A Perturbative dS S-matrix for massive scalar fields

Donald Marolf, UCSB
4/25/2013

Work with
Ian Morrison and Mark Srednicki

See arxiv: **1209.6039**
and also **1010.5327**, **1104.4343**, **1104.4343**,

Motivations:

1) We can do it!

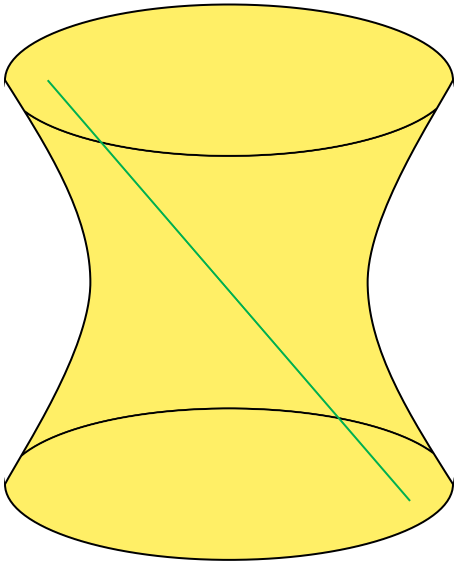
2) Better understand dS QFT. Develop gauge-invariant technology to study gravitons in the IR.

3) Address long-standing concerns about large IR effects & possible instabilities in both massive scalar QFTs (e.g. Nachtmann, Meyrvold, Polyakov) and perturbative gravity (e.g. Starobinski, Tsamis &Woodard, Giddings & Sloth).

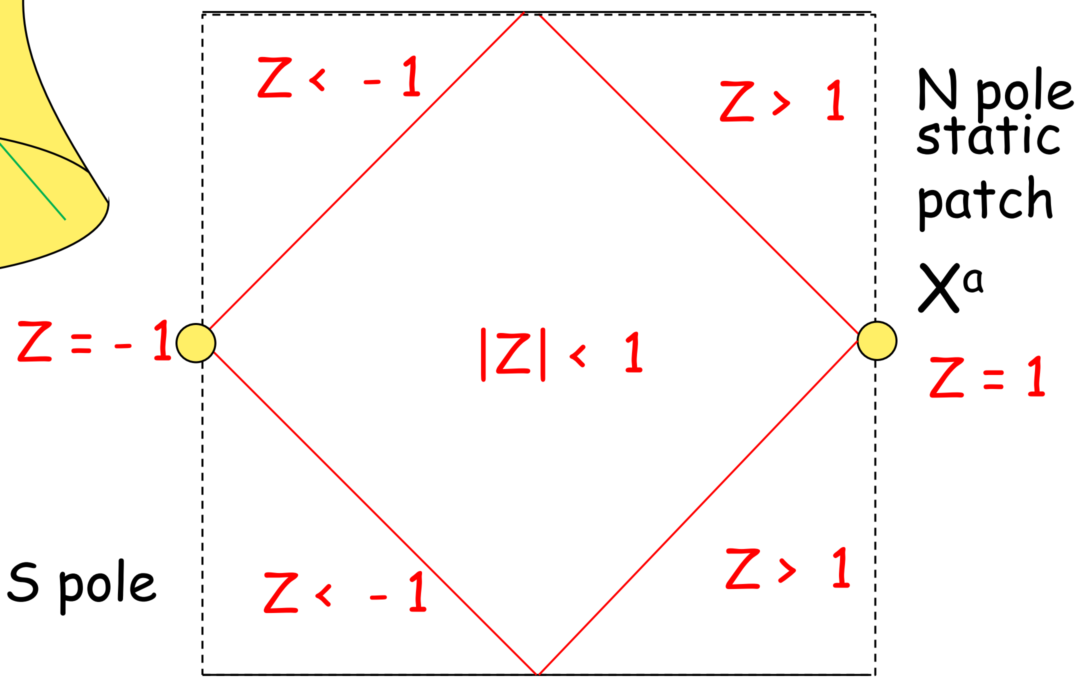
4) Define an observable for stringy models of dS, and perhaps for cosmology more generally.

This talk: massive scalar fields only.
Assume minimal coupling.

I. Free fields in dS



Given X^a, Y^a in $M^{d,1}$,
define $Z = X^a Y^b \eta_{ab} / l^2$.



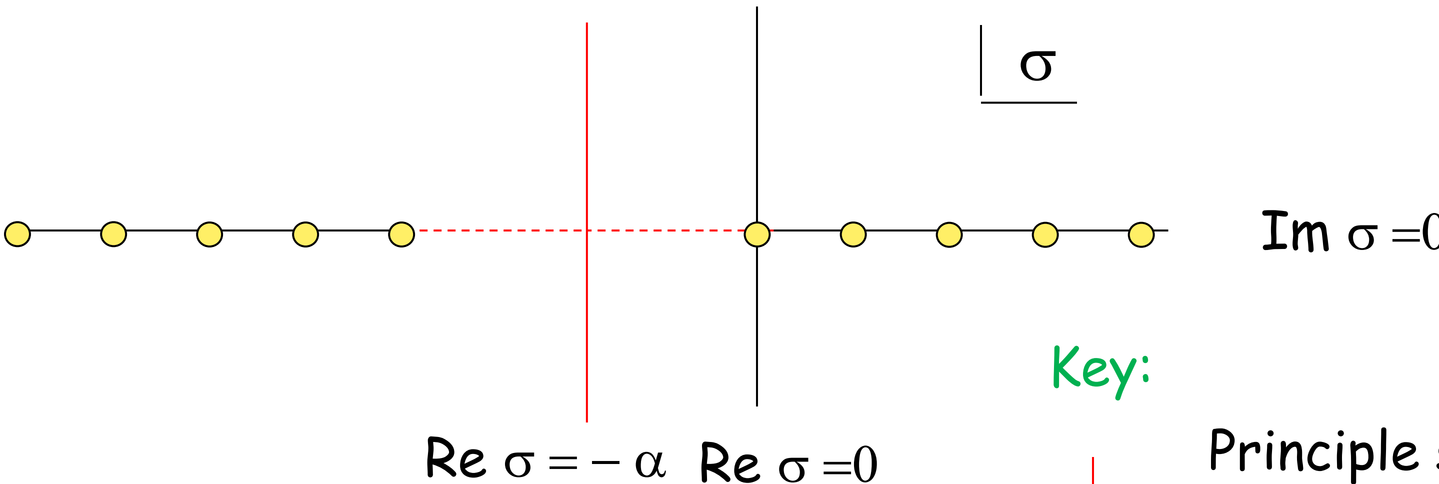
Makes 2 pt functions easy.
Also useful for higher n-pt functions.

Free fields and 2-pt functions

$|Z| \gg 1$: $\Delta_\sigma(Z) \sim A_\sigma (-Z)^\sigma + B_\sigma (-Z)^{-\sigma-(d-1)}$

$m^2 l^2 = -\sigma(\sigma + d-1)$

$$\sigma_\pm = -\frac{d-1}{2} \pm \sqrt{\left(\frac{d-1}{2}\right)^2 - m^2 l^2}$$



Key:

Principle series, $m^2 l^2 > [(d-1)/2]^2$.

Complimentary series, $[(d-1)/2]^2 \geq m^2 l^2 > 0$

Analytic continuation of $SO(d+1)$, $m^2 \leq 0$ @ integer σ_+ (& images)

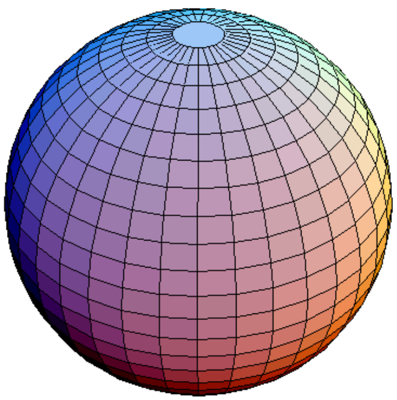
II. Interacting fields

Naïve S-matrix or in/out correlators are IR divergent

No big deal… compute in/in correlators as in thermal field theory

E.g., by analytic continuation* from Euclidean perturbation theory

⇨ interacting HH state (manifestly free of IR divergences).



Key Question:

Can we make IR decay of Lorentz-signature correlators manifest as well?

*Agrees w/Lorentzian Schwinger-Keldysh (aka in/in) for massive scalars if "t=0" is the dS horizon and the initial state is the Bunch-Davies (aka free Euclidean or free HH vacuum).

dS Polology I

Analytic continuation is nicer in momentum space.

But momentum space is discrete on Euclidean dS (S^d).

Watson-Sommerfeld transformations!

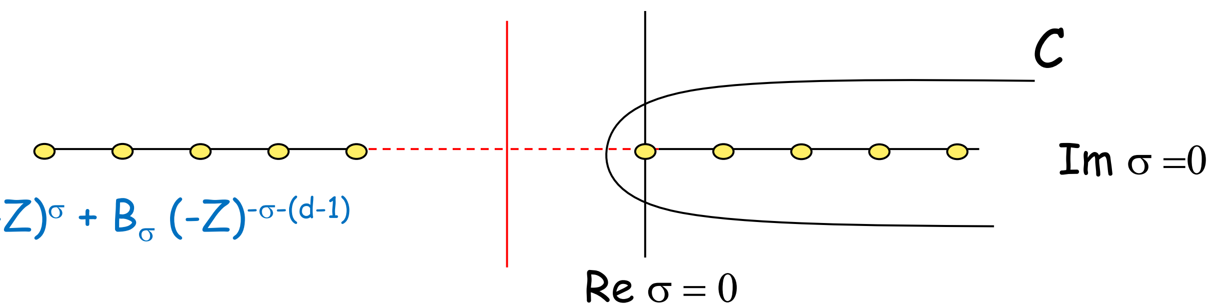
E.g., compute interacting 2-pt function on S^d as an expansion in spherical harmonics:

$$\Delta_{int}(Z) = \sum_{L=1}^{\infty} F_L C_L^{(d-1)/2}(-Z)$$

And recall free 2pt functions:

$$\Delta_\sigma(Z) = -\ell^{2-d} \frac{\Gamma(d/2)}{(4\pi)^{(d-1)/2}\sin(\pi\sigma)} C_\sigma^{(d-1)/2}(-Z)$$

$\Delta_\sigma(Z) \sim A_\sigma (-Z)^\sigma + B_\sigma (-Z)^{-\sigma-(d-1)}$



A Källen-Lehmann-like representation in the complex σ-plane!

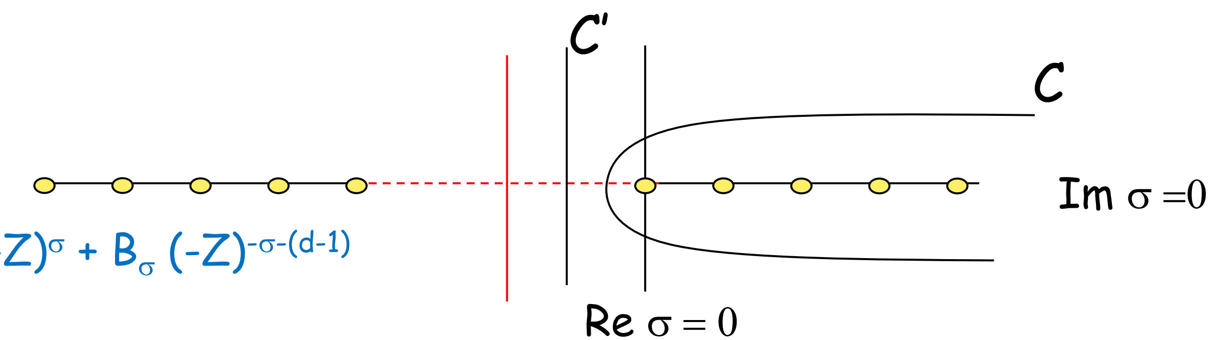
$$\Delta_{int}(Z) = -\ell^{d-2} \frac{(4\pi)^{d/2}}{\Gamma(d/2)} \int_C F_\sigma \Delta_\sigma(Z)$$

dS Polology II

$$\Delta_{int}(Z) = -\ell^{d-2}\frac{(4\pi)^{d/2}}{\Gamma(d/2)}\int_C F_\sigma \Delta_\sigma(Z)$$

$$\Delta_\sigma(Z) = -\ell^{2-d}\frac{\Gamma(d/2)}{(4\pi)^{(d-1)/2}\sin(\pi\sigma)} C_\sigma^{(d-1)/2}(-Z)$$

Decay at large |Z|
set by Re σ;
move left for faster decay.



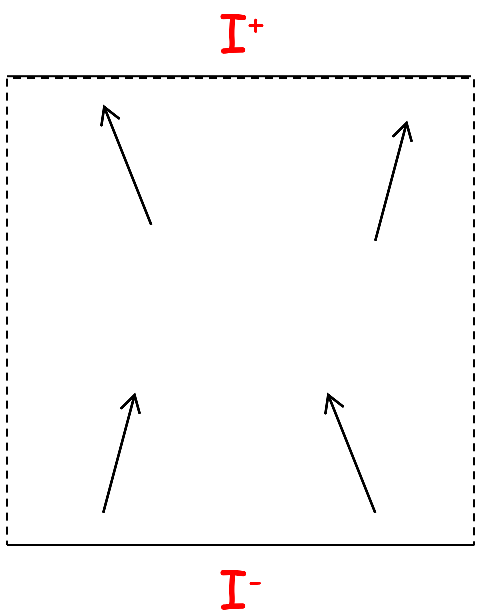
$\Delta_\sigma(Z) \sim A_\sigma\,(-Z)^\sigma + B_\sigma\,(-Z)^{-\sigma-(d-1)}$

So, straighten C to C' and deform to the left…

If good convergence along C', large |Z| behavior controlled by right-most singularity (poles!) in F_σ.

Carlson's Thm: Analytic extension from positive integers is unique if rapidly decreasing at large Im σ.

III. A global dS S-matrix



Analytic structure above is similar to flat-space S-matrix.

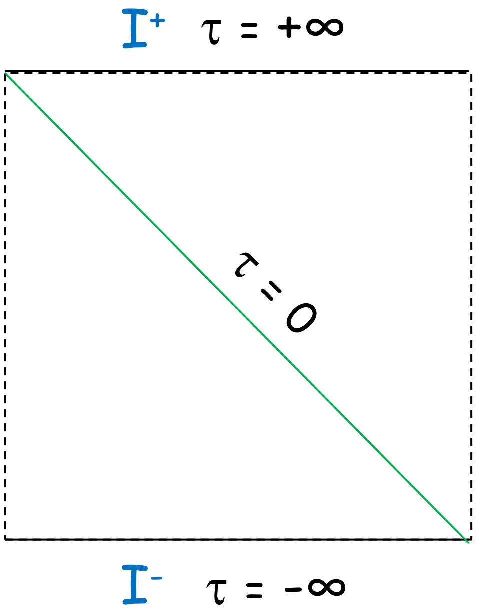
So let's use residues at poles to *define* a dS S-matrix.

Unitary, invariant under field redefinitions, and (sufficiently) finite.

- Correct flat-space limit as $\ell \to \infty$.
- Useful for porting over results from flat space & understanding the implications of unitarity. (dS Optical theorem!)

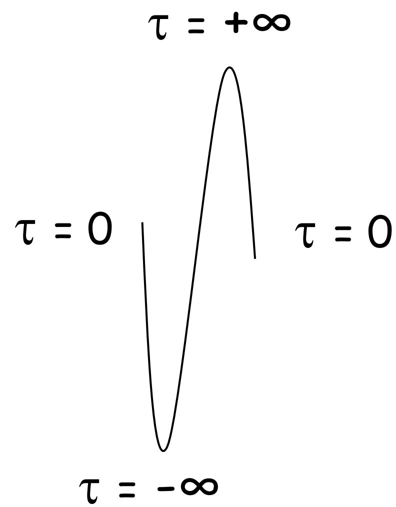
Does not refer to experiments causally accessible to a single observer.
Relates "asymptotic particle states" on I^- to those on I^+.

III. A global dS S-matrix



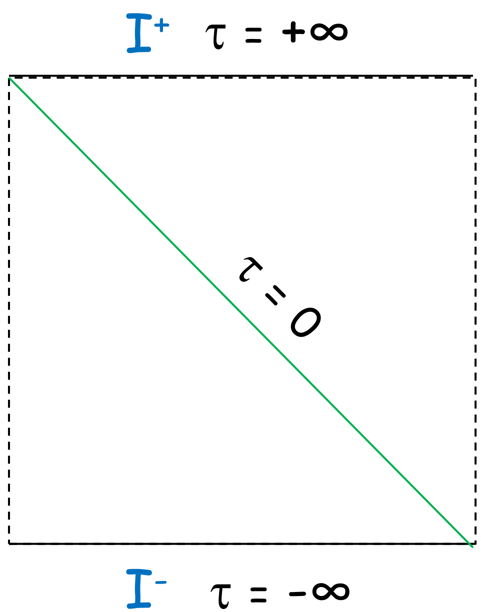
- In/out formalism is IR divergent

Fix by using out/out-in/in
"double closed time path" path integral



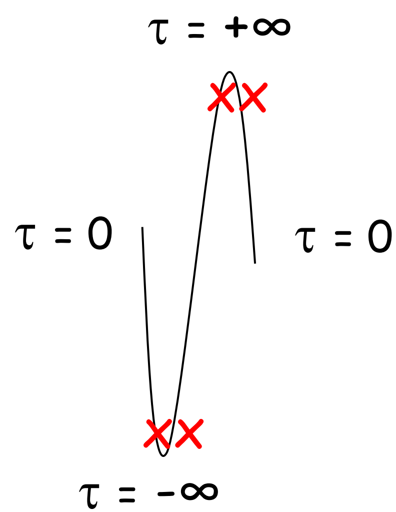
Equivalently: Deform integral over Euclidean S^d to pass near $\tau = \pm\infty$

III. A global dS S-matrix



- In/out formalism is IR divergent

Fix by using out/out-in/in
"double closed time path" path integral

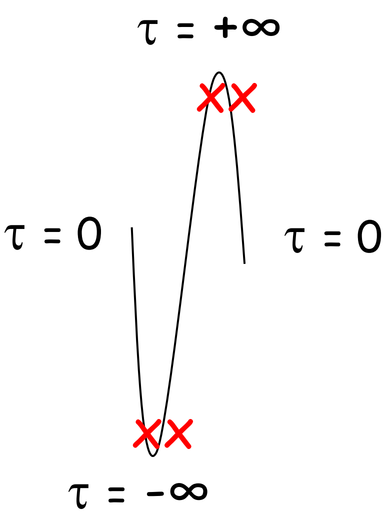


- Insert operators near $\tau = \pm\infty$.

- Define S-matrix by picking out pole-terms and using Klein-Gordon inner product with free field modes to label states. (Requires some technology, esp. for light fields.)

- Note: vacuum $|\Omega\rangle$ is interacting $|HH\rangle$.

Features



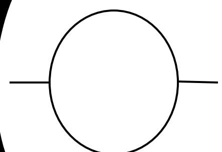
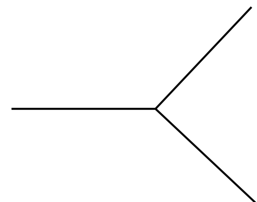
- By construction, $S|\Omega\rangle_i = |\Omega\rangle_f$
- But usual asymptotic states not orthogonal
  E.g., while ${}_i\langle\Omega|1\rangle_i = 0 = {}_i\langle\Omega|2\rangle_i$,
  for ϕ^3 ${}_i\langle\Omega|3\rangle_i \neq 0$.
- Generic (all?) particles decay, even for small masses. I.e. $S|1\rangle_i = \Sigma|n\rangle_f$
  Boyanovsky vs. Bros, Epstein, et al.
- Finite at tree level but,
  as with unstable particles in flat space,
  loops can give IR divergences (at least in the norms of asymptotic states).

These divergences are associated with self-energy corrections.

IV. Optical theorem

$$S = 1 - iT;\; S^\dagger S = 1$$

$$\longrightarrow \quad 2\,\mathrm{Im}\,T = T^\dagger T$$

2 Re ( + ...) = |  + ... |$^2 \geq 0$

Principle Series: $\propto \mathrm{Im}\,\Pi(\sigma_+) = \frac{1}{2}\,(\,\Pi(\sigma_+) - \Pi(\sigma_-)\,)$

Complementary Series: $\propto \frac{1}{2}\,(\,\Pi(\sigma_+) - \Pi(\sigma_-)\,)$

$$\longrightarrow \quad (\sigma_+ + \sigma_-)_{\text{shifted}} \leq -(d-1)$$

All examples so far: strictly < unless free

Summary

- The IR behavior of in/in dS correlators is controlled by an analytic structure reminiscent of the flat-space S-matrix.

- Leads to a (perturbative) notion of a dS S-matrix with many familiar features (Unitarity, Invariance under field redefinitions, flat-space limit) and some unfamiliar ones (vacuum and multi-particle asymptotic states not orthogonal).

- Can probably also define an S-matrix on just the future (expanding) half of dS.

- To what extent does it define an observable for gauge fields, gravitons, and strings?

  What does it teach us about them?