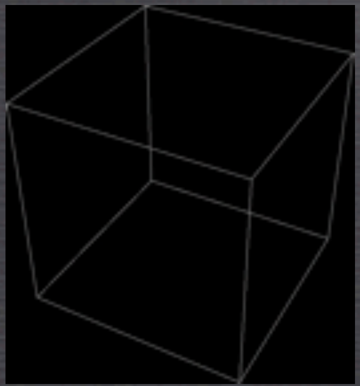
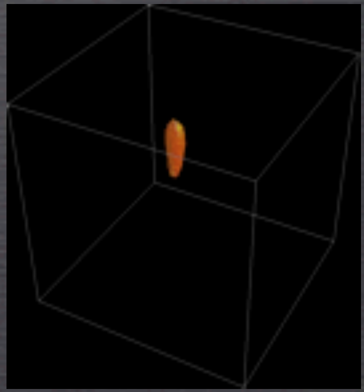
NONLINEAR FIELD DYNAMICS AFTER INFLATION



MUSTAFA AMIN



2013.04.17

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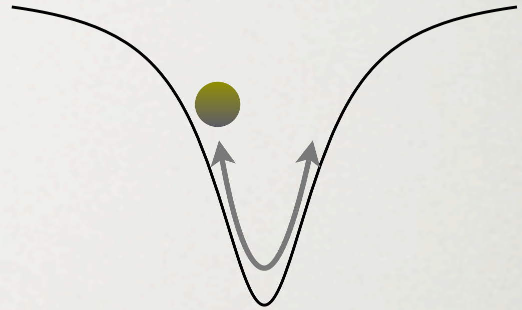
nonlinear scalar field evolution

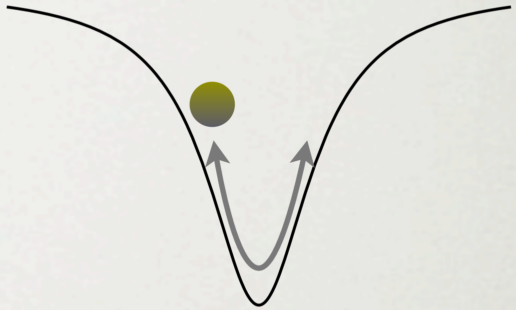
$$\Box\varphi = V'(\varphi)$$



MA (2010)

Also see: *Khlopov, Molamed and Zeldovich (1985)*

nonlinear scalar field evolution



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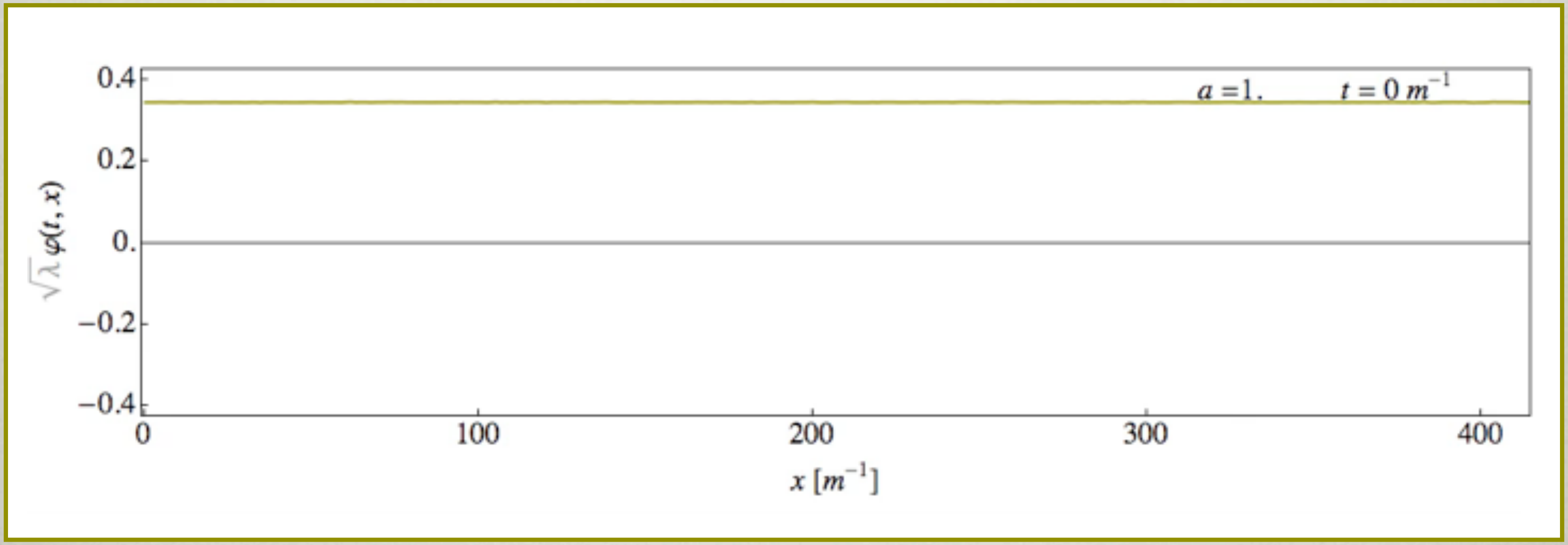

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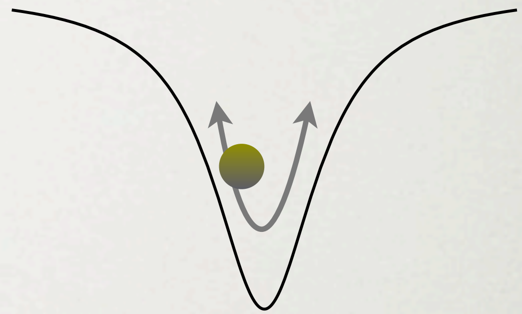

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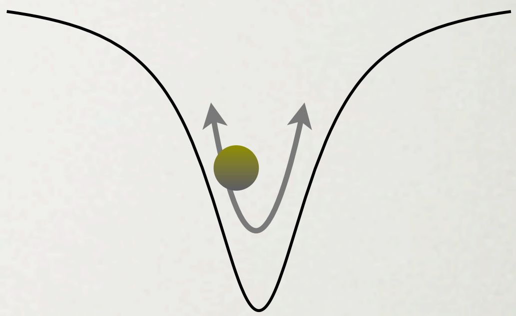
nonlinear scalar field evolution



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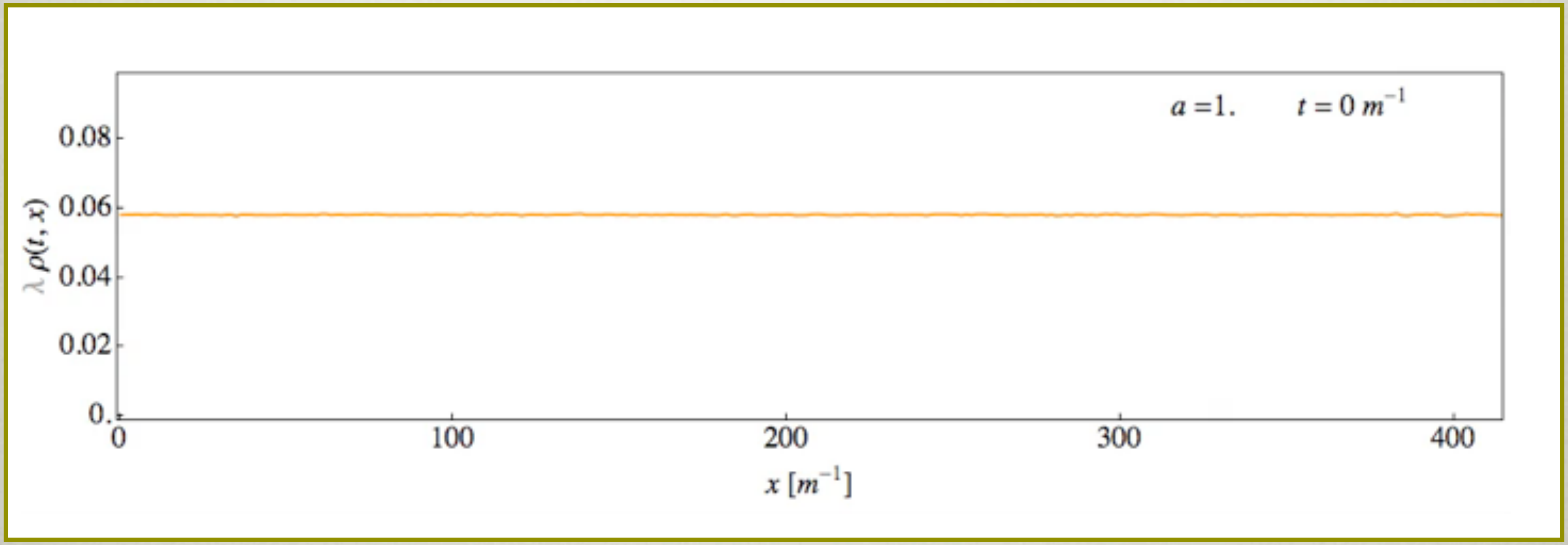
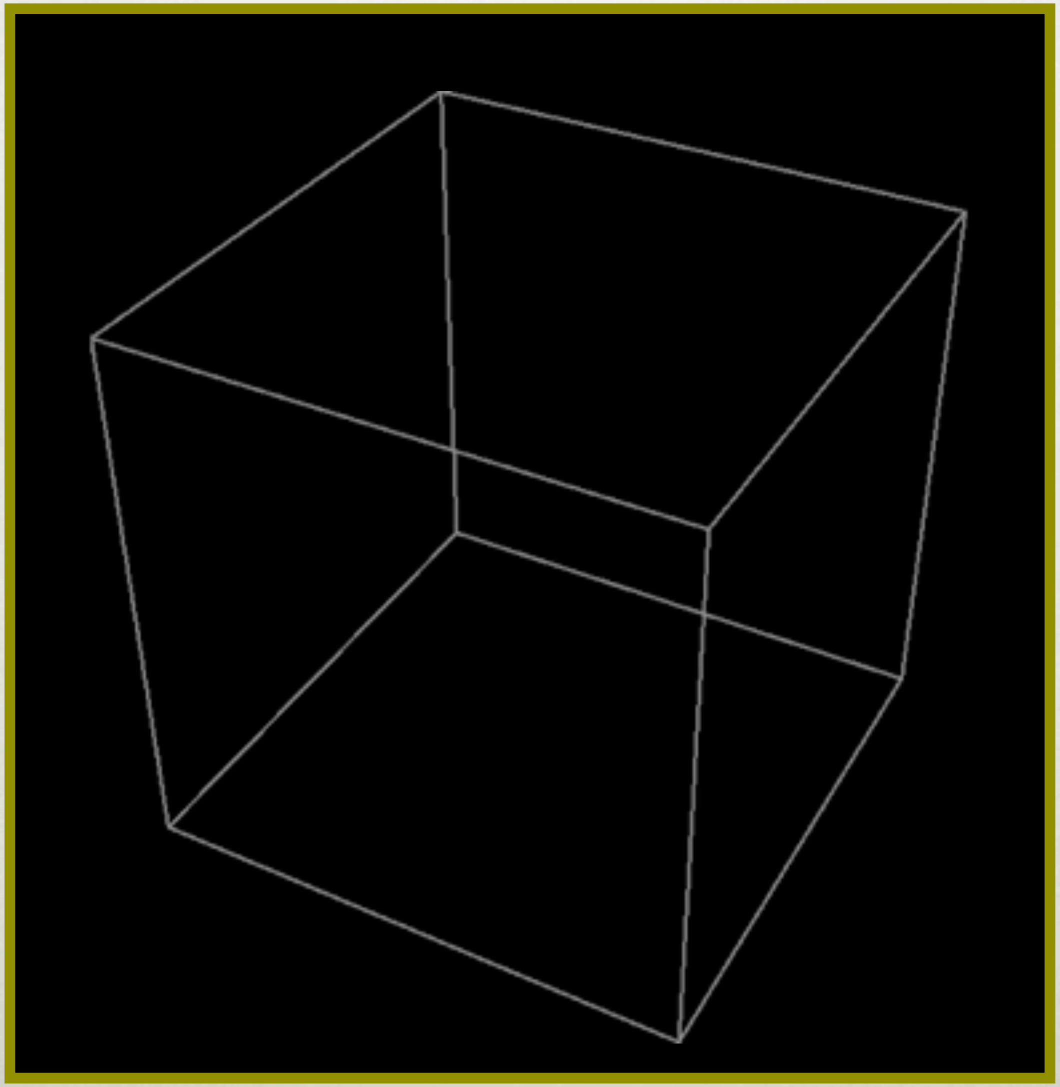


now in 3D

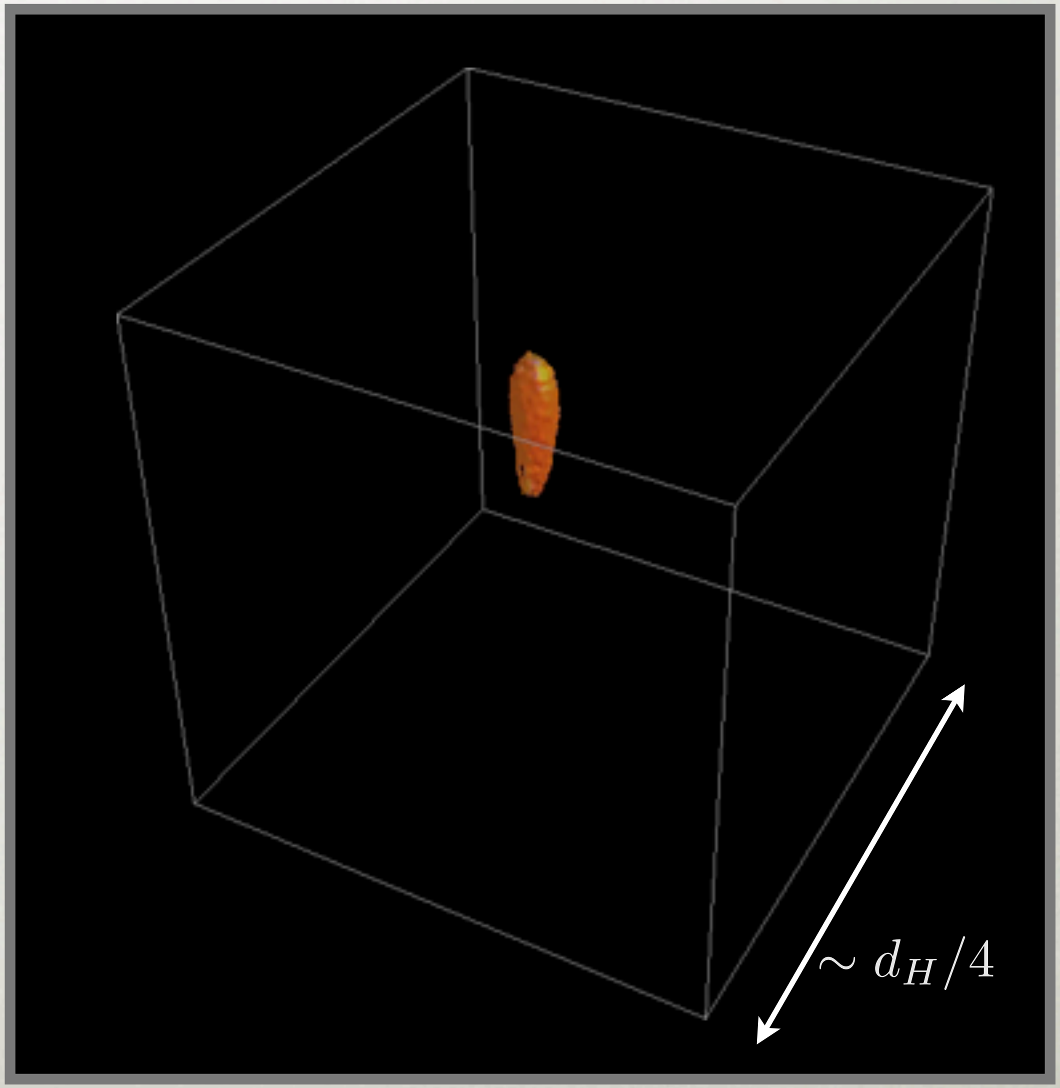


$\sim d_H/4$

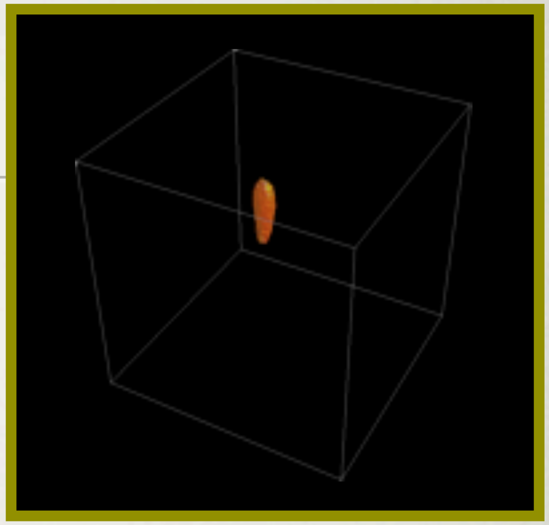
MA, Easther & Finkel (2010)

inflaton fragmentation into oscillons

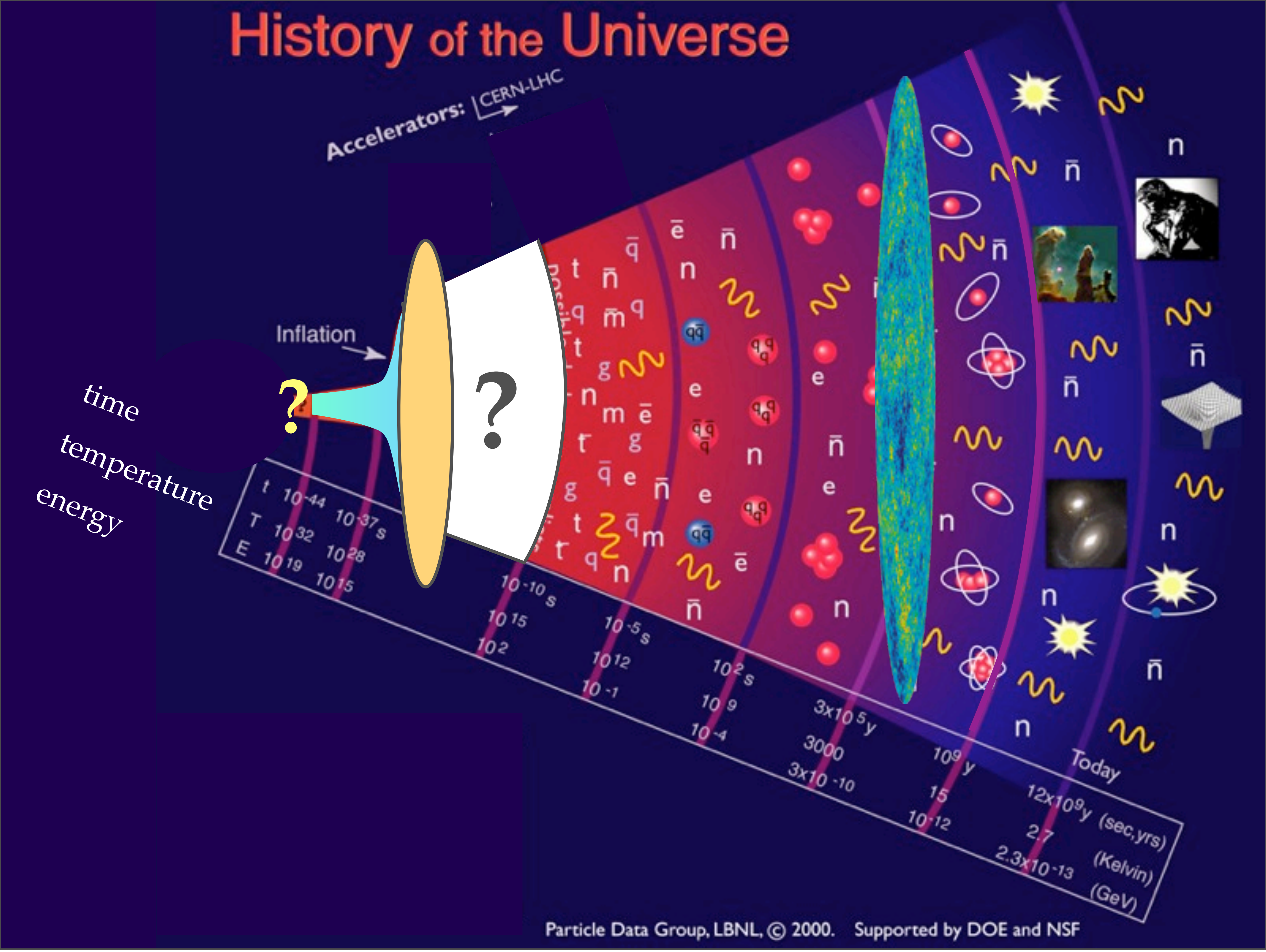
- observationally consistent
- theoretically motivated
- dominate the energy density of the post inflationary universe
- assumption: self couplings dominate over others.



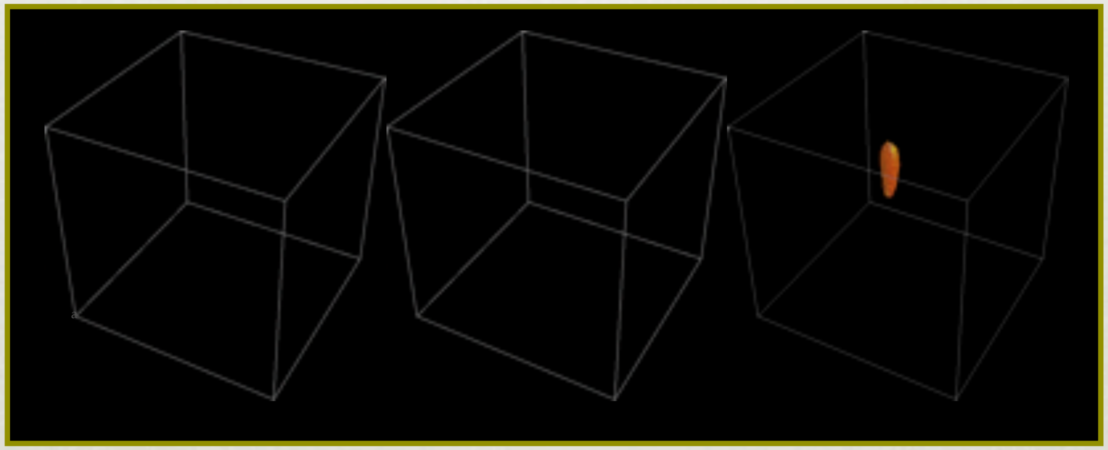
synopsis



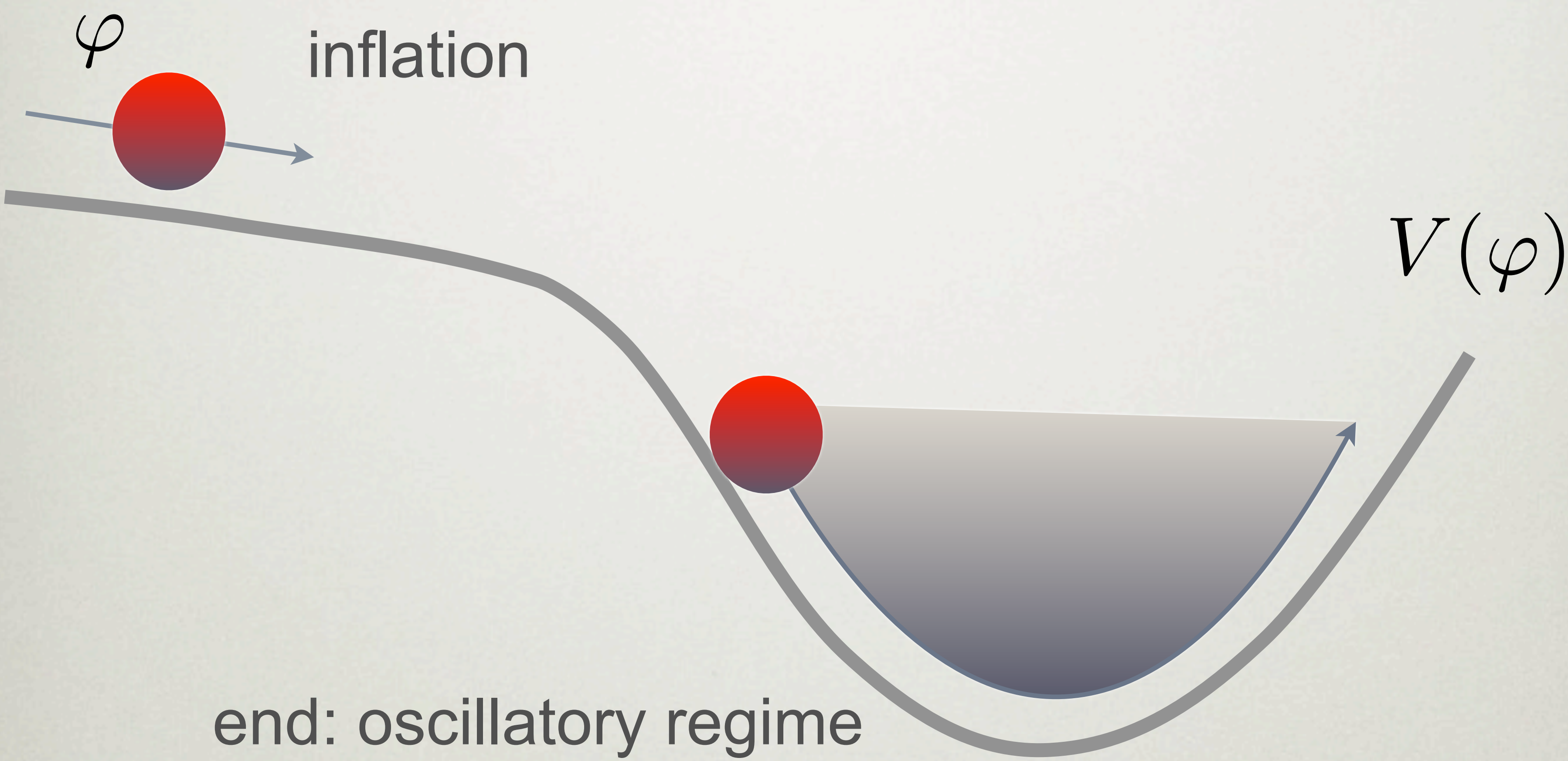
- end of inflation: simple scenarios
- fragmentation ⟶ lumps
  - motivation, emergence and consequences
- details of lumps (*oscillons*)
  - solutions, stability and interactions
- summary


History of the Universe
Accelerators:
CERN-LHC
Inflation
time
temperature
energy
?
?
t 10-44 10-37 s
T 10 32 10 28
E 10 19 10 15
10-10 s
10 15
10 2
10-5 s
10 12
10-1
10 2 s
10 9
10-4
3x10 5 y
3000
3x10-10
10 9 y
15
10-12
Today
12x10 9 y (sec,yrs)
2.7 (Kelvin)
2.3x10-13 (GeV)
Particle Data Group, LBNL, © 2000. Supported by DOE and NSF

what does the universe
look like at the end of inflation?



inflation and its end



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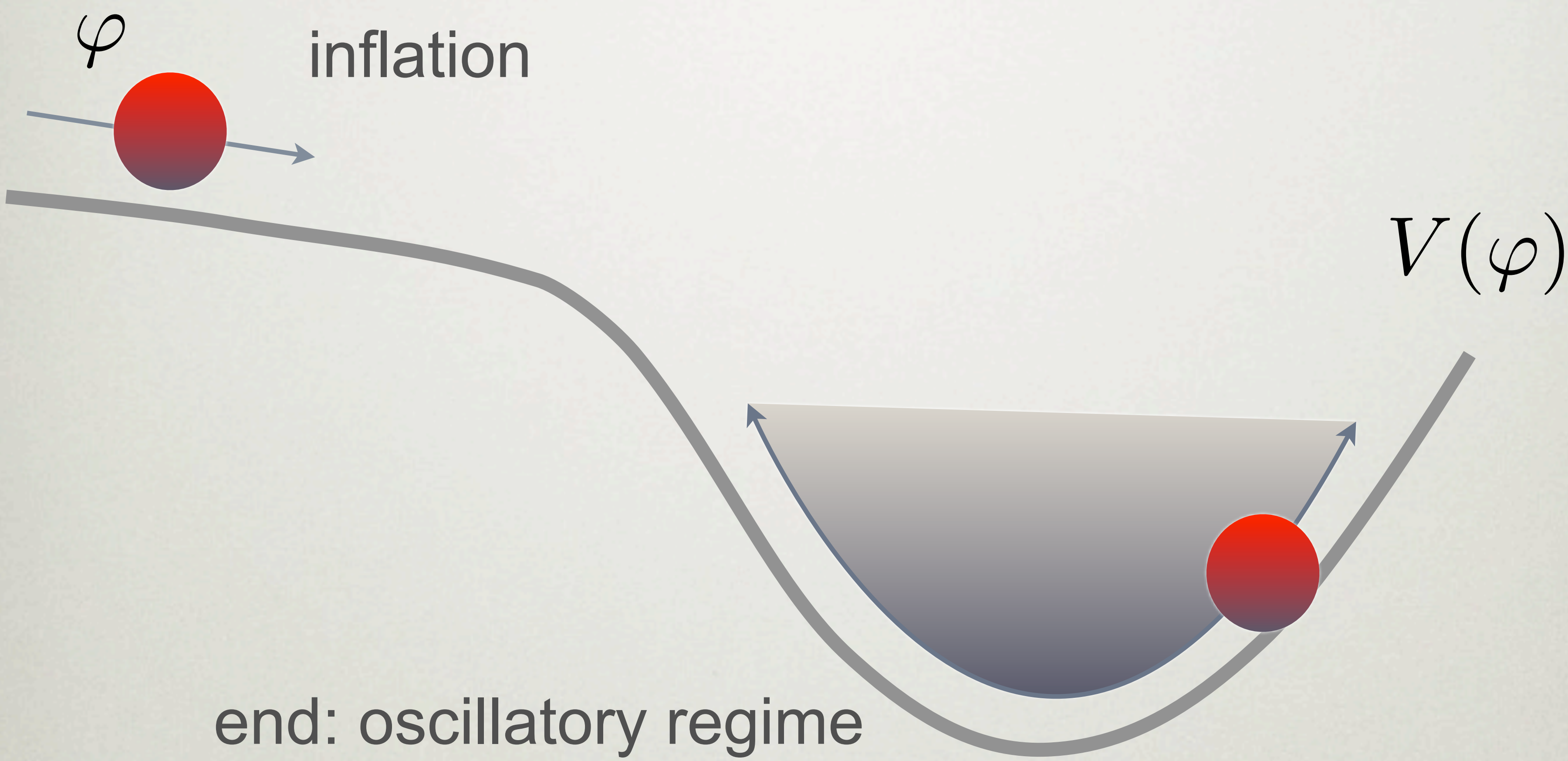
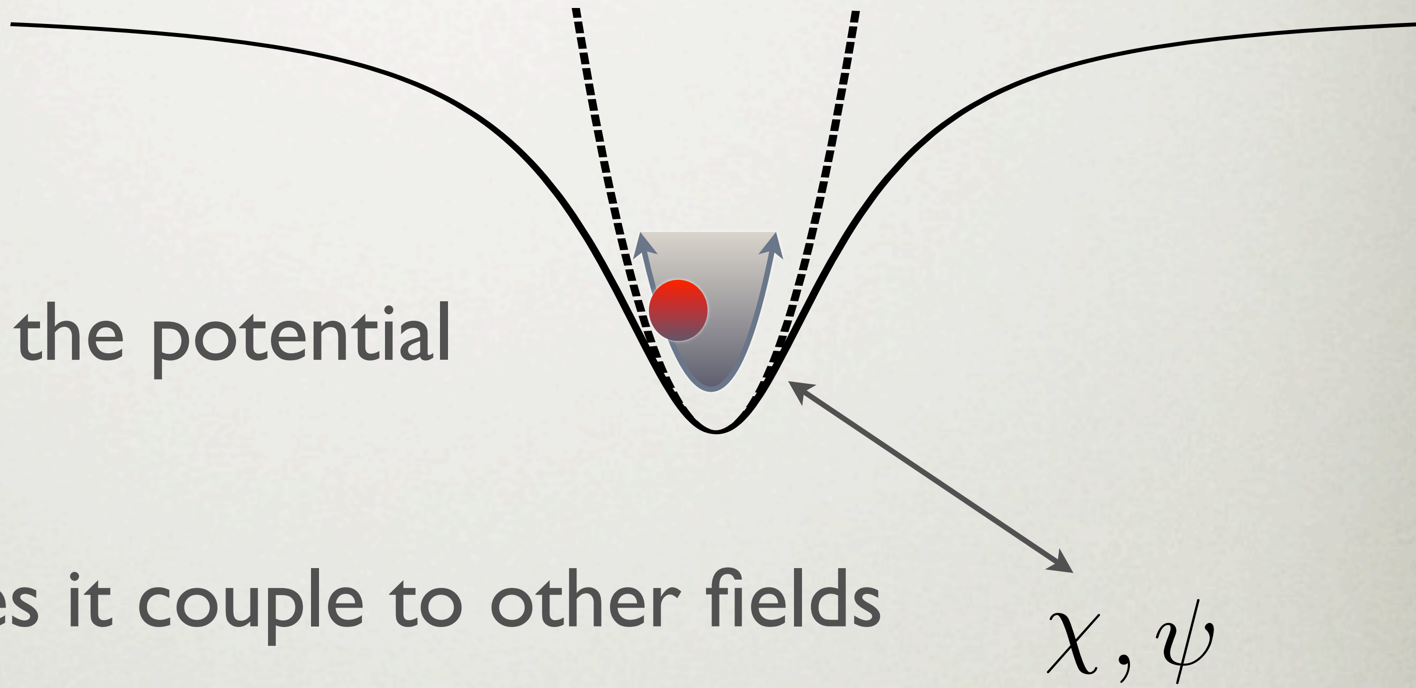


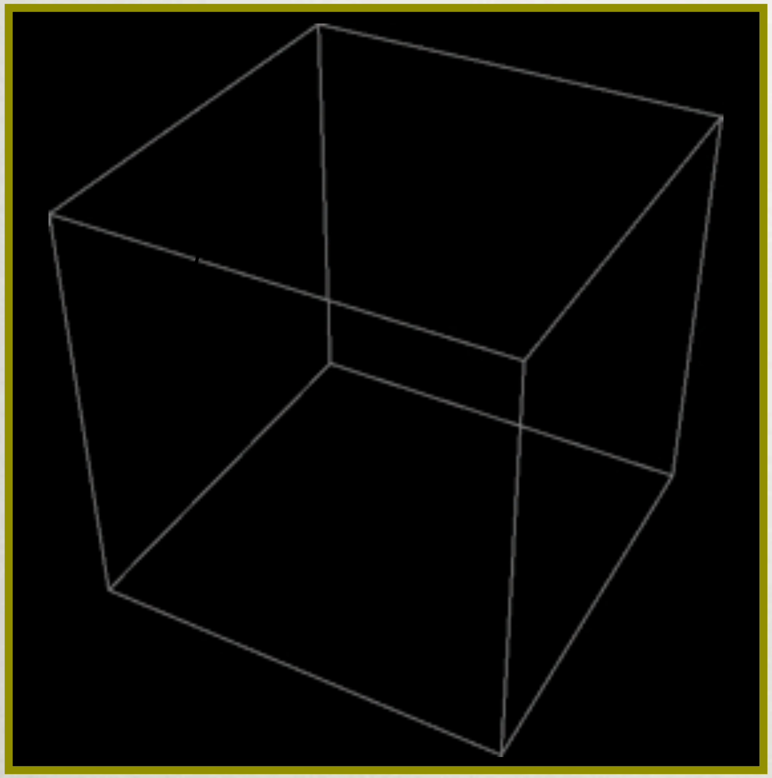
field dynamics at the end of inflation

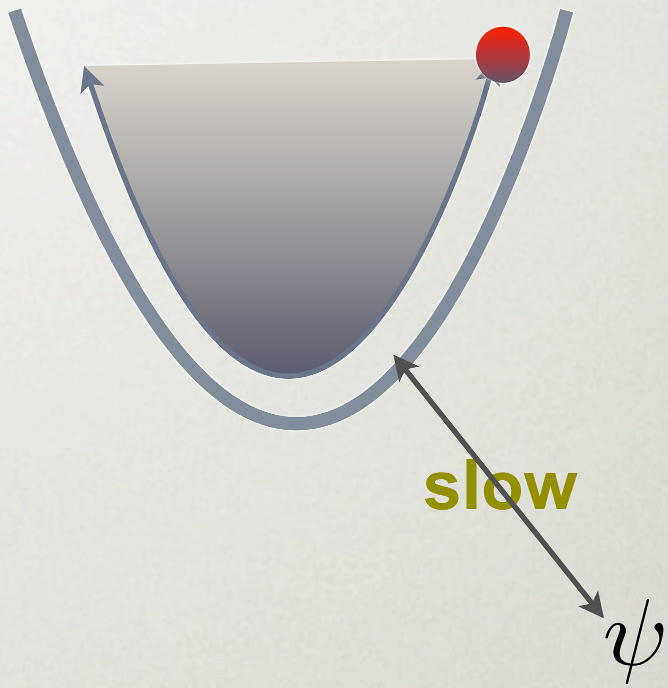
- shape of the potential
- how does it couple to other fields



1 : slow decay

$$V \sim \frac{1}{2} m_\varphi^2 \varphi^2 + h\varphi\bar{\psi}\psi + \ldots$$

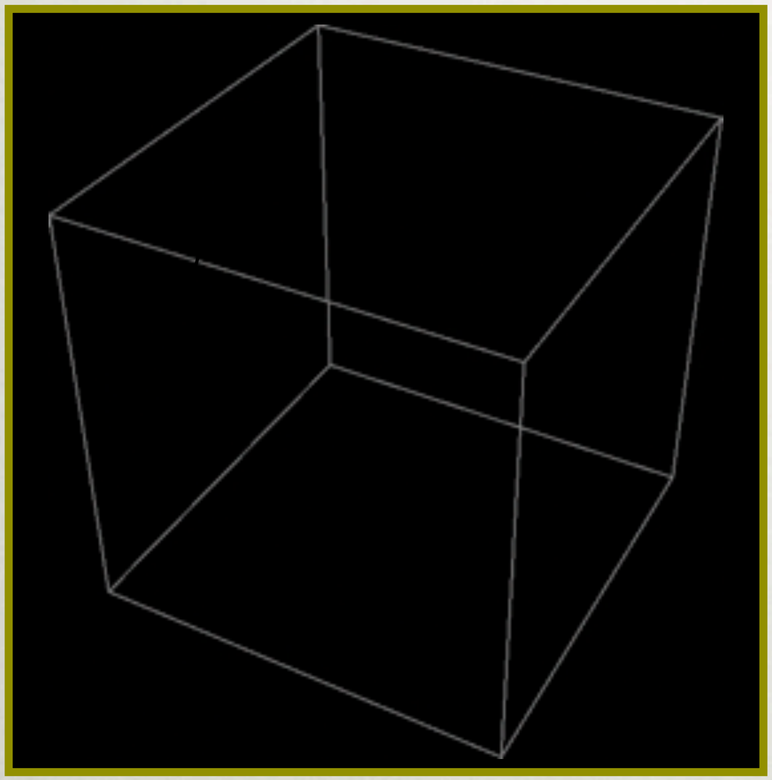


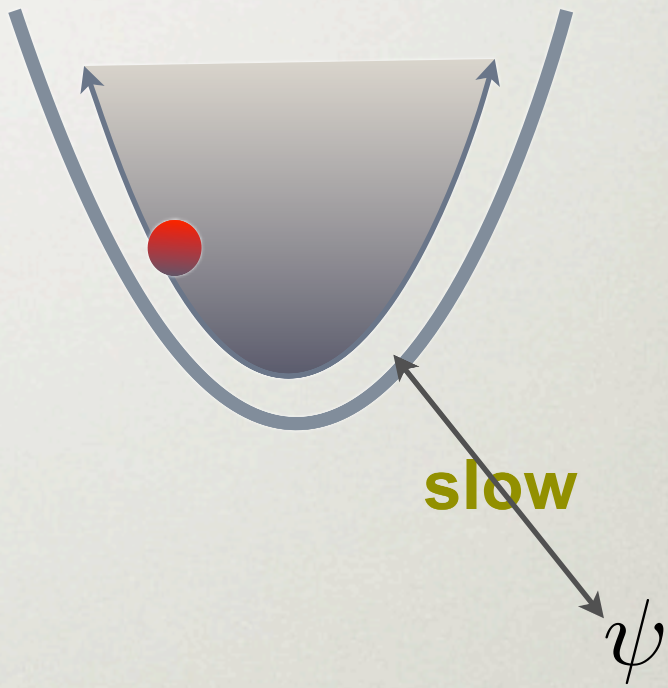


$$\Gamma(\varphi \to \psi\psi) \ll H$$

I : slow decay

$$V \sim \frac{1}{2} m_\varphi^2 \varphi^2 + h\varphi\bar{\psi}\psi + \dots$$



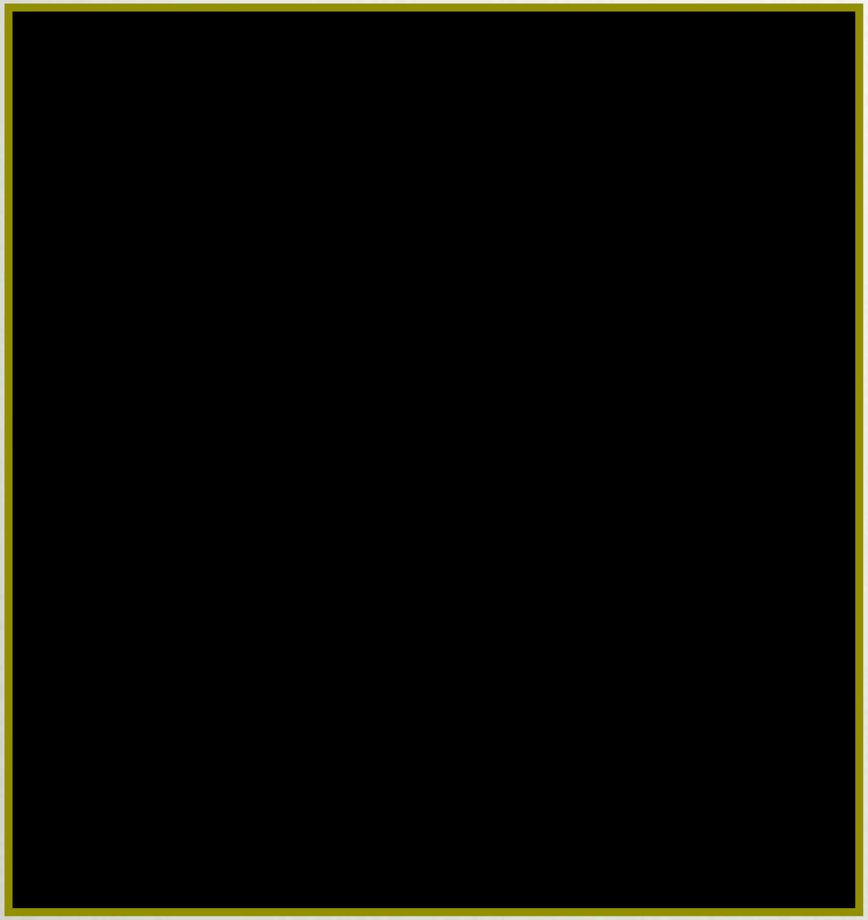


$$\Gamma(\varphi \to \psi\psi) \ll H$$

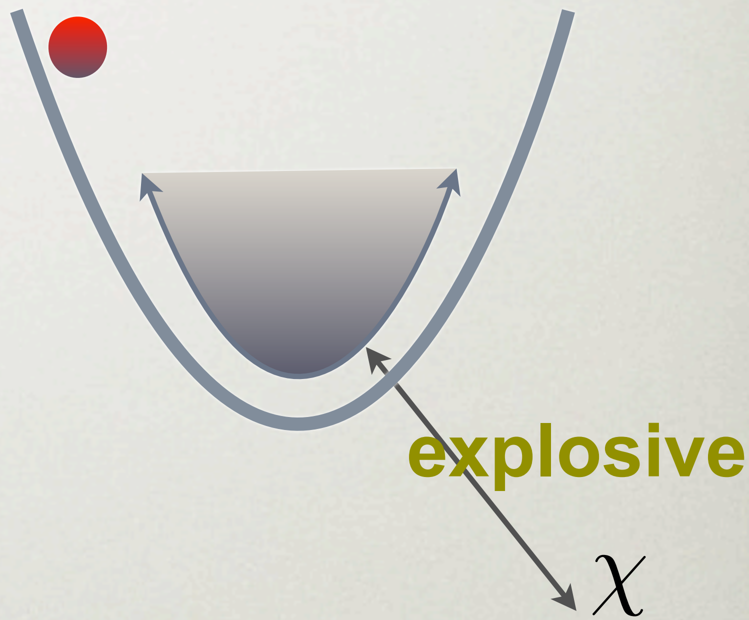
2 : explosive decay (scattering)

$$V \sim \frac{1}{2} m_\varphi^2 \varphi^2 + g^2 \varphi^2 \chi^2 + h \varphi \bar{\psi} \psi + \ldots$$

$$\Box \chi = V_{,\chi} = g^2 \varphi^2 \chi$$



Movie: courtesy of R. Easther (code used: pspectre)



decay rate >> H

Trachen & Brandenberger (1990), **Kofman, Linde, Starobinsky et. al (1994)** ... also Tachyonic cases

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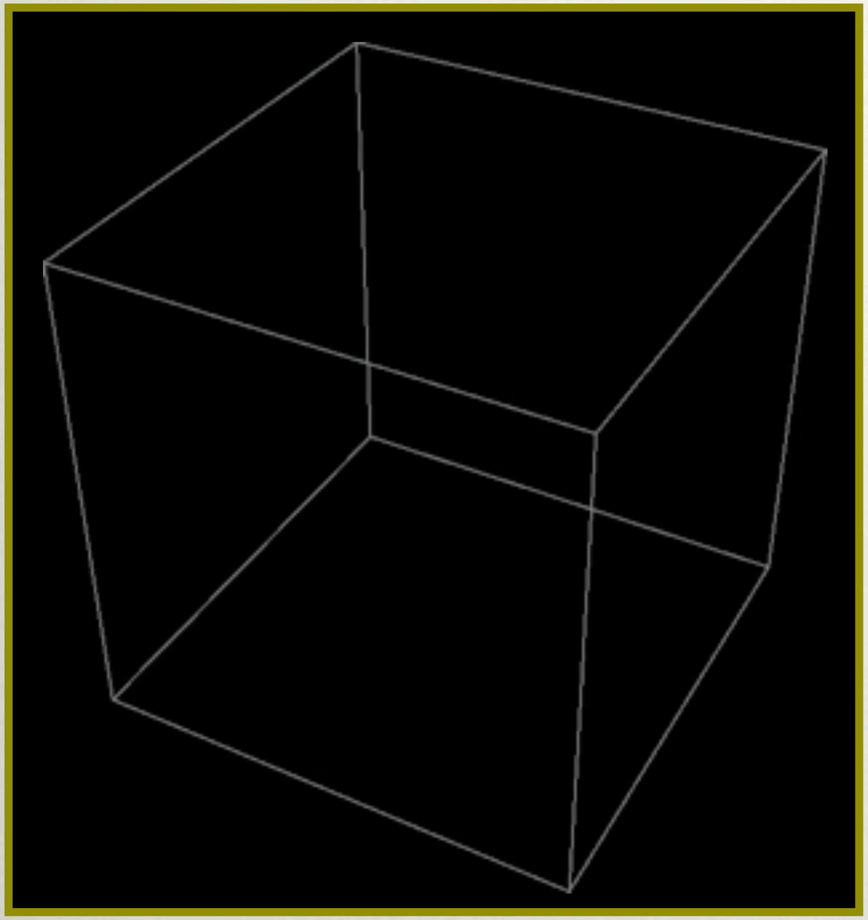

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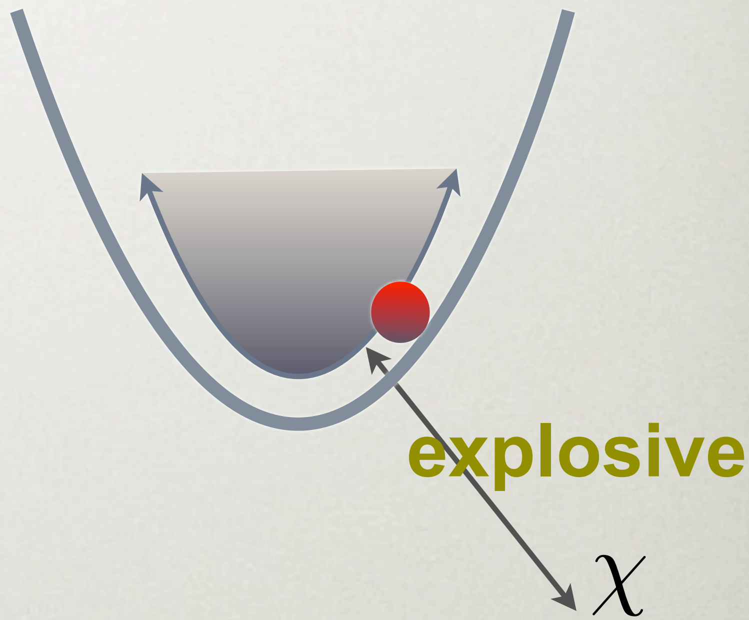

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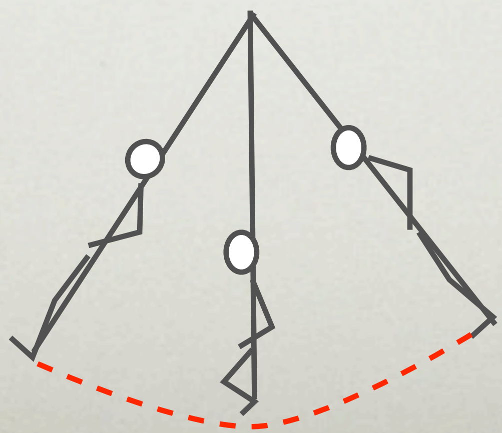
parametric resonance

inflaton

daughter fields

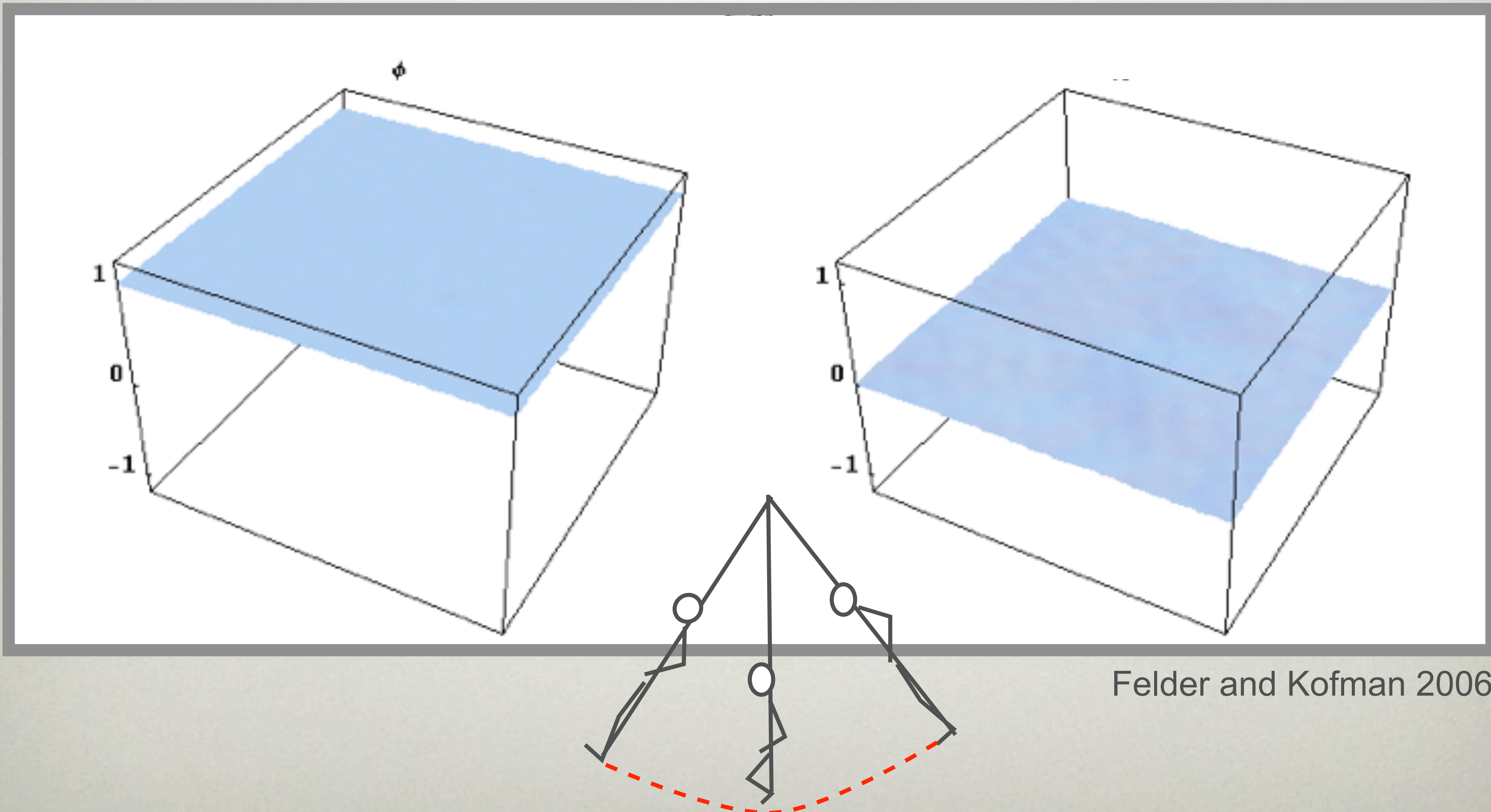


Felder and Kofman 2006

parametric resonance

inflaton

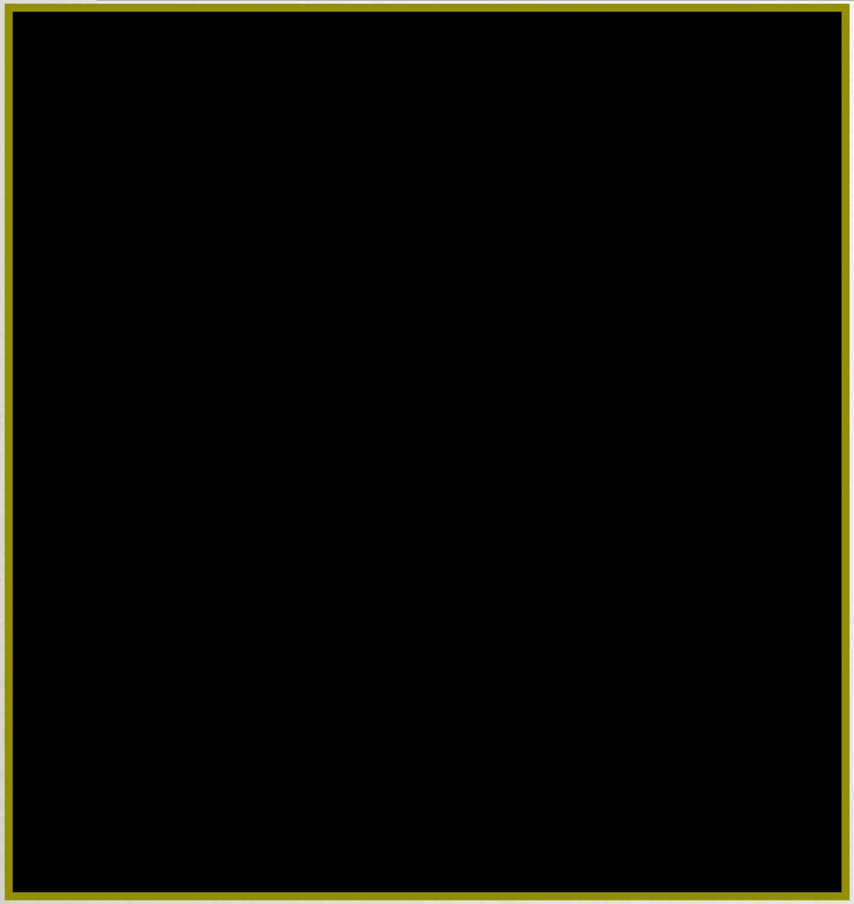
daughter fields



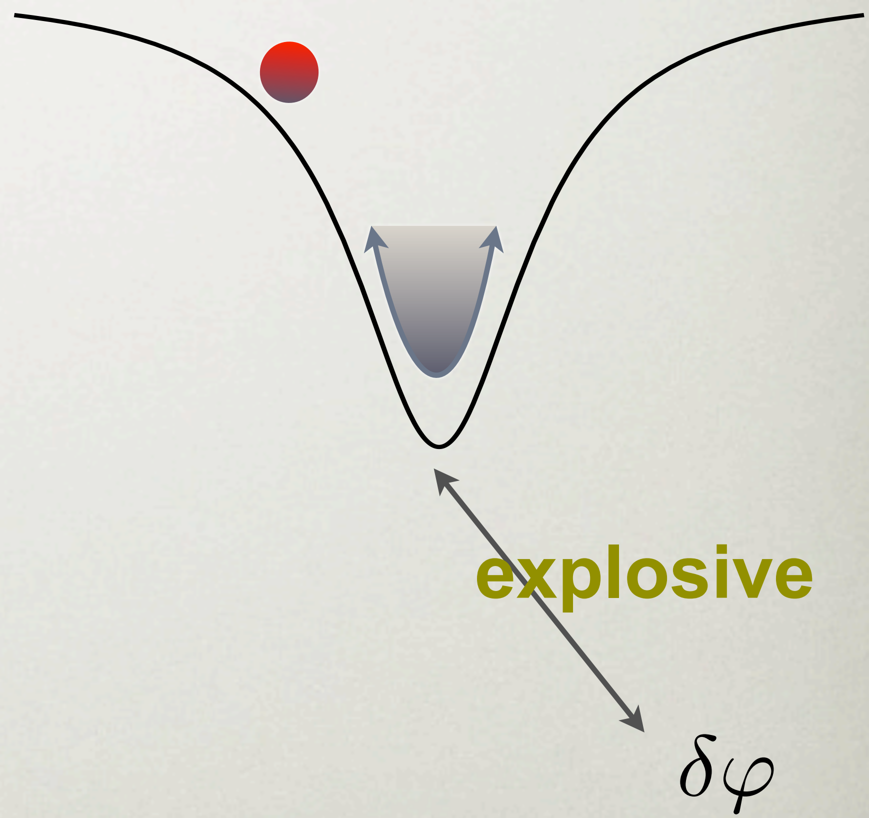
Felder and Kofman 2006

“usual” inflationary potential

3 : explosive fragmentation



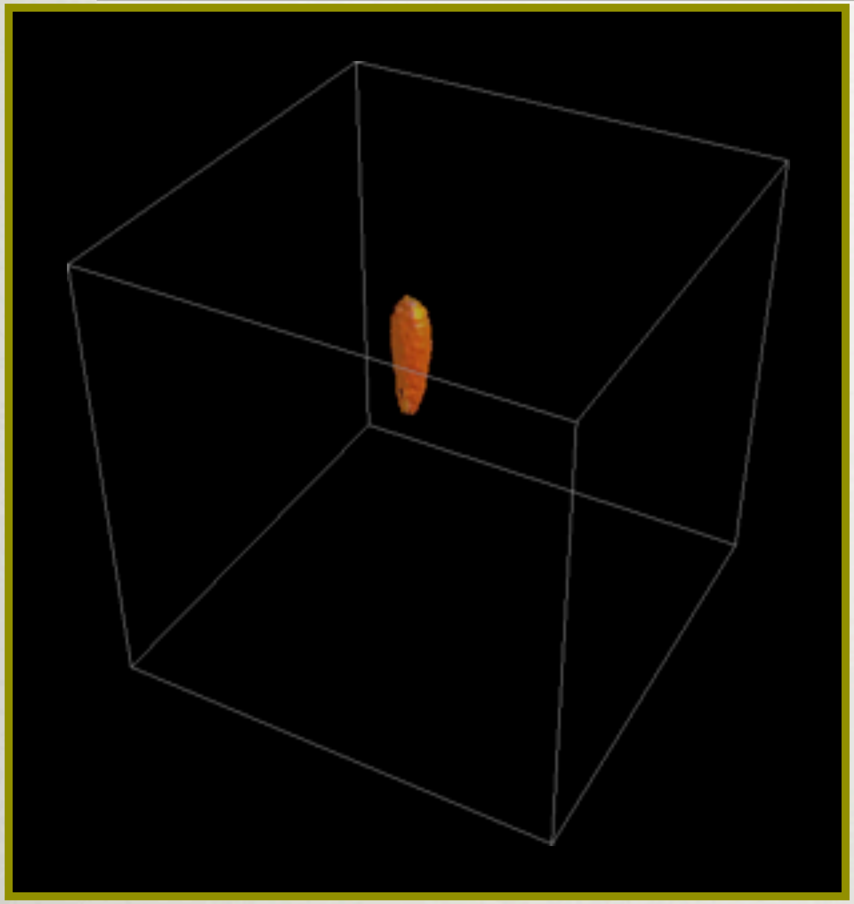
$$V \sim \frac{1}{2}m^2\varphi^2 + \frac{\lambda_4}{4}\varphi^4 + \ldots$$



MA 2010
MA, Finkel, Easther 2010
MA, Easther, Finkel, Flauger, Hertzberg 2011

Also see: McDonald & Broadhead, Rajantie & Copeland, Gleiser et. al. and the Q-ball literature

3 : explosive fragmentation



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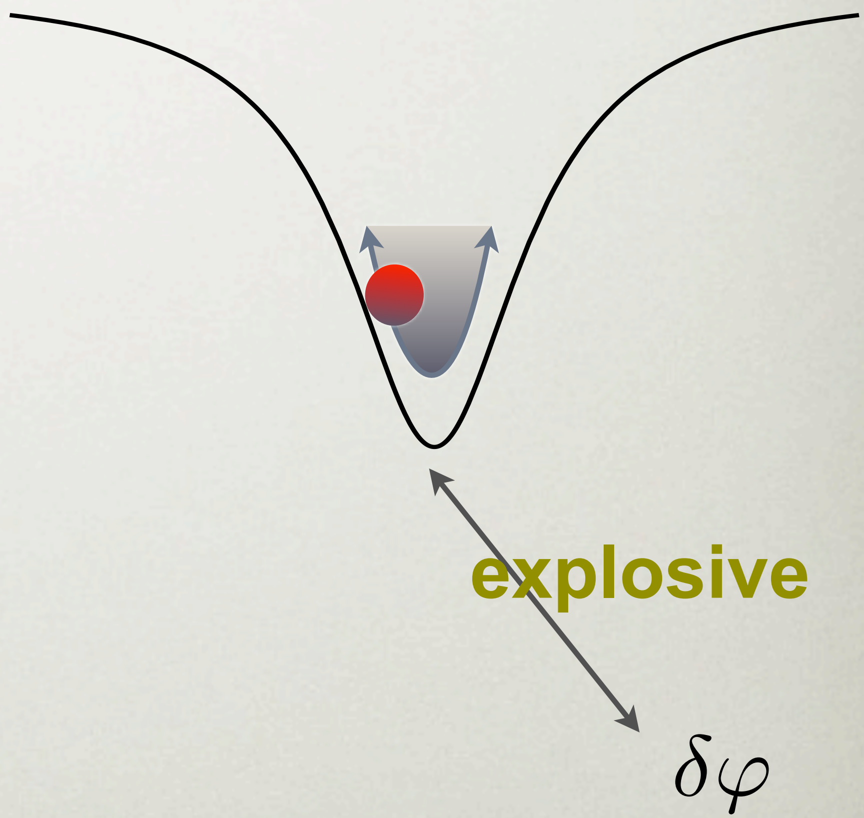
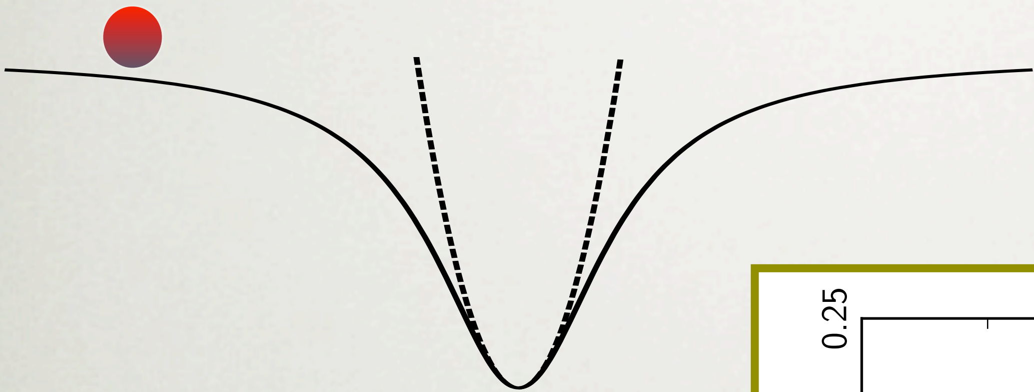


Also see: McDonald & Broadhead, Rajantie & Copeland, Gleiser et. al. and the Q-ball literature

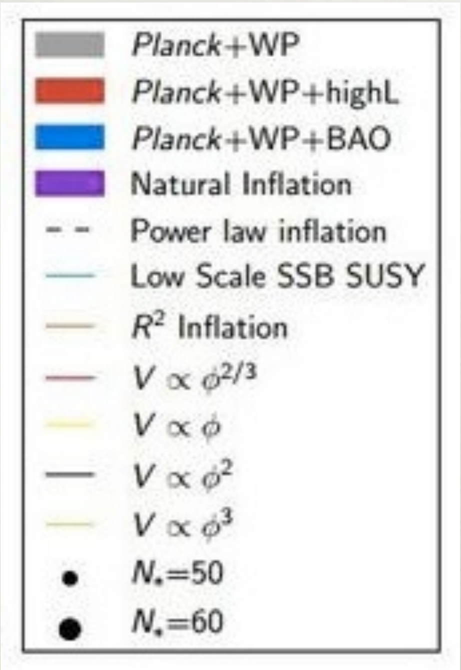
observations prefer shallow potentials

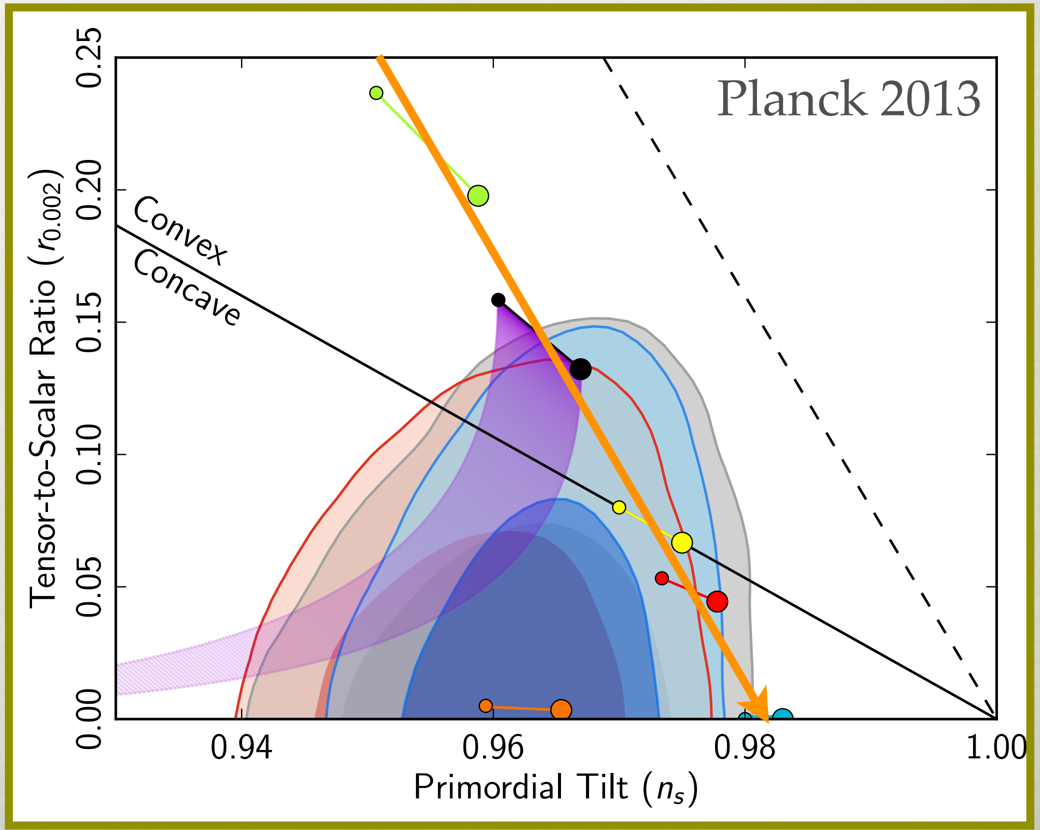
$V(\varphi) \propto \varphi^{2\alpha}$



$$r \approx \frac{8\alpha}{N}$$

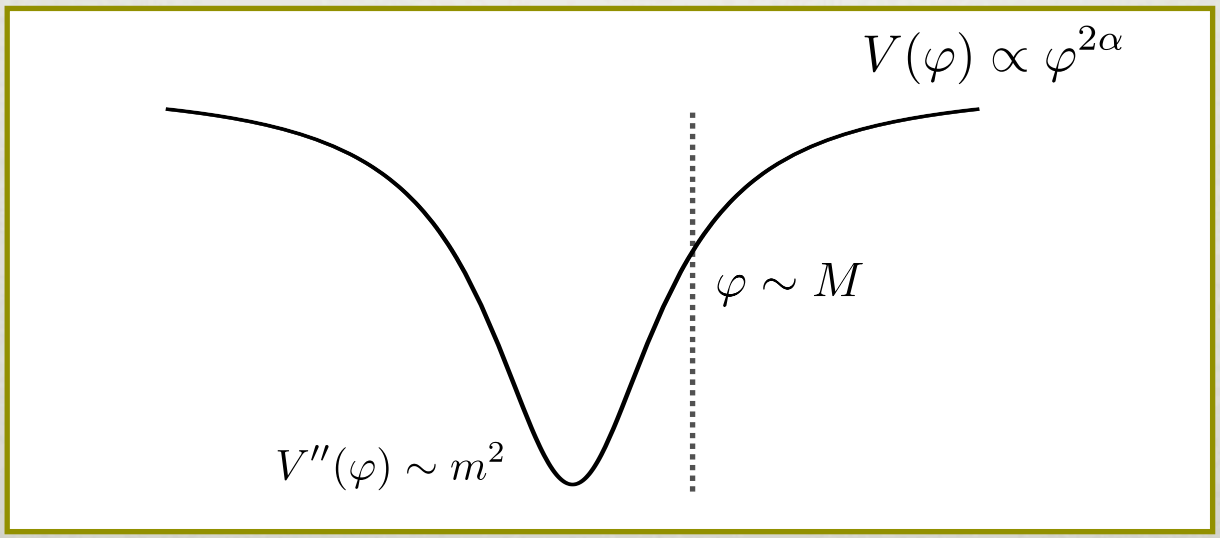
$$n_s \approx 1 - \frac{\alpha + 1}{N}$$





theoretical motivation: shallow potentials

- Silverstein & Weshtpal 2008 $\alpha = 1/3$
- McAllister, Silverstein & Westphal 2008 $\alpha = 1/2$
- generic flattening of potentials (Dong et. al 2010) $\alpha < 1$



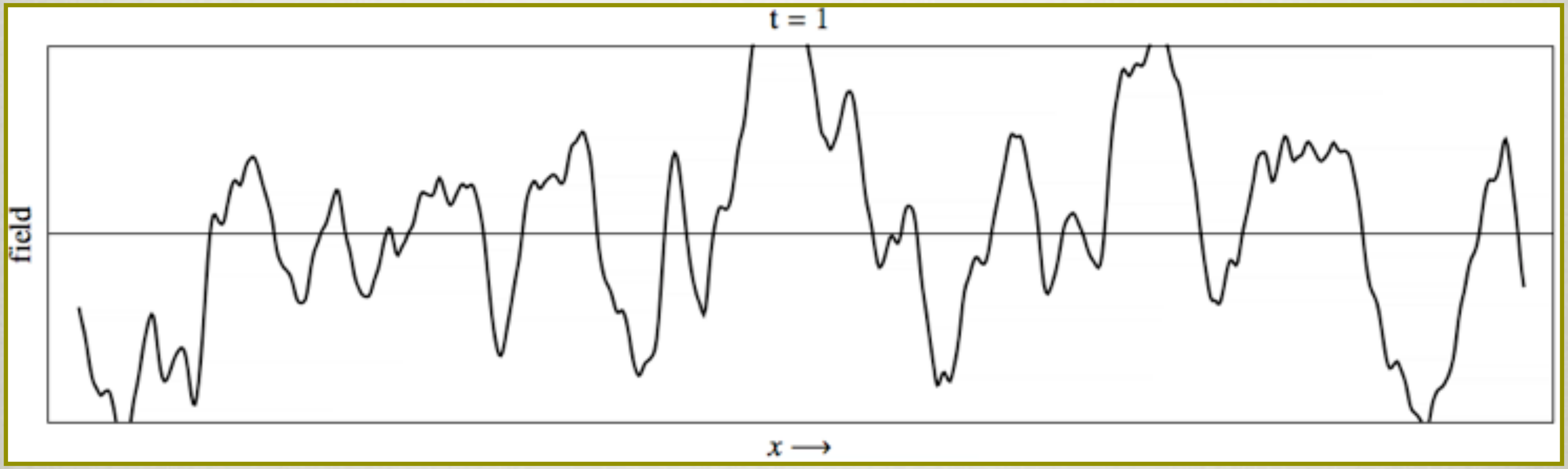
COSMOLOGICAL EMERGENCE

generic emergence

> 80% energy density in oscillons !

Farhi et. al 2008

generic emergence



> 80% energy density in oscillons !

Farhi et. al 2008

post-inflationary initial conditions

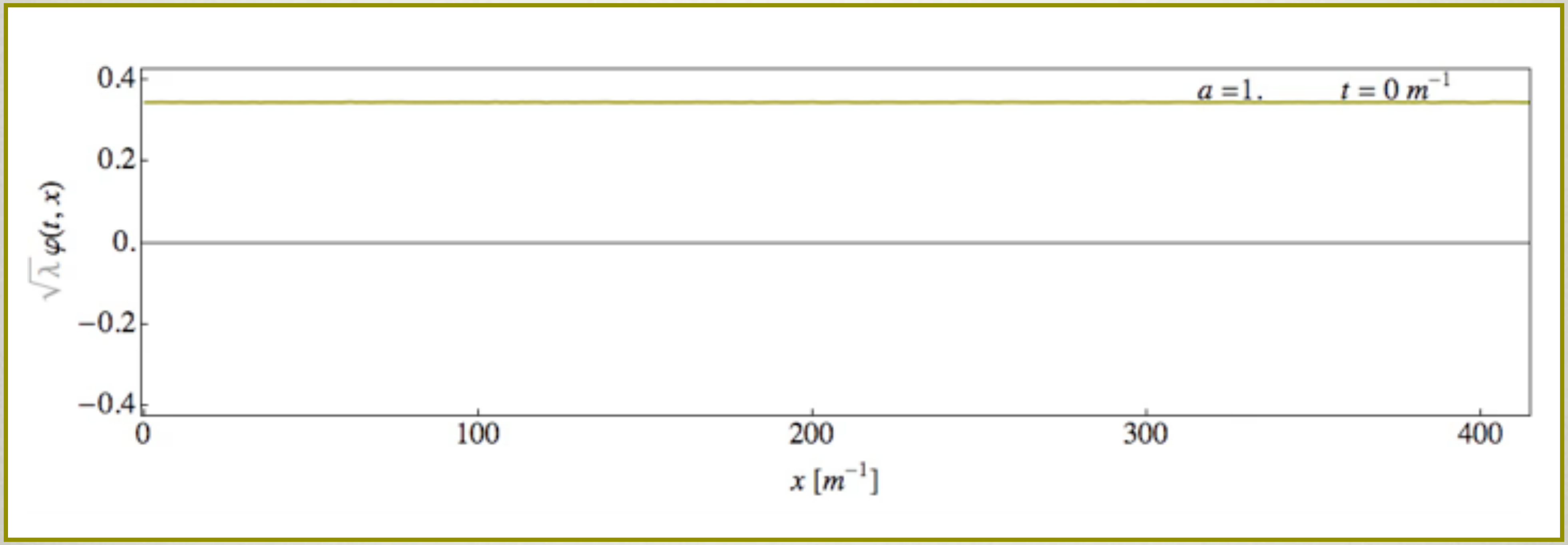
- start with tiny ‘quantum’ fluctuations
- how can we get large non-linear blobs?

how do they form?
self resonance

MA (2010)

“anharmonic” oscillations of the homogeneous inflaton are unstable, causing a resonant growth of fluctuations

how do they form?
self resonance



MA (2010)

“anharmonic” oscillations of the homogeneous inflaton are unstable, causing a resonant growth of fluctuations

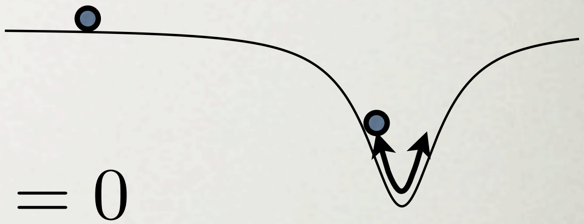
requires efficient growth

- growth rate μ_k
- expansion rate H

$$\mu_k \gg H$$

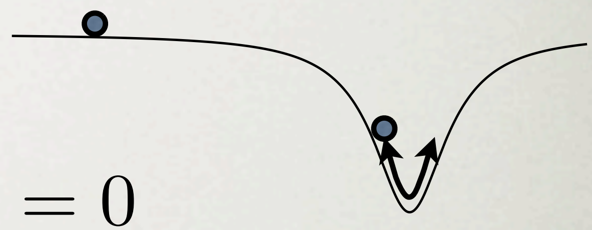
μ_k depends on the parameters, and can be calculated easily

growth rate



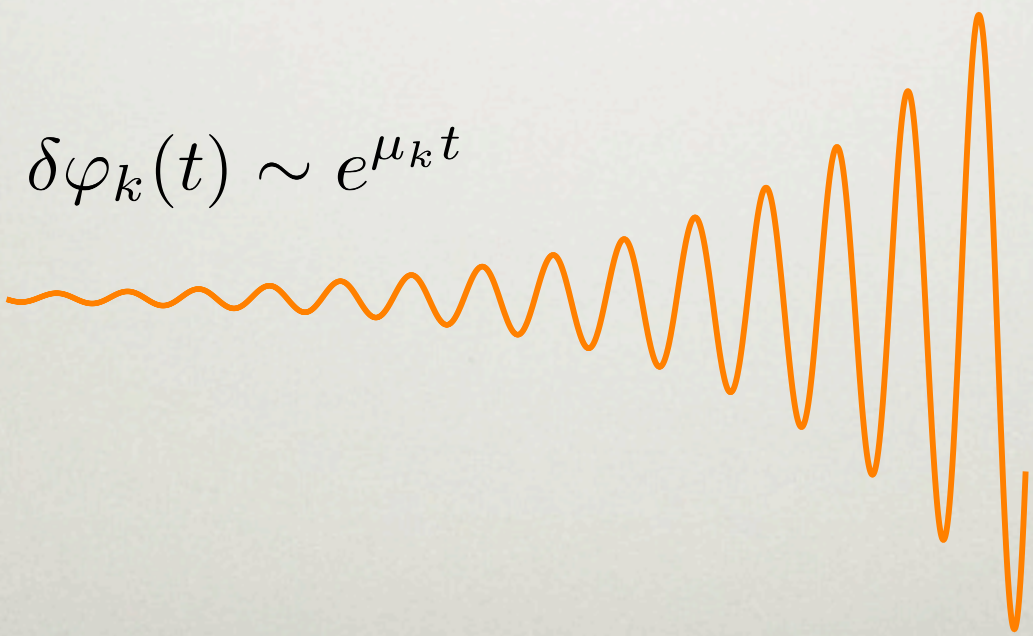
$$\partial_t^2 \delta\varphi_k + \left(k^2 + U''(\bar{\varphi})\right) \delta\varphi_k = 0$$

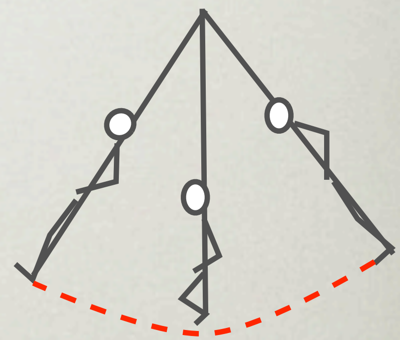
growth rate



$$\partial_t^2 \delta\varphi_k + \left(k^2 + U''(\bar{\varphi})\right) \delta\varphi_k = 0$$

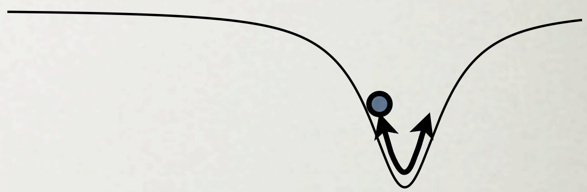
$$\delta\varphi_k(t) \sim e^{\mu_k t}$$

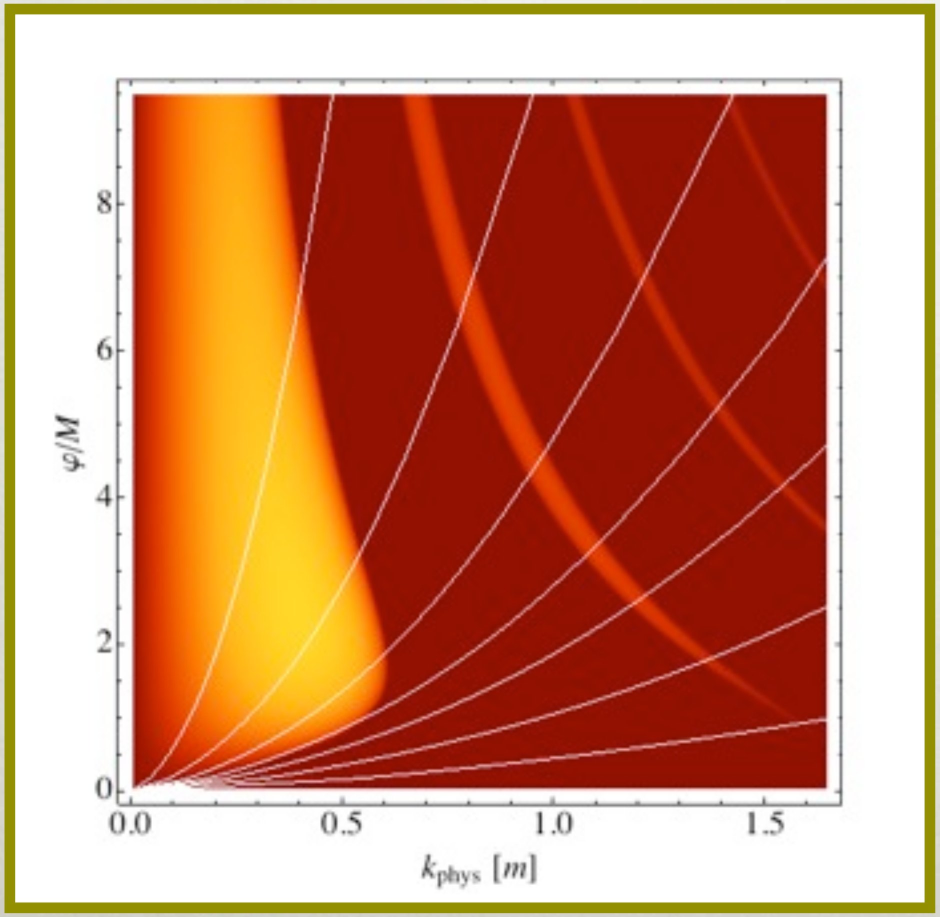




Floquet analysis

$$\partial_t^2 \delta\varphi_k + \left(k^2 + U''(\bar{\varphi})\right) \delta\varphi_k = 0$$

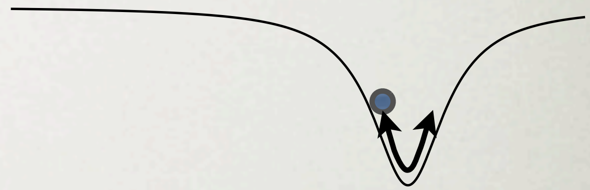


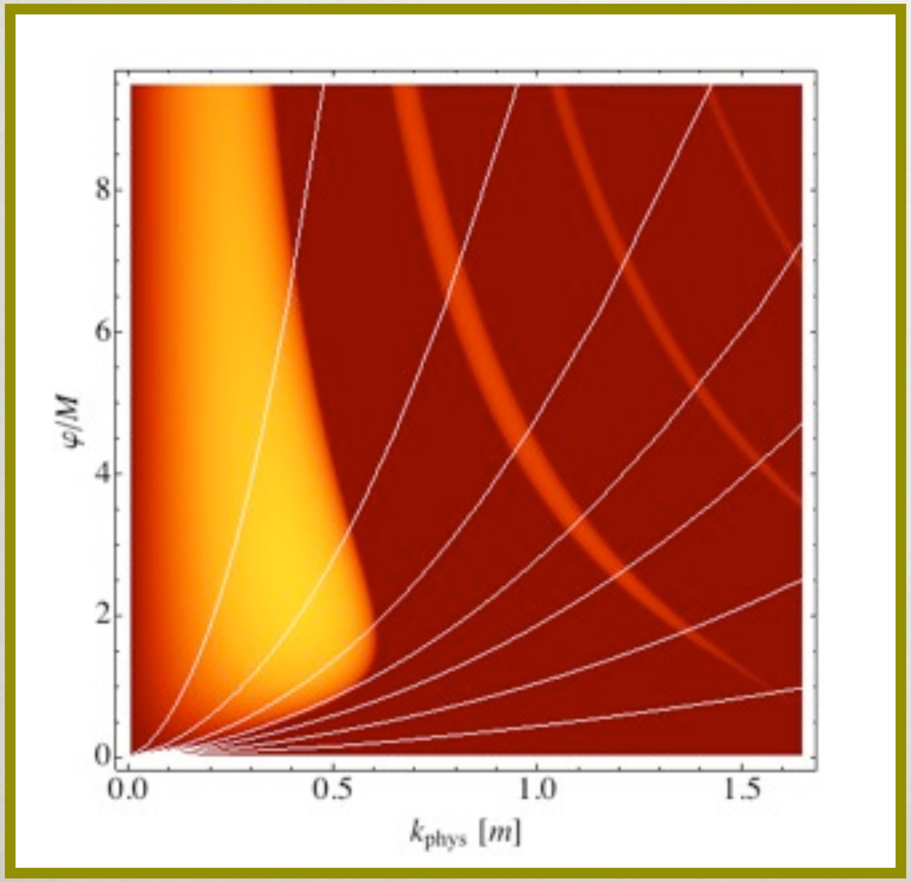


$$\delta\varphi_k(t) \sim e^{\mu_k t}$$

including expansion

$$\partial_t^2 \delta\varphi_k + 3H\partial_t\delta\varphi_k + \left(\frac{k^2}{a^2} + U''(\bar{\varphi})\right)\delta\varphi_k = 0$$





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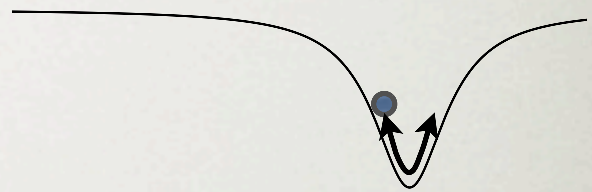
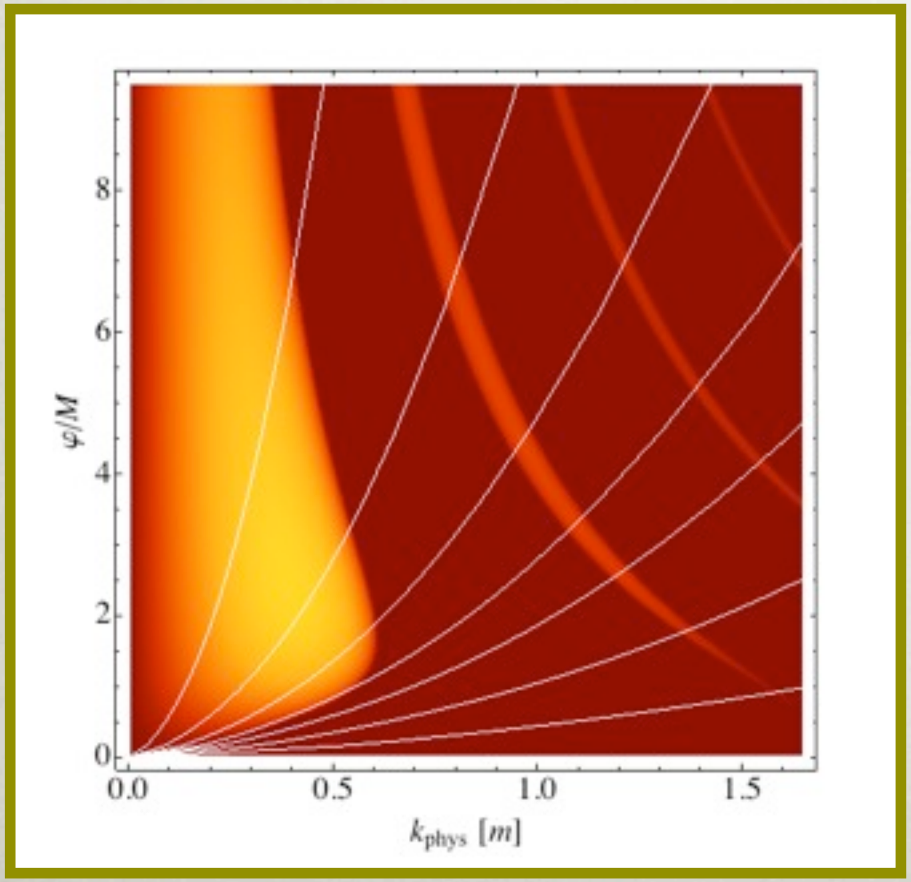




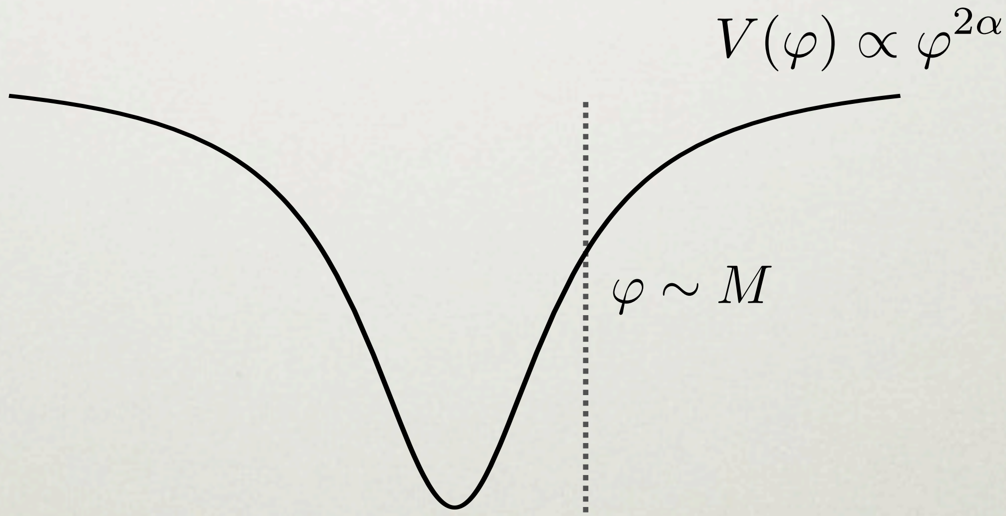
$$\delta\varphi_k \approx \frac{\delta\varphi_k(t_i)}{a^{3/2}(t)} \exp\left[\int dt \mu_k(t)\right]$$

$$= \frac{\delta\varphi_k(a_i)}{a^{3/2}} \exp\left[\int d\ln a \frac{\mu_k(a)}{H(a)}\right]$$

$$\Re(\mu_k) \gg H$$

emergence condition

$$\left[\frac{|\Re(\mu_k)|}{H}\right]_{\max} \approx f(\alpha)\frac{m_{\rm pl}}{M} \gg 1$$



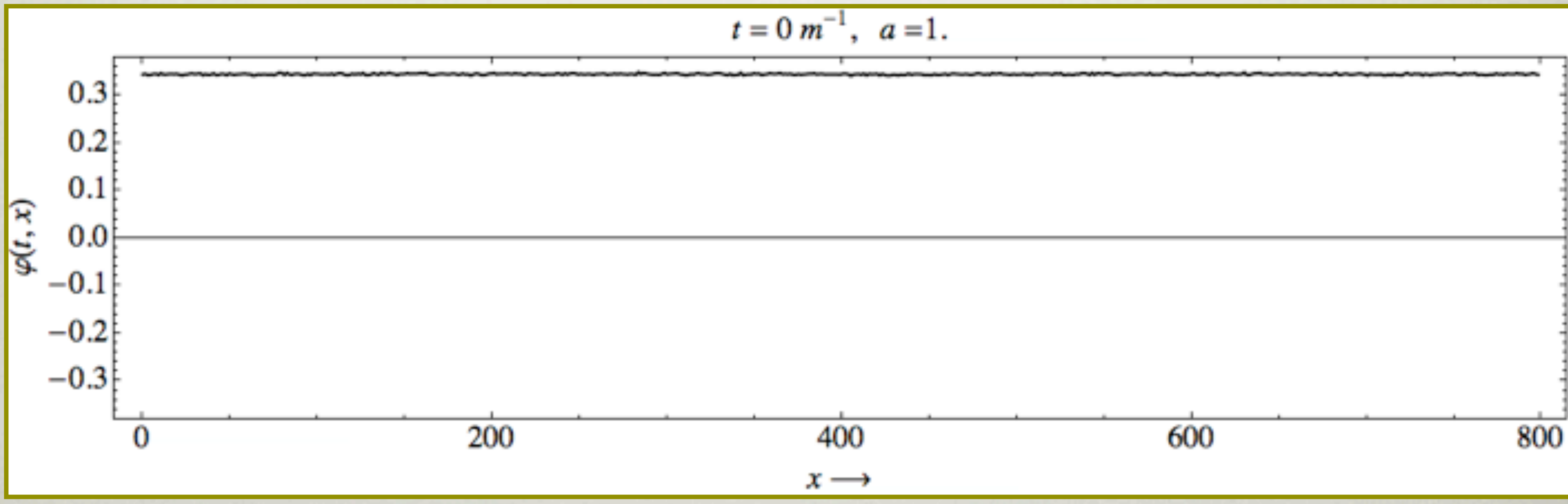
number density

$$n_{osc}a^3 \sim \left(\frac{k_{nl}}{2\pi}\right)^3$$

MA (2010)

number density

$$n_{osc}a^3 \sim \left(\frac{k_{nl}}{2\pi}\right)^3$$



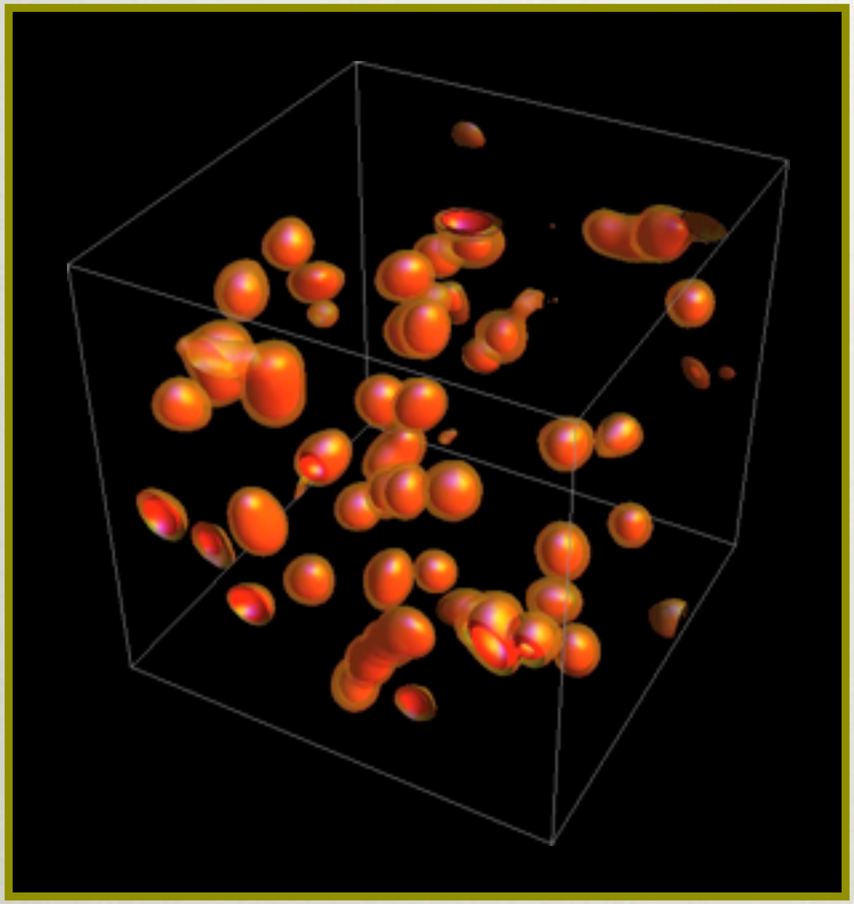
MA (2010)

IMPLICATIONS

energy fraction in lumps?

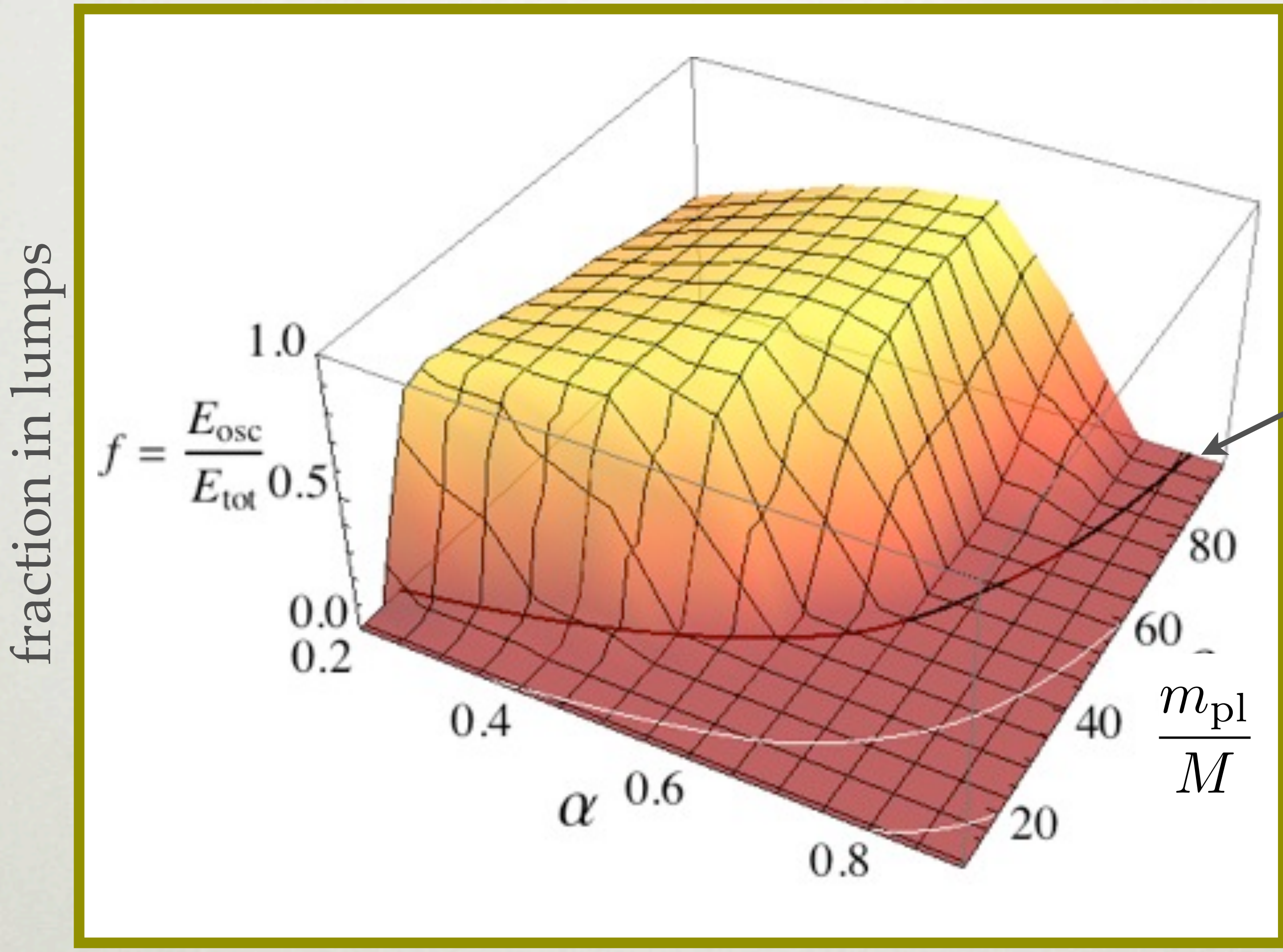
(as a function of parameters)

simulations



- pseudo spectral code:256^3 (**pspectre**: Easther, Finkel and Roth). Also tested with **defrost** (Frolov) and **LatticeEasy** (Felder and Tkachev)
- initial conditions at end of inflation(small zero point fluctuations)
- fraction of energy in oscillons (overdense by a few)

energy fraction: >> 50%



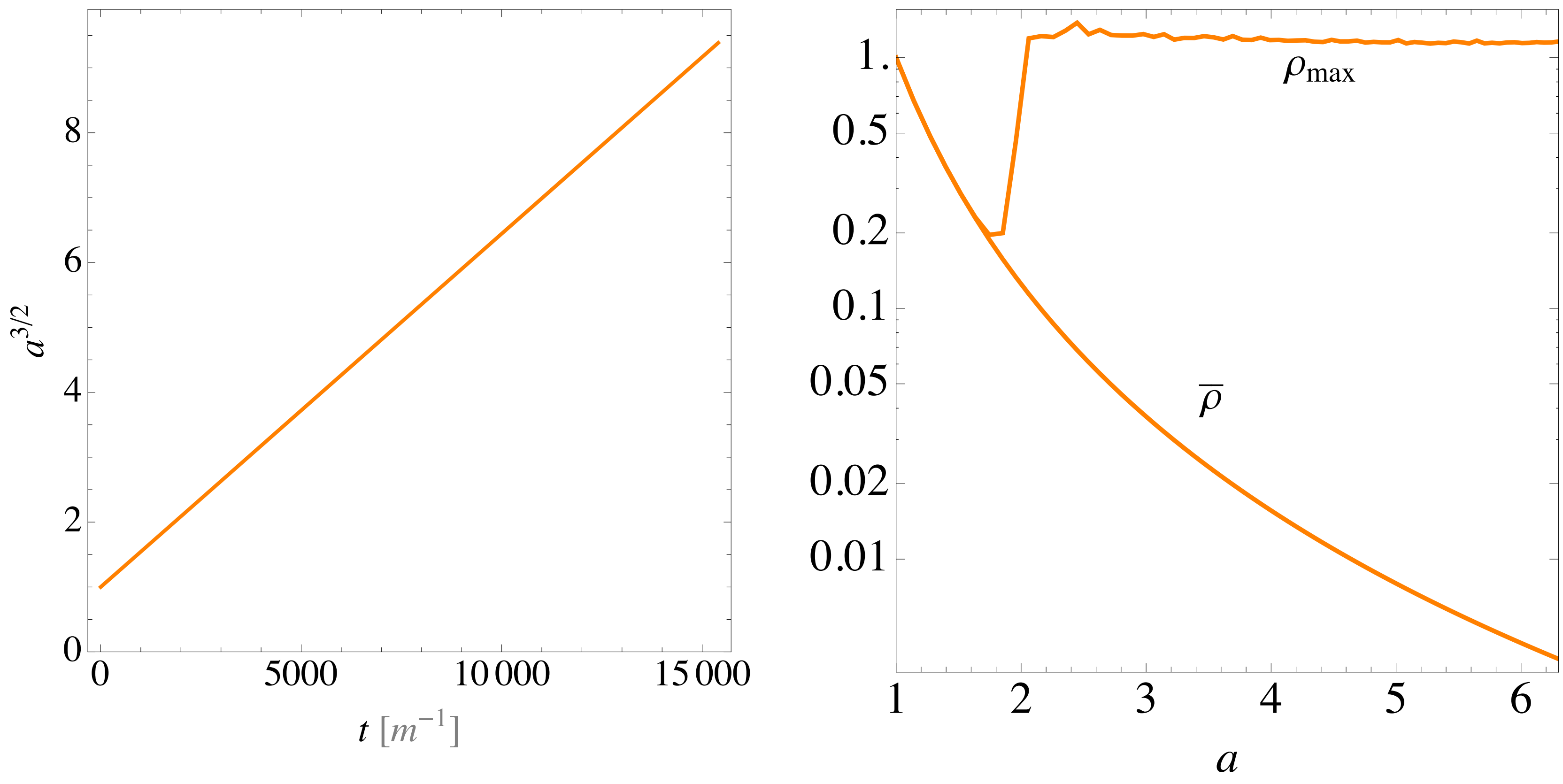
analytic curves from Floquet analysis

$$\left[\frac{|\Re(\mu_k)|}{H}\right]_{\max} \gtrsim 10$$

(inverse)scale where potential changes shape $m_{pl}/M \gg 1$

asymptotic slope $\alpha \lesssim 1$

expansion history



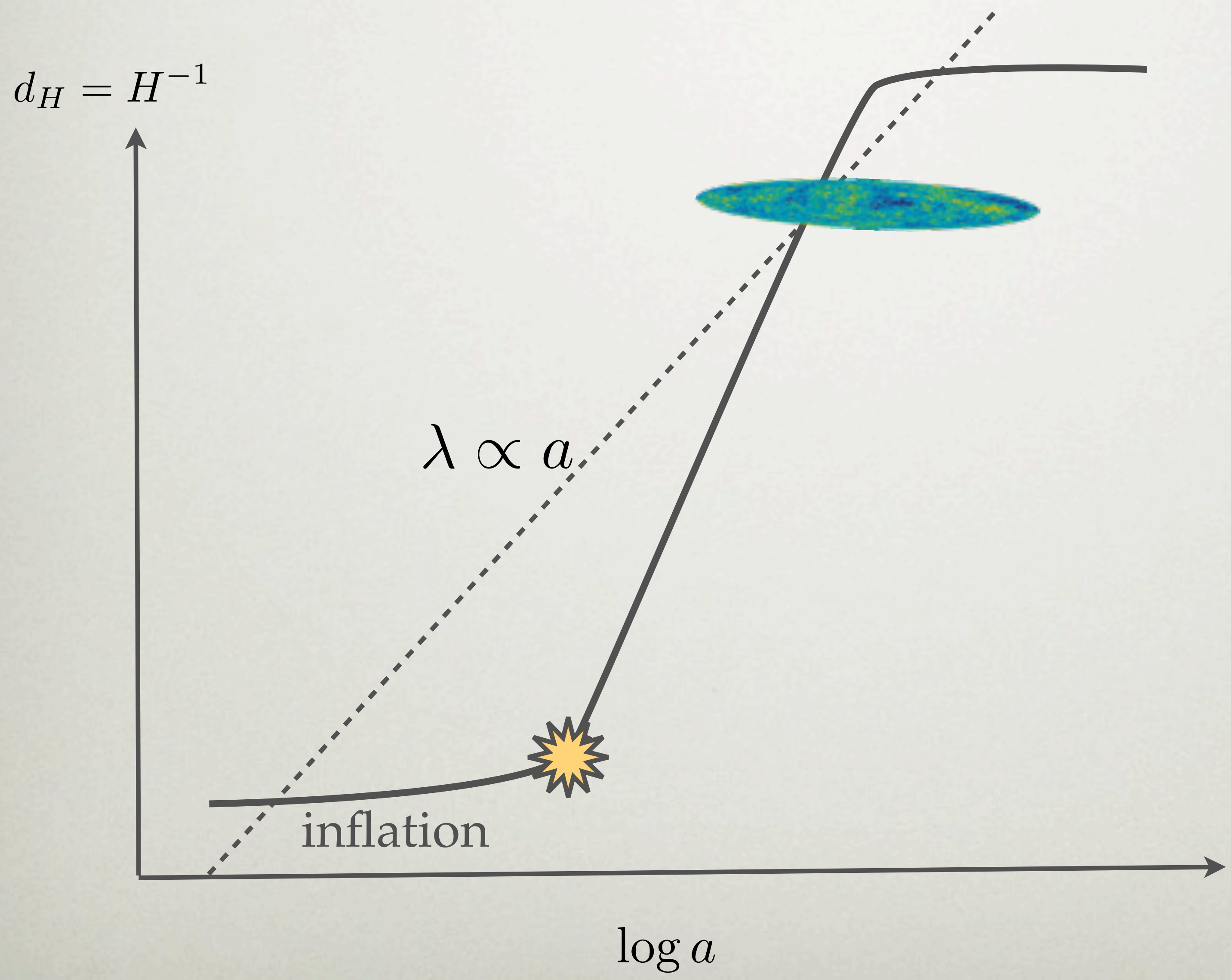
MA, Easther & Finkel (2010)

what to do with them?

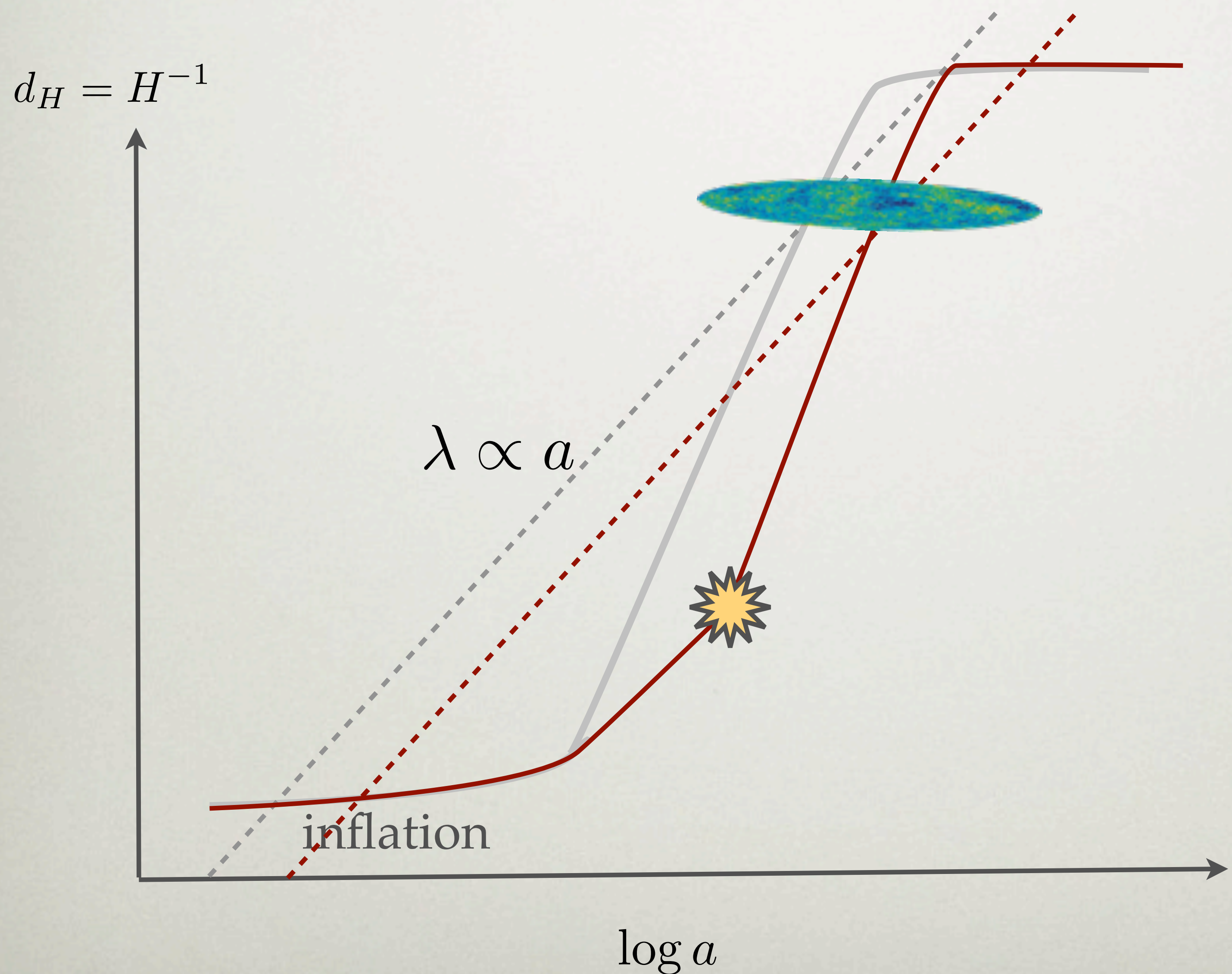
- **delayed reheating** (*MA, Giblin & Child in (progress)*)

- **gravitational effects:**
  - primodial black holes ? *(difficult!)*
  - expansion history and influence on inflationary observables ? *(Adshead et. al 2011)*
  - g-waves ? (Zhou et. al (in progress))
  - number density modulation -- non-gaussianity
    - *(related to non-gaussianity from preheating Bond et. al 2009 and Jonathan's talk)*

expansion history effects



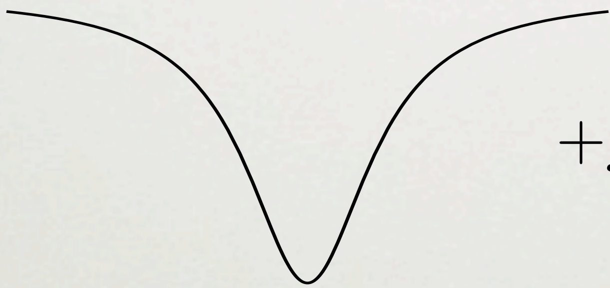
expansion history effects



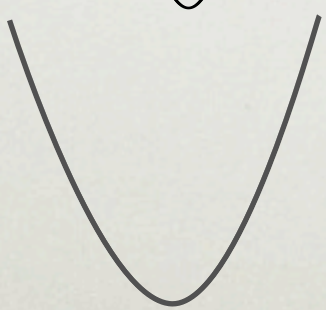
delayed reheating

measure time for inflaton to decay into massless field

$$t_1(\varphi \to \chi) > t_2(\varphi \to \chi)$$

1. 

$$+g^2\varphi^2\chi^2 + c\varphi\chi^2$$

2. 

$$+g^2\varphi^2\chi^2 + c\varphi\chi^2$$

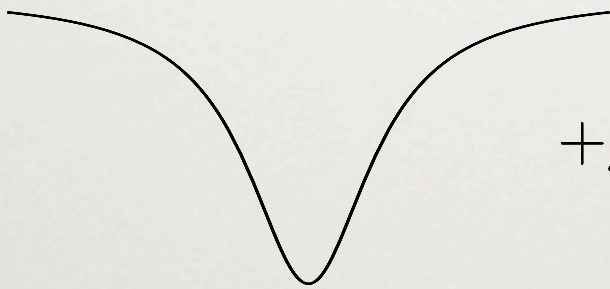
MA, Child, Giblin (in progress)

work in progress

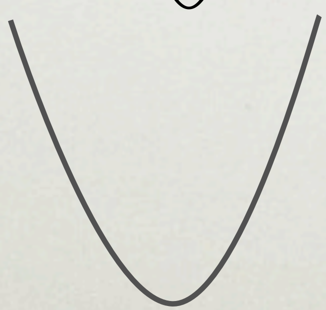
delayed reheating

measure time for inflaton to decay into massless field

$$t_1(\varphi \to \chi) > t_2(\varphi \to \chi)$$

1. 

$+g^2\varphi^2\chi^2 + c\varphi\chi^2$

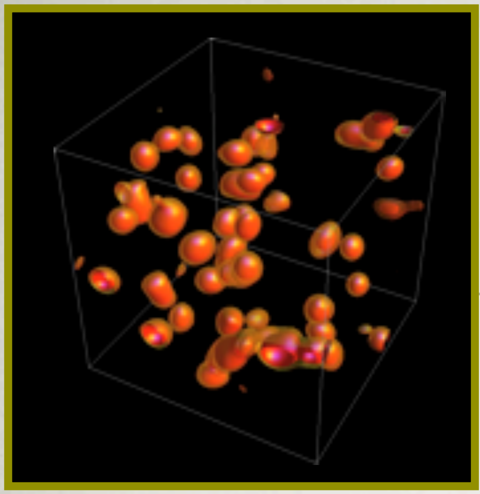
2. 

$+g^2\varphi^2\chi^2 + c\varphi\chi^2$

MA, Child, Giblin (in progress)

LUMPS DETAILS

37



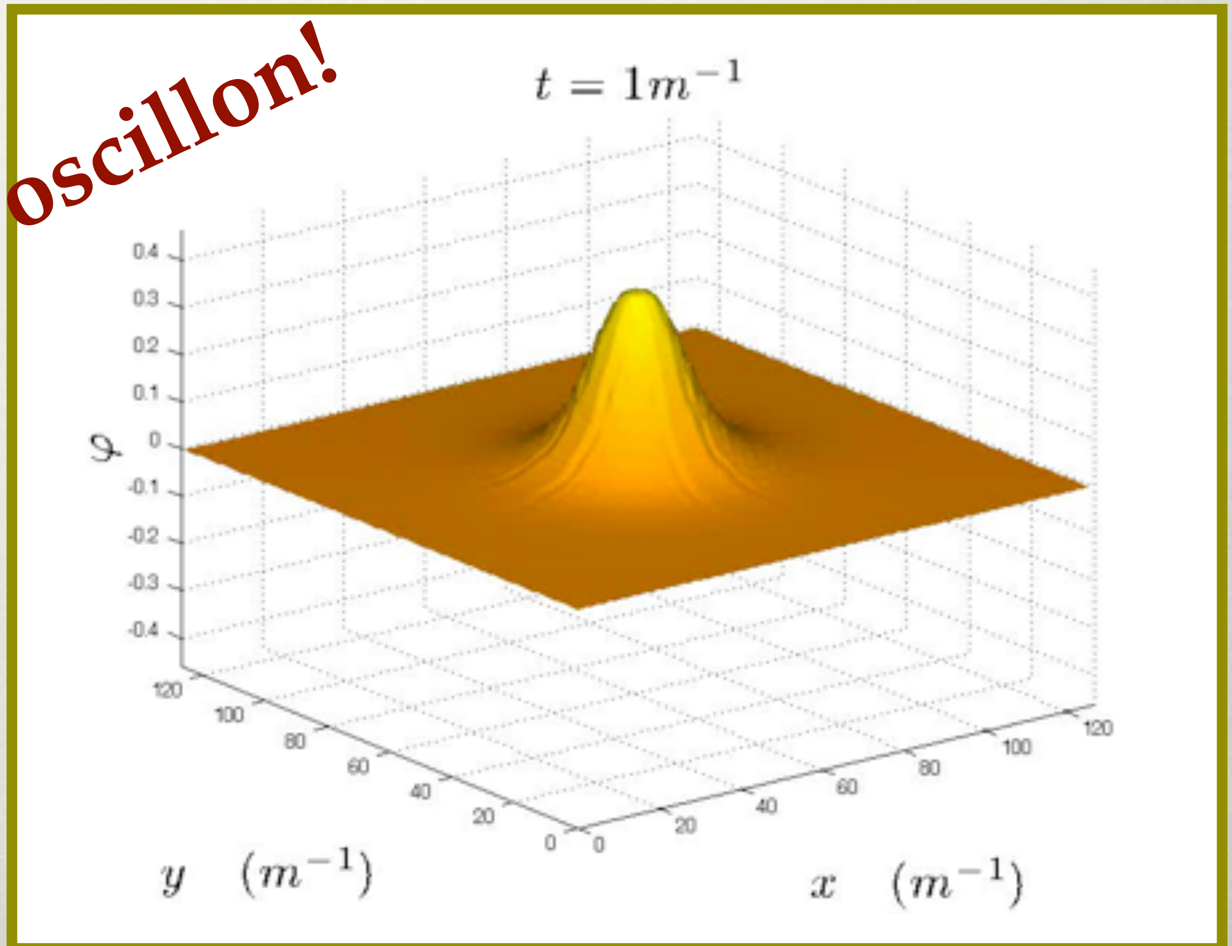
lumps?

(1) oscillatory (2) spatially localized (3) **very long lived**

necessary:

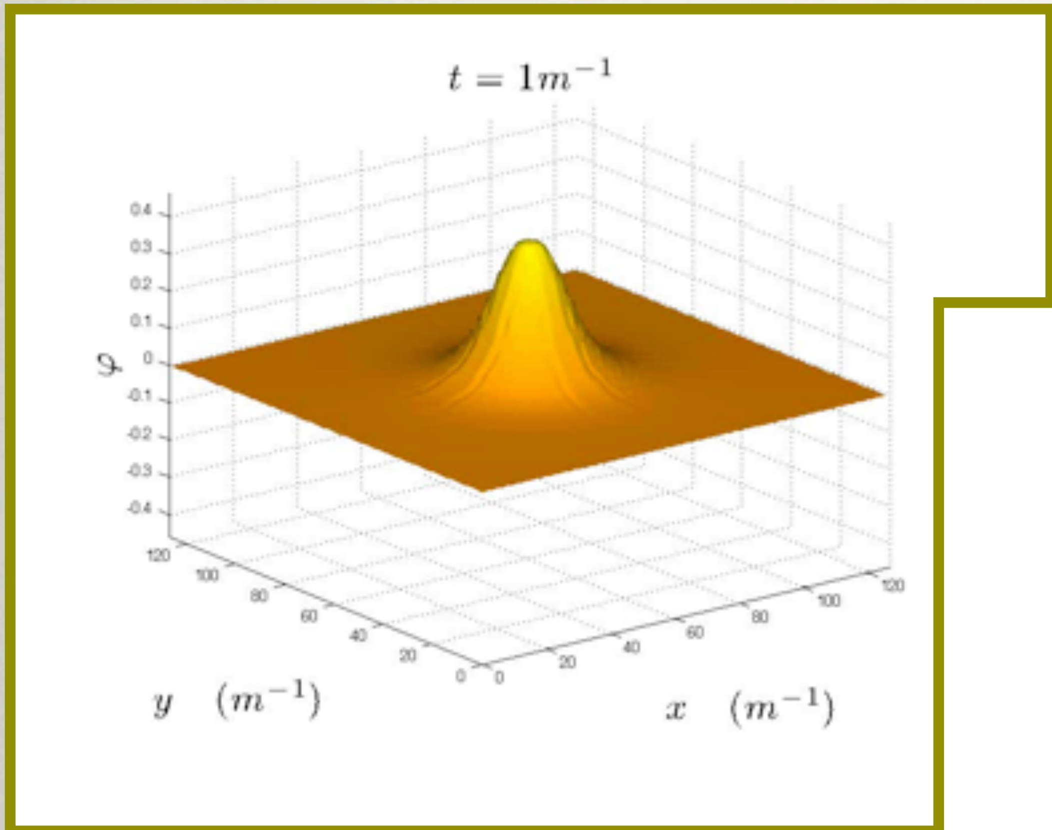
$$V(\varphi) - \frac{1}{2}m^2\varphi^2 < 0$$

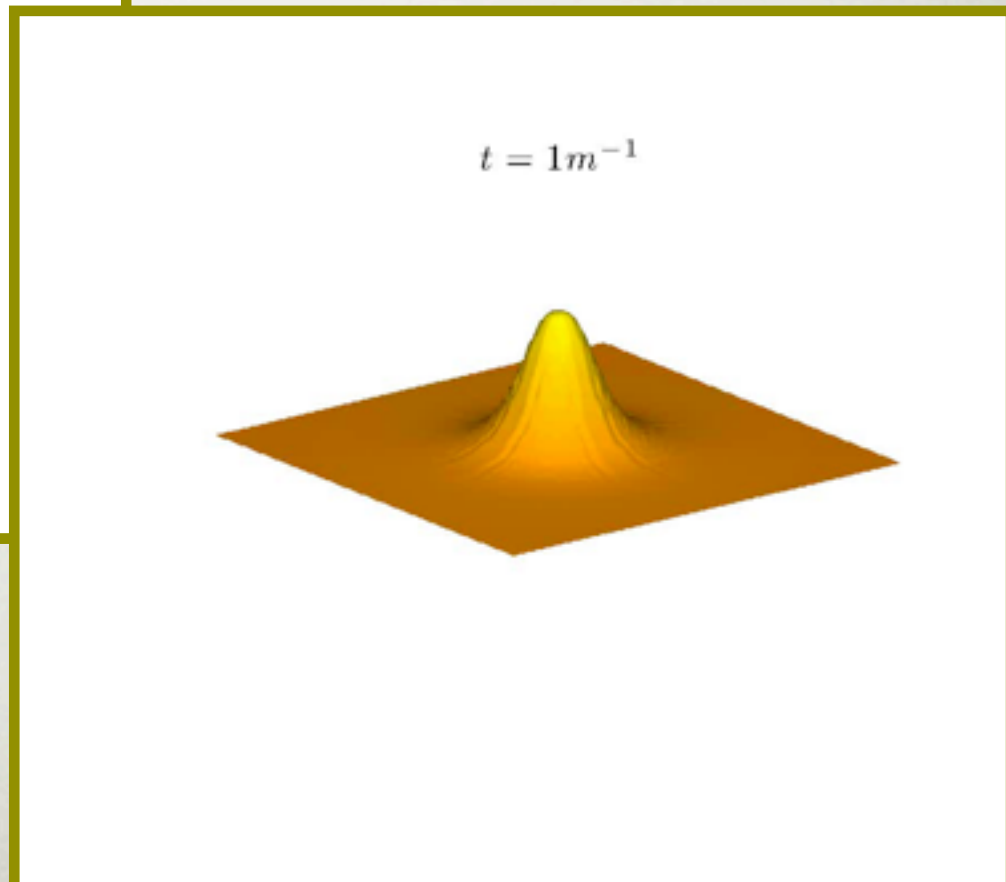
for some range of φ



Bogolubsky & Makhankov 1976, *Gleiser* 1994, *Copeland* et al. 1995, ..., MA & Shirokoff 2010, MA 2013

this is unusual!





Movie: courtesy of A Speranza (undergrad@MIT)

why ?

$$\partial_t^2\varphi - \partial_x^2\varphi + V'(\varphi) = 0$$

localized periodic solution: $\varphi(t,x) \sim \Phi(x)\cos\omega t$

why ?

$$\partial_t^2\varphi - \partial_x^2\varphi + V'(\varphi) = 0$$

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localization: $\omega^2 < m^2$

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localization: $\omega^2 < m^2$

$$\left[(m^2 - \omega^2)\varphi\right] + \left[-\partial_x^2 \varphi\right] + \left[V'(\varphi) - m^2 \varphi\right] \sim 0$$

frequency curvature non-linearity

why ?

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+ve

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frequency curvature non-linearity

+ve +ve

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frequency curvature non-linearity

$+$ve $+$ve

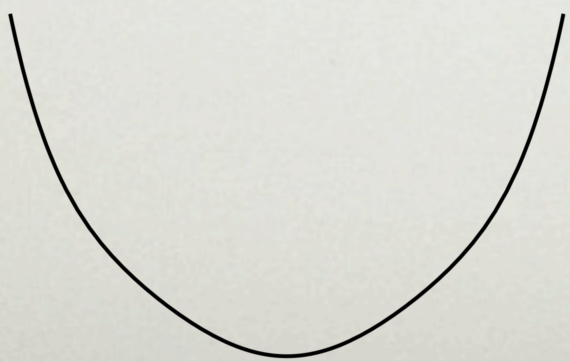
Need, at least somewhere $V'(\varphi) - m^2\varphi < 0$

necessary & sufficient condition
(for small amplitudes)

$$V(\varphi) = \frac{1}{2}\varphi^2 + \frac{1}{3}\lambda_3\varphi^3 + \frac{1}{4}\lambda_4\varphi^4 + \ldots$$

$$\Delta \equiv -\lambda_4 + \frac{10}{9}\lambda_3^2 > 0$$

symmetry breaking



axions, axion monodromy

necessary & sufficient condition

(for small amplitudes)

$$\mathcal{L} = T(X, \varphi) - V(\varphi)$$

$$T(X, \varphi) = X + \xi_2 X^2 + \xi_3 \varphi X^2 + \dots$$

$$V(\varphi) = \frac{1}{2}\varphi^2 + \frac{\lambda_3}{3}\varphi^3 + \frac{\lambda_4}{4}\varphi^4 + \frac{\lambda_5}{5}\varphi^5 + \dots$$

$$\boxed{\Delta = \xi_2 - \lambda_4 + \frac{10}{9}\lambda_3^2 > 0.}$$

MA (2013)

numerical simulations for DBI case in progress: Michael Pearce, undergrad@MIT

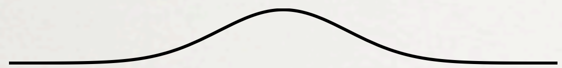
controlled analytic solution

small amplitude

$$\varphi(t,r) = \epsilon f(\epsilon r)\cos\left[\sqrt{1-\epsilon^2}t\right] + \mathcal{O}[\epsilon^2]$$

controlled analytic solution

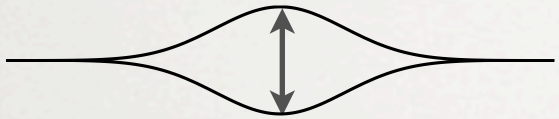
small amplitude



$$\varphi(t,r) = \epsilon f(\epsilon r) \cos\left[\sqrt{1-\epsilon^2}t\right] + \mathcal{O}[\epsilon^2]$$

controlled analytic solution

small amplitude



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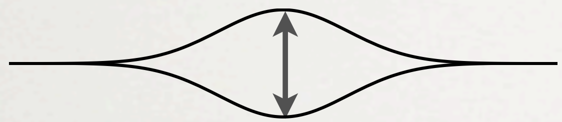
controlled analytic solution

small amplitude



$$\varphi(t,r) = \epsilon f(\epsilon r) \cos\left[\sqrt{1-\epsilon^2} t\right] + \mathcal{O}[\epsilon^2]$$

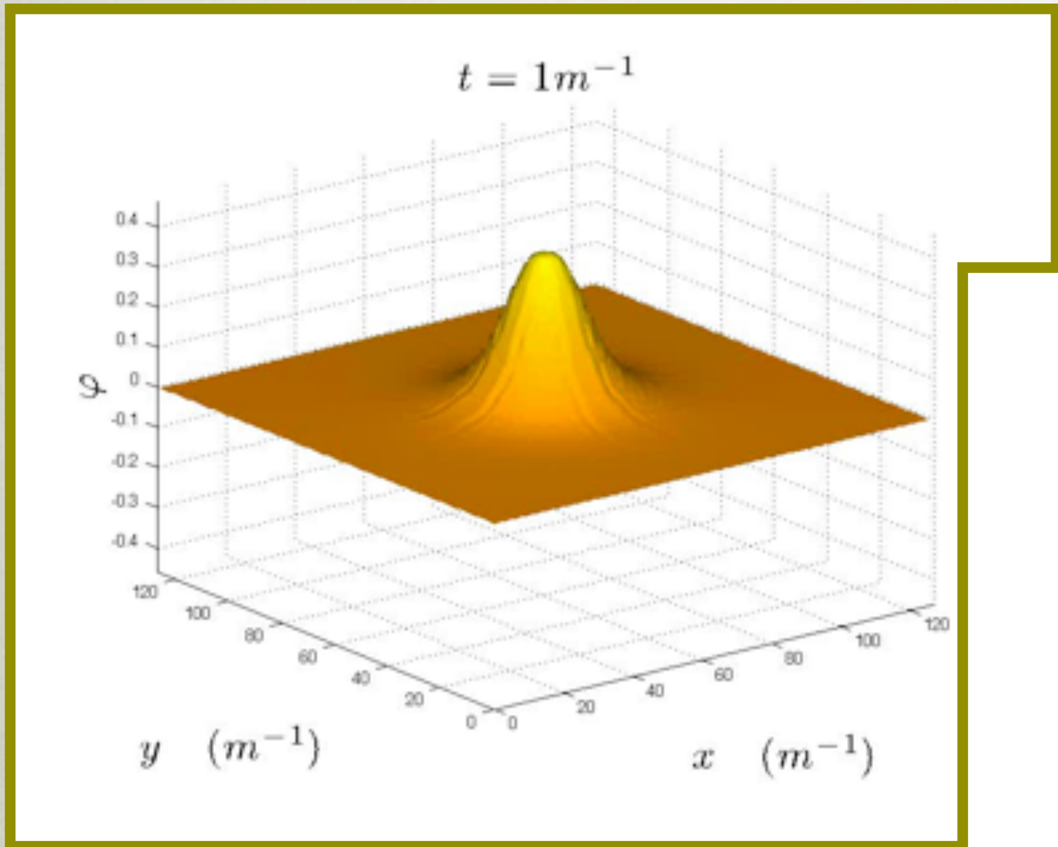
$$f_1(\epsilon r) = \sqrt{\frac{8}{3\Delta}} \operatorname{sech}(\epsilon r)$$ 1+1

$$f_d(\epsilon r) = \sqrt{\frac{8}{3\Delta}} F(\epsilon r)$$ d+1

$$\Delta = \xi_2 - \lambda_4 + \frac{10}{9}\lambda_3^2 > 0$$

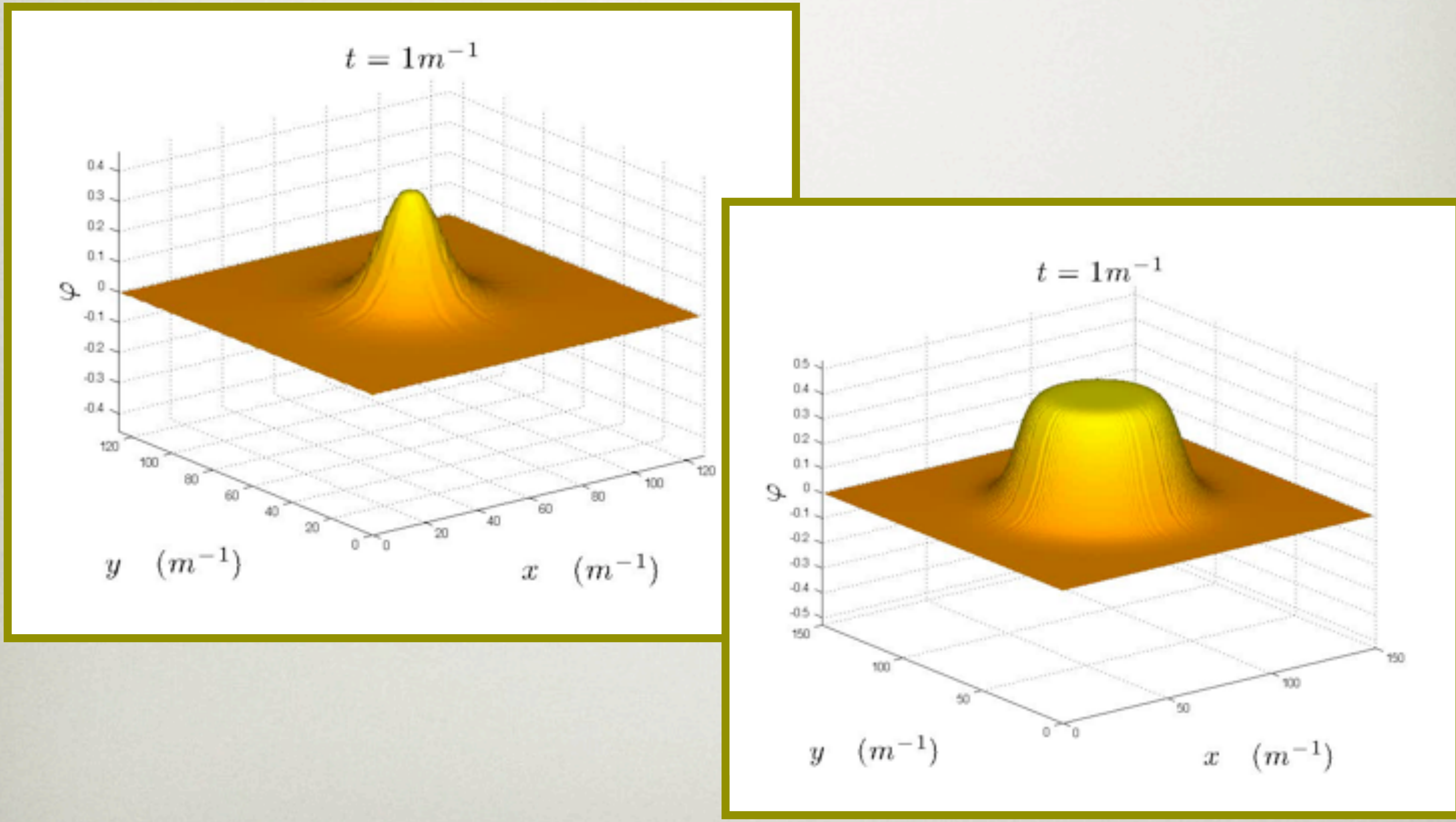
approximate solutions possible for larger amplitudes but not under analytic control, exception: flat tops

solutions



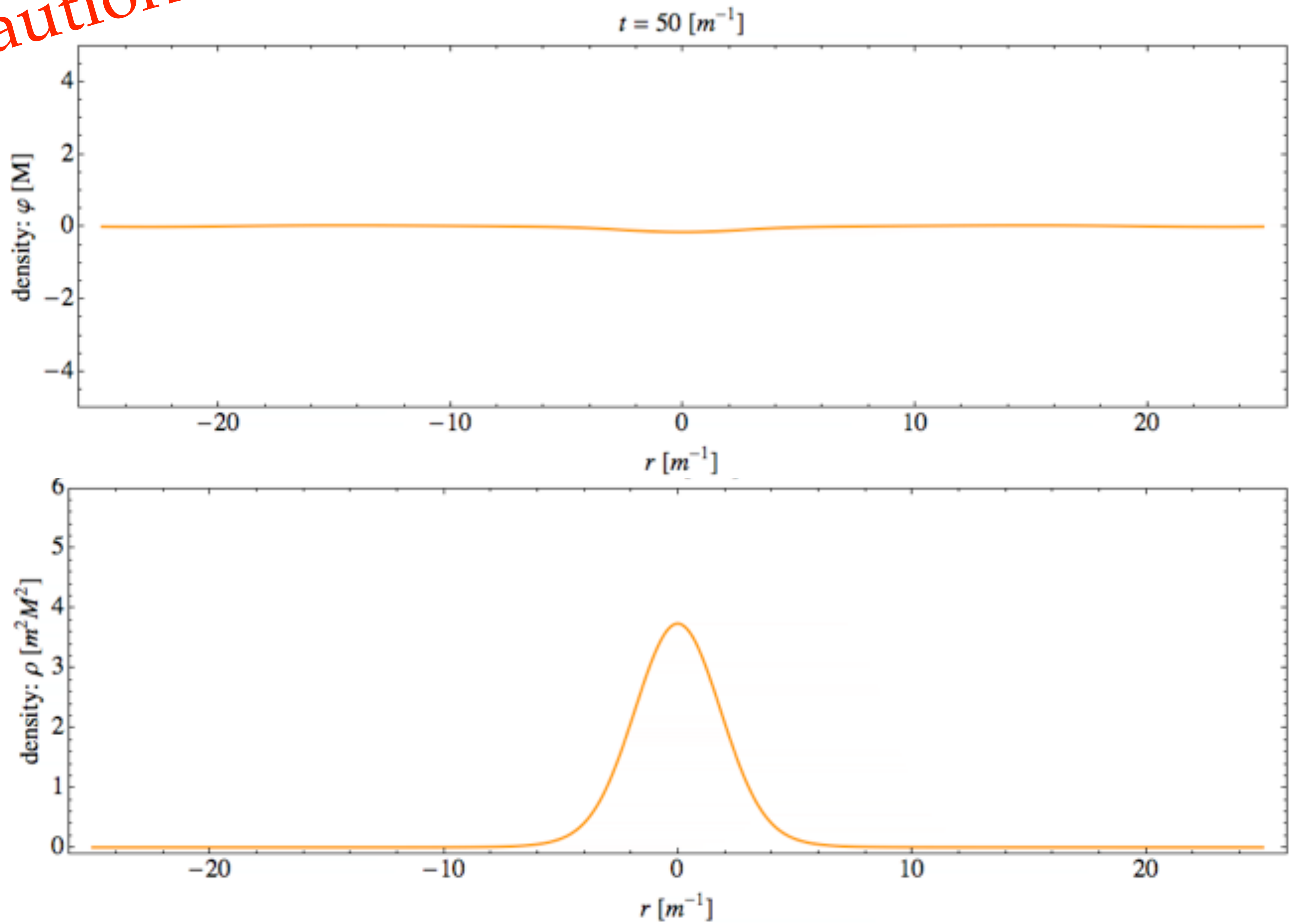
MA &. D.Shirokoff (2010)

solutions



MA &. D.Shirokoff (2010)

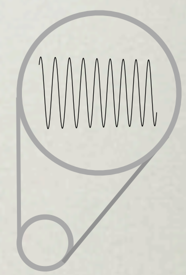
caution


$t = 50\ [m^{-1}]$
density: φ [M]
4
2
0
−2
−4
−20
−10
0
10
20
$r\ [m^{-1}]$
density: $\rho\ [m^2M^2]$
6
5
4
3
2
1
0
−20
−10
0
10
20
$r\ [m^{-1}]$

outgoing radiation

- classical : *Kruskal and Segur (1987)*

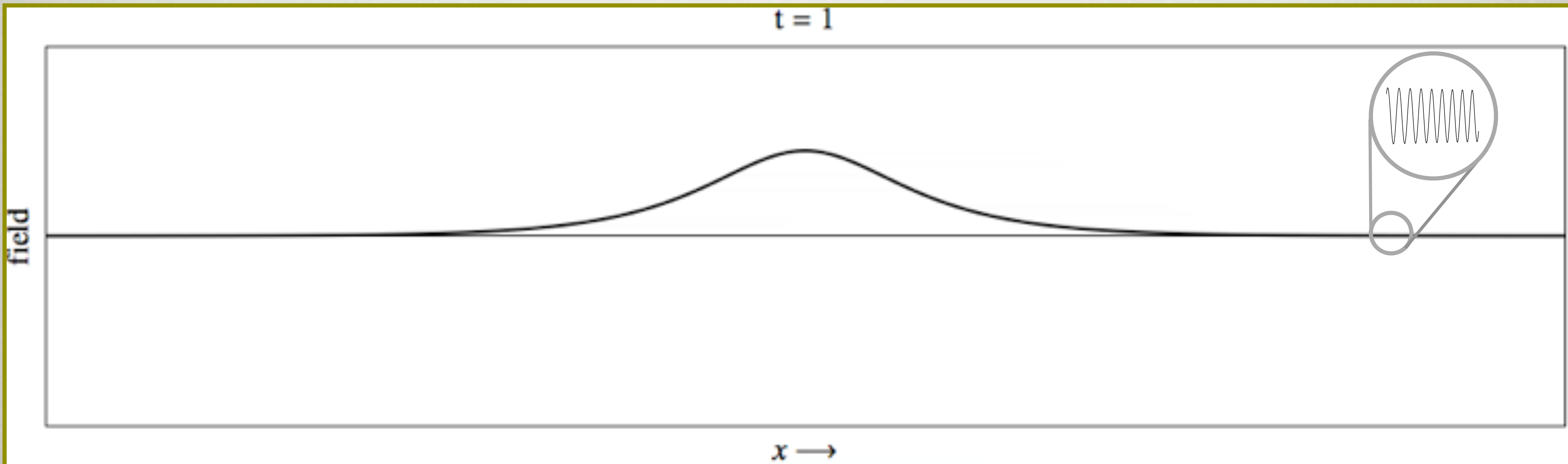
radiative tail



outgoing radiation

- classical : *Kruskal and Segur (1987)*

radiative tail



decay rate extremely small!

$$\Gamma = \frac{1}{E_{osc}} \frac{dE_{osc}}{dt} \sim \frac{1}{\epsilon} e^{-\frac{a}{\epsilon}} \times m$$

$\epsilon =$ central amplitude of oscillons

$a =$ order one number

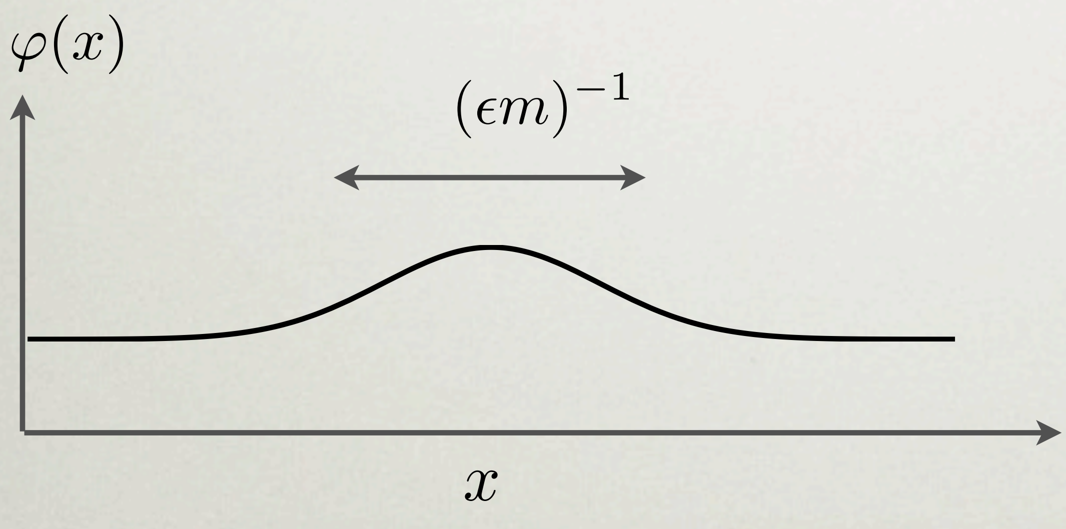
valid for small amplitude, classical only

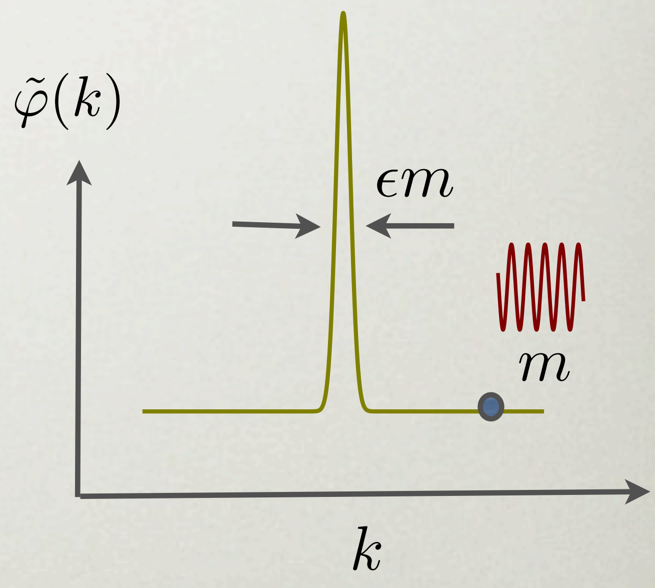
Kruskal and Segur (1987)

small decay rate: why?

$$\frac{dE_{osc}}{dt} \sim |\tilde{\varphi}(k_{rad})|^2$$

$$k_{rad} \sim m$$

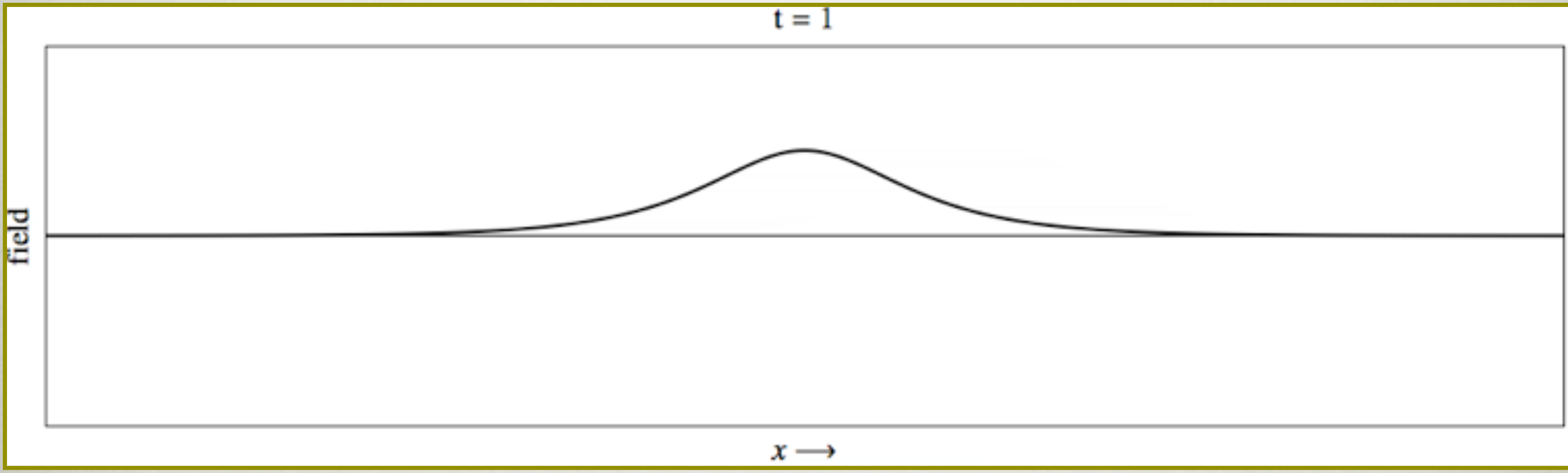




Hertzberg 2010

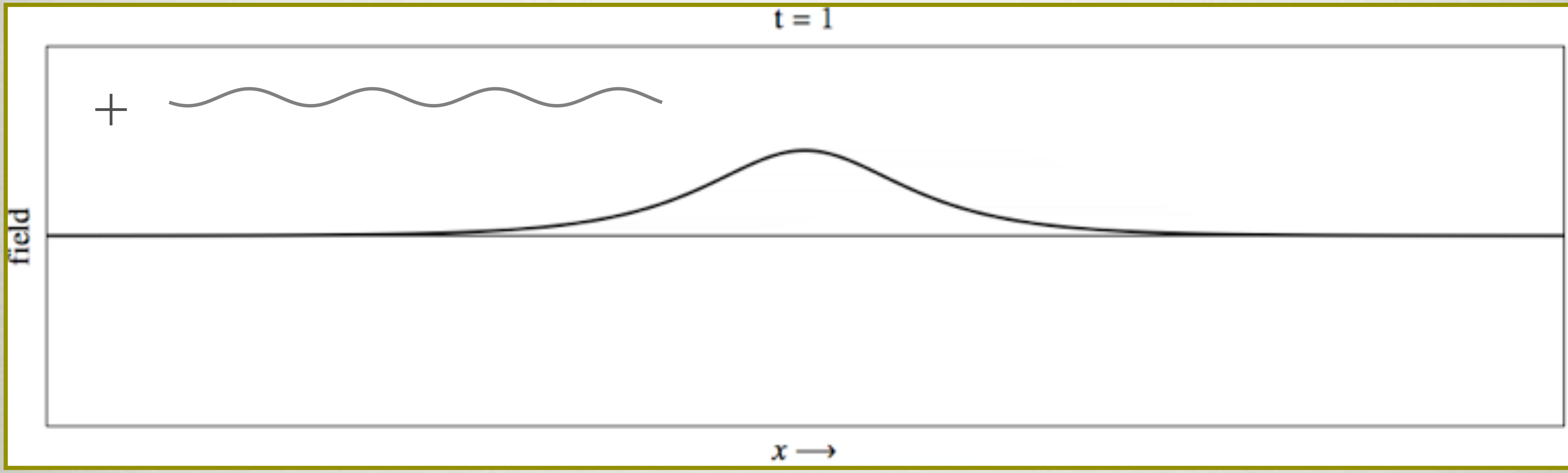
what about stability ?

what about stability ?



what about stability ?

perturbations



linear stability analysis

- *collapse* instability $\quad \text{wavelength} \sim \text{width}$
- *Floquet* instability ? $\quad \text{wavelength} \ll \text{width}$

MA &. D.Shirokoff (2010)

linear stability analysis

small amplitude oscillons stable iff

$$\frac{d}{d\epsilon}\int \Phi^2(\epsilon, r)d^d r > 0$$

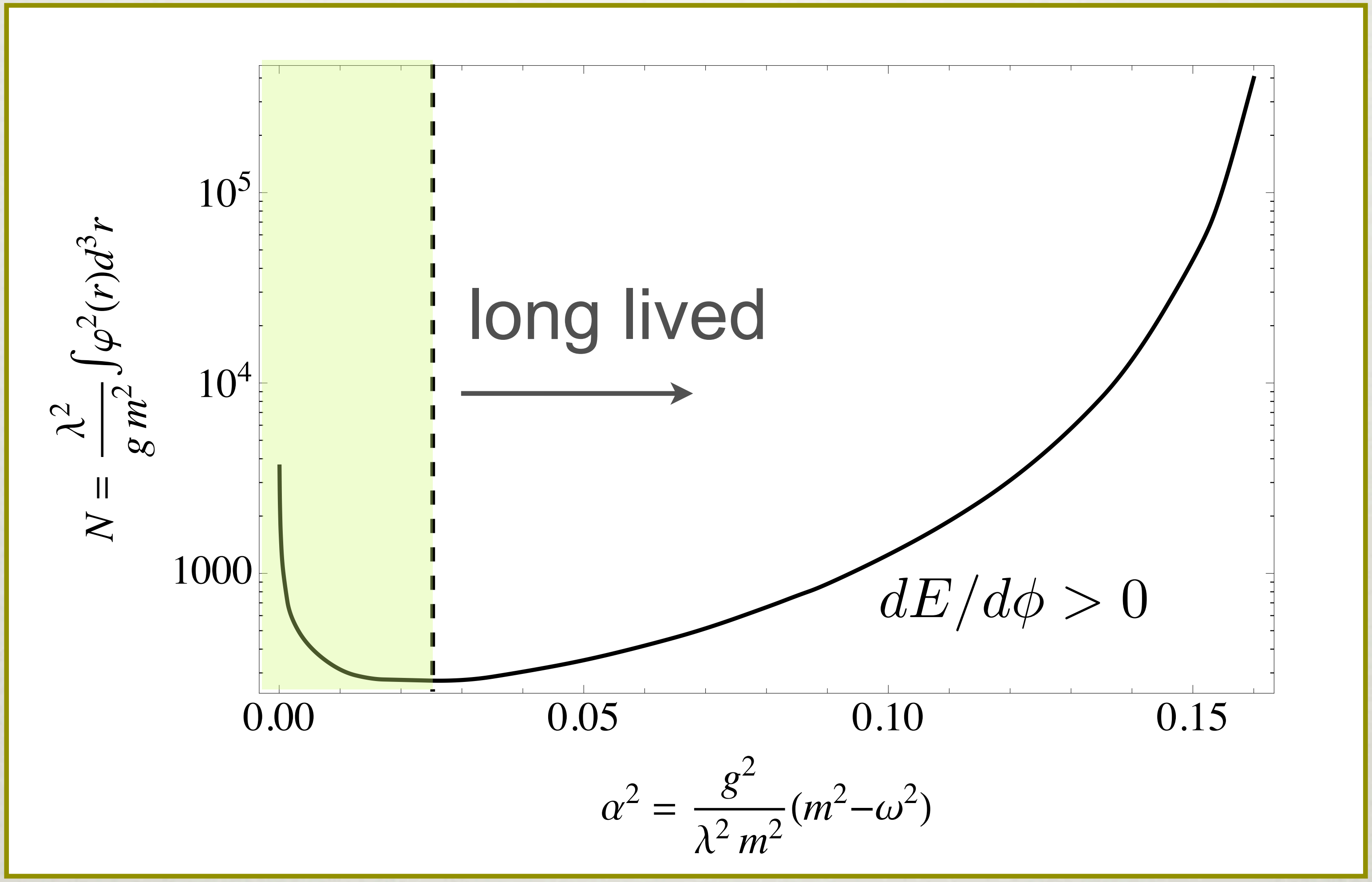
$$\varphi(t,r) = \Phi(\epsilon, r)\cos\left[\sqrt{1-\epsilon^2}t\right] + \ldots$$

MA &. D.Shirokoff (2010)
MA (2013)

Note: Proof is the same as Vakhitov & Kolokolov (1975) who were investigating light focusing in a nonlinear medium.

stable iff

~ energy

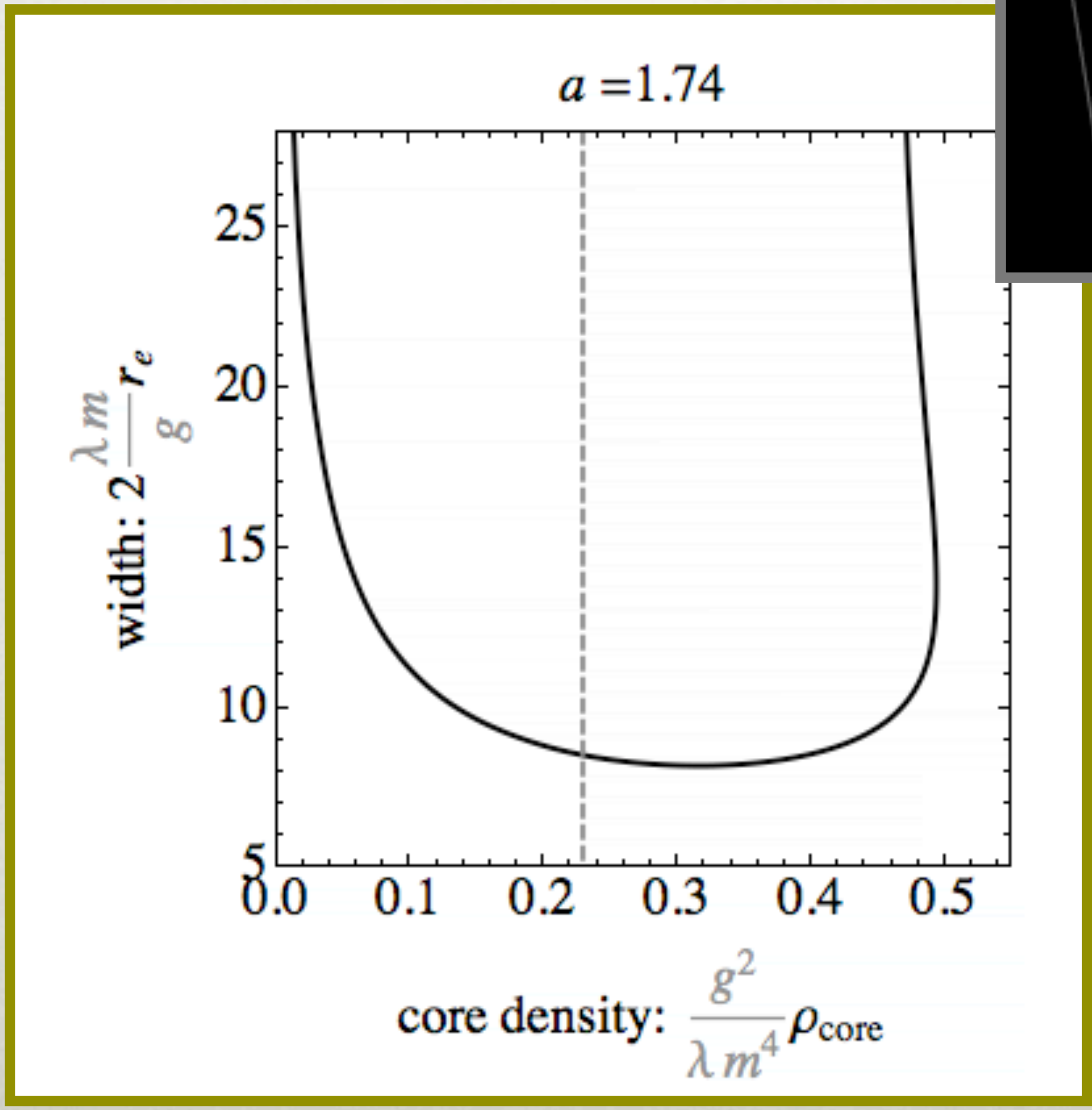


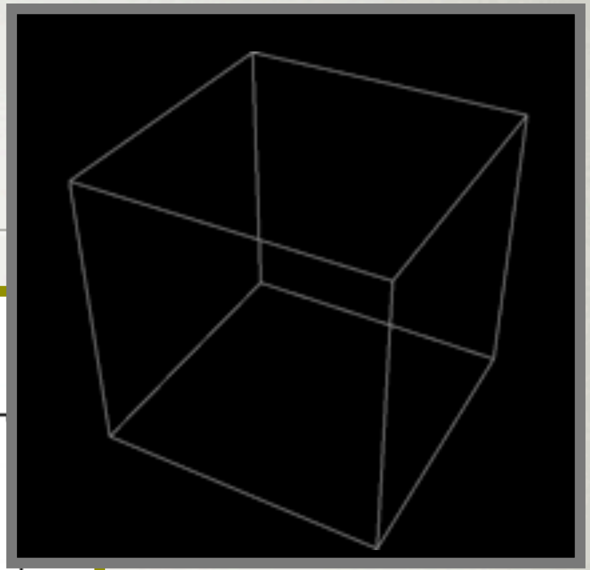
~ amplitude

caution! dimension dependent

MA &. D.Shirokoff (2010)

stability





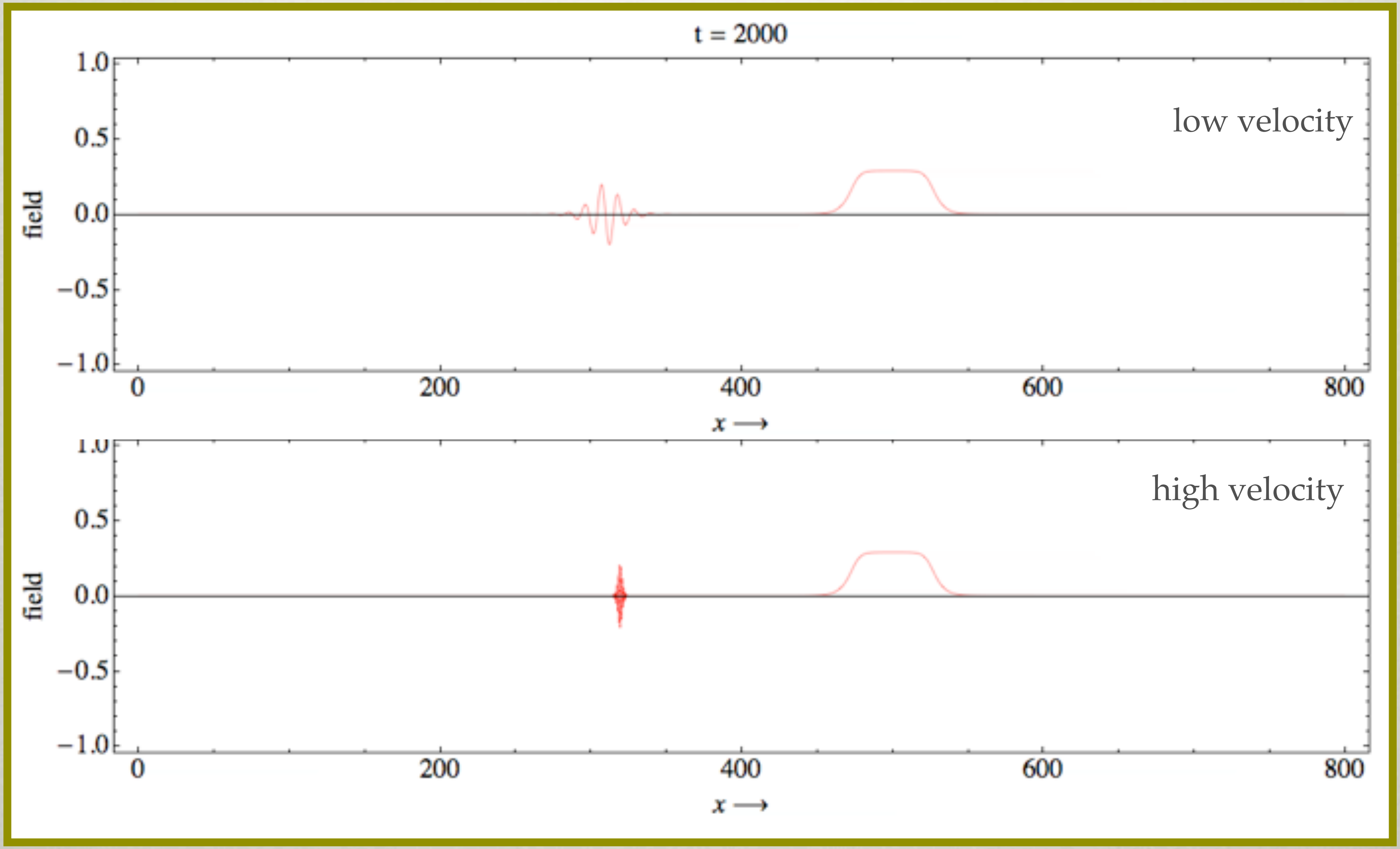
MA, Easther and Finkel (2010)

social life of oscillons

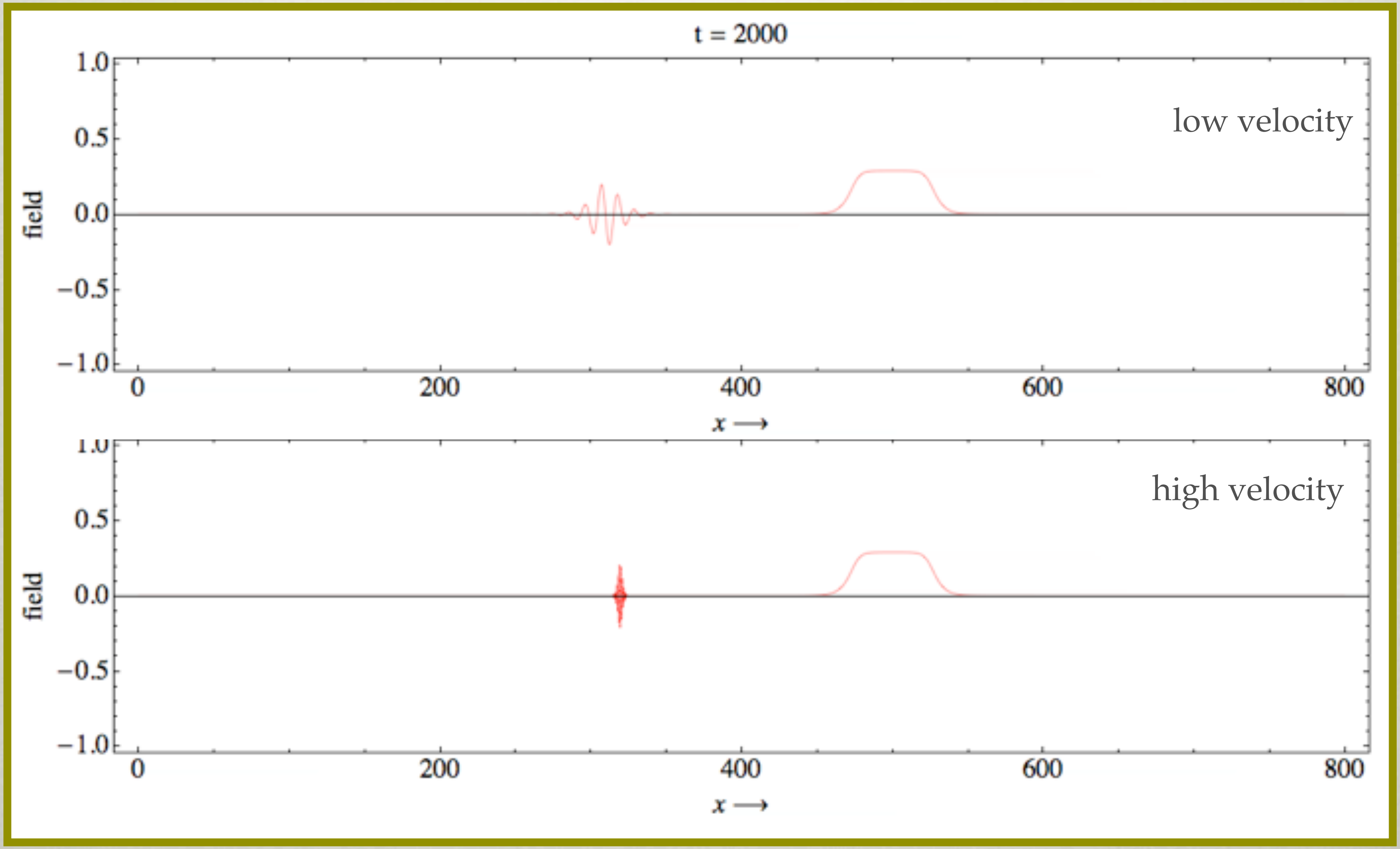
low velocity

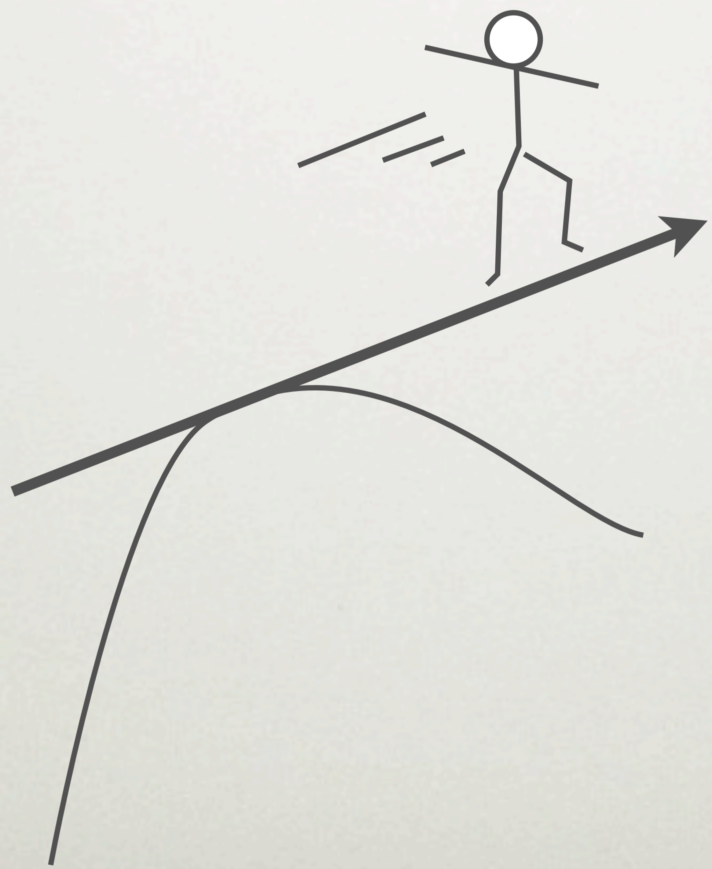
high velocity

social life of oscillons

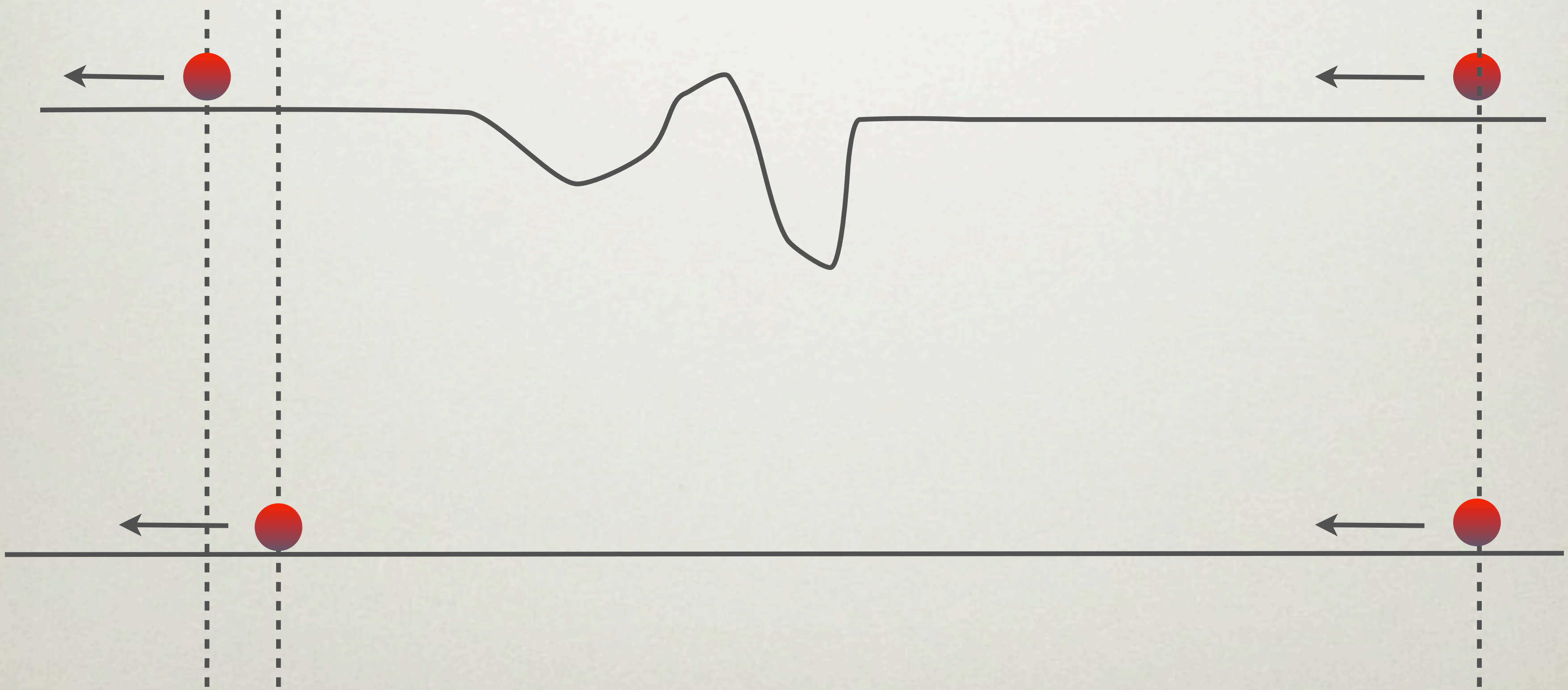


social life of oscillons



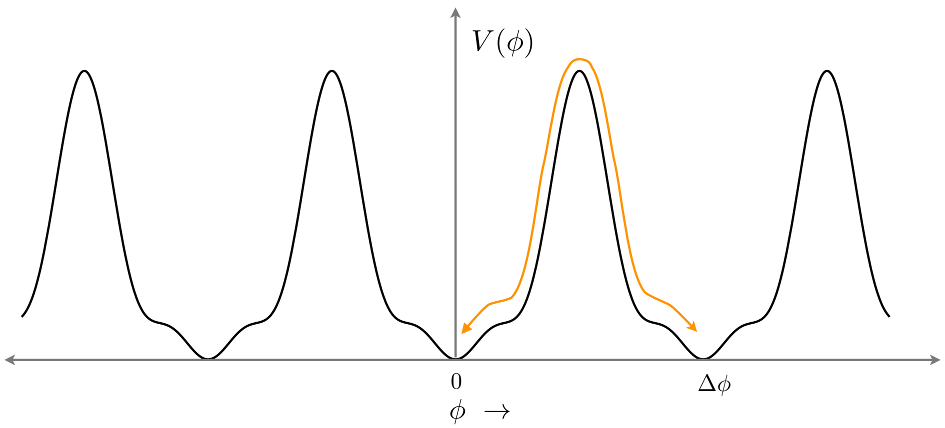


elastic interactions



a scattering theory of "solitons"

in progress

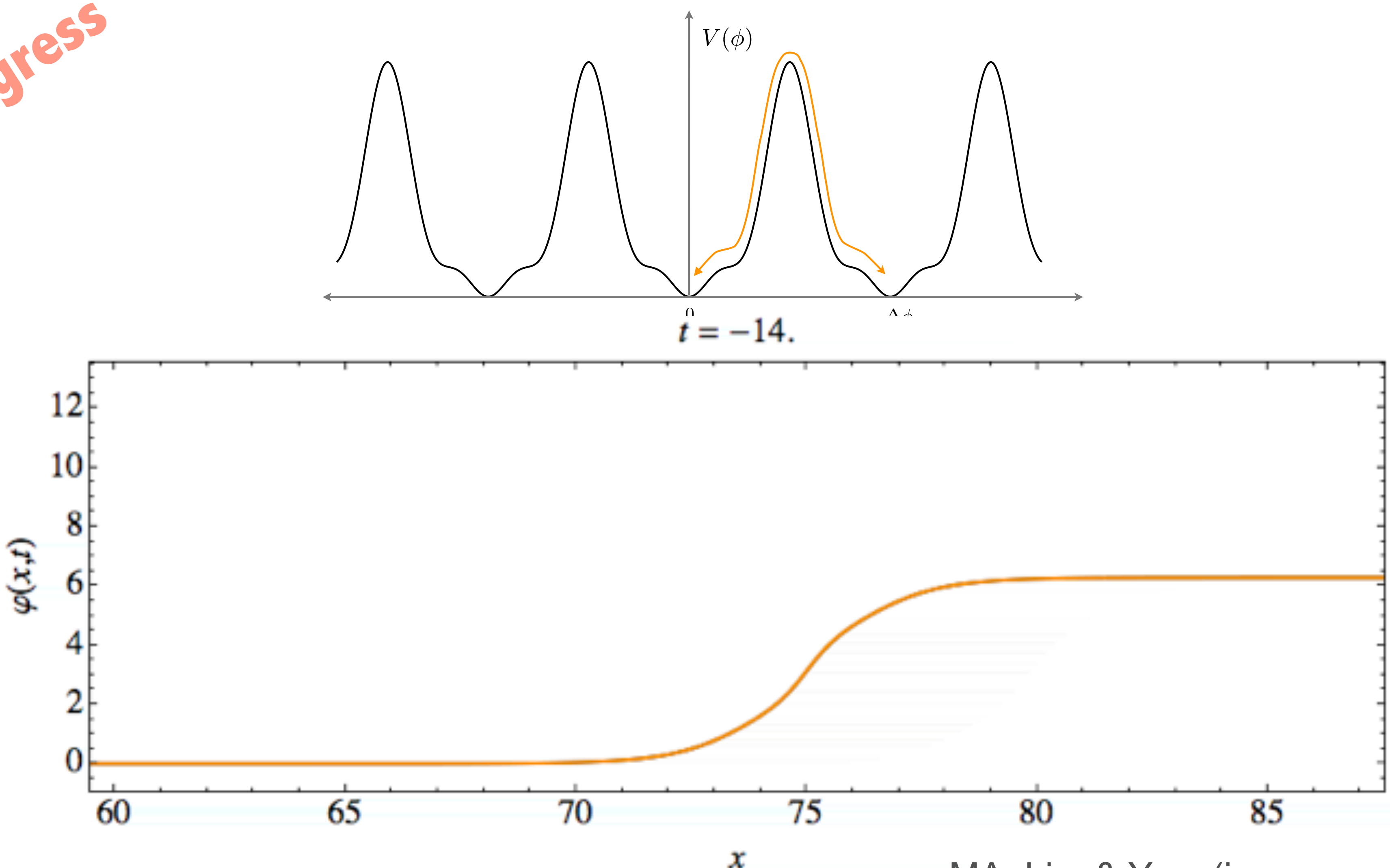


MA, Lim & Yang(in progress)

related: "bubble collisions" in Giblin et. al 2010

a scattering theory of “solitons”

in progress



MA, Lim & Yang(in progress)

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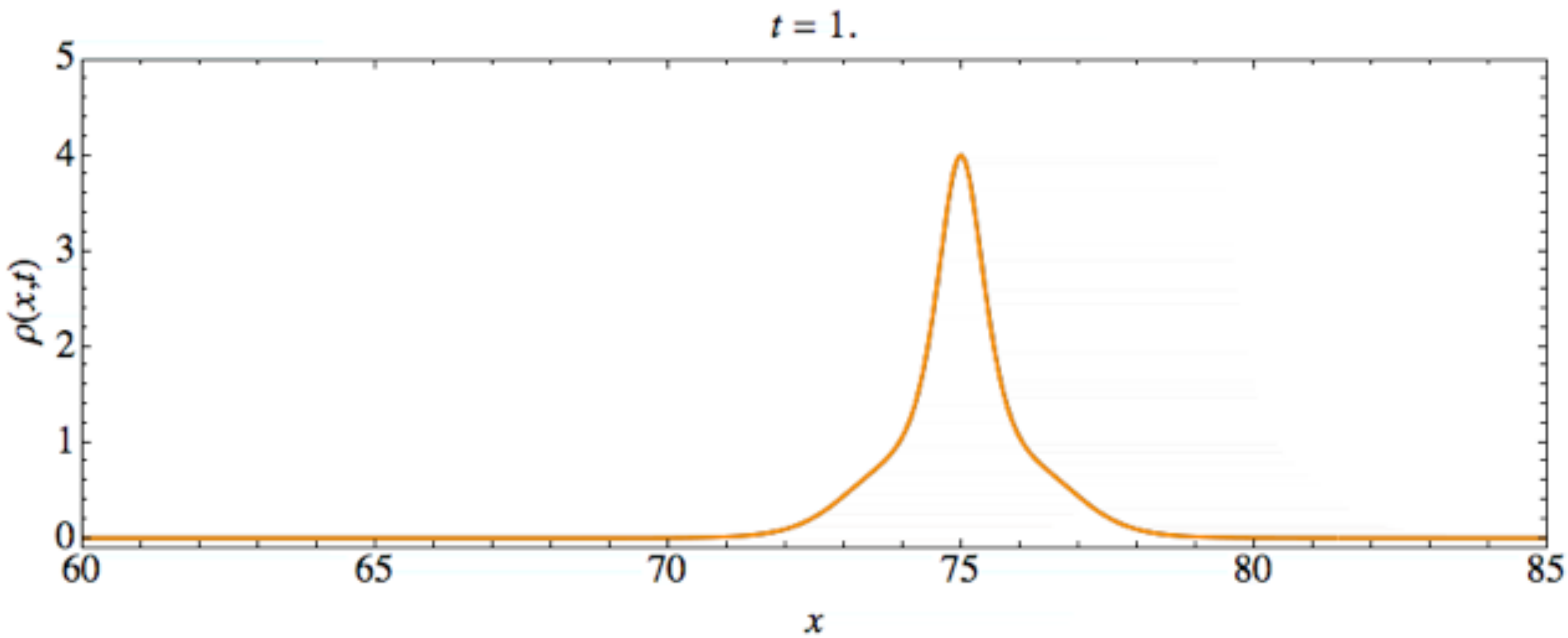
MA, Lim & Yang(in progress)

a scattering theory of "solitons"



in progress

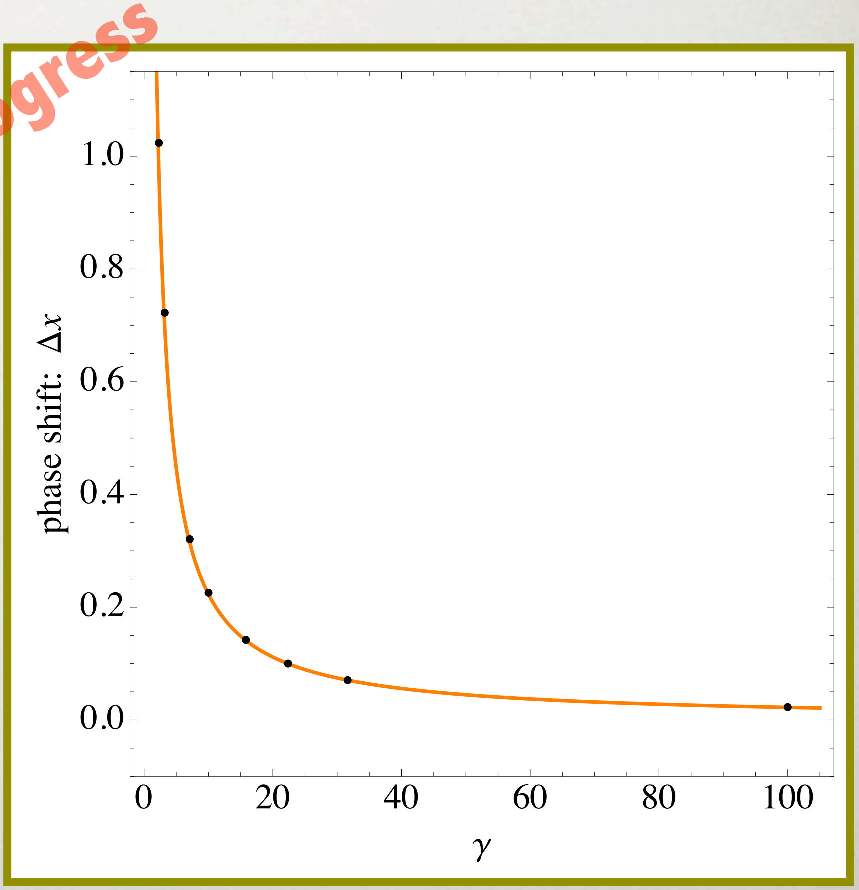
MA, Lim & Yang(in progress)

a scattering theory of "solitons"

$$\Delta x = \frac{1}{2\gamma M}\int_0^{\Delta\phi}\int_0^{\Delta\phi} d\phi_1 d\phi_2 \left[\frac{V(\phi_1)+V(\phi_2)-V(\phi_1+\phi_2)}{\sqrt{V(\phi_1)V(\phi_2)}}\right] + \mathcal{O}[\gamma^{-2}]$$

$$\Delta v = 0 + \mathcal{O}[\gamma^{-2}]$$

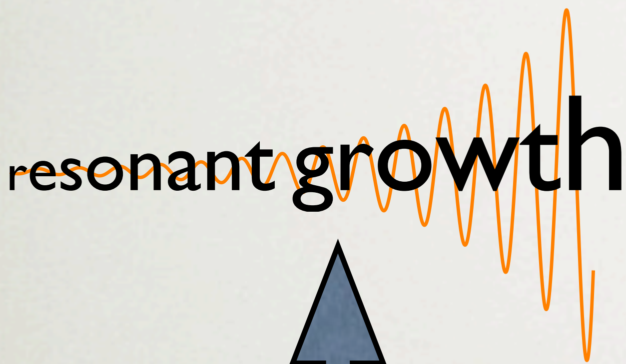
rate of convergence might depend on the models



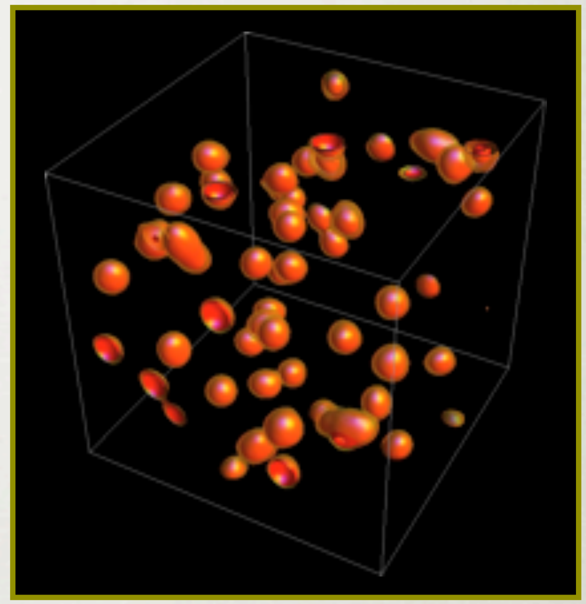
Note: checked for kinks not oscillons

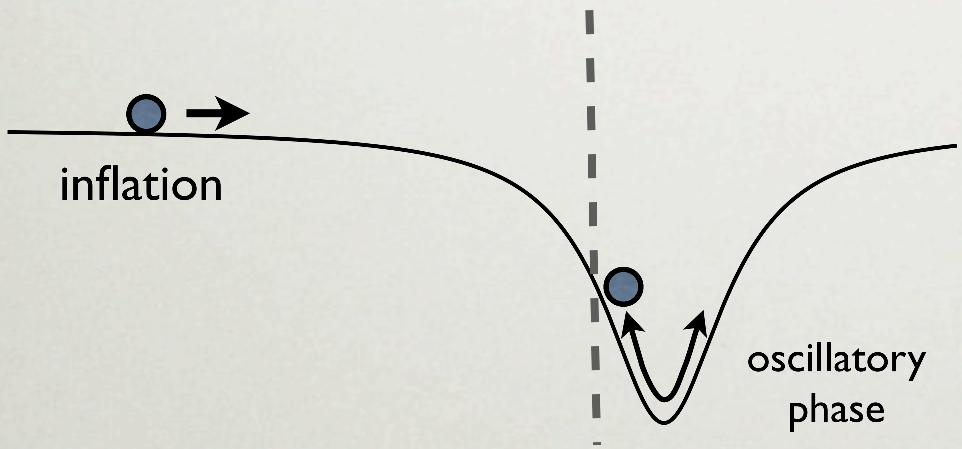
MA, Lim & Yang(in progress)

summary



>50%!





implications:

1. bottleneck for reheating ??
2. g-waves (high-frequency)
3. expansion history (degenerate)
4. non-gaussianity ??

papers and collaborators

1002.3380 MA and D. Shirokoff (solutions and stability)
1006.3075 MA (emergence & number density: 1d)

1009.2505 MA, Easther and Finkel (emergence & energy fraction: 3d)
1106.3335 MA, Easther, Finkel, Flaugher and Hertzberg (flattened potentials)

1303.1102 MA (non-canonical kinetic terms)

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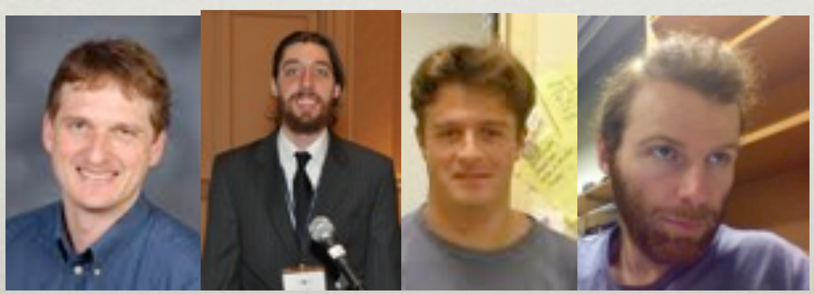
1305.???? MA, Lim and Yang (interactions)

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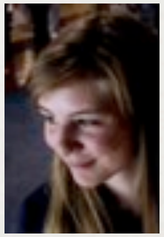
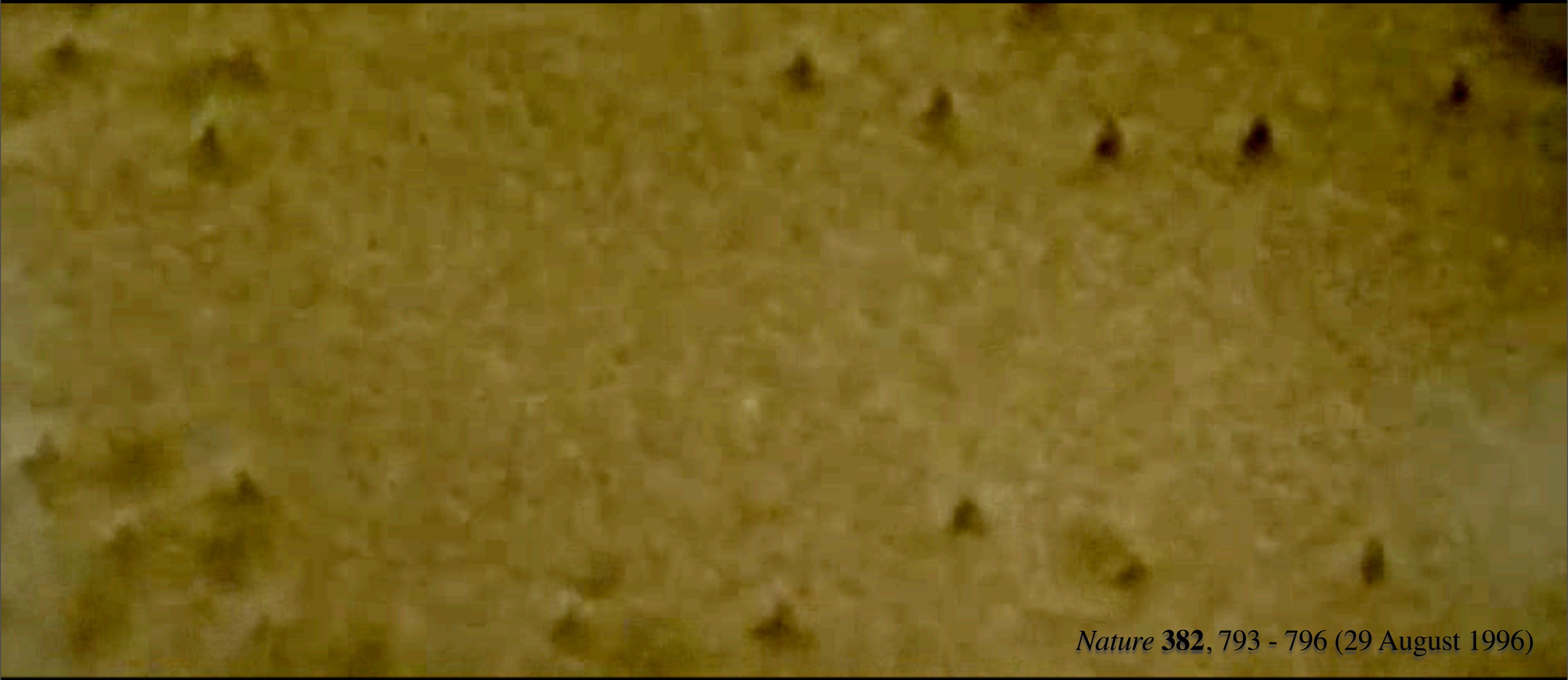
undergraduate collaborators

1) Delay in radiation domination
with H. Child (undergrad @ Kenyon) & J. Giblin



2) simulating DBI oscillons
M Pearce (undergrad @ MIT)

3) Oscillon lifetimes and interactions:
Thomas Roxlo & Antony Speranza (undergrads @ MIT)


Nature **382**, 793 - 796 (29 August 1996)

Paola Rebusco


Paola Rebusco