Planck2013 and Superconformal Symmetry

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Santa Barbara, June 19, 2013

Outline

Spontaneously broken superconformal symmetry is useful to describe inflationary models favored by the data

Universality of Conformal Inflation (T-Model and beyond)

Superconformal Models

Small non-minimal coupling is `technically natural'

$$\frac{\lambda}{4}\phi^4 - \frac{\xi}{2}\phi^2 R$$

Universality of the Superconformal T-Model

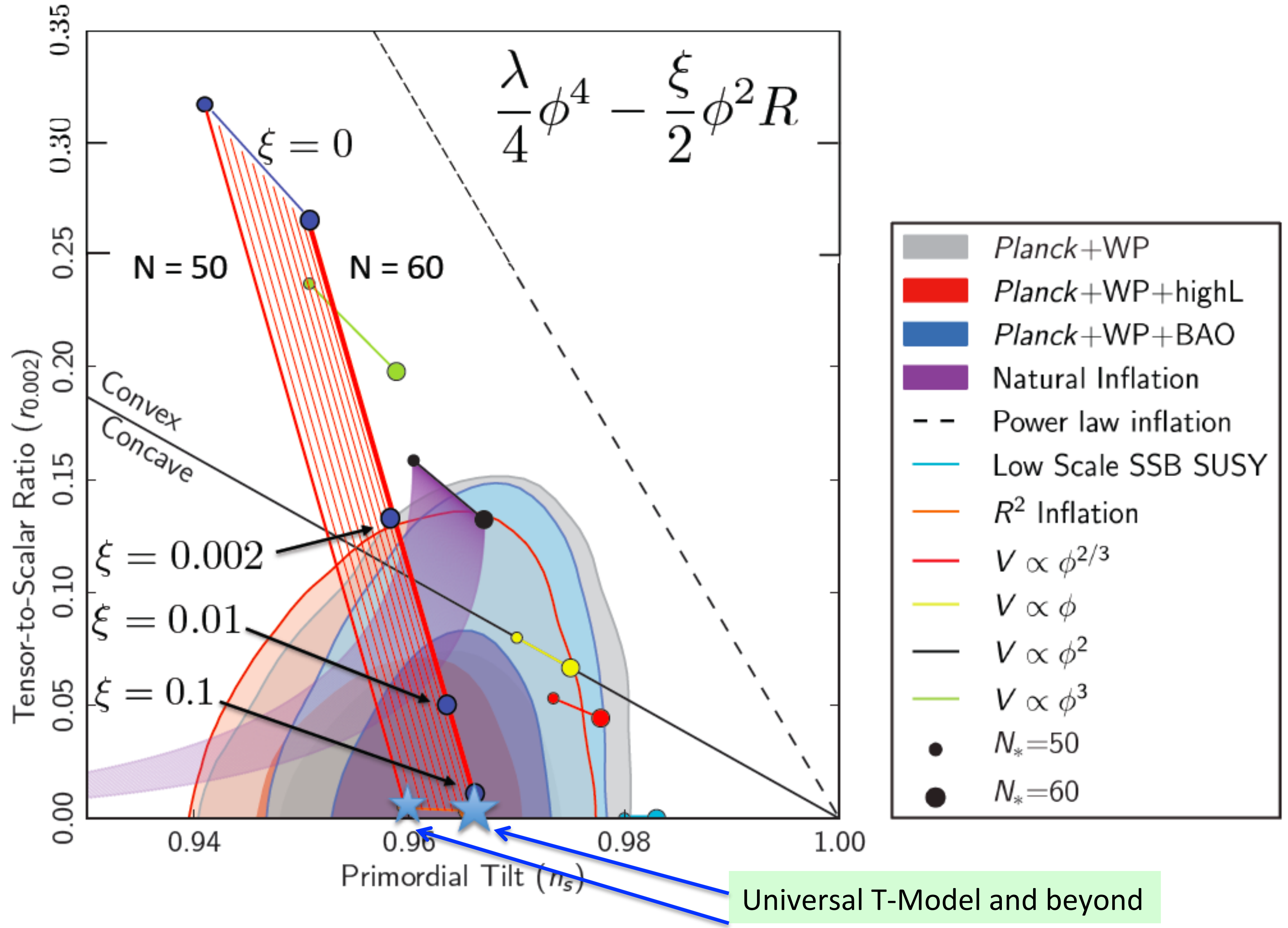
Superconformal critical phenomena in Cosmology

Universality of conformal inflation

We develop a new class of chaotic inflation models with spontaneously broken conformal invariance. Observational consequences of a broad class of such models are stable with respect to strong deformations of the scalar potential.

In this class of models, inflation is possible even in the theories with very steep potentials because of their exponential flattening at the boundary of the moduli space. This effect can be described as inflation of moduli space, which exponentially stretches and flattens the inflaton potential.

In this sense, slow roll inflation **in** the landscape is facilitated by inflation **of** the landscape.



$\frac{\lambda}{4}\phi^4 - \frac{\xi}{2}\phi^2 R$
$\xi = 0$
N = 50
N = 60
Convex
Concave
$\xi = 0.002$
$\xi = 0.01$
$\xi = 0.1$
Tensor-to-Scalar Ratio ($r_{0.002}$)
Primordial Tilt (n_s)
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.94
0.96
0.98
1.00
Planck+WP
Planck+WP+highL
Planck+WP+BAO
Natural Inflation
Power law inflation
Low Scale SSB SUSY
R^2 Inflation
$V \propto \phi^{2/3}$
$V \propto \phi$
$V \propto \phi^2$
$V \propto \phi^3$
N_*=50
N_*=60
Universal T-Model and beyond

De Sitter from spontaneously broken conformal symmetry

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}\partial_\mu\chi\partial_\nu\chi\, g^{\mu\nu} + \frac{\chi^2}{12}R(g) - \frac{\lambda}{4}\chi^4\right]$$

This theory is locally conformal invariant

$$\tilde{g}_{\mu\nu} = \mathrm{e}^{-2\sigma(\mathrm{x})}\mathrm{g}_{\mu\nu}\,, \qquad \tilde{\chi} = \mathrm{e}^{\sigma(\mathrm{x})}\chi$$

The field chi(x) is referred to as a conformal compensator, which we will call `conformon.' It has negative sigh kinetic term, but this is not a problem because it can be removed from the theory by fixing the gauge symmetry, for example

$$\chi = \sqrt{6}$$

This gauge fixing can be interpreted as a spontaneous breaking of conformal invariance due to existence of a classical field $\chi = \sqrt{6}$

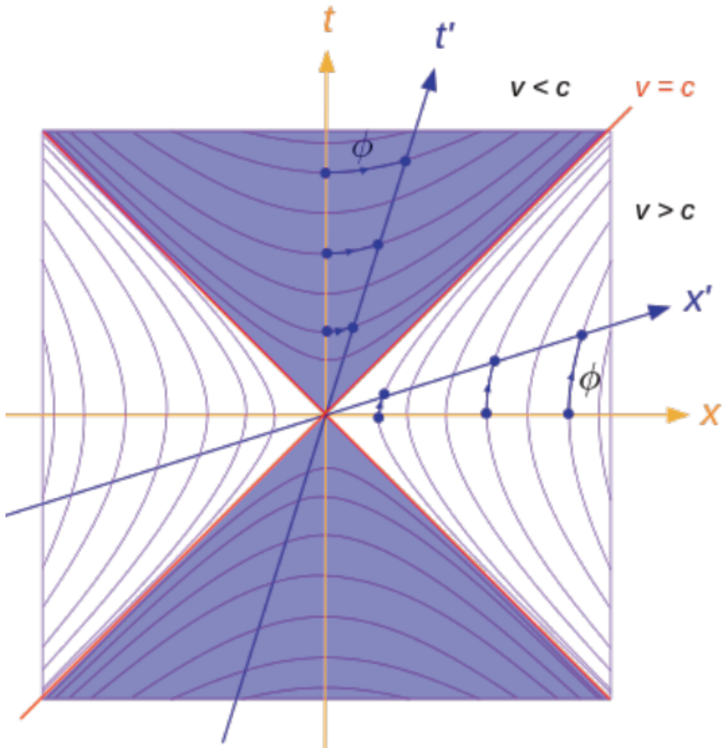
The action in this gauge: dS or AdS

$$\mathcal{L} = \sqrt{-g}\left[\frac{R(g)}{2} - 9\lambda\right]$$

SPECIAL RELATIVITY REMINDER

$$\gamma \frac{v}{c} = \sinh\varphi$$

$$\gamma = \cosh\varphi$$



$$v = 0\,, \qquad \cosh\varphi = 1\,, \qquad \varphi = 0$$

$$v < c\,, \qquad \cosh\varphi > 1\,, \qquad |\varphi| > 0$$

$$v \to c\,, \qquad \cosh\varphi \to \infty\,, \qquad |\varphi| \to \infty$$

$$\tanh\varphi = \frac{v}{c}$$

φ is called rapidity

$$ct' = ct\cosh\varphi - x\sinh\varphi$$

$$x' = -ct\sinh\varphi + x\cosh\varphi$$

The simplest conformally invariant two-field model of dS or AdS space and the SO(1,1) invariant conformal gauge

$$\mathcal{L}_{\rm toy} = \sqrt{-g}\left[\frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \frac{\chi^2}{12}R(g) - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\phi^2}{12}R(g) - \frac{\lambda}{4}(\phi^2-\chi^2)^2\right]$$

Local conformal symmetry

$$\tilde{g}_{\mu\nu} = \mathrm{e}^{-2\sigma(\mathrm{x})}\mathrm{g}_{\mu\nu}\,, \qquad \tilde{\chi} = \mathrm{e}^{\sigma(\mathrm{x})}\chi\,, \qquad \tilde{\phi} = \mathrm{e}^{\sigma(\mathrm{x})}\phi$$

The global SO(1,1) transformation is a boost between these two fields.

SO(1,1) invariant conformal gauge $\chi^2 - \phi^2 = 6$ Rapidity gauge

This gauge condition represents a hyperbola which can be parameterized by a canonically normalized field φ

$$\chi = \sqrt{6}\cosh\frac{\varphi}{\sqrt{6}}\,, \qquad \phi = \sqrt{6}\sinh\frac{\varphi}{\sqrt{6}}$$

The action in this gauge, dS/AdS

$$L = \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - 9\lambda\right]$$

Chaotic inflation from conformal theory: T-Model

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \frac{\chi^2}{12}R(g) - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\phi^2}{12}R(g) - \frac{1}{36}F\left(\phi/\chi\right)\left(\phi^2-\chi^2\right)^2\right]$$

Here F is an arbitrary function of the ratio phi/chi. When this function is present, it breaks the SO(1,1) symmetry of the de Sitter model. Note that this is the only possibility to keep local conformal symmetry and to deform the SO(1,1) symmetry!

In rapidity gauge it becomes

$$L = \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - F(\tanh\varphi)\right]$$

The attractor behavior near a critical point where SO(1,1) symmetry is restored is the following: start with generic F(tanh), always get

$$n_s \approx 0.967 \qquad r \approx 0.0032$$

T-Model

$$F\left(\phi/\chi\right) = \lambda\left(\phi/\chi\right)^{2n}$$

$$V(\varphi) = \lambda_n \tanh^{2n}(\varphi/\sqrt{6})$$

Functions $\tanh^{2n}(\varphi/\sqrt{6})$ are symmetric with respect to $\varphi \to -\varphi$, but to study inflationary regime in this model at $\varphi \gg 1$, it is convenient to represent them as follows:

$$V(\varphi) = \lambda\left(\frac{1 - e^{-\sqrt{2/3}\,\varphi}}{1 + e^{-\sqrt{2/3}\,\varphi}}\right)^{2n} = \lambda\left(1 - 4n\, e^{-\sqrt{2/3}\,\varphi} + O\left(n^2\, e^{-2\sqrt{2/3}\,\varphi}\right)\right)$$

The slow-roll equation for the field φ at $\varphi \gg 1$ in terms of the large e-folding number N is

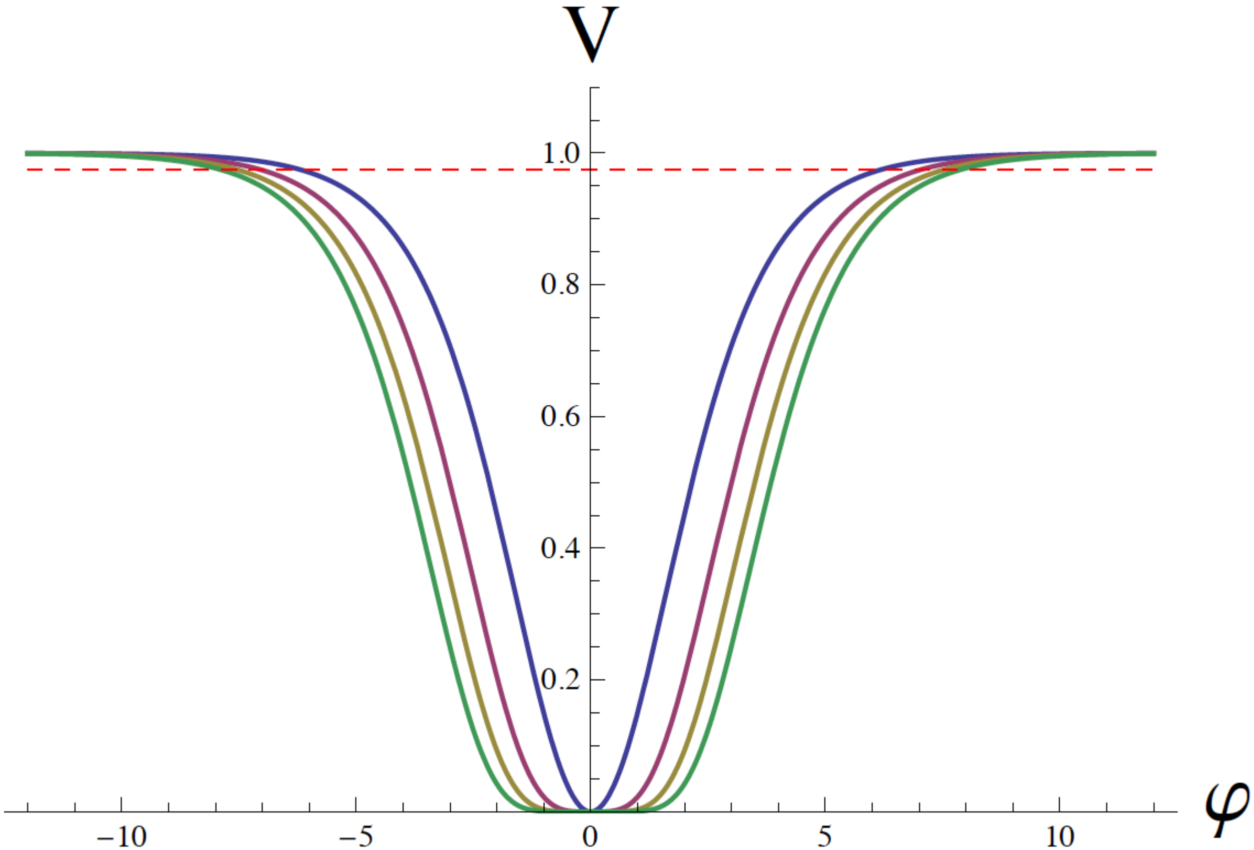
$$\frac{d\varphi}{dN} = \frac{V'}{V} = 4n\sqrt{\frac{2}{3}}\; e^{-\sqrt{2/3}\,\varphi}$$

For large N, this leads to a relation

$$e^{-\sqrt{2/3}\,\varphi(N)} = \frac{3}{8n\,N}$$

Therefore for a given N, one has

$$\frac{V'}{V} = 4n\sqrt{\frac{2}{3}}\; e^{-\sqrt{2/3}\,\varphi(N)} = \frac{3}{2N}\sqrt{\frac{2}{3}}$$



T-Model

Figure 1: Potentials for the T-Model inflation $\tanh^{2n}(\varphi/\sqrt{6})$ for $n = 1, 2, 3, 4$ (blue, red, brown and green, corresponding to increasingly wider potentials). We took $\lambda_n = 1$ for each of the potential for convenience of comparison. As we see, these potentials differ from each other quite considerably, especially at $\varphi \lesssim 1$: at small ϕ they behave as φ^{2n}. Nevertheless all of these models predict the same values $n_s = 1 - 2/N$, $r = 12/N^2$, in the leading approximation in $1/N$, where $N \sim 60$ is the number of e-foldings. The points where each of these potentials cross the red dashed line $V = 1 - 3/2N = 0.96$ correspond to the points where the perturbations are produced in these models on scale corresponding to $N = 60$.

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 = \frac{1}{2}\Big(4n\sqrt{\frac{2}{3}}\, e^{-\sqrt{2/3}\,\varphi(N)}\Big)^2 = \frac{1}{2}\Big(\frac{3}{2N}\sqrt{\frac{2}{3}}\Big)^2$$

This result, in the leading order in $1/N$, is valid for any n. The same is true for the slow roll parameter η, and, consequently, for the parameters n_s and r: for the set of T-Models described in this section,

$$1 - n_s = 2/N\,, \qquad r = 12/N^2$$

ATTRACTOR

in the leading approximation in $1/N$

T-Model

$$V(\varphi) = \lambda_n \tanh^{2n}(\varphi/\sqrt{6})$$

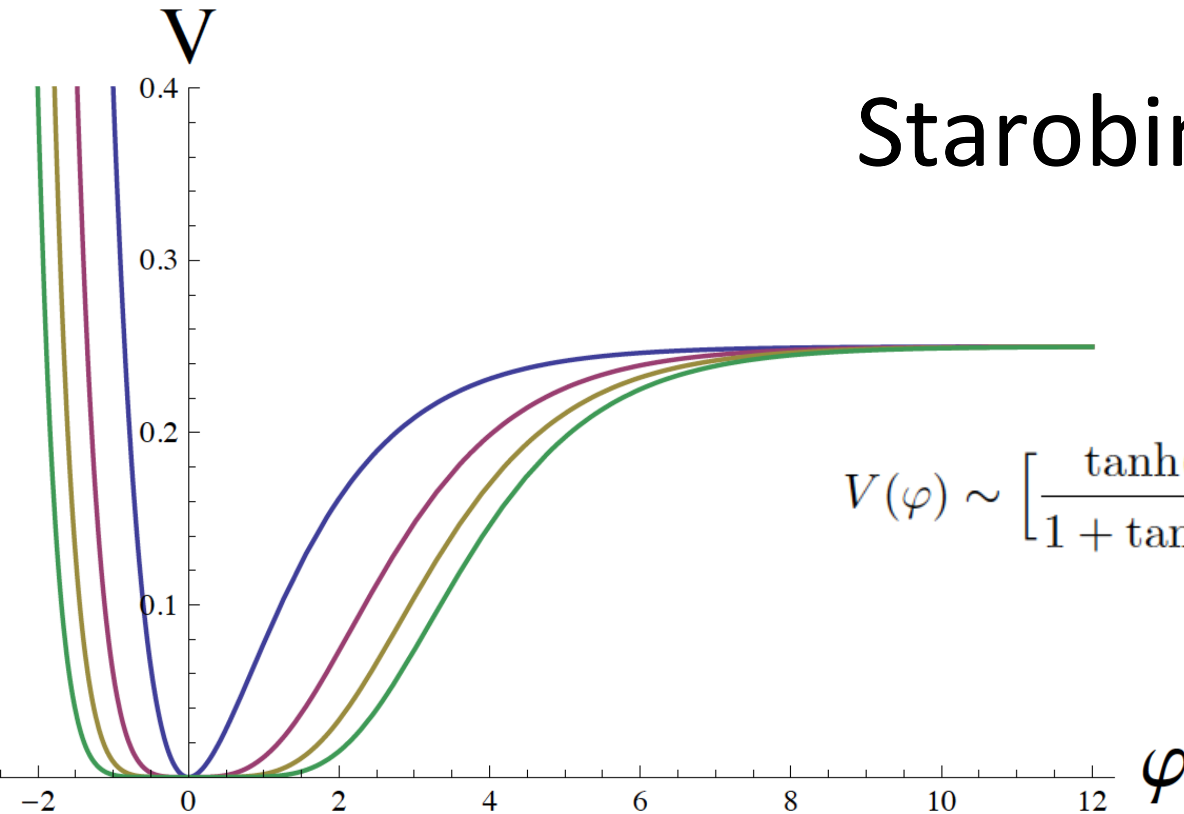
One could expect that these results may become increasingly unreliable for large N, but in fact the expansion parameter is $1/N$ for each model, so the difference of the predictions for $1 - n_s = 2/N$ and $r = 12/N^2$ in the slow-roll approximation is indeed $O(1/N^2)$. We checked this statement by explicitly comparing models $\tanh^2(\varphi/\sqrt{6})$ and $\tanh^{20}(\varphi/\sqrt{6})$, and we found in both cases $n_s \approx 0.967$ and $r \approx 0.0032$.

Any customer can have a car painted any color that he wants so long as it is black

Henry Ford

UNIVERSALITY of T-Model

Starobinsky model

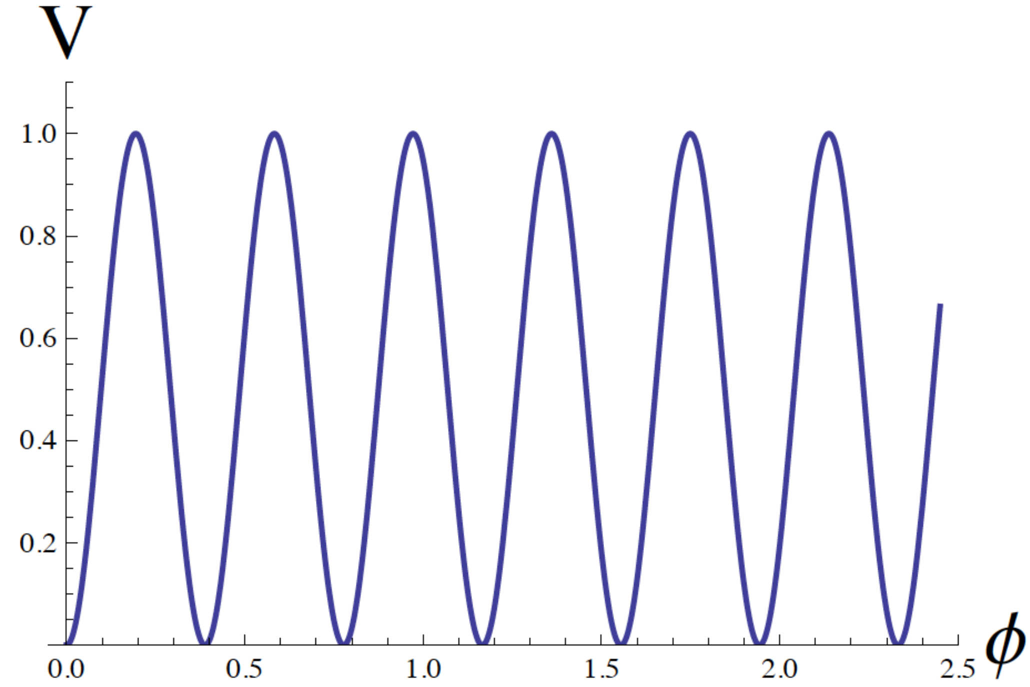


$$V(\varphi) \sim \left[\frac{\tanh(\varphi/\sqrt{6}))}{1+\tanh(\varphi/\sqrt{6}))}\right]^2 \sim \left(1 - e^{-\sqrt{2/3}\,\varphi}\right)^2$$

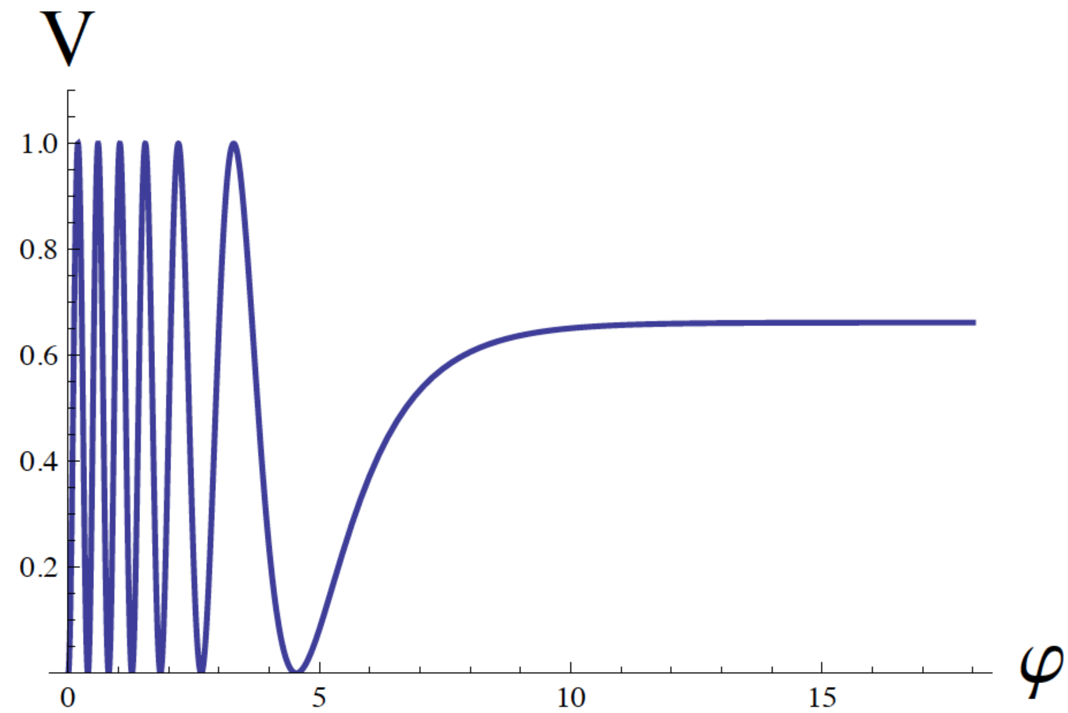
Figure 2: Models of conformal inflation based on generalizations of the Starobinsky model, with $F(\phi/\chi) \sim \frac{\phi^{2n}}{\chi^{2n-2}(\phi+\chi)^2}$, $n = 1, 2, 3, 4$.

↑ n=1

Original potential



Potential in terms of the canonical field φ



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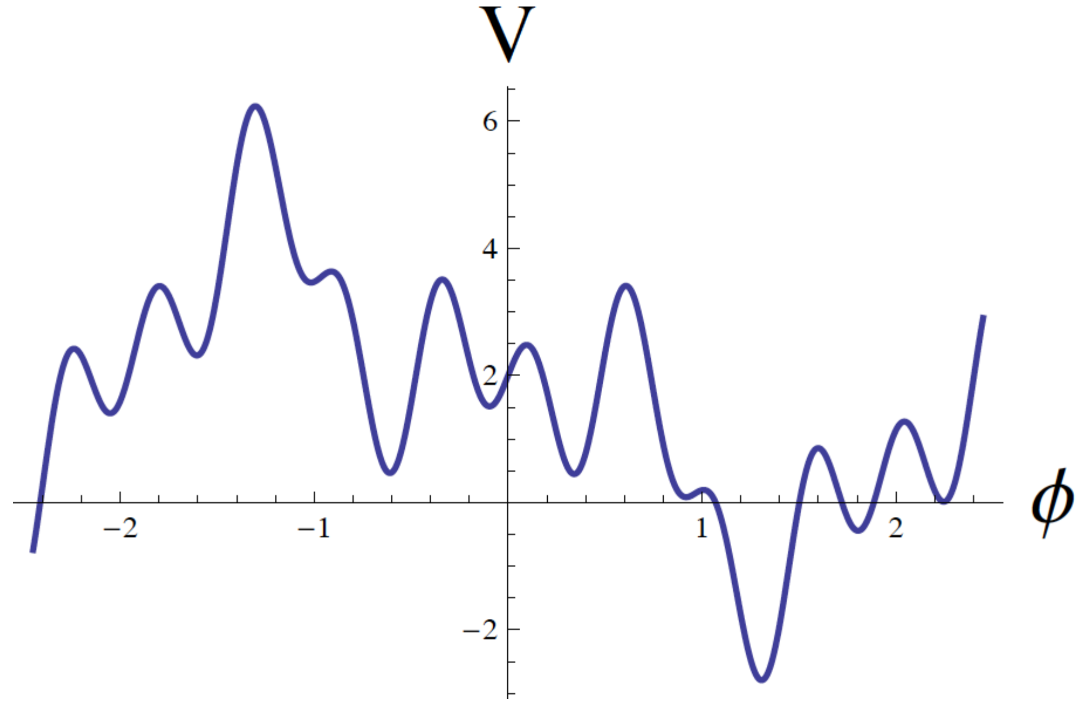

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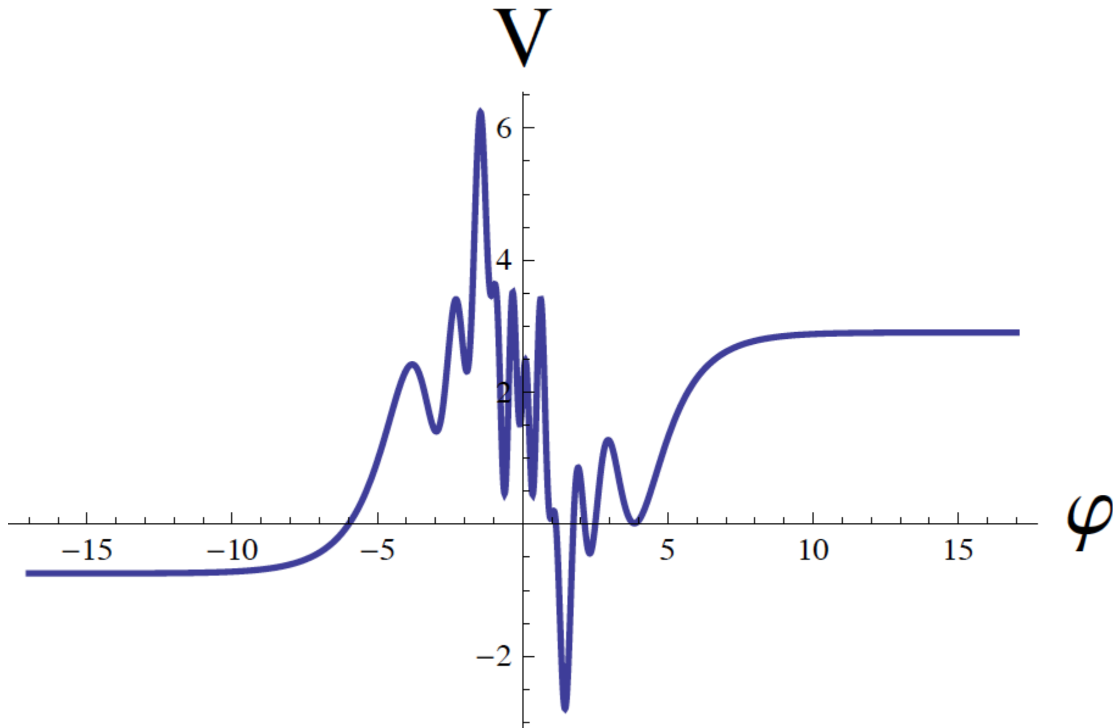


Universality of conformal inflation

Distance from the boundary of the moduli space

$$x = \sqrt{6} - \phi = \sqrt{6}(1 - \tanh\frac{\varphi}{\sqrt{6}}) \approx 2\sqrt{6}\; e^{-\sqrt{2/3}\,\varphi}$$

Expanding the potential in **X**:

$$V(\phi) = V(\sqrt{6})\,(1 - \sum c_n x^n) = V(\sqrt{6})\,(1 - \sum c_n \left(2\sqrt{6}\; e^{-\sqrt{2/3}\,\varphi}\right)^n)$$

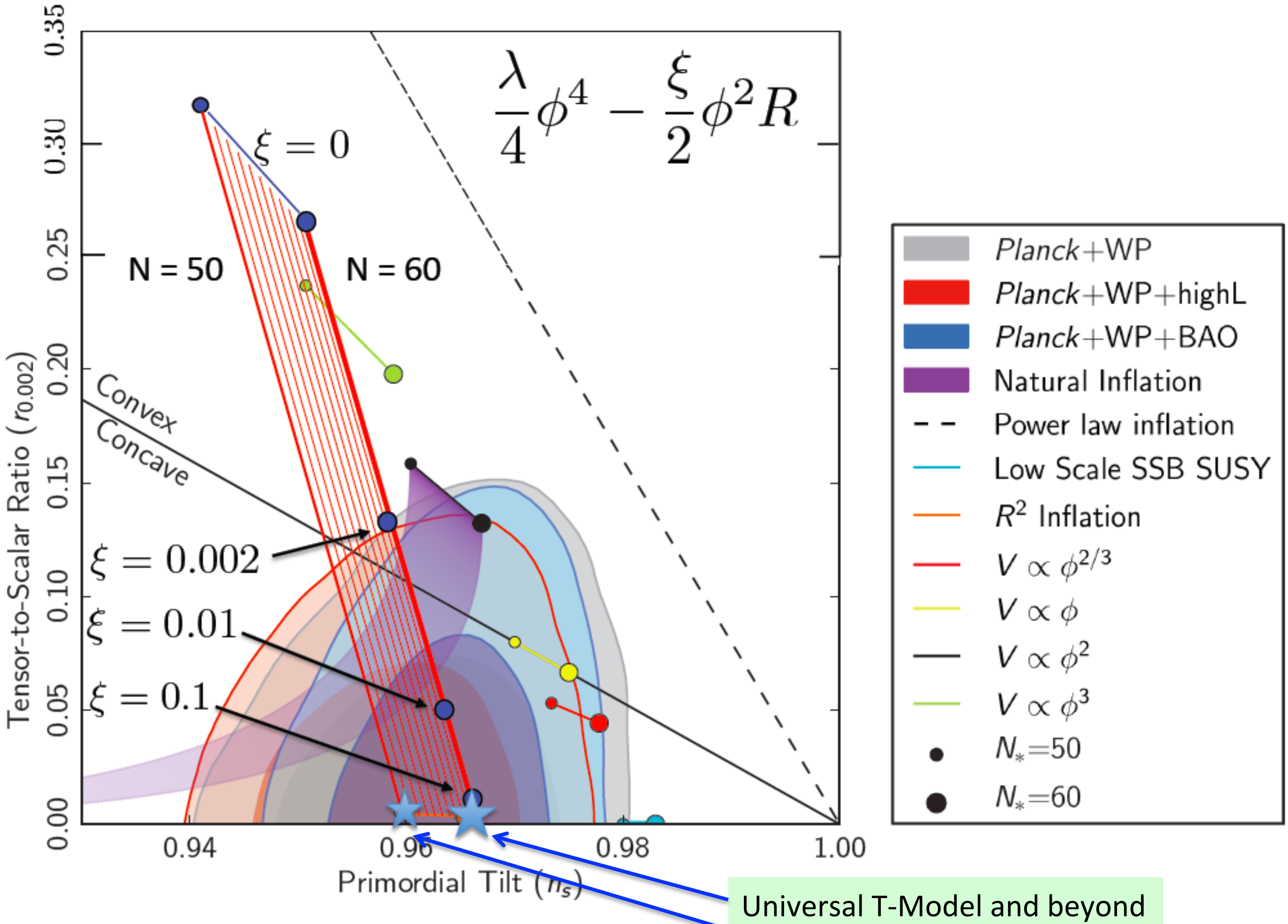
Take the first term in the expansion:

$$V(\phi) \approx V(\sqrt{6})\,(1 - 2\sqrt{6}\,c_1\; e^{-\sqrt{2/3}\,\varphi})$$

Solving equations of motion gives $2\sqrt{6}\,c_1 e^{-\sqrt{2/3}\,\varphi(N)} = \dfrac{3}{2\,N}$

Therefore it is indeed OK to ignore higher order corrections:

$$V(\phi_N) = V(\sqrt{6})\,(1 - \frac{3}{2\,N} - O(1/N^2))$$



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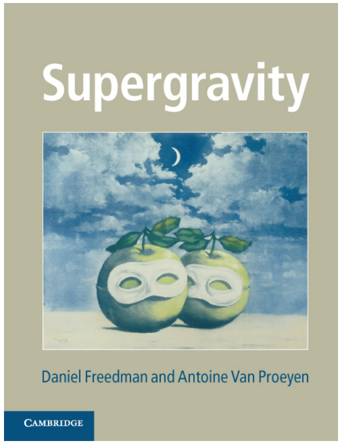
Universal T-Model and beyond

Textbook version:
Einstein frame
Supergravity

New book, contains
Superconformal
Supergravity



Einstein frame, by gauge fixing of the conformal compensator

RK, Kofman, AL and Van Proeyen 2000

$$-\frac{1}{6}\mathcal{N}(X,\bar{X})R \quad \Longrightarrow \quad \frac{1}{2}M_p^2 R$$

New version:
Jordan frame

Ferrara, RK, AL, Marrani and Van Proeyen, 2010

$$-\frac{1}{6}\mathcal{N}(X,\bar{X})\,R \quad \Longrightarrow \quad -\frac{1}{6}\Phi(z,\bar{z})\,R$$

Jordan frame
CSS

CSS (canonical superconformal supergravity) is a special class of SUGRA models with canonical kinetic terms and a potential as in a global SUSY

$SU(2,2|1)$ invariant superconformal action

Framework for describing the critical points in superconformal theories of interest

We start with Canonical Superconformal Supergravity and make various deformations of it.

We use 3 superfields: a conformon, an inflaton, and a goldstino

$$\frac{1}{\sqrt{-g}}\mathcal{L}_{\rm sc}^{\rm scalar-grav} = -\tfrac{1}{6}\mathcal{N}(X,\bar{X})R - G_{I\bar{J}}\mathcal{D}^{\mu}X^{I}\mathcal{D}_{\mu}\bar{X}^{\bar{J}} - G^{I\bar{J}}\mathcal{W}_{I}\bar{\mathcal{W}}_{\bar{J}}\,, \qquad I,\bar{I}=0,1,2.$$

Kahler potential of an embedding manifold

Derivative of the superpotential

CSS

$$\mathcal{N}(X,\bar{X}) = -|X^0|^2 + |\Phi|^2 + |S|^2$$

$$\mathcal{W}_0 \equiv \frac{\partial\mathcal{W}}{\partial X^0} = 0$$

Flat Kahler

Conformon-independent superpotential

List of critical points in superconformal theories of interest

We start with Canonical Superconformal Supergravity Model: flat Kahler, conformon independent superpotential

$$\Delta = 0$$

$$\mathcal{W}_0 \equiv \frac{\partial \mathcal{W}}{\partial X^0} = 0$$

Totally boring physics, needs to be deformed

$$\frac{\lambda}{4}\phi^4 - \frac{\xi}{2}\phi^2 R$$

Parameter of deformation of the flatness of the embedding Kahler manifold

$$\Delta_{\rm cr} = \pm 1/6$$

Double point of enhanced symmetry, mixing Inflaton with conformon, small deviation

$$\xi = \Delta - \Delta_{\rm cr}$$

$$\xi \gtrsim 0.002$$

Fit Planck2013

Technically natural model: small dimensionless parameter, such that when it vanishes, there is an extra symmetry

T-Model and beyond Universality Class

Kahler manifold remains flat conformal coupling

$$\Delta = 0$$

Superpotential which depends on the conformon preserves SO(1,1) : critical point F=const

$$\mathcal{W} = F\left(\frac{X^1}{X^0}\right)[(X^0)^2 - (X^1)^2]$$

F(tanh) landscape

Stabilization of goldstino during inflation in the superconformal T-Model