A nearly Gaussian Hubble-patch in a non-Gaussian universe

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arXiv: 1303.3549

Idea:

statistics of curvature perturbation ζ (i.e. inhomogeneities) are primary means to learn about inflation

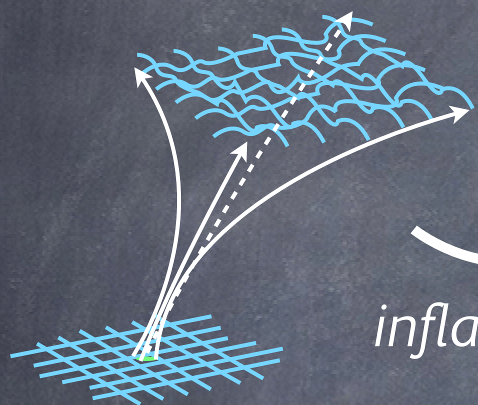
but we only observe ζ in our Hubble patch and *local statistics may not be representative*

Outline

- How are local and global statistics related?
- Three Examples:
  - single-source weakly non-Gaussian IC's
  - single-source strongly non-Gaussian IC's
  - multi-source initial conditions
- Conclusions

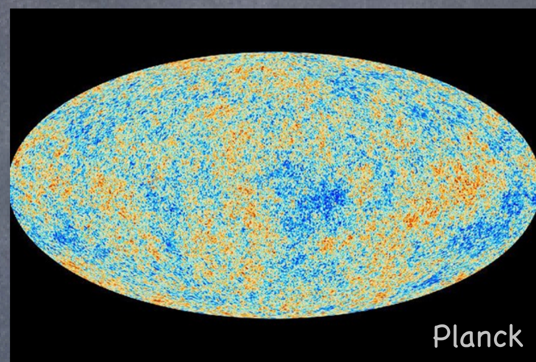
Inflation as the origin of structure

quantum fluctuations

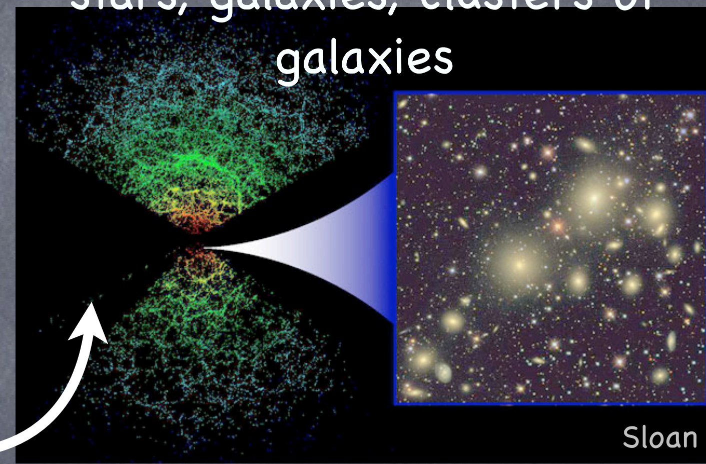


inflation

small matter & energy fluctuations



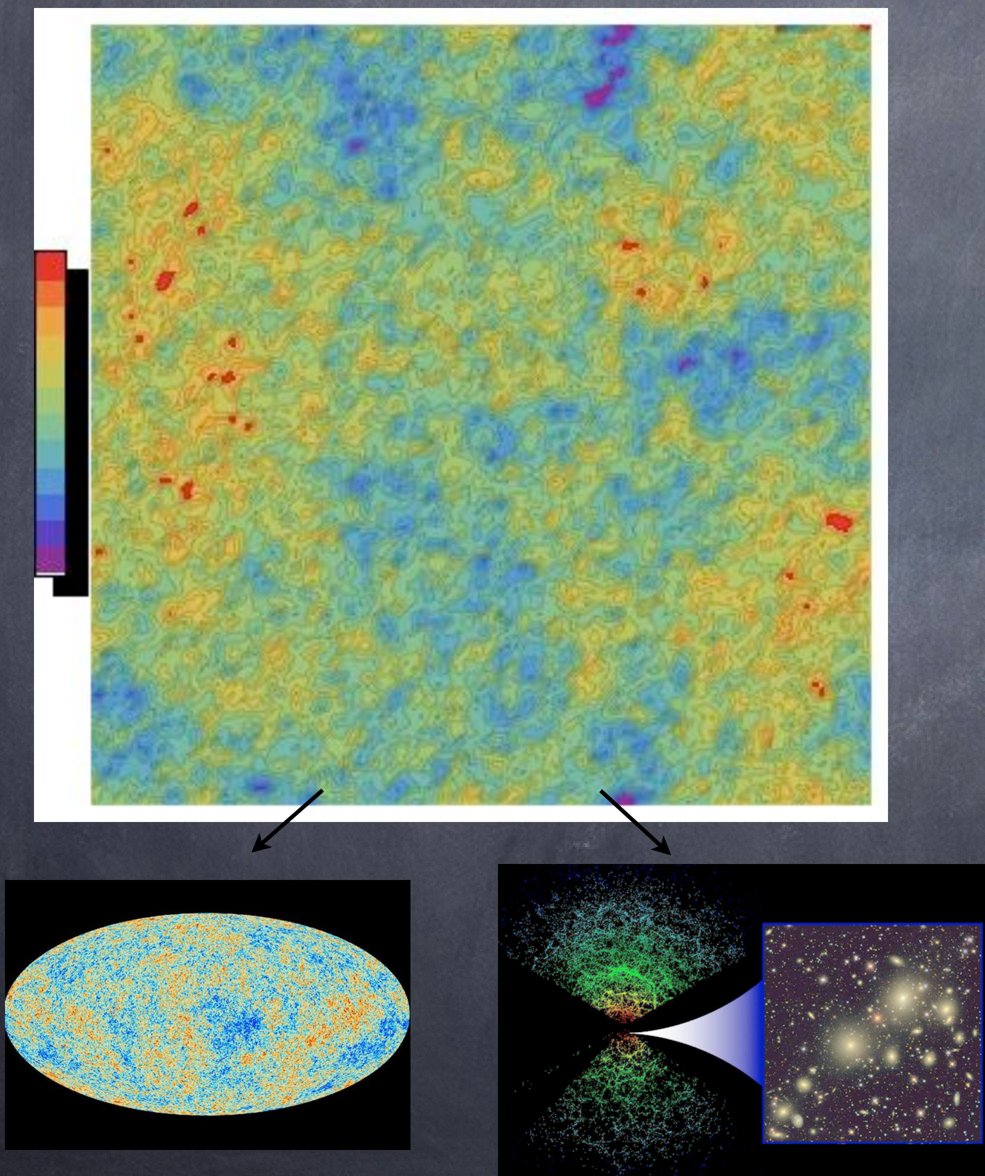
stars, galaxies, clusters of galaxies



gravitational collapse

$$\delta\ \phi_{\text{inflaton}} \longrightarrow \zeta_{\text{curvature}} \rightsquigarrow \begin{array}{l} \delta\, T_{\text{CMB}} \\ \delta\ \rho_{\text{matter}} \\ \delta\, n_{\text{galaxies}} \end{array}$$

For example, we can measure the power spectrum



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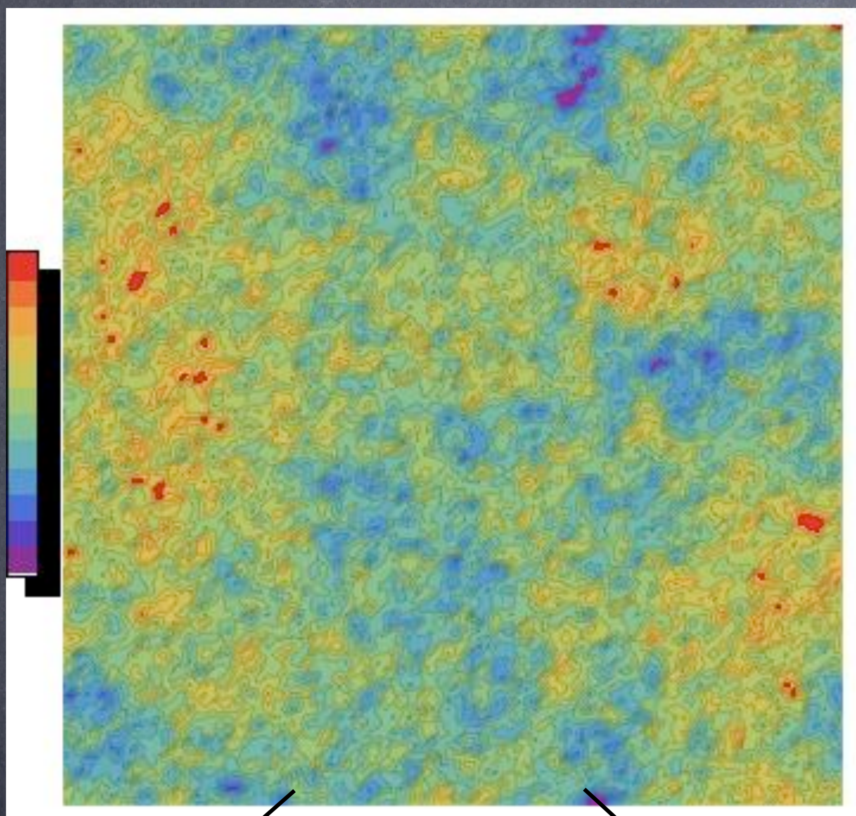
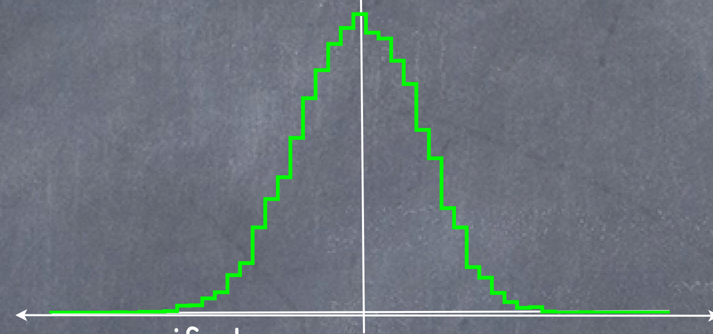
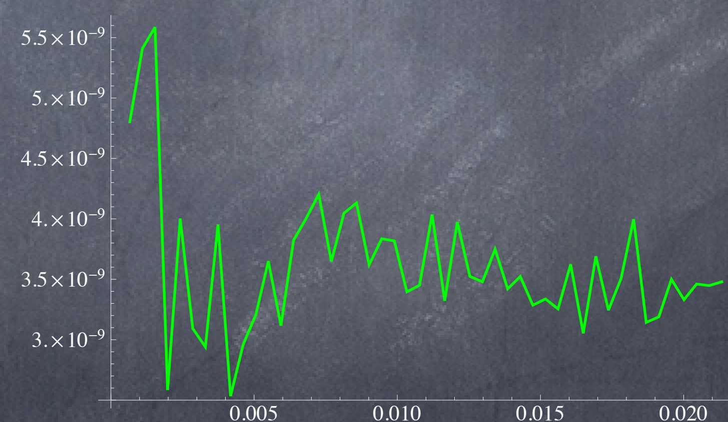




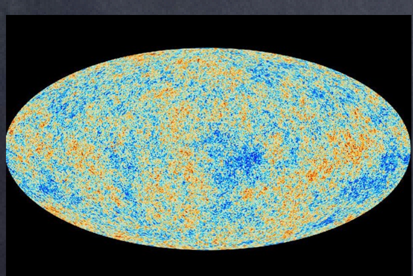
two-point function ↔ Power spectrum:

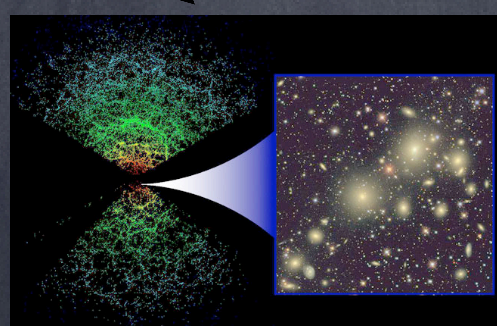
$$\langle \zeta(x)\,\zeta(y) \rangle \leftrightarrow P_\zeta(k)$$



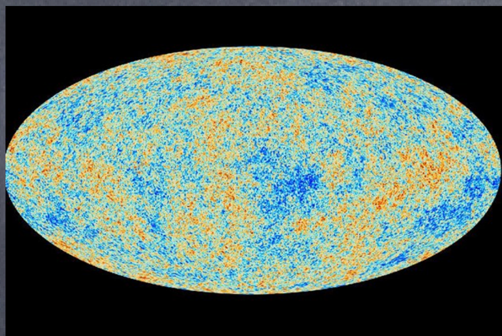
more large spots ↔ more small spots

power spectrum ~ typical spot size

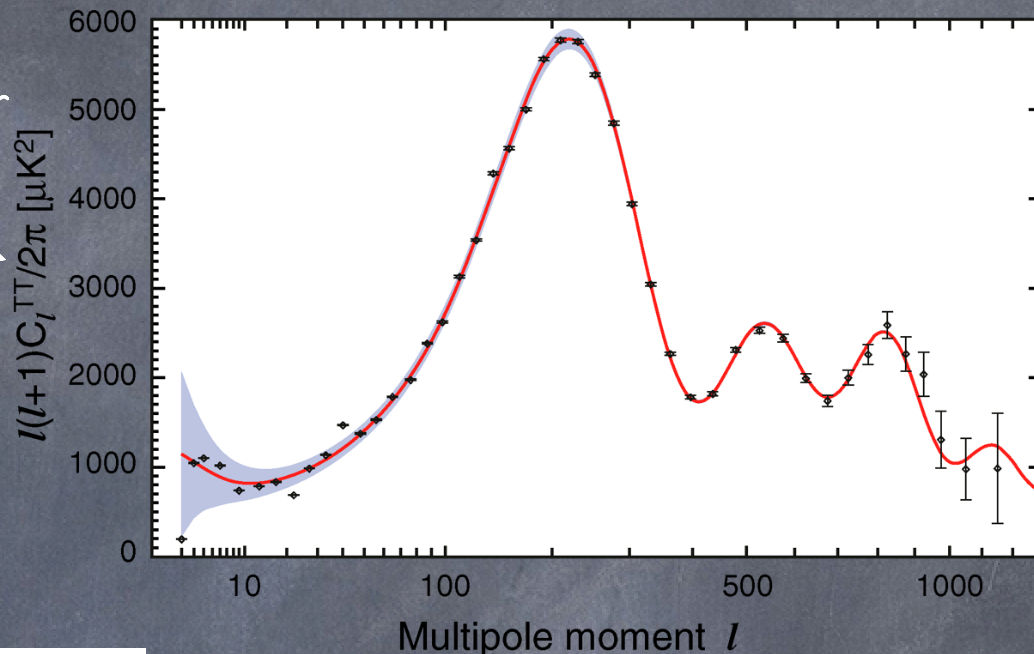




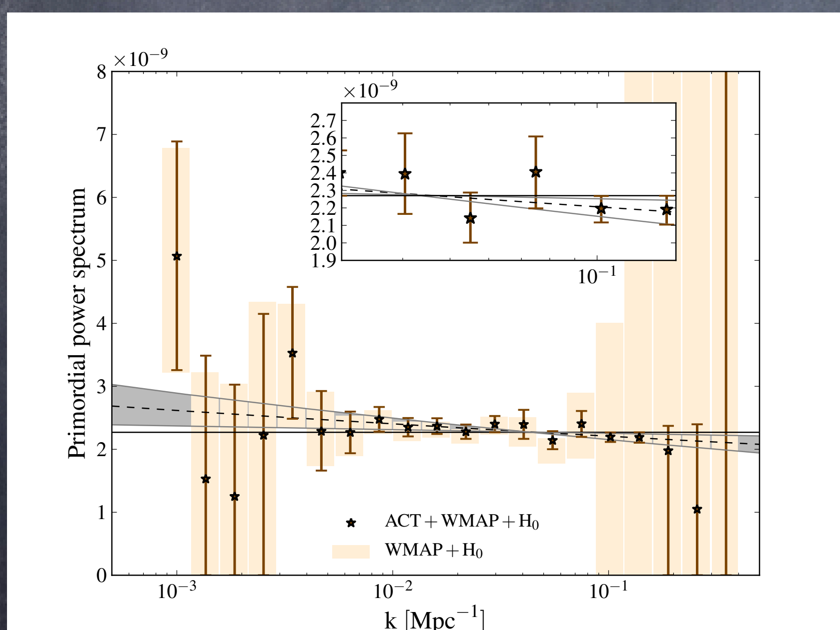
For instance, with CMB data



analyze power spectrum



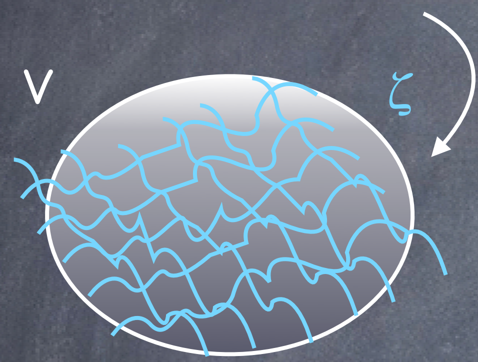
undo post-inflationary evolution of perturbations to recover primordial power spectrum



Hlozek et al 2011

How do the statistics we observe in our Hubble volume relate to what's predicted from inflation?

What's ζ ?

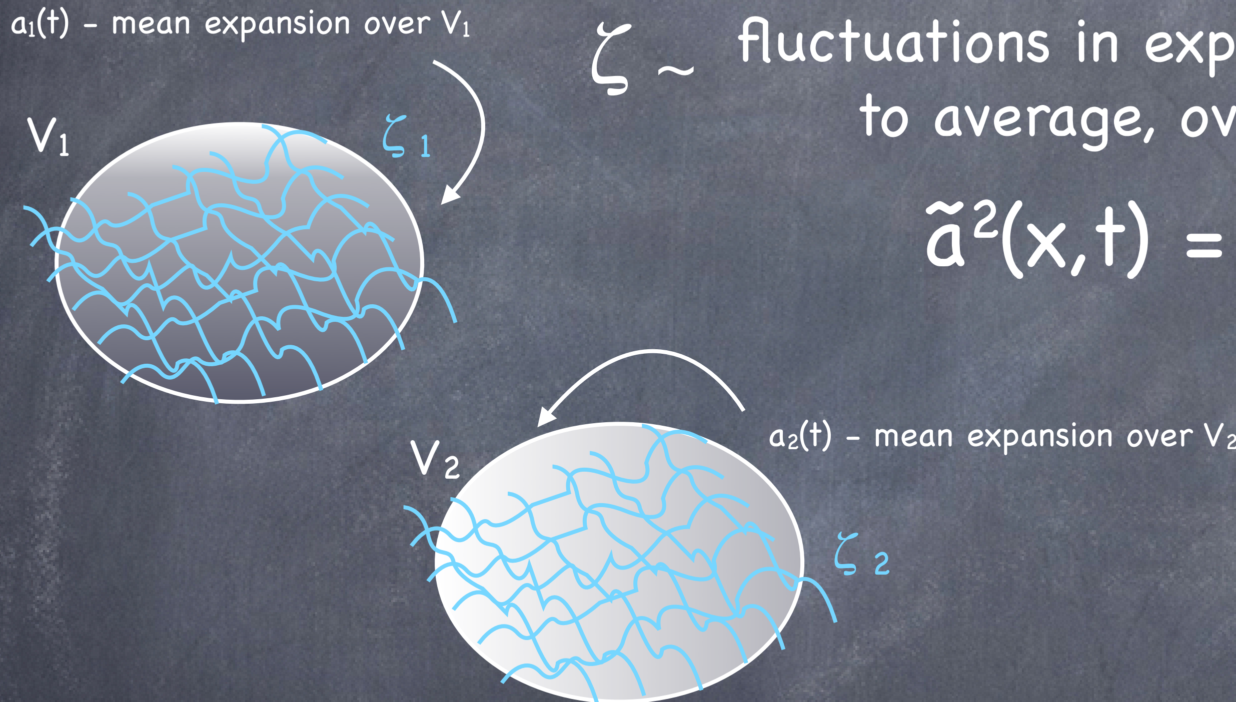


$\zeta \sim$ fluctuations in expansion history relative to average, over some volume V

$$\tilde{a}^2(x,t) = a(t)e^{\zeta(x,t)}$$

see e.g. Wands, Malik, Lyth, and Liddle 2000

Local and Global ζ



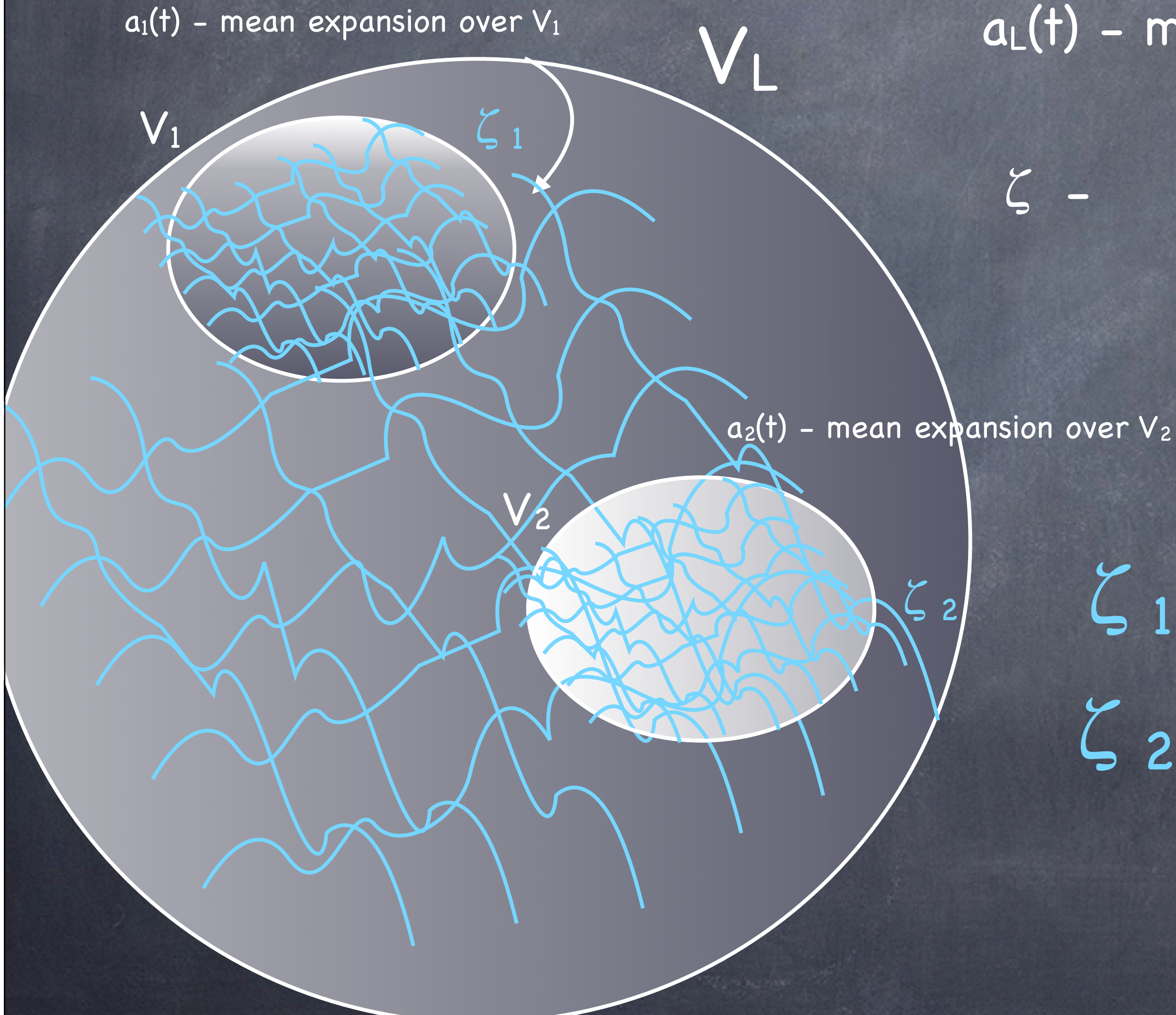
$\zeta \sim$ fluctuations in expansion history relative to average, over some volume V

$$\tilde{a}^2(x,t) = a(t)e^{\zeta(x,t)}$$

different regions may have different fluctuations and different average expansion histories

see e.g. Wands, Malik, Lyth, and Liddle 2000

Local and Global ζ



$a_L(t)$ - mean expansion over V_L

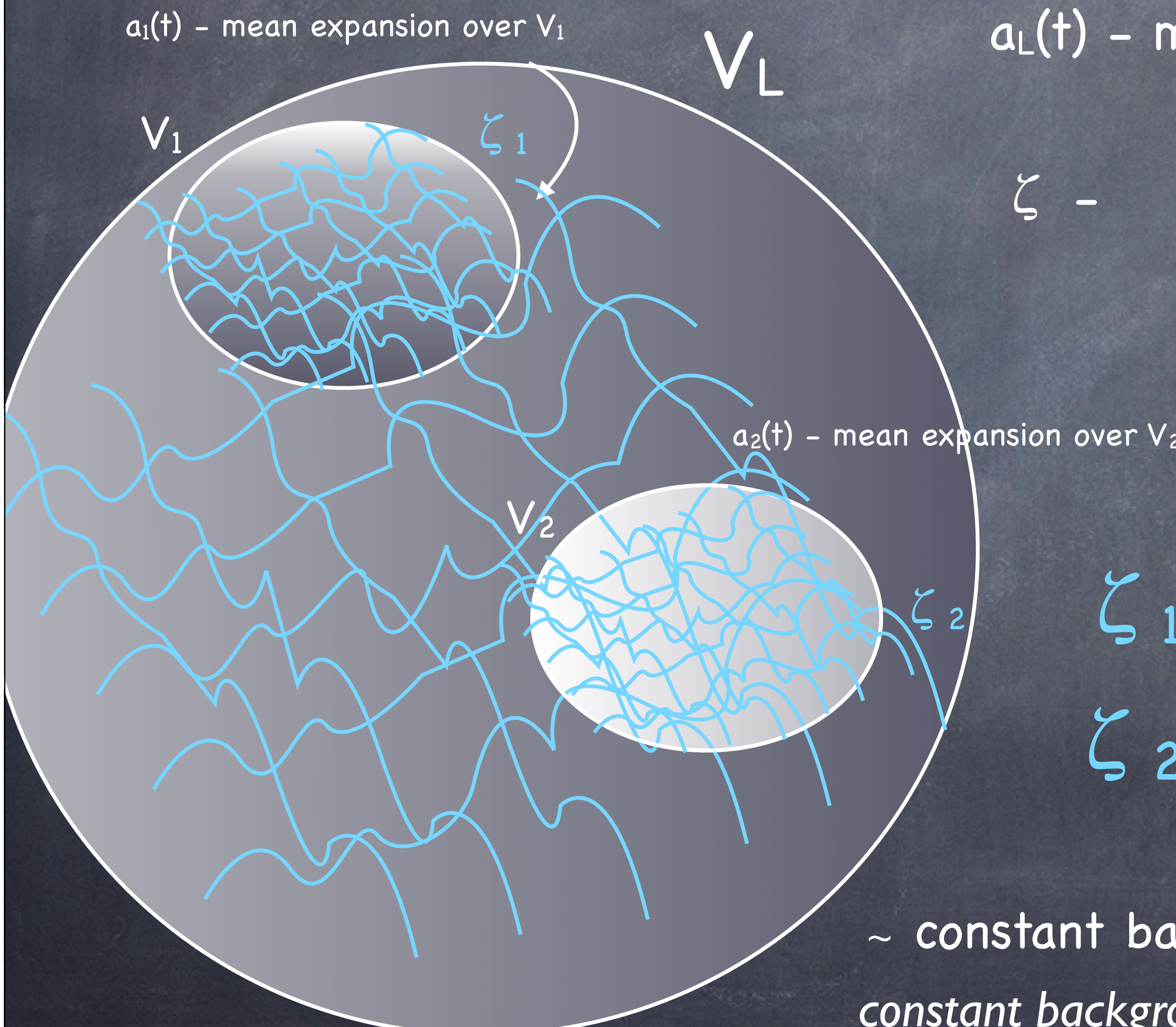
ζ - perturbation with respect to average expansion in V_L

$$\zeta_1(x) = \zeta(x) - \langle \zeta \rangle_1$$

$$\zeta_2(x) = \zeta(x) - \langle \zeta \rangle_2$$

see e.g. Wands, Malik, Lyth, and Liddle 2000

Local and Global ζ



$a_L(t)$ - mean expansion over V_L

ζ - perturbation with respect to average expansion in V_L

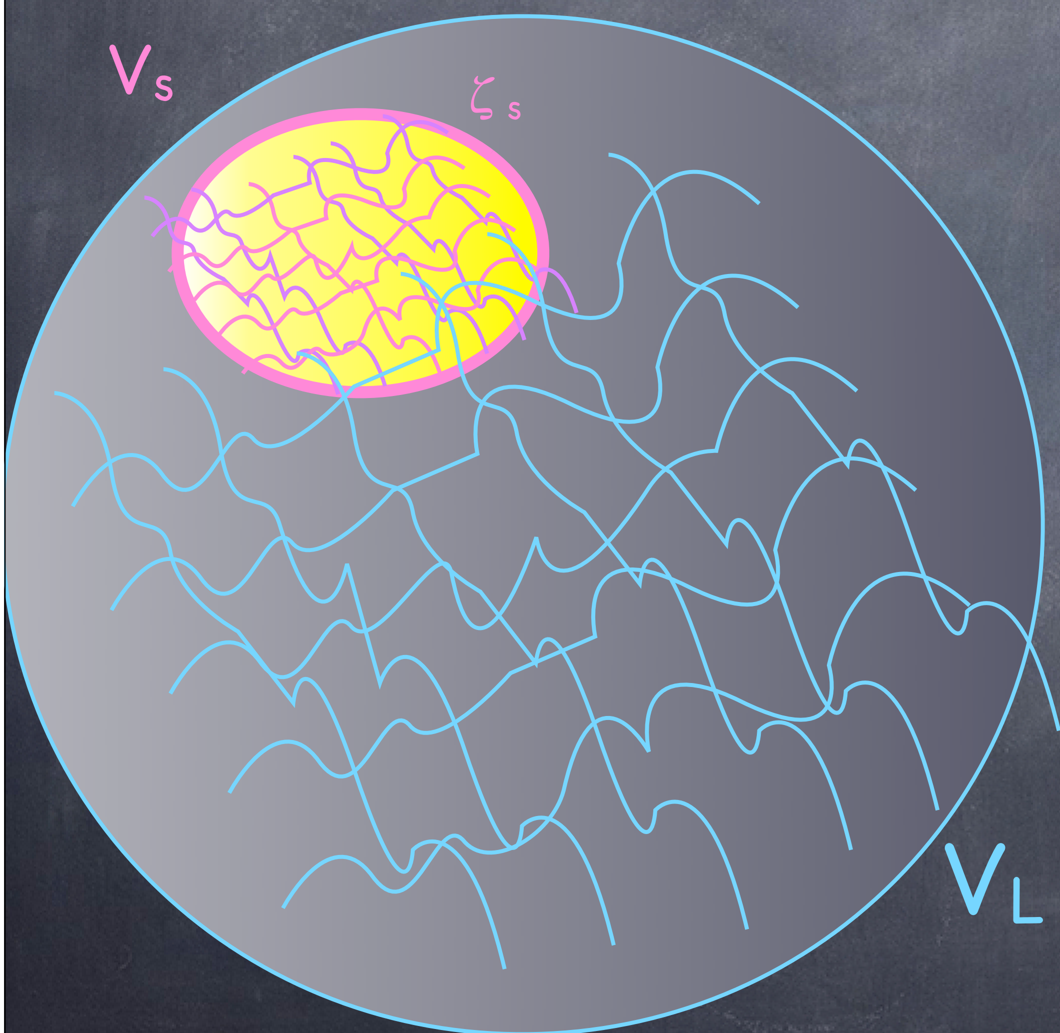
$$\zeta_1(x) = \zeta(x) - \langle \zeta \rangle_1$$

$$\zeta_2(x) = \zeta(x) - \langle \zeta \rangle_2$$

~ constant background modes in V_1 and V_2

constant background mode is not locally observable

Local and Global ζ



DEFINITIONS

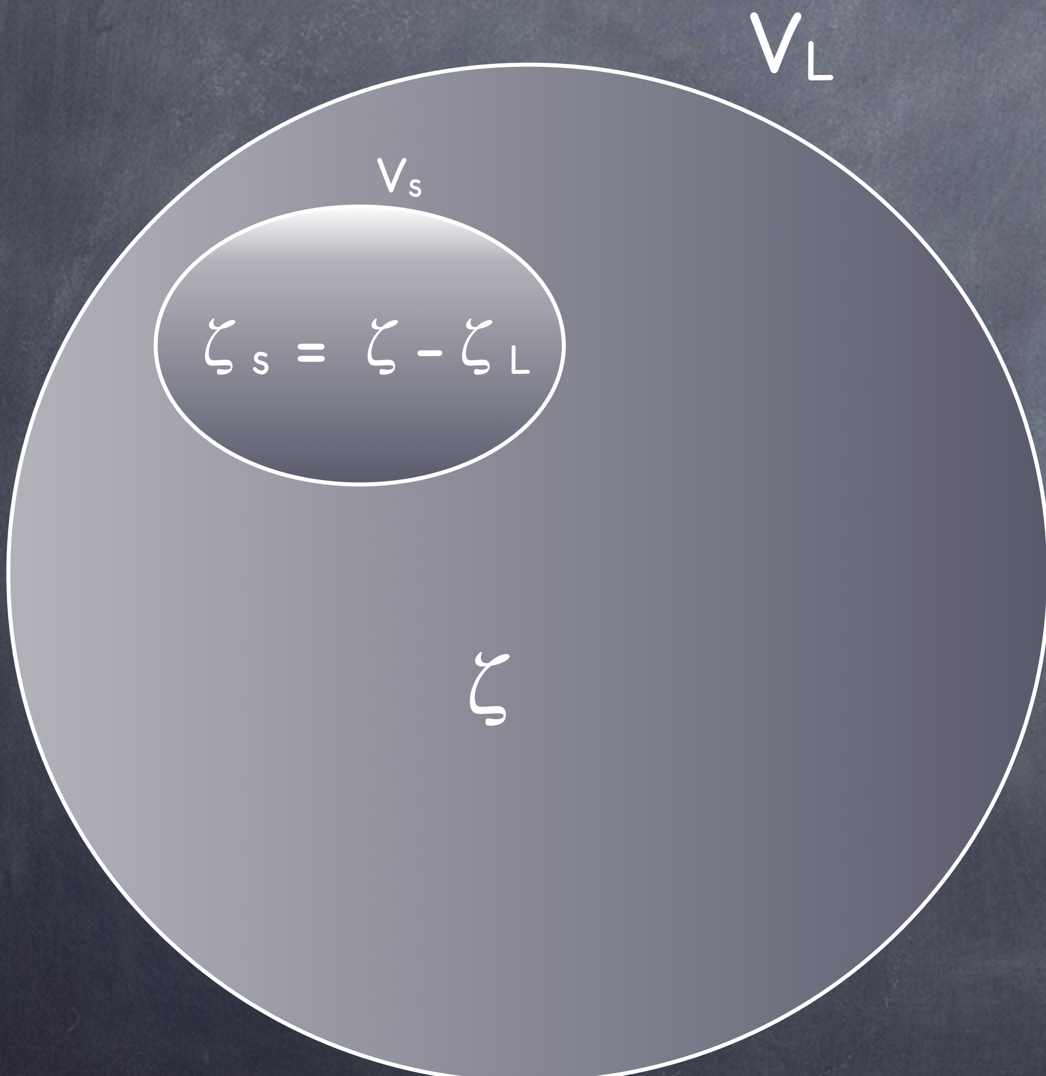
local curvature perturbation

$$\zeta_s(x) \equiv \zeta(x) - \zeta_L$$

$$\zeta_L \equiv \langle \zeta \rangle_{V_s}$$

average over volume V_s

Local and Global ζ



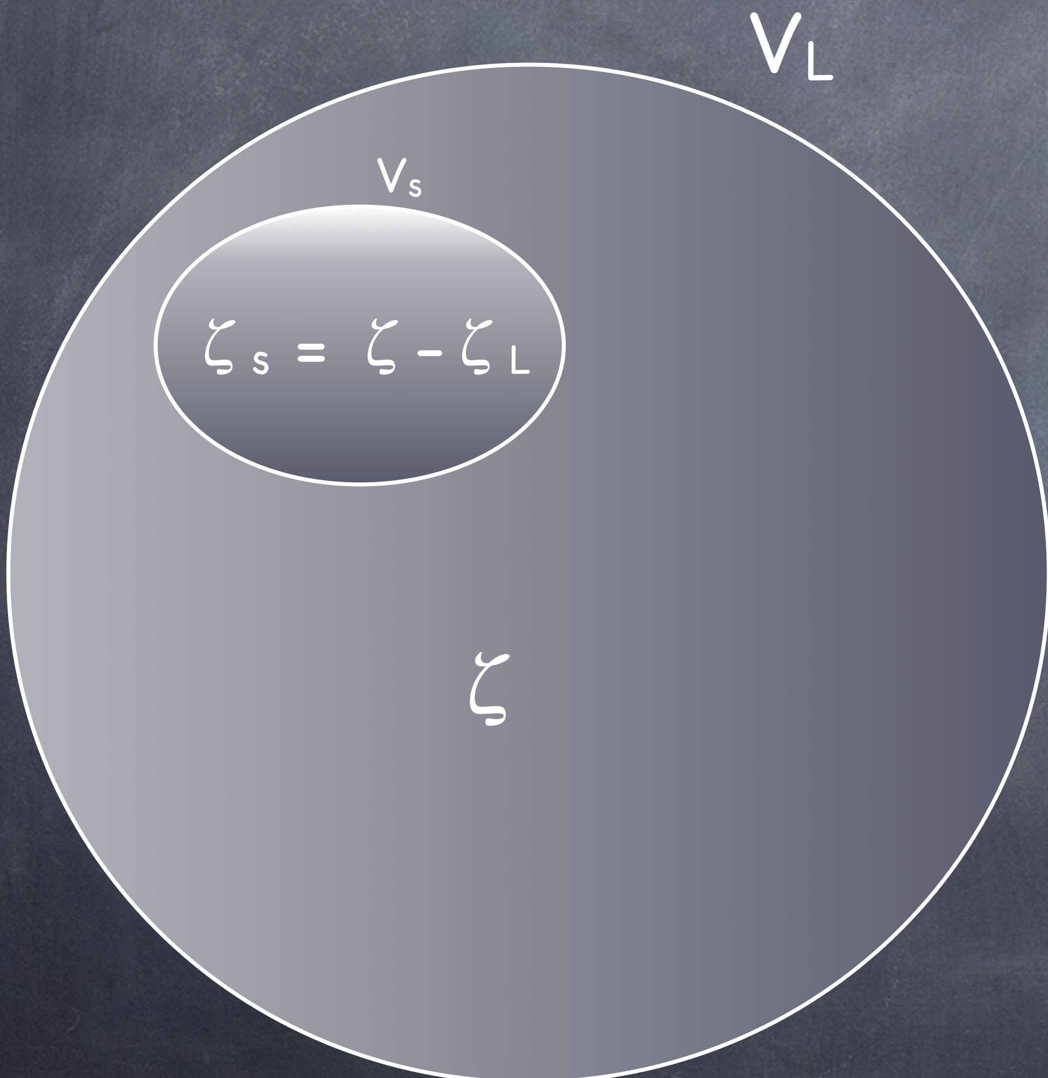
if ζ is Gaussian,

ζ_s and ζ_L

are uncorrelated*

* ok, strictly speaking this is only true in Fourier space. corrections depend on how we define volumes, but we can calculate them

Local and Global ζ



if ζ is Gaussian,

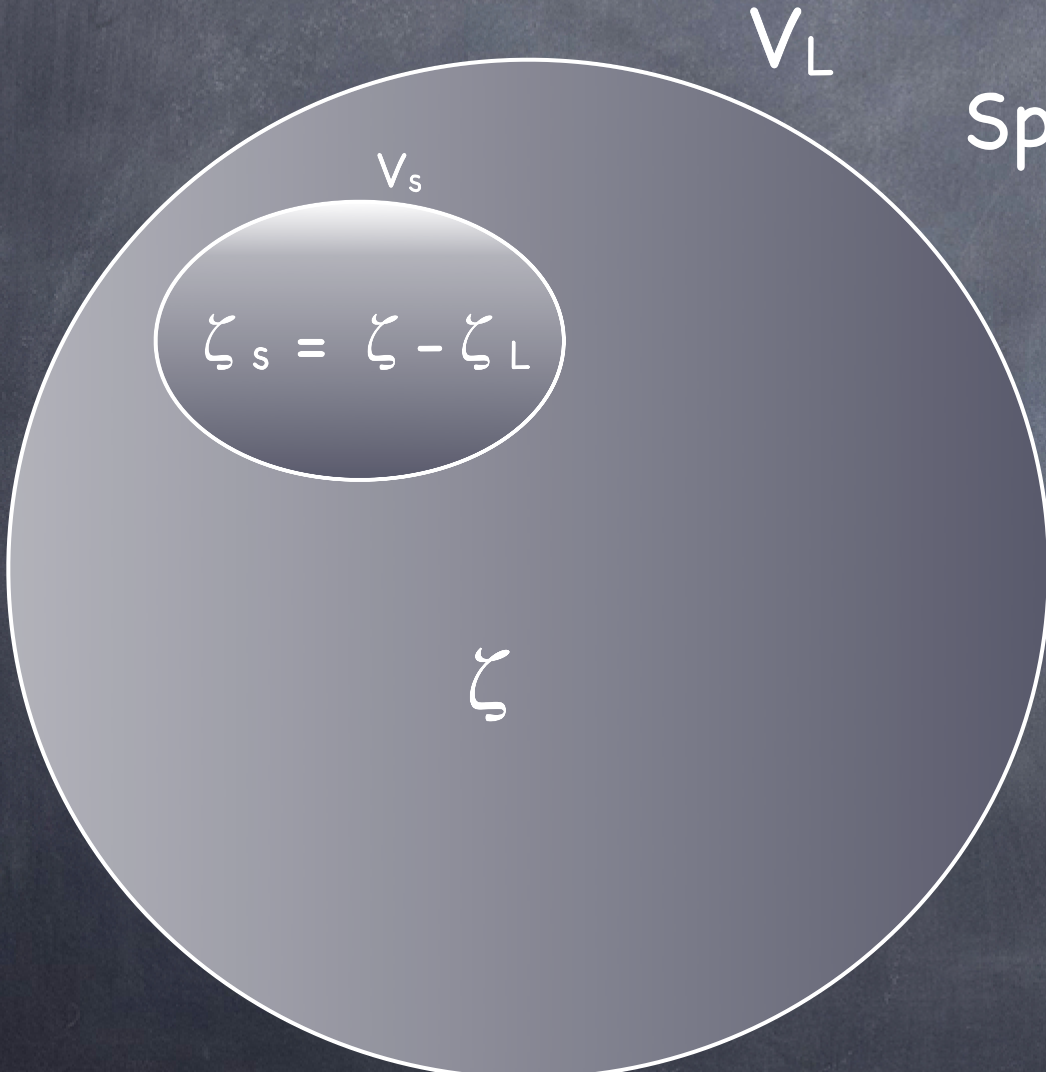
ζ_s and ζ_L

are uncorrelated*

BUT if ζ is non-Gaussian *ζ_s can depend on the value of ζ_L*

* ok, strictly speaking this is only true in Fourier space. corrections depend on how we define volumes, but we can calculate them

Non-linear couplings



Specifically, if

$$\zeta = F(\zeta_G(x)) - \langle F(\zeta_G) \rangle$$

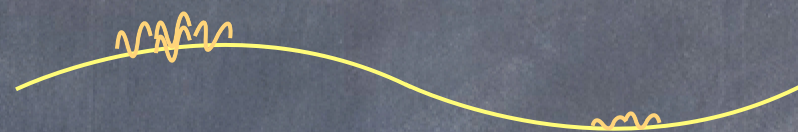
ζ_s can have a strong dependence on ζ_L

possibly familiar example:

Non-linear couplings

For instance, the quadratic local ansatz: $\zeta = \zeta_G + f_{NL} \zeta_G^2$

$$\langle \zeta_s^2 \rangle = \langle \zeta_{G,s}^2 \rangle (1 + 4 f_{NL} \zeta_{G,L}(x))$$



small-scale power depends on large-scale fluctuations

Dalal, Doré, Huterer, Shirokov 2007

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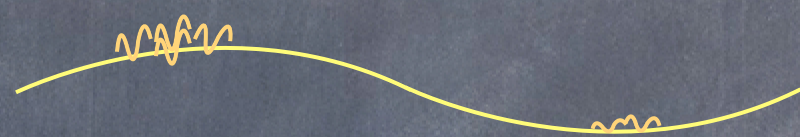


small-scale power depends on large-scale fluctuations

In our Hubble volume this gives rise to the scale-dependent halo bias of Dalal et al

$$\delta n = \frac{\partial n}{\partial \delta} \delta_l + 4 f_{NL} \frac{\partial n}{\partial \sigma_8} \Phi_l \ldots \quad \longrightarrow \quad \delta n \sim \left(\frac{\partial n}{\partial \delta} + \frac{4 f_{NL}}{k^2} \frac{\partial n}{\partial \sigma_8} \right) \delta_l$$

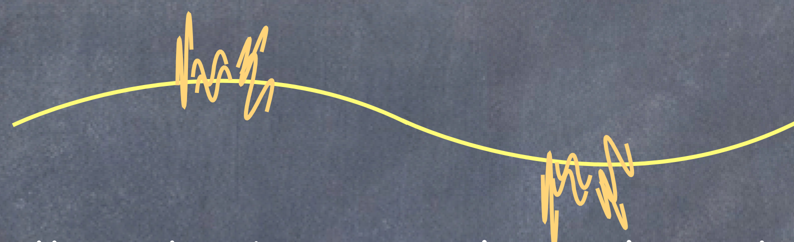
Dalal, Doré, Huterer, Shirokov 2007

possibly familiar example:

Non-linear couplings

similarly, the cubic local ansatz: $\zeta = \zeta_G + g_{NL}\,\zeta_G^3$

$$\langle \zeta_s^3 \rangle = 18\, g_{NL} \langle \zeta_{G,s}^2 \rangle^2 \zeta_{G,l}(x) \equiv 6 f_{NL}^{eff}(x) \langle \zeta_{G,s}^2 \rangle$$



small-scale skewness depends on large-scale fluctuations

Desjacques & Seljak 2009; Smith, Ferraro, ML 2011

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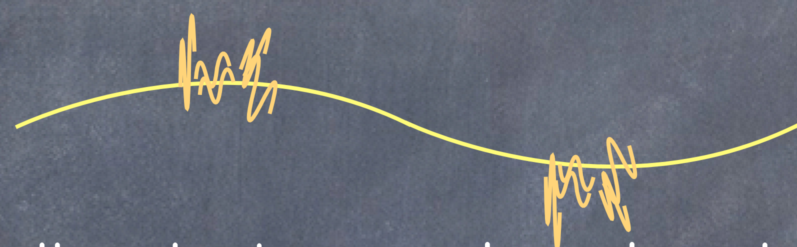


small-scale skewness depends on large-scale fluctuations

which has been shown to give a similar, scale-dependent bias in halo clustering

$$\delta n = \frac{\partial n}{\partial \delta} \delta_l + 18 g_{NL} \frac{\partial n}{\partial S_3} \Phi_l \ldots \longrightarrow \left(\frac{\partial n}{\partial \delta} + 18 g_{NL} \frac{\partial n}{\partial S_3} / k^2 \right) \delta_l(k) \ldots$$

Desjacques & Seljak 2009; Smith, Ferraro, ML 2011

So, this large-small scale coupling is somewhat familiar

$$\zeta = \zeta_G + f_{NL}\,\zeta_G^2 + g_{NL}\,\zeta_G^3 + \ldots$$

$$\downarrow$$

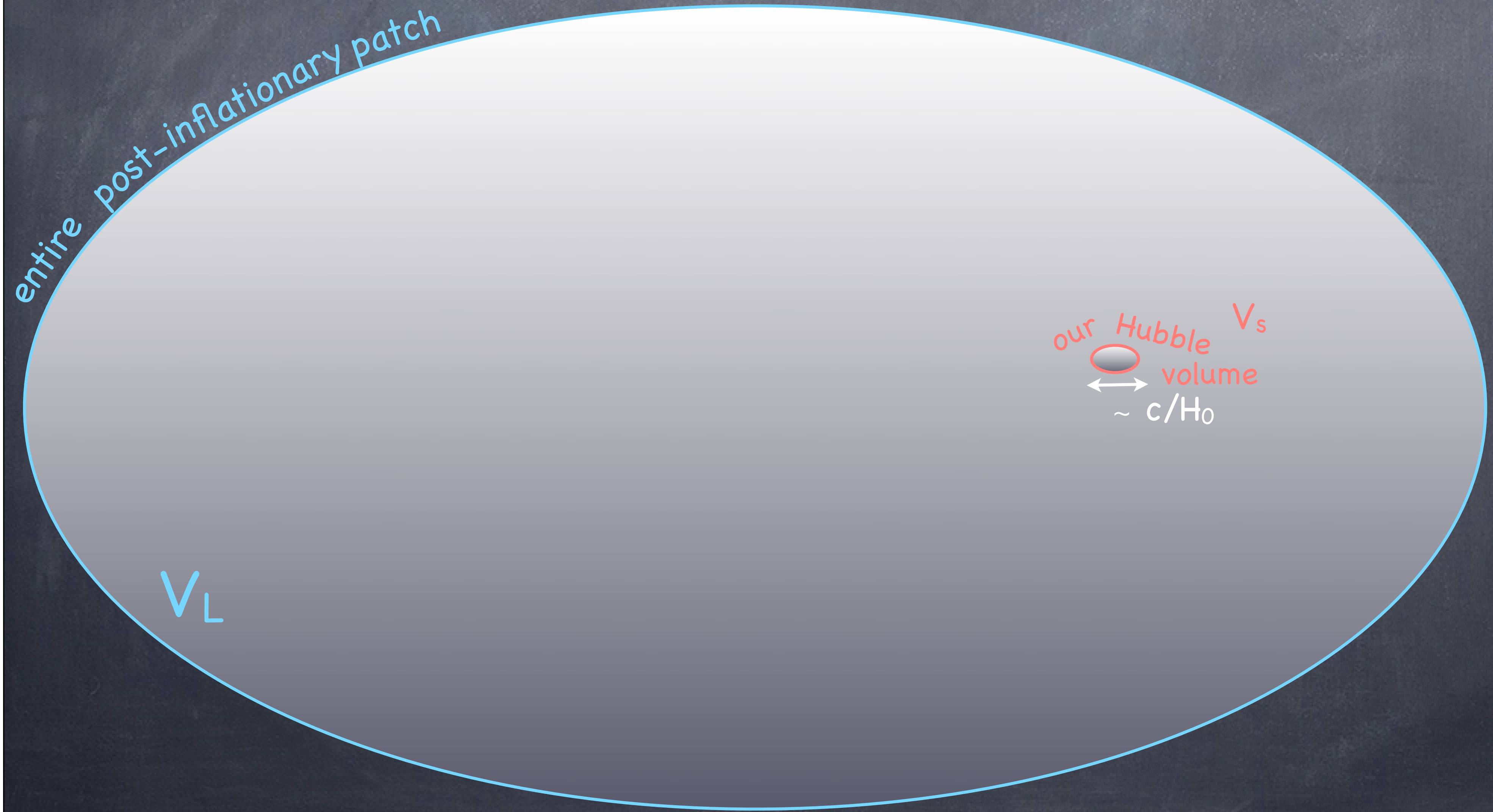
$$\langle \zeta_s^2 \rangle = \langle \zeta_{G,s}^2 \rangle \,(1 + 4\, f_{NL}\, \zeta_{G,L}(x))$$

$$\langle \zeta_s^3 \rangle = 18\, g_{NL}\, \langle \zeta_{G,s}^2 \rangle^2\, \zeta_{G,l}(x) \equiv 6 f_{NL}^{eff}(x)$$

. . .

What are the consequences of this mode coupling?

Suppose our Hubble volume is small compared with the entire post-inflationary patch



Suppose our Hubble volume is small compared with the entire post-inflationary patch

entire post-inflationary patch

If, $\zeta = F(\zeta_G(x)) - \langle F(\zeta_G) \rangle$

our Hubble volume V_s

the small-scale statistics (power spectrum, bispectrum, trispectrum) of ζ measured in our Hubble patch depend on the amplitude of $\zeta_{G,L}$ in our Hubble patch

V_L

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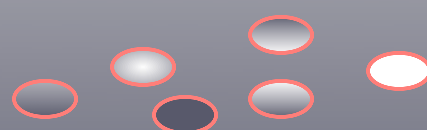
V_L



other Hubble volumes with different $\zeta_{G,L}$ values

typical size of $\zeta_{G,L}$?

$$\zeta_{G,L} = \int_{V_s \sim H_0^{-3}} d^3x \, \zeta_G(x)$$

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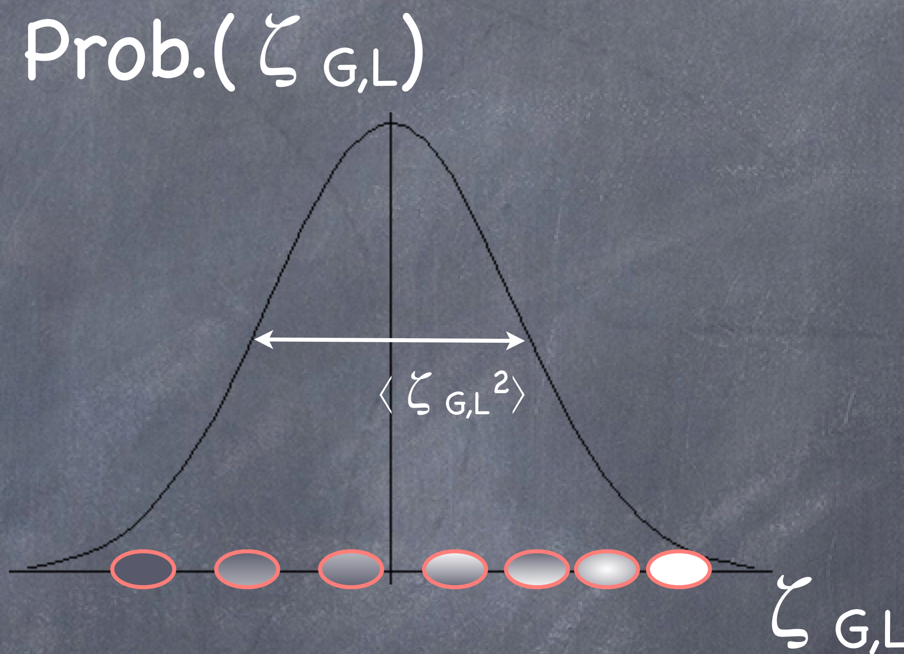
random (unknown) variable
in each Hubble-size patch

typical size of $\zeta_{G,L}$?

$$\zeta_{G,L} = \int_{V_s \sim H_0^{-3}} d^3x\, \zeta_G(x)$$

variance:

$$\langle \zeta_{G,L}^2 \rangle = \int_{2\pi/V_L^{1/3}}^{H_0} \frac{d^3k}{(2\pi)^3} \Delta^2_{\zeta_G}(k)$$



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sum over all modes with $k \lesssim H_0$

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depends on power spectrum outside horizon! which we don't know

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depends on power spectrum outside horizon! which we don't know

$$\langle \zeta_{G,L}^2 \rangle \sim \Delta^2_{\zeta_G} N$$ for scale invariant ($n_s=1$)

$$N \equiv \frac{1}{3} \ln \frac{V_L}{V_s}$$

typical size of $\zeta_{G,L}$?

$$\zeta_{G,L} = \int_{V_s \sim H_0^{-3}} d^3x \, \zeta_G(x)$$

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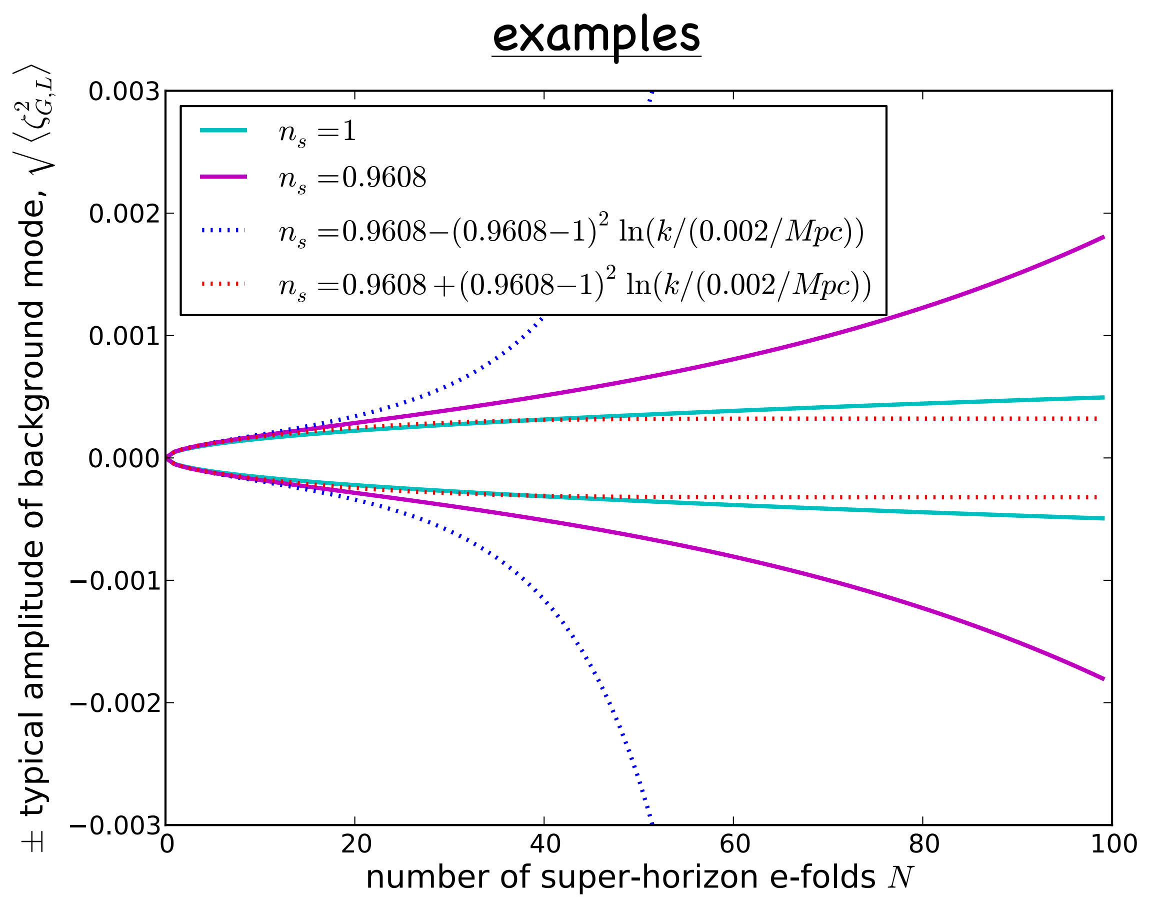
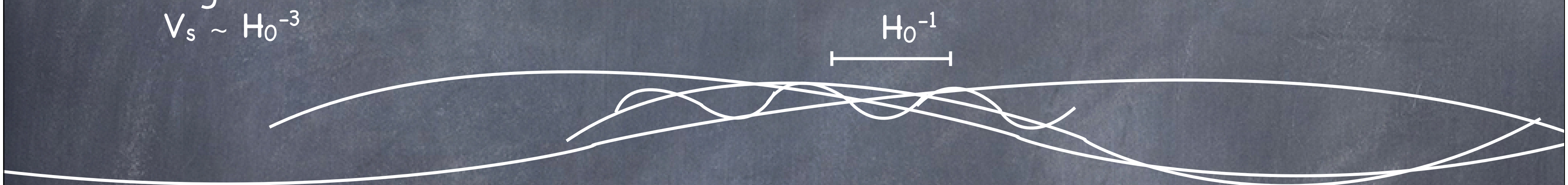


$$N \equiv \frac{1}{3} \ln \frac{V_L}{V_s}$$

ML, Nelson, Shandera 2013

Super-horizon perturbations?

$$\Omega_k \sim \int_{V_s \sim H_0^{-3}} d^3x \, \nabla^2 \zeta(x)$$



only modes with $k \sim H_0$ contribute

see e.g. Knox 2006, Erickeck et al 2008, Waterhouse 2008, Vardanyan et al 2009, Guth & Nomura 2012, Kleban & Schillo 2012

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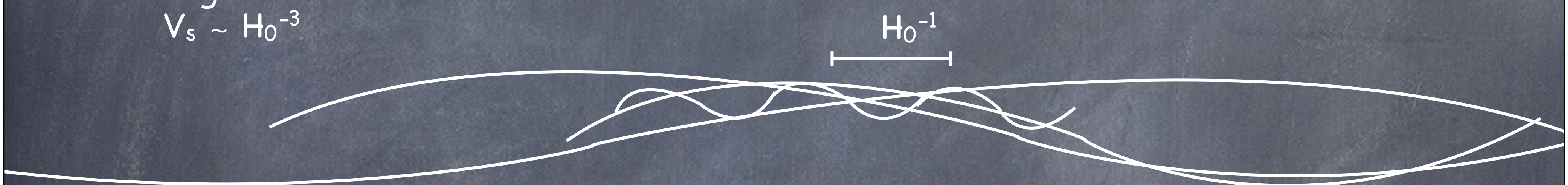


only modes with $k \sim H_0$ contribute

ζ_L is not something we can observe

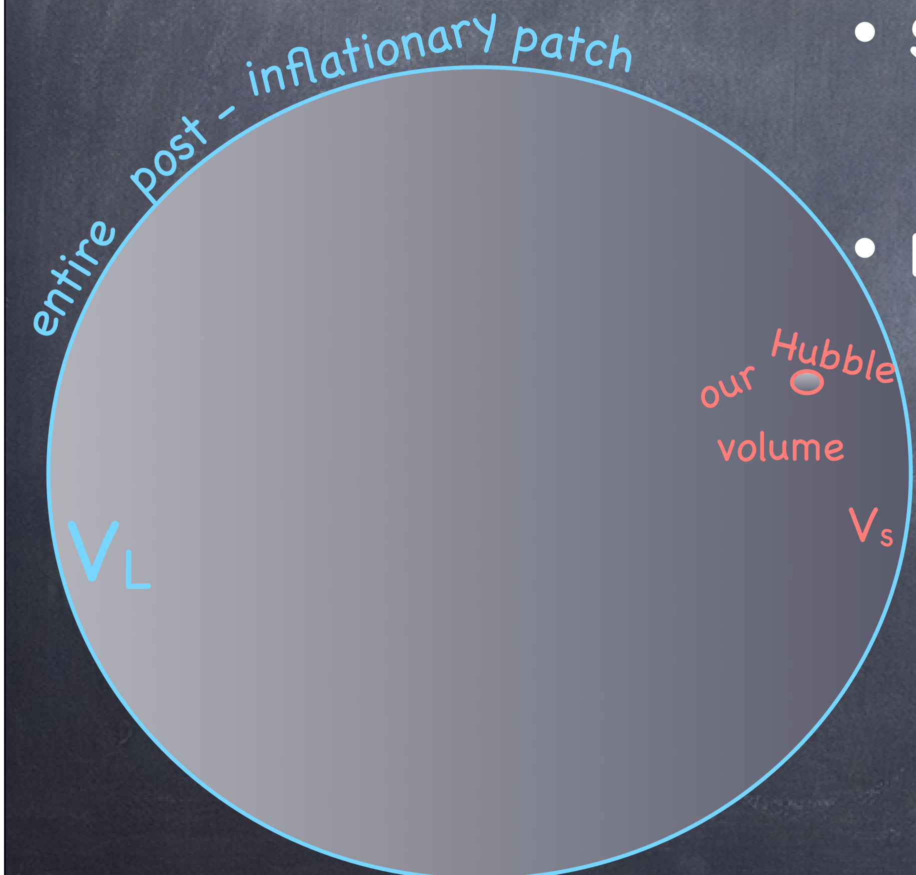
see e.g. Knox 2006, Erickeck et al 2008, Waterhouse 2008, Vardanyan et al 2009, Guth & Nomura 2012, Kleban & Schillo 2012

ML, Nelson, Shandera 2013

What are the consequences?

Consider three examples of statistics for ζ in V_L :

- weakly non-Gaussian
  Nurmi, Byrnes, Tasinato 2013
- strongly non-Gaussian
  Nelson & Shandera 2012
- multi-source non-Gaussian



ML, Nelson, Shandera 2013

Examples

- Single-source weakly non-Gaussian (usual local ansatz)
- Single-source strongly non-Gaussian
- Multi-source weakly non-Gaussian

ML, Nelson, Shandera 2013

Single-source weak NG

Single-source weak NG

globally,

$$\zeta = \zeta_G + f_{NL}\,(\zeta_G{}^2 - \langle\zeta_G{}^2\rangle) + g_{NL}\,(\zeta_G{}^3 - 3\,\zeta_G\langle\zeta_G{}^2\rangle)\;.\;.\;.$$

Nurmi, Byrnes, Tasinato 2013

ML, Nelson, Shandera 2013

Single-source weak NG

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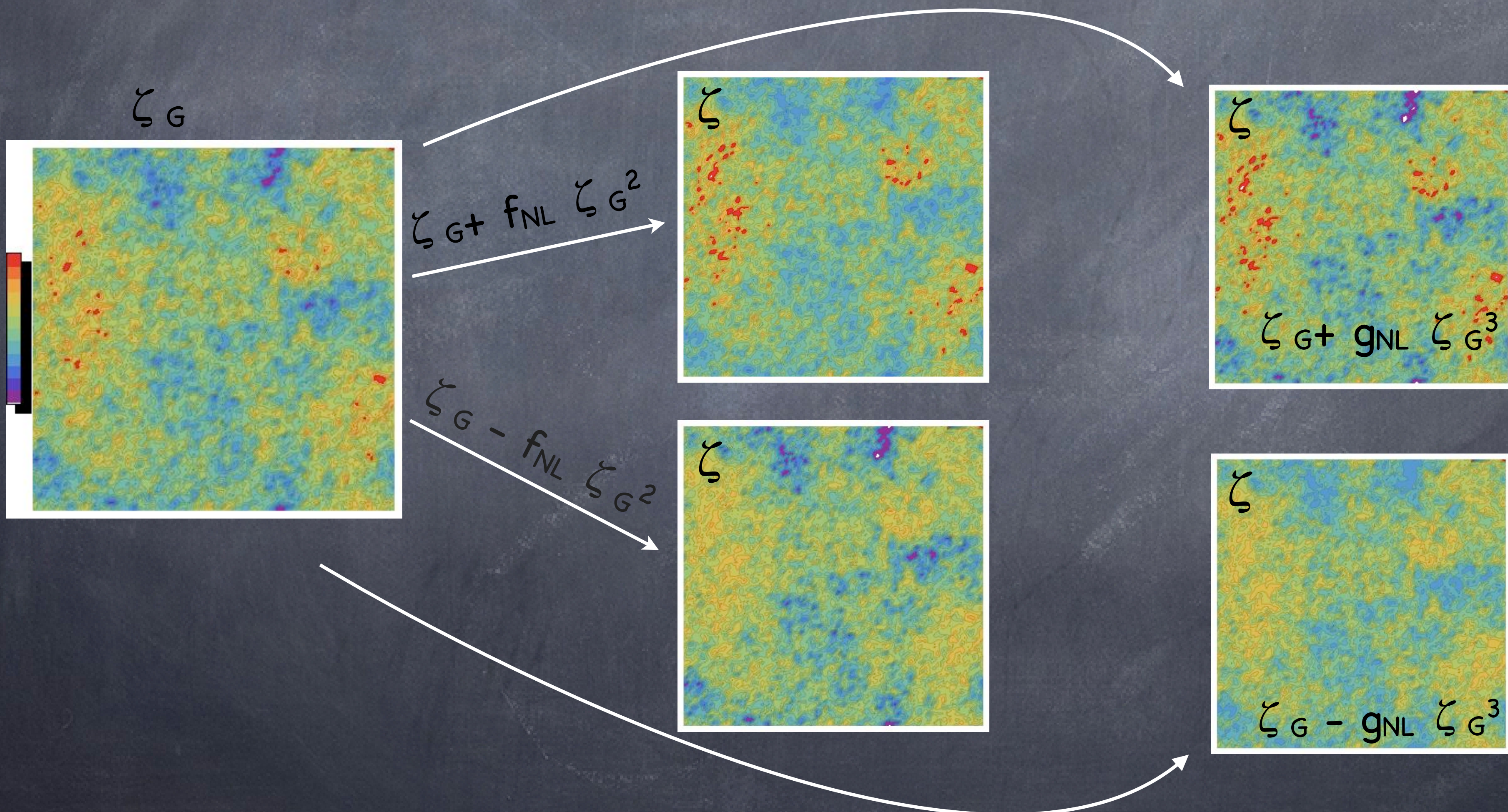

Nurmi, Byrnes, Tasinato 2013

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power spectrum

$$\langle \zeta \zeta \rangle \sim \langle \zeta_G \zeta_G \rangle$$

bispectrum

$$\langle \zeta \zeta \zeta \rangle \sim f_{NL} \langle \zeta_G \zeta_G \rangle \langle \zeta_G \zeta_G \rangle$$

trispectrum

$$\langle \zeta \zeta \zeta \zeta \rangle - 3 \langle \zeta \zeta \rangle^2 \sim f_{NL}^2 \langle \zeta_G \zeta_G \rangle \langle \zeta_G \zeta_G \rangle \langle \zeta_G \zeta_G \rangle$$
$$+ g_{NL} \langle \zeta_G \zeta_G \rangle \langle \zeta_G \zeta_G \rangle \langle \zeta_G \zeta_G \rangle$$

Nurmi, Byrnes, Tasinato 2013

ML, Nelson, Shandera 2013

Single-source weak NG

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$$\Downarrow$$

locally,

$$\zeta_s = \zeta_{G,s}(1 + 2f_{NL}\zeta_{G,L}) + (f_{NL} + \frac{9}{5} g_{NL}\zeta_{G,L})(\zeta_G^2 - \langle \zeta_G^2 \rangle) + \ldots$$

Nurmi, Byrnes, Tasinato 2013

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Single-source weak NG

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↓

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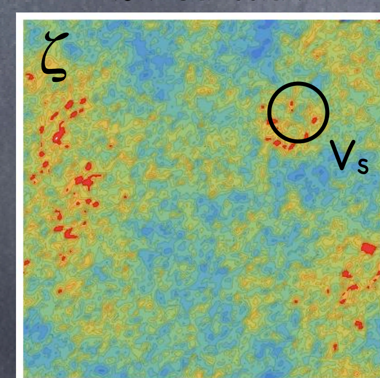
non-Gaussian



Nurmi, Byrnes, Tasinato 2013

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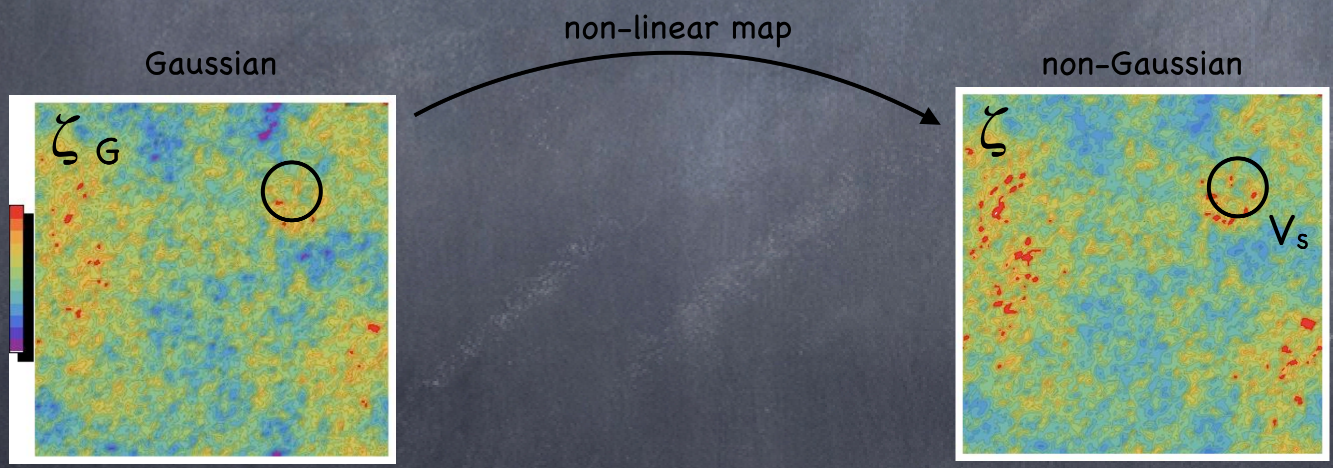

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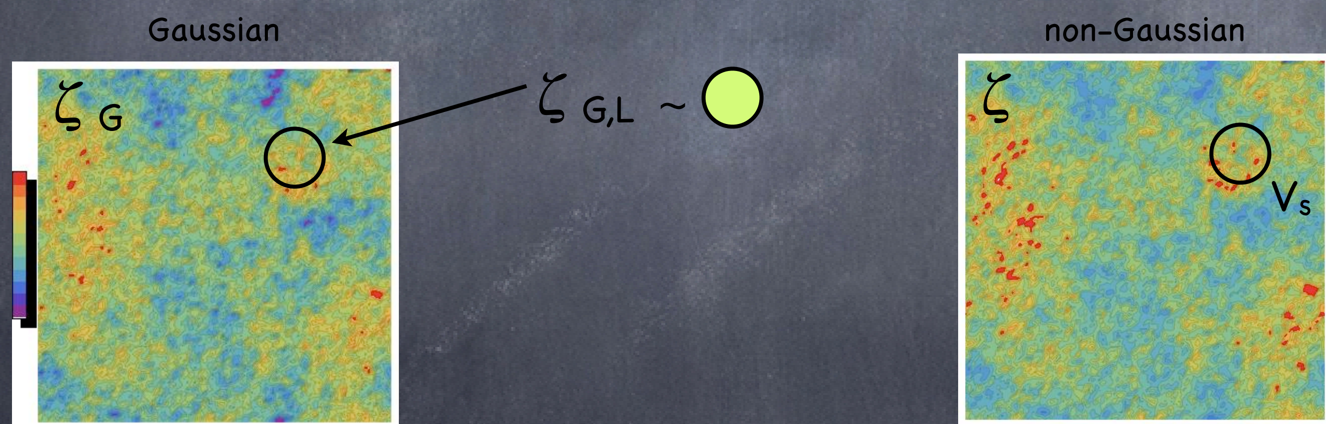

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Single-source weak NG

globally,

$$\zeta = \zeta_G + f_{NL}(\zeta_G^2 - \langle \zeta_G^2 \rangle) + g_{NL}(\zeta_G^3 - 3\zeta_G \langle \zeta_G^2 \rangle) \ldots$$

$$\Downarrow$$

locally,

$$\zeta_s = \zeta_{G,s}(1 + 2f_{NL}\zeta_{G,L}) + (f_{NL} + \frac{9}{5} g_{NL}\zeta_{G,L})(\zeta_G^2 - \langle \zeta_G^2 \rangle) + \ldots$$

$$\Downarrow$$

$$P_\zeta \Big|_{\text{in } V_s} = P_\zeta (1 + 2f_{NL}\zeta_{G,L})$$

$$f_{NL} \Big|_{\text{in } V_s} = f_{NL} - \frac{12}{5} f_{NL}^2 \zeta_{G,L} + \frac{9}{5} g_{NL}\zeta_{G,L} + \ldots$$

$$g_{NL} \Big|_{\text{in } V_s} = g_{NL} - \frac{18}{5} f_{NL} g_{NL}\zeta_{G,L} + \frac{12}{5} h_{NL}\zeta_{G,L}$$

Nurmi, Byrnes, Tasinato 2013

ML, Nelson, Shandera 2013

Single-source weak NG

globally,

$$\zeta = \zeta_G + f_{NL}(\zeta_G^2 - \langle\zeta_G^2\rangle) + g_{NL}(\zeta_G^3 - 3\zeta_G\langle\zeta_G^2\rangle)\ ...$$

↓

locally,

$$\zeta_s = \zeta_{G,s}(1 + 2f_{NL}\zeta_{G,L}) + (f_{NL} + \frac{9}{5}g_{NL}\zeta_{G,L})(\zeta_G^2 - \langle\zeta_G^2\rangle) + \ ...$$

↓

$$P_\zeta\Big|_{\text{in } V_s} = P_\zeta(1 + 2f_{NL}\zeta_{G,L})$$

$$f_{NL}\Big|_{\text{in } V_s} = f_{NL} - \frac{12}{5}f_{NL}^2\zeta_{G,L} + \frac{9}{5}g_{NL}\zeta_{G,L} + \ ...$$

$$g_{NL}\Big|_{\text{in } V_s} = g_{NL} - \frac{18}{5}f_{NL}g_{NL}\zeta_{G,L} + \frac{12}{5}h_{NL}\zeta_{G,L}$$

$$f_{NL}\sqrt{\Delta^2_\zeta} \ll 1$$

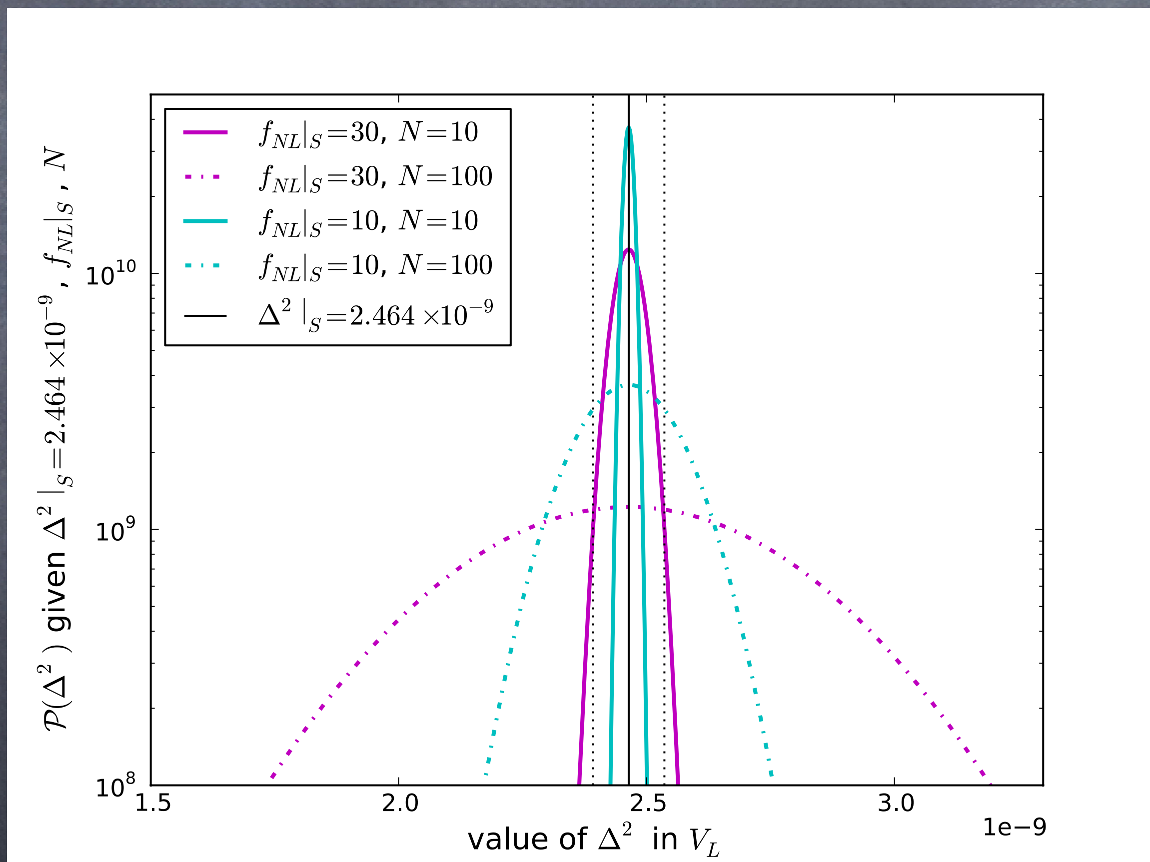
$$g_{NL}\Delta^2_\zeta \ll 1$$

Nurmi, Byrnes, Tasinato 2013

ML, Nelson, Shandera 2013

Single-source weak NG

probabilistic relationship between observations in
$V_s \sim H_0^{-3}$ and V_L



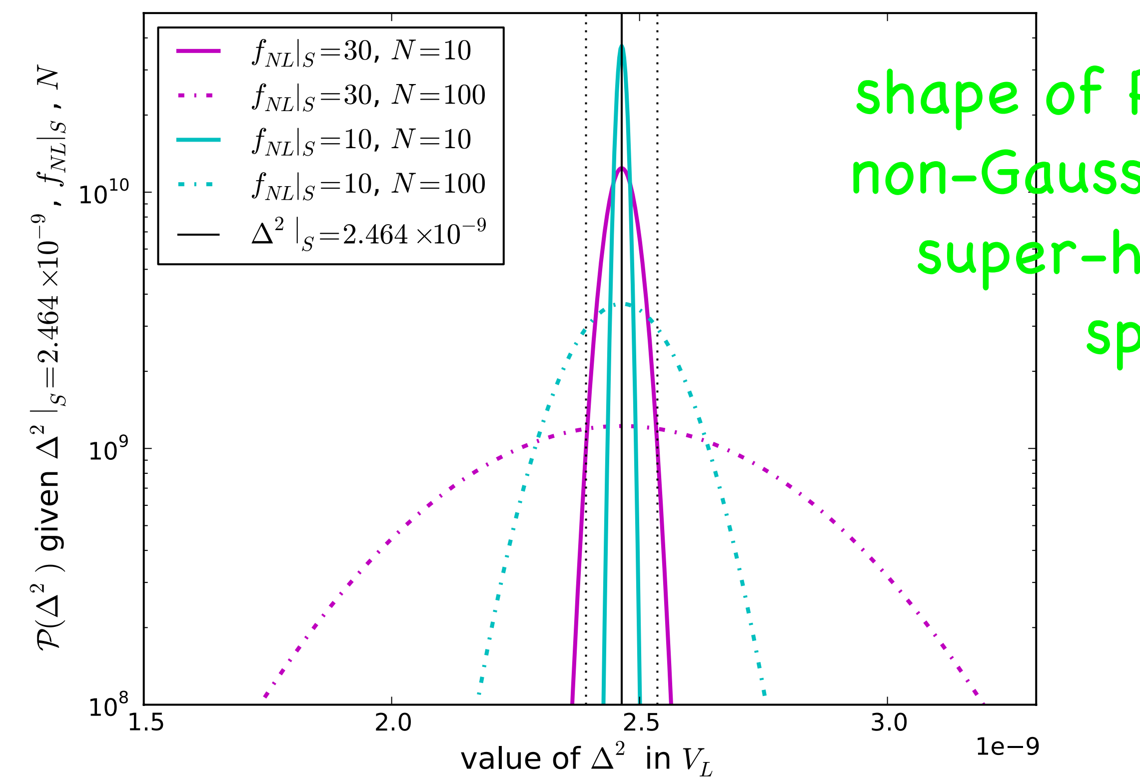
see also Nurmi, Byrnes, Tasinato 2013

ML, Nelson, Shandera 2013

Single-source weak NG

probabilistic relationship between observations in
$V_s \sim H_0^{-3}$ and V_L

Planck: $f_{NL} = 2.7 \pm 5.8$! (planck collaboration 2013)



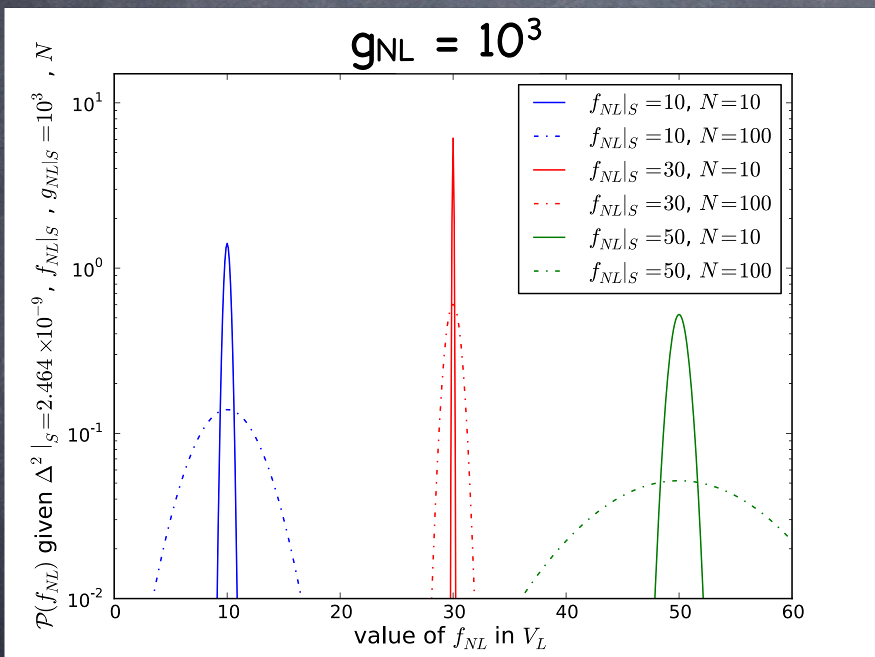
shape of PDF depends on non-Gaussian parameters, super-horizon power spectrum

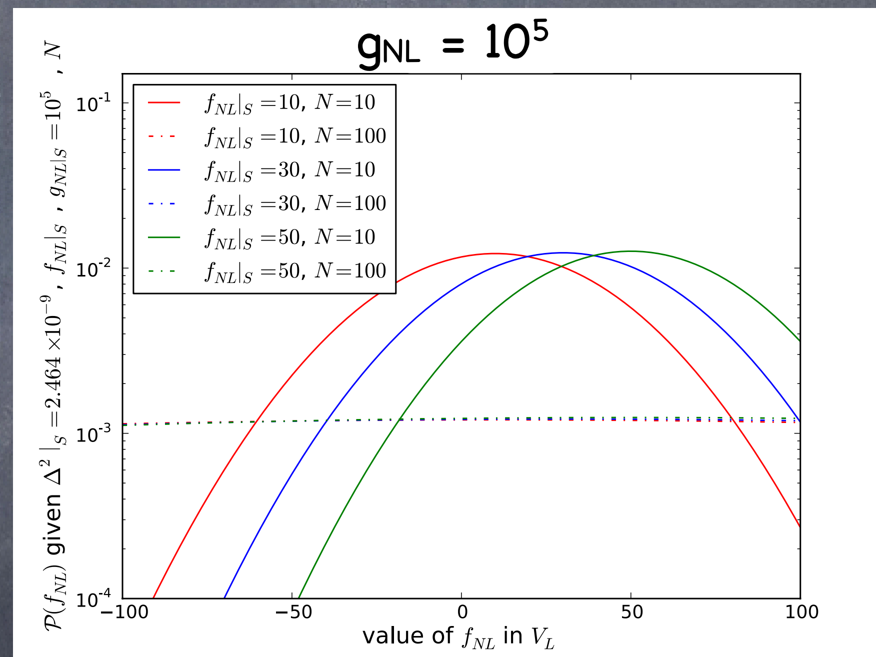
see also Nurmi, Byrnes, Tasinato 2013

ML, Nelson, Shandera 2013

Single-source weak NG

probabilistic relationship between observations in
$V_s \sim H_0^{-3}$ and V_L





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see also Nurmi, Byrnes, Tasinato 2013

ML, Nelson, Shandera 2013

Single-source Strong NG

ML, Nelson, Shandera 2013

Single-source Strong NG

globally:

$$\zeta(x) = \zeta^p_G(x) - \langle \zeta^p_G \rangle$$

ML, Nelson, Shandera 2013

Single-source Strong NG

globally:

$$\zeta(x) = \zeta_G^p(x) - \langle \zeta_G^p \rangle$$

power spectrum $\sim \langle \zeta_G{}^2 \rangle^p$

bispectrum $\sim \langle \zeta_G{}^2 \rangle^{3p/2}$

trispectrum $\sim \langle \zeta_G{}^2 \rangle^{2p}$

ML, Nelson, Shandera 2013

Single-source Strong NG

globally:

$$\zeta(x) = \zeta_G^p(x) - \langle \zeta_G^p \rangle$$

power spectrum $\sim \langle \zeta_G^2 \rangle^p$

bispectrum $\sim \langle \zeta_G^2 \rangle^{3p/2}$

trispectrum $\sim \langle \zeta_G^2 \rangle^{2p}$

$$1 \sim f_{NL}\sqrt{\Delta^2_\zeta} \sim g_{NL}\Delta^2_\zeta$$

strongly non-Gaussian

ML, Nelson, Shandera 2013

Single-source Strong NG

globally:

$$\zeta(x) = \zeta_G^p(x) - \langle \zeta_G^p \rangle$$

power spectrum $\sim \langle \zeta_G^2 \rangle^p$

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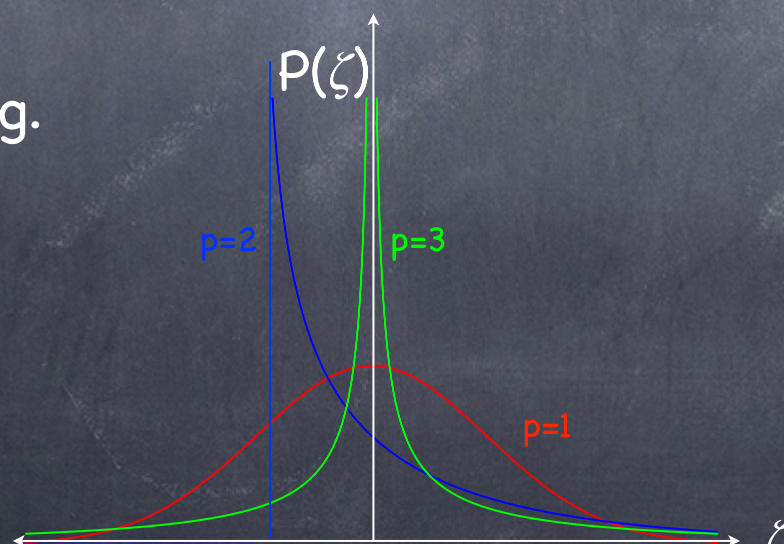
$$1 \sim f_{NL}\sqrt{\Delta^2_\zeta} \sim g_{NL}\Delta^2_\zeta$$

strongly non-Gaussian

e.g.



ML, Nelson, Shandera 2013

Single-source Strong NG

globally:

$$\zeta(x) = \zeta_G^p(x) - \langle \zeta_G^p \rangle$$

Nelson & Shandera 2012; ML, Nelson, Shandera 2013

Single-source Strong NG

globally:

$$\zeta(x) = \zeta^p_G(x) - \langle \zeta^p_G \rangle$$

locally:

$$\zeta_s(x) = p\,\zeta_{G,s}(x)\,\zeta^{p-1}_{G,L} + \binom{p}{2}\zeta^2_{G,s}(x)\,\zeta^{p-2}_{G,L} + \binom{p}{3}\zeta^3_{G,s}(x)\,\zeta^{p-3}_{G,L} + \ldots$$

$$- \binom{p}{2}\langle \zeta^2_{G,s}(x)\rangle\,\zeta^{p-2}_{G,L} + \ldots$$

Nelson & Shandera 2012; ML, Nelson, Shandera 2013

Single-source Strong NG

globally:

$$\zeta(x) = \zeta^{p}_{G}(x) - \langle \zeta^{p}_{G} \rangle$$

locally:

$$\zeta_{s}(x) = p\,\zeta_{G,s}(x)\,\zeta^{p-1}_{G,L} + \binom{p}{2}\zeta^{2}_{G,s}(x)\,\zeta^{p-2}_{G,L} + \binom{p}{3}\zeta^{3}_{G,s}(x)\,\zeta^{p-3}_{G,L} + \ldots$$

$$- \binom{p}{2}\langle \zeta^{2}_{G,s}(x)\rangle\,\zeta^{p-2}_{G,L} + \ldots$$

on average, all terms are equally important

Nelson & Shandera 2012; ML, Nelson, Shandera 2013

Single-source Strong NG

globally:

$$\zeta(x) = \zeta^p_G(x) - \langle \zeta^p_G \rangle$$

locally:

$$\zeta_s(x) = p\,\zeta_{G,s}(x)\,\zeta^{p-1}_{G,L} + \binom{p}{2}\zeta^2_{G,s}(x)\,\zeta^{p-2}_{G,L} + \binom{p}{3}\zeta^3_{G,s}(x)\,\zeta^{p-3}_{G,L} + \ldots$$

$$- \binom{p}{2}\langle \zeta^2_{G,s}(x)\rangle\,\zeta^{p-2}_{G,L} + \ldots$$

on average, all terms are equally important

BUT if: $\zeta_{G,L} \gg \sqrt{\overline{\zeta^2_{G,s}}}$

Nelson & Shandera 2012; ML, Nelson, Shandera 2013

Single-source Strong NG

globally:

$$\zeta(x) = \zeta_G^p(x) - \langle \zeta_G^p \rangle$$

locally:

$$\zeta_s(x) = p\,\zeta_{G,s}(x)\,\zeta_{G,L}^{p-1} + \binom{p}{2}\zeta_{G,s}^2(x)\,\zeta_{G,L}^{p-2} + \binom{p}{3}\zeta_{G,s}^3(x)\,\zeta_{G,L}^{p-3} + \ldots$$

$$- \binom{p}{2}\langle \zeta_{G,s}^2(x)\rangle\,\zeta_{G,L}^{p-2} + \ldots$$

on average, all terms are equally important

BUT if:

$$\zeta_{G,L} \gg \sqrt{\zeta_{G,s}^2}$$

$$\zeta_s = \chi_G + f_{NL}\,(\chi_G^2 - \langle \chi_G^2 \rangle) + g_{NL}\,(\chi_G^3 - 3\chi_G\langle \chi_G^2 \rangle) \ldots$$

Nelson & Shandera 2012; ML, Nelson, Shandera 2013

Single-source Strong NG

globally:

$$\zeta(x) = \zeta^p_G(x) - \langle \zeta^p_G \rangle$$

locally:

$$\zeta_s(x) = p\,\zeta_{G,s}(x)\,\zeta^{p-1}_{G,L} + \binom{p}{2}\zeta^2_{G,s}(x)\,\zeta^{p-2}_{G,L} + \binom{p}{3}\zeta^3_{G,s}(x)\,\zeta^{p-3}_{G,L} + \ldots$$

$$- \binom{p}{2}\langle \zeta^2_{G,s}(x)\rangle\,\zeta^{p-2}_{G,L} + \ldots$$

on average, all terms are equally important

BUT if:

$$\zeta_{G,L} \gg \sqrt{\zeta^2_{G,s}}$$

$$\zeta_s = \chi_G + f_{NL}\,(\chi_G^2 - \langle \chi_G^2\rangle) + g_{NL}\,(\chi_G^3 - 3\chi_G\langle \chi_G^2\rangle)\ \ldots$$

you recover statistics that are only weakly non-Gaussian!

Nelson & Shandera 2012; ML, Nelson, Shandera 2013

Single-source Strong NG

Can $\zeta_{G,L} \gg \sqrt{\zeta^2_{G,s}}$?

Nelson & Shandera 2012; ML, Nelson, Shandera 2013

Single-source Strong NG

$$\text{Can } \zeta_{G,L} \gg \sqrt{\zeta^2_{G,s}}\ ?$$

roughly:

$$\frac{\zeta_{G,L}}{\sqrt{\langle \zeta^2_{G,L} \rangle}} \gg \sqrt{\frac{N_s}{N}} \quad \text{for } n_s = 1$$

Nelson & Shandera 2012; ML, Nelson, Shandera 2013

Single-source Strong NG

$$\text{Can } \zeta_{G,L} \gg \sqrt{\zeta^2_{G,s}}\,?$$

roughly:

$$\frac{\zeta_{G,L}}{\sqrt{\langle \zeta^2_{G,L} \rangle}} \gg \sqrt{\frac{N_s}{N}} \quad \text{for } n_s = 1$$

N_s: number of sub-horizon e-folds ~60?

N: (as before, number of super-horizon e-folds)

Nelson & Shandera 2012; ML, Nelson, Shandera 2013

Single-source Strong NG

Can $\zeta_{G,L} \gg \sqrt{\zeta^2_{G,s}}$?

roughly:

$$\frac{\zeta_{G,L}}{\sqrt{\langle \zeta^2_{G,L} \rangle}} \gg \sqrt{\frac{N_s}{N}} \quad \text{for } n_s = 1$$

(N doesn't have to be as large for $n_s < 1$)

Nelson & Shandera 2012; ML, Nelson, Shandera 2013

Single-source Strong NG

so, we can have $\zeta_{G,L} \gg \sqrt{\zeta^2_{G,s}}$

giving our condition for weak non-Gaussianity in a region with background mode $\zeta_{G,L}$

but our Hubble-patch appears to be really, really Gaussian (f_{NL} really small!)

Nelson & Shandera 2012; ML, Nelson, Shandera 2013

Single-source Strong NG

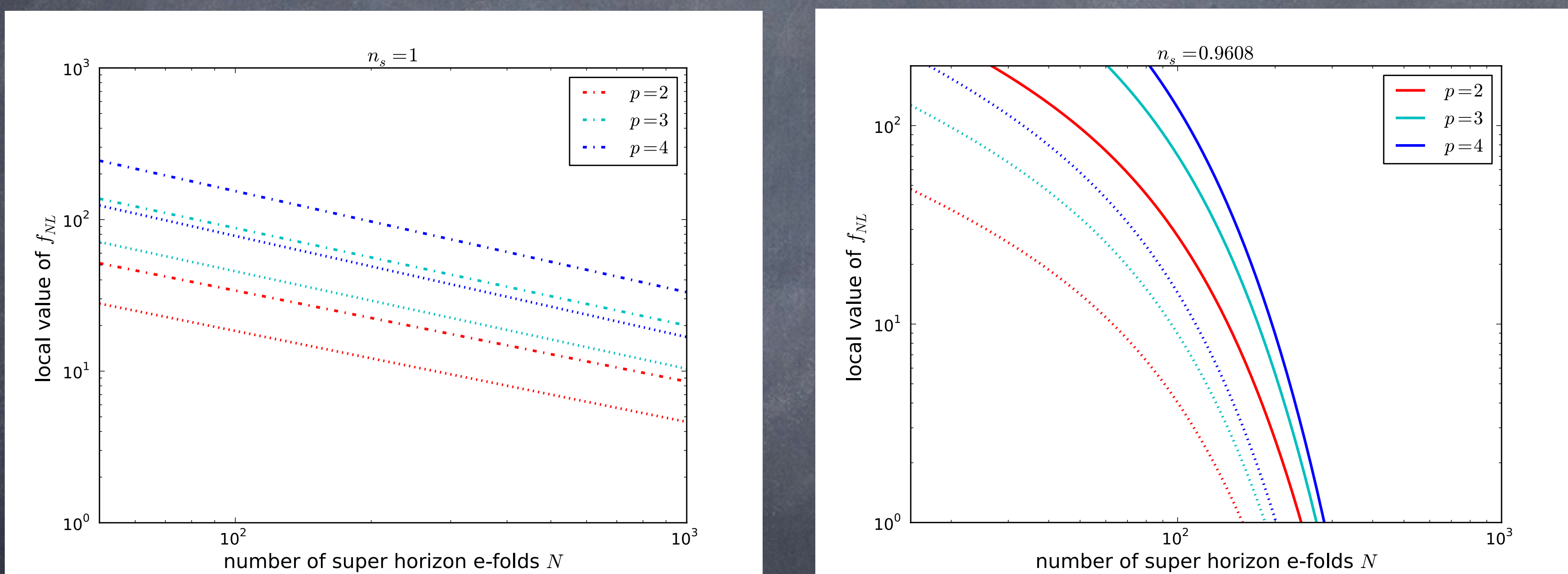
But, for $\zeta(x) = \zeta^p_G(x) - \langle \zeta^p_G \rangle$ in V_L, we can produce $\Delta^2_\zeta \sim 10^{-9}$ and weakly non-Gaussian, in agreement with observations

Nelson & Shandera 2012; ML, Nelson, Shandera 2013

Single-source Strong NG

But, for $\zeta(x) = \zeta^p_G(x) - \langle \zeta^p_G \rangle$ in V_L, we can produce $\Delta^2_\zeta \sim 10^{-9}$

and weakly non-Gaussian, in agreement with observations



$\zeta_{G,L} / \sqrt{\zeta^2_{G,L}}$ = 1 (solid), 3 (dot-dashed), 5 (dotted)

32% 0.3% 0.00006%

Nelson & Shandera 2012; ML, Nelson, Shandera 2013

Multi-source weak NG I

ML, Nelson, Shandera 2013

Multi-source weak NG I

Assume two uncorrelated fields generate ζ in V_L

$$\zeta = \phi_G + \sigma_G + \tilde{f}_{NL}\left(\sigma_G^2 - \langle \sigma_G^2 \rangle\right)$$

ML, Nelson, Shandera 2013

Multi-source weak NG I

Assume two uncorrelated fields generate ζ in V_L

$$\zeta = \phi_G + \sigma_G + \tilde{f}_{NL}(\sigma_G^2 - \langle \sigma_G^2 \rangle)$$

$$\langle \phi_G \sigma_G \rangle = 0$$

$$P_\zeta = P_\phi + P_\sigma$$

$$f_{NL} = \tilde{f}_{NL} / (1 + P_\phi / P_\sigma)^2$$

$$\tau_{NL} = 2\,(1 + P_\phi / P_\sigma)\, f_{NL}^2$$

ML, Nelson, Shandera 2013

Multi-source weak NG I

Assume two uncorrelated fields generate ζ in V_L

$$\zeta = \phi_G + \sigma_G + \tilde{f}_{NL}(\sigma_G^2 - \langle \sigma_G^2 \rangle)$$

$$\langle \phi_G \sigma_G \rangle = 0$$

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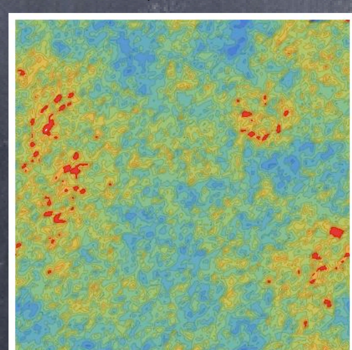
$$f_{NL} = \tilde{f}_{NL} / (1 + P_\phi / P_\sigma)^2$$

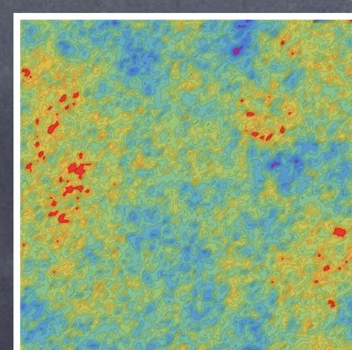
$$\tau_{NL} = 2\,(1 + P_\phi / P_\sigma)\, f_{NL}^2$$

skewness suppressed/kurtosis boosted relative to skewness

$P_\phi = 0$



$P_\phi = P_\sigma$



ML, Nelson, Shandera 2013

Multi-source weak NG I

Assume two uncorrelated fields generate ζ in V_L

$$\zeta = \phi_G + \sigma_G + \tilde{f}_{NL}\left(\sigma_G^2 - \langle \sigma_G^2 \rangle\right)$$

$$\langle \phi_G \sigma_G \rangle = 0$$

$$P_\zeta = P_\phi + P_\sigma$$

↓

locally

$$P_\zeta \Big|_{\text{in } V_s} = P_\zeta \left(1 + \frac{12}{5} f_{NL} \left(1 + P_\phi / P_\sigma\right) \sigma_{G,L}\right)$$

$$f_{NL} \Big|_{\text{in } V_s} = f_{NL} \left(1 + \frac{12}{5} \left(\frac{\tau_{NL} - 2 f_{NL}^2}{f_{NL}}\right)\left(1 + P_\phi / P_\sigma\right) \sigma_{G,L}\right)$$

ML, Nelson, Shandera 2013

Multi-source weak NG I

Assume two uncorrelated fields generate ζ in V_L

$$\zeta = \phi_G + \sigma_G + \tilde{f}_{NL}\left(\sigma_G^2 - \langle \sigma_G^2 \rangle\right)$$

$$\langle \phi_G \sigma_G \rangle = 0$$

$$P_\zeta = P_\phi + P_\sigma$$

↓

only $\sigma_{G,L}$ modulates local stats

locally

$$P_\zeta \Big|_{\text{in } V_s} = P_\zeta \left(1 + \frac{12}{5} f_{NL} \left(1 + P_\phi / P_\sigma\right) \sigma_{G,L}\right)$$

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ML, Nelson, Shandera 2013

Multi-source weak NG I

Assume two uncorrelated fields generate ζ in V_L

$$\zeta = \phi_G + \sigma_G + \tilde{f}_{NL}(\sigma_G^2 - \langle \sigma_G^2 \rangle)$$

$$\langle \phi_G \sigma_G \rangle = 0$$

$$P_\zeta = P_\phi + P_\sigma$$

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only $\sigma_{G,L}$ modulates local stats

locally

$$P_\zeta \Big|_{\text{in } V_s} = P_\zeta\left(1 + \frac{12}{5} f_{NL}\,(1 + P_\phi/P_\sigma)\,\sigma_{G,L}\right)$$

$$f_{NL}\Big|_{\text{in } V_s} = f_{NL}\left(1 + \frac{12}{5}\left(\frac{\tau_{NL} - 2 f_{NL}^2}{f_{NL}}\right)(1 + P_\phi/P_\sigma)\,\sigma_{G,L}\right)$$

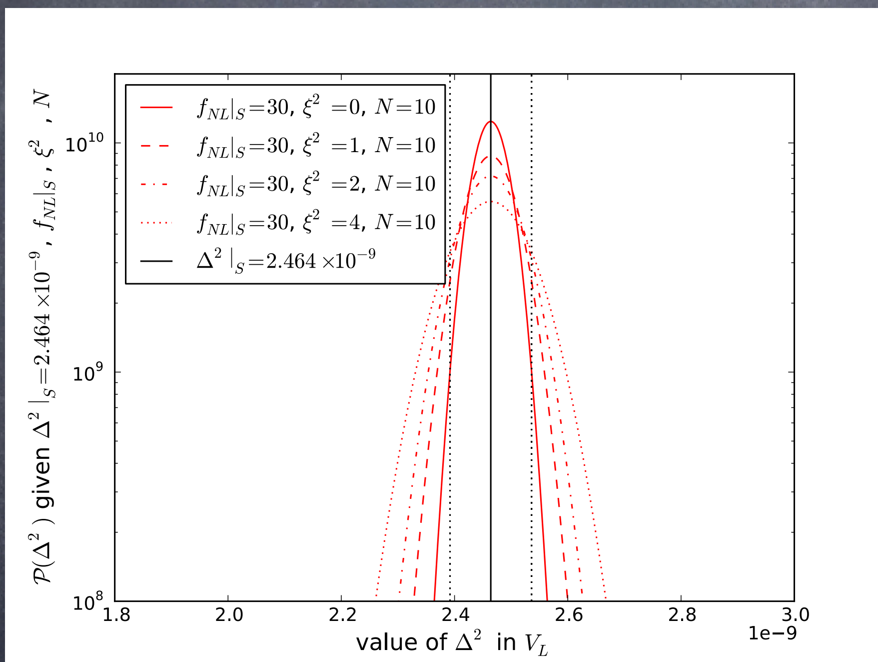
for fixed f_{NL}, P_ζ typical modulation is larger

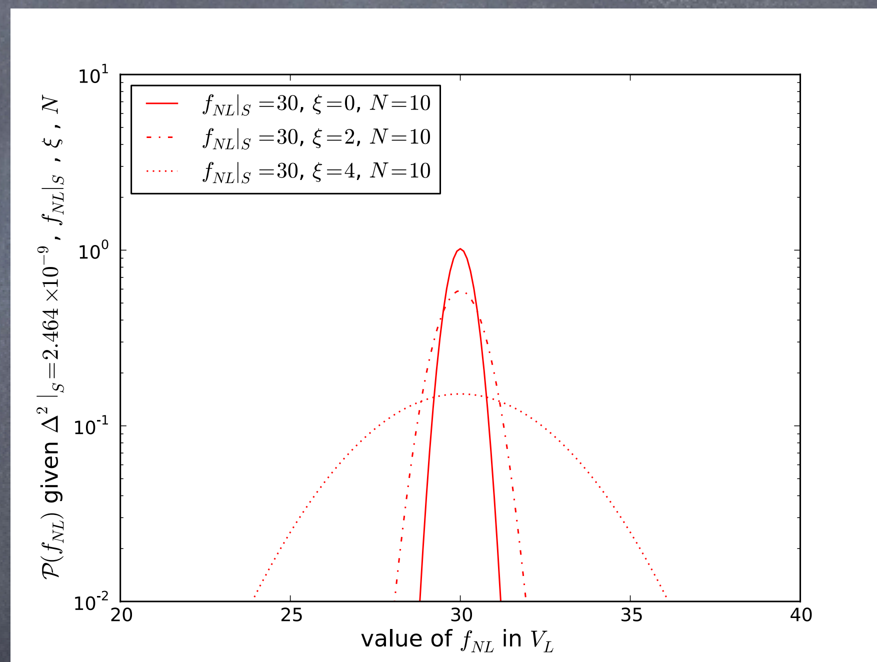
$$\langle (1 + P_\phi/P_\sigma)^2 \sigma_{G,L}^2 \rangle \sim (1 + P_\phi/P_\sigma) \langle \zeta_{G,L}^2 \rangle$$

ML, Nelson, Shandera 2013

Multi-source weak NG I

for fixed f_{NL}, P_ζ typical modulation is larger





ML, Nelson, Shandera 2013

Multi-source weak NG II

ML, Nelson, Shandera 2013

Multi-source weak NG II

$$\zeta = \phi_G + \tilde{f}_{NL} (\sigma_G^2 - \langle \sigma_G^2 \rangle)$$

take $\tilde{f}_{NL} \Delta_\sigma \sim 1$, but $\Delta^2_\phi \gg \Delta^2_\sigma$ so still only weakly non-Gaussian

f_{NL}, τ_{NL} ~ scale-dependent

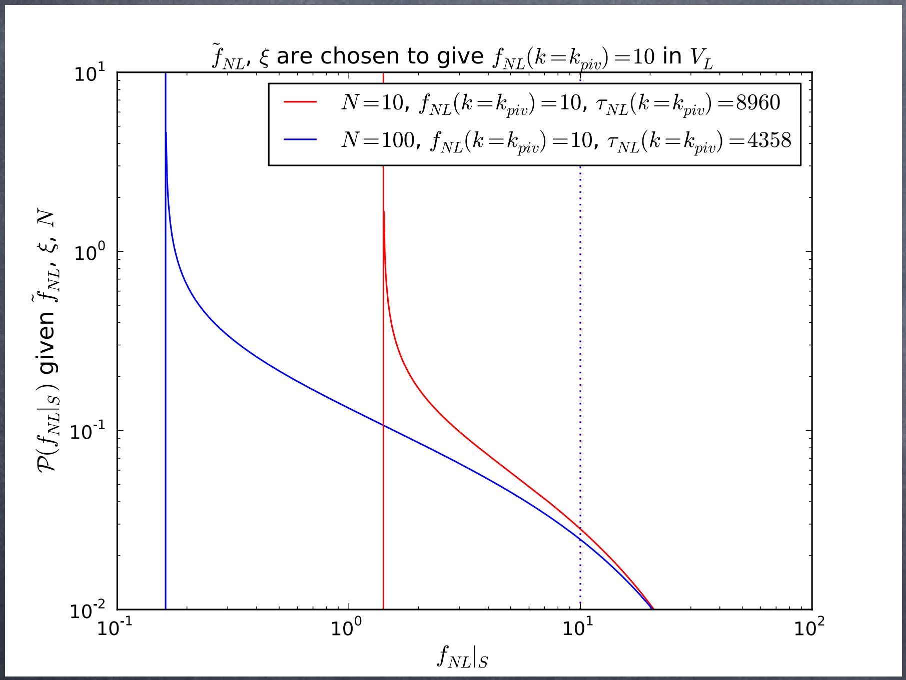
but, for $\sigma_{G,L} \gg \sqrt{\sigma^2_{G,s}}$

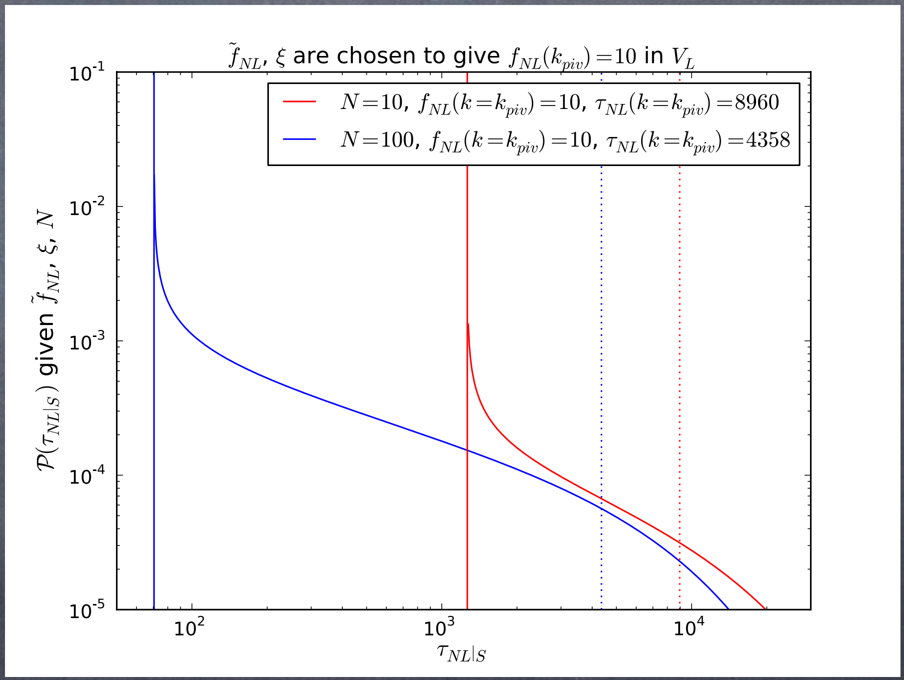
can again recover weakly non-Gaussian statistics
with constant f_{NL}, τ_{NL}

ML, Nelson, Shandera 2013

Multi-source weak NG II

Local parameters are modulated by $\sigma^2_{G,L}$, instead of $\sigma_{G,L}$ so probability distributions are highly skewed!





ML, Nelson, Shandera 2013

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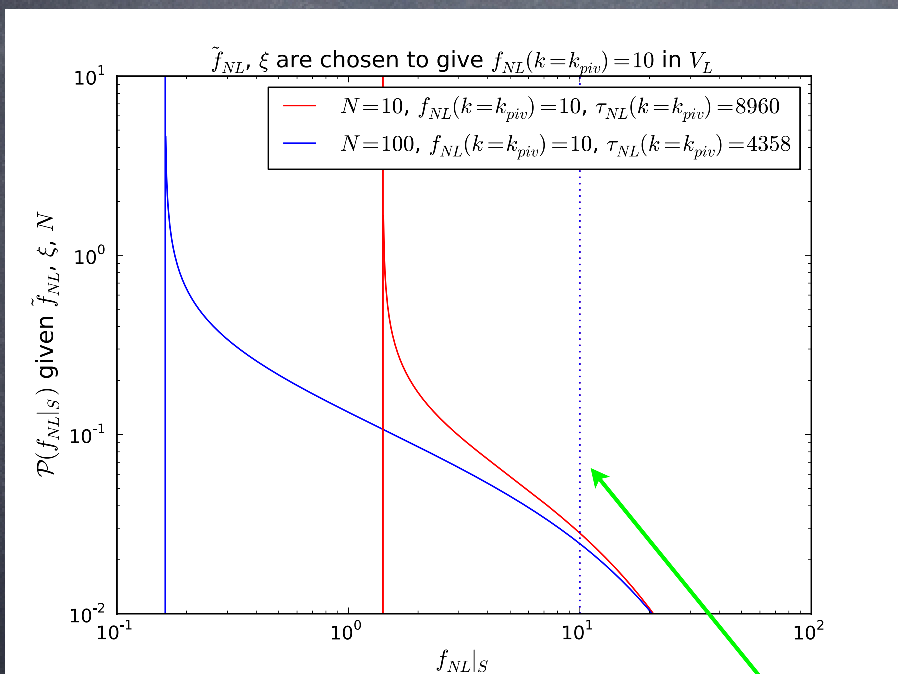
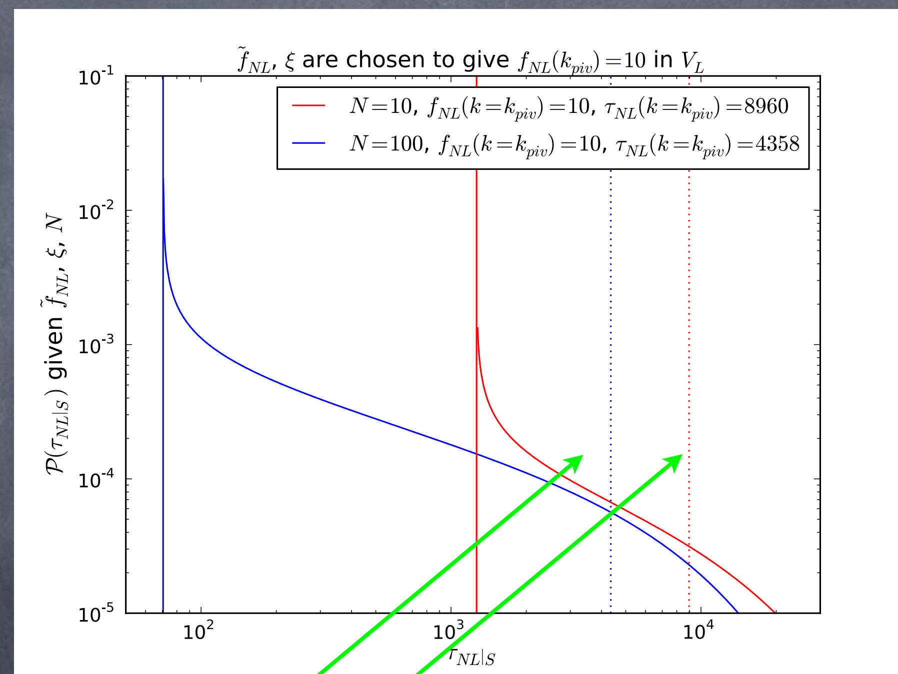




average values in all V_L

ML, Nelson, Shandera 2013

OK, so . . . ?

How does this change inferences about inflationary model?

- What kind of model parameters does this actually change? Or how does this change inferences about models?
- Also, we've assumed that Δ^2, f_{NL}, are free and independent parameters, may not be true in a real model

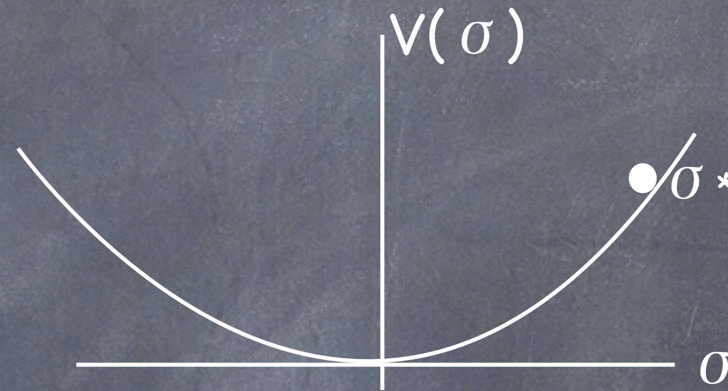
How does this change inferences about inflationary model?

- (I) worked example: curvaton
- (II) your example? thoughts?

How does this change inferences ?

worked example: curvaton

curvaton only at curvaton decay, $V(\sigma) = m^2 \sigma^2$



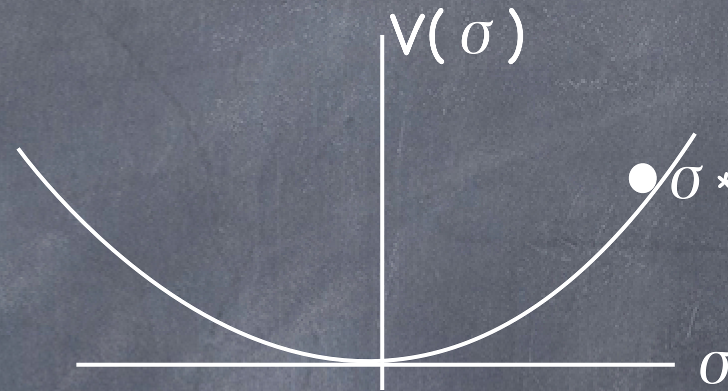
Linde & Mukhanov 1997; Lyth and Wands 2002
Linde & Mukhanov 2006; Demozzi, Linde, Mukhanov 2011

ML, Wayne Hu

How does this change inferences ?

worked example: curvaton, no perturbations from inflaton

curvaton only at curvaton decay, $V(\sigma) = m^2 \sigma^2$



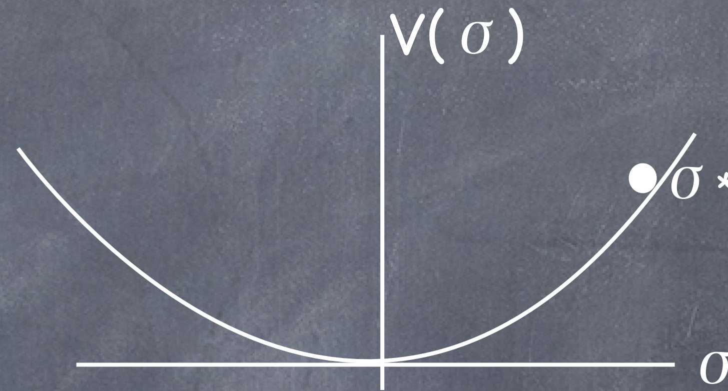
$$\zeta = \frac{2\delta\sigma}{3\sigma_*} - \frac{5}{4}\left(\frac{2\delta\sigma}{3\sigma_*}\right)^2 + \frac{25}{12}\left(\frac{2\delta\sigma}{3\sigma_*}\right)^3 + \ldots$$

Linde & Mukhanov 1997; Lyth and Wands 2002
Linde & Mukhanov 2006; Demozzi, Linde, Mukhanov 2011

How does this change inferences ?

worked example: curvaton, no perturbations from inflaton

curvaton only at curvaton decay, $V(\sigma) = m^2 \sigma^2$



$$\zeta = \frac{2\delta\sigma}{3\sigma_*} - \frac{5}{4}\left(\frac{2\delta\sigma}{3\sigma_*}\right)^2 + \frac{25}{12}\left(\frac{2\delta\sigma}{3\sigma_*}\right)^3 + \ldots$$

$$\Delta_\zeta^2 \rightarrow \Delta_\zeta^2 (1 - 3\zeta_L)$$

$$f_{NL} \rightarrow f_{NL}$$

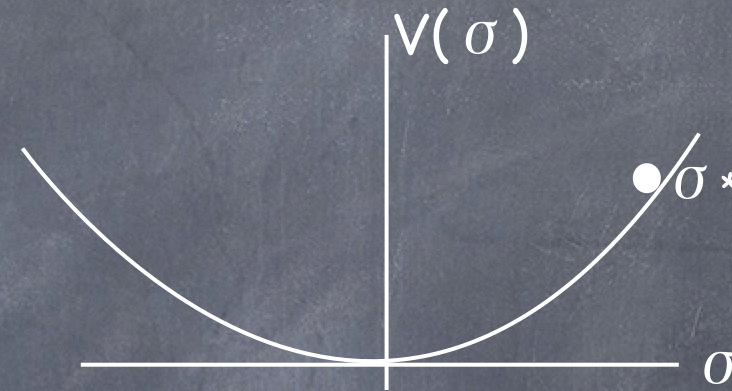
Linde & Mukhanov 1997; Lyth and Wands 2002
Linde & Mukhanov 2006; Demozzi, Linde, Mukhanov 2011

ML, Wayne Hu

How does this change inferences ?

worked example: curvaton, no perturbations from inflaton

curvaton only at curvaton decay, $V(\sigma) = m^2 \sigma^2$



$$\zeta = \frac{2\delta\sigma}{3\sigma_*} - \frac{5}{4}\left(\frac{2\delta\sigma}{3\sigma_*}\right)^2 + \frac{25}{12}\left(\frac{2\delta\sigma}{3\sigma_*}\right)^3 + \ldots$$

$$\Delta_\zeta^2 \rightarrow \Delta_\zeta^2 (1 - 3\,\zeta_L)$$

$$f_{NL} \rightarrow f_{NL}$$

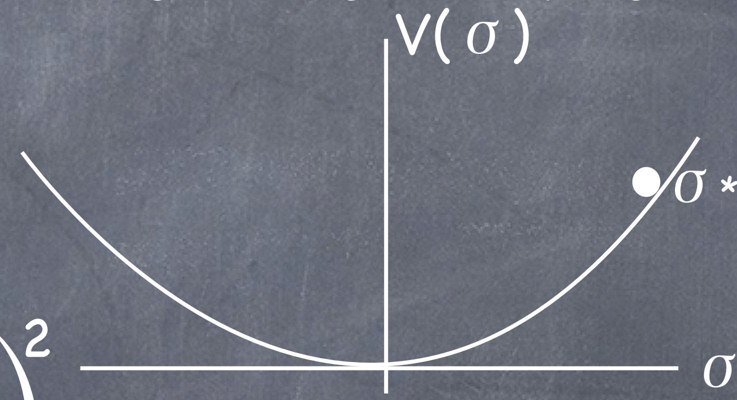
just looks like different value of σ_*

Linde & Mukhanov 1997; Lyth and Wands 2002
Linde & Mukhanov 2006; Demozzi, Linde, Mukhanov 2011

ML, Wayne Hu

How does this change inferences ?

worked example: curvaton, no perturbations from inflaton



curvaton and radiation at curvaton decay, $V(\sigma) = m^2 \sigma^2$

$$\zeta = \frac{2r\delta\sigma}{3\sigma_*} + \left(\frac{5}{4r} - \frac{5}{3} - \frac{5}{6}\right)\left(\frac{2r\delta\sigma}{3\sigma_*}\right)^2 + \ . \ . \ .$$

$$r \equiv \left.\frac{3\Omega_\sigma}{3\Omega_\sigma + 4\ \Omega_r}\right|_{\text{curv. decay}}$$

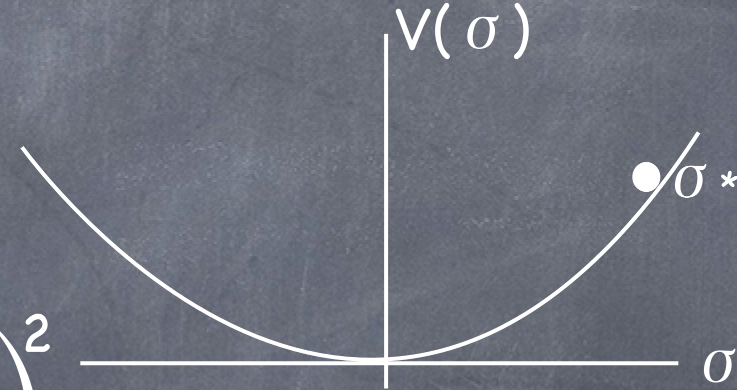
shift in σ_* also shifts r

Linde & Mukhanov 1997; Lyth and Wands 2002
Linde & Mukhanov 2006; Demozzi, Linde, Mukhanov 2011

ML, Wayne Hu

How does this change inferences ?

worked example: curvaton, no perturbations from inflaton



curvaton and radiation at curvaton decay, $V(\sigma) = m^2 \sigma^2$

$$\zeta = \frac{2r\delta\sigma}{3\sigma_*} + \left(\frac{5}{4r} - \frac{5}{3} - \frac{5}{6}\right)\left(\frac{2r\delta\sigma}{3\sigma_*}\right)^2 + \; . \; . \; .$$

$$r \equiv \left.\frac{3\Omega_\sigma}{3\Omega_\sigma + 4\,\Omega_r}\right|_{\text{curv. decay}}$$

shift in σ_* also shifts r

but see

Linde & Mukhanov 1997; Lyth and Wands 2002

Linde & Mukhanov 2006; Demozzi, Linde, Mukhanov 2011

ML, Wayne Hu

Summary

- If the curvature perturbation ζ has local non-Gaussianity (even at a relatively small level) the statistics observed in our Hubble volume may be a biased sample
- We explicitly computed local/global relationship in three simple examples that each give local statistics consistent with observations, even if globally, the statistics are very different and inconsistent with observations