{10z^3+5z^2+20z-16}{(1-z)z^2}\pi^2 \\
&\quad -\frac{1}{16}\frac{8z^3+142z^2+23z-34}{(1-z)z^2}H_0(z)+\frac{1}{120}\frac{5z-36}{1-z}\pi^4-\frac{1}{8}\frac{4z^2-17z+8}{(1-z)z^2}+\frac{2z^2-2z+1}{(1-z)z^2}\Big(\frac{1}{4}(3z+4)H_{0,0,1}(z) \\
&\quad +(3z+2)\left(-\frac{3}{4}\zeta_3-\frac{1}{4}H_{0,-1,0}(z)\right)\Big)+\frac{(2z^2-6z+3)(1+z)}{z^3}\left(\frac{3}{4}H_{1,0,0}(z)+\frac{3}{4}H_{1,0,1}(z)-\frac{1}{8}H_1(z)\pi^2\right) \\
&\quad -\frac{1}{(1-z)z}\left(\frac{1}{8}H_{0,0,0}(z)+\frac{1}{2}(9z^2-8z-2)\zeta_3+\frac{5}{48}H_0(z)\pi^2\right)+\frac{(2z^2-2z+1)(1+z)^2}{(1-z)z^3}\Big(\frac{3}{4}H_{-1,-1,0}(z) \\
&\quad -\frac{3}{4}H_{-1,0,0}(z)-\frac{3}{2}H_{-1,0,1}(z)-H_{-1,-1,-1,0}(z)+H_{-1,-1,0,0}(z)+3H_{-1,-1,0,1}(z)+H_{-1,0,-1,0}(z) \\
&\quad -\frac{1}{2}H_{-1,0,0,1}(z)-\frac{1}{2}H_{0,-1,0,0}(z)-2H_{0,-1,0,1}(z)+\left(\frac{1}{8}H_{-1}(z)-\frac{1}{12}H_{-1,-1}(z)+\frac{1}{12}H_{0,-1}(z)\right)\pi^2+\frac{3}{2}H_{-1}(z)\zeta_3\Big) \\
&\quad +\frac{1}{8}\frac{4z^3+64z^2-z-13}{z^3}H_1(z)+\frac{1}{8}\frac{(5z+3)(2z^2-2z+1)(1+z)}{(1-z)z^3}H_{-1,0}(z)+\frac{z^4-4z^2+z+1}{(1-z)z^3}\Big(H_{0,1,0,0}(z) \\
&\quad +H_{0,1,0,1}(z)-\frac{1}{6}H_{0,1}(z)\pi^2\Big)+\Big[\frac{\sqrt{4z-1}}{8z}\Big(-\frac{10z+3}{1-z}(H_r(z^{-1})-\pi)-(\pi H_0(z)+H_{0,r}(z^{-1}))+\frac{17z+4}{1-z}H_{r,0}(z^{-1}) \\
&\quad -\frac{6z+1}{1-z}(i\pi^2-3i\pi H_r(z^{-1})-3H_{r,1}(z^{-1}))\Big)-\frac{1}{8}\frac{3z+2}{(1-z)z}(H_{r,r}(z^{-1})-\pi H_r(z^{-1}))-\frac{1}{8}\frac{30z^2-20z-1}{(1-z)z}H_{r,r,0}(z^{-1}) \\
&\quad +\frac{1}{8}\frac{1}{(1-z)z}(H_{0,r,r}(z^{-1})-\pi H_{0,r}(z^{-1}))-\frac{1}{8}\frac{6z^2-4z+1}{(1-z)z}(H_{r,0,r}(z^{-1})-\pi H_{r,0}(z^{-1}))+\frac{1}{2}\frac{3z-2}{1-z}\Big(-3H_{r,r,1}(z^{-1}) \\
&\quad -3i\pi H_{r,r}(z^{-1})+i\pi^2H_r(z^{-1})-i\frac{\pi^3}{6}\Big)+\frac{z+2}{1-z}\Big(i\frac{\pi^3}{6}H_0(z)+i\pi^2H_{0,r}(z^{-1})-3i\pi H_{0,r,r}(z^{-1})-3H_{0,r,r,0}(z^{-1}) \\
&\quad -3H_{0,r,r,1}(z^{-1})-4i\pi\zeta_3\Big)\Big]\Bigg]
\end{aligned}
$$

The analytic result is now available and even reasonably compact.

Non-factorising part of finite remainder becomes this simple when expressed in terms of iterated integrals over $z=\frac{m_W^2}{m_Z^2}$

$$H_{a,\vec{b}}(z)=\int_0^z f_a(t)H_{\vec{b}}(t)\,\mathrm{d}t$$

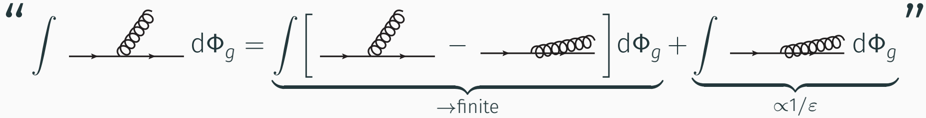
with HPL- and square root letters

$$f_1(t)=\frac{1}{1-t},\quad f_0(t)=\frac{1}{t},$$
$$f_{-1}(t)=\frac{1}{1+t},\quad f_r(t)=\frac{1}{\sqrt{t(4-t)}}$$

Subtraction

Cross-sections develop IR singularities in soft and collinear limits of massless particles
$\rightarrow$ cancel between real and virtual corrections

- Use a subtraction scheme to make poles from real radiation explicit



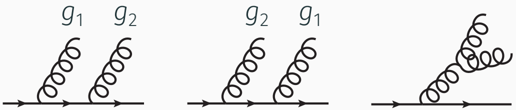
- Build on progress with NNLO QCD subtraction schemes to tackle mixed QCD-EW corrections (here: nested soft-collinear subtraction scheme)
  - Z: Abelianisation of NNLO QCD subtraction is sufficient
  - W: New contributions from radiating W bosons

Subtraction for mixed QCD-EW corrections: triple-collinear limits

We can make use of simplifications compared to NNLO QCD.

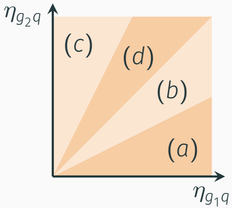
Triple-collinear limits

- NNLO QCD: Overlapping singularities in triple-collinear limits



$\rightarrow$ Needs 4 sectors to disentangle collinear singularities

$\eta_{ij} = \frac{1}{2}(1 - \cos\theta_{ij})$

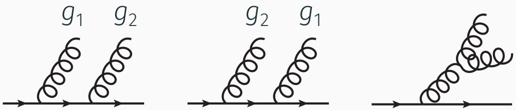


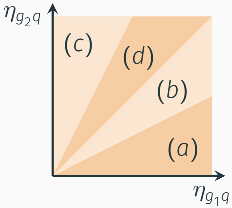
We can make use of simplifications compared to NNLO QCD.

Triple-collinear limits

$$\eta_{ij} = \tfrac{1}{2}(1 - \cos\theta_{ij})$$

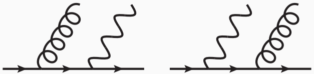
- NNLO QCD: Overlapping singularities in triple-collinear limits

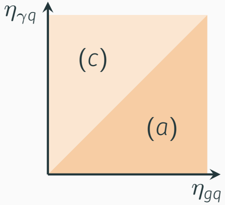




$\rightarrow$ Needs 4 sectors to disentangle collinear singularities

- Mixed QCD-EW: Collinear limit of photon and gluon is not singular





$\rightarrow$ 2 sectors can be dropped in $q\bar{q}$ channel

Overall: No new collinear limits arise compared to NNLO QCD

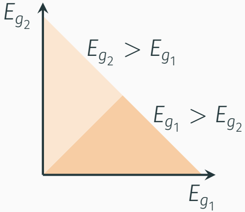
Subtraction for mixed QCD-EW corrections: double-soft limits

We can make use of simplifications compared to NNLO QCD.

Double-soft limits

- NNLO QCD: Overlapping singularities in the double-soft limit
    - Non-trivial double-soft eikonal function
    - Distinguish rates at which energies of soft particles vanish

$$1 = \theta(E_{g_1} - E_{g_2}) + \theta(E_{g_2} - E_{g_1})$$

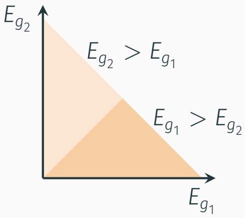


We can make use of simplifications compared to NNLO QCD.

Double-soft limits

- NNLO QCD: Overlapping singularities in the double-soft limit
  - Non-trivial double-soft eikonal function
  - Distinguish rates at which energies of soft particles vanish

$$1 = \theta(E_{g_1} - E_{g_2}) + \theta(E_{g_2} - E_{g_1})$$



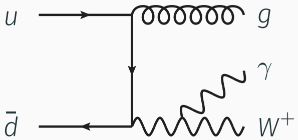
- Mixed QCD-EW: Soft gluons and photons are not entangled
  - Double-soft limit factorises into NLO QCD $\times$ NLO QED

$$\lim_{E_g, E_\gamma \to 0} |\mathcal{M}_{Wg\gamma}|^2 = g_s^2 \mathrm{Eik}_g(p_u, p_{\bar{d}}; p_g)\, e^2 \mathrm{Eik}_\gamma(p_u, p_{\bar{d}}, p_W; p_\gamma) |\mathcal{M}_W|^2$$

$$\mathrm{Eik}_g(p_u, p_{\bar{d}}; p_g) = 2C_F \frac{(p_u \cdot p_{\bar{d}})}{(p_u \cdot p_g)(p_g \cdot p_{\bar{d}})}$$

  - No need to distinguish $E_g > E_\gamma$ vs. $E_\gamma > E_g$

Subtraction for mixed QCD-EW corrections: radiating W bosons



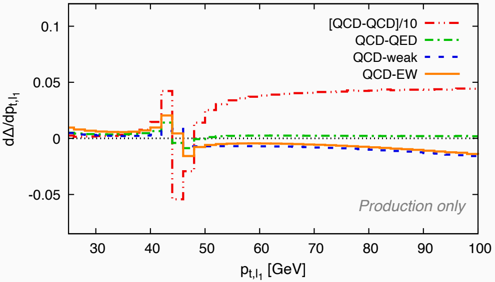
New contribution compared to NNLO QCD: W bosons can radiate photons

- Mass of W boson prevents collinear singularities
- Soft limit of photon is still singular
  - Requires soft eikonal function for massive emitter
  - QCD and QED factorise in soft limit $\rightarrow$ only NLO eikonal functions necessary

$$\mathrm{Eik}_\gamma(p_u, p_{\bar{d}}, p_W; p_\gamma) = \Bigg\{ Q_u Q_d \frac{2(p_u \cdot p_{\bar{d}})}{(p_u \cdot p_\gamma)(p_{\bar{d}} \cdot p_\gamma)} - Q_W^2 \frac{p_W^2}{(p_W \cdot p_\gamma)^2} + Q_W \left(Q_u \frac{2(p_W \cdot p_u)}{(p_W \cdot p_\gamma)(p_u \cdot p_\gamma)} - Q_d \frac{2(p_W \cdot p_{\bar{d}})}{(p_W \cdot p_\gamma)(p_{\bar{d}} \cdot p_\gamma)} \right) \Bigg\}$$

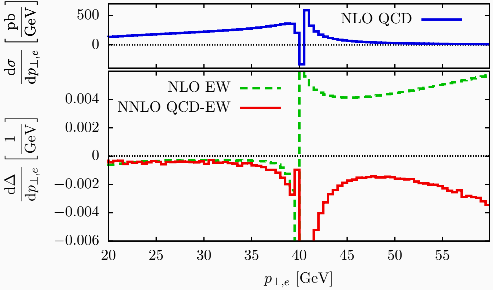
Estimates for impact on W mass

$pp \to Z^0 \to e^+e^-$



[Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch '20]

$pp \to W^+ \to e^+\nu_e$



[AB, Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch '20]

- Our implementation allows to calculate differential distributions including mixed QCD-EW corrections
- Impact on W-mass measurement is not immediately obvious

Estimate W mass shifts from mixed QCD-EW corrections

Objective: Estimate impact of new corrections on W boson mass

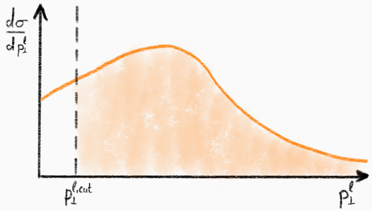
Considerations:

- Should combine W and Z measurements
  - $\rightarrow$ model what is done in experiments
  - $\rightarrow$ make use of available precision for Z mass
- Should be physically and conceptually simple and transparent
- Should be accessible with our calculations

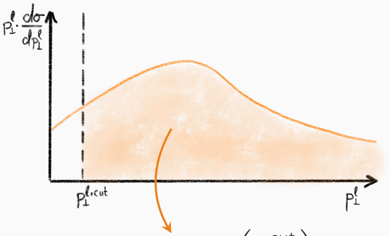
Construction of our observable

We use the average transverse momentum of the charged lepton ($V = W, Z$):

$$\langle p_\perp^{\ell,V} \rangle = \frac{\int \mathrm{d}\sigma_V \times p_\perp^\ell}{\int \mathrm{d}\sigma_V}$$



$$\xrightarrow{\times p_\perp^\ell}$$

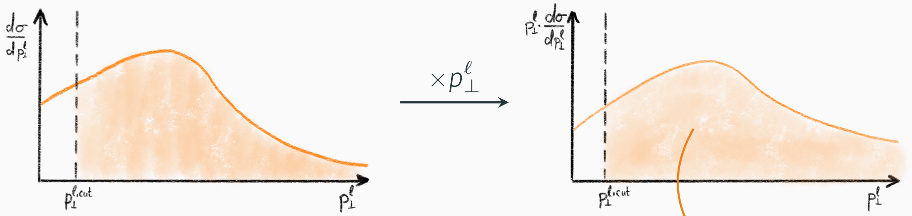


$$\langle p_\perp^{\ell,V} \rangle = m_V\, f\left(\frac{p_\perp^{\mathrm{cut}}}{m_V}\right)$$

with $f^{\mathrm{LO}}(0) = \frac{15\pi}{128}$

We use the average transverse momentum of the charged lepton ($V = W, Z$):

$$\langle p_\perp^{\ell,V} \rangle = \frac{\int d\sigma_V \times p_\perp^\ell}{\int d\sigma_V}$$



$$\langle p_\perp^{\ell,V} \rangle = m_V f\left(\frac{p_\perp^{\mathrm{cut}}}{m_V}\right)$$

with $f^{\mathrm{LO}}(0) = \frac{15\pi}{128}$

Construction of our observable (cont.)

Use the average lepton $p_\perp$ in W and Z production as well as the Z mass to construct an observable for the W mass:

$$m_W^{\text{meas}} = \frac{\langle p_\perp^{\ell,W} \rangle^{\text{meas}}}{\langle p_\perp^{\ell,Z} \rangle^{\text{meas}}}\, m_Z\, C_{\text{th}}$$

Construction of our observable (cont.)

Use the average lepton $p_\perp$ in W and Z production as well as the Z mass to construct an observable for the W mass:

Measurement from LHC

$\propto m_W$

$$m_W^{\text{meas}} = \frac{\boxed{\langle p_\perp^{\ell,W} \rangle^{\text{meas}}}}{\boxed{\langle p_\perp^{\ell,Z} \rangle^{\text{meas}}}} \, m_Z \, C_{\text{th}}$$

$\propto m_Z$

Measurement from LHC

Construction of our observable (cont.)

Use the average lepton $p_\perp$ in W and Z production as well as the Z mass to construct an observable for the W mass:

Measurement from LEP

$$m_W^{\text{meas}} = \frac{\langle p_\perp^{\ell,W} \rangle^{\text{meas}}}{\langle p_\perp^{\ell,Z} \rangle^{\text{meas}}} \, m_Z \, C_{\text{th}}$$

Construction of our observable (cont.)

Use the average lepton $p_\perp$ in W and Z production as well as the Z mass to construct an observable for the W mass:

$$m_W^{\text{meas}} = \frac{\langle p_\perp^{\ell,W} \rangle^{\text{meas}}}{\langle p_\perp^{\ell,Z} \rangle^{\text{meas}}} \, m_Z \, \boxed{C_{\text{th}}}$$

Theoretical correction factor

Construction of our observable (cont.)

Use the average lepton $p_\perp$ in W and Z production as well as the Z mass to construct an observable for the W mass:

$$m_W^{\text{meas}} = \frac{\langle p_\perp^{\ell,W} \rangle^{\text{meas}}}{\langle p_\perp^{\ell,Z} \rangle^{\text{meas}}} \, m_Z \, \boxed{C_{\text{th}}}$$

Theoretical correction factor

$\rightarrow$ Calculate via $C_{\text{th}} = \dfrac{m_W}{m_Z} \dfrac{\langle p_\perp^{\ell,Z} \rangle^{\text{th}}}{\langle p_\perp^{\ell,W} \rangle^{\text{th}}}$

Construction of our observable (cont.)

Use the average lepton $p_\perp$ in W and Z production as well as the Z mass to construct an observable for the W mass:

$$m_W^{\text{meas}} = \frac{\langle p_\perp^{\ell,W} \rangle^{\text{meas}}}{\langle p_\perp^{\ell,Z} \rangle^{\text{meas}}} \, m_Z \, \boxed{C_{\text{th}}}$$

Theoretical correction factor

$\rightarrow$ Calculate via $C_{\text{th}} = \dfrac{m_W}{m_Z} \dfrac{\langle p_\perp^{\ell,Z} \rangle^{\text{th}}}{\langle p_\perp^{\ell,W} \rangle^{\text{th}}}$

Adding a new correction to the theory

$\rightarrow$ changes C_{th}

$\rightarrow$ changes extracted mass m_W^{meas}

$$\frac{\delta m_W^{\text{meas}}}{m_W^{\text{meas}}} = \frac{\delta C_{\text{th}}}{C_{\text{th}}} = \frac{\delta \langle p_\perp^{\ell,Z} \rangle}{\langle p_\perp^{\ell,Z} \rangle} - \frac{\delta \langle p_\perp^{\ell,W} \rangle}{\langle p_\perp^{\ell,W} \rangle}$$

Shifts on W mass (inclusive setup)

Estimated impact of ...

... mixed QCD-EW corrections:

$$\delta m_W \approx -7\,\mathrm{MeV}$$

... NLO electroweak corrections:

$$\delta m_W \approx 1\,\mathrm{MeV}$$

$$\delta m_W = \left(\frac{\delta \langle p_\perp^{\ell,Z} \rangle}{\langle p_\perp^{\ell,Z} \rangle} - \frac{\delta \langle p_\perp^{\ell,W} \rangle}{\langle p_\perp^{\ell,W} \rangle} \right) m_W$$

Shifts on W mass (inclusive setup)

Estimated impact of ...

... mixed QCD-EW corrections:

$$\delta m_W \approx -7\,\mathrm{MeV}$$

... NLO electroweak corrections:

$$\delta m_W \approx 1\,\mathrm{MeV}$$

$$\delta m_W = \left(\frac{\delta \langle p_\perp^{\ell,Z} \rangle}{\langle p_\perp^{\ell,Z} \rangle} - \frac{\delta \langle p_\perp^{\ell,W} \rangle}{\langle p_\perp^{\ell,W} \rangle} \right) m_W$$

Mixed QCD-EW corrections appear to have larger impact than NLO EW corrections

- G_μ input parameter scheme reduces size of NLO EW corrections
- Strong cancellation between changes in Z and W

Shifts on W mass (inclusive setup)

Estimated impact of ...

... mixed QCD-EW corrections:

$$\delta m_W \approx -7\,\text{MeV}$$

... NLO electroweak corrections:

$$\delta m_W \approx 1\,\text{MeV}$$

$$\delta m_W = \left(\cancel{\frac{\delta\langle p_\perp^{\ell,Z}\rangle}{\langle p_\perp^{\ell,Z}\rangle}} - \frac{\delta\langle p_\perp^{\ell,W}\rangle}{\langle p_\perp^{\ell,W}\rangle} \right) m_W$$

$\rightarrow \delta m_W \approx 54\,\text{MeV}$ (mixed QCD-EW)

$\rightarrow \delta m_W \approx -31\,\text{MeV}$ (NLO EW)

Mixed QCD-EW corrections appear to have larger impact than NLO EW corrections

- G_μ input parameter scheme reduces size of NLO EW corrections
- Strong cancellation between changes in Z and W

Shifts on W mass (inclusive setup)

Estimated impact of ...

... mixed QCD-EW corrections:

$$\delta m_W \approx -7\,\text{MeV}$$

... NLO electroweak corrections:

$$\delta m_W \approx 1\,\text{MeV}$$

$$\delta m_W = \left(\cancel{\frac{\delta\langle p_\perp^{\ell,Z}\rangle}{\langle p_\perp^{\ell,Z}\rangle}} - \frac{\delta\langle p_\perp^{\ell,W}\rangle}{\langle p_\perp^{\ell,W}\rangle} \right) m_W$$

$\rightarrow$ $\delta m_W \approx 54\,\text{MeV}$ (mixed QCD-EW)
$\rightarrow$ $\delta m_W \approx -31\,\text{MeV}$ (NLO EW)
$\rightarrow$ Changes are more correlated between Z and W for NLO EW

Mixed QCD-EW corrections appear to have larger impact than NLO EW corrections

- G_μ input parameter scheme reduces size of NLO EW corrections
- Strong cancellation between changes in Z and W

Shifts on W mass (inclusive setup)

Estimated impact of ...

... mixed QCD-EW corrections:

$$\delta m_W \approx -7\,\text{MeV}$$

... NLO electroweak corrections:

$$\delta m_W \approx 1\,\text{MeV}$$

$$\delta m_W = \left(\frac{\delta \langle p_\perp^{\ell,Z} \rangle}{\langle p_\perp^{\ell,Z} \rangle} - \frac{\delta \langle p_\perp^{\ell,W} \rangle}{\langle p_\perp^{\ell,W} \rangle} \right) m_W$$

Minor influence of PDFs:

- Tested with specialised minimal PDF sets provided by NNPDF collaboration (based on `NNPDF3.1luxQED`)
- Mixed QCD-EW corrections: About $\mathcal{O}(1)$ MeV changes

Scale variation: $\mathcal{O}(\pm 2)$ MeV

The influence of fiducial cuts

Repeat calculation with fiducial cuts (inspired by [ATLAS ’17] analysis)

Estimated impact of ...

... mixed QCD-EW corrections:

$$\delta m_W \approx -17\,\text{MeV}$$

... NLO electroweak corrections:

$$\delta m_W \approx 3\,\text{MeV}$$

W production:

- $p_\perp^{e^+} > 30\,\text{GeV}$
- $p_\perp^{\text{miss}} > 30\,\text{GeV}$
- $|\eta_{e^+}| < 2.4$
- $m_T^W > 60\,\text{GeV}$

Z production:

- $p_\perp^{e^\pm} > 25\,\text{GeV}$
- $|\eta_{e^\pm}| < 2.4$

$\rightarrow$ Shifts are larger than for inclusive setup

The influence of fiducial cuts

Repeat calculation with fiducial cuts (inspired by [ATLAS '17] analysis)

Estimated impact of ...

... mixed QCD-EW corrections:

$$\delta m_W \approx -17\,\text{MeV}$$

... NLO electroweak corrections:

$$\delta m_W \approx 3\,\text{MeV}$$

W production:

- $p_\perp^{e^+} > 30\,\text{GeV}$
- $p_\perp^{\text{miss}} > 30\,\text{GeV}$
- $|\eta_{e^+}| < 2.4$
- $m_T^W > 60\,\text{GeV}$

Z production:

- $p_\perp^{e^\pm} > 25\,\text{GeV}$
- $|\eta_{e^\pm}| < 2.4$

$\rightarrow$ Shifts are larger than for inclusive setup

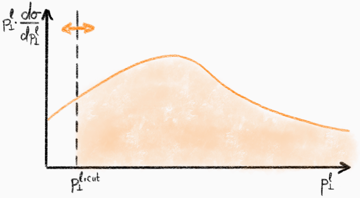
Key reason:

- Relevant transverse momenta: $p_\perp^{e^+}/M_V$
- ATLAS applies larger $p_\perp^{e^+}$ cuts to (lighter) W bosons than to (heavier) Z bosons
- Leads to small decorrelation of corrections to W and Z bosons

Tuning the cuts

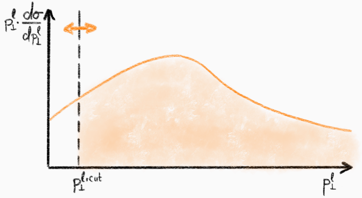
Can we "tune" the cuts to reduce the impact of mixed QCD-EW corrections?

- Start from ATLAS-inspired cuts as baseline
- Decrease cuts on $p_\perp^{e^+}$ for W^+ case until $C_{\text{th}} = 1$ at LO
- Leads to $p_\perp^{e^+} > 25.44\,\text{GeV}$



Can we “tune” the cuts to reduce the impact of mixed QCD-EW corrections?

- Start from ATLAS-inspired cuts as baseline
- Decrease cuts on $p_\perp^{e^+}$ for W^+ case until $C_{\text{th}} = 1$ at LO
- Leads to $p_\perp^{e^+} > 25.44\,\text{GeV}$



Estimated impact of ...

... mixed QCD-EW corrections:

$$\delta m_W \approx -1\,\text{MeV}$$

... NLO electroweak corrections:

$$\delta m_W \approx -3\,\text{MeV}$$

$\rightarrow$ Strong cut dependence of δm_W allows to “tune away” QCD-EW corrections in our setup

Conclusions

Conclusions

- We calculate mixed QCD-EW corrections to fully-differential on-shell W and Z production at the LHC.
  $\rightarrow$ Possible thanks to progress on amplitude calculations and subtraction schemes.
- Size of mixed QCD-EW corrections to the production part is $\mathcal{O}(0.5)$‰.
  $\rightarrow$ Corrections are small but in line with expectations.
- Experimental measurements of m_W rely on similarity between W and Z distributions. Based on this, we build a transparent and simple model to estimate shifts on m_W via

$$\delta m_W = \left(\frac{\delta \langle p_\perp^{\ell,Z} \rangle}{\langle p_\perp^{\ell,Z} \rangle} - \frac{\delta \langle p_\perp^{\ell,W} \rangle}{\langle p_\perp^{\ell,W} \rangle} \right) m_W \, .$$

- We find that mixed QCD-EW corrections induce shifts on m_W that are comparable or larger than the target precision of $\mathcal{O}(10)$ MeV.
- Further investigations on the impact of mixed QCD-EW corrections on m_W are clearly warranted. They should reflect all relevant details of experimental analyses.

Backup

Input parameters

Input parameters used:

$$G_F = 1.166\,39 \times 10^{-5}\,\mathrm{GeV}^{-2}$$
$$m_Z = 91.1876\,\mathrm{GeV}$$
$$m_W = 80.398\,\mathrm{GeV}$$
$$m_H = 125\,\mathrm{GeV}$$
$$m_t = 173.2\,\mathrm{GeV}$$

- We use the G_μ input parameter scheme.
- PDFs: NNLO set `NNPDF3.1luxQED` with $\alpha_s(m_Z) = 0.118$
- Simulations for 13 TeV LHC
- Central scale: $\mu_R = \mu_F = m_V/2$

Detailed results for cross-sections and moments

Results for the cross-sections and average transverse momentum of the charged lepton for the inclusive setup of $pp \to Z \to e^+e^-$ and $pp \to W^+ \to e^+\nu_e$ (corrections only to the production part)

$$\mathrm{d}\sigma_{Z,W} = \sum_{i,j=0} \alpha_s^i \alpha_W^i \mathrm{d}\sigma_{Z,W}^{i,j} \qquad F_{Z,W}(i,j,\mathcal{O}) = \alpha_s^i \alpha_W^i \int \mathrm{d}\sigma_{Z,W}^{i,j} \times \mathcal{O}$$

	$V = Z$			$V = W^+$		
	$\mu = m_Z/4$	$\mu = m_Z/2$	$\mu = m_Z$	$\mu = m_W/4$	$\mu = m_W/2$	$\mu = m_W$
$F_V(0,0;1)$, [pb]	1273	1495	1700	7434	8810	10083
$F_V(1,0;1)$, [pb]	570.2	405.4	246.9	3502	2533	1580
$F_V(0,1;1)$, [pb]	$-5810 \cdot 10^{-3}$	$-6146 \cdot 10^{-3}$	$-6073 \cdot 10^{-3}$	$-1908 \cdot 10^{-3}$	$3297 \cdot 10^{-3}$	$10971 \cdot 10^{-3}$
$F_V(1,1;1)$, [pb]	$-2985 \cdot 10^{-3}$	$-2033 \cdot 10^{-3}$	$-1236 \cdot 10^{-3}$	$-8873 \cdot 10^{-3}$	$-7607 \cdot 10^{-3}$	$-7556 \cdot 10^{-3}$
$F_V(0,0;p_\perp^e)$ [GeV pb]	42741	50191	57073	220031	260772	298437
$F_V(1,0;p_\perp^e)$ [GeV pb]	23418	17733	12221	124487	95132	66090
$F_V(0,1;p_\perp^e)$ [GeV pb]	-182.85	-192.77	-189.11	74.53	243.54	484.82
$F_V(1,1;p_\perp^e)$ [GeV pb]	-163.87	-125.22	-92.05	-553.87	-482.0	-448.0

Detailed results for W mass shifts

Detailed results for the shifts δm_W for different setups, orders and scales

δm_W [MeV]		$\mu = m_V/4$	$\mu = m_V/2$	$\mu = m_V$
Inclusive	NLO EW	-0.1	0.3	0.2
	QCD-EW	-5.1	-7.5	-9.3
Fiducial	NLO EW	0.2	2.3	4.2
	QCD-EW	-16	-17	-19
Tuned fiducial	NLO EW	-4.4	-2.5	-0.8
	QCD-EW	3.9	-1.0	-5.7