On-shell SM EFT(s)

Yael Shadmi

Technion

YS, Yaniv Weiss 1809.09644
Gauthier Durieux, Teppei Kitahara, YS, Yaniv Weiss 1909.10551
Gauthier Durieux, Teppei Kitahara, Camila Machado, YS, Yaniv Weiss 2008.09652
Reuven Balkin, Gauthier Durieux, Teppei Kitahara, YS, Yaniv Weiss, in progress

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EFfective Theories:
quantifying our (finally acknowledged..) ignorance

ad-hoc parametrization of EWSB in terms of the SM Higgs
why $\mu^2 < 0$?
where does scale come from and how is it stabilized?
is Naturalness (& natural extensions of the SM) dead? (tuning? $10^{-3} \neq 10^{-32}$)

turn to bottom-up approach:
• precision measurements of electroweak sector
• EFTs: a well defined-framework for interpreting these measurements (model independently*)
* Different possible SM EFTs:
- SM only? SM+X?
- SU(2)$\times$U(1) realized linearly? Higgs as part of an SU(2) doublet?

[see talks by Sanz, Craig, Sutherland]

Outline

- Motivation for going on-shell
- Amplitude essentials
  - massless and massive spinors
  - massless $\leftrightarrow$ massive: high-energy limit, (un)bolding
- Warm-up example to demonstrate the construction of amplitudes: $X + 3$ gluons; $X =$ massive spin-0 (SM or new particle)
- The electroweak EFT: Amplitude Bases: (in terms of LG covariant massive spinor formalism)
  - Bottom-up bootstrap [more in Gauthier's talk]
  - Top-down: "Higgsing" massless (unbroken phase) amplitudes

Current massive toolbox

general 3- and 4-point amplitudes of massive/massless particles of various spins, classified by dimension Durieux Kitahara YS Weiss; Durieux Kitahara Machado YS Weiss

electroweak: spins 0, 1/2, 1

- detailed gluing prescription (massive spin 1/2, 1)
- bases for all massive 3-points
- bases for all massive 4-points (any dimension): generic amplitudes, contact-terms
- matching to broken-phase SMEFT 3-point (+) couplings

beyond electroweak:

- bases for all massive 3-point spin≤ 3
- bases for all 4-point massless contact terms spin ≤ 2; higher points for spins 0,1/2,1,2, dim ≤ 8

See also: Christensen Field; Christensen Field Moore Pinto; Herderschee Koren Trott; Aoude Machado; Bachu Yelleshpur..

• On-shell operator basis for all masses and spins. Dong, Ma, Shu, [general construction, op dim not manifest]

• Pairwise spinors. Csaki, Hong, Telem, Terning, Shirman, Waterbury [LG-neutral combinations for pairs of particles, may be useful for EFT constructions]

On-shell Effective theories: motivation

Effective Field Theory

input:

- SM fields [+ possibly: some BSM fields]
- SM symmetry (global, gauge)
- + Lorentz, locality

$\to$ most general Lagrangian

$$\mathcal{L}_{\rm EFT} = \sum_i \frac{c_i}{\Lambda^{n_i}} \mathcal{O}_i$$

On-shell Effective theories: motivation

$\rightarrow$ **bottom-up** (bootstrap) on-shell approach is very natural:

input:

- SM particles [+ possibly: some BSM particles]
- symmetry (global, gauge)
- + Lorentz, locality

$\rightarrow$ 3-points + $n > 3$ contact-terms
$\rightarrow$ "glue" to get higher-point/higher-order amplitudes

gauge symmetry emerges from the requirement of consistent interactions of spin-1 particles (Bose statistics, factorization..)

On-shell Effective theories: motivation

1st step of EFT construction: identify basis of operators $\mathcal{O}_i$ (complete, independent)

modulo field redefinitions, EOMs, gauge redundancies

mathematically: polynomials of operators subject to a set of constraints
beautifully solved using Hilbert Series   Jenkins Manohar; Lehman Martin; Henning Lu Melia Murayama

On-shell Effective theories: motivation

on-shell construction: only deal with physical quantities
$\rightarrow$ this 1st step is circumvented (field redefinitions, gauge redundancies)

the remainder simplifies considerably:

polynomials of operators subject to a set of constraints (EOM, momentum conservation)

maps to:
polynomials in the Mandelstam invariants (just numbers..) subject to a set of kinematical constraints, e.g., $s + t + u = \sum m^2$

also used in Henning Lu Melia Murayama

by-product: counting & classification of EFT operators

On-shell Effective theories: motivation

SM applications:

$$\mathcal{L}_{\rm EFT} = \sum_i \frac{c_i}{\Lambda^{n_i}} \mathcal{O}_i$$

typically: start with full SU(2)$\times$U(1) (SMEFT)

EWSB: $\mathcal{O}_i$ modify masses, SM couplings
$\rightarrow$redefine input parameters

on-shell construction: relate physical observables (largely), directly measurable

[computation: avoid Feynman diagrams, particularly complicated with EFT vertices]

on-shell EFTs for the SM

How on-shell do you want to be?

semi on-shell: work in unbroken electroweak symmetric phase
massless amplitudes
impose full SU(2)$\times$U(1)

- good approximation at high-energies
- all you care about for running, anomalous dimensions
- mostly used, many powerful results: classification of operators, selection rules, operator mixings, anomalous dimensions, positivity..

Chueng Shen; Azatov Contino Machado Riva; Bern Parra-Martinez Sawyer; Ma Shu Xiao; Baratella Fernandez von Harling Pomarol; Gu Wang Zhang; Falkowski; ..

on-shell EFTs for the SM

or: fully on-shell:

instead of

$$\mathcal{L}_{\rm EFT} = \sum_i \frac{c_i}{\Lambda^{n_i}} \mathcal{O}_i \;\to\; \mathcal{L}_{\rm EFT}\Big|_{\langle H\rangle = v} \;\to\; A(p_1,\ldots p_n;\{c_i\}) \;\to\; \text{experiment}$$

look at $A(p_1,\ldots p_n;\{c_i\})$ directly; working with SM particles: massive W, Z, h..

want to exploit full power of on-shell approach in relating purely physical observables

on-shell EFTs for the SM

bottom-up construction captures general EFTs with the SM particle content:

in particular: in a bottom-up construction: the Higgs is just the “PDG Higgs”:

$$A(H^0(\mathbf{p_1}), \ldots,): \quad H^0: \;\; J = 0 \;\; m = 125.10 \pm 0.14\,\mathrm{GeV}$$

cover both SMEFT, HEFT (+ and all orders in v/Λ expansion)

more on this in Gauthier’s talk

(MASSIVE) EFT AMPLITUDE BASICS

EFT AMPLITUDE BASICS

- Little Group + spin-statistics determine all n-point Contact Terms (CT) (≥ 3)

  3-points: generically complex momenta

- Remaining parts of amplitudes determined by factorization/generalized unitarity

$\rightarrow$ bootstrap higher point amplitudes by gluing together lower-point amplitudes + adding CTs

so: derive 3-point amplitudes
get 4-point amplitudes by gluing two 3-points + adding 4-pt CT $\cdots$

CTs $\leftrightarrow$ EFT operators

unknown coefficients of CTs $\leftrightarrow$ unknown Wilson coefficients

0th order task: find CTs

Writing amplitudes: massless

Write amplitudes in terms of massless spinors

- momenta:

$$k_{\alpha\dot{\alpha}} \equiv k_\mu \sigma^\mu_{\alpha\dot{\alpha}} = |k\rangle\,[k|\,, \qquad \bar{k}^{\dot{\alpha}\alpha} \equiv k_\mu \bar{\sigma}^{\mu\dot{\alpha}\alpha} = |k]\,\langle k|,$$

- external polarizations: spin 1/2:

$$|k\rangle \quad = \lambda_\alpha(k)\,, \qquad |k] \quad = \tilde{\lambda}^{\dot{\alpha}}(k)$$

- external polarizations: spin-1:

$$\varepsilon^+_{\alpha\dot{\alpha}}(k) = \sqrt{2}\frac{|r\rangle\,[k|}{\langle kr\rangle} \quad \varepsilon^-_{\alpha\dot{\alpha}}(k) = \sqrt{2}\frac{|k\rangle\,[r|}{[kr]}$$

r is reference momentum

Writing amplitudes: massless

$\rightarrow$ amplitude written in terms of $\langle ij \rangle$, $[ij]$

can use the power of the little-group:

for a (massless) particle of momentum k:
k is invariant under little group [U(1)], polarizations are not:

assigning LG charges: $|k\rangle \;\; (1)$
$\rightarrow \quad |k] \;\; (-1) \quad \varepsilon^+ \;\; (+2) \quad \varepsilon^- \;\; (-2)$

$\rightarrow$ selection rules: external particle helicities determine LG weights of amplitude

note:
each massless fermion i: one spinor $|i]$ (or $|i\rangle$)
each massless vector i: two such spinors: $|i]\,|i]$ (or $|i\rangle|i\rangle$)

Writing amplitudes: massive

LG covariant massive formalism   Arkani-Hamed Huang Huang; Conde Marzolla

- momenta: decompose in terms of two massless momenta:

$$p = p^{I=1} + p^{I=2} \qquad 2p^1 \cdot p^2 = m^2$$

LG [SU(2)] rotates:

$$|\boldsymbol{p}^I\rangle \to W^I_{\ J}|\boldsymbol{p}^J\rangle\,, \quad \text{and} \quad [\boldsymbol{p}_I| \to \left(W^{-1}\right)_I^{\ J}[\boldsymbol{p}_J|\,.$$

$$I, J = 1, 2$$

- polarizations: spin 1/2:

$$u^I(p) = \begin{pmatrix} |p^I\rangle \\ |p^I] \end{pmatrix} \quad \ldots \tag{1}$$

$$\mathbf{p}|p^I\rangle = m\,|p^I] \tag{2}$$

Writing amplitudes: massive

- polarizations: spin 1:

$$\varepsilon^{\{IJ\}}_{\alpha\dot\alpha} = \sqrt{2}\frac{|\boldsymbol{p}\rangle\,[\boldsymbol{p}|}{m} \qquad I{=}J{=}1{:}\ +1\,,\ \ I{=}J{=}2{:}\ -1\,,\ \ I{\neq}J{:}\ 0$$

bold denotes symmetrization over LG indices, *e.g.* $|\boldsymbol{p}\rangle\,[\boldsymbol{p}| = |p^{\{I}\rangle\left[p^{J\}}\right|$

will use boldface for massive momenta too to distinguish them from massive ones

$\rightarrow$ amplitudes are expressed in terms of spinor products $\langle \boldsymbol{ij}\rangle$, $[\boldsymbol{ij}]$
(possibly with momentum insertions)

note:
each massive fermion i: one spinor $|\boldsymbol{i}]$ (or: $|\boldsymbol{i}\rangle$)
each massive vector i: two such spinors: $|\boldsymbol{i}]\,|\boldsymbol{i}]$ (or: $|\boldsymbol{i}\rangle|\boldsymbol{i}\rangle$, $|\boldsymbol{i}]\,|\boldsymbol{i}\rangle$:)

Writing amplitudes: massive

more generally:
since a spin s rep can be obtained as a symmetric combination of spin-1/2 reps
$\rightarrow$ external leg of spin s:

$$\mathcal{M}^{I_1 \ldots I_{2s}} = |\boldsymbol{p}\rangle_{\alpha_1}^{I_1} \ldots |\boldsymbol{p}\rangle_{\alpha_{2s}}^{I_{2s}} M^{\{\alpha_1 \ldots \alpha_{2s}\}} = |\boldsymbol{p}]_{\dot{\alpha}_1}^{I_1} \ldots |\boldsymbol{p}]_{\dot{\alpha}_{2s}}^{I_{2s}} \tilde{M}^{\{\dot{\alpha}_1 \ldots \dot{\alpha}_{2s}\}}$$

Why have we suffered through this?

with the amplitudes written in terms of the LG covariant massive spinor formalism:

- can use the power of the LG: now SU(2) selection rules
- beautiful connection between massive and massless amplitudes via unbolding (and bolding) see Gauthier's talk

$$\boldsymbol{p} = p^{I=1} + p^{I=2} \equiv k + q \qquad 2k \cdot q = m^2$$

can choose: high-energy limit: $k = \mathcal{O}(E), \quad q = \mathcal{O}(m^2/E); \quad \boldsymbol{p} \sim k$

roughly: unbolding massive amplitude: $\mathbf{p}] \to p]$ to get massless amplitude

(and bolding works too)

High-energy limit: massive $\to$ massless (unbolding)

$p = p^1 + p^2$

choosing the direction of p^1 = choosing spin polarization axis

convenient to recover helicity states in high-energy limit:

$$p^1_\mu = \frac{E+p}{2}(1,0,0,1) \equiv k_\mu\,, \quad p^2_\mu = \frac{E-p}{2}(1,0,0,-1) \equiv q_\mu$$

[$k = \mathcal{O}(E), \quad q = \mathcal{O}(m^2/E)$]

High-energy limit: massive $\rightarrow$ massless (unbolding)

in HE limit: $p \rightarrow k \quad q \rightarrow 0$

then *e.g.*, for a vector of positive polarization $I = J = 1$:

$$|\boldsymbol{p}]\,|\boldsymbol{p}] \rightarrow |k]\,|k]$$

(all others vanish, *e.g.* $|\boldsymbol{p}]\,|\boldsymbol{p}\rangle \rightarrow 0$)

for a vector of zero polarization $(I, J) = (1, 2)$:

$$|\boldsymbol{p}]\,|\boldsymbol{p}\rangle \rightarrow |k]\,|k\rangle$$

$\rightarrow$get massless amplitudes by unbolding

Let's put this to work

Example: $h + 3g$ amplitude

YS Weiss 2018

1st example: SM singlet + 3 gluons   effectively massless
h is spin-0 SM singlet
all $+$ helicities:

LG $\rightarrow$ $[12][23][13]$ up to function of invariants   1,2,3 label gluon momenta

Contact Terms: no poles, any polynomial of s_{12}, s_{23}, s_{13} subject to
$s_{12} + s_{23} + s_{13} = m_h^2$

$$\mathcal{M}\big(h; g^{a+}(p_1) g^{b+}(p_2) g^{c+}(p_3)\big) = \frac{[12][13][23]}{\Lambda} \Bigg[f^{abc} \left(+ \frac{c_7}{\Lambda^2} + \frac{c_{11}}{\Lambda^6}(s_{12}s_{23} + s_{13}s_{23} + s_{12}s_{13}) + \frac{c_{13}}{\Lambda^8} s_{12}s_{13}s_{23} \right) + d^{abc} \frac{c'_{13}}{\Lambda^8} (s_{12} - s_{13})(s_{12} - s_{23})(s_{13} - s_{23}) \Bigg] + \cdots$$

Example: $h + 3g$ amplitude

$$\mathcal{M}\big(h; g^{a+}(p_1) g^{b+}(p_2) g^{c+}(p_3)\big) = \frac{[12][13][23]}{\Lambda} \Bigg[$$

$$f^{abc} \left(+ \frac{c_7}{\Lambda^2} + \frac{c_{11}}{\Lambda^6}(s_{12}s_{23} + s_{13}s_{23} + s_{12}s_{13}) + \frac{c_{13}}{\Lambda^8} s_{12}s_{13}s_{23} \right)$$

$$+ d^{abc} \frac{c'_{13}}{\Lambda^8} \left(s_{12} - s_{13}\right)\left(s_{12} - s_{23}\right)\left(s_{13} - s_{23}\right) \Bigg]$$

$$+ \cdots$$

here shown for dim≤ 13 but trivial to extend: polynomials in s_{ij}'s (symmeric or antisymmetric) — derivative expansion

but coefficients c_i: all orders in v/Λ

Example: $h + 3g$ amplitude

adding in factorizable part (from “gluing” ggg and hgg):

$$\mathcal{M}\big(h; g^{a+}(p_1)g^{b+}(p_2)g^{c+}(p_3)\big) = \frac{[12][13][23]}{\Lambda}\Bigg[f^{abc}\left(-i\,\frac{m^4\,g_s\,c_5^{hgg}}{s_{12}s_{13}s_{23}} + \frac{c_7}{\Lambda^2} + \frac{c_{11}}{\Lambda^6}(s_{12}s_{23}+s_{13}s_{23}+s_{12}s_{13}) + \frac{c_{13}}{\Lambda^8}s_{12}s_{13}s_{23}\right) + d^{abc}\,\frac{c'_{13}}{\Lambda^8}\,(s_{12}-s_{13})\,(s_{12}-s_{23})\,(s_{13}-s_{23})\Bigg] + \cdots$$

dim	operators $\mathcal{M}(+++)$	operators $\mathcal{M}(++-)$
5	—	—
7	$h\, G_{\rm SD}^3\, [1, f^{abc}]$	—
9	—	$\mathcal{D}^2\, G_{\rm SD}^2\, G_{\rm ASD}\, h\, [1, f^{abc}]$
11	$\mathcal{D}^4\, G_{\rm SD}^3\, h\, [1, f^{abc}]$	$\mathcal{D}^4\, G_{\rm SD}^2\, G_{\rm ASD}\, h\, [1, f^{abc}; 1, d^{abc}]$
13	$\mathcal{D}^6\, G_{\rm SD}^3\, h\, [1, f^{abc}; 1, d^{abc}]$	$\mathcal{D}^6\, G_{\rm SD}^2\, G_{\rm ASD}\, h\, [2, f^{abc}; 1, d^{abc}]$

check with Mathematica notebook of Henning Lu Melia Murayama

ELECTROWEAK EFTs

- Bottom-up construction of electroweak amplitudes [Gauthier's talk; here rough sketch]

  Durieux Kitahara YS Weiss

  Durieux Kitahara Machado YS Weiss

- Top-down: starting from UV (unbroken) massless amplitudes and "Higgsing"

  Balkin Durieux Kitahara YS Weiss, in progress

ON-SHELL EFTs: BASES (working directly in massive theory)

Durieux Kitahara Machado YS Weiss

bases of $n \geq 4$-amplitudes

3 types of bases:

1) Spinor structure basis: minimal set of spinor structures $\boldsymbol{S}^{\{I\}}$ spanning the amplitude

spinor structure $\boldsymbol{S}^{\{I\}}$: product of *e.g.*, $[\boldsymbol{ij}]$,$\langle\boldsymbol{ijk}]$,$[\boldsymbol{ijki}]$, .. $\{I\}$= all LG indices
no prefactors of invariants s_{ij}

analogous constructions in terms of usual polarizations

eg, Bonifacio Hinterbichler

a structure is redundant if it can be written as a linear combination of other $\boldsymbol{S}^{\{I\}}$'s
with coefficients = rational functions of the invariants

number of elements: $\Pi_i(2s_i+1)$ Schomerus Sobko Isachenkov; Kravchuk Simmons-Duffin

[inner products of spinor-structures:
$(\boldsymbol{S}_1, \boldsymbol{S}_2) \equiv \sum_{\{I\}} \boldsymbol{S}_1^{\{I\}^*} \boldsymbol{S}_2^{\{I\}}$= function of the invariants]

bases of $n \geq 4$-amplitudes

EFT amplitude: contact terms only: no poles:

to get a manifestly local amplitude: don't want negative powers of the invariants

$\rightarrow$ a structure is redundant if it can be written as a linear combination of other $\boldsymbol{S}^{\{I\}}$'s with coefficients = polynomials of the invariants (as opposed to rational functions)

$\rightarrow$2) Stripped Contact Term (SCT) basis: minimal set of spinor structures of this type

From this it's easy to construct the 3) Contact Term basis: (SCT)$\times$ polynomials of s_{ij}'s

massless $\leftrightarrow$ massive

the massless case is easy, so let's use it:

• classify spinor structures according to "helicity categories" = helicities of high-energy limits = unbolded versions

e.g., in $ffvs$:

$$[\mathbf{13}][\mathbf{23}] \text{ is } (\,+++0)$$
$$[\mathbf{13}]\langle\mathbf{23}\rangle \text{ is } (+-00)$$

number of helicity categories: $\Pi_i(2s_i+1)$

• massless limit: spinor-structures of different helicities are independent (=inner product vanishes)

massless $\leftrightarrow$ massive

$\rightarrow$ simple prescription for obtaining the spinor-structure basis:
take the massless amplitude in each helicity category and **bold** it

bolding = covariantizing wrt massive little group

(stripped) contact term bases

but we are interested in EFTs: need CT basis:
again can get a lot of information from massless amplitudes, but not as immediate:

eg, if a spinor-structure is redundant in massless case: it is also redundant in massive case
main subtlety:
the normalization of longitudinal vectors

(stripped) contact term bases

$|\boldsymbol{i}\rangle\,[\boldsymbol{i}| \rightarrow |\boldsymbol{i}\rangle\,[\boldsymbol{i}|/m_i$

different ways to see this:

• massive polarization is $|\boldsymbol{i}\rangle\,[\boldsymbol{i}|/m_i$: so with $1/m_i$ correctly identify dimension of operator

• these are the Goldstone amplitudes $\rightarrow$ scale as p_i/m_i

massless limit: $p_i = |i\rangle\,[i|$

covariantizing wrt massive little-group: $\rightarrow |\boldsymbol{i}\rangle\,[\boldsymbol{i}|$

with these inverse masses included can get rid of additional structures (in favor of lower-dim structures)

EFT BASES FROM ON-SHELL HIGGSING

Balkin, Durieux, Kitahara, YS, Weiss, in progress

EFT BASES FROM ON-SHELL HIGGSING

Balkin, Durieux, Kitahara, YS, Weiss, in progress

On-shell version of Higgs mechanism: involves: bolding; “freezing” the Higgs momentum [“soft Higgs”]

EFTs: easy starting point: no propagators, no poles

$N = 4$ SUSY Coulomb branch analysis of Craig, Elvang, Kiermaier, Slatyer (also “soft Higgs”: Dixon, Glover, Khoze, MHV rules for Higgs plus multi-gluon amplitudes)

EFTs from on-shell Higgsing (SMEFT)

mentioned: longitudinal vector amplitudes: Goldstone amplitudes $\rightarrow$ scale as p_i/m_i

massless limit: $p_i = |i\rangle\,[i|$

covariantizing wrt massive little-group: $\rightarrow |\boldsymbol{i}\rangle\,[\boldsymbol{i}|$

Higgsing (equivalence theorem) $\leftrightarrow$ Lorentz

Massive EFTs from on-shell Higgsing (SMEFT)

assume: masses originate from Higgsing $(m \lesssim v \ll \Lambda)$ [as in SMEFT]

Low-E: an n-point CT $\leftrightarrow$ independent Wilson coefficient [not given by 3-,..., $(n-1)$-point couplings][1]

Number of independent LE Wilson coefficients = Number of independent HE Wilson coefficients (possibly dressed by $v^{\#}$)

$\rightarrow$start with HE theory ($v < E < \Lambda$)
write down all CTs (easy)
each of these gives LE (massive) CT

how?

- bold to get correct massive LG (=covariantize wrt to SU(2) LG)
- freeze Higgs momentum q: $q + k = \mathbf{p}$ to get lower-point amplitude(s)

[1] Also: mass-suppressed contributions from lower point couplings required to restore unitarity (UV gauge theory)

Massive EFTs from on-shell Higgsing (SMEFT)

example: want low-E: $A_4(\bar{u}dWh)$ amplitude $[(1/2, 1/2, 1, 0)]$

need HE massless amplitudes:

- $M_4(\bar{Q}DWH)$, .. $[M_4(1/2, 1/2, 1, 0)]$

[4-pts: same spins as desired amplitude]

- $M_4(\bar{Q}QH^\dagger H)$, .. $[M_4(1/2, 1/2, 0, 0)]$

[4-pts: longitudinal vector from scalar]

- $M_5(\bar{Q}QWH^\dagger H)$, .. $[M_5(1/2, 1/2, 1, 0, 0)]$

[5-pts: same spins; extra Higgs required by gauge symmetry]

- $M_5(\bar{Q}DHHH^\dagger)$, .. $[M_5(1/2, 1/2, 0, 0, 0)]$

[5-pts: longitudinal vector from scalar; extra Higgs required by gauge symmetry]

helicities implicit

$A_4(\bar{u}dWh)$

(i) HE amplitude features same spin as LE amplitude

$$M_4(1/2, 1/2, 1, 0): \quad \text{independent CTs}: [13][23], \quad [12]\langle 3123\rangle$$

just need to bold to get massive CTs:

$$[\mathbf{13}][\mathbf{23}], \quad [\mathbf{12}]\langle \mathbf{3123}\rangle$$

showing “half” of fermion, vector helicities: opposite helicities by exchanging squares and brackets

$A_4(\bar{u}dWh)$

(ii) LE longitudinal vector $\leftrightarrow$ HE scalar:

$$M_4(1/2, 1/2, 0, 0) : \; [132\rangle$$

bold:

$$[132\rangle = [13]\langle 32\rangle \to \mathbf{[13]}\langle\mathbf{23}\rangle$$

3-scalar $\rightarrow$ 3-vector

$A_4(\bar{u}dWh)$

(iii) LE amplitude $\leftrightarrow$ HE amplitude with same spin content plus additional scalar leg(s) required to restore gauge invariance.

$$M_5(1/2, 1/2, 1, 0, 0) : [13]\langle 243] \;\; [13]\langle 253] \, , \;\; [1243]\langle 243]$$

set Higgs (say 5) to its VEV $\rightarrow$ freeze momentum:
identify: $5 \rightarrow q_3$, $3 \rightarrow k_3$: $\rightarrow 3 + 5 \rightarrow \mathbf{5}$

$$[13]\langle 243] \rightarrow \mathbf{[13]\langle 243]}$$

others redundant (using momentum conservation $p_4 = -p_1 - p_2 - p_3$)
very familiar: 5-pt with extra Higgs leg $\rightarrow$ 4-pt with Higgs set to its VEV

EFT Lagrangian: $v + h$

Note dimensions: $A_4 = vA_5\Big|_{frozen\ higgs}$

$A_4(\bar{u}dWh)$

(iv) LE amplitude: longitudinal vector $\leftrightarrow$ HE amplitude: scalar
plus additional scalar leg(s) required to restore gauge invariance

$$M_5(1/2, 1/2, 0, 0, 0) : [1342], \quad [12]$$

$$[1342] \to [13]\langle 342] \to \mathbf{[13]\langle 342]}$$

$$[12]s_{13} \to \mathbf{[12]\langle 313]} ..$$

note:

- above: just used Lorentz (LG): general
- specifying to SMEFT: can also match couplings: get LE broken phase couplings in terms of SMEFT (unbroken) Wilson coefficients

Conclusions

EFTs are taking center-stage
reflecting our ignorance (very healthy)

Conclusions [the perennial skeptic's version]

EFTs are taking center-stage
reflecting our ignorance (very healthy)

?? the new supersymmetry

recall super-split supersymmetry [hep-th/0503249]
let's hope we don't end up with
super-effective field theory ($\Lambda = \infty$ where EFTs are super reliable)

Conclusions

on-shell approach is natural for constructing EFT extensions of low-energy SM
= all possible couplings of SM particles consistent with Lorentz, global symmetries, locality and unitarity

To really see this: Gauthier's talk

Constructing massive amplitude bases: either directly, or "Higgsing" massless EFT amplitudes
Beyond EFTs: develop understanding of "on-shell Higgsing"? apply to massive recursion relations..

THANK YOU!

BACK UP

recall fermion-fermion-Z-higgs amplitude:

$$\mathcal{M}^{\text{contact}}(\mathbf{1}_{\psi^c}, \mathbf{2}_{\psi}, \mathbf{3}_Z, \mathbf{4}_h) = \frac{C_6^{(1)}}{\bar{\Lambda}^2}[\mathbf{13}][\mathbf{23}] + \frac{C_6^{(2)}}{\bar{\Lambda}^2}[\mathbf{13}]\langle\mathbf{23}\rangle + \frac{C_7^{(1)}}{\bar{\Lambda}^3}[\mathbf{312}\rangle[\mathbf{13}] + \frac{C_7^{(2)}}{\bar{\Lambda}^3}[\mathbf{321}\rangle[\mathbf{23}] + \text{angle} \leftrightarrow \text{square}$$

in red = basic independent spinor-structures—stripped of any $\tilde{s}_{ij}$

then full contact term basis given by

$$[\mathbf{13}][\mathbf{23}]\,, \quad \tilde{s}_{12}\,[\mathbf{13}][\mathbf{23}]\,, \quad \tilde{s}_{12}^2\,[\mathbf{13}][\mathbf{23}]\,, \quad \cdots$$

recall fermion-fermion-Z-higgs amplitude:

$$\mathcal{M}^{\text{contact}}(\mathbf{1}_{\psi^c}, \mathbf{2}_{\psi}, \mathbf{3}_Z, \mathbf{4}_h) = \frac{C_6^{(1)}}{\bar{\Lambda}^2}[\mathbf{13}][\mathbf{23}] + \frac{C_6^{(2)}}{\bar{\Lambda}^2}[\mathbf{13}]\langle\mathbf{23}\rangle + \frac{C_7^{(1)}}{\bar{\Lambda}^3}[\mathbf{312}\rangle[\mathbf{13}] + \frac{C_7^{(2)}}{\bar{\Lambda}^3}[\mathbf{321}\rangle[\mathbf{23}] + \text{angle} \leftrightarrow \text{square}$$

in red = basic independent spinor-structures—stripped of any $\tilde{s}_{ij}$

then full contact term basis given by

$$[\mathbf{13}][\mathbf{23}]\,, \quad \tilde{s}_{12}\,[\mathbf{13}][\mathbf{23}]\,, \quad \tilde{s}_{12}^2\,[\mathbf{13}][\mathbf{23}]\,, \quad \cdots$$

used the spinor-structure identity:

$$[\mathbf{12}]\langle\mathbf{3123}\rangle = [\mathbf{12}]\left[\tilde{s}_{23}\,\langle\mathbf{313}] - \tilde{s}_{13}\,\langle\mathbf{323}]\right]/m_3 - \tilde{s}_{12}[\mathbf{13}][\mathbf{23}] - m_1[\mathbf{321}\rangle[\mathbf{23}] - m_2[\mathbf{312}\rangle[\mathbf{13}]$$

bases

simple example: massless 4-fermion $++++$ amplitude:

spinor-structure basis: $[14][23]$

spans generic amplitude:

$$[13][24] = -\frac{s_{13}}{s_{14}}[14][23] \qquad + \text{Schouten } [12][34] = [13][24] - [14][23]$$

— —

stripped-contact term basis: $[14][23]$, $[13][24]$

spans the contact-term (EFT) amplitude): manifestly local

— —

contact-term basis:

$$\dim - 6: [14][23]\,, [13][24]$$

$$\dim - 8: s_{13}\,[14][23]\,, s_{14}\,[14][23]\,, s_{13}\,[13][24] \qquad \text{but no } s_{14}\,[13][24]$$

Example: vector + 3-gluon amplitude

$$\mathcal{M}\big(Z'; g^{a-}(p_1); g^{b-}(p_2); g^{c+}(p_3)\big)$$
$$= d^{abc}\,\langle 12\rangle^2 \times \Big[[34]^2\,A + [13][23]\langle 14\rangle\langle 24\rangle\,B + [34]\Big([31]\langle 14\rangle - [32]\langle 24\rangle\Big)\,C\Big]$$
$$+ f^{abc}\,\langle 12\rangle^2 \times \Big[[34]^2\,D + [13][23]\langle 14\rangle\langle 24\rangle\,E + [34]\Big([31]\langle 14\rangle - [32]\langle 24\rangle\Big)\,F\Big]$$

up to dim ≤ 12:

$$A = \frac{d_8}{\Lambda^4} + \frac{d_{10}^{(1)}}{\Lambda^6}\,s_{12} + \frac{d_{12}^{(1)}\,s_{12}^2 + d_{12}^{(2)}\,s_{13}s_{23}}{\Lambda^8} \qquad B = \frac{m^2\,s_{12}\,d_{12}^{(6)}}{\Lambda^8}$$

$$C = (s_{13} - s_{23})\frac{m\,d_{12}^{(5)}}{\Lambda^8} \qquad D = (s_{23} - s_{13})\left(\frac{d_{10}^{(3)}}{\Lambda^6} + \frac{d_{12}^{(4)}}{\Lambda^8}s_{12}\right)$$

$$E = 0 \qquad F = \frac{m\,d_{10}^{(2)}}{\Lambda^6} + \frac{m\,d_{12}^{(3)}}{\Lambda^8}s_{12}$$

Example: vector + 3-gluon amplitude

unbolding $\rightarrow$massless amplitudes:

$$\mathcal{M}\big(Z'; g^{a-}(p_1); g^{b-}(p_2); g^{c+}(p_3)\big)$$
$$= d^{abc}\,\langle 12\rangle^2 \times \Big[[34]^2\,A + [13][23]\langle 14\rangle\langle 24\rangle\,B + [34]\Big([31]\langle 14\rangle - [32]\langle 24\rangle\Big)\,C\Big]$$
$$+ f^{abc}\,\langle 12\rangle^2 \times \Big[[34]^2\,D + [13][23]\langle 14\rangle\langle 24\rangle\,E + [34]\Big([31]\langle 14\rangle - [32]\langle 24\rangle\Big)\,F\Big]$$

POSITIVE NEGATIVE ZERO

$$[\mathbf{12}]\langle\mathbf{3123}\rangle = [\mathbf{12}]\,\left[\tilde{s}_{23}\,\langle\mathbf{313}] - \tilde{s}_{13}\,\langle\mathbf{323}]\right]/m_3$$
$$- \tilde{s}_{12}[\mathbf{13}][\mathbf{23}] - m_1[\mathbf{321}\rangle[\mathbf{23}] - m_2[\mathbf{312}\rangle[\mathbf{13}]$$

inverse mass in $\langle\mathbf{3}\cdots\mathbf{3}]$: longitudinal vector

Bottom-up construction of electroweak EFTs:

Durieux Kitahara YS Weiss

Constructed all relevant 3-points
Provided a detailed prescription for gluing
One full 4-point example ($ffVh$)
Perturbative unitarity $\rightarrow$ SU(2)$\times$U(1)
Related work: Aoude Machado; Bachu Yelleshpur (SU(2)$\times$U(1) imposed)
See talk by Gauthier
+ bases for general 4-point contact-terms

Durieux, Machado, Kitahara, YS, Weiss

[A construction of 3-point bases was given by Arkani-Hamed Huang and Huang: gives over-complete bases in some cases (*e.g.*, 3 vectors)]

RESULTS:

- Stripped Contact Term bases for all 4-point amplitudes of massive spin 0, 1/2, 1 (all dimensions)
- Stripped Contact Term bases for all 4-point amplitudes of massless spin 0, 1/2, 1 (all dimensions)
  complementing results of Durieux, Machado '19 for dim ≤ 8
- + all 3-point amplitudes for spin ≤ 3
  complementing results of Durieux, Kitahara, YS, Weiss '18 for spin ≤ 1
- Dimension of corresponding operator manifest

4-point SCT basis

spins	$n_{\rm SCT}$	n_s	hel. cat.	spinor structures	$n_{\rm perm}$	$\min\{d_{\rm op}\}$
$ssss$	1	1	(0000)	constant	1	4
$vsss$	$4 \to 3$	3	(0000)	$[121\rangle, [131\rangle$	1	5
			(+000)	$[1231] \to [1231] - \langle 1231\rangle$	$\not 2 \to 1$	7
$ffss$	4	4	(++00)	$[12]$	2	5
			(+−00)	$[132\rangle$	2	6
$vvss$	$10 \to 9$	9	(0000)	$[12]\langle 12\rangle, [131\rangle[232\rangle$	1	4,6
			(+000)	$[12][132\rangle$	4	6
			(++00)	$[12]^2$	2	6
			(+−00)	$[132\rangle^2 \to [132\rangle^2 - \langle 132]^2$	$\not 2 \to 1$	8
$ffvs$	$14 \to 12$	12	(++00)	$[12]\{[313\rangle, [323\rangle\}$	2	6
			(+−00)	$[13]\langle 23\rangle$	2	5
			(+++0)	$[13][23]$	2	6
			(++−0)	$[12]\langle 3123\rangle \to \varnothing$	$\not 2 \to 0$	8
			(+−+0)	$[13][312\rangle$	4	7
$ffff$	18	16	(++++)	$[12][34], [13][24]$	2	6
			(++−−)	$[12]\langle 34\rangle$	6	6
			(+++−)	$[12][324\rangle$	8	7
$vvvs$	$35 \to 29$	27	(0000)	$[12][343\rangle\langle 12\rangle, [13][242\rangle\langle 13\rangle, [23][141\rangle\langle 23\rangle$	1	5
			(+000)	$[12][13]\langle 23\rangle$	6	5
			(++00)	$[12]^2\left\{[313\rangle, [323\rangle\right\}$	6	7
			(+−00)	$[13][132\rangle\langle 23\rangle$	6	7
			(+++0)	$[12][13][23]$	2	7
			(++−0)	$[12]^2\langle 3123\rangle \to \varnothing$	$\not 6 \to 0$	9
$vvff$	$46 \to 38$	36	(00++)	$\langle 12\rangle \times \{[12][34], [13][24]\}$	2	5
			(00+−)	$\langle 14\rangle\langle 231][23], \langle 24\rangle\langle 132][13]$	2	6
			(0−++)	$\langle 12\rangle[34]\langle 241] \to \langle 12\rangle[34](\langle 241]/m_1 - \langle 142]/m_2)$	$\not 4 \to 2$	7
			(0+++)	$\langle 132] \times \{[12][34], [13][24]\}$	4	7
			(0++−)	$\langle 14\rangle[12][23]$	8	6
			(+++−)	$[12]^2[314\rangle$	4	8
			(++++)	$[12] \times \{[12][34], [13][24]\}$	2	7
			(−+++)	$\langle 1231\rangle[23][24] \to \varnothing$	$\not 4 \to 0$	9
			(++−−)	$[12]^2\langle 34\rangle$	2	7
			(+−−+)	$[14][132\rangle\langle 23\rangle \to [14][132\rangle\langle 23\rangle - [24][231\rangle\langle 13\rangle$	$\not 4 \to 2$	8
$vvvv$	$116 \to 85$	81	(0000)	$\{[12][34], [13][24]\} \times \{\langle 12\rangle\langle 34\rangle, \langle 13\rangle\langle 24\rangle\}$	1	4
			(+000)	$\{[12][34], [13][24]\} \times [142\rangle\langle 34\rangle \to \cdots$	$\not 8 \to 6$	6
			(++00)	$\{[12][34], [13][24]\} \times [12]\langle 34\rangle$	12	6
			(+−00)	$[13][14]\langle 23\rangle\langle 24\rangle$	12	6
			(+++0)	$\{[12][34], [13][24]\} \times [23][134\rangle$	8	8
			(++−0)	$[12]^2\langle 34\rangle\langle 324] \to [12]^2\langle 34\rangle(\langle 324]/m_4 - \langle 423]/m_3) \to \cdots$	$\not{24} \to \not{12} \to 5$	8
			(++++)	$[12]^2[34]^2, [12][13][24][34], [13]^2[24]^2$	2	8
			(+++−)	$[12][13][23]\langle 4124\rangle \to \varnothing$	$\not 8 \to 0$	10
			(++−−)	$[12]^2\langle 34\rangle^2$	6	8

On-shell SM EFTs

Gauthier Durieux
(CERN)

JHEP 01 (2020) 119, [1909.10551]
with Teppei Kitahara, Yael Shadmi, Yaniv Weiss

Phys. Rev. D101 (2020) 095021, [1912.08827]
with Camila Machado

JHEP 12 (2020) 175, [2008.09652]
with Teppei Kitahara, Camila Machado, Yael Shadmi, Yaniv Weiss

KITP 'Precision' program, 22 Apr 2021

A bottom-up approach

I. Bootstrapping EFT amplitudes
   a. Massless bases
   b. Massive bases

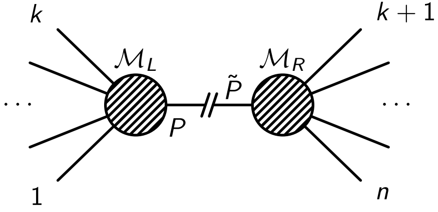
II. Massive electroweak EFT(s)
   w/ Electroweak symmetry emergence
   a. Three-points
   b. Full four-point example

I. Bootstrapping EFT amplitudes

Factorization and gluing

- Higher-point amplitudes factorize into on-shell lower-point ones, on single poles:

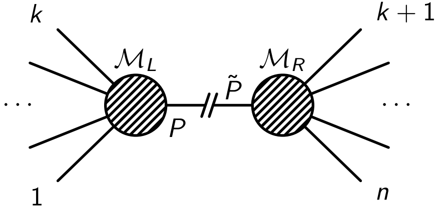
$$\underset{P^2\to m^2}{\text{Res}}\,\mathcal{M}(1,...k,...n)=\mathcal{M}_L(1,...k,P)\,\mathcal{M}_R(\tilde{P},k+1,...n)$$



Factorization and gluing

- Higher-point amplitudes factorize into on-shell lower-point ones, on single poles:

$$\underset{P^2\to m^2}{\text{Res}}\ \mathcal{M}(1,\ldots k,\ldots n)=\mathcal{M}_L(1,\ldots k,P)\,\mathcal{M}_R(\tilde{P},k+1,\ldots n)$$



- Conversely, amplitudes can be tree-level reconstructed from lower-point ones (*gluing*), up to contact terms:

$$\mathcal{M}^{\text{tree}}(1,\ldots k,\ldots n)=\sum_{\text{channels}}\frac{\mathcal{M}_L^{\text{tree}}(1,\ldots k,P)\,\mathcal{M}_R^{\text{tree}}(\tilde{P},k+1,\ldots n)}{P^2-m^2}$$

$$+\,\mathcal{M}^{\text{contact}}(1,\ldots k,\ldots n)$$

a. Massless contact terms

Massless three-points

fully determined by little-group covariance

$$\mathcal{M}(1^{h_1}, 2^{h_2}, 3^{h_3}) = g \begin{cases} [12]^{h_1+h_2-h_3} \quad [23]^{h_2+h_3-h_1} \quad [31]^{h_3+h_1-h_2} & \text{for } h_1+h_2+h_3>0 \\ \langle 12\rangle^{-h_1-h_2+h_3} \langle 23\rangle^{-h_2-h_3+h_1} \langle 31\rangle^{-h_3-h_1+h_2} & \text{for } h_1+h_2+h_3<0 \end{cases}$$

f^+f^+s $[12]$

v^+v^+s $[12]^2$

$f^+f^-v^+$ $[13]^2/[12]$

$v^+v^+v^-$ $[12]^3/[23][31]$

$t^+t^+t^-$ $\big([12]^3/[23][31]\big)^2$

$$\langle ij\rangle \equiv \epsilon_{\alpha\beta}\lambda_i^\alpha\lambda_j^\beta$$
$$[ij] \equiv \epsilon^{\dot\alpha\dot\beta}\tilde\lambda^i_{\dot\alpha}\tilde\lambda^j_{\dot\beta}$$

Massless higher-points

Multiple independent structures for given helicities

satisfying · little group constraints
· momentum conservation
· Schouten identities e.g. $[12][34] - [13][24] + [14][23] = 0$

Non-trivial Lorentz invariants

$\tilde{s}_{ij} = 2\, p_i \cdot p_j, \quad \epsilon(p_i, p_j, p_k, p_l)$

factorize out positive powers in *stripped contact terms* (SCTs)

leave aside *non-local* relations

e.g. at four-point $[12][34]/\tilde{s}_{12} = -[13][24]/\tilde{s}_{13} = [14][23]/\tilde{s}_{14}$

Construction

- *harmonics* (distinguishable particles) [Henning, Melia '19]
- *twistors* trivializing momentum conservation [Falkowski '19]
- systematic algorithm and explicit construction [GD, Machado '19]

Massless higher-points

Multiple independent structures for given helicities

satisfying · little group constraints

· momentum conservation

· Schouten identities   e.g. $[12][34]-[13][24]+[14][23]=0$

Non-trivial Lorentz invariants

$\tilde{s}_{ij} = 2\, p_i \cdot p_j, \quad \epsilon(p_i, p_j, p_k, p_l)$

factorize out positive powers in *stripped contact terms* (SCTs)

leave aside *non-local* relations

e.g. minimal dimension of operators contributing to any helicity amplitude:

$$\dim\{\text{operator}\} \geq n - \sum_i \max(0, \text{ceil}\{|h_i|-1\}) + \sum_i |h_i| + 2\max\begin{bmatrix} \{\sum_{h_i>0} 2h_i\} \mod 2 \\ 2\max_{h_i>0}\{|h_i|\} - \sum_{h_i>0} |h_i| \\ 2\max_{h_i<0}\{|h_i|\} - \sum_{h_i<0} |h_i| \end{bmatrix}$$

[GD, Machado '19]

Construction

· *harmonics* (distinguishable particles) [Henning, Melia '19]

· *twistors* trivializing momentum conservation [Falkowski '19]

· systematic algorithm and explicit construction [GD, Machado '19]

Massless operator bases

adding gauge structures, treating identical particles

- $h/Z+$ gluons [Shadmi, Weiss '18]
- SMEFT at dim-6 [Ma, Shu, Xiao '19]
- SMEFT at dim-8 and 9 [Li, Ren, Shu, Xiao, Yu, Zheng '20]
  [see also Murphy '20]
  [Li, Ren, Xiao, Yu, Zheng '20]
- LEFT at dim-8 and 9 [Li Ren, Xiao, Yu, Zheng '20]
  [see also Murphy '20]
- GR-SMEFT at dim-8 [GD, Machado '19]
  [see also Ruhdorfer, Serra, Weiler '19]

Massless operator bases

adding ga

e.g. GR-SMEFT [GD, Machado '19]

mult.	min. dim.	helicity conf.	spinor structures	SM gauge	spin stat.	Hilbert series
3-pt	dim-5	t^+t^+s	$[12]^4$	✗		
	dim-6	$t^+t^+t^+$	$[12]^2[13]^2[23]^2$			C_R^3
		$t^+t^+v^+$	$[12]^3[23][13]$		✗	
		$t^+v^+v^+$	$[12]^2[13]^2$			$(B_R^2, W_R^2, G_R^2)C_R$
4-pt	dim-6	$\underline{t}^+\underline{t}^+ss$	$[12]^4$; $[12]^4 s_{12}$			$HC_R^2H^\dagger$, $HD^2H^\dagger C_R^2$
	dim-7	$t^+t^+t^+s$	$[12]^2[13]^2[23]^2$	✗		
		$t^+t^+v^+s$	$[12]^3[13][23]$	✗		
		$\underline{t}^+\underline{t}^+f^+f^+$	$[12]^4[34]$	✗		
		$t^+t^+f^-f^-$	$[12]^4\langle 34\rangle$	✗		
		$t^+v^+v^+s$	$[12]^2[13]^2$	✗		
		$t^+v^+f^+f^+$	$[12]^2[13][14]$	✗		
	dim-8	$\underline{t}^+\underline{t}^+\underline{t}^+\underline{t}^+$	$[12]^4[34]^4+[13]^4[24]^4+[14]^4[23]^4$			C_R^4
		$t^+t^+t^+v^+$	$[12]^3[13][23][34]^2$, $[12][13]^3[23][24]^2$, $[12][13][23]^3[14]^2$		✗	
		$t^+t^+t^-t^-$	$[12]^4\langle 34\rangle^4$			$C_R^2C_L^2$
		$\underline{t}^+\underline{t}^+v^+v^+$	$[12]^4[34]^2$, $[12]^2[13][14][24][23]$			$2(B_R^2, W_R^2, G_R^2)C_R^2$
		$t^+t^+v^-v^-$	$[12]^4\langle 34\rangle^2$			$(B_L^2, W_L^2, G_L^2)C_R^2$
		$t^+t^+f^+f^-$	$[12]^4[324\rangle$		✗	
		$t^+v^+\underline{v}^+\underline{v}^+$	$[12][13][14]([13][24]+[14][23])$			$(W_R^2, G_R^2)B_RC_R$
		$t^+v^+f^+f^-$	$[12]^2[13][124\rangle$			$(QQ^\dagger, uu^\dagger, dd^\dagger, LL^\dagger, ee^\dagger)DB_RC_R$, $(QQ^\dagger, LL^\dagger)DW_RC_R$, $(QQ^\dagger, uu^\dagger, dd^\dagger)DG_RC_R$
		t^+v^+ss	$[12]^2[1231]$			$(B_R, W_R)HH^\dagger D^2C_R$
		$t^+f^+f^+s$	$[12][13][1231]$			$(Q^\dagger u^\dagger H^\dagger, Q^\dagger d^\dagger H, L^\dagger e^\dagger H)D^2C_R$
	...					
5-pt	dim-7	t^+t^+sss	$[12]^4$	✗		
	dim-8	$t^+t^+t^+ss$	$[12]^2[13]^2[23]^2$			$HH^\dagger C_R^3$
		$t^+t^+v^+ss$	$[12]^3[13][23]$		✗	
		$\underline{t}^+\underline{t}^+f^+f^+s$	$[12]^4[34]$			$(Q^\dagger u^\dagger H^\dagger, Q^\dagger d^\dagger H, L^\dagger e^\dagger H)C_R^2$
		$t^+t^+f^-f^-s$	$[12]^4\langle 34\rangle$			$(QuH, QdH^\dagger, LeH^\dagger)C_R^2$
		$t^+v^+v^+ss$	$[12]^2[13]^2$			$(B_R^2, B_RW_R, W_R^2, G_R^2)HH^\dagger C_R$
		$t^+v^+f^+f^+s$	$[12]^2[13][14]$			$(Q^\dagger u^\dagger H^\dagger, Q^\dagger d^\dagger H, L^\dagger e^\dagger H)(B_R, W_R)C_R$, $(Q^\dagger u^\dagger H^\dagger, Q^\dagger d^\dagger H)G_RC_R$
		$t^+f^+f^+f^+f^+$	$[12][13][14][15]$			$Q^\dagger Q^\dagger Q^\dagger L^\dagger C_R, d^\dagger e^\dagger u^{\dagger 2}C_R$, $d^\dagger Q^{\dagger 2}u^\dagger C_R, e^\dagger L^\dagger Q^\dagger u^\dagger C_R$
	...					
6-pt	dim-8	t^+t^+ssss	$[12]^4$			$H^2H^{\dagger 2}C_R^2$

[Shadmi, Weiss '18]

[Ma, Shu, Xiao '19]

, Xiao, Yu, Zheng '20]

[see also Murphy '20]

, Xiao, Yu, Zheng '20]

, Xiao, Yu, Zheng '20]

[see also Murphy '20]

[GD, Machado '19]

rfer, Serra, Weiler '19]

b. Massive contact terms

Massive three-points

Counting from angular momentum

number of irreps in the spin addition: [Costa, Penedones, Poland, Rychkov '11]

$$(2s_1+1)(2s_2+1)-p(p+1) \quad \text{with} \quad \begin{cases} p \equiv \max\{0, s_1+s_2-s_3\} \\ s_1 \le s_2 \le s_3 \end{cases}$$

Construction by correcting a massless-like ansatz

[GD, Kitahara, Machado, Shadmi, Weiss '20]

$$(\mathbf{12})^{s_1+s_2-\tilde{s}_3}\,(\mathbf{23})^{-s_1+s_2+\tilde{s}_3}\,(\mathbf{13})^{s_1-s_2+\tilde{s}_3}\,[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle^{s_3-\tilde{s}_3}$$

with

$$(\boldsymbol{ij})^k \equiv \text{ any } \langle \boldsymbol{ij}\rangle^{k-l}[\boldsymbol{ij}]^l \quad \text{for } l=0,...,k$$

$$s_1 \le s_2 \le s_3$$

$$\tilde{s}_3 \equiv s_3 - \max\{0, s_3-s_2-s_1\}$$

removing occurrences of $\epsilon(\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2, \boldsymbol{\varepsilon}_3, \boldsymbol{p_1}+\boldsymbol{p_2}+\boldsymbol{p_3})$

$$\begin{aligned} & m_1\langle\mathbf{12}\rangle\langle\mathbf{13}\rangle[\mathbf{23}] + m_2\langle\mathbf{12}\rangle[\mathbf{13}]\langle\mathbf{23}\rangle + m_3[\mathbf{12}]\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle \\ = {} & m_1[\mathbf{12}]\,[\mathbf{13}]\langle\mathbf{23}\rangle + m_2[\mathbf{12}]\langle\mathbf{13}\rangle[\mathbf{23}] + m_3\langle\mathbf{12}\rangle[\mathbf{13}]\,[\mathbf{23}] \end{aligned}$$

Massive t

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land, Rychkov '11]

$s_3\}$

Constr

Kitahara, Machado, Shadmi, Weiss '20]

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wit

rem

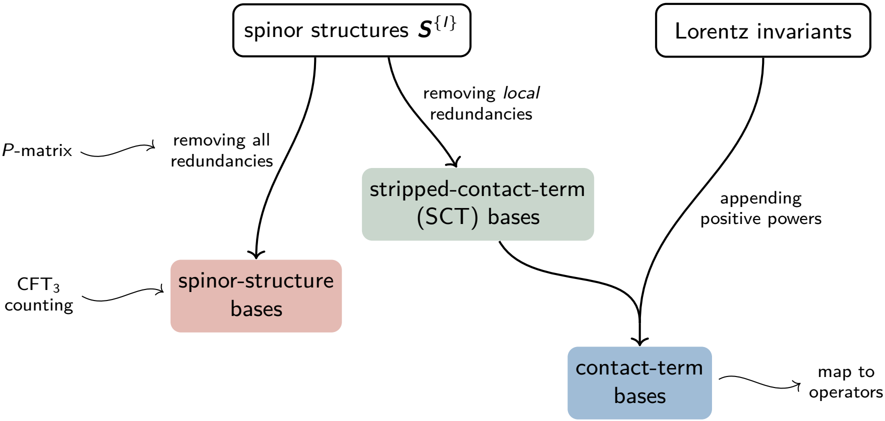
$+ p_2 + p_3)$

s_1	s_2	s_3	$n^{\text{3-pt}}$	n_{rel}	spinor structures
0	0	0	1		constant
0	0	1	1		$[\mathbf{3(1-2)3}\rangle$
0	0	2	1		$[\mathbf{3(1-2)3}\rangle^2$
0	0	3	1		$[\mathbf{3(1-2)3}\rangle^3$
0	1/2	1/2	2		$([\mathbf{23}], \langle\mathbf{23}\rangle)$
0	1/2	3/2	2		$[\mathbf{3(1-2)3}\rangle \otimes ([\mathbf{23}], \langle\mathbf{23}\rangle)$
0	1/2	5/2	2		$[\mathbf{3(1-2)3}\rangle^2 \otimes ([\mathbf{23}], \langle\mathbf{23}\rangle)$
0	1	1	3		$([\mathbf{23}]^2, \langle\mathbf{23}\rangle[\mathbf{23}], \langle\mathbf{23}\rangle^2)$
0	1	2	3		$[\mathbf{3(1-2)3}\rangle \otimes ([\mathbf{23}]^2, \langle\mathbf{23}\rangle[\mathbf{23}], \langle\mathbf{23}\rangle^2)$
0	1	3	3		$[\mathbf{3(1-2)3}\rangle^2 \otimes ([\mathbf{23}]^2, \langle\mathbf{23}\rangle[\mathbf{23}], \langle\mathbf{23}\rangle^2)$
0	3/2	3/2	4		$([\mathbf{23}]^3, \langle\mathbf{23}\rangle[\mathbf{23}]^2, \langle\mathbf{23}\rangle^2[\mathbf{23}], \langle\mathbf{23}\rangle^3)$
0	3/2	5/2	4		$[\mathbf{3(1-2)3}\rangle \otimes ([\mathbf{23}]^3, \langle\mathbf{23}\rangle[\mathbf{23}]^2, \langle\mathbf{23}\rangle^2[\mathbf{23}], \langle\mathbf{23}\rangle^3)$
0	2	2	5		$([\mathbf{23}]^4, \langle\mathbf{23}\rangle[\mathbf{23}]^3, \langle\mathbf{23}\rangle^2[\mathbf{23}]^2, \langle\mathbf{23}\rangle^3[\mathbf{23}], \langle\mathbf{23}\rangle^4)$
0	2	3	5		$[\mathbf{3(1-2)3}\rangle \otimes ([\mathbf{23}]^4, \langle\mathbf{23}\rangle[\mathbf{23}]^3, \langle\mathbf{23}\rangle^2[\mathbf{23}]^2, \langle\mathbf{23}\rangle^3[\mathbf{23}], \langle\mathbf{23}\rangle^4)$
0	5/2	5/2	6		$([\mathbf{23}]^5, \langle\mathbf{23}\rangle[\mathbf{23}]^4, \langle\mathbf{23}\rangle^2[\mathbf{23}]^3, \langle\mathbf{23}\rangle^3[\mathbf{23}]^2, \langle\mathbf{23}\rangle^4[\mathbf{23}], \langle\mathbf{23}\rangle^5)$
0	3	3	7		$([\mathbf{23}]^6, \langle\mathbf{23}\rangle[\mathbf{23}]^5, \langle\mathbf{23}\rangle^2[\mathbf{23}]^4, \langle\mathbf{23}\rangle^3[\mathbf{23}]^3, \langle\mathbf{23}\rangle^4[\mathbf{23}]^2, \langle\mathbf{23}\rangle^5[\mathbf{23}], \langle\mathbf{23}\rangle^6)$
1/2	1/2	1	4		$([\mathbf{23}], \langle\mathbf{23}\rangle) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1/2	1/2	2	4		$[\mathbf{3(1-2)3}\rangle \otimes ([\mathbf{23}], \langle\mathbf{23}\rangle) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1/2	1/2	3	4		$[\mathbf{3(1-2)3}\rangle^2 \otimes ([\mathbf{23}], \langle\mathbf{23}\rangle) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1/2	1	3/2	6		$([\mathbf{23}]^2, \langle\mathbf{23}\rangle[\mathbf{23}], \langle\mathbf{23}\rangle^2) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1/2	1	5/2	6		$[\mathbf{3(1-2)3}\rangle \otimes ([\mathbf{23}]^2, \langle\mathbf{23}\rangle[\mathbf{23}], \langle\mathbf{23}\rangle^2) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1/2	3/2	2	8		$([\mathbf{23}]^3, \langle\mathbf{23}\rangle[\mathbf{23}]^2, \langle\mathbf{23}\rangle^2[\mathbf{23}], \langle\mathbf{23}\rangle^3) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1/2	3/2	3	8		$[\mathbf{3(1-2)3}\rangle \otimes ([\mathbf{23}]^3, \langle\mathbf{23}\rangle[\mathbf{23}]^2, \langle\mathbf{23}\rangle^2[\mathbf{23}], \langle\mathbf{23}\rangle^3) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1/2	2	5/2	10		$([\mathbf{23}]^4, \langle\mathbf{23}\rangle[\mathbf{23}]^3, \langle\mathbf{23}\rangle^2[\mathbf{23}]^2, \langle\mathbf{23}\rangle^3[\mathbf{23}], \langle\mathbf{23}\rangle^4) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1/2	5/2	3	12		$([\mathbf{23}]^5, \langle\mathbf{23}\rangle[\mathbf{23}]^4, \langle\mathbf{23}\rangle^2[\mathbf{23}]^3, \langle\mathbf{23}\rangle^3[\mathbf{23}]^2, \langle\mathbf{23}\rangle^4[\mathbf{23}], \langle\mathbf{23}\rangle^5) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1	1	1	7	1	$([\mathbf{12}], \langle\mathbf{12}\rangle) \otimes ([\mathbf{23}], \langle\mathbf{23}\rangle) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1	1	2	9		$([\mathbf{23}]^2, \langle\mathbf{23}\rangle[\mathbf{23}], \langle\mathbf{23}\rangle^2) \otimes ([\mathbf{13}]^2, \langle\mathbf{13}\rangle[\mathbf{13}], \langle\mathbf{13}\rangle^2)$
1	1	3	9		$[\mathbf{3(1-2)3}\rangle \otimes ([\mathbf{23}]^2, \langle\mathbf{23}\rangle[\mathbf{23}], \langle\mathbf{23}\rangle^2) \otimes ([\mathbf{13}]^2, \langle\mathbf{13}\rangle[\mathbf{13}], \langle\mathbf{13}\rangle^2)$
1	3/2	3/2	10	2	$([\mathbf{12}], \langle\mathbf{12}\rangle) \otimes ([\mathbf{23}]^2, \langle\mathbf{23}\rangle[\mathbf{23}], \langle\mathbf{23}\rangle^2) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1	3/2	5/2	12		$([\mathbf{23}]^3, \langle\mathbf{23}\rangle[\mathbf{23}]^2, \langle\mathbf{23}\rangle^2[\mathbf{23}], \langle\mathbf{23}\rangle^3) \otimes ([\mathbf{13}]^2, \langle\mathbf{13}\rangle[\mathbf{13}], \langle\mathbf{13}\rangle^2)$
1	2	2	13	3	$([\mathbf{12}], \langle\mathbf{12}\rangle) \otimes ([\mathbf{23}]^3, \langle\mathbf{23}\rangle[\mathbf{23}]^2, \langle\mathbf{23}\rangle^2[\mathbf{23}], \langle\mathbf{23}\rangle^3) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1	2	3	15		$([\mathbf{23}]^4, \langle\mathbf{23}\rangle[\mathbf{23}]^3, \langle\mathbf{23}\rangle^2[\mathbf{23}]^2, \langle\mathbf{23}\rangle^3[\mathbf{23}], \langle\mathbf{23}\rangle^4) \otimes ([\mathbf{13}]^2, \langle\mathbf{13}\rangle[\mathbf{13}], \langle\mathbf{13}\rangle^2)$
1	5/2	5/2	16	4	$([\mathbf{12}], \langle\mathbf{12}\rangle) \otimes ([\mathbf{23}]^4, \langle\mathbf{23}\rangle[\mathbf{23}]^3, \langle\mathbf{23}\rangle^2[\mathbf{23}]^2, \langle\mathbf{23}\rangle^3[\mathbf{23}], \langle\mathbf{23}\rangle^4) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
1	3	3	19	5	$([\mathbf{12}], \langle\mathbf{12}\rangle) \otimes ([\mathbf{23}]^5, \langle\mathbf{23}\rangle[\mathbf{23}]^4, \langle\mathbf{23}\rangle^2[\mathbf{23}]^3, \langle\mathbf{23}\rangle^3[\mathbf{23}]^2, \langle\mathbf{23}\rangle^4[\mathbf{23}], \langle\mathbf{23}\rangle^5) \otimes ([\mathbf{13}], \langle\mathbf{13}\rangle)$
3/2	3/2	2	14	4	$([\mathbf{12}], \langle\mathbf{12}\rangle) \otimes ([\mathbf{23}]^2, \langle\mathbf{23}\rangle[\mathbf{23}], \langle\mathbf{23}\rangle^2) \otimes ([\mathbf{13}]^2, \langle\mathbf{13}\rangle[\mathbf{13}], \langle\mathbf{13}\rangle^2)$
3/2	3/2	3	16		$([\mathbf{23}]^3, \langle\mathbf{23}\rangle[\mathbf{23}]^2, \langle\mathbf{23}\rangle^2[\mathbf{23}], \langle\mathbf{23}\rangle^3) \otimes ([\mathbf{13}]^3, \langle\mathbf{13}\rangle[\mathbf{13}]^2, \langle\mathbf{13}\rangle^2[\mathbf{13}], \langle\mathbf{13}\rangle^3)$
3/2	2	5/2	18	6	$([\mathbf{12}], \langle\mathbf{12}\rangle) \otimes ([\mathbf{23}]^3, \langle\mathbf{23}\rangle[\mathbf{23}]^2, \langle\mathbf{23}\rangle^2[\mathbf{23}], \langle\mathbf{23}\rangle^3) \otimes ([\mathbf{13}]^2, \langle\mathbf{13}\rangle[\mathbf{13}], \langle\mathbf{13}\rangle^2)$
3/2	5/2	3	22	8	$([\mathbf{12}], \langle\mathbf{12}\rangle) \otimes ([\mathbf{23}]^4, \langle\mathbf{23}\rangle[\mathbf{23}]^3, \langle\mathbf{23}\rangle^2[\mathbf{23}]^2, \langle\mathbf{23}\rangle^3[\mathbf{23}], \langle\mathbf{23}\rangle^4) \otimes ([\mathbf{13}]^2, \langle\mathbf{13}\rangle[\mathbf{13}], \langle\mathbf{13}\rangle^2)$
2	2	2	19	8	$([\mathbf{12}]^2, \langle\mathbf{12}\rangle[\mathbf{12}], \langle\mathbf{12}\rangle^2) \otimes ([\mathbf{23}]^2, \langle\mathbf{23}\rangle[\mathbf{23}], \langle\mathbf{23}\rangle^2) \otimes ([\mathbf{13}]^2, \langle\mathbf{13}\rangle[\mathbf{13}], \langle\mathbf{13}\rangle^2)$
2	2	3	23	9	$([\mathbf{12}], \langle\mathbf{12}\rangle) \otimes ([\mathbf{23}]^3, \langle\mathbf{23}\rangle[\mathbf{23}]^2, \langle\mathbf{23}\rangle^2[\mathbf{23}], \langle\mathbf{23}\rangle^3) \otimes ([\mathbf{13}]^3, \langle\mathbf{13}\rangle[\mathbf{13}]^2, \langle\mathbf{13}\rangle^2[\mathbf{13}], \langle\mathbf{13}\rangle^3)$
2	5/2	5/2	24	12	$([\mathbf{12}]^2, \langle\mathbf{12}\rangle[\mathbf{12}], \langle\mathbf{12}\rangle^2) \otimes ([\mathbf{23}]^3, \langle\mathbf{23}\rangle[\mathbf{23}]^2, \langle\mathbf{23}\rangle^2[\mathbf{23}], \langle\mathbf{23}\rangle^3) \otimes ([\mathbf{13}]^2, \langle\mathbf{13}\rangle[\mathbf{13}], \langle\mathbf{13}\rangle^2)$
2	3	3	29	16	$([\mathbf{12}]^2, \langle\mathbf{12}\rangle[\mathbf{12}], \langle\mathbf{12}\rangle^2) \otimes ([\mathbf{23}]^4, \langle\mathbf{23}\rangle[\mathbf{23}]^3, \langle\mathbf{23}\rangle^2[\mathbf{23}]^2, \langle\mathbf{23}\rangle^3[\mathbf{23}], \langle\mathbf{23}\rangle^4) \otimes ([\mathbf{13}]^2, \langle\mathbf{13}\rangle[\mathbf{13}], \langle\mathbf{13}\rangle^2)$
5/2	5/2	3	30	18	$([\mathbf{12}]^2, \langle\mathbf{12}\rangle[\mathbf{12}], \langle\mathbf{12}\rangle^2) \otimes ([\mathbf{23}]^3, \langle\mathbf{23}\rangle[\mathbf{23}]^2, \langle\mathbf{23}\rangle^2[\mathbf{23}], \langle\mathbf{23}\rangle^3) \otimes ([\mathbf{13}]^3, \langle\mathbf{13}\rangle[\mathbf{13}]^2, \langle\mathbf{13}\rangle^2[\mathbf{13}], \langle\mathbf{13}\rangle^3)$
3	3	3	37	27	$([\mathbf{12}]^3, \langle\mathbf{12}\rangle[\mathbf{12}]^2, \langle\mathbf{12}\rangle^2[\mathbf{12}], \langle\mathbf{12}\rangle^3) \otimes ([\mathbf{23}]^3, \langle\mathbf{23}\rangle[\mathbf{23}]^2, \langle\mathbf{23}\rangle^2[\mathbf{23}], \langle\mathbf{23}\rangle^3) \otimes ([\mathbf{13}]^3, \langle\mathbf{13}\rangle[\mathbf{13}]^2, \langle\mathbf{13}\rangle^2[\mathbf{13}], \langle\mathbf{13}\rangle^3)$

$\tilde{s}_{ij} \equiv 2p_i \cdot p_j$

Massive four-points: *strip and conquer*

e.g. $W^+W^+W^-W^-$: $\dfrac{[\mathbf{13}][\mathbf{24}]\langle\mathbf{13}\rangle\langle\mathbf{24}\rangle - [\mathbf{14}][\mathbf{23}]\langle\mathbf{14}\rangle\langle\mathbf{23}\rangle}{m_1 m_2 m_3 m_4}$ $(\tilde{s}_{13} - \tilde{s}_{14} - \tilde{s}_{23} + \tilde{s}_{24})$



[GD, Kitahara, Machado, Shadmi, Weiss '20]

An aside: massive spinor structures

Counting from 3D conformal correlators [Henning, Lu, Melia, Murayama '17] [Shomerus, Sobko, Isachenkov '16] [Kravchuk, Dimmons-Duffin '16]

$$\prod_i (2s_i + 1)$$

Independence test with P-matrix determinant [Bonifacio, Hinterbichler '18]

inner product between spinor structures of identical spin content

$$P_{m,n} \equiv \sum_{\{I\},\{J\}} \boldsymbol{S}_m^{\{I\}*} \boldsymbol{S}_n^{\{J\}} \; \delta_{\{I\},\{J\}}$$

Construction by bolding massless amps [GD, Kitahara, Machado, Shadmi Weiss '20]

- Pick spinor structures with leading high-energy limits contributing to different helicity amplitudes.
- Their P-matrix is diagonal in the massless limit as they do not interfere. So they are independent.
- There are $\prod_i (2s_i + 1)$ of them. So they form a basis.

Massive four-points

[GD, Kitahara, Machado, Shadmi, Weiss '20]

Some relations between massless spinor structures are *non-local*:

$$[12][34]/\tilde{s}_{12} = -[13][24]/\tilde{s}_{13} = [14][23]/\tilde{s}_{14}$$

Massless relations are easily identified and (not so easily) *mass-completed*.

Inverse masses are allowed in $\boldsymbol{i}\rangle[\boldsymbol{i}/m_i$ polarization vectors:

$$\begin{aligned}[\mathbf{12}]\langle\mathbf{3123}\rangle = &([\mathbf{12}][\mathbf{313}\rangle\tilde{s}_{23} - [\mathbf{12}][\mathbf{323}\rangle\tilde{s}_{13})/m_3 \\ &- \tilde{s}_{12}[\mathbf{13}][\mathbf{23}] - m_1[\mathbf{321}\rangle[\mathbf{23}] - m_2[\mathbf{312}\rangle[\mathbf{13}]\end{aligned}$$

Massive four-points

[GD, Kitahara, Machado, Shadmi, Weiss '20]

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Exhausting all *local* relations, one obtains a basis of *stripped contact terms*.

Contact terms are obtained by appending positive powers of Lorentz invariants.

Massive four-points

[GD, Kitahara, Machado, Shadmi, Weiss '20]

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Exhausting all *local* relations, one obtains a basis of *stripped contact terms*.

Contact terms are obtained by appending positive powers of Lorentz invariants.

Few remaining redundancies are finally removed:

$$\begin{aligned}\tilde{s}_{12}\,[\mathbf{13}][\mathbf{24}] = -(\tilde{s}_{24} + m_2^2)\,[\mathbf{12}][\mathbf{34}] &- m_1\langle\mathbf{123}][\mathbf{24}] - m_2\langle\mathbf{213}][\mathbf{14}] \\ &+ m_3\langle\mathbf{324}][\mathbf{12}] + m_4\langle\mathbf{423}][\mathbf{12}]\end{aligned}$$

Massive four-points

spins	$n_{\rm SCT}$	$n_{\rm s}$	hel. cat.	spinor structures	$n_{\rm perm}$	$\min\{d_{\rm op}\}$
$ssss$	1	1	(0000)	constant	1	4
$vsss$	$4\to3$	3	(0000)	$[\mathbf{121}\rangle, [\mathbf{131}\rangle$	1	5
			(+000)	$[\mathbf{1231}] \to [\mathbf{1231}] - \langle\mathbf{1231}\rangle$	$\not{2}\to 1$	7
$ffss$	4	4	(++00)	$[\mathbf{12}]$	2	5
			(+−00)	$[\mathbf{132}\rangle$	2	6
$vvss$	$10\to9$	9	(0000)	$[\mathbf{12}]\langle\mathbf{12}\rangle, [\mathbf{131}\rangle[\mathbf{232}\rangle$	1	4,6
			(+000)	$[\mathbf{12}][\mathbf{132}\rangle$	4	6
			(++00)	$[\mathbf{12}]^2$	2	6
			(+−00)	$[\mathbf{132}\rangle^2 \to [\mathbf{132}\rangle^2 - \langle\mathbf{132}]^2$	$\not{2}\to 1$	8
$ffvs$	$14\to12$	12	(++00)	$[\mathbf{12}]\{[\mathbf{313}\rangle, [\mathbf{323}\rangle\}$	2	6
			(+−00)	$[\mathbf{13}]\langle\mathbf{23}\rangle$	2	5
			(+++0)	$[\mathbf{13}][\mathbf{23}]$	2	6
			(++−0)	$[\mathbf{12}]\langle\mathbf{3123}\rangle \to \varnothing$	$\not{2}\to 0$	8
			(+−+0)	$[\mathbf{13}][\mathbf{312}\rangle$	4	7
$ffff$	18	16	(++++)	$[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]$	2	6
			(++−−)	$[\mathbf{12}]\langle\mathbf{34}\rangle$	6	6
			(+++−)	$[\mathbf{12}][\mathbf{324}\rangle$	8	7
$vvvs$	$35\to29$	27	(0000)	$[\mathbf{12}][\mathbf{343}\rangle\langle\mathbf{12}\rangle, [\mathbf{13}][\mathbf{242}\rangle\langle\mathbf{13}\rangle, [\mathbf{23}][\mathbf{141}\rangle\langle\mathbf{23}\rangle$	1	5
			(+000)	$[\mathbf{12}][\mathbf{13}]\langle\mathbf{23}\rangle$	6	5
			(++00)	$[\mathbf{12}]^2\big\{[\mathbf{313}\rangle, [\mathbf{323}\rangle\big\}$	6	7
			(+−00)	$[\mathbf{13}][\mathbf{132}\rangle\langle\mathbf{23}\rangle$	6	7
			(+++0)	$[\mathbf{12}][\mathbf{13}][\mathbf{23}]$	2	7
			(++−0)	$[\mathbf{12}]^2\langle\mathbf{3123}\rangle \to \varnothing$	$\not{6}\to 0$	9
$vvff$	$46\to38$	36	(00++)	$\langle\mathbf{12}\rangle \times \{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\}$	2	5
			(00+−)	$\langle\mathbf{14}\rangle\langle\mathbf{231}][\mathbf{23}], \langle\mathbf{24}\rangle\langle\mathbf{132}][\mathbf{13}]$	2	6
			(0−++)	$\langle\mathbf{12}\rangle[\mathbf{34}]\langle\mathbf{241}] \to \langle\mathbf{12}\rangle[\mathbf{34}](\langle\mathbf{241}]/m_1 - \langle\mathbf{142}]/m_2)$	$\not{4}\to 2$	7
			(0+++)	$\langle\mathbf{132}] \times \{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\}$	4	7
			(0++−)	$\langle\mathbf{14}\rangle[\mathbf{12}][\mathbf{23}]$	8	6
			(+++−)	$[\mathbf{12}]^2[\mathbf{314}\rangle$	4	8
			(++++)	$[\mathbf{12}] \times \{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\}$	2	7
			(−+++)	$\langle\mathbf{1231}\rangle[\mathbf{23}][\mathbf{24}] \to \varnothing$	$\not{4}\to 0$	9
			(++−−)	$[\mathbf{12}]^2\langle\mathbf{34}\rangle$	2	7
			(+−−+)	$[\mathbf{14}][\mathbf{132}\rangle\langle\mathbf{23}\rangle \to [\mathbf{14}][\mathbf{132}\rangle\langle\mathbf{23}\rangle - [\mathbf{24}][\mathbf{231}\rangle\langle\mathbf{13}\rangle$	$\not{4}\to 2$	8
$vvvv$	$116\to85$	81	(0000)	$\{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\} \times \{\langle\mathbf{12}\rangle\langle\mathbf{34}\rangle, \langle\mathbf{13}\rangle\langle\mathbf{24}\rangle\}$	1	4
			(+000)	$\{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\} \times [\mathbf{142}\rangle\langle\mathbf{34}\rangle \to \cdots$	$\not{8}\to 6$	6
			(++00)	$\{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\} \times [\mathbf{12}]\langle\mathbf{34}\rangle$	12	6
			(+−00)	$[\mathbf{13}][\mathbf{14}]\langle\mathbf{23}\rangle\langle\mathbf{24}\rangle$	12	6
			(+++0)	$\{[\mathbf{12}][\mathbf{34}], [\mathbf{13}][\mathbf{24}]\} \times [\mathbf{23}][\mathbf{134}\rangle$	8	8
			(++−0)	$[\mathbf{12}]^2\langle\mathbf{34}\rangle\langle\mathbf{324}] \to [\mathbf{12}]^2\langle\mathbf{34}\rangle(\langle\mathbf{324}]/m_4 - \langle\mathbf{423}]/m_3) \to \cdots$	$\not{24}\to\not{12}\to 5$	8
			(++++)	$[\mathbf{12}]^2[\mathbf{34}]^2, [\mathbf{12}][\mathbf{13}][\mathbf{24}][\mathbf{34}], [\mathbf{13}]^2[\mathbf{24}]^2$	2	8
			(+++−)	$[\mathbf{12}][\mathbf{13}][\mathbf{23}]\langle\mathbf{4124}\rangle \to \varnothing$	$\not{8}\to 0$	10
			(++−−)	$[\mathbf{12}]^2\langle\mathbf{34}\rangle^2$	6	8

Massive harmonics

[Dong, Ma, Shu '21]

Recipe

- use eoms to keep one type of spinors: $|\boldsymbol{i}] = \boldsymbol{p_i}|\boldsymbol{i}\rangle/m_i$
- treat separately the spinors and momenta

$$\left(\langle\mathbf{1}|^{2s_1}\ldots\langle\boldsymbol{n}|^{2s_n}\right)\otimes\left(\boldsymbol{p_1}\ldots\boldsymbol{p_n}\right)$$

- decompose $\left(\langle\mathbf{1}|^{2s_1}\ldots\langle\boldsymbol{n}|^{2s_n}\right)_{\{\alpha^1_1\cdots\alpha^1_{2s_1}\}\cdots\{\alpha^n_1\cdots\alpha^n_{2s_n}\}}$ in SU(2) irreps
- treat $\left(\boldsymbol{p_1}\ldots\boldsymbol{p_n}\right)_{\{\alpha^1_1\cdots\alpha^1_{2s_1}\}\cdots\{\alpha^n_1\cdots\alpha^n_{2s_n}\}}$ as if massless
  and enumerate independent elements using harmonics [Henning, Melia '19]

Complications

- produces lengthy linear combinations of elementary structures
- obscures the corresponding operator dimension

Bootstrapping EFT contact terms

- ✓ massless, any-point, any spin
- ✓ massive, three-points, any spin
- ✓ massive, four-points, spins$\leq$1 (explicitly)
- (✓) massive, higher-points or higher spins (implicitly)

II. Massive electroweak EFT(s)

[GD, Kitahara, Shadmi, Weiss '19]

Assumptions

Poincaré, locality, unitarity

electroweak spectrum: $\psi, \psi', \gamma, Z, W^{\pm}, h$
(all massive but γ)

global electric charge & fermion number
$$|Q_\psi - Q_{\psi'}| = |Q_W|$$

Electroweak symmetry emergence

Given a particle content,
from perturbative unitarity up to $\bar{\Lambda} \gg m$!

[Llewelly Smith '73]
[Joglekar '73]
[Conwall et al. '73, '74]

HIGH ENERGY BEHAVIOUR AND GAUGE SYMMETRY

C.H. LLEWELLYN SMITH
CERN, Geneva, Switzerland

Received 13 May 1973

The imposition of unitarity bounds is shown to lead to a Yang-Mills structure in a wide class of theories involving vector mesons. Scalar fields are needed and, at least in simple cases, the unique unitary theory is of the Higgs type

S-Matrix Derivation of the Weinberg Model[1]

SATISH D. JOGLEKAR

Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11790

Received June 18, 1973

Uniqueness of Spontaneously Broken Gauge Theories*

John M. Cornwall,† David N. Levin, and George Tiktopoulos
Department of Physics, University of California, Los Angeles, California 90024
(Received 25 April 1973)

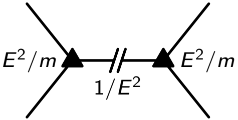
We have made a systematic search for theories of interacting heavy vector mesons which have unitarily bound trees. In simple cases (four vector mesons and one scalar particle) the *only* unitarily bound models are spontaneously broken gauge theories. Evidently, a unitarity bound, which controls high-energy behavior, imposes internal symmetry on heavy-vector-boson interactions.

a. Massive EW three-points

Perturbative unitarity for three-points?

Rule of thumb:

Three-points growing as E^2/m or faster (for complex momenta) lead to unacceptable E^2/m^2 four-point growths.



$\sim E^2/m^2$

not considered here
(the dependence of the spinor structure coefficient on the propagator virtuality could cancel the E growth)

$\rightarrow$ E^2 three-points only arise · at the loop level

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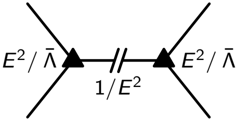


$\sim E^2/\bar{\Lambda}^2$

not considered here

(the dependence of the spinor structure coefficient on the propagator virtuality could cancel the E growth)

$\rightarrow$ E^2 three-points only arise
- · at the loop level
- · at the $1/\bar{\Lambda}$ level

$\psi^c \psi Z$

$$\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle,\ \langle\mathbf{13}\rangle[\mathbf{23}],\ [\mathbf{13}]\langle\mathbf{23}\rangle,\ [\mathbf{13}][\mathbf{23}]$$

four terms expected from ang. mom.: $2 \otimes 2 \otimes 3 = 1 \oplus 3 \oplus 3 \oplus 5$

$\psi^c \psi Z$

$$\langle \mathbf{13} \rangle \langle \mathbf{23} \rangle,\ \langle \mathbf{13} \rangle [\mathbf{23}],\ [\mathbf{13}] \langle \mathbf{23} \rangle,\ [\mathbf{13}][\mathbf{23}]$$

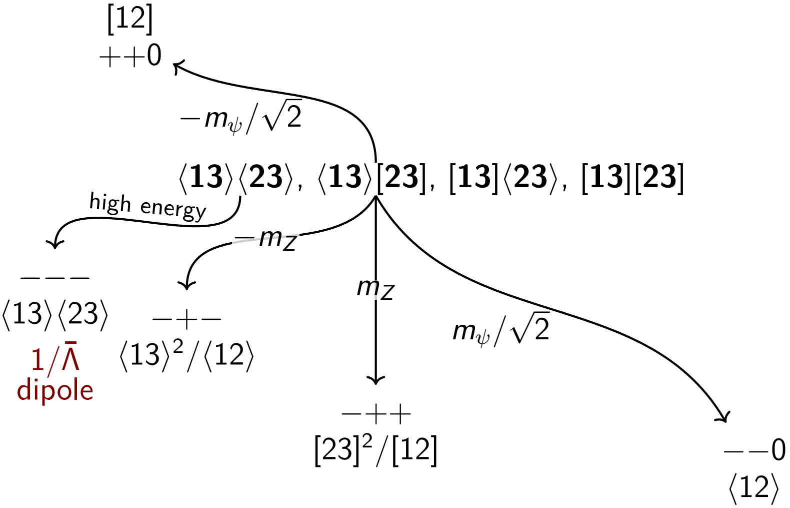
high energy

$---$

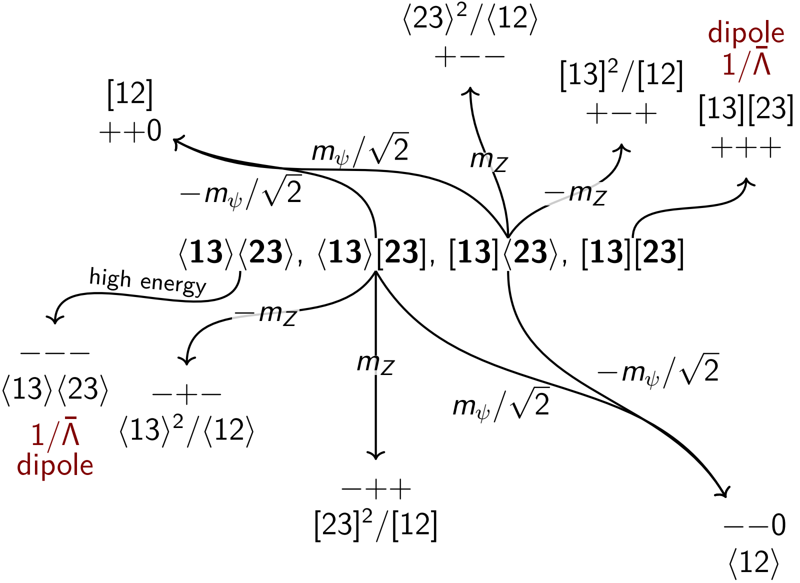
$\langle 13 \rangle \langle 23 \rangle$

$1/\bar{\Lambda}$
dipole

$\psi^c\psi Z$


[12]
++0
$-m_\psi/\sqrt{2}$
⟨13⟩⟨23⟩, ⟨13⟩[23], [13]⟨23⟩, [13][23]
high energy
$-m_Z$
m_Z
$m_\psi/\sqrt{2}$
−−−
⟨13⟩⟨23⟩
$1/\bar{\Lambda}$
dipole
−+−
⟨13⟩²/⟨12⟩
−++
[23]²/[12]
−−0
⟨12⟩

$\psi^c \psi Z$



$\langle 23\rangle^2/\langle 12\rangle$
$+--$
$[13]^2/[12]$
$+-+$
dipole
$1/\bar{\Lambda}$
$[13][23]$
$+++$
$[12]$
$++0$
$m_\psi/\sqrt{2}$
m_Z
$-m_\psi/\sqrt{2}$
$-m_Z$
$\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle$, $\langle\mathbf{13}\rangle[\mathbf{23}]$, $[\mathbf{13}]\langle\mathbf{23}\rangle$, $[\mathbf{13}][\mathbf{23}]$
high energy
$-m_Z$
$---$
$\langle 13\rangle\langle 23\rangle$
$1/\bar{\Lambda}$
dipole
$-+-$
$\langle 13\rangle^2/\langle 12\rangle$
m_Z
$m_\psi/\sqrt{2}$
$-m_\psi/\sqrt{2}$
$-++$
$[23]^2/[12]$
$--0$
$\langle 12\rangle$

$\psi^c \psi Z$

$$\left.\frac{\text{longitudinal}}{\text{transverse}}\right|_{\text{high } E} \sim \quad \frac{g^L - g^R}{g^{L,R}} \; \frac{m_\psi}{\sqrt{2} m_Z}$$

$\rightarrow \sim y_\psi$

$\rightarrow$ pseudo-scalar

$\psi^c\psi Z$

$$\left.\frac{\text{longitudinal}}{\text{transverse}}\right|_{\text{high } E} \sim \quad \frac{g^L - g^R}{g^{L,R}}\;\frac{m_\psi}{\sqrt{2}m_Z}$$

→ $\sim y_\psi$

→ pseudo-scalar

reproduces expectations
from the Higgs mechanism
and the Goldstone equivalence theorem

more in the four-point discussion

W^+W^-Z

- 8 combinations of $\begin{matrix}\langle\mathbf{12}\rangle\\ [\mathbf{12}]\end{matrix} \otimes \begin{matrix}\langle\mathbf{23}\rangle\\ [\mathbf{23}]\end{matrix} \otimes \begin{matrix}\langle\mathbf{31}\rangle\\ [\mathbf{31}]\end{matrix}$

- one non-trivial relation between them:

$$m_1\langle\mathbf{12}\rangle\langle\mathbf{13}\rangle[\mathbf{23}] + m_2\langle\mathbf{12}\rangle[\mathbf{13}]\langle\mathbf{23}\rangle + m_3[\mathbf{12}]\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle$$
$$= m_1[\mathbf{12}]\,[\mathbf{13}]\langle\mathbf{23}\rangle + m_2[\mathbf{12}]\langle\mathbf{13}\rangle[\mathbf{23}] + m_3\langle\mathbf{12}\rangle[\mathbf{13}]\,[\mathbf{23}]$$

7 combinations expected from angular momentum
$3 \otimes 3 \otimes 3 = 1 \oplus 3 \oplus 3 \oplus 3 \oplus 5 \oplus 5 \oplus 7$

- combinations growing like E^3 and E^2
can only arise at the non-renormalizable (tree) level

W^+W^-Z

C is $\mathbf{1} \leftrightarrow \mathbf{2}$

P is $\cdot] \leftrightarrow \cdot\rangle$

$$
\begin{aligned}
&\mathcal{M}(\mathbf{1}_W, \mathbf{2}_W, \mathbf{3}_Z) = \\
&\quad + \langle\mathbf{12}\rangle\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle\ c_{WWZ}^{LLL}/\bar{\Lambda}^2 \\
&\quad + \langle\mathbf{12}\rangle(\langle\mathbf{13}\rangle[\mathbf{23}] - [\mathbf{13}]\langle\mathbf{23}\rangle)\ c_{WWZ}^{[L0]0}/m_Z\bar{\Lambda} \\
&\quad + \langle\mathbf{12}\rangle(\langle\mathbf{13}\rangle[\mathbf{23}] + [\mathbf{13}]\langle\mathbf{23}\rangle)\ c_{WWZ}^{\{L0\}0}/m_Z\bar{\Lambda} \\
&\quad + \big\{[\mathbf{12}]\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle\ m_Z/m_W + \langle\mathbf{12}\rangle[\mathbf{13}]\langle\mathbf{23}\rangle + \langle\mathbf{12}\rangle\langle\mathbf{13}\rangle[\mathbf{23}] \\
&\quad + \langle\mathbf{12}\rangle\,[\mathbf{13}]\,[\mathbf{23}]\ m_Z/m_W + [\mathbf{12}]\,\langle\mathbf{13}\rangle[\mathbf{23}] + [\mathbf{12}]\,[\mathbf{13}]\,\langle\mathbf{23}\rangle\big\}\ c_{WWZ}/m_Z m_W \\
&\quad + [\mathbf{12}](\langle\mathbf{13}\rangle[\mathbf{23}] - [\mathbf{13}]\langle\mathbf{23}\rangle)\ c_{WWZ}^{[R0]0}/m_Z\bar{\Lambda} \\
&\quad + [\mathbf{12}](\langle\mathbf{13}\rangle[\mathbf{23}] + [\mathbf{13}]\langle\mathbf{23}\rangle)\ c_{WWZ}^{\{R0\}0}/m_Z\bar{\Lambda} \\
&\quad + [\mathbf{12}][\mathbf{13}][\mathbf{23}]\ c_{WWZ}^{RRR}/\bar{\Lambda}^2\,,
\end{aligned}
$$

- one single (rather non-trivial) renormalizable structure!

W^+W^-Z

C is $\mathbf{1} \leftrightarrow \mathbf{2}$

P is $\cdot] \leftrightarrow \cdot\rangle$

$$
\begin{aligned}
\mathcal{M}(\mathbf{1}_W, \mathbf{2}_W, \mathbf{3}_Z) = \\
&+ \langle\mathbf{12}\rangle\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle\; c_{WWZ}^{LLL}/\bar{\Lambda}^2 \\
&+ \langle\mathbf{12}\rangle(\langle\mathbf{13}\rangle[\mathbf{23}] - [\mathbf{13}]\langle\mathbf{23}\rangle)\; c_{WWZ}^{[L0]0}/m_Z\bar{\Lambda} \\
&+ \langle\mathbf{12}\rangle(\langle\mathbf{13}\rangle[\mathbf{23}] + [\mathbf{13}]\langle\mathbf{23}\rangle)\; c_{WWZ}^{\{L0\}0}/m_Z\bar{\Lambda} \\
&+ \big\{[\mathbf{12}]\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle\; m_Z/m_W + \langle\mathbf{12}\rangle[\mathbf{13}]\langle\mathbf{23}\rangle + \langle\mathbf{12}\rangle\langle\mathbf{13}\rangle[\mathbf{23}] \\
&+ \langle\mathbf{12}\rangle\,[\mathbf{13}]\,[\mathbf{23}]\; m_Z/m_W + [\mathbf{12}]\,\langle\mathbf{13}\rangle[\mathbf{23}] + [\mathbf{12}]\,[\mathbf{13}]\,\langle\mathbf{23}\rangle\big\}\; c_{WWZ}/m_Z m_W \\
&+ [\mathbf{12}](\langle\mathbf{13}\rangle[\mathbf{23}] - [\mathbf{13}]\langle\mathbf{23}\rangle)\; c_{WWZ}^{[R0]0}/m_Z\bar{\Lambda} \\
&+ [\mathbf{12}](\langle\mathbf{13}\rangle[\mathbf{23}] + [\mathbf{13}]\langle\mathbf{23}\rangle)\; c_{WWZ}^{\{R0\}0}/m_Z\bar{\Lambda} \\
&+ [\mathbf{12}][\mathbf{13}][\mathbf{23}]\; c_{WWZ}^{RRR}/\bar{\Lambda}^2\,,
\end{aligned}
$$

- one single (rather non-trivial) renormalizable structure!
- for identical vectors ($m_Z/m_W \to 1$)
  - no fully symmetric combination $\rightarrow$ ZZZ vanishes
  - only fully antisymmetric combinations $\rightarrow$ $W^a W^b W^c$ requires ϵ_{abc}

All three-points

$$\psi^c \psi Z$$

$$\psi^c \psi \gamma$$

$$\psi^c \psi' W$$

$$\psi^c \psi h$$

$$ZZh,\ WWh$$

$$\gamma\gamma h,\ \gamma Z h$$

$$hhZ,\ hh\gamma$$

$$hhh$$

$$WWZ$$

$$WW\gamma$$

$$ZZZ,\ ZZ\gamma,\ Z\gamma\gamma,\ \gamma\gamma\gamma$$

All three-points

$$\psi^c \psi Z$$

$$\psi^c \psi \gamma$$

$$\psi^c \psi' W$$

$$\psi^c \psi h$$

$$ZZh,\ WWh$$

some vanish identically
some only arise at the non-renormalizable (tree) level
some can be obtained from the massless limit of others

$$\gamma\gamma h,\ \gamma Zh$$

$$hhZ,\ hh\gamma$$

$$hhh$$

$$WWZ$$

$$WW\gamma$$

$$ZZZ,\ ZZ\gamma,\ Z\gamma\gamma,\ \gamma\gamma\gamma$$

[see also renormalizable discussion in Christensen et al. '18]

Matching to SMEFT

Extending results from [Aoude, Machado '19]
Warsaw basis conventions from [Dedes et al. '17]

Three-point amplitude coefficients capture all orders in v/Λ.

e.g. $\mathcal{M}(\mathbf{1}_{\psi^c}, \mathbf{2}_{\psi}, \mathbf{3}_Z) = \frac{c^{LLL}_{\psi^c\psi Z}}{\bar\Lambda}\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle + \frac{c^{LR0}_{\psi^c\psi Z}}{m_Z}\langle\mathbf{13}\rangle[\mathbf{23}] + \frac{c^{RL0}_{\psi^c\psi Z}}{m_Z}[\mathbf{13}]\langle\mathbf{23}\rangle + \frac{c^{RRR}_{\psi^c\psi Z}}{\bar\Lambda}[\mathbf{13}][\mathbf{23}]$

at dim-6:

$$c^{LR0}_{\psi^c\psi Z} = -\sqrt{2}Q_\psi \frac{\bar g'^2}{\sqrt{\bar g^2+\bar g'^2}} + \frac{v^2}{\Lambda^2}\left[-\sqrt{2}\frac{\bar g^3 \bar g'}{(\bar g^2+\bar g'^2)^{3/2}} Q_\psi C_{\varphi WB} - \frac{1}{\sqrt{2}}\sqrt{\bar g^2+\bar g'^2}\,C_{\varphi\psi_R}\right]$$

$$c^{RL0}_{\psi^c\psi Z} = \sqrt{2}I_\psi \frac{\bar g^2}{\sqrt{\bar g^2+\bar g'^2}} - \sqrt{2}Y_\psi \frac{\bar g'^2}{\sqrt{\bar g^2+\bar g'^2}}$$

$$+ \frac{v^2}{\Lambda^2}\left[\sqrt{2}\frac{\bar g \bar g'}{(\bar g^2+\bar g'^2)^{3/2}}\left(-Y_\psi \bar g^2 + I_\psi \bar g'^2\right) C_{\varphi WB} - \frac{1}{\sqrt{2}}\sqrt{\bar g^2+\bar g'^2}\left(C^1_{\varphi\psi_L} - 2I_\psi C^3_{\varphi\psi_L}\right)\right]$$

$$\frac{c^{RRR}_{\psi^c\psi Z}}{\bar\Lambda} = \frac{v}{\Lambda^2}\left(-4I_\psi \frac{\bar g}{\sqrt{\bar g^2+\bar g'^2}} C_{\psi W} + 2\frac{\bar g'}{\sqrt{\bar g^2+\bar g'^2}} C_{\psi B}\right)$$

$$c^{LLL}_{\psi^c\psi Z} = \left(c^{RRR}_{\psi^c\psi Z}\right)^*$$

$I_{u,d,e,\nu} \equiv 1/2,\ -1/2,\ -1/2,\ 1/2$
$Y_{u,d,e,\nu} \equiv 1/6,\ 1/6,\ -1/2,\ -1/2$

b. Full four-point example: $\psi^c \psi Zh$

contact + factorizable terms

Contact terms

captures all EFT orders both in HEFT and SMEFT

- Twelve independent SCTs:

$$\begin{aligned}\mathcal{M}^{\text{nf}}(\mathbf{1}_{\psi^c},\mathbf{2}_{\psi},\mathbf{3}_Z,\mathbf{4}_h) = {} & \frac{c^{RRR}_{\psi^c\psi Zh}}{\bar\Lambda^2}[\mathbf{13}][\mathbf{23}] + \frac{[\mathbf{12}]}{\bar\Lambda^3}\langle\mathbf{3}\ \{c^{RR0_A}_{\psi^c\psi Zh}(\mathbf{1}+\mathbf{2}) + c^{RR0_S}_{\psi^c\psi Zh}(\mathbf{1}-\mathbf{2})\}\ \mathbf{3}] \\ & + \frac{c^{RL0}_{\psi^c\psi Zh}}{\bar\Lambda^2}[\mathbf{13}]\langle\mathbf{23}\rangle + \frac{c^{RLR}_{\psi^c\psi Zh}}{\bar\Lambda^3}[\mathbf{312}\rangle[\mathbf{13}] + \frac{c^{RLL}_{\psi^c\psi Zh}}{\bar\Lambda^3}\langle\mathbf{321}]\langle\mathbf{23}\rangle \\ & + \frac{c^{LR0}_{\psi^c\psi Zh}}{\bar\Lambda^2}\langle\mathbf{13}\rangle[\mathbf{23}] + \frac{c^{LRR}_{\psi^c\psi Zh}}{\bar\Lambda^3}[\mathbf{321}\rangle[\mathbf{23}] + \frac{c^{LRL}_{\psi^c\psi Zh}}{\bar\Lambda^3}\langle\mathbf{312}]\langle\mathbf{13}\rangle \\ & + \frac{c^{LLL}_{\psi^c\psi Zh}}{\bar\Lambda^2}\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle + \frac{\langle\mathbf{12}\rangle}{\bar\Lambda^3}\langle\mathbf{3}\ \{c^{LL0_A}_{\psi^c\psi Zh}(\mathbf{1}+\mathbf{2}) + c^{LL0_S}_{\psi^c\psi Zh}(\mathbf{1}-\mathbf{2})\}\ \mathbf{3}]\end{aligned}$$

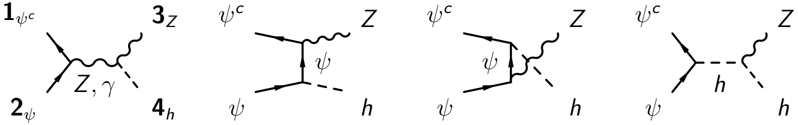
where $c_{\psi^c\psi Zh}$'s are expansions in $\tilde{s}_{ij} \equiv 2p_i\cdot p_j$

(negative power of $p_i\cdot p_j$ would cover factorizable contributions, non-rational functions would cover loop contributions)

- Perturbative unitarity up to $\bar\Lambda \gg m$ forbids e.g. $[\mathbf{13}]\langle\mathbf{23}\rangle/m_3\bar\Lambda$.

- For $p_h \to 0$ (aka *soft Higgs limit*), one recovers $\psi^c\psi Z$ amplitudes.

Factorizable terms



[skipping algebra & definitions]

Leading high-energy amplitudes $\sim E/m$: see also [Mantani et al. '19]

$$(--0): \quad -\langle 12\rangle \,(c^{RL0}_{\psi^c\psi Z} - c^{LR0}_{\psi^c\psi Z})\,(c^{00}_{ZZh}\, m_\psi/2m_Z - c^{LL}_{\psi^c\psi h})/\sqrt{2}m_Z$$

$$(++0): \quad +[12]\,(c^{RL0}_{\psi^c\psi Z} - c^{LR0}_{\psi^c\psi Z})\,(c^{00}_{ZZh}\, m_\psi/2m_Z - c^{RR}_{\psi^c\psi h})/\sqrt{2}m_Z$$

Perturbative unitarity up to $\bar\Lambda \gg m$ requires:

either vector-like fermion: $c^{RL0}_{\psi^c\psi Z} = c^{LR0}_{\psi^c\psi Z}$

or Higgs mechanism: $c^{RR}_{\psi^c\psi h} = c^{00}_{ZZh}\dfrac{m_\psi}{2m_Z} = c^{LL}_{\psi^c\psi h}$

up to $\mathcal{O}(m/\bar\Lambda)$

On-shell SM EFTs

Going on-shell avoids gauge and field-redefinition redundancies.

Massless contact terms can be bootstrapped
and replace massless operator enumerations.

Massive contact terms can be constructed systematically,
although $n>4$ and spins>1 are presently cumbersome.

Gauge symmetries emerge from perturbative unitarity,
for a given particle spectrum.

The machinery is in place for massive SM EFT(s) applications.

New insights and computations are awaited!
more four-points, loops, massive recursions, double copy, positivity, etc.