

# Comments on Taxonomy of Dark Energy Models

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## References:

arXiv:1304.4840 Gleyzes, Langlois, Piazza, Vernizzi

arXiv:1211.7054 w/ Bloomfield, Flanagan, and Park

arXiv:1210.0201 Gubitosi, Piazza, Vernizzi

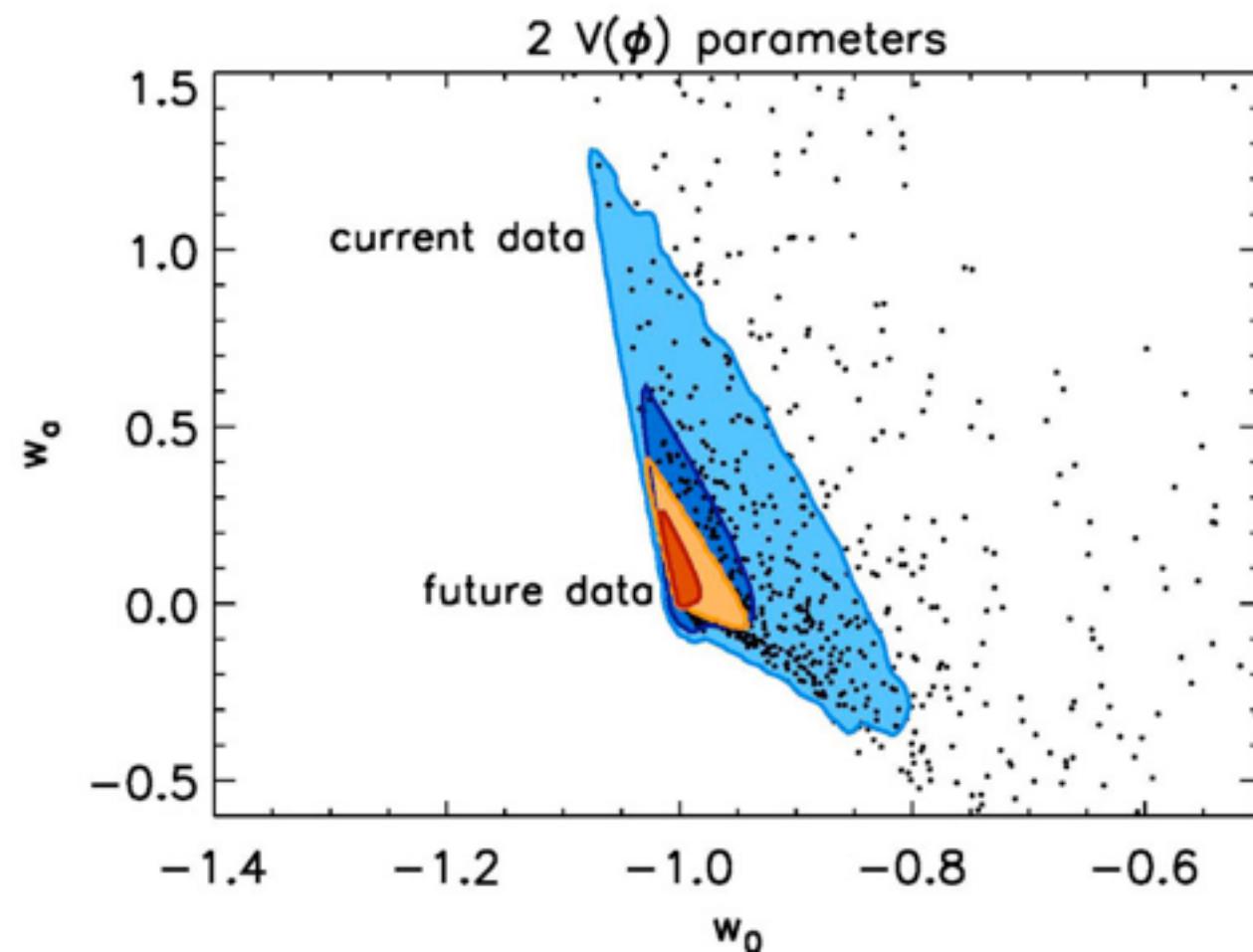
arXiv:1209.2706 w/ Mueller and Bean

arXiv:1112.0303 Bloomfield and Flanagan

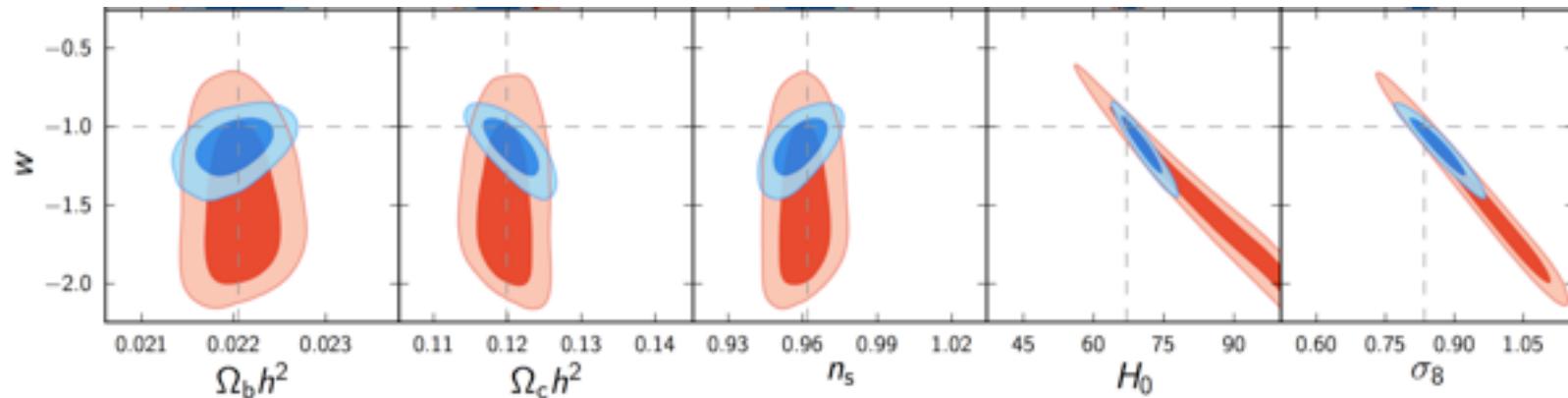
arXiv:1003.1722 w/ Park and Zurek

arXiv:0811.0827 Creminelli, D'Amico, Noreña, Vernizzi

Can we establish a theoretical framework to help guide “realistic” model building?



# Nothing wrong with $\Lambda CDM$



We want to systematically parametrize all consistent alternatives to a Cosmological Constant in a single framework that can then be compared with observations.



# Effective Field Theory and Inflation

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## Approach One

Weinberg, “Effective Field Theory for Inflation”, 0804.4291

Expand background (inflaton and metric) in derivative expansion

## Approach Two

Cheung, et. al. “The Effective Field Theory of Inflation”, 0709.0293

Expand perturbations about spontaneously broken background  
(broken by time dependence)

### Advantages of approach two:

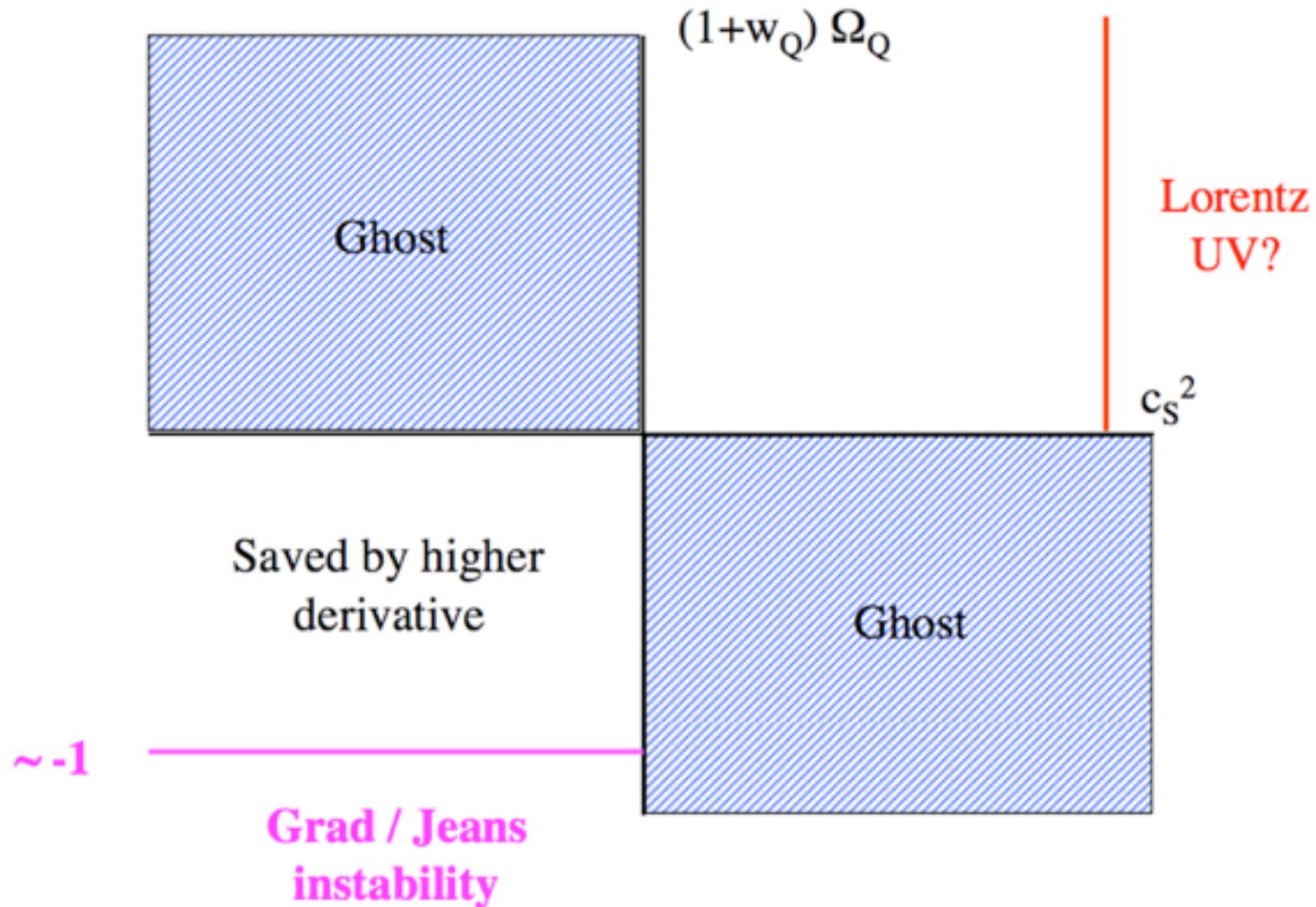
- Inflationary background is not directly observable = focus on perturbations
- Non-linearly realized symmetry establishes relationships between parameters.
- Many inflation models don't admit derivative expansion of background  
( e.g. DBI )

**Disadvantage:** Background is taken a priori.

# EFT of Quintessence

(useful to address stability issues)

arXiv:0811.0827 Creminelli, D'Amico, Noreña, Vernizzi



# Simple First Step towards EFT of Dark Energy

Adapt approach of Weinberg

arXiv:1003.1722 w/ Park and Zurek

arXiv:1112.0303 Bloomfield and Flanagan

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## Approach One

Weinberg, “Effective Field Theory for Inflation”, 0804.4291

Expand background (inflaton and metric) in derivative expansion

$$\begin{aligned} \Delta S = & \int d^4x \sqrt{-g} \left\{ \frac{\alpha_1}{\Lambda^4} (g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi)^2 + \frac{\alpha_2}{\Lambda^3} \square \varphi g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + \frac{\alpha_3}{\Lambda^2} (\square \varphi)^2 \right. \\ & + \frac{b_1}{\Lambda^2 \Lambda_m^2} T^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + \frac{b_2}{\Lambda^2 \Lambda_m^2} T g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + \frac{b_3}{\Lambda \Lambda_m^2} T \square \varphi \\ & + \frac{c_1}{\Lambda^2} R^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + \frac{c_2}{\Lambda^2} R g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + \frac{c_3}{\Lambda} R \square \varphi \\ & + d_1 W^{\mu\nu\lambda\rho} W_{\mu\nu\lambda\rho} + d_2 \epsilon^{\mu\nu\lambda\rho} W_{\mu\nu}{}^{\alpha\beta} W_{\lambda\rho\alpha\beta} + d_3 R^{\mu\nu} R_{\mu\nu} + d_4 R^2 \\ & \left. + \frac{e_1}{\Lambda_m^4} T^{\mu\nu} T_{\mu\nu} + \frac{e_2}{\Lambda_m^4} T^2 + \frac{e_3}{\Lambda_m^2} R_{\mu\nu} T^{\mu\nu} + \frac{e_4}{\Lambda_m^2} R T \right\}, \end{aligned}$$

Already many subtleties...

## Hierarchy of Scales, Additional matter sector complicates picture

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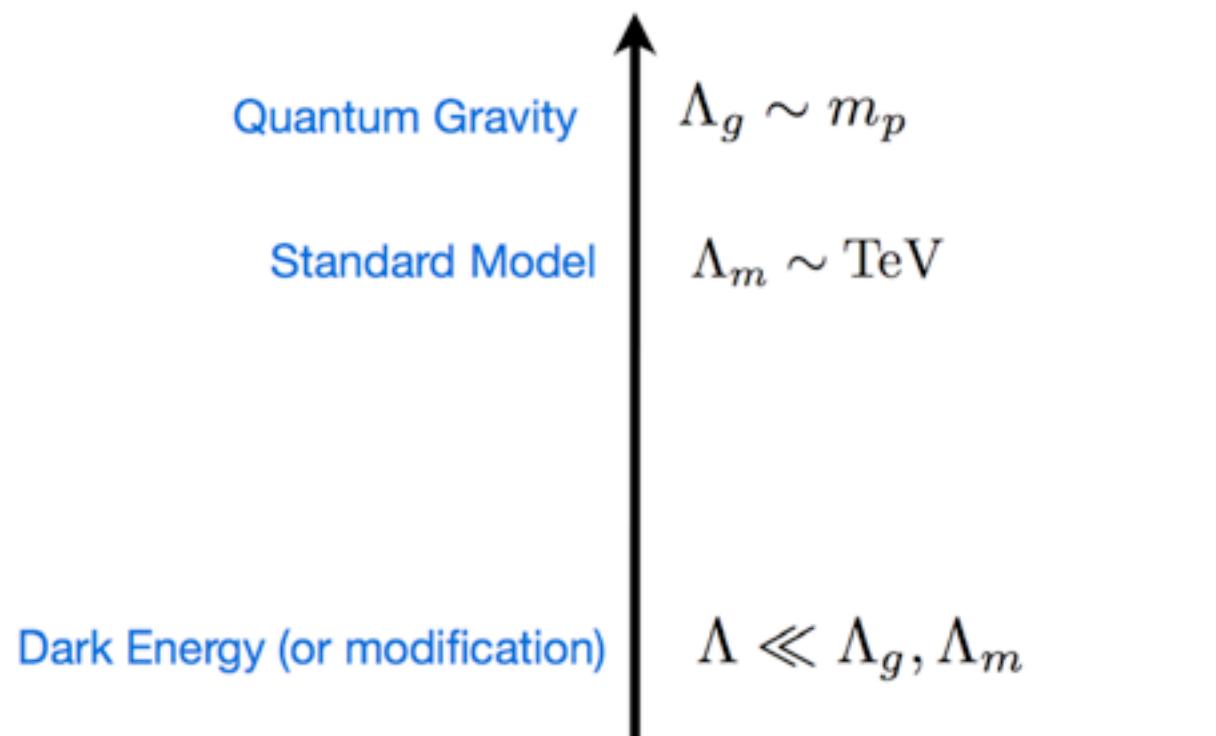
Examples at the Four Derivative level

$$\frac{1}{\Lambda^2} f_1(\varphi) (\square\varphi)^2$$

$$\frac{1}{\Lambda_m^4} f_2(\varphi) T^2$$

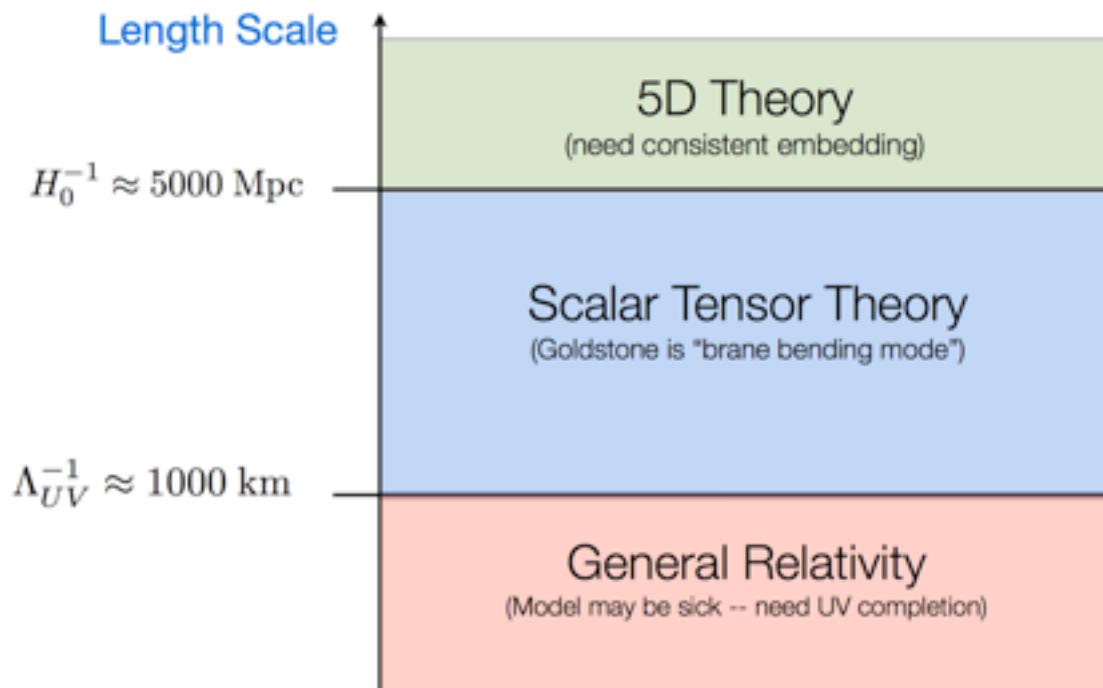
$$f_3(\varphi) \frac{T}{\Lambda_m^2} \frac{\square\varphi}{\Lambda^2}$$

$$\frac{1}{\Lambda^4} f_4(\varphi) (\partial\varphi)^4$$



Already many subtleties...

## How to capture modified gravity models?



### Scalar-tensor theory

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{16\pi G_N} \Omega^2(\varphi) R + \mathcal{L}_\varphi + \mathcal{L}_m \right]$$

### Non-canonical scalar with "K-essence" type correction

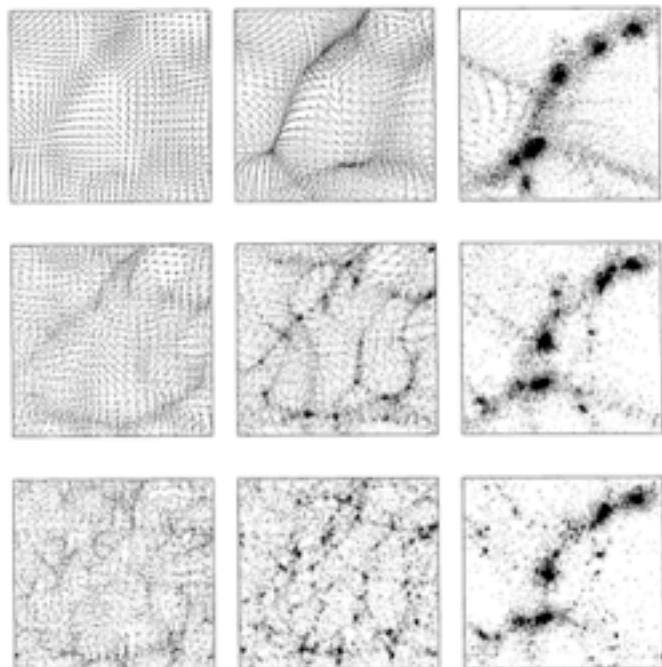
$$\mathcal{L}_\varphi = -\frac{1}{2} Z(\varphi) \partial_\mu \varphi \partial^\mu \varphi - U(\varphi) + \frac{1}{\Lambda^4} \alpha(\varphi) (\partial \varphi)^4$$

Already many subtleties...

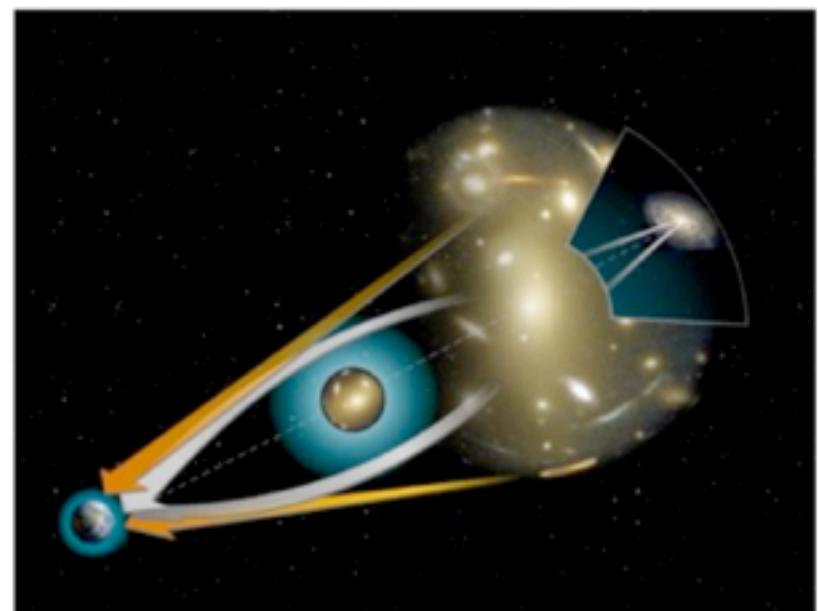
## How to capture modified gravity models?

$$ds^2 = - (1 + 2\Phi) dt^2 + a^2 (1 - 2\Psi) d\vec{x}^2$$

$\Phi$  Growth of Structure



$\Phi + \Psi$  Gravitational Lensing



→ Time

Already many subtleties...

## How to capture modified gravity models?

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Scalar - Tensor / Modified Gravity leads to shear

$$\Psi - \Phi = 2 \frac{\Omega'}{\Omega} \delta\varphi$$

Growth modified by change of effective coupling

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{eff}\bar{\rho}\delta_m \approx 0 \quad \delta_m \sim \frac{\delta\rho}{\rho}$$

$$G_{eff} = G_N \Omega_0^{-2} \frac{Z_0 + 8m_p^2 \Omega'_0{}^2}{Z_0 + 6m_p^2 \Omega'_0{}^2}$$

# Approach One: EFT of Background

arXiv:1003.1722 w/ Park and Zurek

- What observational properties might the most general action predict?

$$S = \int d^4x \sqrt{-g} \left\{ \frac{M_p^2}{2} R - \frac{1}{2} (\nabla \phi)^2 - V(\phi) \right\}$$

Canonical scalar field

Quartic kinetic

$$+ f_{quartic}(\phi) (\nabla \phi)^4$$

Coupling to curvature

$$+ f_{curv}(\phi) G^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi$$

Gauss-Bonnet (GB) term

$$+ f_{GB}(\phi) (R^2 - 4R^{\mu\nu}R_{\mu\nu} + R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho}) \}$$

$$+ S_m \left[ e^{\alpha(\phi)} g_{\mu\nu} (1 + f_{kin}(\phi) (\nabla \phi)^2), \psi_m \right]$$

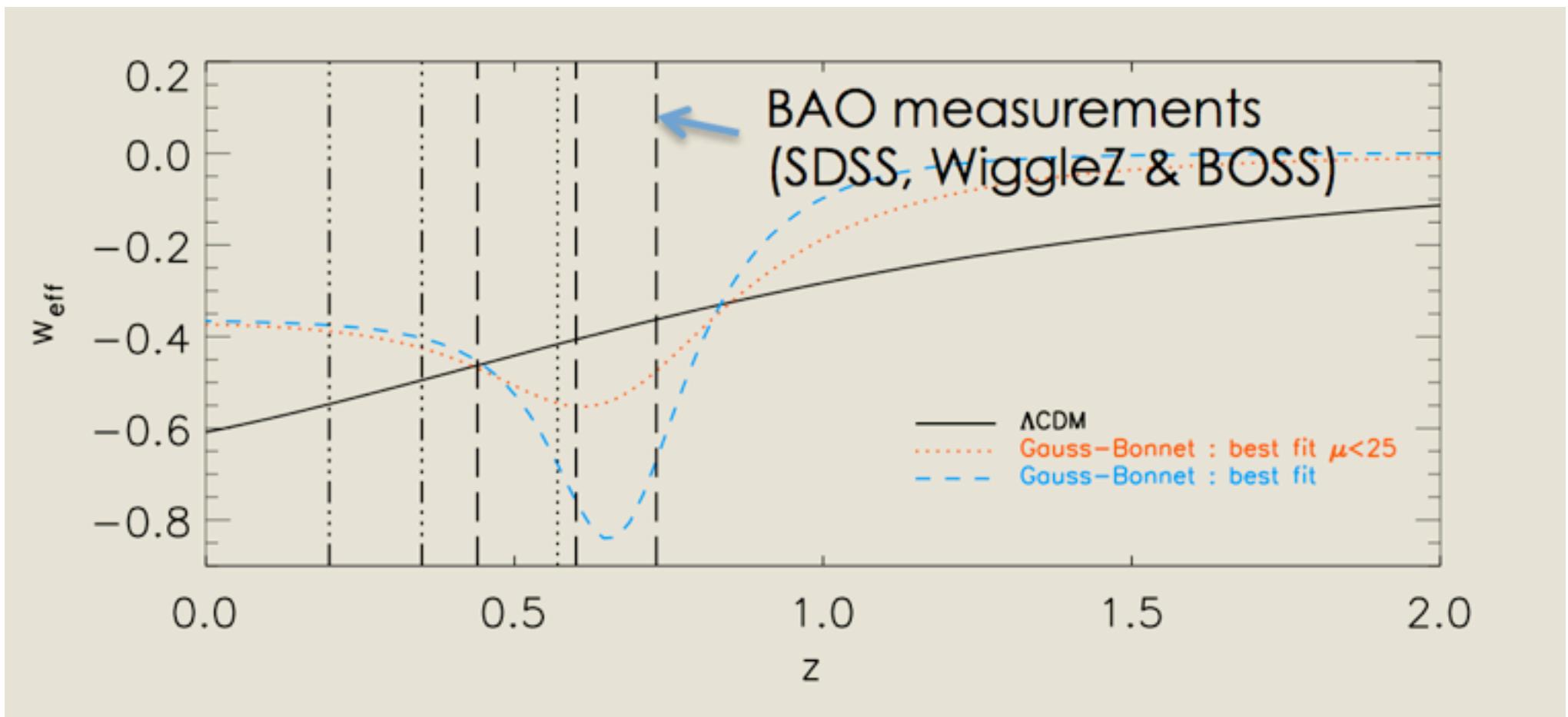
Non-minimally coupling to matter

Weak Equivalence principle

## Approach One Application:

Requiring attractor behavior leads to constraints on coefficients of the EFT

arXiv:1209.2706 w/ Mueller and Bean



Can establish constraints on the background,  
but we miss a large class of non-linear models

## Approach Two:

Assume  $\Lambda CDM$  Background,  
construct most general theory of perturbations

arXiv:1304.4840 Gleyzes, Langlois, Piazza, Vernizzi  
arXiv:1211.7054 w/ Bloomfield, Flanagan, and Park  
arXiv:1210.0201 Gubitosi, Piazza, Vernizzi

$$S = \int d^4x \sqrt{-g} \left[ \frac{m_0^2}{2} \Omega(t) R + \Lambda(t) - c(t) \delta g^{00} \right. \\ \left. + \frac{M_2^4(t)}{2} (\delta g^{00})^2 + \frac{M_3^4(t)}{3!} (\delta g^{00})^3 + \dots \right. \\ \left. - \frac{\bar{M}_1^3(t)}{2} \delta g^{00} \delta K^\mu_{\mu} - \frac{\bar{M}_2^2(t)}{2} (\delta K^\mu_{\mu})^2 - \frac{\bar{M}_3^2(t)}{2} \delta K^\mu_{\nu} \delta K^\nu_{\mu} + \dots \right. \\ \left. + \lambda_1(t) (\delta R)^2 + \lambda_2(t) \delta R_{\mu\nu} \delta R^{\mu\nu} + \gamma_1(t) C_{\mu\nu\sigma\lambda} C^{\mu\nu\sigma\lambda} + \gamma_2(t) \epsilon^{\mu\nu\sigma\lambda} C_{\mu\nu}{}^{\rho\theta} C_{\sigma\lambda\rho\theta} + \dots \right. \\ \left. + m_1^2(t) n^\mu n^\nu \partial_\mu g^{00} \partial_\nu g^{00} + m_2^2(t) (g^{\mu\nu} + n^\mu n^\nu) \partial_\mu g^{00} \partial_\nu g^{00} + \dots \right] + S_m[g_{\mu\nu}]. \quad (2.1)$$

**Framework now compatible with Vainshtein mechanism**

# Adapt EFT of Inflation (Cheung, et. al. 0709.0293) to dark energy

arXiv:1304.4840 Gleyzes, Langlois, Piazza, Vernizzi  
 arXiv:1211.7054 w/ Bloomfield, Flanagan, and Park  
 arXiv:1210.0201 Gubitosi, Piazza, Vernizzi

## Must include Dark Matter Sector

### Background taken a priori as Lambda-CDM

**Messy!**

$$S_\pi = \int d^4x a^3 \sqrt{\tilde{g}} \left[ c \left( \dot{\pi}^2 - \frac{\tilde{g}^{ij}}{a^2} \tilde{\nabla}_i \pi \tilde{\nabla}_j \pi \right) + \left( 3\dot{H}c - \frac{m_0^2}{4} \dot{\Omega} \dot{R}^{(0)} \right) \pi^2 - c \dot{h} \pi + \frac{m_0^2}{2} \dot{\Omega} \pi \delta R \right. \\ + 2\bar{M}_2^4 \dot{\pi}^2 + \bar{M}_1^3 \dot{\pi} \left( -\frac{\dot{h}}{2} + 3\dot{H}\pi + \frac{\tilde{\nabla}^2 \pi}{a^2} \right) - \frac{\bar{M}_2^2}{2} \left( -\frac{\dot{h}}{2} + 3\dot{H}\pi + \frac{\tilde{\nabla}^2 \pi}{a^2} \right)^2 \\ - \frac{\bar{M}_3^2}{2} \left( -\frac{\dot{h}^i_j}{2} + \dot{H}\pi \delta^i_j + \frac{\tilde{g}^{ik}}{a^2} \tilde{\nabla}_k \tilde{\nabla}_j \pi \right) \left( -\frac{\dot{h}^j_i}{2} + \dot{H}\pi \delta^j_i + \frac{\tilde{g}^{jl}}{a^2} \tilde{\nabla}_l \tilde{\nabla}_i \pi \right) \\ \left. + \bar{M}_3^2(t) H^2 \frac{\tilde{g}^{ij}}{a^2} \tilde{\nabla}_i \pi \tilde{\nabla}_j \pi - \hat{M}^2 \dot{\pi} \delta R^{(3)} + \dots \right],$$

### However, adequately reproduces existing models...

Operator	$\Omega$	$\Lambda$	$c$	$M_2^4$	$\bar{M}_1^3$	$\bar{M}_2^2$	$\bar{M}_3^2$	$\hat{M}^2$
Model	R		$\delta g^{00}$	$(\delta g^{00})^2$	$\delta g^{00} \delta K^\mu_\mu$	$(\delta K^\mu_\mu)^2$	$\delta K^\mu_\nu K^\nu_\mu$	$\delta g^{00} \delta R^{(3)}$
$\Lambda$ CDM	1	✓	0	-	-	-	-	-
Quintessence	1/✓	✓	✓	-	-	-	-	-
$F(R)$	✓	✓	0	-	-	-	-	-
$k$ -essence	1/✓	✓	✓	✓	-	-	-	-
Galileon Kinetic Braiding	1/✓	✓	✓	✓	✓	-	-	-
DGP	✓	✓†	✓†	✓†	✓	-	-	-
Ghost Condensate	1/✓	✓	0	-	-	✓	✓	-
Horndeski	✓	✓	✓	✓	✓	✓†	✓†	✓†

## **Is such an approach worth exploring further?**

Many experimentalists / observers seem to think yes.

### **Some Challenges:**

- Hierarchy of many scales**
- Non-locality at equal matter/DE?**
- Complicated equations (Just a Numerical challenge?)**
- Can we measure these coefficients well?**
- Is there really a good model of modified gravity?**
- ... ?**

Should we all just accept  $w = -1$  and move on?

**“Don’t modify gravity, understand it.” -- Nima**

<https://webcast.stsci.edu/webcast/detail.xhtml?talkid=3398>