Bosonic String Formation in QCD

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QCD and String Theory
November 15, 2004
KITP, Santa Barbara

Collaborators:

Jimmy Juge	Dublin
Francesca Maresca	Utah
Colin Morningstar	Carnegie Mellon
Mike Peardon	Dublin
James Drummond	Dublin

string counselor at UCSD: Ken Intriligator

Early work: Polyakov
Luscher
Polchinski, Strominger
Baker et al.
Michael
Teper
Gliozzi et al.
Hasenbusch, Pinn
JKM (old)
Munster
…

This talk: report on the excitation spectrum of the fixed end string and the string-soliton with unit winding

New work:

Juge, JK, Morningstar → fixed end spectrum with fine structure
HEP-LAT 0207004, PRL 90 (2003) 161601

Juge, JK, Maresca, Morningstar, Peardon → closed winding string with fine structure
HEP-LAT 0309180, Nucl.Phys.Proc.129:703-705,2004

Luscher, Weisz → ground state Casimir energy and theoretical work on spectrum
JHEP 0207 (2002) 049, JHEP 0407:014,2004

Casimir energy paradox ?

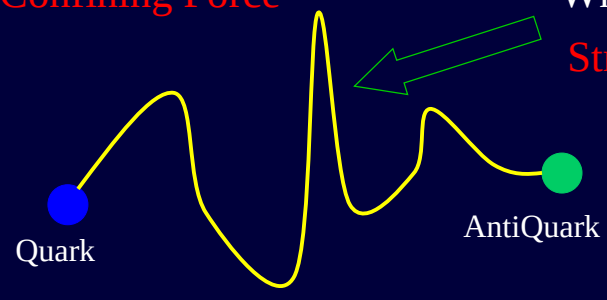
OUTLINE

1. String formation in field theory
   - picture in space and time
   - main physical properties of the string
2. QCD String in D=3,4 dimensions
   - fixed end spectrum
   - Casimir energy
   - paradox ?
   - closed string spectrum (one unit winding)
3. Z(2) string in 2+1 dimension
   - spectrum
   - Casimir energy
   - paradox ?
   - closed string spectrum (one unit winding)
4. Insight from 1+1 dimensional model (left for discussion)
   - exercise in quantum mechanics
5. Conclusions

Confining Force

What is this stuff?

String in QCD?



1. On-lattice QCD string spectrum

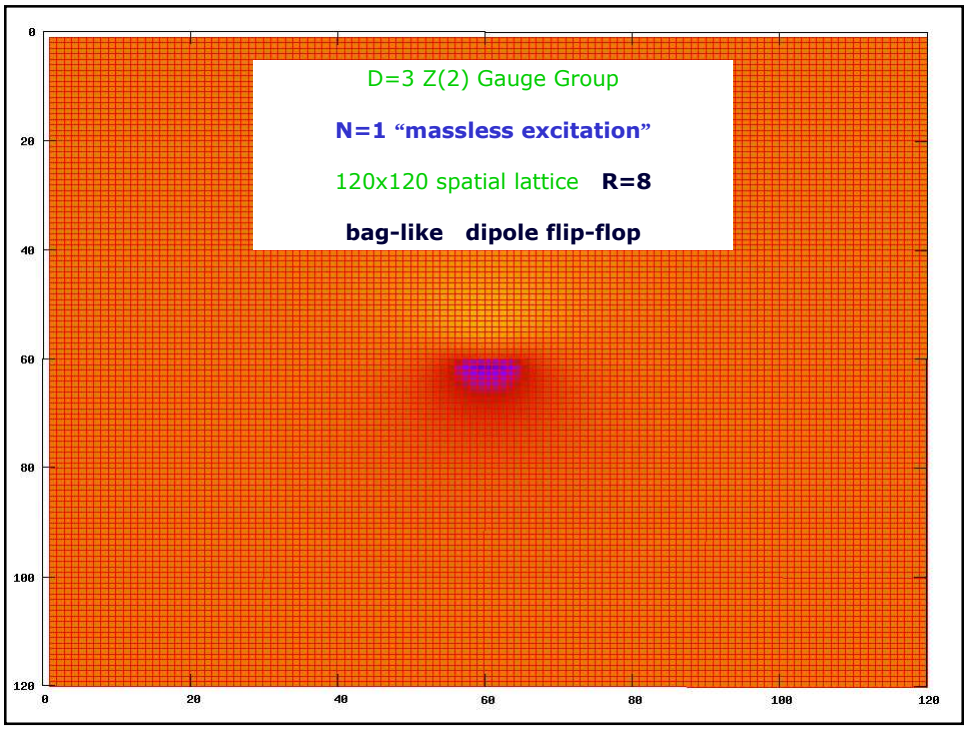
2. D=3 Z(2) gauge model
   microscopic loop equations (Polyakov)
   macroscopic string duality 3d Ising interface

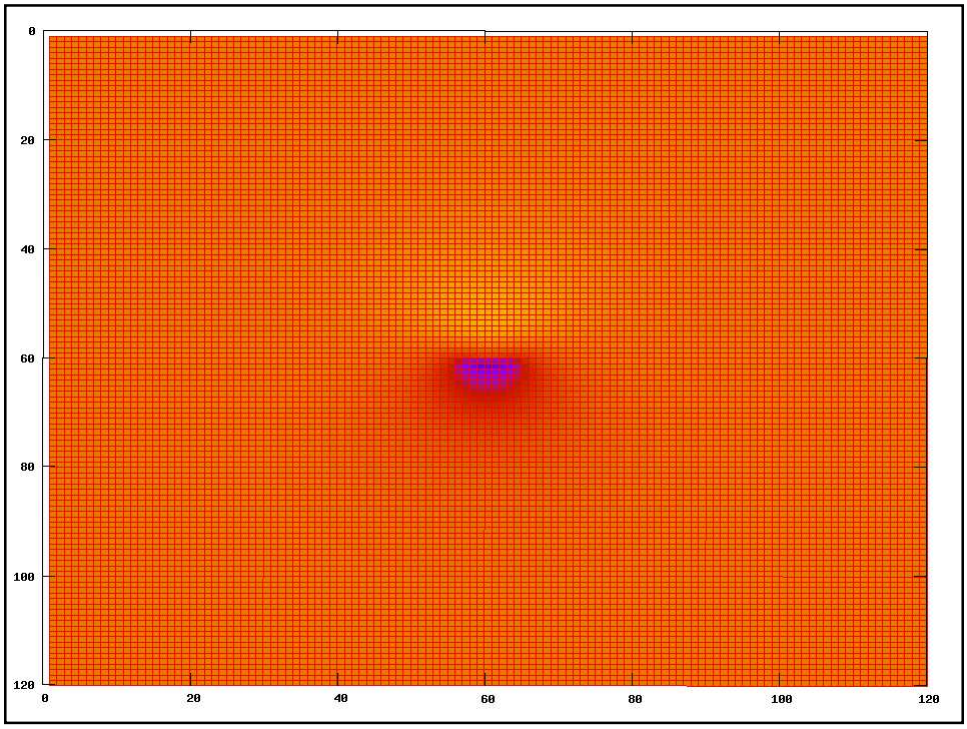
Casimir energy of ground state
Excitation spectrum
Goldstone modes and collective variables
Effective theory?
Microscopic variables (loop equation)?
Geometric interpretation?

String theorists interested in QCD string problem

Quenched, but relevant in confinement/string and large N

- Massless Goldstone modes?
- Local derivative expansion for their interactions?
  from fine structure in the spectrum
- Massive excitations?
- Breathing modes in effective Lagrangian?
- String properties ? Bosonic, NG, rigid, …?

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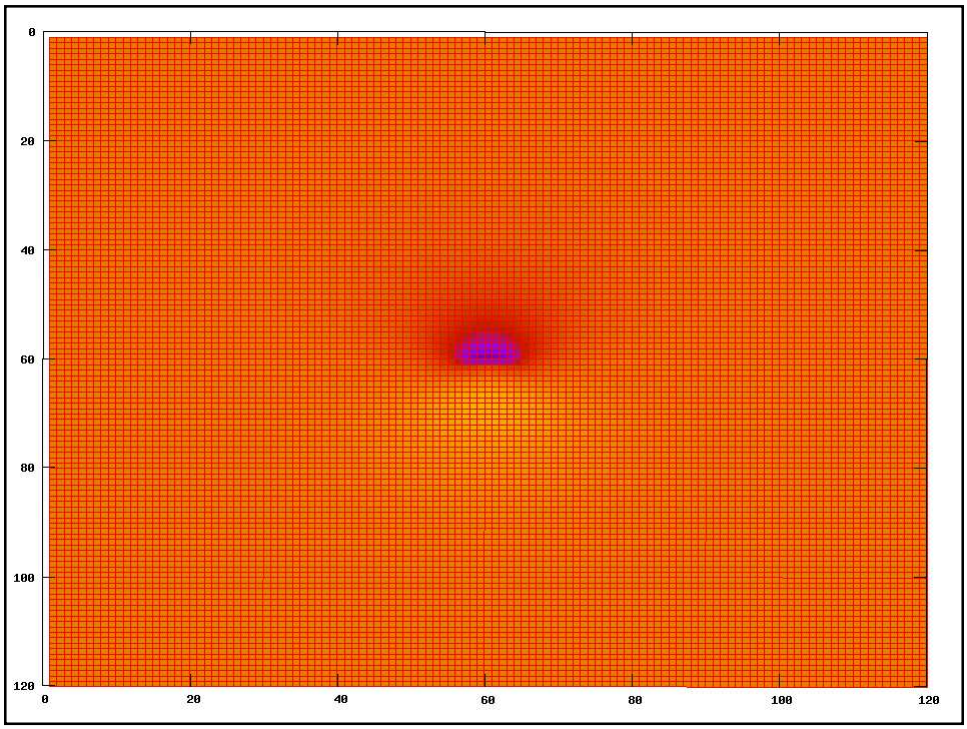
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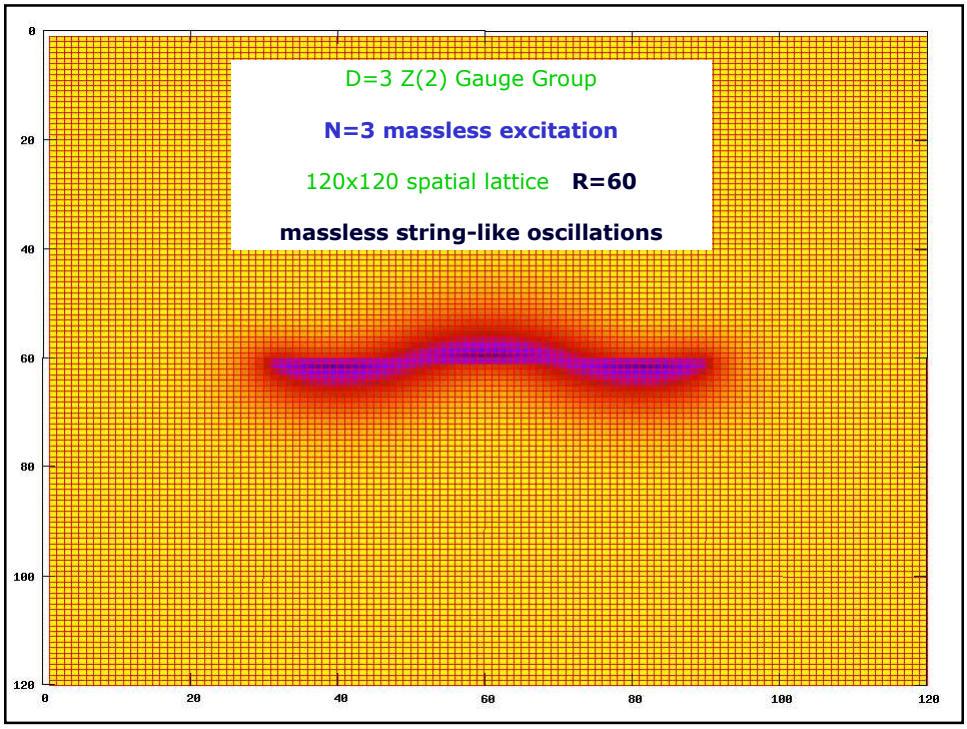
D=3 Z(2) Gauge Group
N=3 massless excitation
120x120 spatial lattice R=60
massless string-like oscillations
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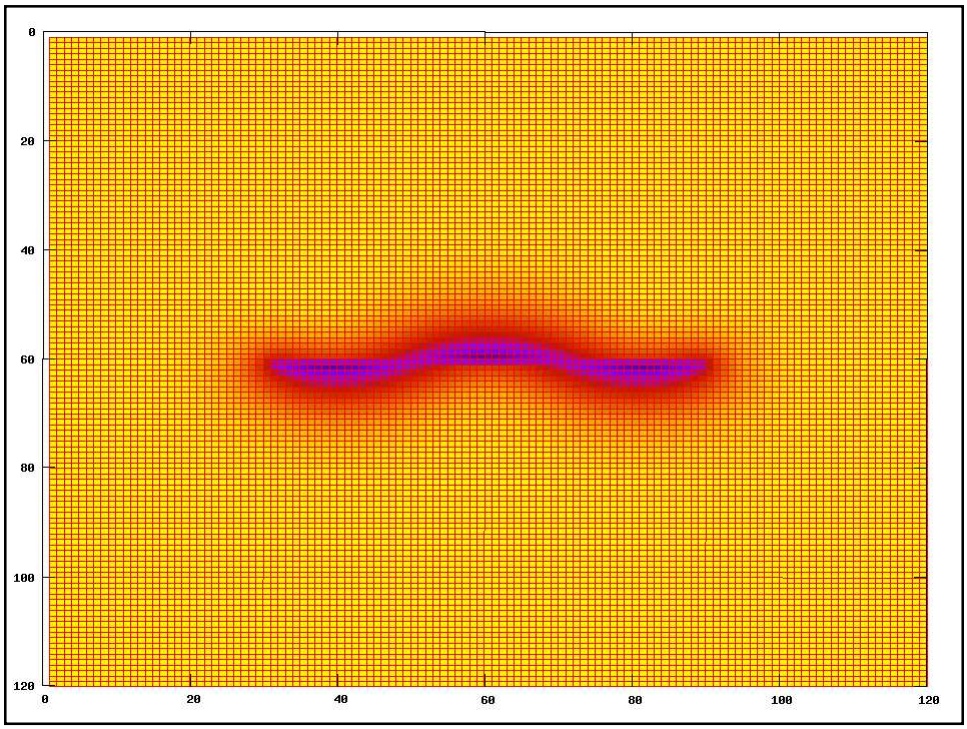
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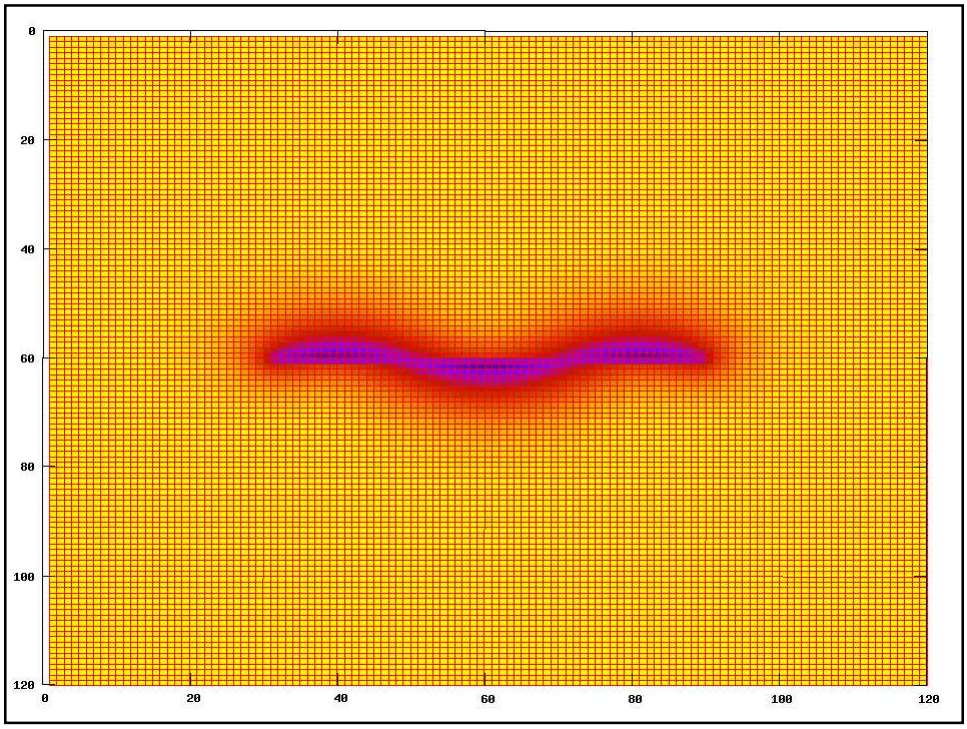
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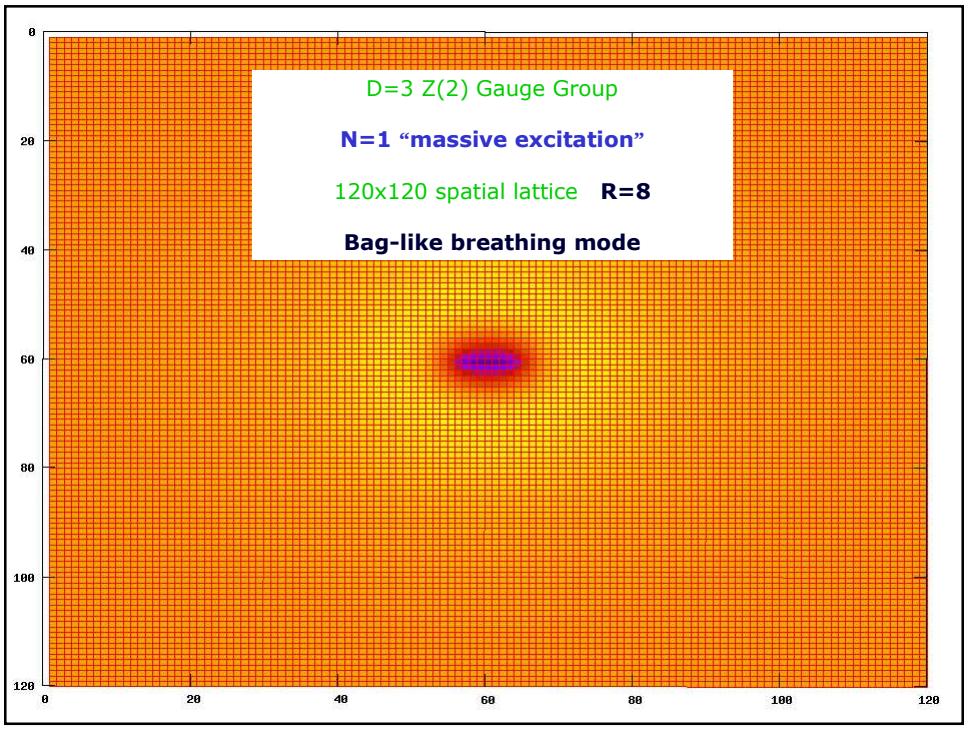
D=3 Z(2) Gauge Group
N=1 "massive excitation"
120x120 spatial lattice R=8
Bag-like breathing mode
0
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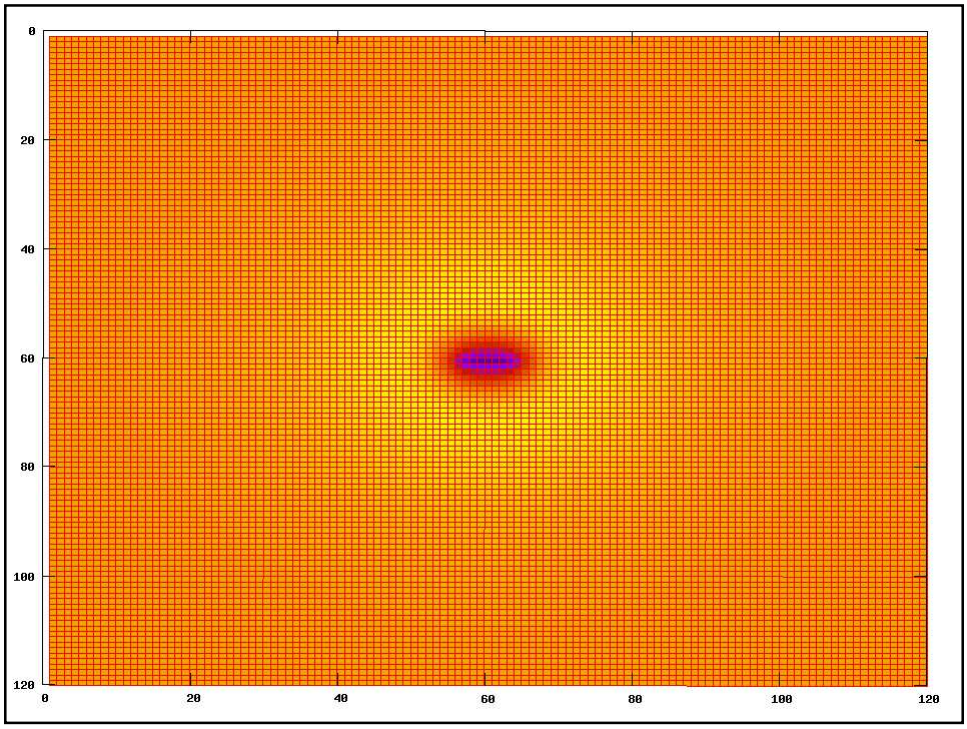
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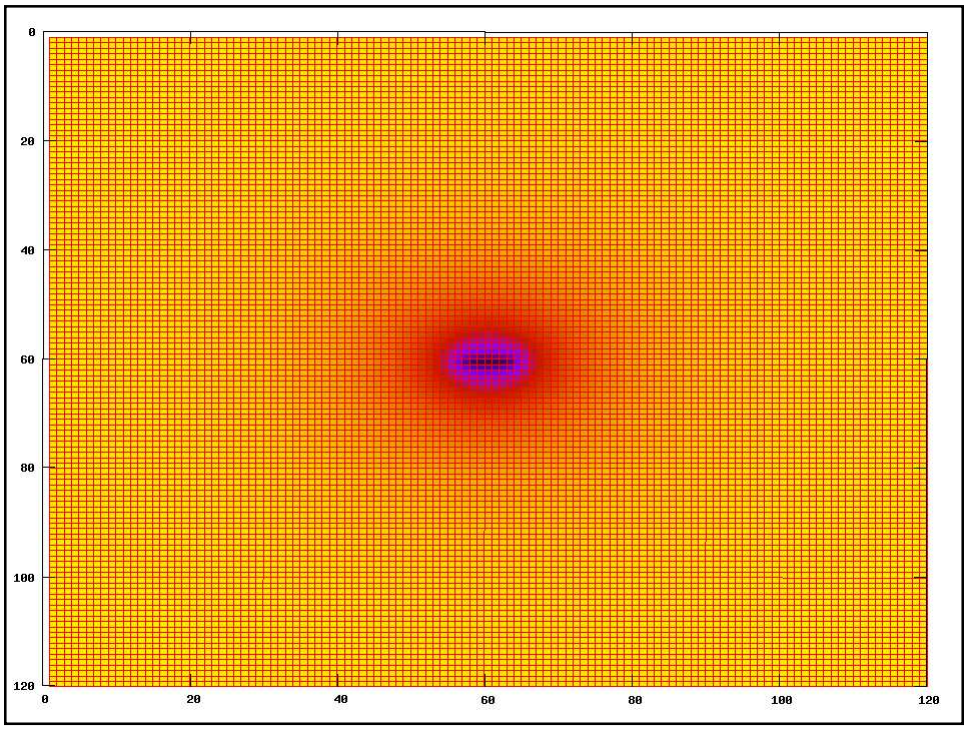
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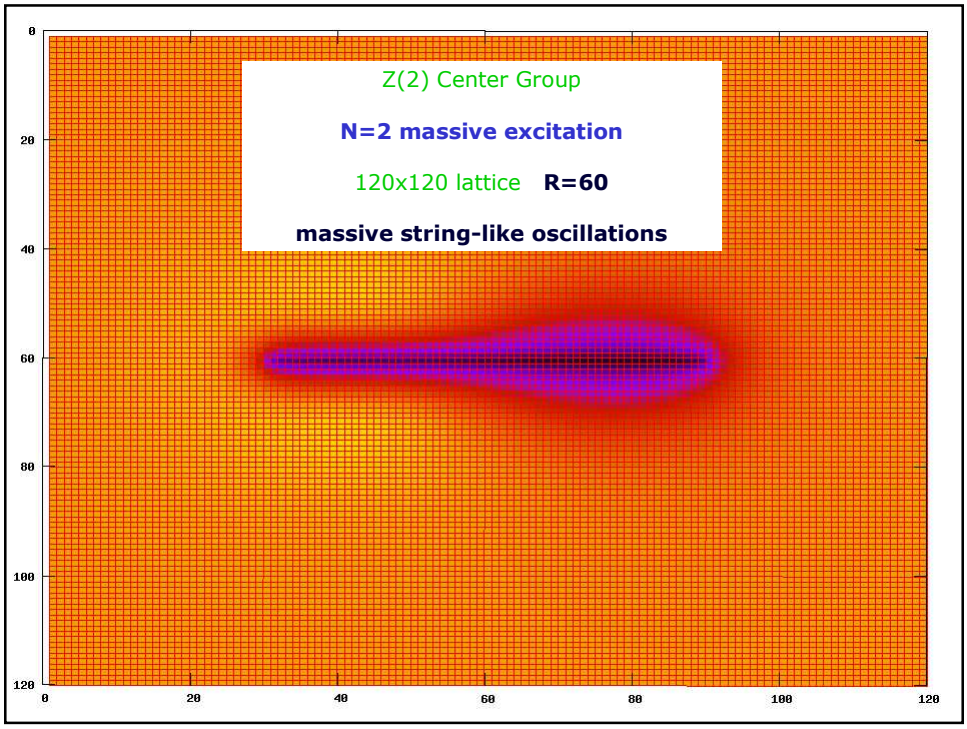
Z(2) Center Group
N=2 massive excitation
120x120 lattice R=60
massive string-like oscillations
0
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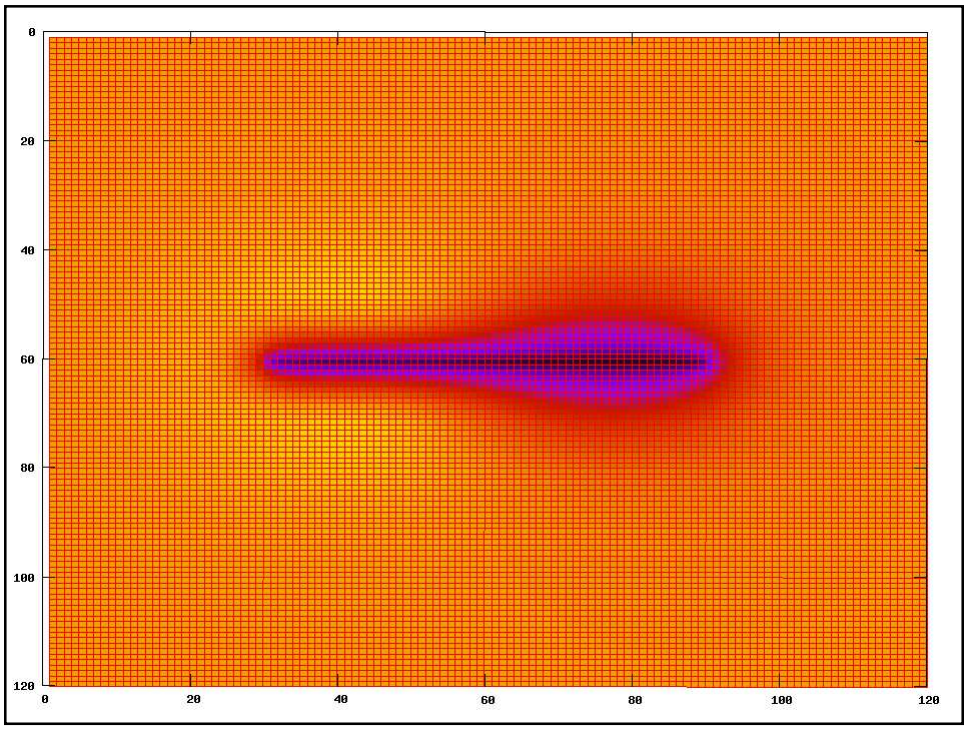
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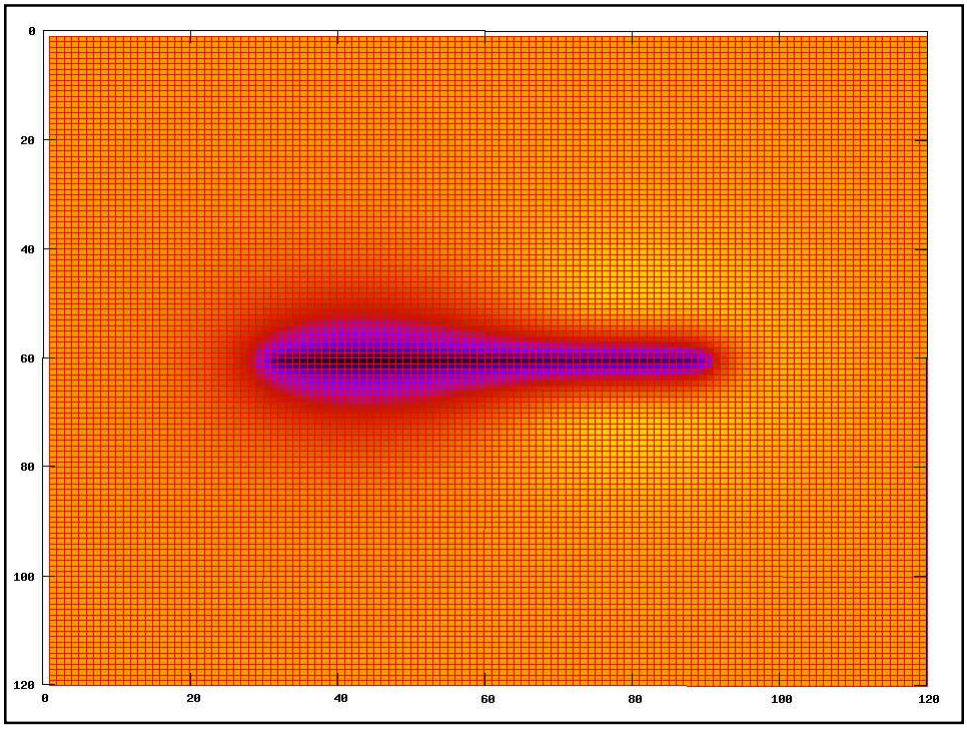

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   - Casimir energy
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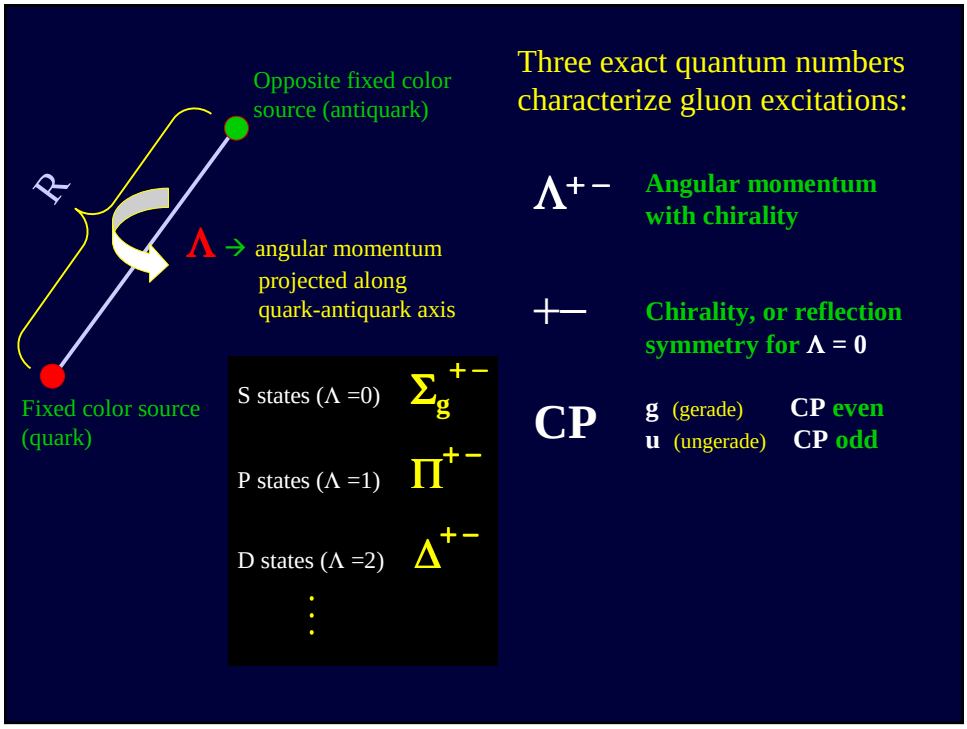
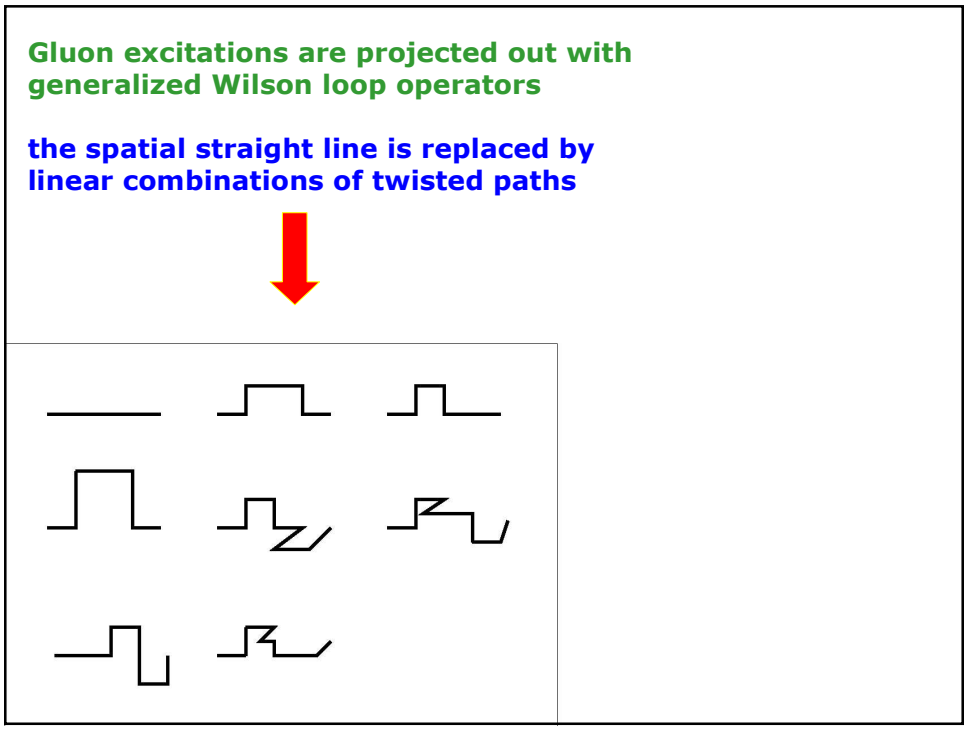
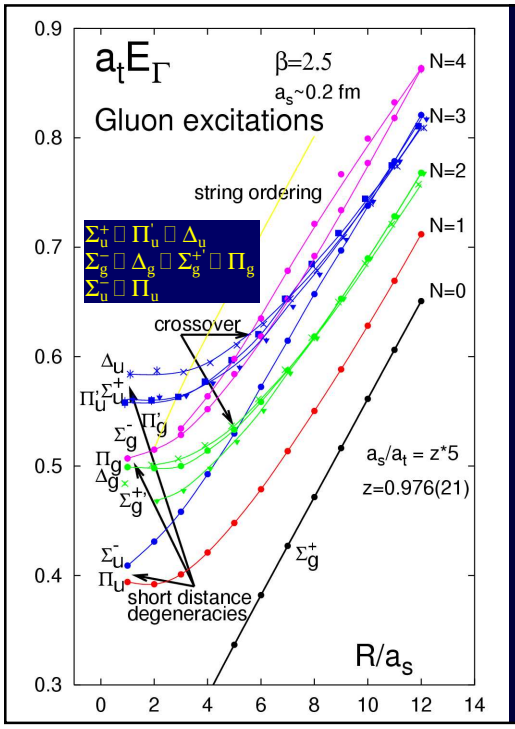

Opposite fixed color source (antiquark)
R
Λ → angular momentum projected along quark-antiquark axis
Fixed color source (quark)
S states (Λ =0) Σ_g^{+-}
P states (Λ =1) Π^{+-}
D states (Λ =2) Δ^{+-}
Three exact quantum numbers characterize gluon excitations:
Λ^{+-} Angular momentum with chirality
+− Chirality, or reflection symmetry for Λ = 0
CP
g (gerade) CP even
u (ungerade) CP odd


Gluon excitations are projected out with generalized Wilson loop operators
the spatial straight line is replaced by linear combinations of twisted paths

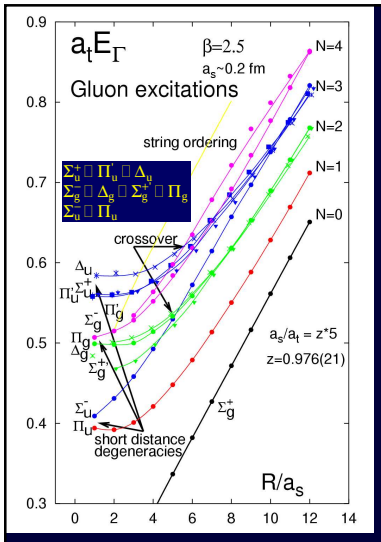


Three length scales in energy spectrum:

R < 0.5 fm
Short distance QCD
Bag-like nearly spherical symmetry
OPE (Soto et al.)

R ~ 0.5 fm – 1.5 fm
Crossover (model sensitive)

R ~ 2 fm - 3 fm
Onset of string ordering



Crossover transition region
difficult to interpret
model dependent

Short distance region

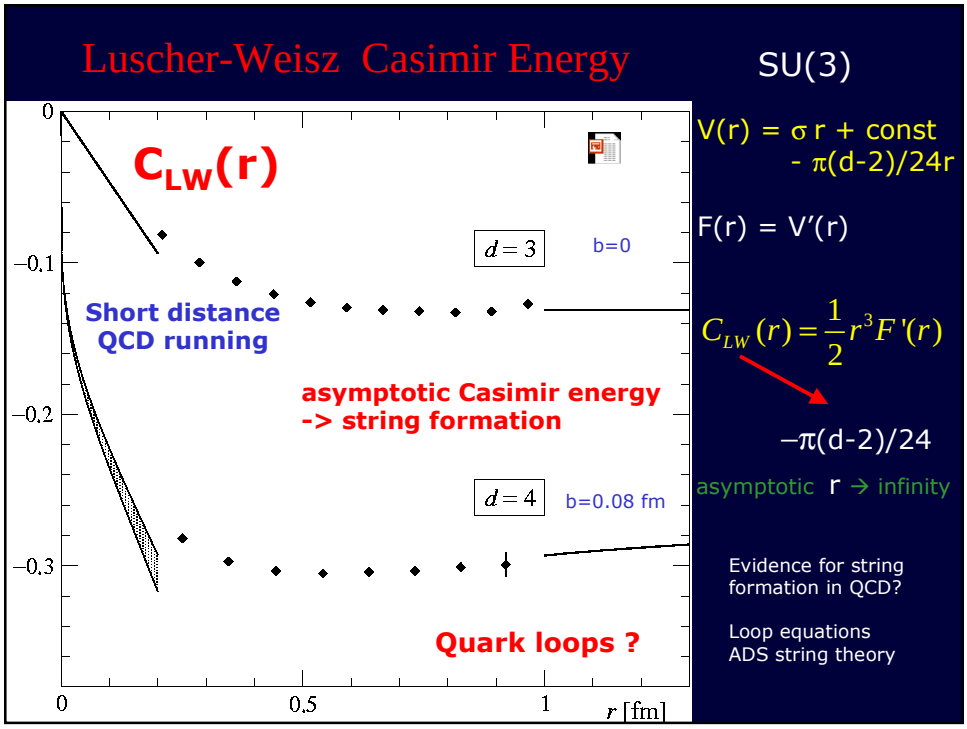
R<0.5fm approximate spherical symmetry
Bag-like "non-string" picture

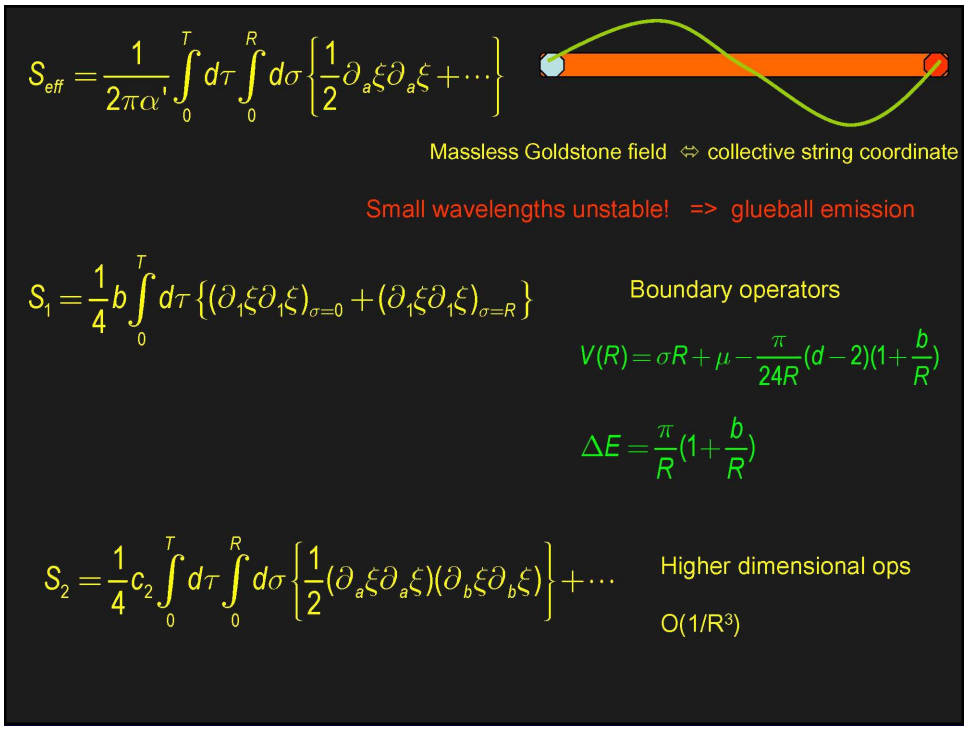
Multipole operator product expansion
of $\vec{A}(\vec{r},t)$ and $\vec{J}(\vec{r},t)$:

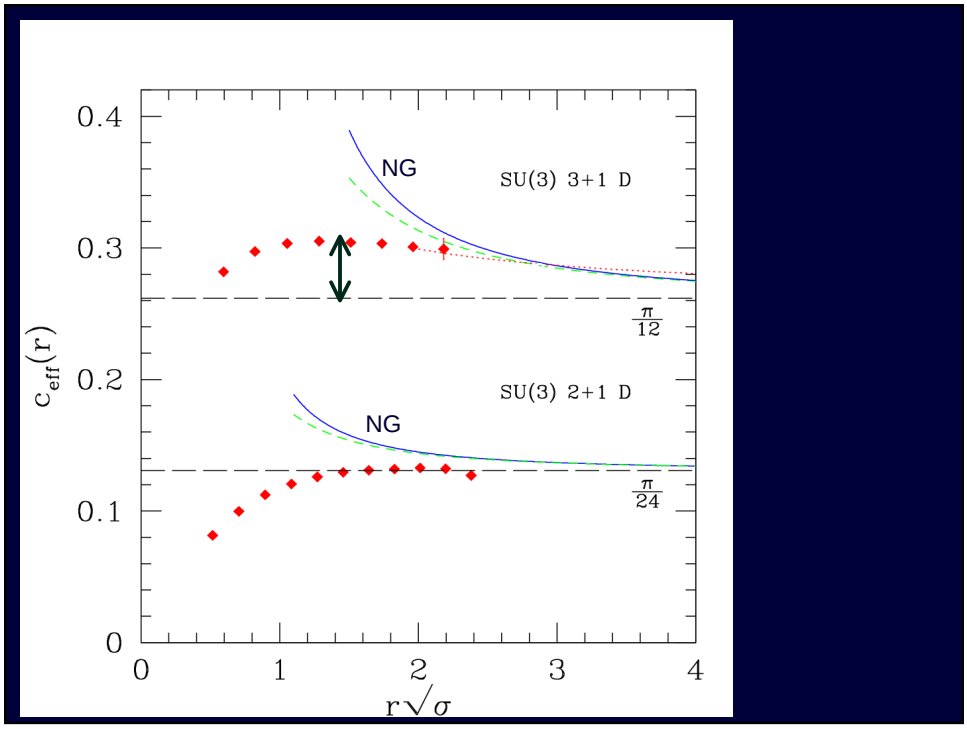
$$H_{Cb} = H_{gauge} + \frac{g^2 Q\cdot\bar{Q}}{4\pi|\vec{r}_1-\vec{r}_2|} - g^3(Q\times\bar{Q})_a \int d^3\vec{r}\,\vec{A}_a(\vec{r},t)\vec{J}(\vec{r},t)$$

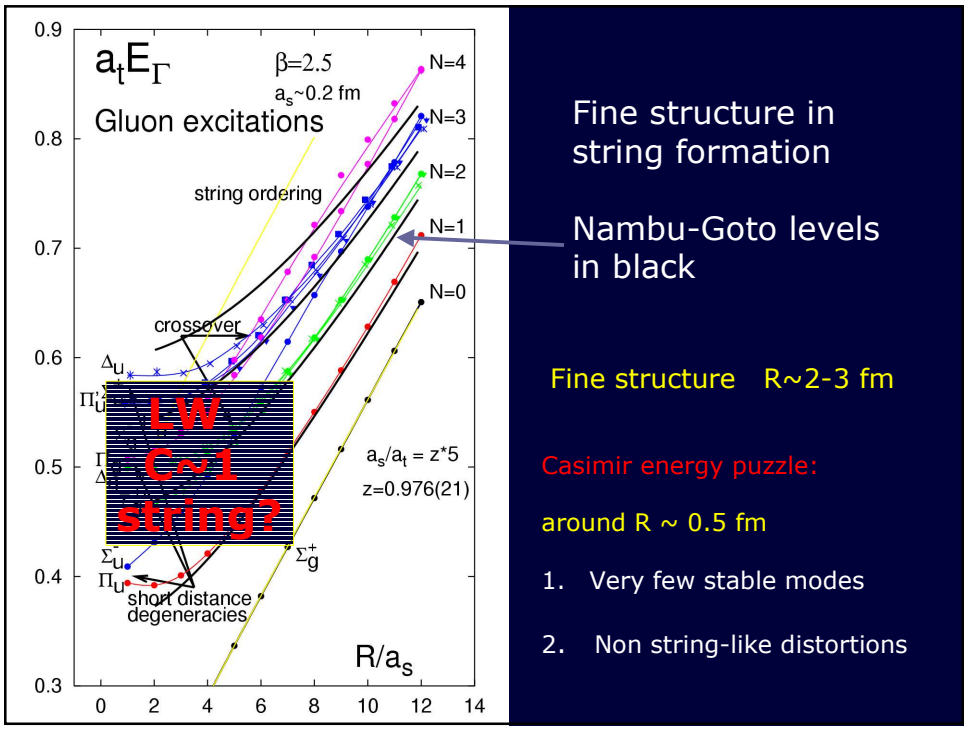
$$4\pi\vec{J}(\vec{r}) = \left(\frac{1}{|\vec{r}-\vec{r}_1|} - \frac{1}{|\vec{r}_1-\vec{r}_2|}\right)\vec{\nabla}\left(\frac{1}{|\vec{r}-\vec{r}_2|} - \frac{1}{|\vec{r}_1-\vec{r}_2|}\right)$$

$$Q\cdot\bar{Q} = \begin{cases} -\frac{4}{3} & \text{singlet} \\ +\frac{1}{6} & \text{octet} \end{cases}$$


Luscher-Weisz Casimir Energy
SU(3)
$C_{LW}(r)$
$d=3$
b=0
Short distance
QCD running
asymptotic Casimir energy
-> string formation
$d=4$
b=0.08 fm
Quark loops ?
0
−0.1
−0.2
−0.3
0
0.5
1
r [fm]
V(r) = σ r + const
- π(d-2)/24r
F(r) = V'(r)
$C_{LW}(r)=\frac{1}{2}r^3F'(r)$
−π(d-2)/24
asymptotic r → infinity
Evidence for string
formation in QCD?
Loop equations
ADS string theory


$S_{eff}=\frac{1}{2\pi\alpha'}\int_0^T d\tau\int_0^R d\sigma\left\{\frac{1}{2}\partial_a\xi\partial_a\xi+\cdots\right\}$
Massless Goldstone field ⇔ collective string coordinate
Small wavelengths unstable! => glueball emission
$S_1=\frac{1}{4}b\int_0^T d\tau\{(\partial_1\xi\partial_1\xi)_{\sigma=0}+(\partial_1\xi\partial_1\xi)_{\sigma=R}\}$
Boundary operators
$V(R)=\sigma R+\mu-\frac{\pi}{24R}(d-2)(1+\frac{b}{R})$
$\Delta E=\frac{\pi}{R}(1+\frac{b}{R})$
$S_2=\frac{1}{4}c_2\int_0^T d\tau\int_0^R d\sigma\left\{\frac{1}{2}(\partial_a\xi\partial_a\xi)(\partial_b\xi\partial_b\xi)\right\}+\cdots$
Higher dimensional ops
O(1/R³)





Fine structure in string formation

Nambu-Goto levels in black

Fine structure R~2-3 fm

Casimir energy puzzle:

around R ~ 0.5 fm

1. Very few stable modes
2. Non string-like distortions

Two basic questions:

- Why the precocious onset of $C_{eff} \sim 1$?
- Where does the central charge C=1 reside?

  On a geometric string?

  Or distributed between massless Goldstone modes and the bulk?

  Answer to second question will determine whether early onset of $C_{eff} \sim 1$ is a true signal of string formation, or just an accident

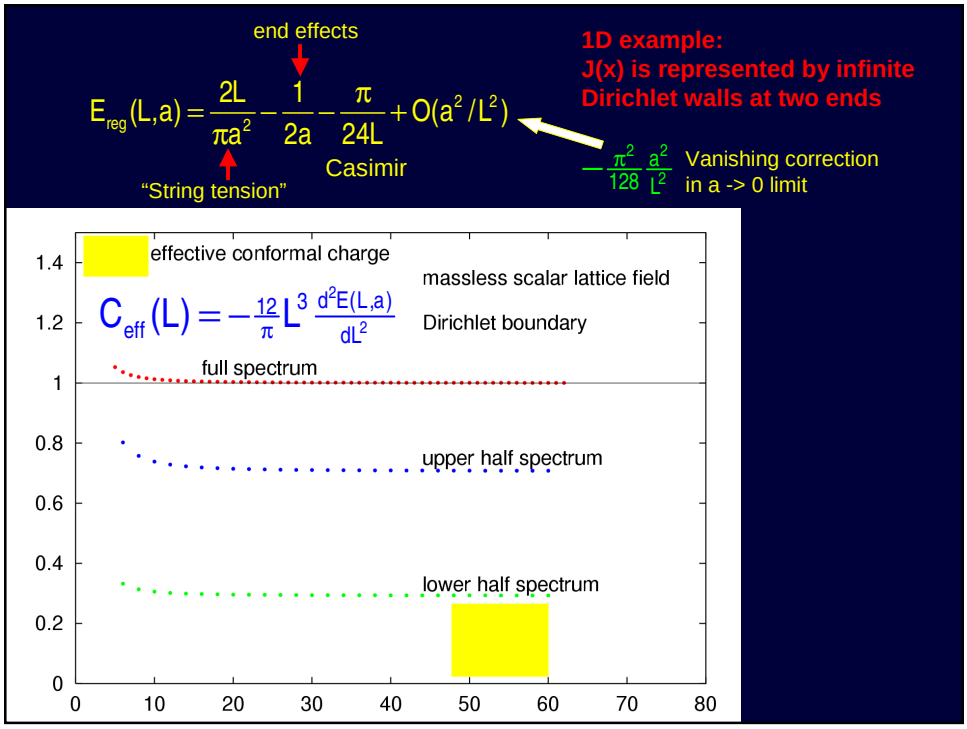
$$\frac{1}{2}\sum_{n=1}^{\infty} n = -\frac{\pi}{24}$$

smart enough for string theory?

This is NOT a paradox

We turn to the D=1+1 lattice for learning how to do the sum:

$$E_{reg}(L,a) = \frac{2L}{\pi a^2} - \frac{1}{2a} - \boxed{\frac{\pi}{24L}} + O(a^2)$$

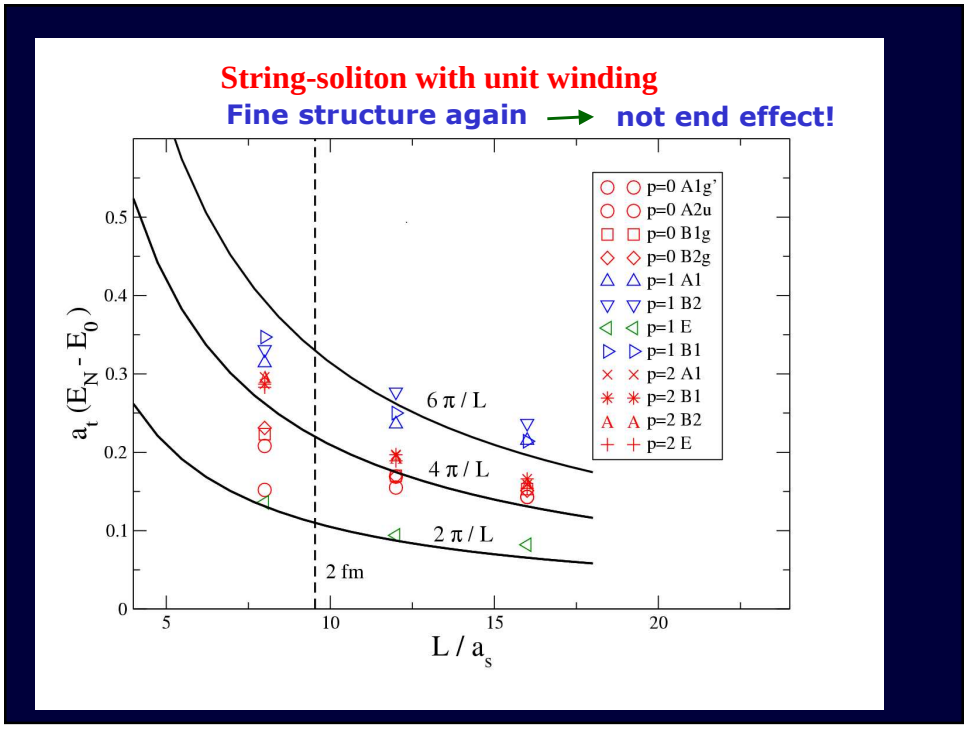


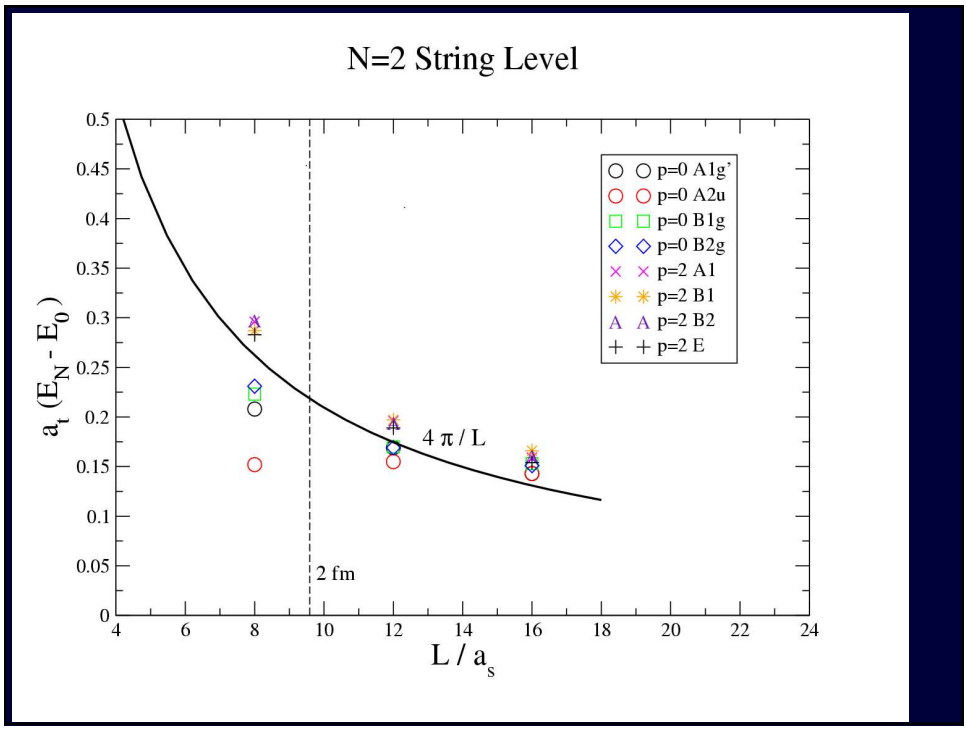
$$\frac{1}{2}\sum_{n=1}^{3} n = -\frac{\pi}{24}$$

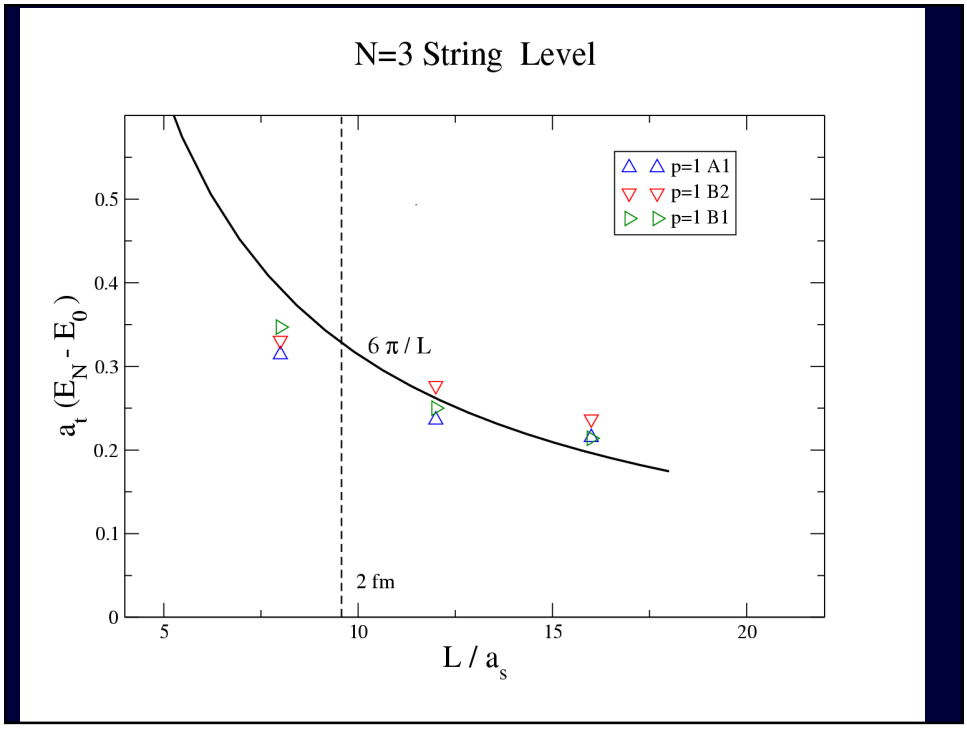
This IS a paradox

But what to do about the killer end effects?

Look for string-solitons with winding


String-soliton with unit winding
Fine structure again → not end effect!
p=0 A1g'
p=0 A2u
p=0 B1g
p=0 B2g
p=1 A1
p=1 B2
p=1 E
p=1 B1
p=2 A1
p=2 B1
p=2 B2
p=2 E
6 π / L
4 π / L
2 π / L
2 fm
a_t (E_N - E_0)
L / a_s


N=2 String Level
p=0 A1g'
p=0 A2u
p=0 B1g
p=0 B2g
p=2 A1
p=2 B1
p=2 B2
p=2 E
4 π / L
2 fm
a_t (E_N - E_0)
L / a_s



String check list

- Massless Goldstone modes ✓
- Local derivative expansion for their interactions ?
  from fine structure in the spectrum ✓
- Massive excitations
- Breathing modes in effective Lagrangian
- String properties ? Bosonic, NG, rigid, …

?

Onto D=3 Z(2) gauge group to learn more

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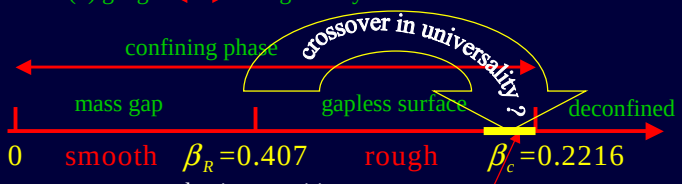
Wilson Surface of 3d Z(2) Gauge Model

Similar picture expected in QCD

$\overline{\beta} = -\frac{1}{2}\ln(\tanh\beta)$ Z(2) gauge ⟷ Ising duality



roughening transition
Kosterlitz-Thouless
universality class

critical region
continuum limit (QCD)
effective ϕ^4 field theory

Semiclassical Loop Expansion
Soliton Quantization (string)

Wilson line of Z(2) gauge group becomes a

seam of flipped links in dual (Ising) representation

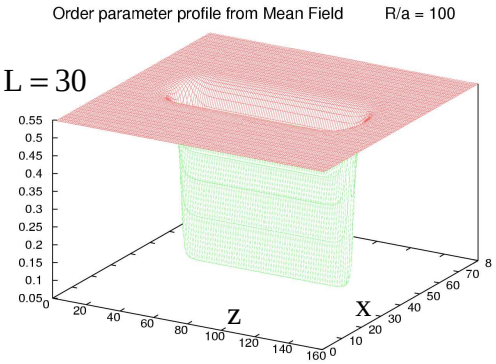
Multispin coding of Ising bits in Monte Carlo

Parallel * Parallel

Effective Schrodinger equation
based on fluctuation matrix of string soliton

$$M = -\nabla^2 + U''(\phi_{soliton})$$

V_{eff}



-in long flux limit spectrum is expected to factorize

- translational zero mode of soliton

- Goldstone spectrum

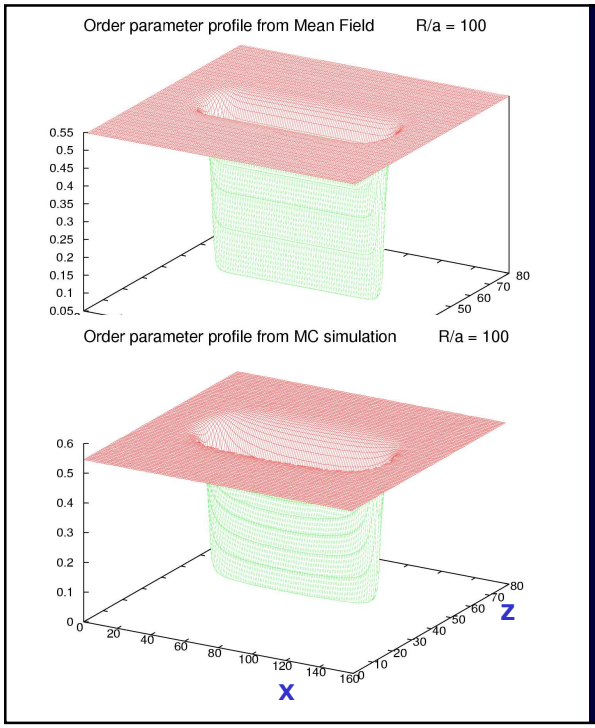
$\phi(z) \cdot \exp(iqx)$

zero energy bound state

quantized momenta of Goldstone modes in box of length L

$$q = \frac{\pi}{L} n, \quad n = 1, 2, 3...$$

shape and end effects distort!



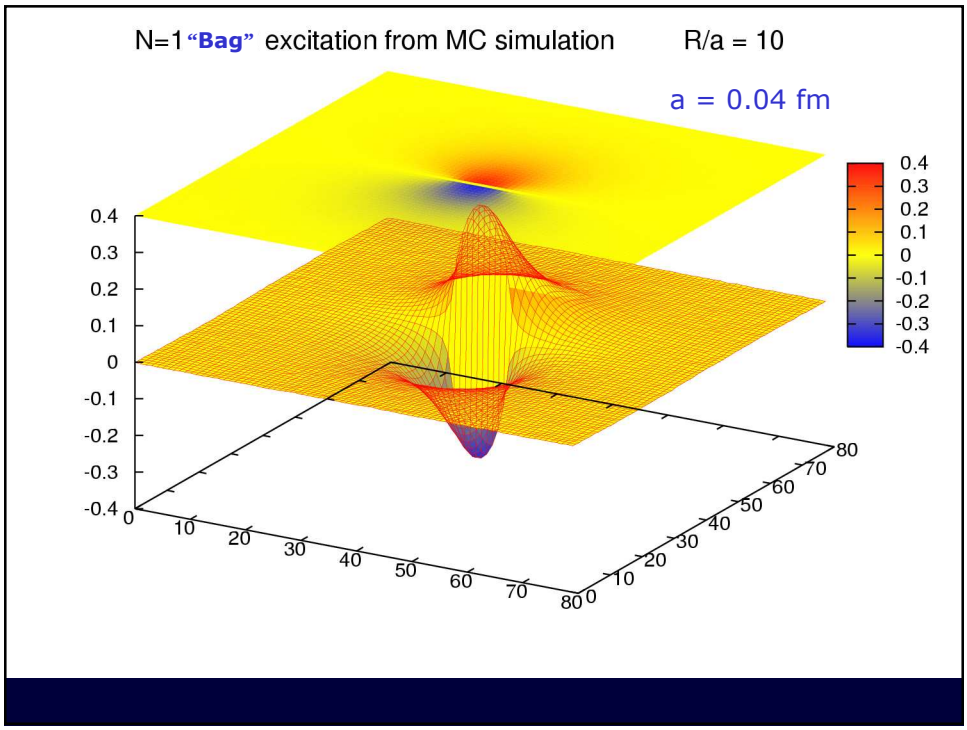
good analytic/numerical handle on the Z(2) model

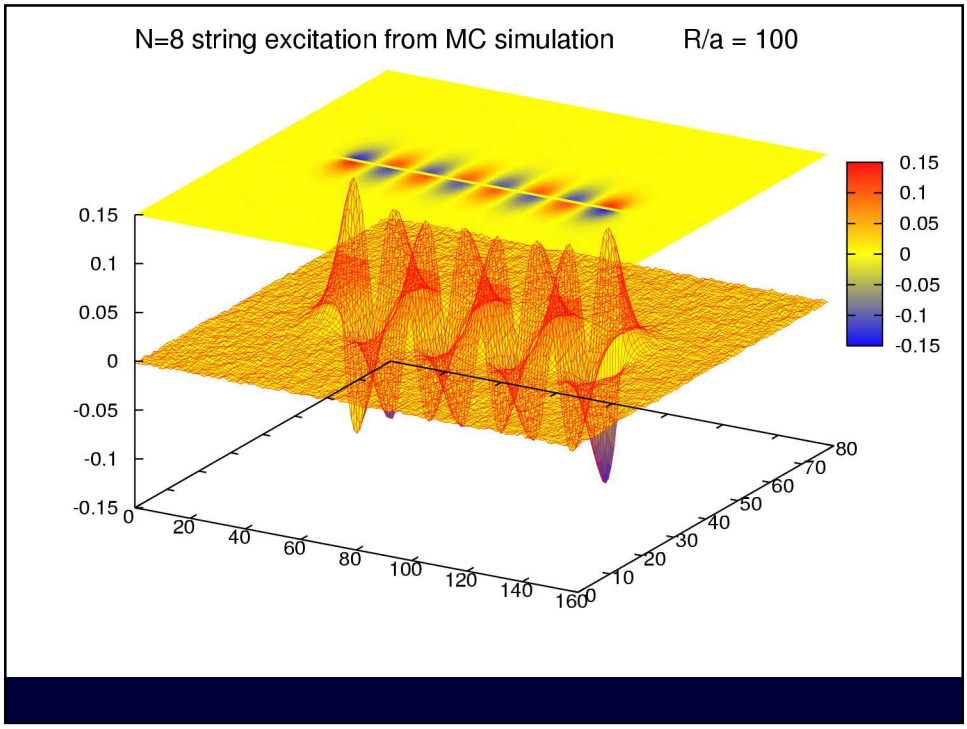
in addition to MC

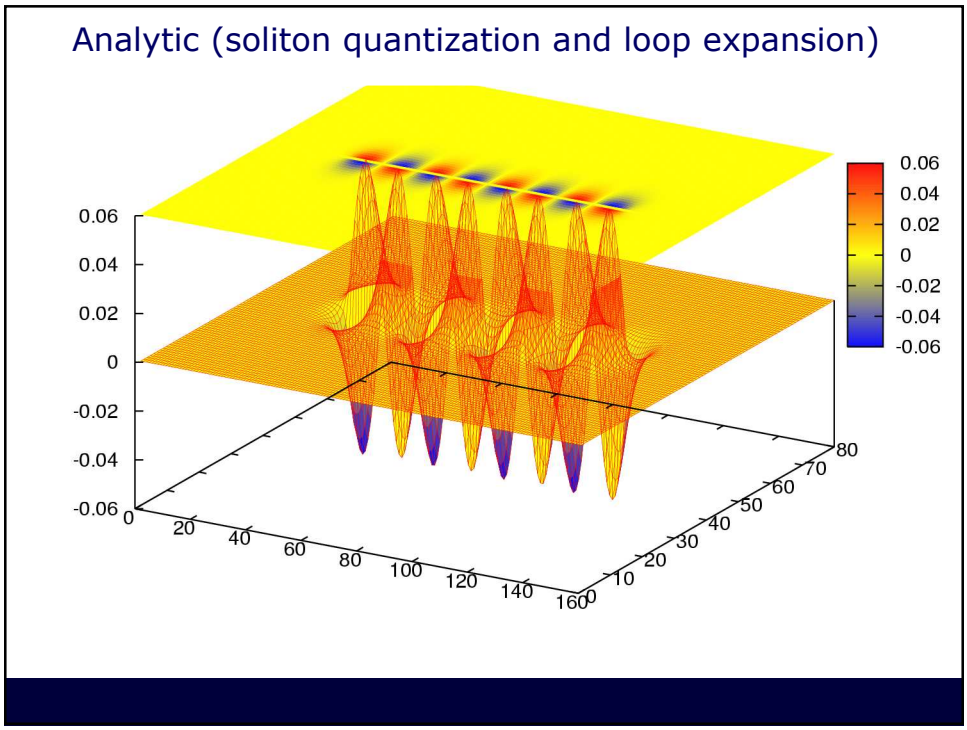
Effective Schrodinger potential

P_x = +-1 and P_z = +-1

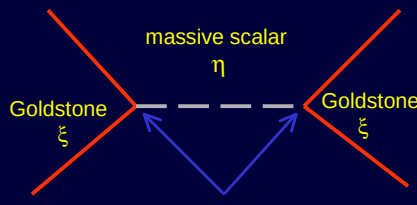
two symmetry quantum numbers




N=8 string excitation from MC simulation
R/a = 100
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
0
20
40
60
80
100
120
140
160
0
10
20
30
40
50
60
70
80


Analytic (soliton quantization and loop expansion)
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
0
20
40
60
80
100
120
140
160
0
10
20
30
40
50
60
70
80

Most important step in deriving correction terms in effective action of Goldstone modes in Z(2) D=3 gauge model:



$$\partial_\mu\xi\partial_\mu\xi\eta$$

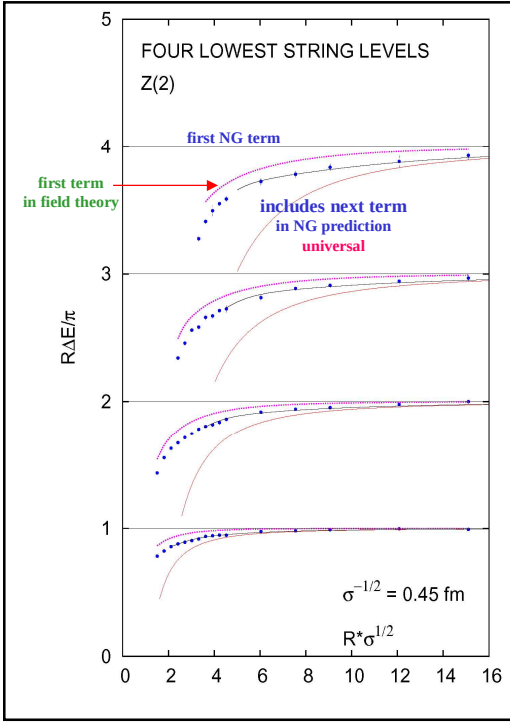
$$\partial_\mu\xi\partial_\mu\xi\frac{1}{q^2+M^2}\partial_\mu\xi\partial_\mu\xi$$

when $q=\frac{\pi}{R}n$ M

n R·M/π

resonance
non-local terms

$$S_2=\frac{1}{4}c_2\int_0^T dt\int_0^R ds\left\{\frac{1}{2}(\partial_a x\partial_a x)(\partial_b x\partial_b x)\right\}+\cdots$$



Summary of main results on the spectrum of the fixed end Z(2) string

$x=\sqrt{\sigma}R$ dimensionless scale variable

$$E_N/\sqrt{\sigma}=x\sqrt{1-\frac{D-2}{12x^2}\pi+\frac{2\pi N}{x^2}}$$ **NG**

Expand energy gaps for large x

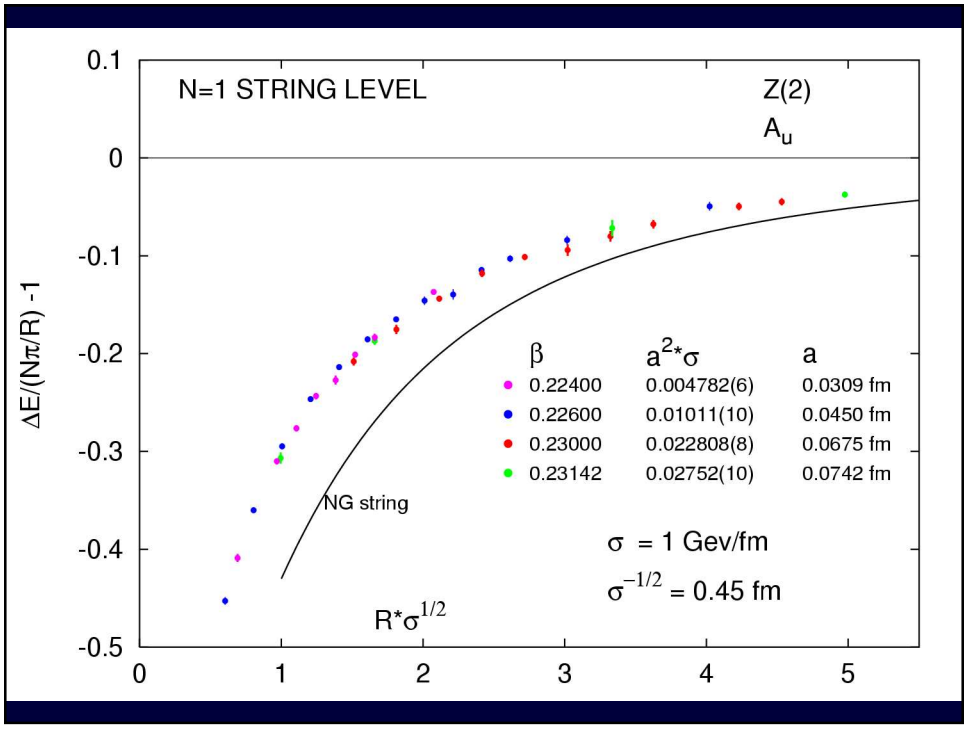
$$\frac{R(E_N-E_0)}{\pi}=N+\frac{a(N)}{x^2}+\frac{b(N)}{x^4}+\ldots$$

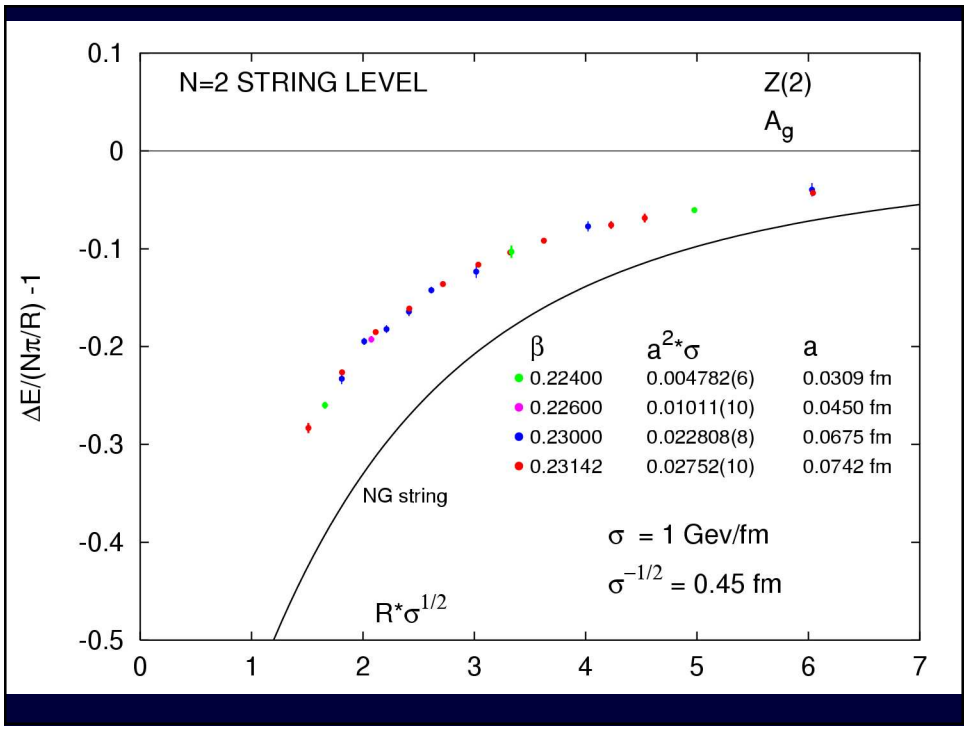
First correction to asymptotic spectrum appears to be universal

Higher corrections code new physics like string rigidity, etc.

Similar expansion for string-soliton with unit winding

Data for R < 4 fermi prefers field theory description which incorporates end effects naturally


N=1 STRING LEVEL
Z(2)
A_u
ΔE/(Nπ/R) -1
R*σ^1/2
NG string
β
a²*σ
a
0.22400 0.004782(6) 0.0309 fm
0.22600 0.01011(10) 0.0450 fm
0.23000 0.022808(8) 0.0675 fm
0.23142 0.02752(10) 0.0742 fm
σ = 1 Gev/fm
σ^−1/2 = 0.45 fm


N=2 STRING LEVEL
Z(2)
A_g
ΔE/(Nπ/R) -1
R*σ^1/2
NG string
β
a²*σ
a
0.22400 0.004782(6) 0.0309 fm
0.22600 0.01011(10) 0.0450 fm
0.23000 0.022808(8) 0.0675 fm
0.23142 0.02752(10) 0.0742 fm
σ = 1 Gev/fm
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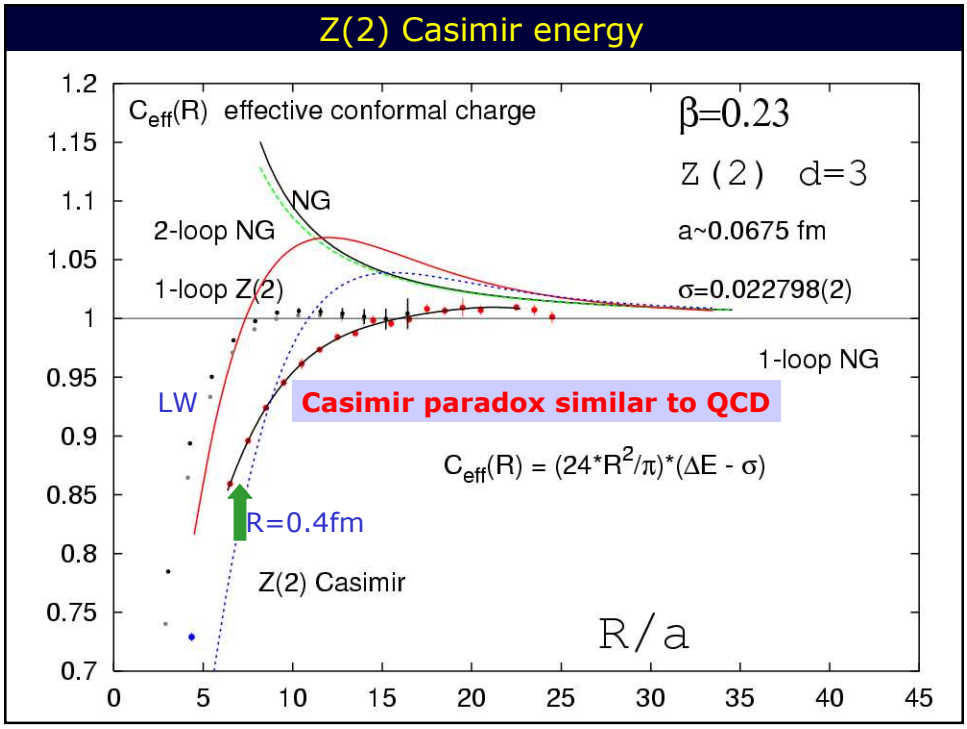

- Massless Goldstone modes
- Local derivative expansion for their interactions
  from fine structure in the spectrum
- Massive excitations
- Breathing modes in effective Lagrangian
- String properties ? Bosonic, NG, rigid, …?

WITHIN OUR REACH

Conclusions:

1. Fine structure in QCD string spectrum
   Progress on string-soliton spectrum

neither was seen before

2. Casimir energy paradox: low energy Goldstone modes → geometric string theory?

3. Is bulk behavior and related resonance spectrum the clue? Origin of central charge?

4. Effective low-energy string theory?
   Universality class of QCD string ?