Exploring Two Axes at Colliders: From Precision to Novel Observables

intro by Iain Stewart (MIT)

Part (1): "The Higgs pT Spectrum and Total Cross Section with Fiducial Cuts at $\mathrm{N^3LL'} + \mathrm{N^3LO}$"

talk by Johannes Michel (MIT)

Part (2): "Pure Quark and Gluon Observables with Collinear Drop"

talk by Xiaojun Yao (MIT)

Exploring Two Axes at Colliders: From Precision to Novel Observables

The Higgs p_T Spectrum and Total Cross Section with Fiducial Cuts at $\mathrm{N^3LL'}$+$\mathrm{N^3LO}$

Johannes Michel

MIT CTP

Joint session with Iain Stewart and Xiaojun Yao

New Physics from Precision at High Energies
KITP, 11 May

Exploring Two Axes at Colliders: From Precision to Novel Observables

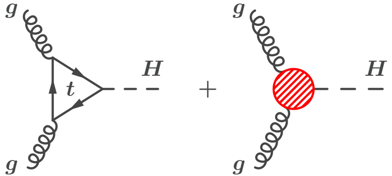
The Higgs p_T Spectrum and Total Cross Section with Fiducial Cuts at N^{3}LL$'$+N^{3}LO

based on
[2102.08039]

in collaboration with
G. Billis, B. Dehnadi, M. Ebert, F. Tackmann

- Measure fiducial & differential Higgs cross sections at the LHC
  - Most basic thing to do after discovering the Higgs
  - Most model-independent way we have to search for BSM in the Higgs sector
- Total fiducial cross section measures deviations from SM gluon-fusion rate:



$$= \left(\frac{\alpha_s}{12\pi v} C_t + \frac{v}{\Lambda^2} C_{HG}\right) H\, G^a_{\mu\nu} G^{a,\mu\nu}$$

Motivation

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  - Most basic thing to do after discovering the Higgs
  - Most model-independent way we have to search for BSM in the Higgs sector
- Total fiducial cross section measures deviations from SM gluon-fusion rate:

	Individual			Marginalised		
SMEFT Coeff.	Best fit [$\Lambda = 1$ TeV]	95% CL range	Scale $\frac{\Lambda}{\sqrt{C}}$ [TeV]	Best fit [$\Lambda = 1$ TeV]	95% CL range	Scale $\frac{\Lambda}{\sqrt{C}}$ [TeV]
C_{Hq}	0.00	[-0.017, +0.012]	8.3	-0.05	[-0.11, +0.012]	4.1
$C_{Hq}^{(1)}$	0.02	[-0.1, +0.14]	2.9	-0.04	[-0.27, +0.18]	2.1
C_{Hd}	-0.03	[-0.13, +0.071]	3.1	-0.39	[-0.91, +0.13]	1.4
C_{Hu}	0.00	[-0.075, +0.073]	3.7	-0.19	[-0.63, +0.25]	1.5
$C_{H\Box}$	-0.27	[-1, +0.47]	1.2	-0.9	[-3, +1.2]	0.69
C_{HG}	0.00	[-0.0034, +0.0032]	17.0	0.00	[-0.014, +0.0086]	9.4
C_{HW}	0.00	[-0.012, +0.006]	11.0	0.12	[-0.38, +0.62]	1.4
C_{HB}	0.00	[-0.0034, +0.002]	19.0	0.07	[-0.09, +0.22]	2.5

[Ellis, Madigan, Mimasu, Sanz, You, 2012.02779; Tab. 6]

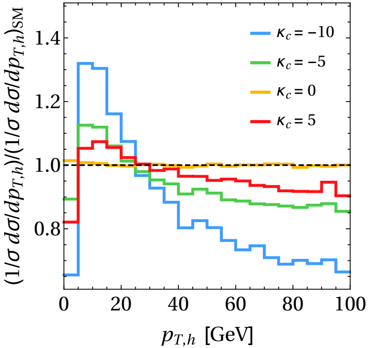
Motivation

- Next-to-most basic thing: measure the Higgs transverse momentum
- High $p_T^H \sim \sqrt{\hat{s}} \gg m_H$ increases sensitivity to new operators

  [...see yesterday's detailed discussion]

- Focus of this talk: $p_T^H \lesssim m_H \sim \sqrt{\hat{s}} \ll 2m_t$ (or p_T^H integrated over)
  - Measure or put bounds on anomalous b, c, and light quark Yukawa couplings

    [Bishara, Haisch, Monni, Re, 1606.09253; Soreq, Zhu, Zupan, 1606.09621]



- Uncertainty $\Delta\sigma$ on SM prediction translates into discovery reach:

$$\frac{\Delta\sigma}{\sigma} \sim \frac{v^2}{\Lambda_{\mathrm{BSM}}^2} \quad \Leftrightarrow \quad \Lambda_{\mathrm{BSM}} \sim v\sqrt{\frac{\sigma}{\Delta\sigma}}$$

Challenges for theory

- QCD corrections to $gg \to H$ are large: $\sigma/\sigma_{\mathrm{LO}} \approx 3$
  - Calculation of inclusive cross section has been pushed to N^{3}LO
    [Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger '15-'18]
- But LHC experiments apply kinematic selection cuts on Higgs decay products
  - Need complete interplay of QCD corrections and $\mathcal{O}(1)$ fiducial acceptance

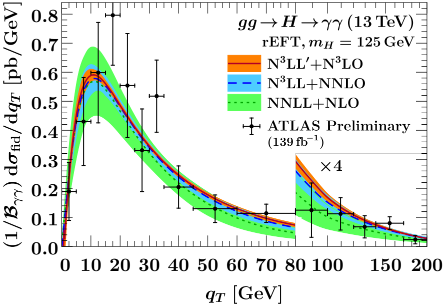
[See talk by Haider Abidi on 27 April about issues with model-dependent acceptance in SMEFT interpretations of $H \to 4\ell$ – this will be similar, but for the SM baseline...]

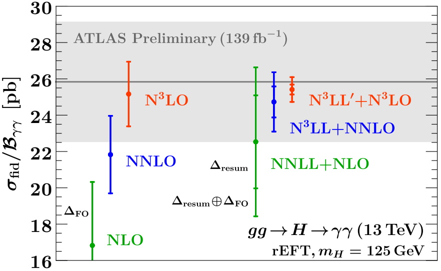
Consider $gg \to H \to \gamma\gamma$ with ATLAS fiducial cuts:

$$p_T^{\gamma 1} \geq 0.35\, m_H\,, \quad p_T^{\gamma 2} \geq 0.25\, m_H\,, \quad |\eta^\gamma| \leq 2.37\,, \quad |\eta^\gamma| \notin [1.37, 1.52]$$

Goal

- Compute fiducial spectrum in $q_T \equiv p_T^H = p_T^{\gamma\gamma}$ at N^{3}LL′+N^{3}LO
- Compute total fiducial cross section at N^{3}LO, and improved by resummation





Goals of this talk

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- Previous state of the art was N^{3}LL(+NNLO$_1$) and NNLO, respectively
  [Chen et al. '18; Bizoń et al. '18; Gutierrez-Reyes et al. '19; Becher, Neumann '20]

Kicked off recent push for fiducial color singlet at complete three-loop accuracy:

- Complementary N^{3}LO results for fiducial $Y_{\gamma\gamma}, \eta_{\gamma 1}, \Delta\eta_{\gamma\gamma}$ (with different method)
  [Chen, Gehrmann, Glover, Huss, Mistlberger, 2102.07607]
- Fiducial N^{3}LL′ results for Drell-Yan (and Higgs) q_T spectrum
  [Camarda, Cieri, Ferrera, 2103.04974; Re, Rottoli, Torrielli, 2104.07509]

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$\Gamma_H \ll m_H \Rightarrow$ production and decay (acceptance) factorize point by point in q_T, Y:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T} = \int \mathrm{d}Y\, A(q_T, Y; \Theta)\, W(q_T, Y)\,, \quad A_{\mathrm{incl}} = 1\,, \quad W(q_T, Y) = \frac{\mathrm{d}\sigma_{\mathrm{incl}}}{\mathrm{d}q_T\, \mathrm{d}Y}$$

Takeaway

$\sigma_{\mathrm{incl}} = \int \mathrm{d}q_T\, W(q_T)$ — resummation effects from $q_T \ll m_H$ formally cancel

$\sigma_{\mathrm{fid}} = \int \mathrm{d}q_T\, A(q_T)\, W(q_T)$ — *derived* quantity sensitive to resummation effects

Power expansion in $q_T \ll m_H$ is organizing principle of the calculation:

$$\frac{d\sigma}{dq_T} = \frac{d\sigma^{(0)}}{dq_T} + \frac{d\sigma^{(1)}}{dq_T} + \frac{d\sigma^{(2)}}{dq_T} + \cdots$$

$$\sim \frac{1}{q_T}\left[\mathcal{O}(1) + \mathcal{O}\left(\frac{q_T}{m_H}\right) + \mathcal{O}\left(\frac{q_T^2}{m_H^2}\right) + \cdots\right]$$

$$\frac{d\sigma^{(0)}}{dq_T} = \sigma_{\mathrm{LO}}\,\delta(q_T) + \sum_n \alpha_s^n \left\{\sigma_n^V\,\delta(q_T) + \sum_m \sigma_{n,m}^{(0)}\left[\frac{\ln^m(q_T/m_H)}{q_T}\right]_+\right\}$$

- Contains LO contribution, virtual corrections, and log-enhanced singular terms

$$\frac{d\sigma^{(1)}}{dq_T} = \sum_n \alpha_s^n \sum_m \sigma_{n,m}^{(1)}\frac{1}{m_H}\ln^m(q_T/m_H)$$

- Still logarithmically divergent, intimately connected to fiducial cuts

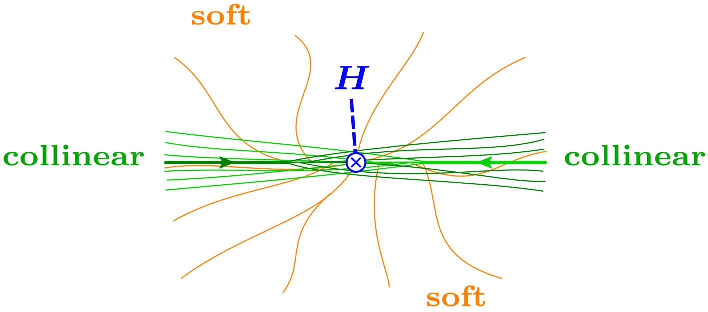
$$\frac{d\sigma^{(2)}}{dq_T} = \sum_n \alpha_s^n \sum_m \sigma_{n,m}^{(2)}\frac{q_T}{m_H^2}\ln^m(q_T/m_H) + \cdots$$

- Finite as $q_T \to 0$, extract from fixed-order $H + 1j$ calculation

Leading-power factorization & resummation

At leading power in $q_T \ll m_H$, the hadronic dynamics factorize as:

$$W^{(0)}(q_T, Y) = H(m_H^2, \mu) \int \mathrm{d}^2\vec{k}_a\, \mathrm{d}^2\vec{k}_b\, \mathrm{d}^2\vec{k}_s\, \delta\big(q_T - |\vec{k}_a + \vec{k}_b + \vec{k}_s|\big) \times B_g^{\mu\nu}(x_a, \vec{k}_a, \mu, \nu)\, B_{g\,\mu\nu}(x_b, \vec{k}_b, \mu, \nu)\, S(\vec{k}_s, \mu, \nu)$$



Leading-power factorization & resummation to N^{3}LL$'$

At leading power in $q_T \ll m_H$, the hadronic dynamics factorize as:

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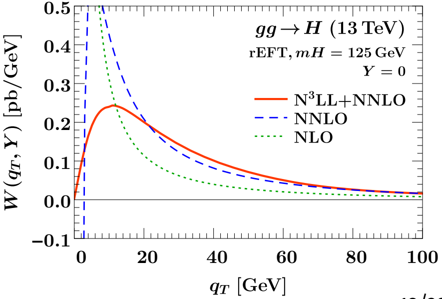
$$\times B_g^{\mu\nu}(x_a, \vec{k}_a, \mu, \nu)\, B_{g\,\mu\nu}(x_b, \vec{k}_b, \mu, \nu)\, S(\vec{k}_s, \mu, \nu)$$

Ingredients satisfy 2D renormalization group equations, e.g. soft function:

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \ln \tilde{S}(\vec{b}_T, \mu, \nu) = \tilde{\gamma}_S^g(\mu, \nu) \qquad \nu \frac{\mathrm{d}}{\mathrm{d}\nu} \ln \tilde{S}(\vec{b}_T, \mu, \nu) = \tilde{\gamma}_\nu^g(b_T, \mu)$$

- Solve recursively at fixed order
  - Complete log structure of $\mathrm{d}\sigma^{(0)}$
- Closed-form all-order solution
  - Resummed Sudakov peak
- Resummation order specified by perturbative order of anom. dims. and boundary conditions



At leading power in $q_T \ll m_H$, the hadronic dynamics factorize as:

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$$\times\, B_g^{\mu\nu}(x_a, \vec{k}_a, \mu, \nu)\, B_{g\,\mu\nu}(x_b, \vec{k}_b, \mu, \nu)\, S(\vec{k}_s, \mu, \nu)$$

To reach N^{3}LL′ for $W^{(0)}$, implemented in `SCETlib`:

- Three-loop soft and hard function …includes in particular the three-loop virtual form factor
  [Li, Zhu, '16] [Baikov et al. '09; Lee et al. '10; Gehrmann et al. '10]
- Three-loop unpolarized and two-loop polarized beam functions
  [Ebert, Mistlberger, Vita '20; Luo, Yang, Zhu, Zhu '20]
  [Luo, Yang, Zhu, Zhu '19; Gutierrez-Reyes, Leal-Gomez, Scimemi, Vladimirov '19]
- Four-loop cusp, three-loop noncusp anomalous dimensions
  [Brüser, Grozin, Henn, Stahlhofen '19; Henn, Korchemsky, Mistlberger '20; v. Manteuffel, Panzer, Schabinger '20] [Li, Zhu, '16; Moch, Vermaseren, Vogt '05; Idilbi, Ma, Yuan '06; Vladimirov '16]
- N^{3}LL solutions to virtuality/rapidity RGEs in b_T space
- Hybrid profile scales for fixed-order matching
  [Lustermans, JM, Tackmann, Waalewijn '19]

At leading power in $q_T \ll m_H$, the hadronic dynamics factorize as:

$$W^{(0)}(q_T, Y) = H(m_H^2, \mu) \int \mathrm{d}^2\vec{k}_a\, \mathrm{d}^2\vec{k}_b\, \mathrm{d}^2\vec{k}_s\, \delta\big(q_T - |\vec{k}_a + \vec{k}_b + \vec{k}_s|\big) \\ \times B_g^{\mu\nu}(x_a, \vec{k}_a, \mu, \nu)\, B_{g\,\mu\nu}(x_b, \vec{k}_b, \mu, \nu)\, S(\vec{k}_s, \mu, \nu)$$

- Use $\mu_{\mathrm{FO}} = \mu_R = \mu_F = m_H$ for central predictions
- Gluon form factor contains large "timelike" logarithms $\ln \dfrac{-m_H^2 - \mathrm{i}0}{\mu^2}$
  [Ahrens, Becher, Neubert, Yang '08]

▸ Resummed by hard evolution from $\mu_H = -\mathrm{i} m_H$:

$$W(q_T, Y) = H(m_H^2, \mu_H)\, U_H(Q, \mu_H, \mu_{\mathrm{FO}}) \left[\frac{W(q_T, Y)}{H(m_H^2, \mu_{\mathrm{FO}})}\right]_{\mathrm{FO}}$$

[Ebert, JM, Tackmann '17]

Treating fiducial power corrections right

Fiducial power corrections (in this case)

... are all the power corrections from the q_T-dependent acceptance:

$$\frac{\mathrm{d}\sigma^{\mathrm{fpc}}}{\mathrm{d}q_T} \equiv \int \mathrm{d}Y \Big[A(q_T, Y; \Theta) - A^{(0)}(Y; \Theta) \Big] W^{(0)}(q_T, Y)$$

- Uniquely predict all linear power corrections $\mathrm{d}\sigma^{(1)}$ because

$$W(q_T, Y) = W^{(0)}(q_T, Y) \left[1 + \mathcal{O}\left(\frac{q_T^2}{m_H^2} \right) \right]$$
$$A(q_T, Y; \Theta) = A^{(0)}(Y; \Theta) \quad \left[1 + \mathcal{O}\left(\frac{q_T}{m_H} \right) \right]$$

- Also capture enhanced corrections $\sim q_T / p_L$
  when approaching edges $p_L \to 0$ of Born phase space ...example coming up
- Resummed to the same accuracy as leading-power terms
  by resumming $W^{(0)}$ and keeping *exact* $A(q_T, Y; \Theta)$

[Presence of linear terms pointed out in Ebert, Tackmann '20]

[Factorization demonstrated in Ebert, JM, Stewart, Tackmann '20; see talk yesterday at QCD Evolution '21]

Key point

Fiducial power corrections induce resummation effects *in the total cross section*

Compare fixed-order series, isolating the effect of $\int \mathrm{d}q_T \, \frac{\mathrm{d}\sigma^{\mathrm{fpc}}}{\mathrm{d}q_T}$:

$$
\begin{aligned}
\sigma^{\mathrm{FO}}_{\mathrm{incl}} &= 13.80\,[1 + 1.291 \quad + 0.783 \quad + 0.299]\,\mathrm{pb} \\
\sigma^{\mathrm{FO}}_{\mathrm{fid}} &= 6.928\,[1 + 1.429 \quad + 0.723 \quad + 0.481]\,\mathrm{pb} \\
&= 6.928\,[1 + (1.300 + 0.129_{\mathrm{fpc}}) + (0.784 - 0.061_{\mathrm{fpc}}) + (0.331 + 0.150_{\mathrm{fpc}})]\,\mathrm{pb}
\end{aligned}
$$

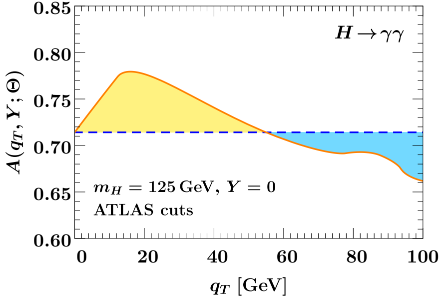
- Fiducial power corrections show no convergence, remainder is similar to inclusive

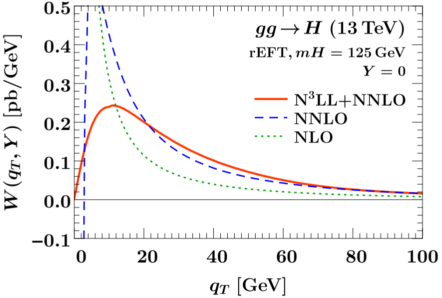
Key point

Fiducial power corrections induce resummation effects *in the total cross section*

Two ways to understand this:

1. Acceptance acts as a weight in the q_T integral





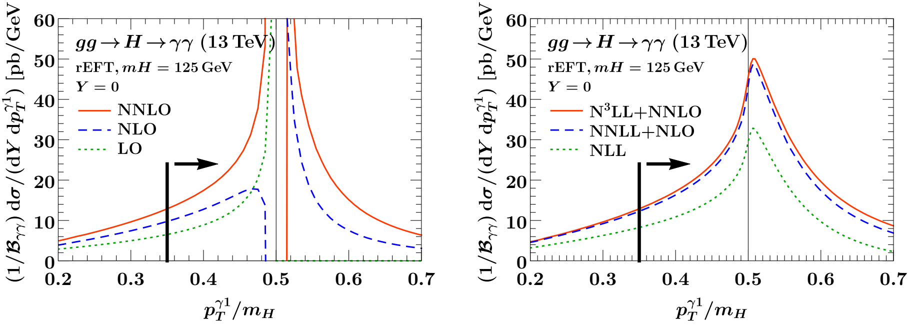
$$\sigma_{\mathrm{incl}} = \int \mathrm{d}q_T\, W(q_T) \qquad \sigma_{\mathrm{fid}} = \int \mathrm{d}q_T\, A(q_T)\, W(q_T)$$

Key point

Fiducial power corrections induce resummation effects *in the total cross section*

Two ways to understand this:

1. Acceptance acts as a weight in the q_T integral
2. We're cutting on the resummation-sensitive photon p_T



- Leaves behind logarithms of $\dfrac{p_L}{m_H} = \dfrac{p_T^{\text{cut}} - m_H/2}{m_H} = 0.15$

Key point

Fiducial power corrections induce resummation effects *in the total cross section*

Compare fixed-order series, isolating the effect of $\int \mathrm{d}q_T \, \frac{\mathrm{d}\sigma^{\mathrm{fpc}}}{\mathrm{d}q_T}$:

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\end{aligned}
$$

- Fiducial power corrections show no convergence, remainder is similar to inclusive

After resummation of $\sigma^{(0)} + \sigma^{\mathrm{fpc}}$, at successive matched orders:

$$
\begin{aligned}
\sigma_{\mathrm{incl}}^{\mathrm{res}} &= 24.16\,[1 + 0.756 + 0.207 + 0.024]\,\mathrm{pb} \\
\sigma_{\mathrm{fid}}^{\mathrm{res}} &= 12.89\,[1 + 0.749 + 0.171 + 0.053]\,\mathrm{pb}
\end{aligned}
$$

Note Checked explicitly that in our profile scale setup, $\sigma_{\mathrm{incl}}^{\mathrm{res}}$ and $\sigma_{\mathrm{incl}}^{\mathrm{FO}}$ *agree* within Δ_{resum}

- *Differ* in the fiducial case $\Rightarrow$ resummation effect is resolved

Extracting the nonsingular cross section

So we dealt with this …

$$\frac{\mathrm{d}\sigma^{\mathrm{sing}}}{\mathrm{d}q_T} = \int \mathrm{d}Y\, A(q_T, Y; \Theta)\, W^{(0)}(q_T, Y) = \frac{\mathrm{d}\sigma^{(0)}}{\mathrm{d}q_T} + \frac{\mathrm{d}\sigma^{\mathrm{fpc}}}{\mathrm{d}q_T}$$

To match to FO and be able to integrate to the total cross section, we still need:

$$\frac{\mathrm{d}\sigma^{\mathrm{nons}}_{\mathrm{FO}}}{\mathrm{d}q_T} = \int \mathrm{d}Y\, A(q_T, Y; \Theta) \left[W^{(2)}_{\mathrm{FO}}(q_T, Y) + \cdots\right] = \left[\frac{\mathrm{d}\sigma_{\mathrm{FO}_1}}{\mathrm{d}q_T} - \frac{\mathrm{d}\sigma^{\mathrm{sing}}_{\mathrm{FO}}}{\mathrm{d}q_T}\right]_{q_T>0}$$

$$\Rightarrow \quad \sigma = \int_0^{q_T^{\mathrm{off}}} \mathrm{d}q_T \left[\frac{\mathrm{d}\sigma^{\mathrm{sing}}}{\mathrm{d}q_T} + \frac{\mathrm{d}\sigma^{\mathrm{nons}}_{\mathrm{FO}}}{\mathrm{d}q_T}\right] + \int_{q_T^{\mathrm{off}}} \mathrm{d}q_T\, \frac{\mathrm{d}\sigma_{\mathrm{FO}_1}}{\mathrm{d}q_T}$$

Challenges:

- Obtaining stable $H + 1j$ results for $q_T \to 0$ is *hard* …in particular at NNLO_1
- Dropping the nonsingular below $q_T \leq q_T^{\mathrm{cut}}$ is not viable, either …as we'll see shortly
  - In the context of q_T subtractions: crucial to use differential subtraction, not slicing

Setup

So we dealt with this ...

$$\frac{d\sigma^{\text{sing}}}{dq_T} = \int dY\, A(q_T, Y; \Theta)\, W^{(0)}(q_T, Y) = \frac{d\sigma^{(0)}}{dq_T} + \frac{d\sigma^{\text{fpc}}}{dq_T}$$

To match to FO and be able to integrate to the total cross section, we still need:

$$\frac{d\sigma^{\text{nons}}_{\text{FO}}}{dq_T} = \int dY\, A(q_T, Y; \Theta) \left[W^{(2)}_{\text{FO}}(q_T, Y) + \cdots\right] = \left[\frac{d\sigma_{\text{FO}_1}}{dq_T} - \frac{d\sigma^{\text{sing}}_{\text{FO}}}{dq_T}\right]_{q_T>0}$$

Key idea

Fit nonsingular data to known form at subleading power and integrate *analytically*:

$$q_T \frac{d\sigma^{\text{nons}}_{\text{FO}}}{dq_T}\bigg|_{\alpha_s^n} = \frac{q_T^2}{m_H^2} \sum_{k=0}^{2n-1} \left(a_k + b_k \frac{q_T}{m_H} + c_k \frac{q_T^2}{m_H^2} + \cdots\right) \ln^k \frac{q_T^2}{m_H^2}$$

- Include higher-power b_k, c_k to get unbiased a_k
  - Allows us to use more precise data at higher q_T as lever arm in the fit

So we dealt with this ...

$$\frac{\mathrm{d}\sigma^{\mathrm{sing}}}{\mathrm{d}q_T} = \int \mathrm{d}Y\, A(q_T, Y; \Theta)\, W^{(0)}(q_T, Y) = \frac{\mathrm{d}\sigma^{(0)}}{\mathrm{d}q_T} + \frac{\mathrm{d}\sigma^{\mathrm{fpc}}}{\mathrm{d}q_T}$$

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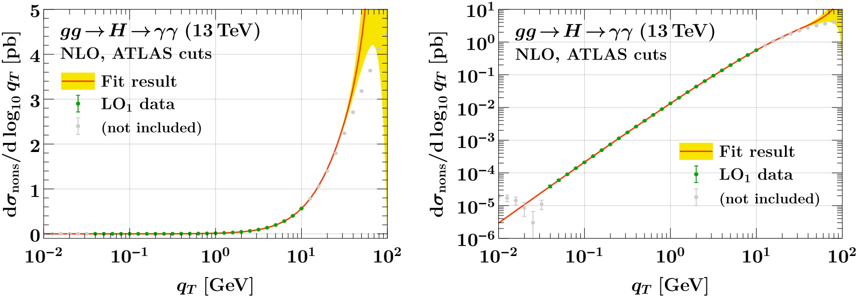
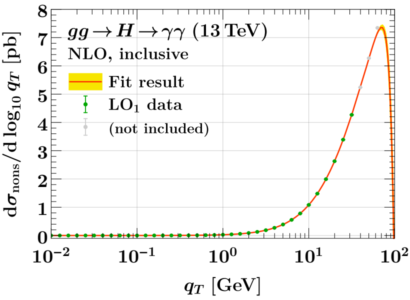
Fixed-order inputs:

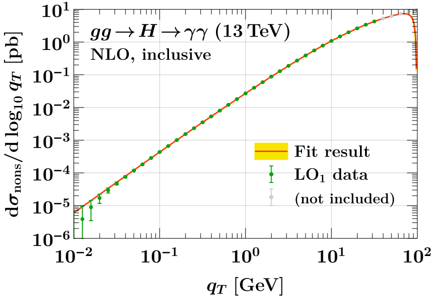
- NLO contribution to $W(q_T, Y)$ at $q_T > 0$ is easy
- At NNLO, renormalize & implement bare analytic results for $W(q_T, Y)$
  [Dulat, Lionetti, Mistlberger, Pelloni, Specchia ’17]
- At N^{3}LO, use existing binned NNLO$_1$ results from `NNLOjet`
  [Chen, Cruz-Martinez, Gehrmann, Glover, Jaquier ’15-16; as used in Chen et al. ’18; Bizoń et al. ’18]
- Use N^{3}LO total inclusive cross section as additional fit constraint on underflow
  [Mistlberger ’18]



Fit procedure:

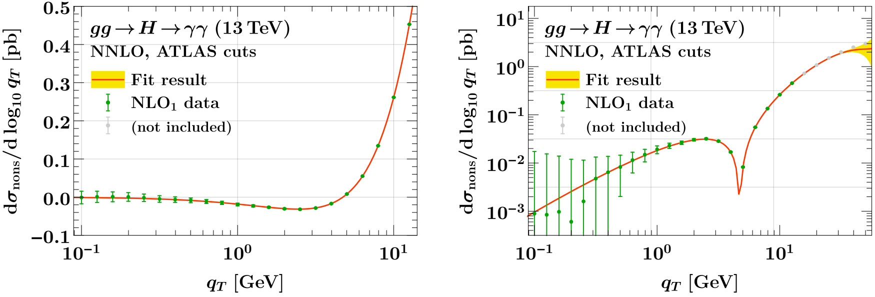
- Perform separate χ^2 fits of $\{a_k^{\mathrm{incl,fid}}\}$ to inclusive and fiducial nonsingular data
  [generated by our analytic implementation]
- Increase fit window to larger q_T until p value decreases
- Include subleading log coefficients at next higher power until p value decreases
- Also test intermediate combination to ensure fit is stable
  [procedure follows Moult, Rothen, Stewart, Tackmann, Zhu ’15-’16]





Fit procedure:

- Perform separate χ^2 fits of $\{a_k^{\rm incl,fid}\}$ to inclusive and fiducial nonsingular data [generated by our analytic implementation]
- Increase fit window to larger q_T until p value decreases
- Include subleading log coefficients at next higher power until p value decreases
- Also test intermediate combination to ensure fit is stable [procedure follows Moult, Rothen, Stewart, Tackmann, Zhu '15-'16]



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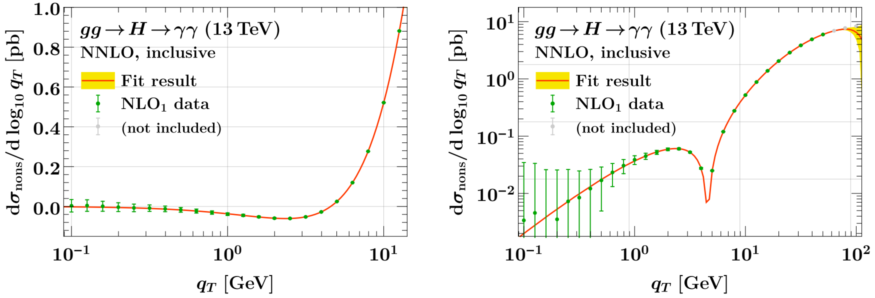

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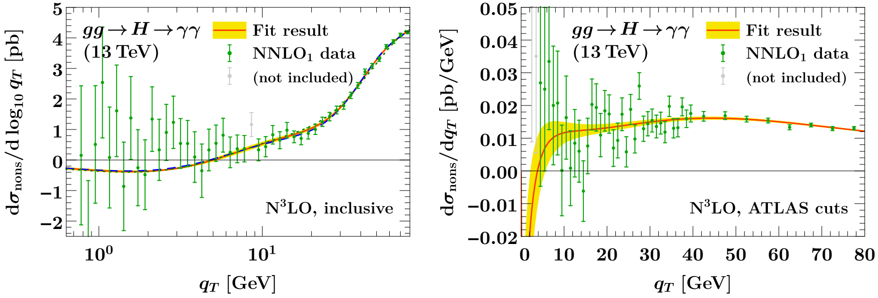
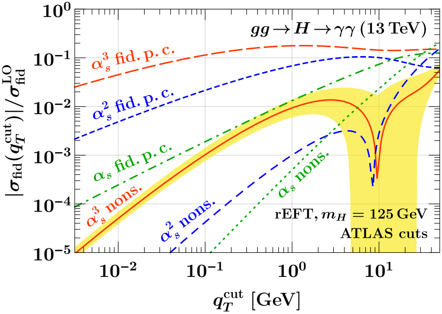


Setup:

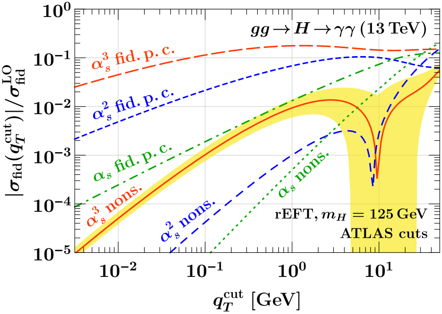
- Perform a combined fit to all inclusive and fiducial data
  [NNLO$_1$: Chen, Cruz-Martinez, Gehrmann, Glover, Jaquier ’15-16; as used in Chen et al. ’18; Bizoń et al. ’18]
  [Incl. N^{3}LO: Mistlberger ’18]
- Empirically find $0.4 \leq a_k^{\mathrm{fid}}/a_k^{\mathrm{incl}} \leq 0.55$ at (N)NLO $\Rightarrow$ use as weak 1σ constraint
  - Makes sense, $a_k^{\mathrm{fid,incl}}$ are same underlying $W^{(2)}$ in slightly different Y range
  - Note that we are *not* just rescaling any part of the cross section by an acceptance
- Add $\sigma_{\mathrm{incl}}(q_T \leq q_T^{\mathrm{cut}}) = \sigma_{\mathrm{incl}}^{\mathrm{N^3LO}} - \sigma_{\mathrm{incl}}(q_T > q_T^{\mathrm{cut}})$ as additional incl. data point



Most general form of q_T subtractions:

$$\sigma = \sigma^{\rm sing}(q_T^{\rm off}) + \sigma^{\rm nons}(q_T^{\rm cut}) + \int_{q_T^{\rm cut}}^{q_T^{\rm off}} \mathrm{d}q_T \left[\frac{\mathrm{d}\sigma}{\mathrm{d}q_T} - \frac{\mathrm{d}\sigma^{\rm sing}}{\mathrm{d}q_T}\right] + \int_{q_T^{\rm off}} \mathrm{d}q_T \, \frac{\mathrm{d}\sigma}{\mathrm{d}q_T}$$

- We literally take $q_T^{\rm cut} = 0$, second term *identically* vanishes
- Slicing calculation would use finite $q_T^{\rm cut} \sim 2\,\mathrm{GeV}$ and take $\sigma^{\rm nons}(q_T^{\rm cut}) \approx 0$
- That would be a bad (catastrophic) approximation with (without) $\sigma^{\rm fpc} \subset \sigma_{\rm sing}$

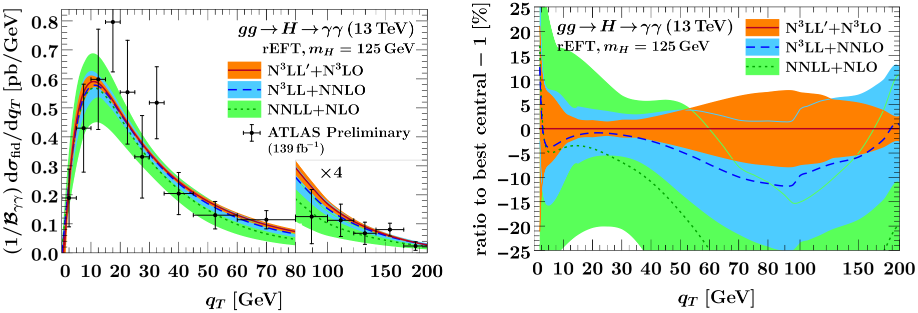


A word of numerical caution:

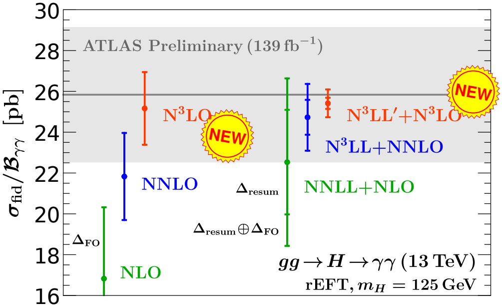
- Contributions from $\sigma^{\rm fpc}(q_T \lesssim 0.1 {\rm GeV})$ can be as high as $\mathcal{O}(10\%) \times \sigma_{\rm LO}$
- If evaluated by MC, as e.g. in projection-to-Born method, unbiased integration at these low q_T will be challenging (generation cuts, stability of amplitudes, ...)

Results

The fiducial q_T spectrum at N^{3}LL$'$+N^{3}LO



- Total uncertainty is $\Delta_{\rm tot} = \Delta_{q_T} \oplus \Delta_\varphi \oplus \Delta_{\rm match} \oplus \Delta_{\rm FO} \oplus \Delta_{\rm nons}$
  [See also Ebert, JM, Stewart, Tackmann, 2006.11382 for details]
- Observe excellent perturbative convergence & uncertainty coverage
  - Crucial to consider *every* variation to probe all parts of the prediction
  - Three-loop beam function has noticeable efffect on central value and band
- Divide $H \to \gamma\gamma$ branching ratio $\mathcal{B}_{\gamma\gamma}$ out of data [LHC Higgs Cross Section WG, 1610.07922]
- Data are corrected for other production channels, photon isolation efficiency
  [ATLAS, 1802.04146]



- Large N^{3}LO correction to fiducial cross section (worse than inclusive)
  - Caused by fiducial power corrections, *not* captured by rescaling
- Resummation restores convergence
  - Needs both q_T and timelike resummation (different effects, neither is sufficient)

Interesting: Infrared sensitivity observed e.g. in $\Delta\eta_{\gamma\gamma}$ spectrum at N^{3}LO
[Chen, Gehrmann, Glover, Huss, Mistlberger, 2102.07607]

$\Leftrightarrow$ Precisely the fiducial p.c.'s we can deal with and resum

- Presented N^{3}LL′+N^{3}LO and N^{3}LO predictions for the fiducial p_T^H spectrum and the total fiducial cross section for $gg \to H \to \gamma\gamma$ at the LHC
  - First direct comparison to LHC data at this order and level of precision
- Observed, explained, and resummed large fiducial power corrections induced by the experimental acceptance
  - Even *total* fiducial cross sections are sensitive to q_T resummation effects
- Nonsingular extraction and matching to total cross section made possible by combining all information from N^{3}LO σ_{incl}, fixed-order $H + 1j$ data, fiducial power corrections, and known functional form at subleading power

- Reached a new level of theory control for two cornerstone LHC observables

Summary & Outlook

- Presented N^{3}LL′+N^{3}LO and N^{3}LO predictions for the fiducial p_T^H spectrum and the total fiducial cross section for $gg \to H \to \gamma\gamma$ at the LHC
  - First direct comparison to LHC data at this order and level of precision
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  - Even *total* fiducial cross sections are sensitive to q_T resummation effects
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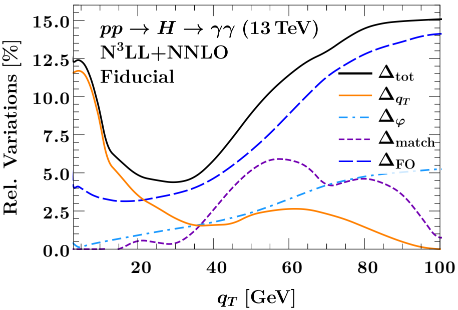
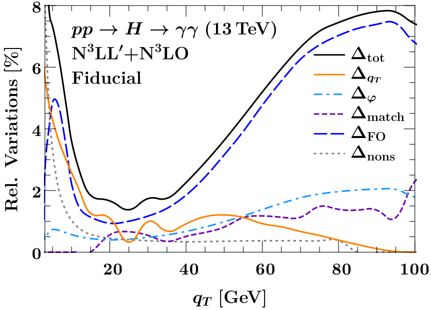
- Reached a new level of theory control for two cornerstone LHC observables

Thank you for your attention!

Backup





N^{3}LO: $\sigma_{fid}/\mathcal{B}_{\gamma\gamma} = (25.16 \pm 1.78_{FO} \pm 0.12_{nons})$ pb

N^{3}LL′+N^{3}LO: $\sigma_{fid}/\mathcal{B}_{\gamma\gamma} = (25.41 \pm 0.59_{FO} \pm 0.21_{q_T} \pm 0.17_{\varphi} \pm 0.06_{match} \pm 0.20_{nons})$ pb

- Δ_{q_T} 36 independent scale variations in $W^{(0)}$ factorization
- Δ_{φ} Vary phase of hard scale over $\arg \mu_H \in \{\pi/4, 3\pi/4\}$
- Δ_{match} Vary transition points governing resummation turn-off
- Δ_{FO} Vary $\mu_R/m_H \in \{1/2, 2\}$ (dominates over μ_F due to overall α_s^2)
- Δ_{nons} Uncertainty on nonsingular extraction

- Beam function kernels are large expressions of HPLs and rational prefactors:

$$I_{ij}^{(n)}(z) = \sum_a \frac{P_a(z)}{Q_a(z)} H_{w_a}(z)\,, \qquad w_a = \left(\begin{smallmatrix}\pm 1\\ 0\end{smallmatrix}, \cdots, \begin{smallmatrix}\pm 1\\ 0\end{smallmatrix}\right) \text{ up to weight 5}$$

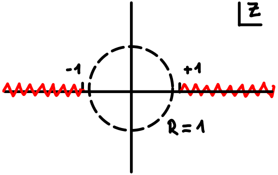
- Many tools for numerically evaluating *individual* HPLs on the market …
  [e.g. Gehrmann, Remiddi '01; Buehler, Duhr '11; Ablinger, Blümlein, Round, Schneider '18]

! But big sum is slow and has uncontrolled floating-point cancellations, in particular in limits $z \to 0, 1$ relevant for convolution $I_{ij}^{(n)} \otimes f_j$ against PDFs

Key idea

Implement the kernels *directly* as smart series expansions, using algebraic methods inspired by those developed for individual HPLs

1. Separate branch cuts by subtractions
   - Much more complex due to rational terms
   - ▶ Treat $Q_a(z)$ as additional primitives
2. Remap variables, push out remaining branch cut
   - Improves convergence radii of series

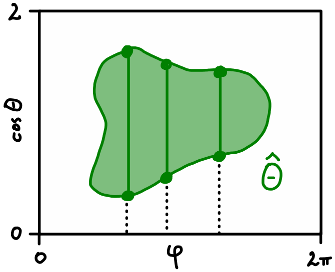


▶ Get $I_{ij}^{(3)}, P_{ij}^{(2)}, \ldots$ at machine precision in $\mathcal{O}(50\text{k})$ cycles for any z, ≥ 100 times faster than naive implementation (and much more precise)

… relies on fast & stable (for $q_T \to 0$) algorithm for evaluating the acceptance:

$$A(q_T, Y; \Theta) = \frac{1}{4\pi} \int \mathrm{d}\cos\theta \, \mathrm{d}\varphi \, \hat{\Theta}(q^\mu, \cos\theta, \varphi)$$

- $\hat{\Theta}(q_T = 0, Y, \cos\theta, \cancel{\varphi})$ is trivial
- For $q_T \neq 0$, analytically solve generic $\hat{\Theta}$ for bounds in θ at given q_T, Y, φ
- Do remaining 1D integral over φ adaptively
- ▶ Takes $\mathcal{O}(1\,\mathrm{ms})$ on $2.50\,\mathrm{GHz}$ CPU for 10^{-7} target precision



Pure Quark and Gluon Observables with Collinear Drop

Xiaojun Yao
MIT

On-going project with Iain W. Stewart

KITP “New Physics from Precision at High Energies” Program
May 11, 2021

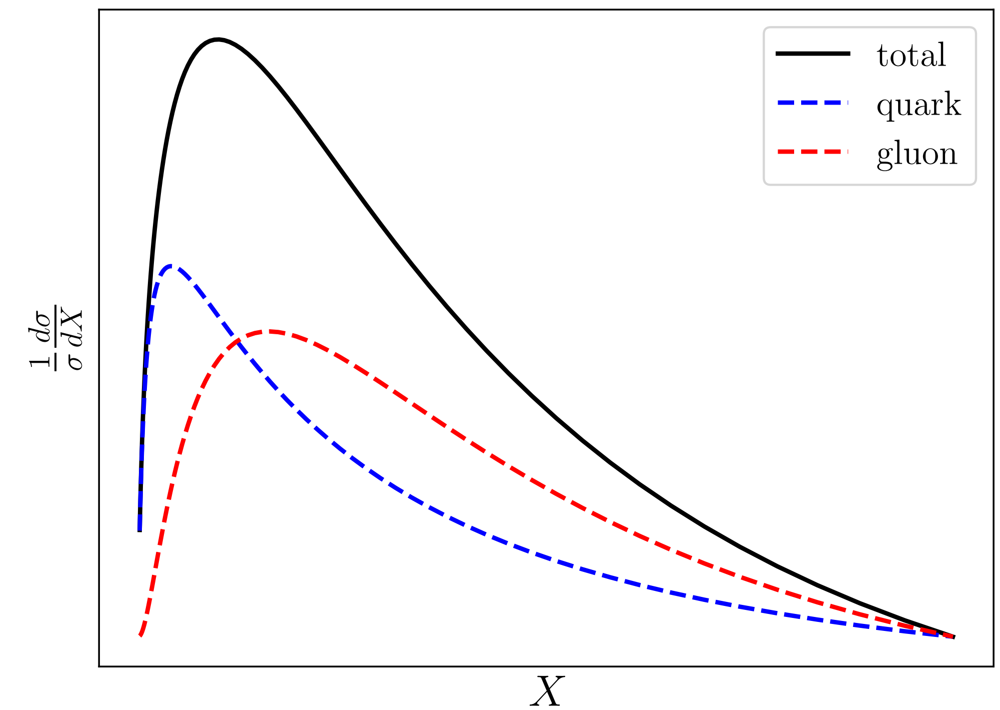
Disentangling Quark- and Gluon-Initiated Jets

- **Jet observables contain quark & gluon contributions**

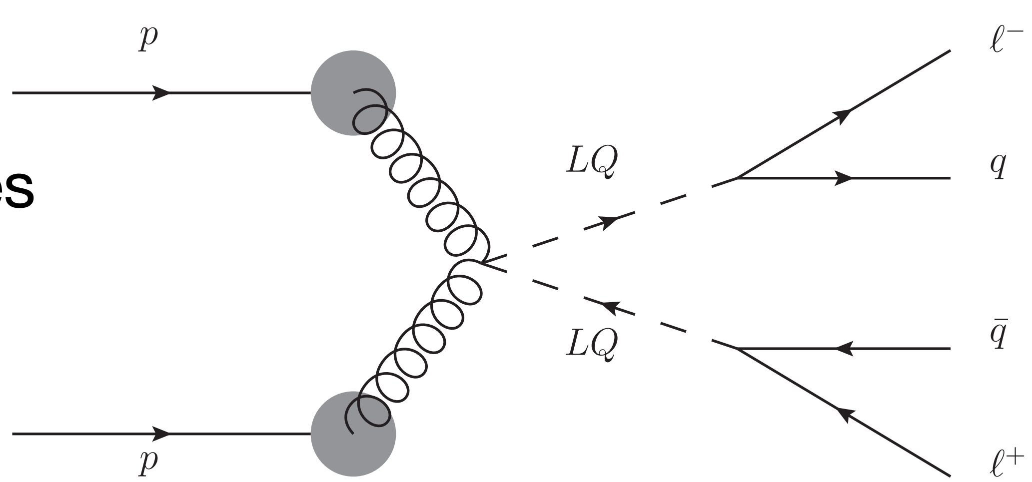
$$X = f_q X_q + f_g X_g$$

$$f_q + f_g = 1$$

Only know total distribution from measurements, want to know individual fraction and distribution



- **Motivations of quark/gluon discrimination**



Increase sensitivity in BSM physics searches

Constrain parton shower generators

Better understand QCD jets

Improve probes of quark-gluon plasma in heavy ion collisions

- **Can we extract quark & gluon fractions (and distributions)?**

Jet Topics: A Data-Driven Technique

E.M.Metodiev, J.Thaler
arXiv:1802.00008

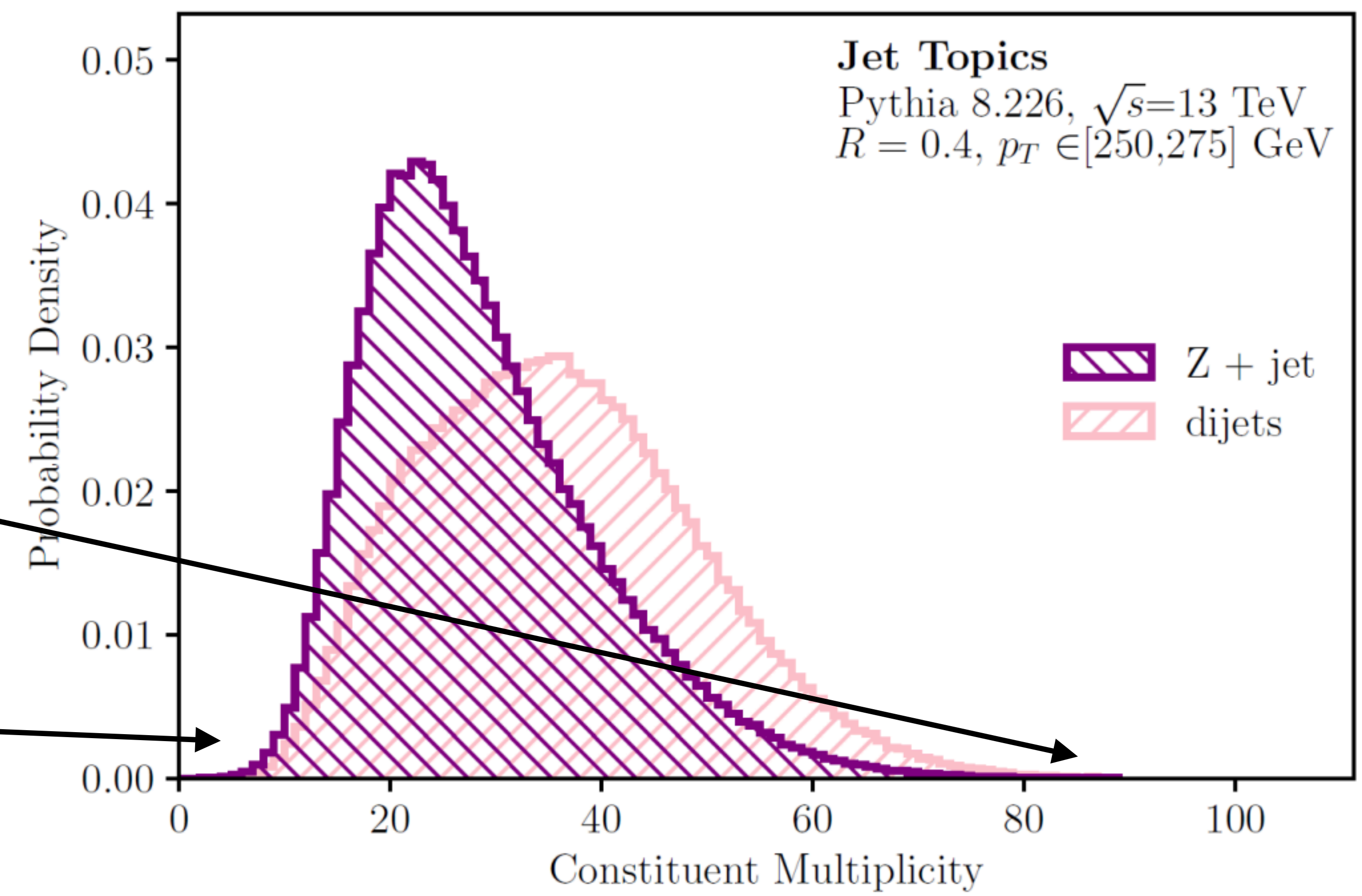
- **Two samples of jets: A = Z + jet, B = dijets**

$$p_A(x) = f_q^A p_q(x) + f_g^A p_g(x)$$
$$p_B(x) = f_q^B p_q(x) + f_g^B p_g(x)$$

- **Obtain reducibility factors**

$$\kappa(A|B) = \min_x \frac{p_A(x)}{p_B(x)}$$

$$\kappa(B|A) = \min_x \frac{p_B(x)}{p_A(x)}$$



- **Jet topics distributions**

$$p_{T1}(x) = \frac{p_A(x) - \kappa(A|B) p_B(x)}{1 - \kappa(A|B)} \xrightarrow{f_q^A > f_q^B} p_q(x)$$

Mutual irreducibility

$$\kappa(q|g) = \kappa(g|q) = 0$$

$$p_{T2}(x) = \frac{p_B(x) - \kappa(B|A) p_A(x)}{1 - \kappa(B|A)} \longrightarrow p_g(x)$$

Jet Topics: A Data-Driven Technique

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E.M.Metodiev, J.Thaler
arXiv:1802.00008

$$p_A(x) = f_q^A p_q(x) + f_g^A p_g(x)$$
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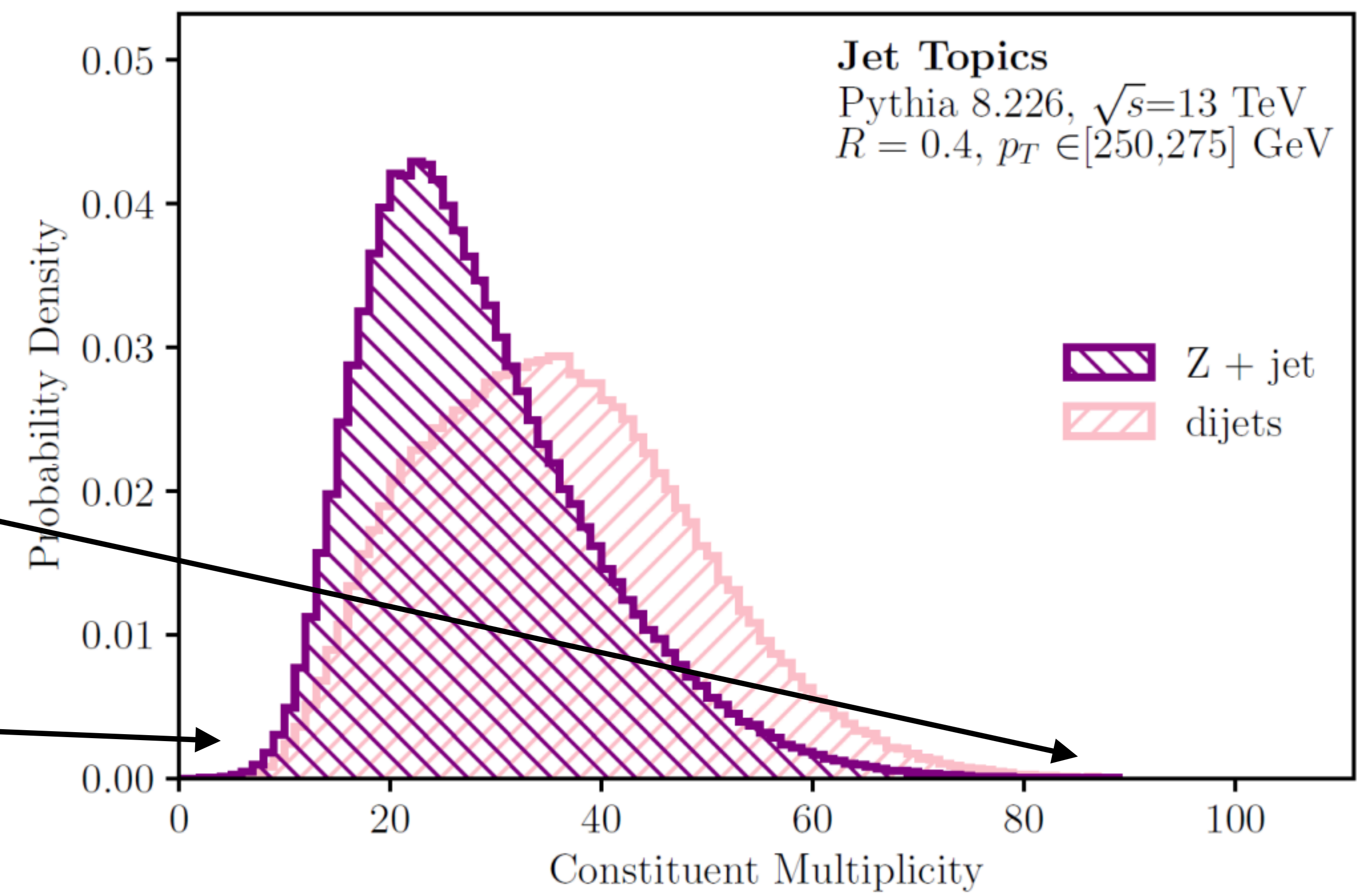


- **Obtain reducibility factors**

$$\kappa(A|B) = \min_x \frac{p_A(x)}{p_B(x)}$$

$$\kappa(B|A) = \min_x \frac{p_B(x)}{p_A(x)}$$

- **Jet topics limitations**

Typical observables do not have mutual irreducibility

E.g. SD jet mass
x = $m_{J_{SD}}$

$$\kappa(g\,|\,q) = 0$$

Relies on Sudakov suppression (significant experimental uncertainty in this region)

$$\kappa(q\,|\,g) = \frac{C_F}{C_A}$$

Relies on Casimir scaling (LL)

Obtain Fractions from Special Observables

- **Two samples of jets**

$$p_A(x) = f_q^A p_q(x) + f_g^A p_g(x)$$
$$p_B(x) = f_q^B p_q(x) + f_g^B p_g(x)$$

- **Assume we had pure quark and gluon observables in different kinematic regions respectively**

$$p_A(x) = f_q^A p_q(x) \qquad p_A(y) = f_g^A p_g(y)$$
$$p_B(x) = f_q^B p_q(x) \qquad p_B(y) = f_g^B p_g(y)$$

- **Can obtain fractions from:**

$$\frac{f_q^A}{f_q^B} = \frac{p_A(x)}{p_B(x)} \qquad \frac{f_g^A}{f_g^B} = \frac{p_A(y)}{p_B(y)}$$

$$f_q^A + f_g^A = 1 \qquad f_q^B + f_g^B = 1$$

And we also obtain $p_q(x)$, $p_g(x)$

- **This motivates finding pure quark and gluon observables, which we will show can be done with collinear drop**

Soft Drop

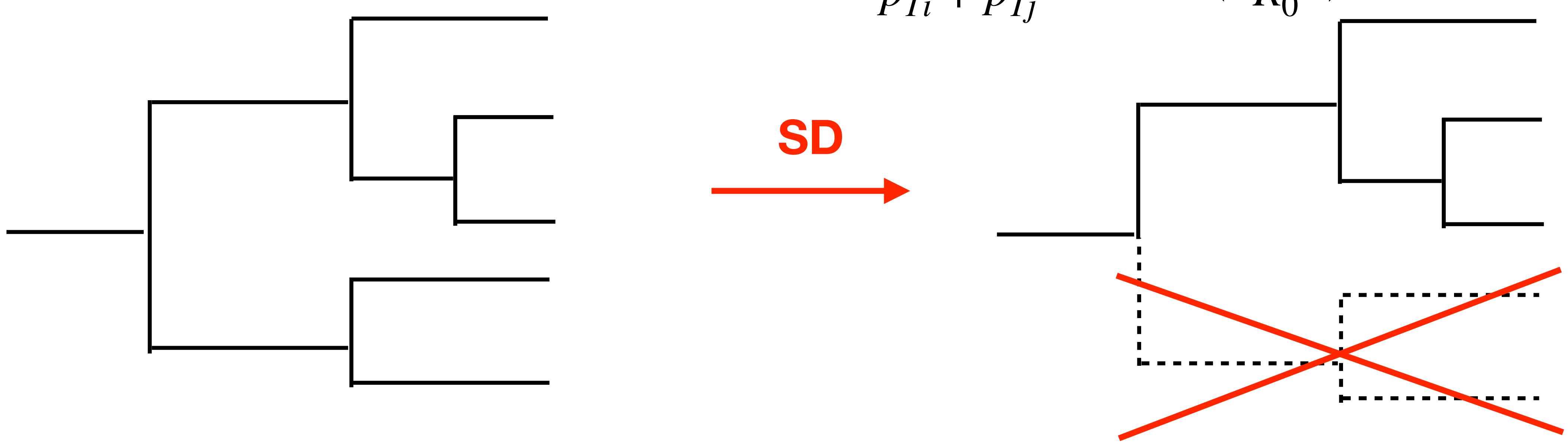
M.Dasgupta,A.Fregoso,S.Marzani
G.P. Salam arXiv:1307.0007

A.J.Larkoski,S.Marzani,G.Soyez
J.Thaler arXiv:1402.2657

- **SD with parameters** $(z_{\rm cut}, \beta)$
  - Start with jet defined by anti-kT with radius R
  - Re-cluster the jet in Cambridge-Aachen algorithm: first combine pairs w/ smallest

$$\Delta R_{ij} = (\phi_i - \phi_j)^2 + (y_i - y_j)^2 \approx \theta_{ij} \cosh \eta_J$$

  - Obtain a tree, consistent with LL branching history: large angle radiated first
  - Keep removing the softer branch until $\dfrac{\min(p_{Ti}, p_{Tj})}{p_{Ti} + p_{Tj}} > z_{\rm cut} \left(\dfrac{\Delta R_{ij}}{R_0} \right)^{\beta} \approx \tilde{z}_{\rm cut}\, \theta_{ij}^{\beta}$



- Define jet observables using the groomed jet (jet mass, …)

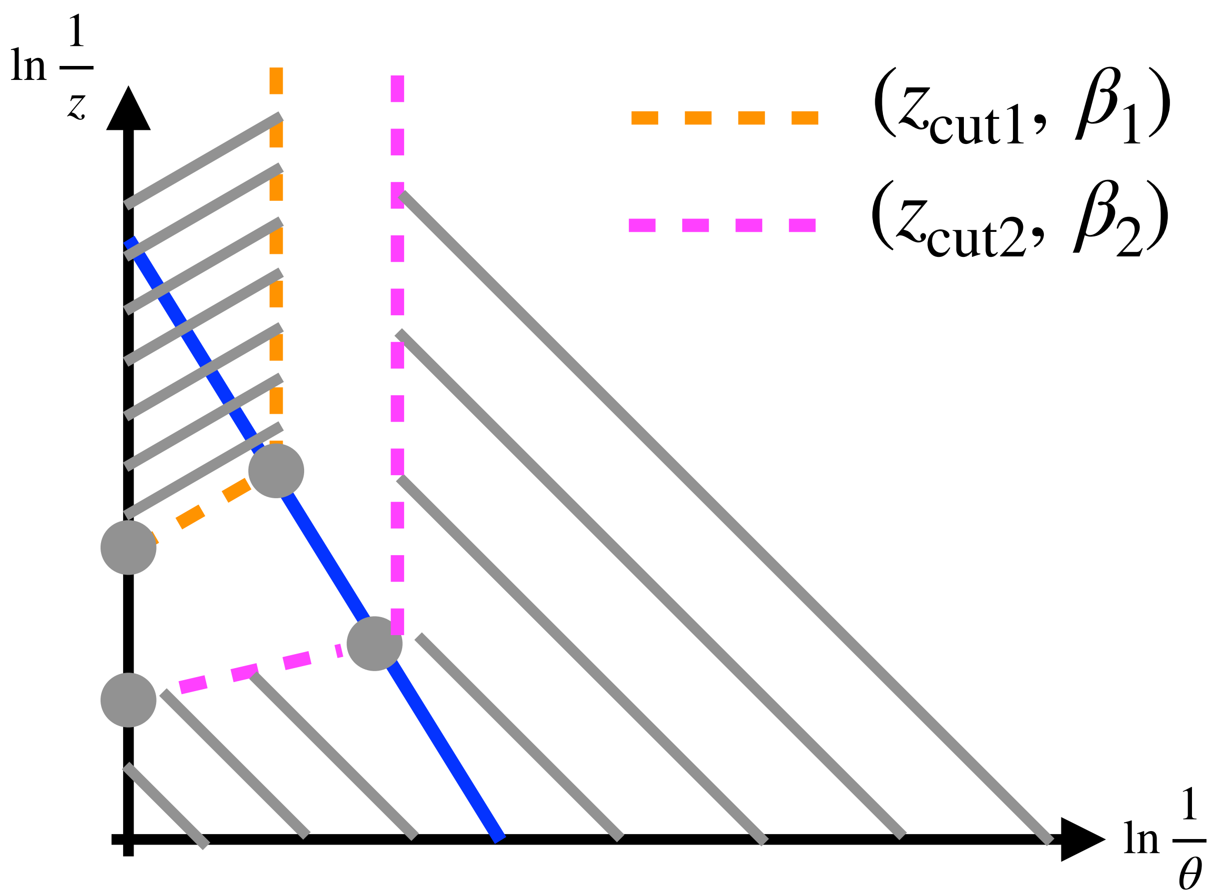
Jet Mass in Collinear Drop

- **CD defined from two SD's, second one more aggressive, IRC safe**
- **Jet mass in CD:**

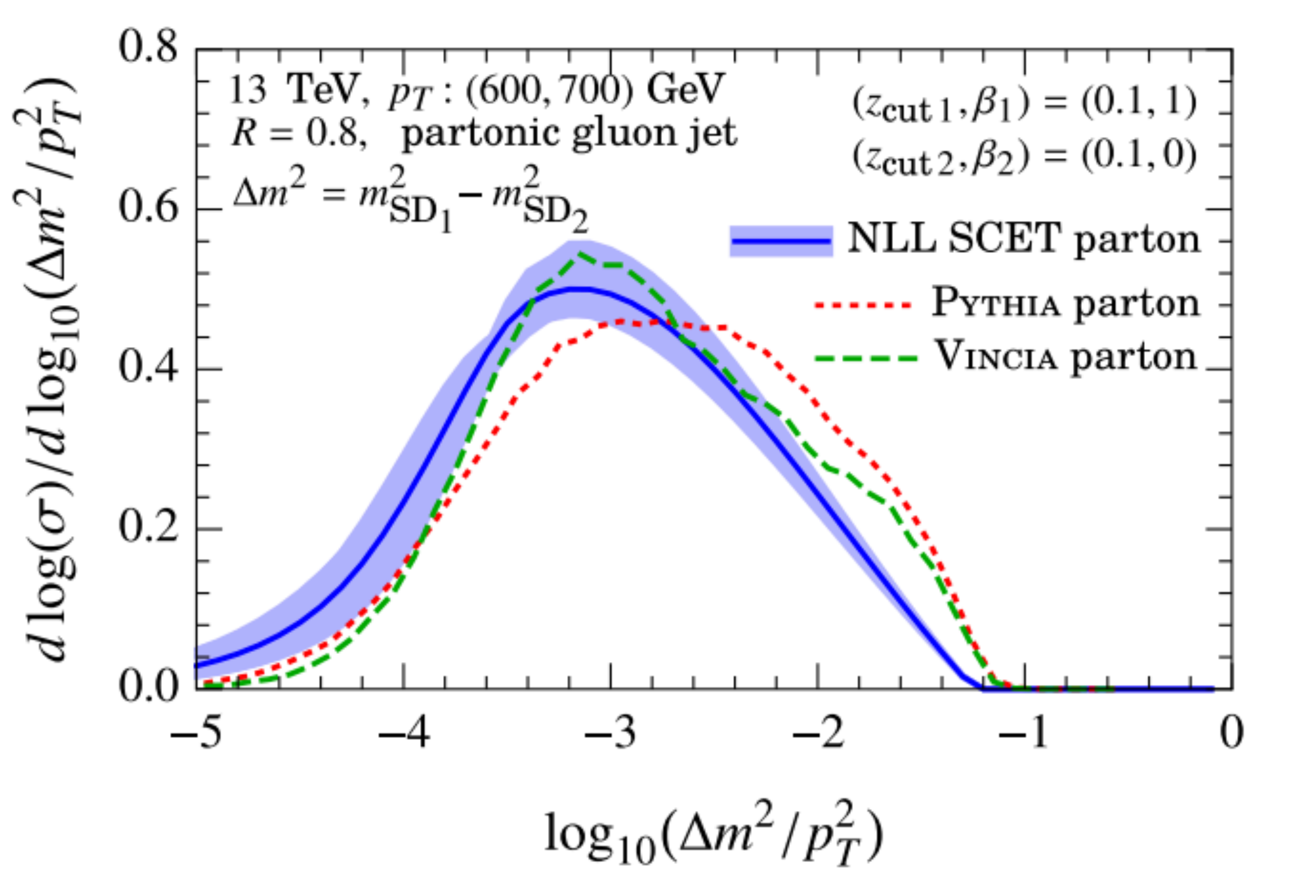
Y.T.Chien, I.W.Stewart
arXiv:1907.11107

$$\Delta m^2 = m^2_{J_{SD1}} - m^2_{J_{SD2}} = \Big(\sum_{i\in J_{SD1}} p_i^\mu\Big)^2 - \Big(\sum_{i\in J_{SD2}} p_i^\mu\Big)^2 = Q\Big(\sum_{i\in J_{SD1}} p_i^+ - \sum_{i\in J_{SD2}} p_i^+\Big) \qquad m_J^2 \ll Q^2$$

$$\frac{d\sigma}{d\Delta m} \sim H \times S_{G1} \times S_{G2} \times S_{C1} \otimes S_{C2}$$



Factorization v.s. Monte Carlo



Cumulative Jet Mass in Collinear Drop in SCET

Cumulative jet mass $\Sigma^{\rm pert}(\Delta m_c^2) = \int_0^{\Delta m_c^2} \mathrm{d}\Delta m^2 \, \frac{\mathrm{d}\sigma}{\mathrm{d}\Delta m^2} = \sum_{j=q,g} H_j \, \Sigma_j^{\rm pert}$

$$\Sigma^{\rm pert}_{j,{\rm NLL}}(\Delta m_c^2) = (e^{-\gamma_E}\Delta m_c^2)^{2C_j\omega(\mu_{\rm cs1},\mu_{\rm cs2})}$$

$$\exp\left(\frac{2C_j}{1+\beta_1}K(\mu_{\rm gs1},\mu) - \frac{2C_j}{1+\beta_2}K(\mu_{\rm gs2},\mu) - 2C_j\frac{2+\beta_1}{1+\beta_1}K(\mu_{\rm cs1},\mu) + 2C_j\frac{2+\beta_2}{1+\beta_2}K(\mu_{\rm cs2},\mu)\right)$$

$$\times\left(\frac{\mu_{\rm gs1}}{Q_{\rm gs1}}\right)^{\frac{2C_j}{1+\beta_1}\omega(\mu_{\rm gs1},\mu)}\left(\frac{\mu_{\rm gs2}}{Q_{\rm gs2}}\right)^{\frac{-2C_j}{1+\beta_2}\omega(\mu_{\rm gs1},\mu)}\left(\frac{Q_{\rm cut1}^{\frac{1}{1+\beta_1}}}{Q\mu_{\rm cs1}^{\frac{2+\beta_1}{1+\beta_1}}}\right)^{2C_j\omega(\mu_{\rm cs1},\mu)}\left(\frac{Q_{\rm cut2}^{\frac{1}{1+\beta_2}}}{Q\mu_{\rm cs2}^{\frac{2+\beta_2}{1+\beta_2}}}\right)^{-2C_j\omega(\mu_{\rm cs2},\mu)}$$

Y.T.Chien, I.W.Stewart
arXiv:1907.11107

Sudakov factors with both negative and positive signs

$$Q_{{\rm gs},i} = p_T R z_{{\rm cut},i}\left(\frac{R}{\cosh\eta_J}\right)^{\beta_i}$$

$$Q_{{\rm cut},i} = Q z_{{\rm cut},i} \qquad i = 1,2$$

$$\mu_{{\rm cs},i} = \left(\frac{\Delta m^2}{Q}\right)^{\frac{1+\beta_i}{2+\beta_i}} Q_{{\rm cut},i}^{\frac{1}{2+\beta_i}}$$

$$K(\mu_1,\mu_2) = \int_{\alpha_s(\mu_1)}^{\alpha_s(\mu_2)} d\alpha \frac{\Gamma_{\rm cusp}(\alpha)}{\beta(\alpha)}\int_{\alpha_s(\mu_1)}^{\alpha}\frac{d\alpha'}{\beta(\alpha')}$$

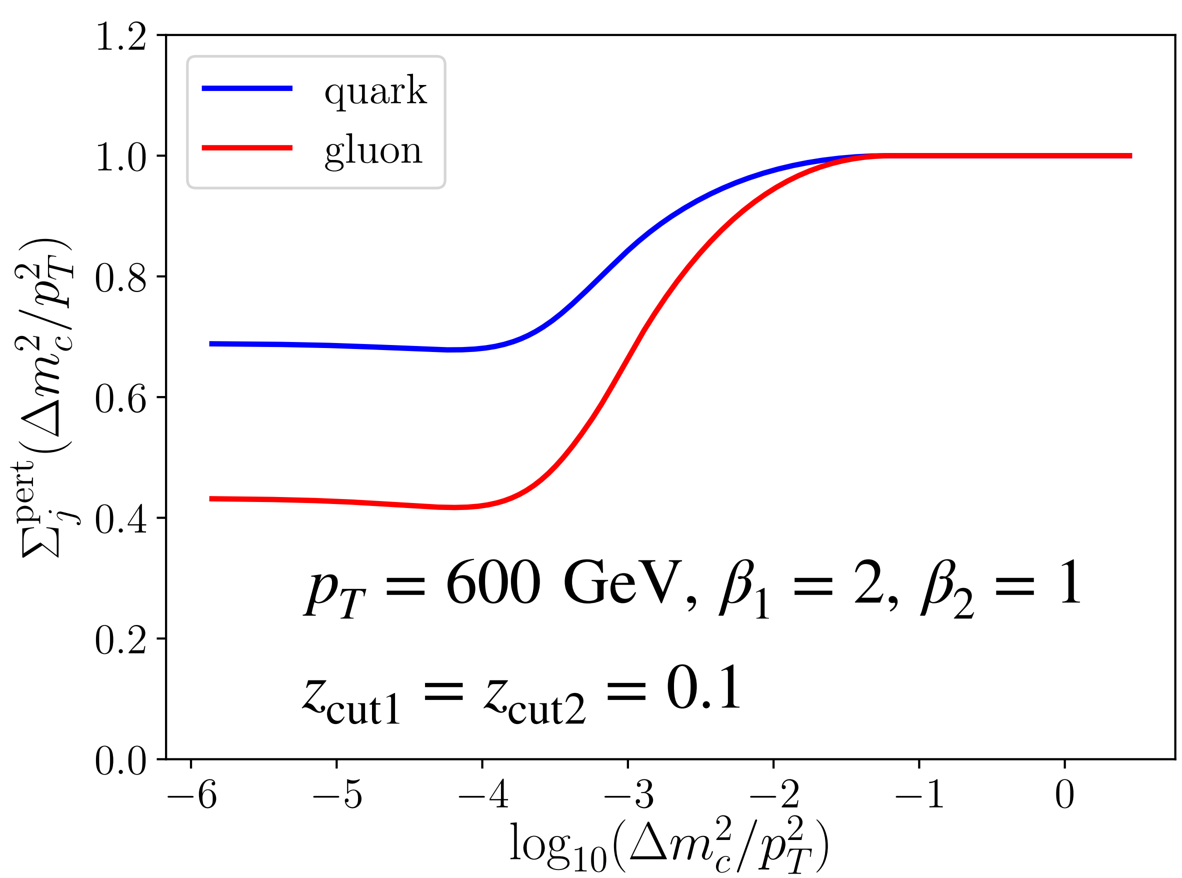
$$\omega(\mu_1,\mu_2) = \int_{\alpha_s(\mu_1)}^{\alpha_s(\mu_2)} d\alpha \frac{\Gamma_{\rm cusp}(\alpha)}{\beta(\alpha)}$$

Cumulative Jet Mass in Collinear Drop in SCET

Cumulative jet mass $\Sigma^{\rm pert}(\Delta m_c^2) = \int_0^{\Delta m_c^2} d\Delta m^2 \, \frac{d\sigma}{d\Delta m^2} = \sum_{j=q,g} H_j \Sigma_j^{\rm pert}$

$$\Sigma^{\rm pert}_{j,\rm NLL}(\Delta m_c^2 \approx 0) =$$
$$\exp\left(\frac{2C_j}{1+\beta_1}K(\mu_{\rm gs1},\mu) - \frac{2C_j}{1+\beta_2}K(\mu_{\rm gs2},\mu) - 2C_j\frac{2+\beta_1}{1+\beta_1}K(\Lambda_{\rm cs1},\mu) + 2C_j\frac{2+\beta_2}{1+\beta_2}K(\Lambda_{\rm cs2},\mu)\right)$$
$$\times \left(\frac{\mu_{\rm gs1}}{Q_{\rm gs1}}\right)^{\frac{2C_j}{1+\beta_1}\omega(\mu_{\rm gs1},\mu)} \left(\frac{\mu_{\rm gs2}}{Q_{\rm gs2}}\right)^{\frac{-2C_j}{1+\beta_2}\omega(\mu_{\rm gs1},\mu)} \left(\frac{Q_{\rm cut1}^{\frac{1}{1+\beta_1}}}{Q\Lambda_{\rm cs1}^{\frac{2+\beta_1}{1+\beta_1}}}\right)^{2C_j\omega(\Lambda_{\rm cs1},\mu)} \left(\frac{Q_{\rm cut2}^{\frac{1}{1+\beta_2}}}{Q\Lambda_{\rm cs2}^{\frac{2+\beta_2}{1+\beta_2}}}\right)^{-2C_j\omega(\Lambda_{\rm cs2},\mu)}$$

Y.T.Chien, I.W.Stewart
arXiv:1907.11107



A significant fraction of events have the two SD jet masses equal

The constant at $\Delta m_c = 0$ gives this fraction (perturbatively)

The constant differs for quark and gluon jets and depends on CD parameters —> we will exploit this in the following construction

Nonperturbative Correction via Shape Function

- **Separating perturbative CS function and nonperturbative shape function**

$$S_{C_j}\left(\ell_1^+ Q_{\mathrm{cut1}}^{\frac{1}{1+\beta_1}}, \beta_1, \mu\right) = \int \mathrm{d}k_1\, S_{C_j}^{\mathrm{pert}}\left(\ell_1^+ Q_{\mathrm{cut1}}^{\frac{1}{1+\beta_1}} - k_1^{\frac{2+\beta_1}{1+\beta_1}}, \beta_1, \mu\right) F_1^j(k_1, \beta_1)$$

Independent of z_{cut1}

A.H.Hoang, S.Mantry, A.Pathak, I.W.Stewart, arXiv:1906.11843

In position space:

$$\widetilde{S}_{C_j}\left(yQQ_{\mathrm{cut1}}^{\frac{-1}{1+\beta_1}}, \beta_1, \mu\right) = \widetilde{S}_{C_j}^{\mathrm{pert}}\left(yQQ_{\mathrm{cut1}}^{\frac{-1}{1+\beta_1}}, \beta_1, \mu\right) \widetilde{F}_1^j(yQQ_{\mathrm{cut1}}^{\frac{-1}{1+\beta_1}}, \beta_1)$$

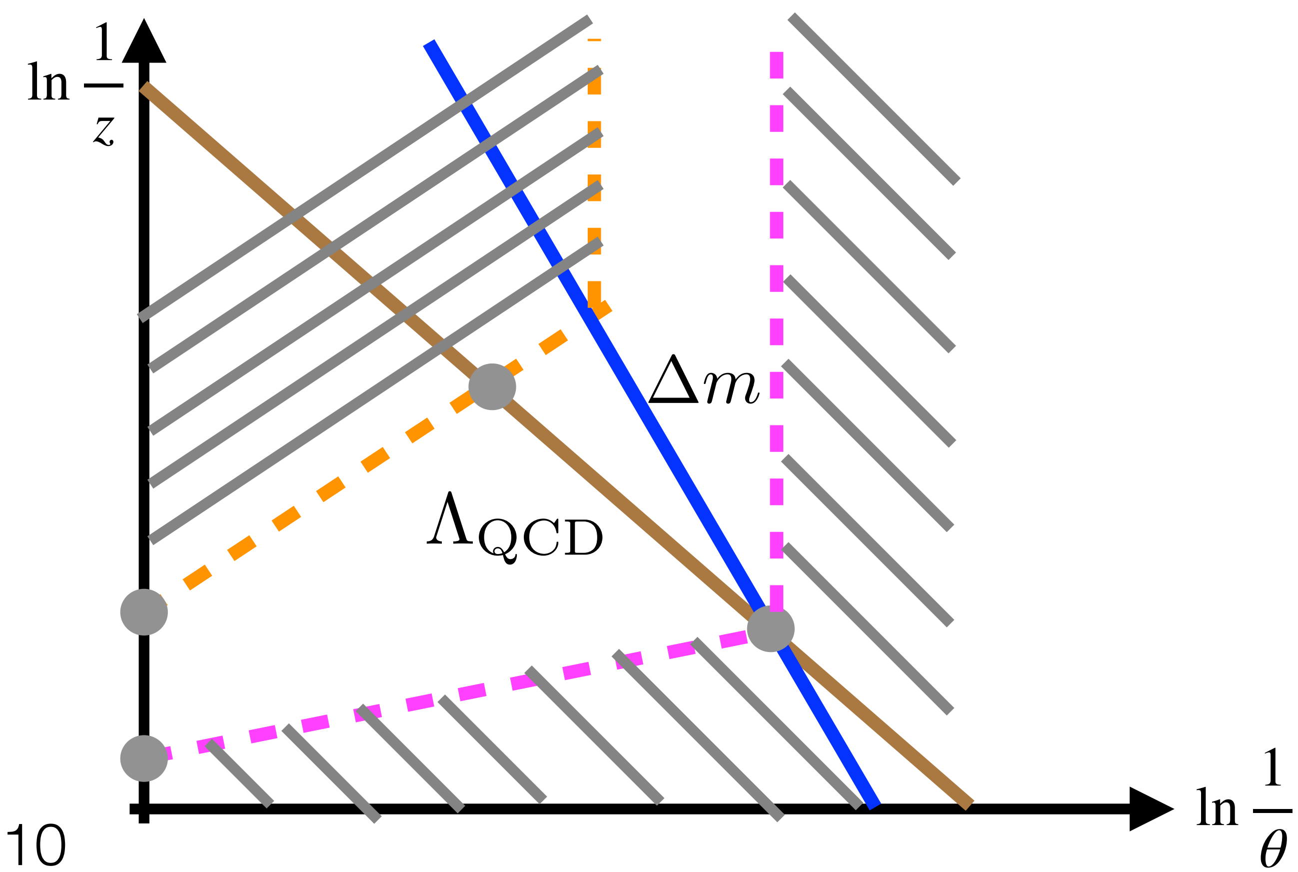
Similarly for CS function associated w/ $(z_{\mathrm{cut2}},\ \beta_2)$

- **Deep nonperturbative regime**

This is the constant region

$$\Delta m_c^2 \lesssim \Lambda_{\mathrm{QCD}}\, Q \left(\frac{\Lambda_{\mathrm{QCD}}}{Q_{\mathrm{cut2}}}\right)^{\frac{1}{1+\beta_2}}$$

From now on focus on this regime



Nonperturbative Correction via Shape Function

- **Separating perturbative CS function and nonperturbative shape function**

$$S_{C_j}\big(\ell_1^+ Q_{\rm cut1}^{\frac{1}{1+\beta_1}},\beta_1,\mu\big) = \int \mathrm{d}k_1\, S_{C_j}^{\rm pert}\big(\ell_1^+ Q_{\rm cut1}^{\frac{1}{1+\beta_1}} - k_1^{\frac{2+\beta_1}{1+\beta_1}},\beta_1,\mu\big) F_1^j(k_1,\beta_1)$$

Independent of $z_{\rm cut1}$

A.H.Hoang, S.Mantry, A.Pathak, I.W.Stewart, arXiv:1906.11843

In position space:

$$\widetilde{S}_{C_j}\big(yQQ_{\rm cut1}^{\frac{-1}{1+\beta_1}},\beta_1,\mu\big) = \widetilde{S}_{C_j}^{\rm pert}\big(yQQ_{\rm cut1}^{\frac{-1}{1+\beta_1}},\beta_1,\mu\big)\,\widetilde{F}_1^j(yQQ_{\rm cut1}^{\frac{-1}{1+\beta_1}},\beta_1)$$

- **Cumulative distribution with shape function**

$$\Sigma(\Delta m_c, p_T, \eta_J, R, z_{{\rm cut}\,i}, \beta_i) = H_q(p_T,\eta_J,R)\Sigma_q + H_g(p_T,\eta_J,R)\Sigma_g \qquad \Sigma_j = \Sigma_j^{\rm pert} \otimes F_j$$

$$\begin{aligned}\Sigma_{j,\rm NLL}(\Delta m_c^2) &= \exp\left(\frac{2C_j}{1+\beta_1}K(\mu_{\rm gs1},\mu) - \frac{2C_j}{1+\beta_2}K(\mu_{\rm gs2},\mu) - 2C_j\frac{2+\beta_1}{1+\beta_1}K(\Lambda_{\rm cs1},\mu) + 2C_j\frac{2+\beta_2}{1+\beta_2}K(\Lambda_{\rm cs2},\mu)\right)\\
&\times\left(\frac{\mu_{\rm gs1}}{Q_{\rm gs1}}\right)^{\frac{2C_j}{1+\beta_1}\omega(\mu_{\rm gs1},\mu)}\left(\frac{\mu_{\rm gs2}}{Q_{\rm gs2}}\right)^{\frac{-2C_j}{1+\beta_2}\omega(\mu_{\rm gs1},\mu)}\left(\frac{Q_{\rm cut1}^{\frac{1}{1+\beta_1}}}{Q\Lambda_{\rm cs1}^{\frac{2+\beta_1}{1+\beta_1}}}\right)^{2C_j\omega(\Lambda_{\rm cs1},\mu)}\left(\frac{Q_{\rm cut2}^{\frac{1}{1+\beta_2}}}{Q\Lambda_{\rm cs2}^{\frac{2+\beta_2}{1+\beta_2}}}\right)^{-2C_j\omega(\Lambda_{\rm cs2},\mu)}\\
&\times \widetilde{F}_1^j\big(QQ_{\rm cut1}^{\frac{-1}{1+\beta_1}}\Delta m_c^{-2}e^{-\frac{\partial}{\partial\eta}},\beta_1\big)\widetilde{F}_2^j\big(QQ_{\rm cut2}^{\frac{-1}{1+\beta_2}}\Delta m_c^{-2}e^{-\frac{\partial}{\partial\eta}},\beta_2\big)\frac{e^{-\gamma_E\eta}}{\Gamma(1+\eta)}\bigg|_{\eta=2C_j\omega(\Lambda_{\rm cs1},\Lambda_{\rm cs2})}\end{aligned}$$

Now depends on Δm_c^2

Construction of Pure Quark/Gluon Observables

- **Form linear combination from two sets of CD parameters:** $z^{(a)}_{\mathrm{cut}\,i},\ z^{(b)}_{\mathrm{cut}\,i},\ i = 1{,}2$

$$\mathscr{Q} = \Sigma(\Delta m_c^{(b)}, p_T, \eta_J, R, z^{(b)}_{\mathrm{cut}\,i}, \beta_i) - \xi_g \Sigma(\Delta m_c^{(a)}, p_T, \eta_J, R, z^{(a)}_{\mathrm{cut}\,i}, \beta_i)$$

$$\mathscr{G} = \Sigma(\Delta m_c^{(b)}, p_T, \eta_J, R, z^{(b)}_{\mathrm{cut}\,i}, \beta_i) - \xi_q \Sigma(\Delta m_c^{(a)}, p_T, \eta_J, R, z^{(a)}_{\mathrm{cut}\,i}, \beta_i)$$

$$\Sigma = \sum_j H_j \widetilde{F}_j \Sigma_j^{\mathrm{pert}}$$

Plan: adjust $\xi_{q/g}$ to make pure quark and gluon observables in small Δm_c^2 regime

E.g. $$\mathscr{Q} = H_q\big(\widetilde{F}_q^{(b)} \Sigma_q^{\mathrm{pert}(b)} - \xi_g \widetilde{F}_q^{(a)} \Sigma_q^{\mathrm{pert}(a)}\big) + H_g\big(\widetilde{F}_g^{(b)} \cancel{\Sigma_g^{\mathrm{pert}(b)} - \xi_g \widetilde{F}_g^{(a)} \Sigma_g^{\mathrm{pert}(a)}}\big)$$

$$\xi_g = \frac{(\Sigma_g^{\mathrm{pert}} \widetilde{F}_g)^{(b)}}{(\Sigma_g^{\mathrm{pert}} \widetilde{F}_g)^{(a)}}$$

$$\widetilde{F}_j(\Delta m_c^2, z_{\mathrm{cut}\,i}, \beta_i) = \widetilde{F}_1^j\big(Q Q_{\mathrm{cut1}}^{\frac{-1}{1+\beta_1}} \Delta m_c^{-2} e^{-\frac{\partial}{\partial\eta}}, \beta_1\big) \widetilde{F}_2^j\big(Q Q_{\mathrm{cut2}}^{\frac{-1}{1+\beta_2}} \Delta m_c^{-2} e^{-\frac{\partial}{\partial\eta}}, \beta_2\big) \frac{e^{-\gamma_E \eta}}{\Gamma(1+\eta)}\bigg|_{\eta=0}$$

- **Problem: ξ_j depends on $\widetilde{F}_j$**

Arguments of shape function depend on Δm_c^2 and $z^{(a,b)}_{\mathrm{cut}\,i}$

Construction of Pure Quark/Gluon Observables

$$\widetilde{F}_j(\Delta m_c^2, z_{\mathrm{cut}\,i}, \beta_i) = \widetilde{F}_1^j\left(QQ_{\mathrm{cut1}}^{\frac{-1}{1+\beta_1}}\Delta m_c^{-2}e^{-\frac{\partial}{\partial\eta}}, \beta_1\right)\widetilde{F}_2^j\left(QQ_{\mathrm{cut2}}^{\frac{-1}{1+\beta_2}}\Delta m_c^{-2}e^{-\frac{\partial}{\partial\eta}}, \beta_2\right)\frac{e^{-\gamma_E\eta}}{\Gamma(1+\eta)}\Bigg|_{\eta=0}$$

- **We can make shape function a common factor by imposing:**

$$\left(\Delta m_c^{(a)}\right)^2\left(Q_{\mathrm{cut1}}^{(a)}\right)^{\frac{1}{1+\beta_1}}Q^{-1} = \left(\Delta m_c^{(b)}\right)^2\left(Q_{\mathrm{cut1}}^{(b)}\right)^{\frac{1}{1+\beta_1}}Q^{-1}$$

$$\left(\Delta m_c^{(a)}\right)^2\left(Q_{\mathrm{cut2}}^{(a)}\right)^{\frac{1}{1+\beta_2}}Q^{-1} = \left(\Delta m_c^{(b)}\right)^2\left(Q_{\mathrm{cut2}}^{(b)}\right)^{\frac{1}{1+\beta_2}}Q^{-1}$$

Solve by picking

$$\frac{(\Delta m_c^{(a)})^2}{(\Delta m_c^{(b)})^2}, \quad z_{\mathrm{cut2}}^{(b)}$$

- **Then we have for $\mathcal{Q}$ (hard process independent of Δm_c^2)**

$$\mathcal{Q} = \Sigma^{(b)} - \xi_g\Sigma^{(a)} = H_q\widetilde{F}_q(\Sigma_q^{\mathrm{pert}(b)} - \xi_g\Sigma_q^{\mathrm{pert}(a)}) + \underbrace{H_g\widetilde{F}_g(\Sigma_g^{\mathrm{pert}(b)} - \xi_g\Sigma_g^{\mathrm{pert}(a)})}_{=0}$$

- **Find value of ξ_g such that gluon contribution to $\mathcal{Q}$ vanishes**

$$\xi_g = \frac{\Sigma_g^{\mathrm{pert}(b)}}{\Sigma_g^{\mathrm{pert}(a)}}$$ is constant, independent of Δm_c^2

Similar procedure for $\mathcal{G}$: $$\xi_q = \frac{\Sigma_q^{\mathrm{pert}(b)}}{\Sigma_q^{\mathrm{pert}(a)}}$$

- **Finally have pure quark and gluon observables:**

$$\mathcal{Q} = H_q\widetilde{F}_q(\Sigma_q^{\mathrm{pert}(b)} - \xi_g\Sigma_q^{\mathrm{pert}(a)}) \qquad \mathcal{G} = H_g\widetilde{F}_g(\Sigma_g^{\mathrm{pert}(b)} - \xi_q\Sigma_g^{\mathrm{pert}(a)})$$

Construction of Pure Quark/Gluon Observables

- **Pure quark and gluon observables:**

$$\mathcal{Q} = H_q \widetilde{F}_q(\Sigma_q^{\mathrm{pert}(b)} - \xi_g \Sigma_q^{\mathrm{pert}(a)}) \qquad \xi_g = \frac{\Sigma_g^{\mathrm{pert}(b)}}{\Sigma_g^{\mathrm{pert}(a)}}$$

$$\mathcal{G} = H_g \widetilde{F}_g(\Sigma_g^{\mathrm{pert}(b)} - \xi_q \Sigma_g^{\mathrm{pert}(a)}) \qquad \xi_q = \frac{\Sigma_q^{\mathrm{pert}(b)}}{\Sigma_q^{\mathrm{pert}(a)}}$$

- **Remaining free parameters are:**

$$(\Delta m_c^{(a)})^2,\ \beta_1,\ \beta_2,\ z_{\mathrm{cut1}}^{(a)},\ z_{\mathrm{cut2}}^{(a)},\ z_{\mathrm{cut1}}^{(b)}$$

We constructed a class of observables

Maximize Disentangling Power

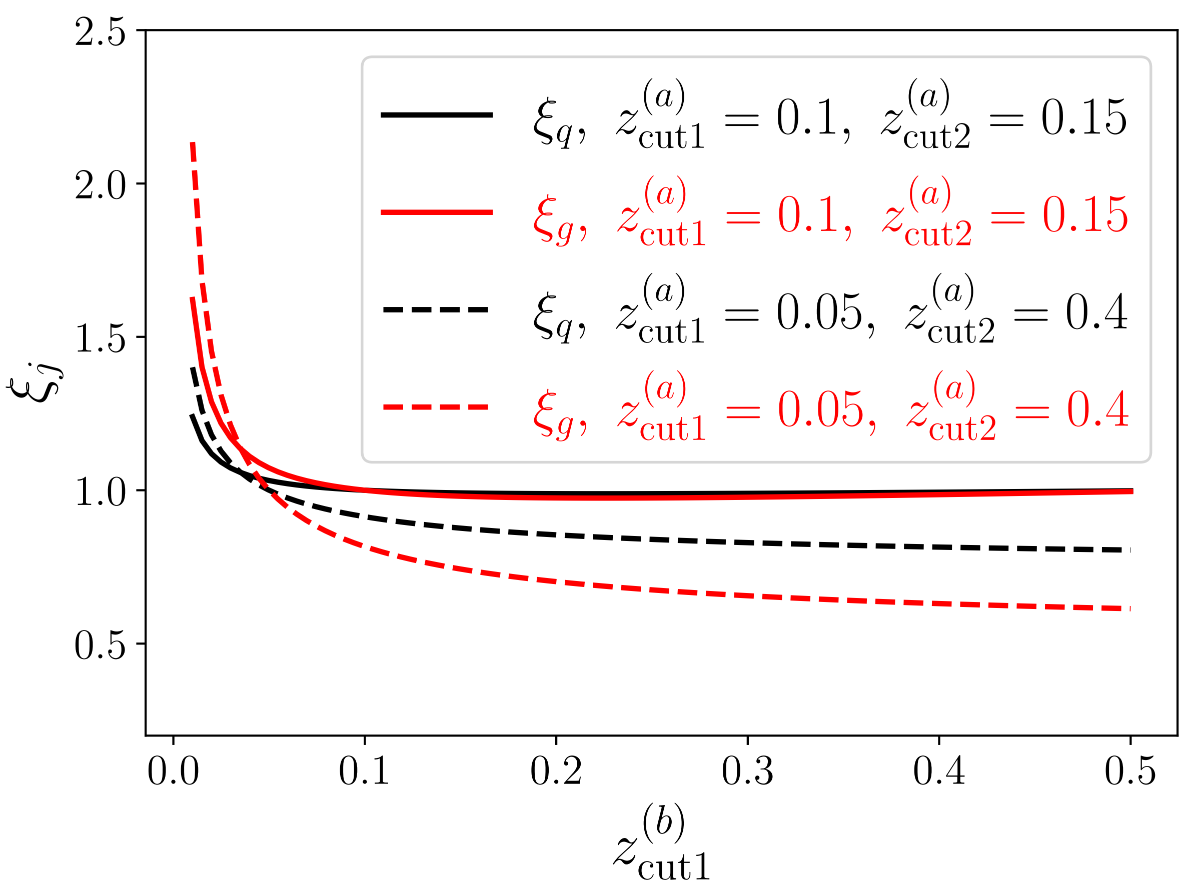
- **NLL expression of ξ_j**

$$\xi_j = \exp\left(\frac{2C_j}{1+\beta_1}\left(K(Q_{\text{gs1}}^{(b)},\mu) - K(Q_{\text{gs1}}^{(a)},\mu)\right) - \frac{2C_j}{1+\beta_2}\left(K(Q_{\text{gs2}}^{(b)},\mu) - K(Q_{\text{gs2}}^{(a)},\mu)\right)\right)$$

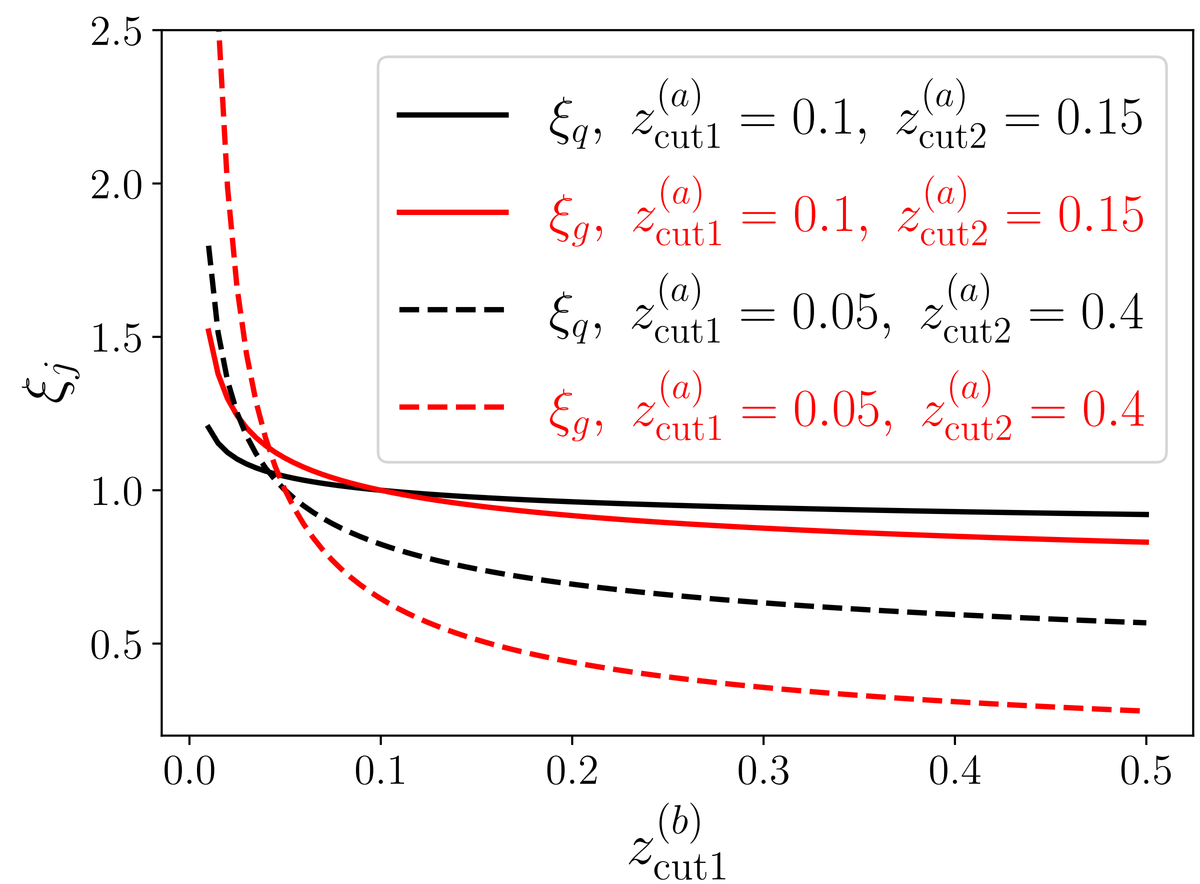
- **Constraint imposed:** $z_{\text{cut2}}^{(b)} = z_{\text{cut2}}^{(a)}\left(\frac{z_{\text{cut1}}^{(b)}}{z_{\text{cut1}}^{(a)}}\right)^{\frac{1+\beta_2}{1+\beta_1}}$

- **If ξ_q and ξ_g close, difficult in experiments; we want to maximize their difference**

$p_T = 300$ GeV $\beta_1 = 1$ $\beta_2 = 0$



$p_T = 300$ GeV $\beta_1 = 0$ $\beta_2 = 0$



Results in Small Δm_c^2 Region (w/o Shape Function)

$p_T = 300$ GeV

$\beta_1 = 1,\ \beta_2 = 0 \quad \dfrac{\Delta m_c^{(a)}}{\Delta m_c^{(b)}} = 0.67$

$z_{\text{cut1}}^{(a)} = 0.05,\ z_{\text{cut2}}^{(a)} = 0.4,\ z_{\text{cut1}}^{(b)} = 0.01,\ z_{\text{cut2}}^{(b)} = 0.179$

$\beta_1 = \beta_2 = 0 \quad \dfrac{\Delta m_c^{(a)}}{\Delta m_c^{(b)}} = 0.45$

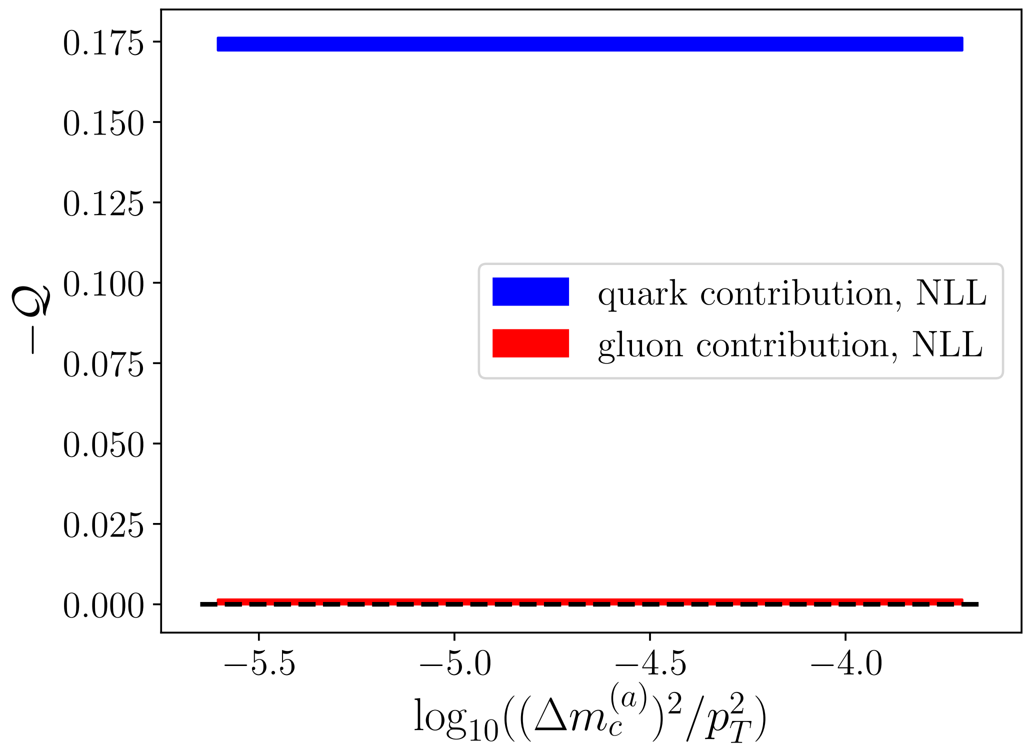
$z_{\text{cut1}}^{(a)} = 0.1,\ z_{\text{cut2}}^{(a)} = 0.4,\ z_{\text{cut1}}^{(b)} = 0.02,\ z_{\text{cut2}}^{(b)} = 0.08$

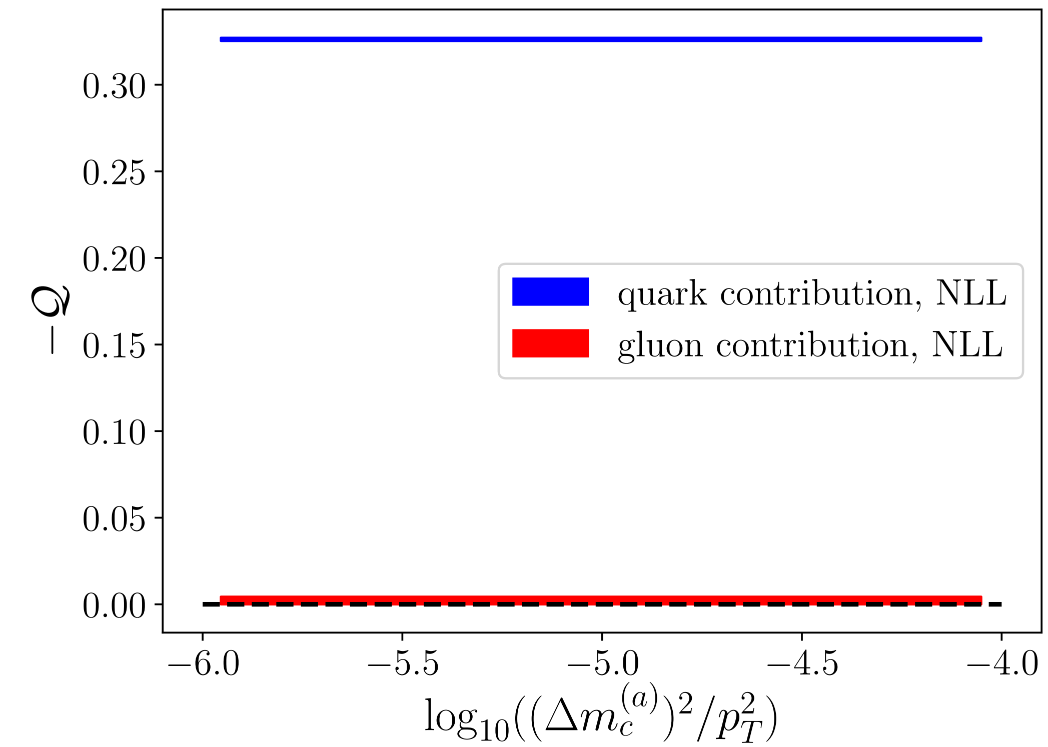
Pure quark

$\xi_g \approx 2.2$

For illustration

$H_q = 1$



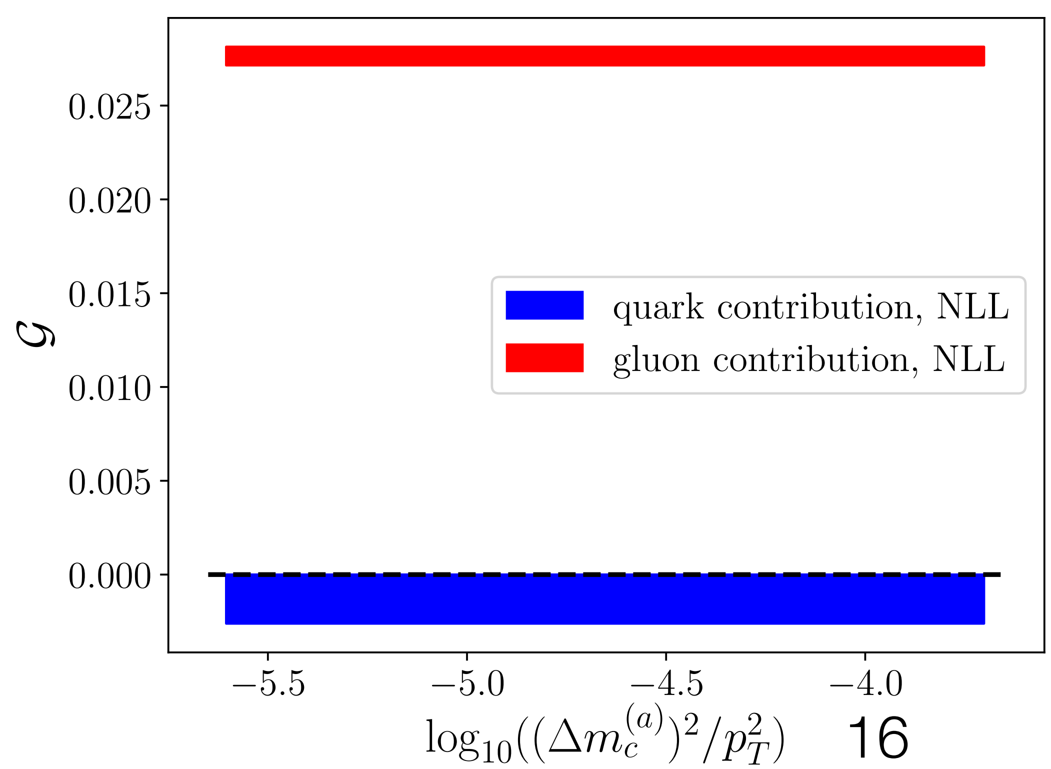


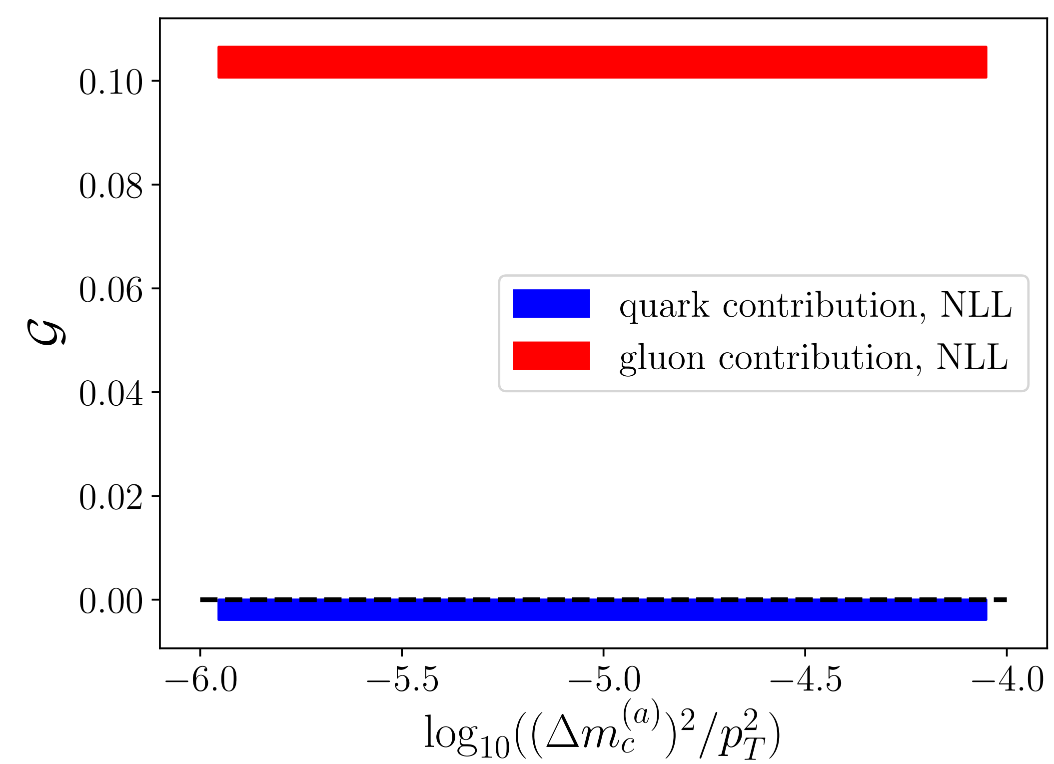
Pure gluon

$\xi_q \approx 1.4$

For illustration

$H_g = 1$





Results in Small Δm_c^2 Region (w Shape Function)

$p_T = 300$ GeV $\quad \beta_1 = 1,\ \beta_2 = 0 \quad \dfrac{\Delta m_c^{(a)}}{\Delta m_c^{(b)}} = 0.67$ $\qquad \beta_1 = \beta_2 = 0 \quad \dfrac{\Delta m_c^{(a)}}{\Delta m_c^{(b)}} = 0.45$

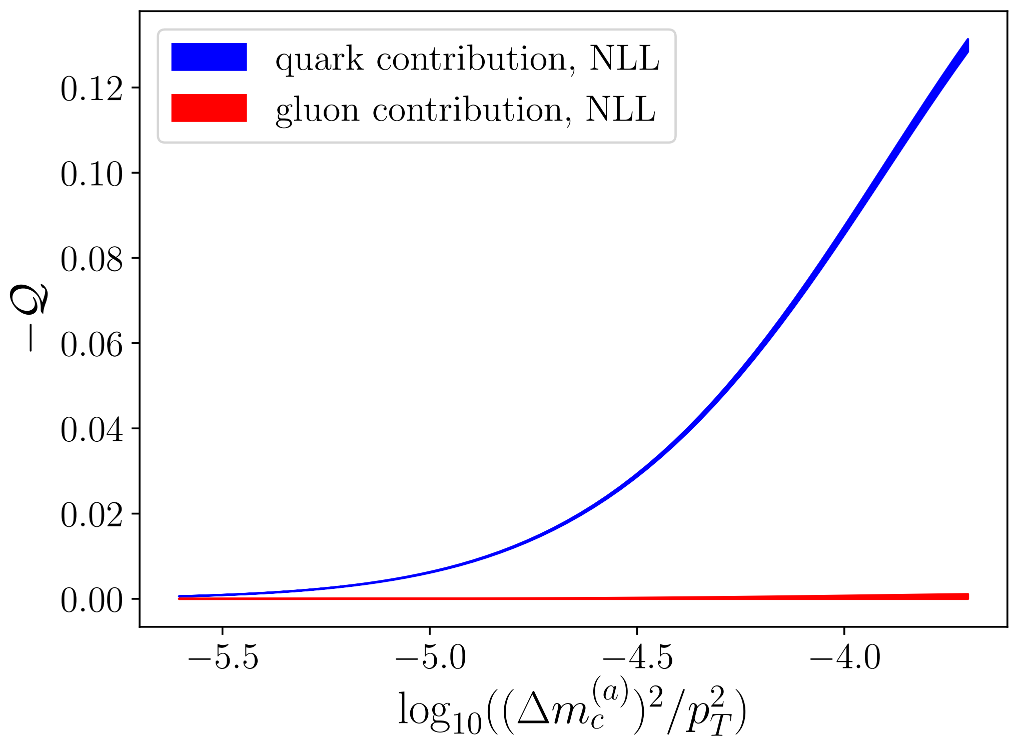
$z_{\text{cut1}}^{(a)} = 0.05,\ z_{\text{cut2}}^{(a)} = 0.4,\ z_{\text{cut1}}^{(b)} = 0.01,\ z_{\text{cut2}}^{(b)} = 0.179$ $\quad z_{\text{cut1}}^{(a)} = 0.1,\ z_{\text{cut2}}^{(a)} = 0.4,\ z_{\text{cut1}}^{(b)} = 0.02,\ z_{\text{cut2}}^{(b)} = 0.08$

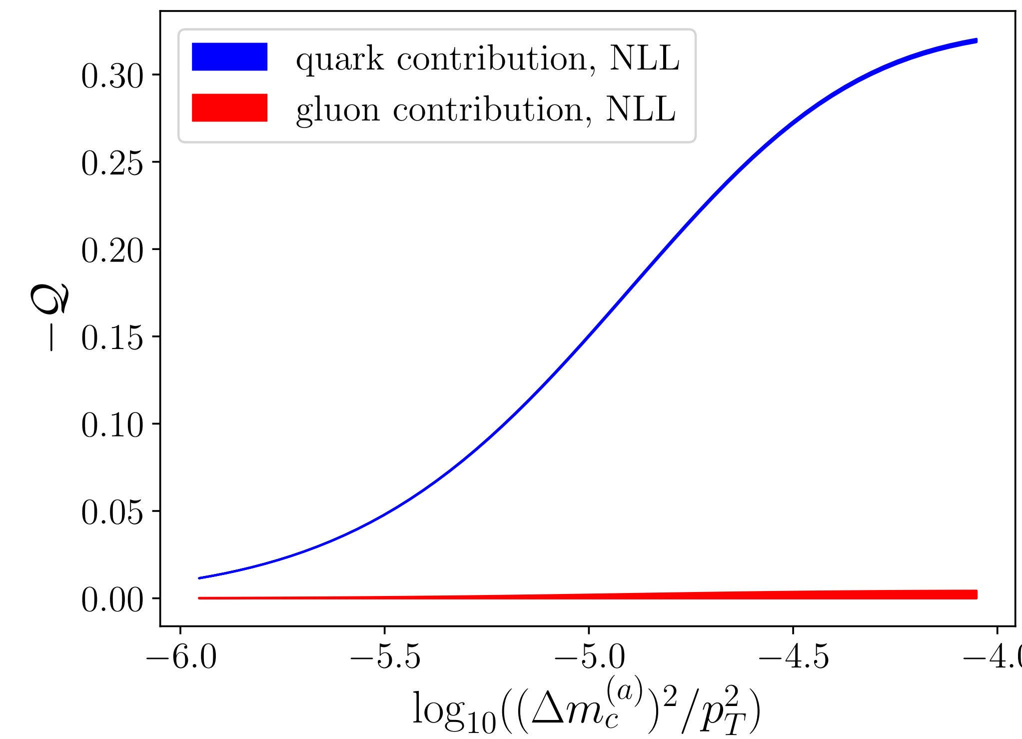
Pure quark

$\xi_g \approx 2.2$

For illustration

$H_q = 1$



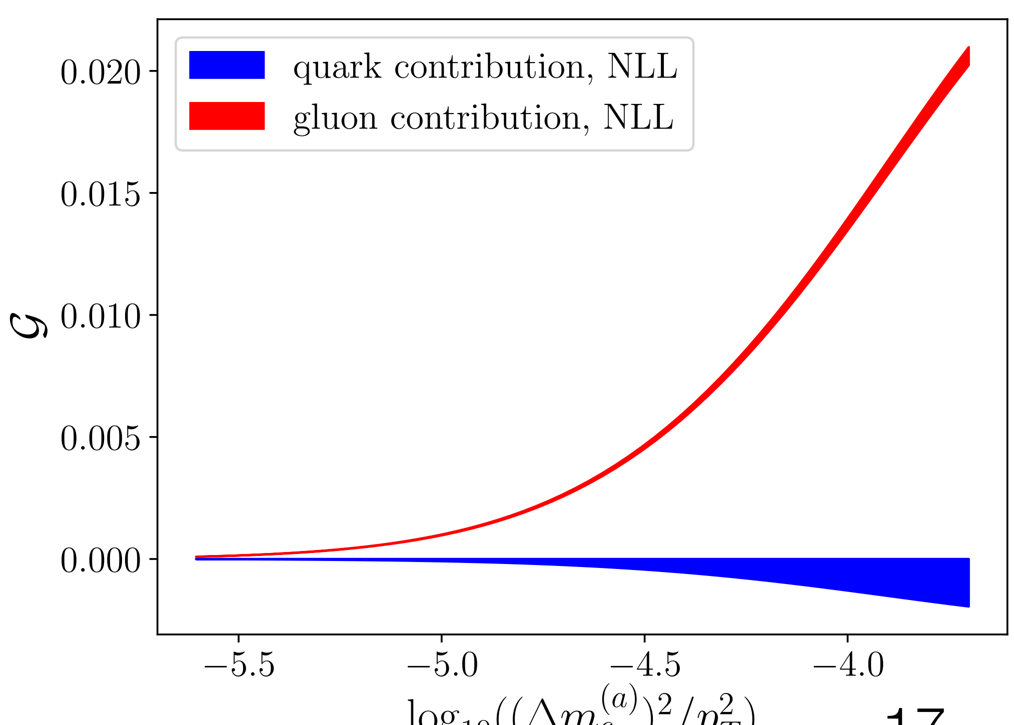


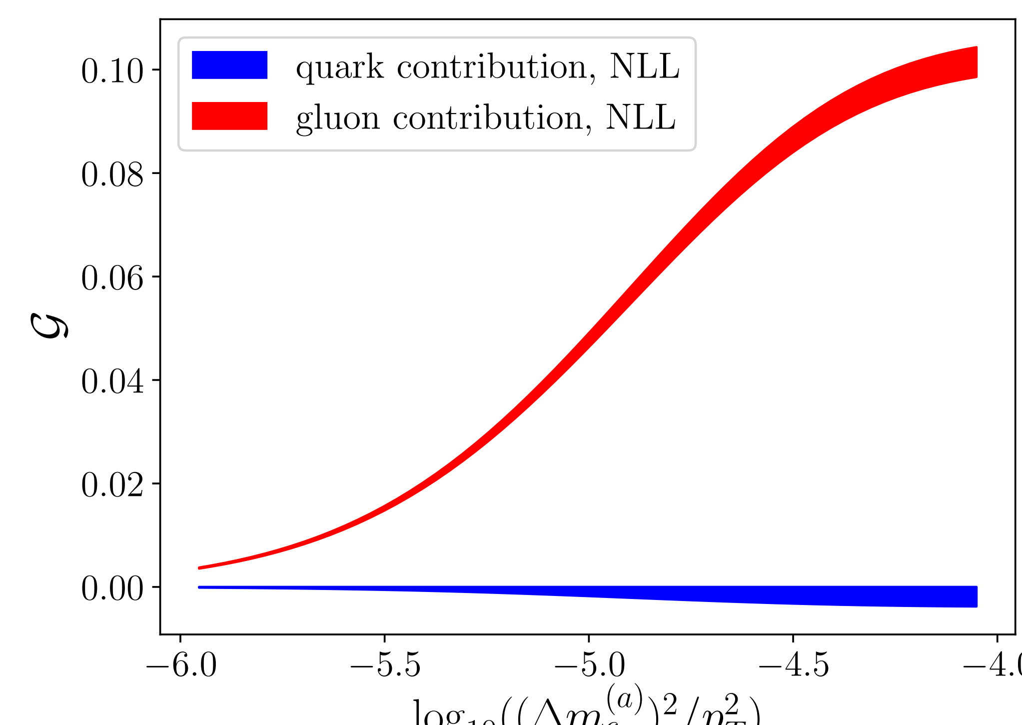
Pure gluon

$\xi_q \approx 1.4$

For illustration

$H_g = 1$





Results in Full Δm_c^2 Region (w Shape Function)

$p_T = 300$ GeV $\quad \beta_1 = 1,\ \beta_2 = 0 \quad \dfrac{\Delta m_c^{(a)}}{\Delta m_c^{(b)}} = 0.67 \qquad \beta_1 = \beta_2 = 0 \quad \dfrac{\Delta m_c^{(a)}}{\Delta m_c^{(b)}} = 0.45$

$z_{\text{cut1}}^{(a)} = 0.05,\ z_{\text{cut2}}^{(a)} = 0.4,\ z_{\text{cut1}}^{(b)} = 0.01,\ z_{\text{cut2}}^{(b)} = 0.179 \qquad z_{\text{cut1}}^{(a)} = 0.1,\ z_{\text{cut2}}^{(a)} = 0.4,\ z_{\text{cut1}}^{(b)} = 0.02,\ z_{\text{cut2}}^{(b)} = 0.08$

Pure quark

$\xi_g \approx 2.2$

For illustration

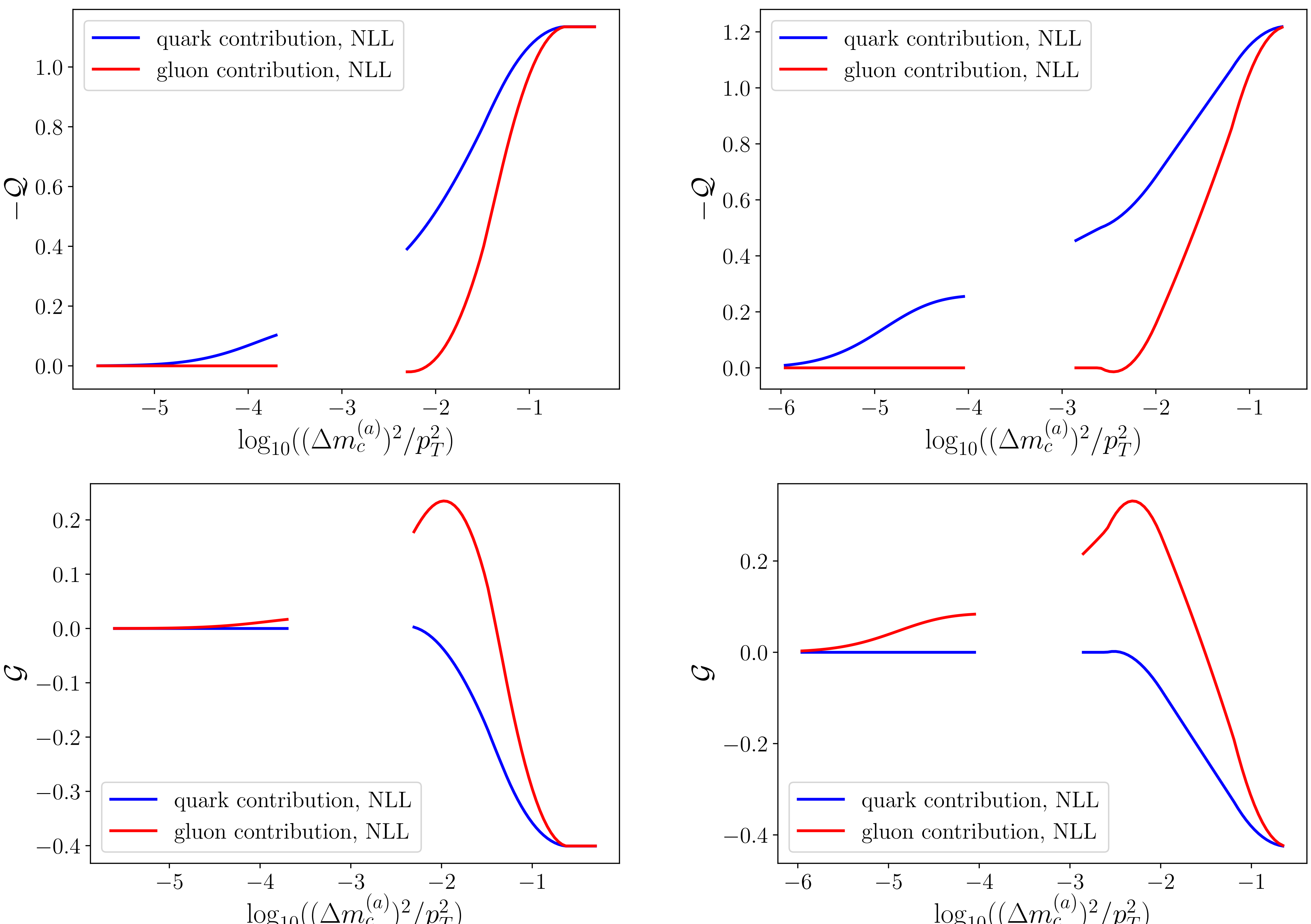
$H_q = 1$

Pure gluon

$\xi_q \approx 1.4$

For illustration

$H_g = 1$



Results in Full Δm_c^2 Region (w Shape Function)

$p_T = 300$ GeV $\quad \beta_1 = 1,\ \beta_2 = 0 \quad \dfrac{\Delta m_c^{(a)}}{\Delta m_c^{(b)}} = 0.67 \qquad \beta_1 = \beta_2 = 0 \quad \dfrac{\Delta m_c^{(a)}}{\Delta m_c^{(b)}} = 0.45$

$z_{\text{cut1}}^{(a)} = 0.05,\ z_{\text{cut2}}^{(a)} = 0.4,\ z_{\text{cut1}}^{(b)} = 0.01,\ z_{\text{cut2}}^{(b)} = 0.179 \qquad z_{\text{cut1}}^{(a)} = 0.1,\ z_{\text{cut2}}^{(a)} = 0.4,\ z_{\text{cut1}}^{(b)} = 0.02,\ z_{\text{cut2}}^{(b)} = 0.08$

Pure quark

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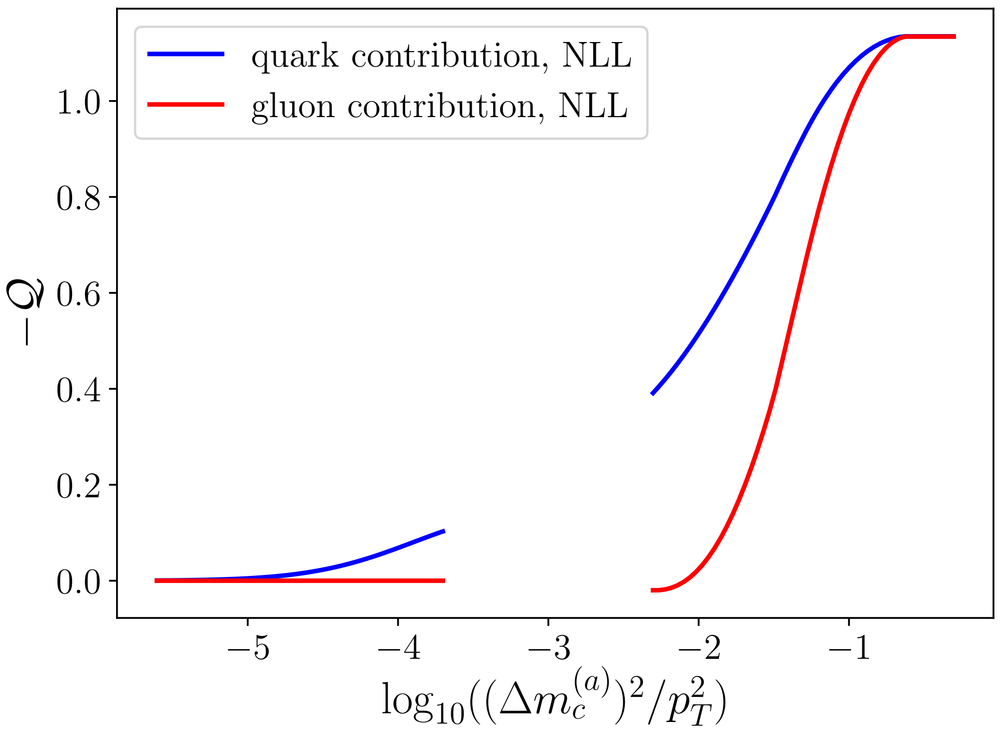
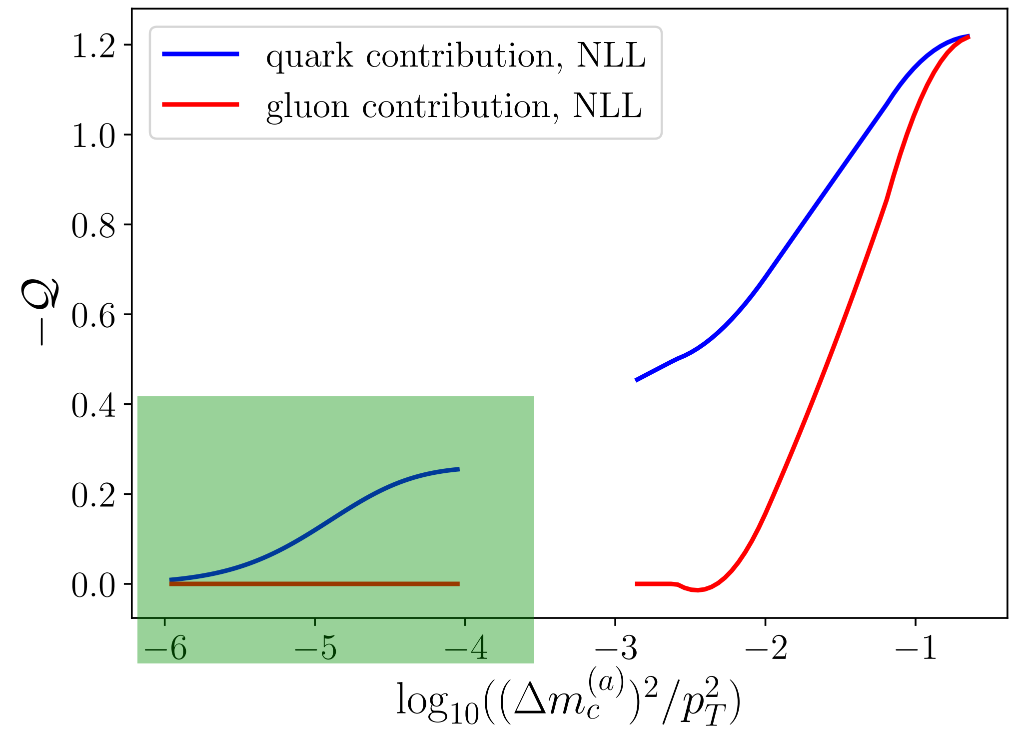
$H_q = 1$



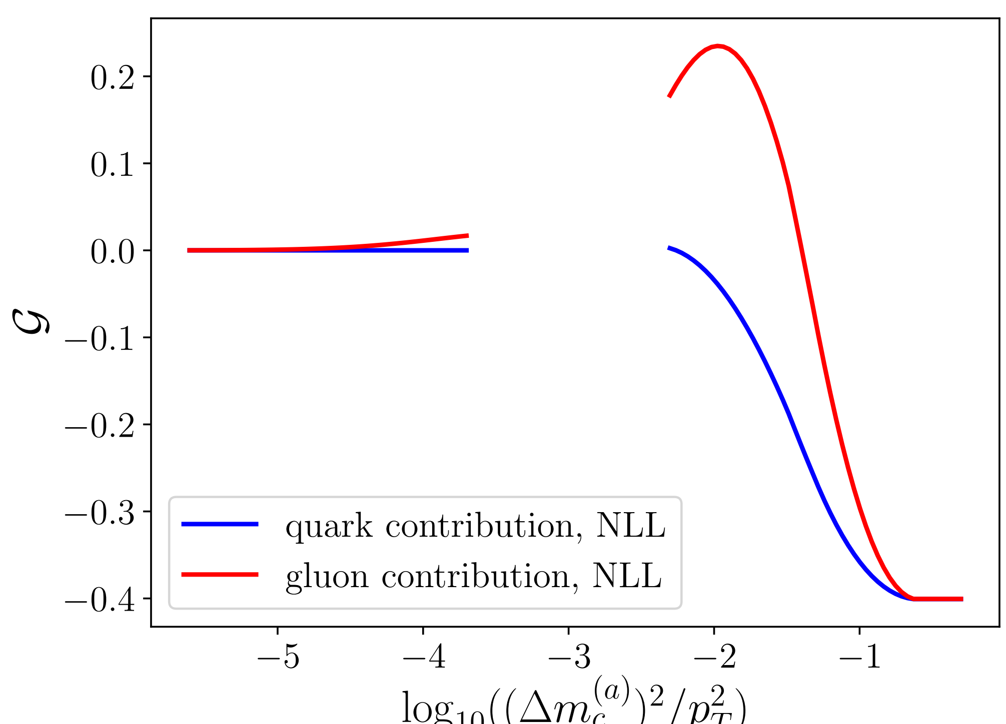


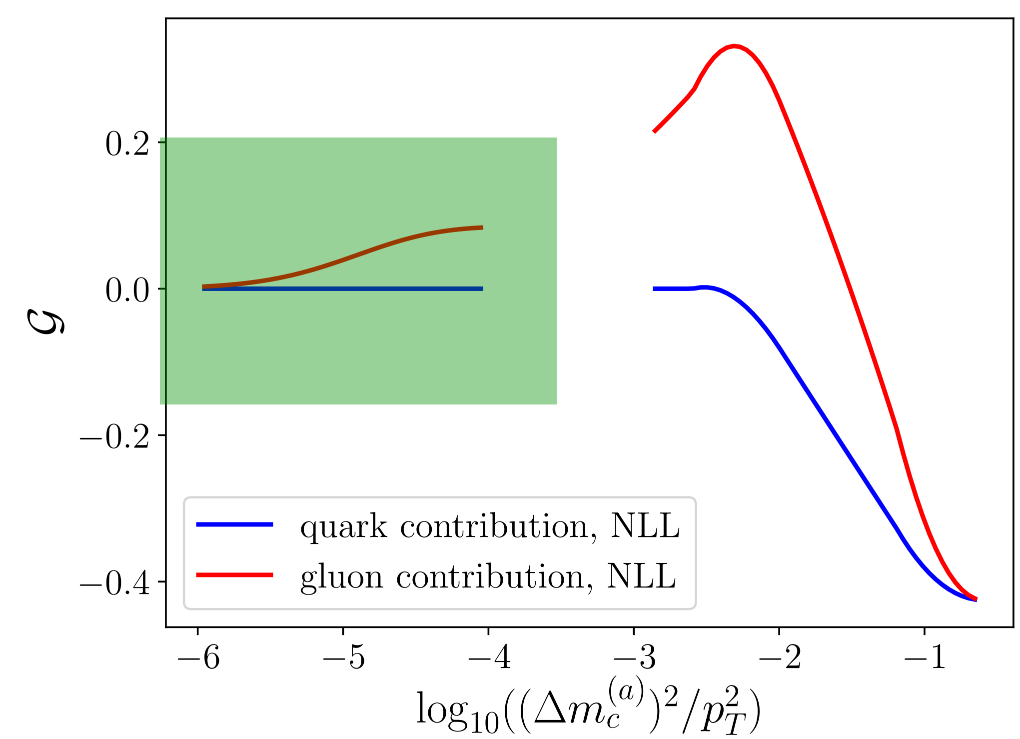
Pure gluon

$\xi_q \approx 1.4$

For illustration

$H_g = 1$





Applications of Pure Quark and Gluon Observables

- **Two samples of jets: A = Z + jet, B = dijets**

Quark and gluon fractions: $f_q^{A/B},\ f_g^{A/B} = 1 - f_q^{A/B}$

- **Construct pure quark and gluon observables for both A and B samples in small Δm_c^2 region**

$$\mathcal{Q}_A = f_q^A \mathcal{Q} \qquad \mathcal{G}_A = f_g^A \mathcal{G}$$

$$\mathcal{Q}_B = f_q^B \mathcal{Q} \qquad \mathcal{G}_B = f_g^B \mathcal{G}$$

- **Can obtain fractions from:**

$$\frac{f_q^A}{f_q^B} = \frac{\mathcal{Q}_A}{\mathcal{Q}_B} \qquad \frac{f_g^A}{f_g^B} = \frac{\mathcal{G}_A}{\mathcal{G}_B}$$

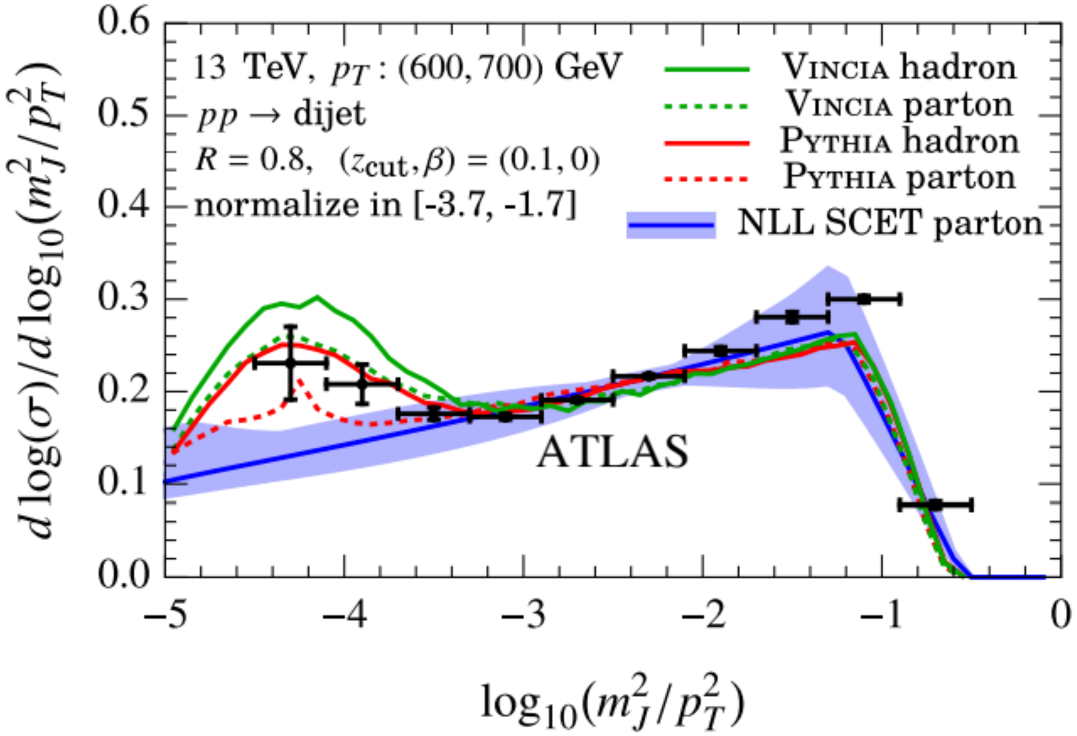
$$f_q^A + f_g^A = 1 \qquad f_q^B + f_g^B = 1$$

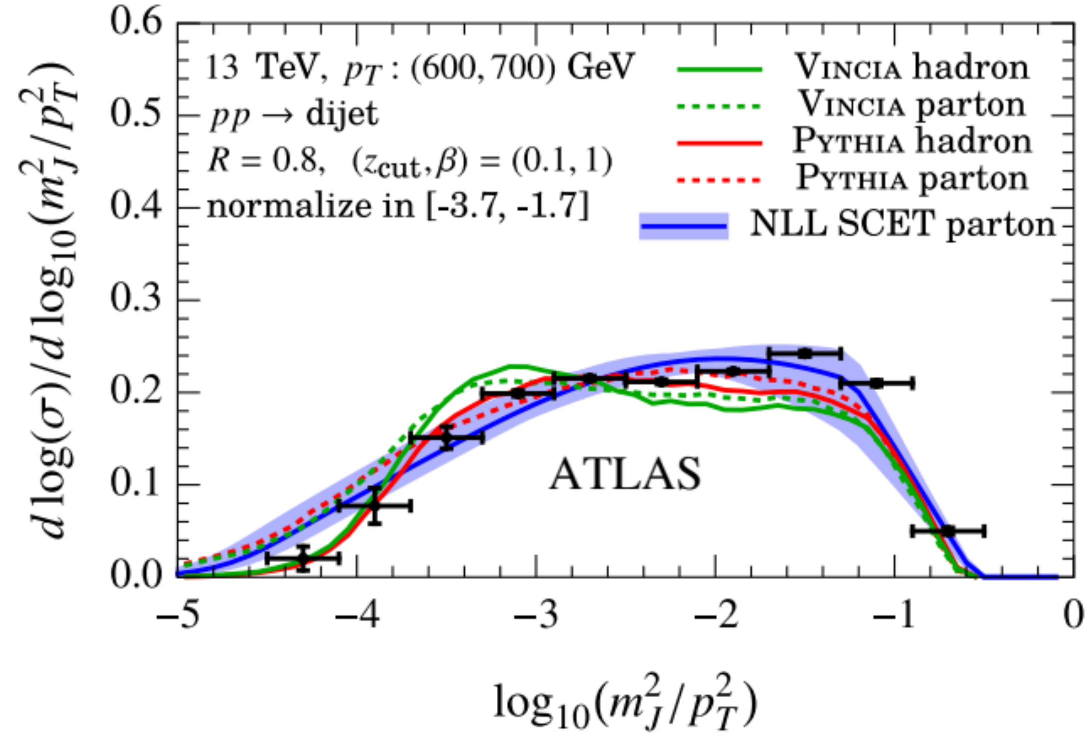
Conclusions

- Construct pure quark/gluon observables with collinear drop
  - Linear combination of cumulative jet mass observables with different CD parameters $z_{\mathrm{cut}\, i}^{(a,b)}$
  - Use two jet mass bins, we construct observables with ξ_q, ξ_g, whose definitions are independent of nonperturbative corrections
  - Application: extracting fractions of quark and gluon jets from jet samples
- Future plans:
  - Theory predictions in the intermediate region to understand the transition to the “constant” regions
  - Monte Carlo studies (compare hadronization model predictions with those from shape functions)

Backup

SD jet mass





Casimir scaling in SD jet mass at LL

$$\Sigma_g(m) = \Sigma_q(m)^{\frac{C_A}{C_F}} \qquad p_j(m) = \frac{d\Sigma_j(m)}{dm}$$

$$\kappa_{qg}^{\text{Cas.}} = \min_m \frac{p_q(m)}{p_g(m)} = \min_m \frac{\frac{d\Sigma_q}{dm}}{\frac{C_A}{C_F}\Sigma_q^{C_A/C_F-1}\frac{d\Sigma_q}{dm}} = \frac{C_F}{C_A}\min_m \Sigma_q^{1-C_A/C_F} = \frac{C_F}{C_A},$$

$$\kappa_{gq}^{\text{Cas.}} = \min_m \frac{p_g(m)}{p_q(m)} = \min_m \frac{\frac{C_A}{C_F}\Sigma_q^{C_A/C_F-1}\frac{d\Sigma_q}{dm}}{\frac{d\Sigma_q}{dm}} = \frac{C_A}{C_F}\min_m \Sigma_q^{C_A/C_F-1} = 0,$$

Backup: SD Jet Mass

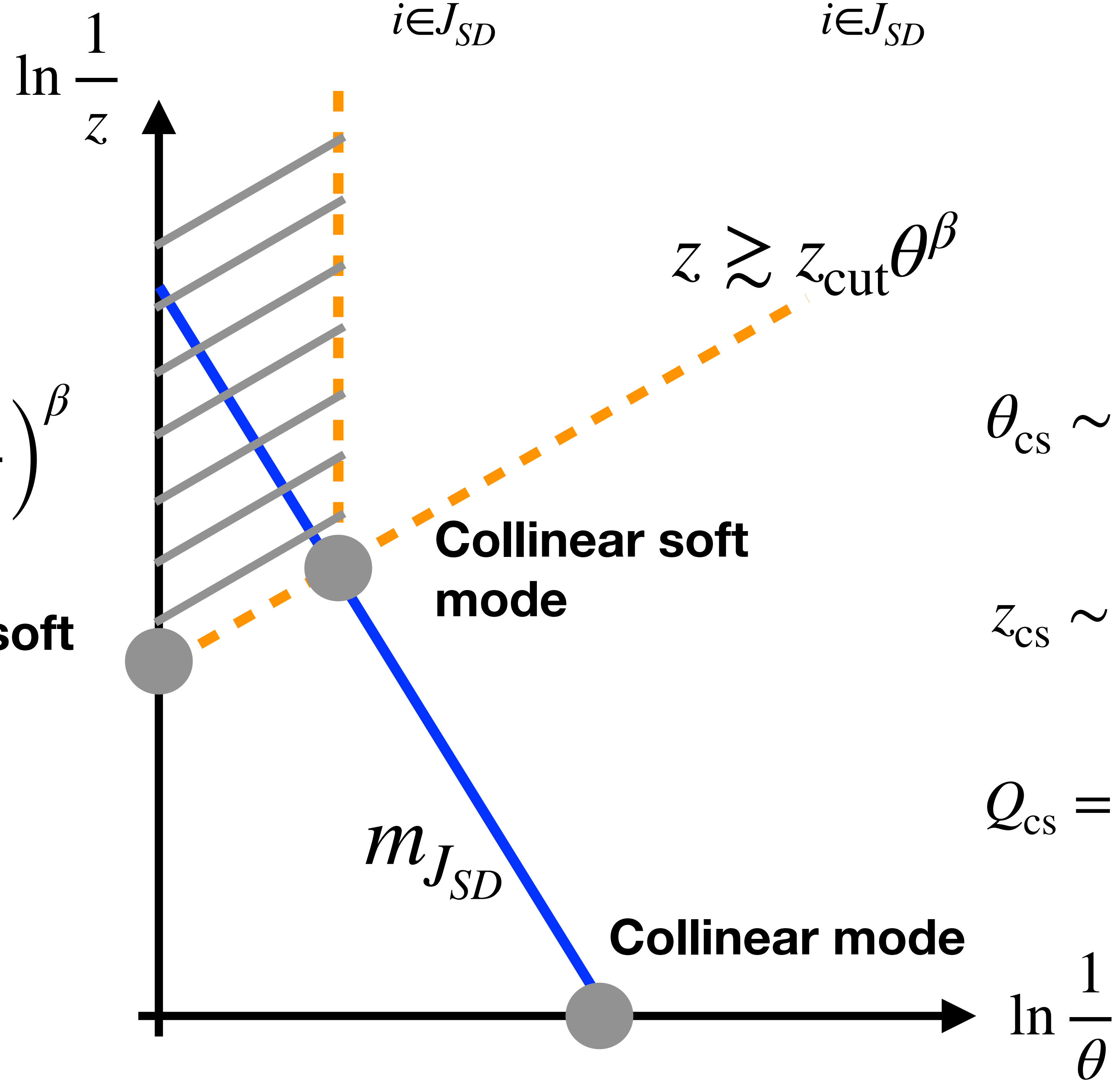
- **Jet mass in SD:** $m_{J_{SD}}^2 = \Big(\sum_{i \in J_{SD}} p_i^\mu\Big)^2 = P_J^- \Big(\sum_{i \in J_{SD}} p_i^+\Big)$

To contribute, must pass SD criterion

$$\theta_{\rm gs} = \frac{R}{\cosh \eta_J}$$

$$z_{\rm gs} = \tilde{z}_{\rm cut}\left(\frac{R}{\cosh \eta_J}\right)^\beta$$

$$Q_{\rm gs} = p_T R z_{\rm gs}$$



$$\theta_{\rm cs} \sim \left(\frac{m_{J_{SD}}^2}{QQ_{\rm cut}}\right)^{\frac{1}{2+\beta}}$$

$$z_{\rm cs} \sim \frac{m_{J_{SD}}^2}{Q^2}\frac{1}{\theta_{\rm cs}^2}$$

$$Q_{\rm cs} = \left(\frac{m_{J_{\rm SD}}^2}{Q}\right)^{\frac{1+\beta}{2+\beta}} Q_{\rm cut}^{\frac{1}{2+\beta}}$$

$$Q_{\rm cut} = Q\tilde{z}_{\rm cut}$$

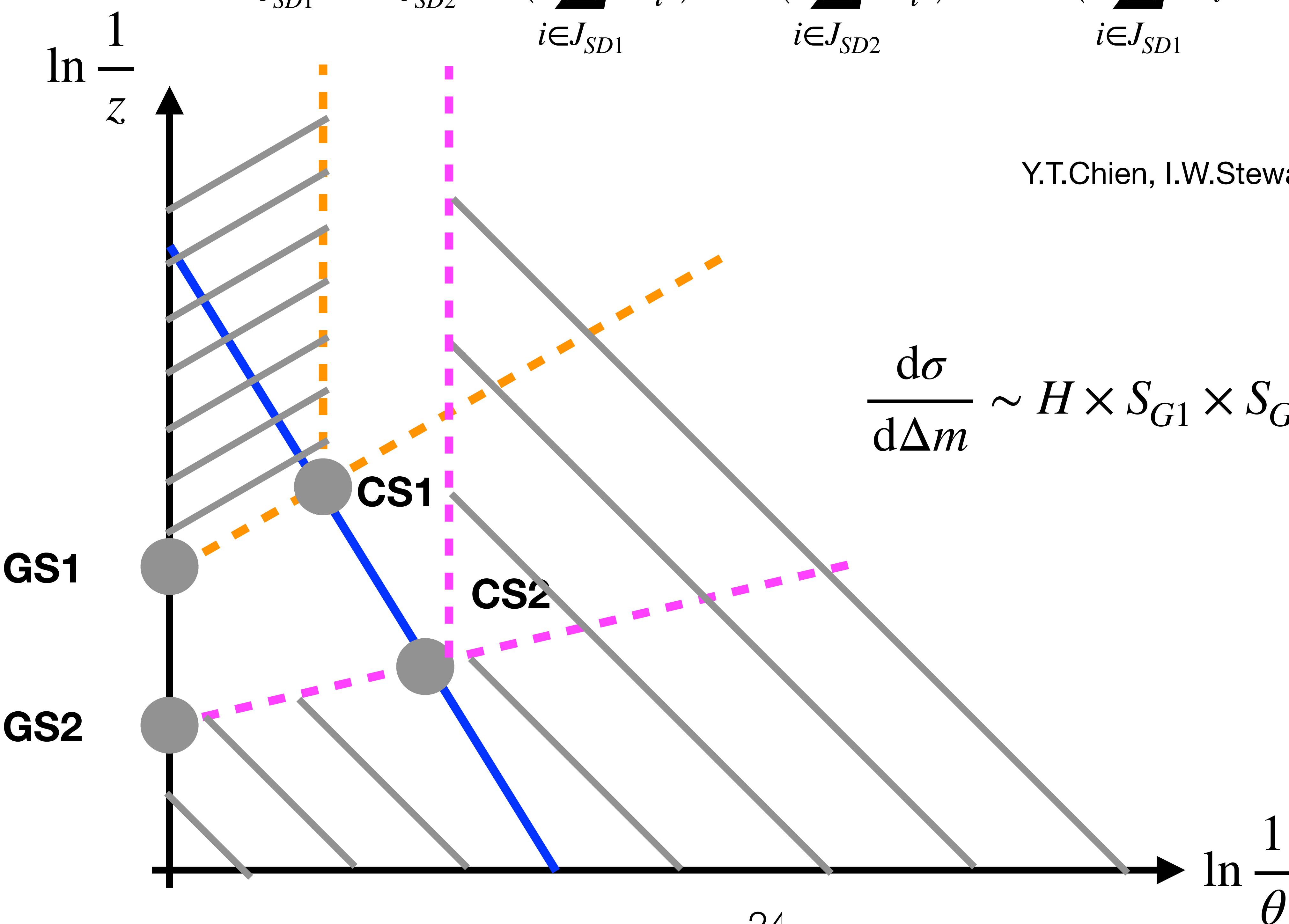
$$\frac{d\sigma}{dm_{J_{SD}}} \sim H \times S_G \times J \otimes S_C$$

C.Frye,A.J.Larkoski,M.D.Schwartz, K.Yan arXiv:1603.09338

Backup: CD Jet Mass

- **Jet mass in CD: CD defined from two SD's, second one more aggressive**

$$\Delta m^2 = m^2_{J_{SD1}} - m^2_{J_{SD2}} = \Big(\sum_{i\in J_{SD1}} p_i^\mu\Big)^2 - \Big(\sum_{i\in J_{SD2}} p_i^\mu\Big)^2 = Q\Big(\sum_{i\in J_{SD1}} p_i^+ - \sum_{i\in J_{SD2}} p_i^+\Big)$$



Y.T.Chien, I.W.Stewart, arXiv:1907.11107

$$\frac{d\sigma}{d\Delta m} \sim H \times S_{G1} \times S_{G2} \times S_{C1} \otimes S_{C2}$$

Backup: Global Soft Functions

- **Global soft functions at one loop in MSbar**

$$S_{G_j}^{\mathrm{ren}}(Q_{\mathrm{gs1}}, \beta_1, \mu) = 1 + \frac{\alpha_s(\mu) C_j}{\pi(1+\beta_1)} \left(\ln^2 \frac{\mu}{Q_{\mathrm{gs1}}} - \frac{\pi^2}{24} \right)$$

$$S_{\overline{G}_j}^{\mathrm{ren}}(Q_{\mathrm{gs2}}, \beta_2, \mu) = 1 - \frac{\alpha_s(\mu) C_j}{\pi(1+\beta_2)} \left(\ln^2 \frac{\mu}{Q_{\mathrm{gs2}}} - \frac{\pi^2}{24} \right)$$

- **RGE of global soft functions**

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} \ln S_{G_j}^{\mathrm{ren}}(Q_{\mathrm{gs1}}, \beta_1, \mu) = \frac{2C_j}{1+\beta_1} \Gamma_{\mathrm{cusp}} \ln \frac{\mu}{Q_{\mathrm{gs1}}} + \gamma_{S_{G_j}}$$

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} \ln S_{\overline{G}_j}^{\mathrm{ren}}(Q_{\mathrm{gs2}}, \beta_2, \mu) = -\frac{2C_j}{1+\beta_2} \Gamma_{\mathrm{cusp}} \ln \frac{\mu}{Q_{\mathrm{gs2}}} - \gamma_{\overline{S}_{G_j}}$$

- **At one loop, non-cusp anomalous dimensions = 0**

Backup: Collinear Soft Functions

- **Collinear soft functions in position space at one loop in MSbar**

$$\widetilde{S}^{\text{ren}}_{C_j}\left(yQQ_{\text{cut}1}^{\frac{-1}{1+\beta_1}},\beta_1,\mu\right) = 1 + \frac{\alpha_s C_j}{\pi}\frac{2+\beta_1}{1+\beta_1}\left(-\ln^2\frac{\mu\, y^{\frac{1+\beta_1}{2+\beta_1}}Q^{\frac{1+\beta_1}{2+\beta_1}}}{Q_{\text{cut}1}^{\frac{1}{2+\beta_1}}} + \frac{\pi^2}{24}\right)$$

$$\widetilde{S}^{\text{ren}}_{\overline{C}_j}\left(yQQ_{\text{cut}2}^{\frac{-1}{1+\beta_2}},\beta_2,\mu\right) = 1 - \frac{\alpha_s C_j}{\pi}\frac{2+\beta_2}{1+\beta_2}\left(-\ln^2\frac{\mu\, y^{\frac{1+\beta_2}{2+\beta_2}}Q^{\frac{1+\beta_2}{2+\beta_2}}}{Q_{\text{cut}2}^{\frac{1}{2+\beta_2}}} + \frac{\pi^2}{24}\right)$$

Laplace transform

$$\tilde{f}(y) = \int_0^\infty \mathrm{d}x\, \exp\left(-ye^{-\gamma_E}\Delta m^2\right) f(\Delta m^2)$$

- **RGE of collinear soft functions in position space**

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\ln\widetilde{S}^{\text{ren}}_{C_j}\left(yQQ_{\text{cut}1}^{\frac{-1}{1+\beta_1}},\beta_1,\mu\right) = 2C_j\Gamma_{\text{cusp}}(\alpha_s)\ln\frac{Q_{\text{cut}1}^{\frac{1}{1+\beta_1}}}{\mu^{\frac{2+\beta_1}{1+\beta_1}}Qy} + \gamma_{S_{C_j}}$$

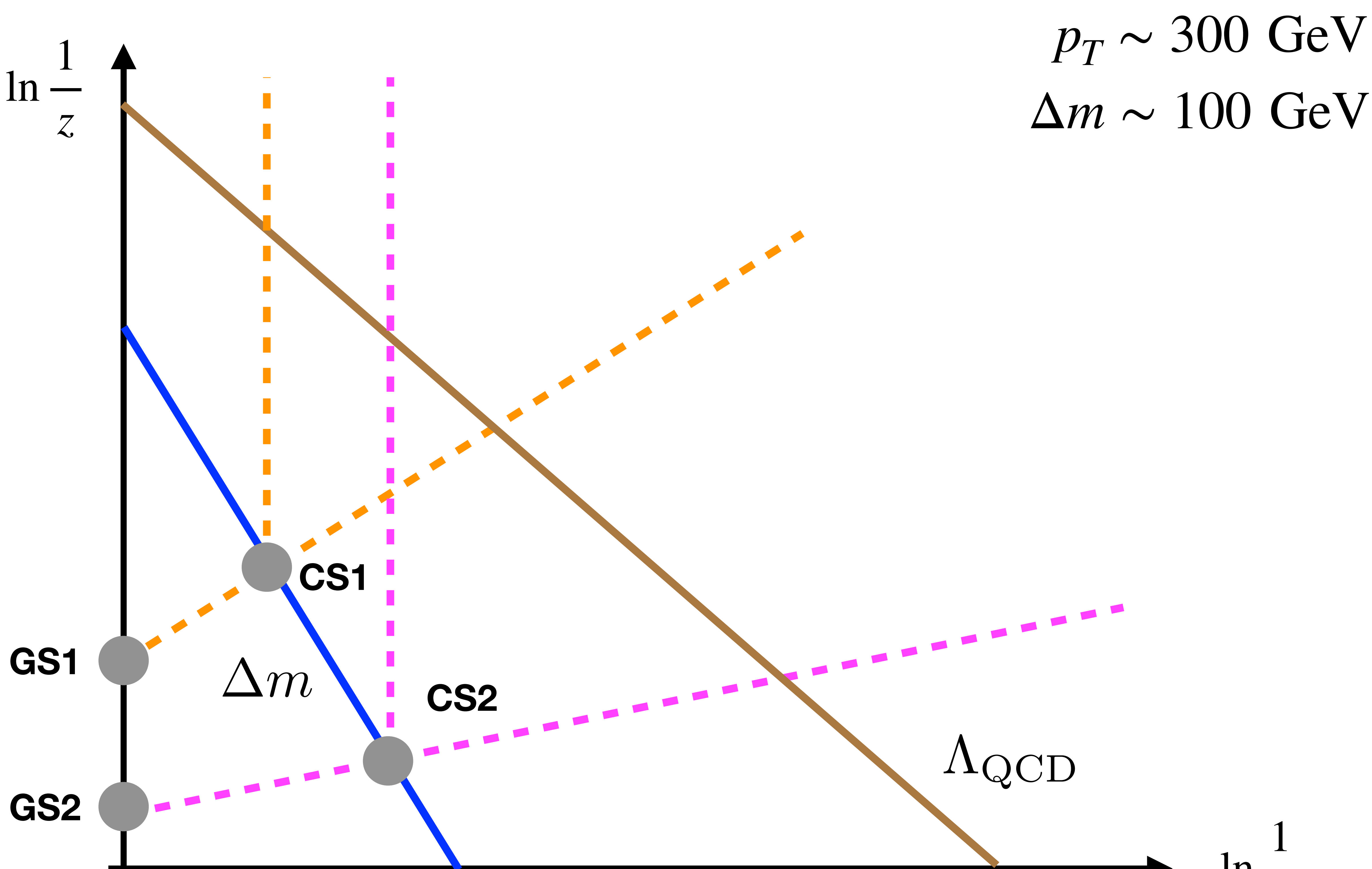
$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\ln\widetilde{S}^{\text{ren}}_{\overline{C}_j}\left(yQQ_{\text{cut}2}^{\frac{-1}{1+\beta_2}},\beta_2,\mu\right) = -2C_j\Gamma_{\text{cusp}}(\alpha_s)\ln\frac{Q_{\text{cut}2}^{\frac{1}{1+\beta_2}}}{\mu^{\frac{2+\beta_2}{1+\beta_2}}Qy} - \gamma_{S_{\overline{C}_j}}$$

- **At one loop, non-cusp anomalous dimensions = 0**

$$K(\mu_1,\mu_2) = \int_{\alpha_s(\mu_1)}^{\alpha_s(\mu_2)}\mathrm{d}\alpha\frac{\Gamma_{\text{cusp}}(\alpha)}{\beta(\alpha)}\int_{\alpha_s(\mu_1)}^{\alpha}\frac{\mathrm{d}\alpha'}{\beta(\alpha')} \qquad \omega(\mu_1,\mu_2) = \int_{\alpha_s(\mu_1)}^{\alpha_s(\mu_2)}\mathrm{d}\alpha\frac{\Gamma_{\text{cusp}}(\alpha)}{\beta(\alpha)}$$

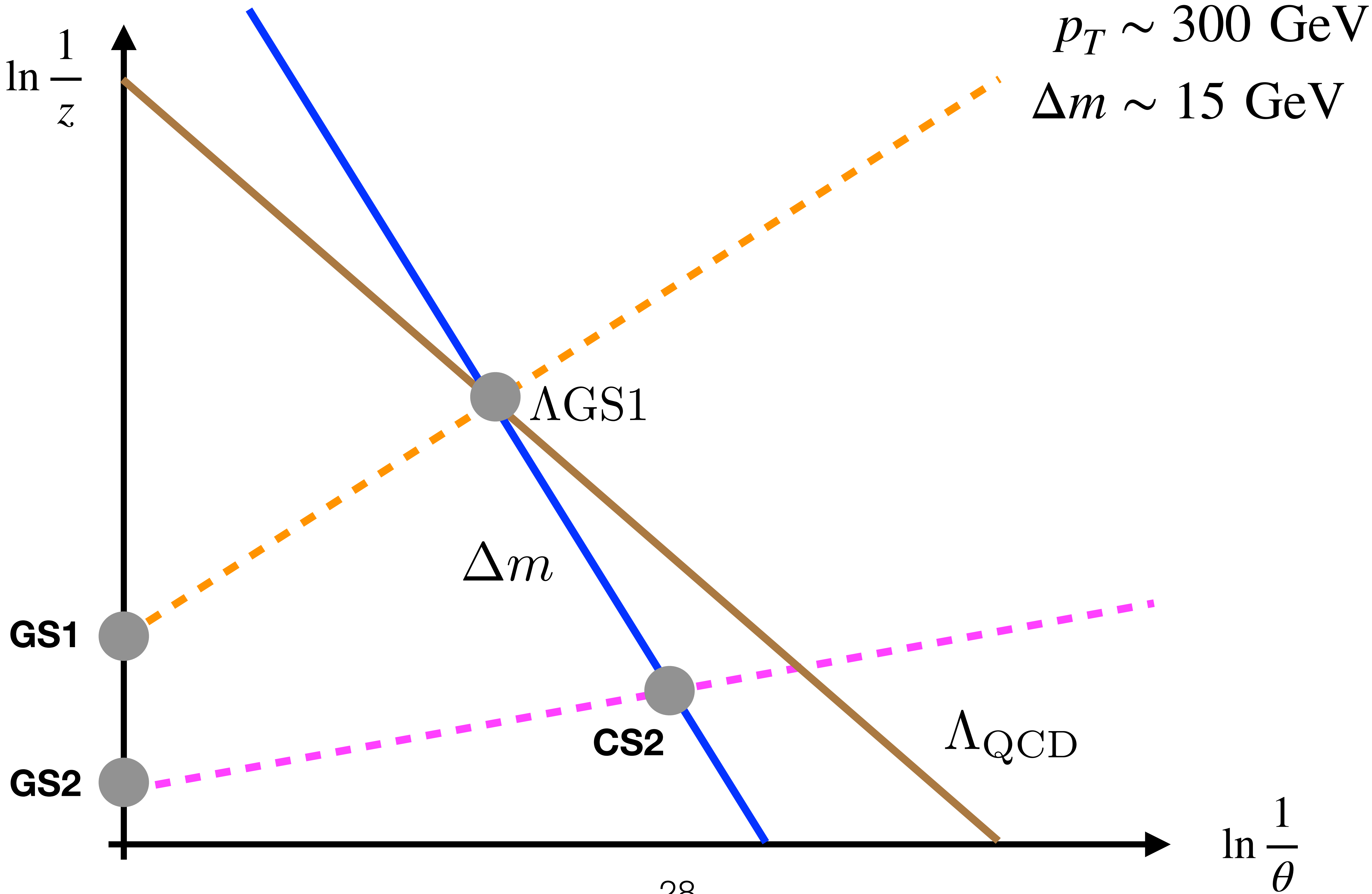
Backup: CD Jet Mass in Perturbative Regime

- **So far, we only consider perturbative case**



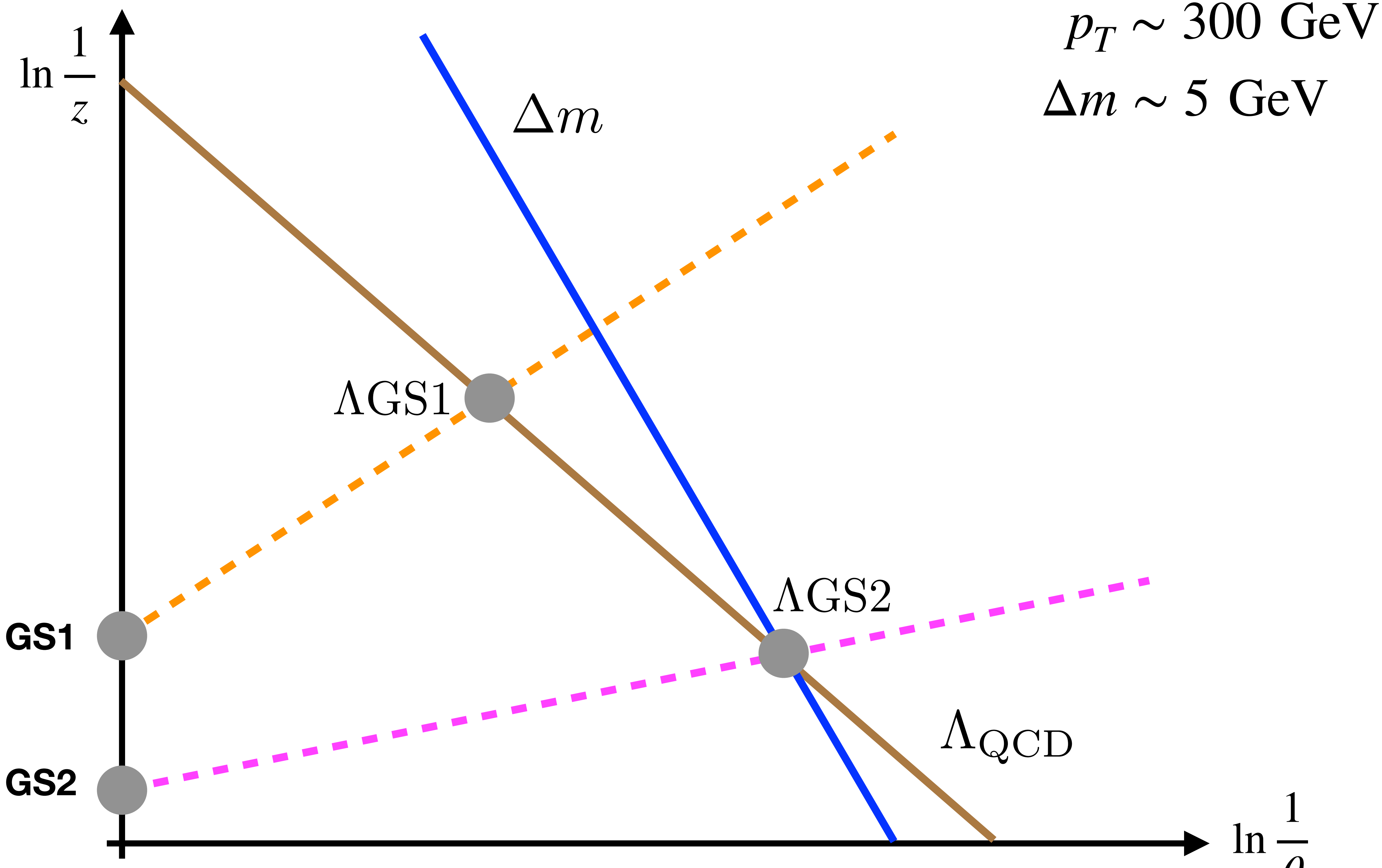
Backup: CD Jet Mass in Perturbative Regime

- **Case when CS1 mode becomes nonperturbative**



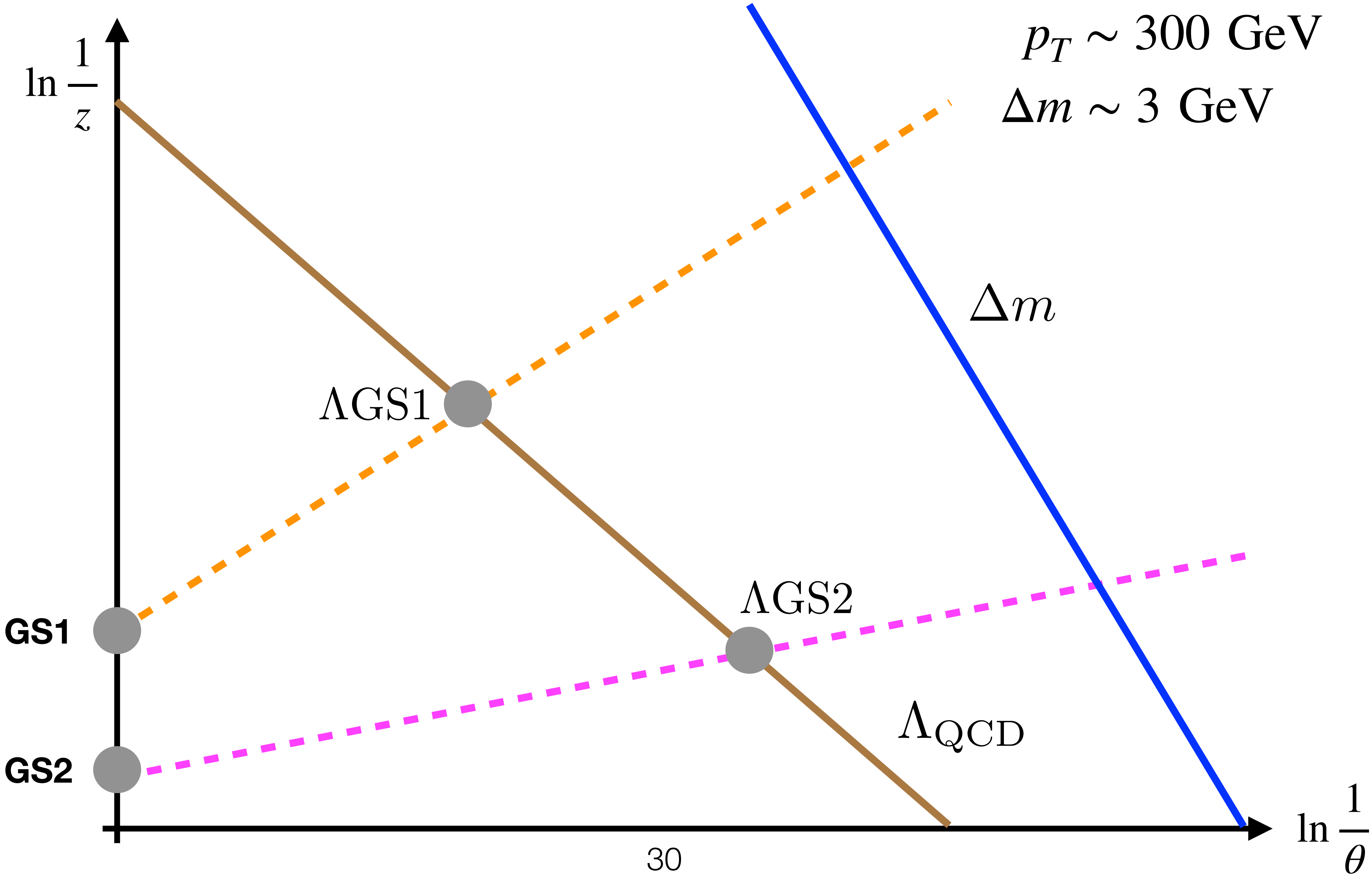
Backup: CD Jet Mass in Perturbative Regime

- **Case when CS2 mode becomes nonperturbative**



Backup: CD Jet Mass in Perturbative Regime

- **Case in deep nonperturbative regime**



Backup: Model of Shape Function

- **Models of shape function**

$$F_i^j(k_i, \beta_i) = \frac{1}{\Lambda}\left(\sum_{n=0}^{\infty} c_n^j(\beta_i) f_n(x, p)\right)^2 \qquad x = \frac{k_i}{\Lambda} \qquad \Lambda \sim \Lambda_{\mathrm{QCD}}$$

Basis function

$$f_n(x, p) = \sqrt{(2n+1)Y(x, p)}\, P_n\big(y(x)\big)$$

$$y(x, p) = -1 + 2\int_0^x \mathrm{d}x'\, Y(x', p)$$

$$Y(x, p) = \frac{(p+1)^{p+1}}{\Gamma(p+1)} x^p e^{-(p+1)x}$$

Legendre polynomial

Z.Ligeti, I.W.Stewart, F.J.Tackmann arXiv:0807.1926