Hadron Tomography

or: Physical Interpretation for the Generalized Parton Distributions $H(x,0,-\mathbf{\Delta}_\perp^2)$ *and* $E(x,0,-\mathbf{\Delta}_\perp^2)$

Matthias Burkardt

burkardt@nmsu.edu

New Mexico State University

Las Cruces, NM, 88003, U.S.A.

(brief) Motivation

- DIS $\stackrel{opt.theorem}{\longrightarrow}$ forward Compton amplitude $\stackrel{Bj-limit}{\longrightarrow}$ $q(x_{Bj})$

$$q(x_{Bj}) = \int \frac{dx^-}{2\pi} \langle p|\overline{q}\left(-\frac{x^-}{2}, \mathbf{0}_\perp\right)\gamma^+ q\left(\frac{x^-}{2}, \mathbf{0}_\perp\right)|p\rangle\, e^{ix^- x_{Bj} P^+}$$

  - Light-cone coordinates $x^\pm = \frac{1}{\sqrt{2}}\left(x^0 \pm x^1\right)$
  - $q(x) =$ light-cone momentum distribution of quarks in the target; $x =$ (light-cone) momentum fraction
  - no information about position of partons!

(brief) Motivation

- generalization to $p' \neq p \quad \Rightarrow \quad$ **G**eneralized **P**arton **D**istributions

$$GPD(x,\xi,t) \equiv \int \frac{dx^-}{2\pi} \langle p'|\bar{q}\left(-\frac{x^-}{2},\mathbf{0}_\perp\right)\gamma^+ q\left(\frac{x^-}{2},\mathbf{0}_\perp\right)|p\rangle\, e^{ix^- xP^+}$$

with $\Delta = p - p'$, $t = \Delta^2$, and $\xi(p^+ + p^{+\prime}) = -2\Delta^+$.

  - can be probed e.g. in **D**eeply **V**irtual **C**ompton **S**cattering (DVCS) at HERMES, JLab@12GeV, eRHIC, ...
  - Interesting observation: X.Ji, PRL**78**,610(1997)

$$\langle J_q \rangle = \frac{1}{2}\int_0^1 dx\, x\left[H_q(x,0,0) + E_q(x,0,0)\right]$$

$$\boxed{\text{DVCS}} \quad \Leftrightarrow \quad \boxed{\text{GPDs}} \quad \Leftrightarrow \quad \boxed{\vec{J}_q}$$

  - But: what other "physical information" about the nucleon can we obtain by measuring/calculating GPDs?

Outline

- DIS $\longrightarrow$ parton distributions
- Deeply virtual Compton scattering (DVCS)

$\hookrightarrow$ Generalized parton distributions (GPDs)

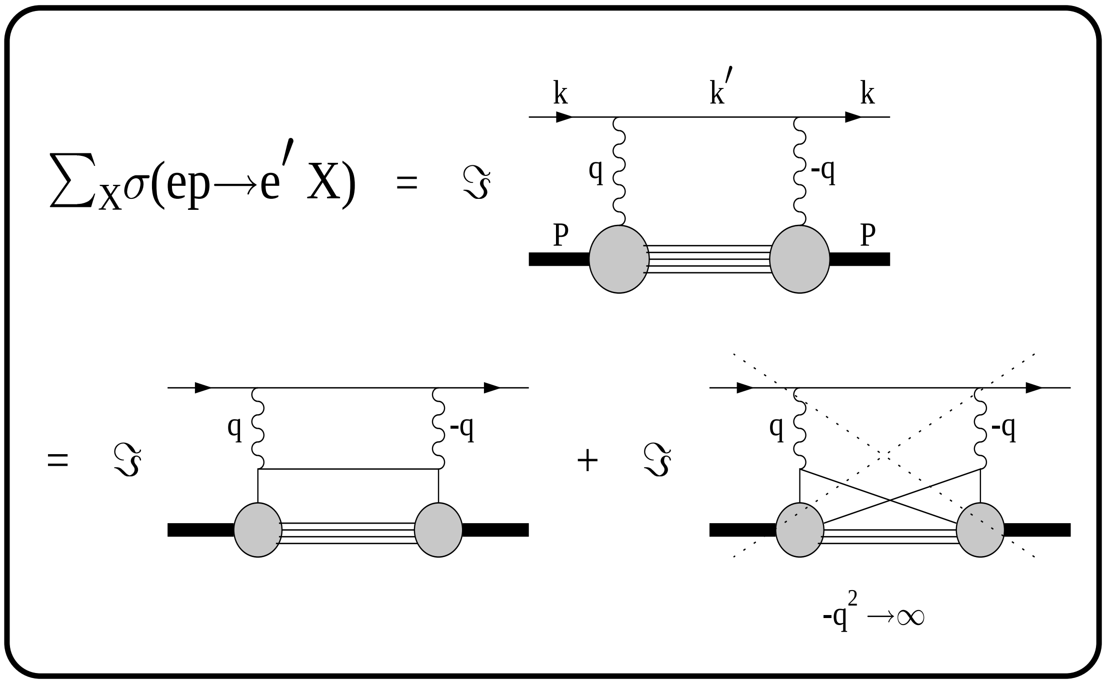
- Probabilistic interpretation of GPDs as Fourier transforms of impact parameter dependent PDFs
  - $H(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow q(x, \mathbf{b}_\perp)$
  - $\tilde{H}(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow \Delta q(x, \mathbf{b}_\perp)$
  - $E(x, 0, -\mathbf{\Delta}_\perp^2) \longrightarrow \perp$ distortion of PDFs when the target is transversely polarized
- Chromodynamik lensing and $\perp$ single-spin asymmetries (SSA)

$$\left.\begin{array}{c}\text{transverse distortion of PDFs}\\ +\ \text{final state interactions}\end{array}\right\} \Rightarrow \quad \perp \text{SSA in} \quad \gamma N \longrightarrow \pi + X$$

- Summary

DIS $\longrightarrow$ light-cone correlations

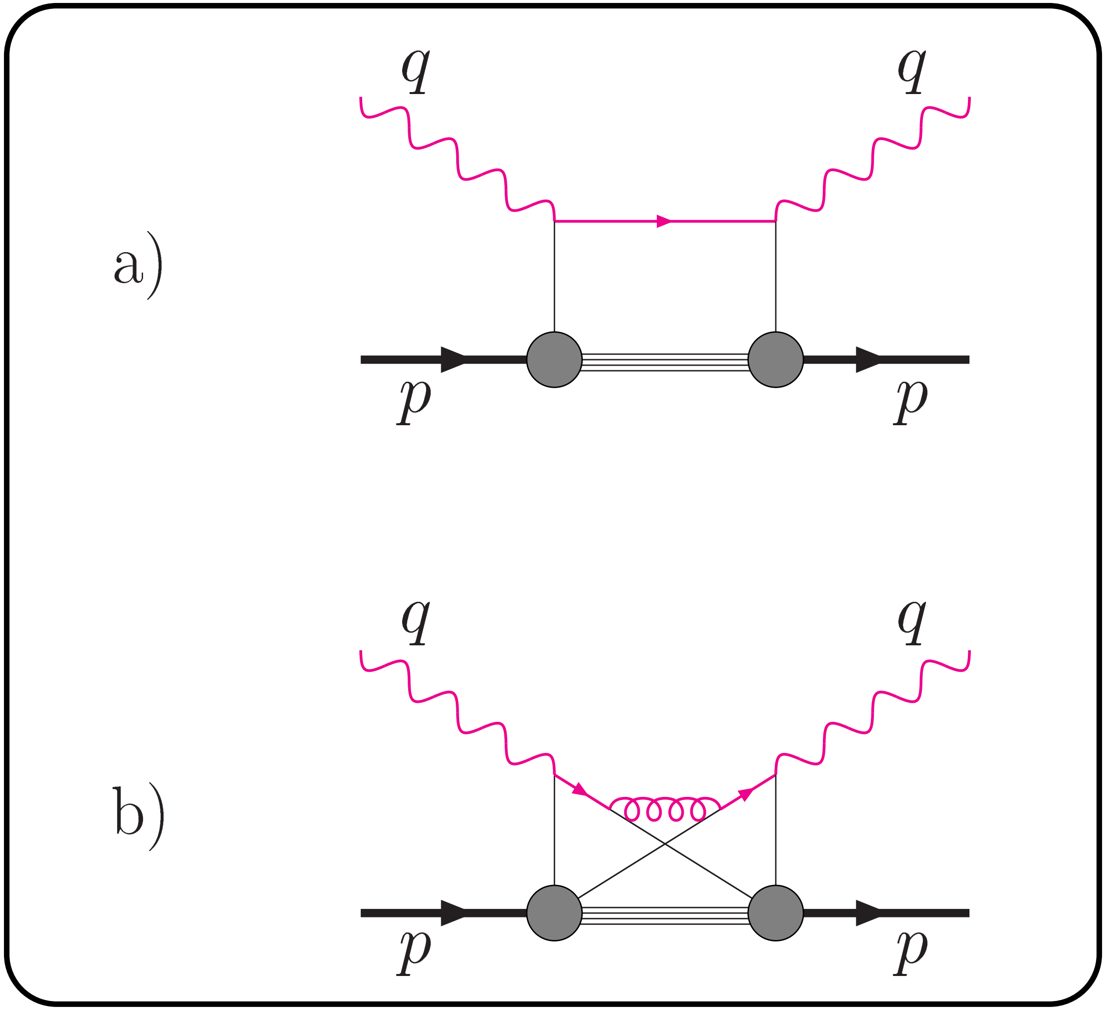
opt. theorem:
inclusive cross–section $\Leftrightarrow$ virtual, forward Compton amplitude



struck quark carries large momentum: $Q^2 \gg \Lambda^2_{\mathrm{QCD}}$

- crossed diagram suppressed (wavefunction!)
- asymptotic freedom $\Rightarrow$ neglect interactions of struck quark
- struck quark propagates along light-cone $x^2 = 0$

suppression of crossed diagrams



Flow of the large momentum q through typical diagrams contributing to the forward Compton amplitude. a) 'handbag' diagrams; b) 'cat's ears' diagram. Diagram b) is suppressed at large q due to the presence of additional propagators.

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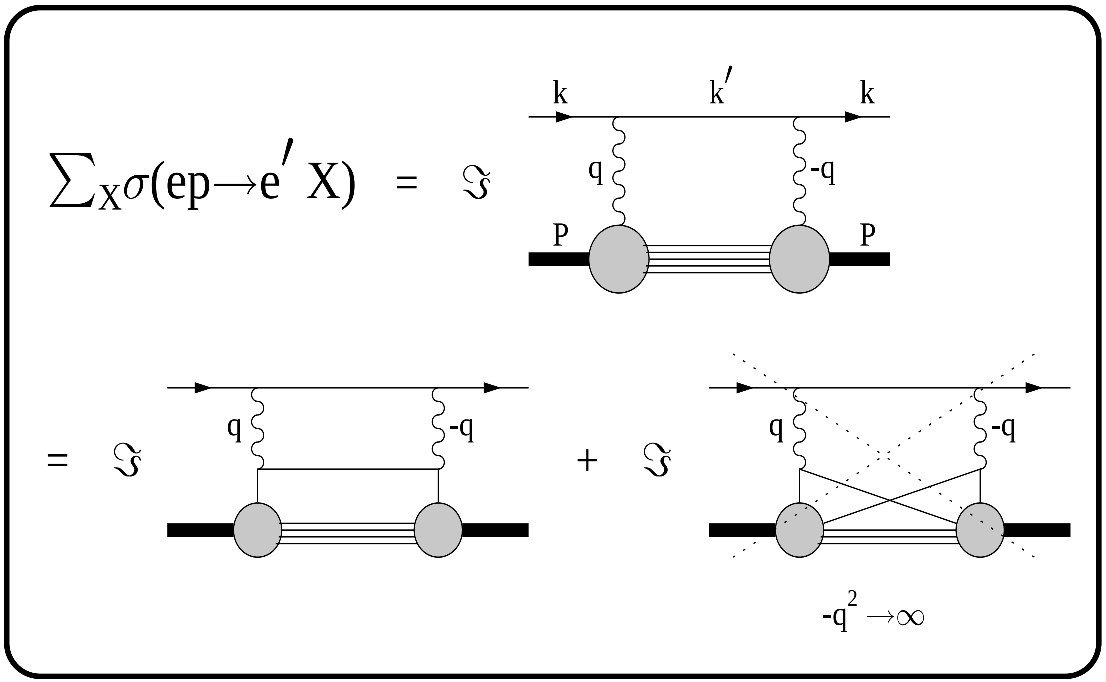
DIS ⟶ light-cone correlations

opt. theorem:

inclusive cross–section ⇔ virtual, forward Compton amplitude



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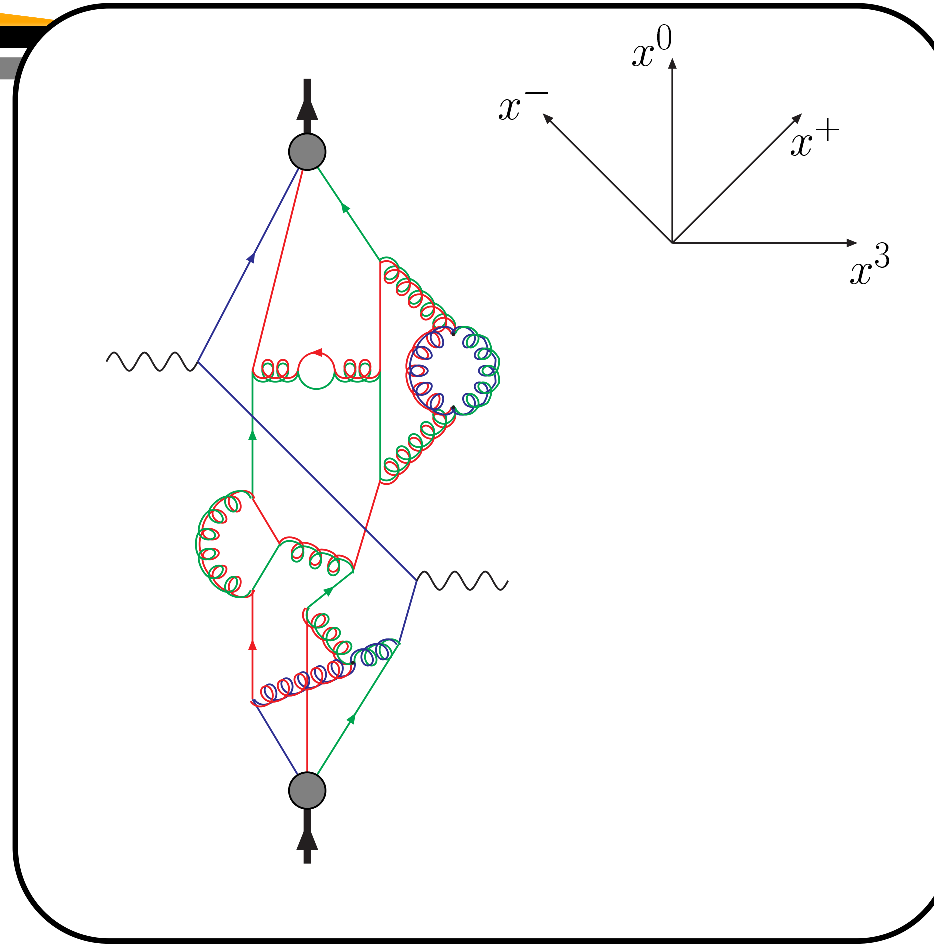


light-cone coordinates:

$$x^+ = \left(x^0 + x^3\right)/\sqrt{2}$$

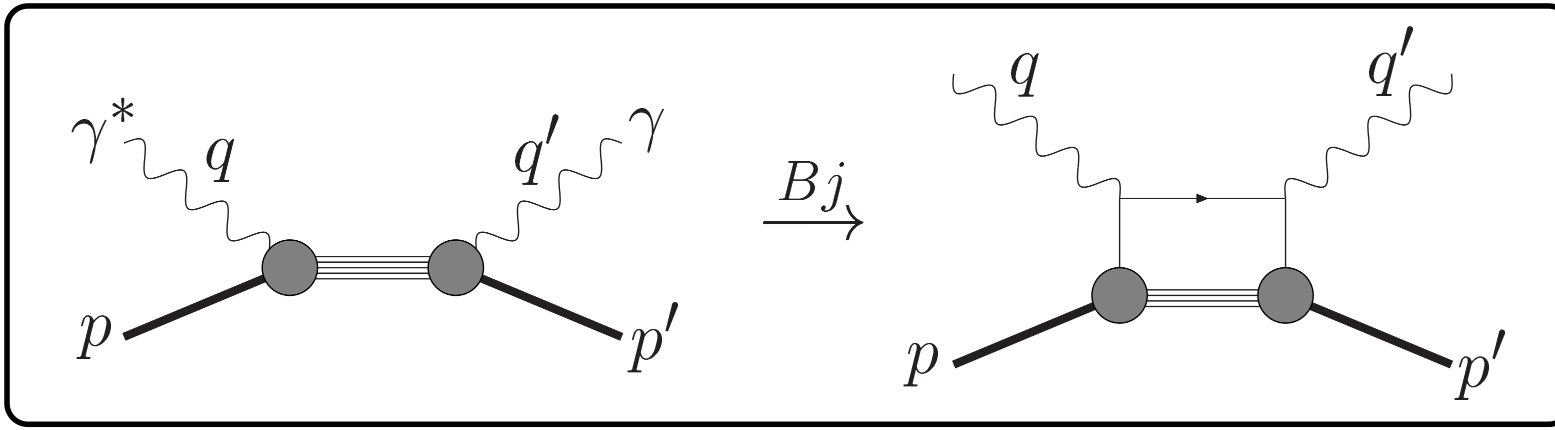
$$x^- = \left(x^0 - x^3\right)/\sqrt{2}$$

DIS related to correlations along light–cone

$$q(x_{Bj}) = \int \frac{dx^-}{2\pi} \langle P|\overline{q}(0^-, \mathbf{0}_\perp)\gamma^+ q(x^-, \mathbf{0}_\perp)|P\rangle \, e^{ix^- x_{Bj} P^+}$$

No information about transverse position of partons!

Deeply Virtual Compton Scattering (DVCS)



$$T^{\mu\nu} = i\int d^4z\, e^{i\bar{q}\cdot z}\left\langle p'\left|TJ^\mu\left(-\frac{z}{2}\right)J^\nu\left(\frac{z}{2}\right)\right|p\right\rangle$$

$$\overset{Bj}{\hookrightarrow} \quad \frac{g_\perp^{\mu\nu}}{2}\int_{-1}^{1}dx\left(\frac{1}{x-\xi+i\varepsilon}+\frac{1}{x+\xi-i\varepsilon}\right)H(x,\xi,\Delta^2)\bar{u}(p')\gamma^+u(p) \quad + \quad \ldots$$

$$\bar{q} = (q+q')/2 \qquad \Delta = p'-p \qquad x_{Bj} \equiv -q^2/2p\cdot q = 2\xi(1+\xi)$$

Generalized Parton Distributions (GPDs)

$$\int \frac{dx^-}{2\pi} e^{ix^-\bar{p}^+x} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right)\gamma^+ q\left(\frac{x^-}{2}\right)\right| p \right\rangle = H(x,\xi,\Delta^2)\bar{u}(p')\gamma^+ u(p) + E(x,\xi,\Delta^2)\bar{u}(p')\frac{i\sigma^{+\nu}\Delta_\nu}{2M}u(p)$$

$$\int \frac{dx^-}{2\pi} e^{ix^-\bar{p}^+x} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right)\gamma^+\gamma_5 q\left(\frac{x^-}{2}\right)\right| p \right\rangle = \tilde{H}(x,\xi,\Delta^2)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}(x,\xi,\Delta^2)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p)$$

where $\Delta = p' - p$ is the momentum transfer and ξ measures the longitudinal momentum transfer on the target $\Delta^+ = \xi(p^+ + p^{+\prime})$.

Parton Interpretation

$$\int \frac{dx^-}{2\pi} e^{ix^-\bar{p}^+x} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right) \right| p \right\rangle = H(x,\xi,\Delta^2)\bar{u}(p')\gamma^+ u(p)$$

$$+E(x,\xi,\Delta^2)\bar{u}(p')\frac{i\sigma^{+\nu}\Delta_\nu}{2M}u(p)$$

- Actually $H = H(x,\xi,\Delta^2,q^2)$, but will not discuss q^2 dependence of GPDs today!
- x is mean long. momentum fraction carried by active quark
- ξ measures longitudinal momentum transfer $\xi = \frac{p^{+\prime}-p^+}{p^+ + p^{+\prime}}$
- In general no probabilistic interpretation since initial and final state not the same $\longrightarrow$ interpretation as transition amplitude
- $\int dx H(x,\xi,\Delta^2) = F_1(\Delta^2)$ and $\int dx E(x,\xi,\Delta^2) = F_2(\Delta^2)$

$\hookrightarrow$ GPDs provide a decomposition of form factor w.r.t. the momentum fraction (in IMF) carried by the active quark

Parton Interpretation

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$\hookrightarrow$ knowledge of GPDs $\Rightarrow$ better understanding of form factors

What is Physics of GPDs ?

- Definition of GPDs resembles that of form factors

$$\left\langle p' \left| \hat{O} \right| p \right\rangle = H(x,\xi,\Delta^2)\bar{u}(p')\gamma^+ u(p) + E(x,\xi,\Delta^2)\bar{u}(p')\frac{i\sigma^{+\nu}\Delta_\nu}{2M}u(p)$$

with $\hat{O} \equiv \int \frac{dx^-}{2\pi} e^{ix^-\bar{p}^+x}\bar{q}\left(-\frac{x^-}{2}\right)\gamma^+ q\left(\frac{x^-}{2}\right)$

$\hookrightarrow$ relation between PDFs and GPDs similar to relation between a charge and a form factor

$\hookrightarrow$ If form factors can be interpreted as Fourier transforms of charge distributions in position space, what is the analogous physical interpretation for GPDs ?

Form Factors vs. GPDs

operator	forward matrix elem.	off-forward matrix elem.	position space
$\bar{q}\gamma^+ q$	Q	$F(t)$	$\rho(\vec{r})$
$\int \frac{dx^- e^{ixp^+x^-}}{4\pi} \bar{q}\left(\frac{-x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right)$	$q(x)$	$H(x,\xi,t)$	?

Form Factors vs. GPDs

operator	forward matrix elem.	off-forward matrix elem.	position space
$\bar{q}\gamma^+ q$	Q	$F(t)$	$\rho(\vec{r})$
$\int \frac{dx^- e^{ixp^+x^-}}{4\pi} \bar{q}\left(\frac{-x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right)$	$q(x)$	$H(x,0,t)$	$q(x,\mathbf{b}_\perp)$

$q(x,\mathbf{b}_\perp)$ = impact parameter dependent PDF

Impact parameter dependent PDFs

- define state that is localized in $\perp$ position:

$$\left|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \lambda\right\rangle \equiv \mathcal{N} \int d^2\mathbf{p}_\perp \left|p^+, \mathbf{p}_\perp, \lambda\right\rangle$$

  Note: $\perp$ boosts in IMF form Galilean subgroup $\Rightarrow$ this state has
  $\mathbf{R}_\perp \equiv \frac{1}{P^+} \int dx^- d^2\mathbf{x}_\perp \, \mathbf{x}_\perp T^{++}(x) = \mathbf{0}_\perp$
  (cf.: working in CM frame in nonrel. physics)

- define impact parameter dependent PDF

$$q(x, \mathbf{b}_\perp) \equiv \int \frac{dx^-}{4\pi} \left\langle p^+, \mathbf{R}_\perp = \mathbf{0}_\perp \right| \bar{q}(-\frac{x^-}{2}, \mathbf{b}_\perp) \gamma^+ q(\frac{x^-}{2}, \mathbf{b}_\perp) \left|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp\right\rangle e^{ixp^+x^-}$$

Impact parameter dependent PDFs

- use translation invariance to relate to same matrix element that appears in def. of GPDs

$$q(x,\mathbf{b}_\perp) \equiv \int dx^- \langle p^+, \mathbf{R}_\perp = \mathbf{0}_\perp | \bar{q}(-\frac{x^-}{2}, \mathbf{b}_\perp)\gamma^+ q(\frac{x^-}{2}, \mathbf{b}_\perp) | p^+, \mathbf{R}_\perp = \mathbf{0}_\perp \rangle e^{ixp^+x^-}$$

$$= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \int dx^- \langle p^+, \mathbf{p}'_\perp | \bar{q}(-\frac{x^-}{2}, \mathbf{b}_\perp)\gamma^+ q(\frac{x^-}{2}, \mathbf{b}_\perp) | p^+, \mathbf{p}_\perp \rangle e^{ixp^+x^-}$$

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$$\times e^{i\mathbf{b}_\perp\cdot(\mathbf{p}_\perp-\mathbf{p}'_\perp)}$$

Impact parameter dependent PDFs

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$$q(x,\mathbf{b}_\perp) \equiv \int dx^- \left\langle p^+, \mathbf{R}_\perp = \mathbf{0}_\perp \right| \bar{q}(-\frac{x^-}{2}, \mathbf{b}_\perp)\gamma^+ q(\frac{x^-}{2}, \mathbf{b}_\perp) \left| p^+, \mathbf{R}_\perp = \mathbf{0}_\perp \right\rangle e^{ixp^+x^-}$$

$$= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \int dx^- \left\langle p^+, \mathbf{p}'_\perp \right| \bar{q}(-\frac{x^-}{2}, \mathbf{b}_\perp)\gamma^+ q(\frac{x^-}{2}, \mathbf{b}_\perp) \left| p^+, \mathbf{p}_\perp \right\rangle e^{ixp^+x^-}$$

$$= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \int dx^- \left\langle p^+, \mathbf{p}'_\perp \right| \bar{q}(-\frac{x^-}{2}, \mathbf{0}_\perp)\gamma^+ q(\frac{x^-}{2}, \mathbf{0}_\perp) \left| p^+, \mathbf{p}_\perp \right\rangle e^{ixp^+x^-} \times e^{i\mathbf{b}_\perp\cdot(\mathbf{p}_\perp - \mathbf{p}'_\perp)}$$

$$= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp H\left(x, 0, -\left(\mathbf{p}'_\perp - \mathbf{p}_\perp\right)^2\right) e^{i\mathbf{b}_\perp\cdot(\mathbf{p}_\perp - \mathbf{p}'_\perp)}$$

$$\hookrightarrow \boxed{q(x,\mathbf{b}_\perp) = \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} H(x,0,-\boldsymbol{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp\cdot\boldsymbol{\Delta}_\perp}}$$

Impact parameter dependent PDFs

- $$q(x,\mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} H(x,0,-\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp}$$

  $(\mathbf{\Delta}_\perp = \mathbf{p}'_\perp - \mathbf{p}_\perp,\ \xi = 0)$
- $q(x,\mathbf{b}_\perp)$ has physical interpretation of a density

$$q(x,\mathbf{b}_\perp) \geq 0 \quad \text{for} \quad x > 0$$
$$q(x,\mathbf{b}_\perp) \leq 0 \quad \text{for} \quad x < 0$$

$$\begin{aligned} q(x,\mathbf{b}_\perp) &\sim \left\langle p^+, \mathbf{0}_\perp \left| b^\dagger(xp^+,\mathbf{b}_\perp) b(xp^+,\mathbf{b}_\perp) \right| p^+, \mathbf{0}_\perp \right\rangle \\ &= \left| b(xp^+,\mathbf{b}_\perp) | p^+, \mathbf{0}_\perp \rangle \right|^2 \geq 0 \end{aligned}$$

Discussion: $GPD \leftrightarrow q(x, \mathbf{b}_\perp)$

- $q(x, \mathbf{b}_\perp)$ has interpretation as density (positivity constraints!) $\hookrightarrow$ positivity constraint on models

Discussion: $GPD \leftrightarrow q(x, \mathbf{b}_\perp)$

- GPDs allow simultaneous determination of longitudinal momentum and transverse position of partons

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} H(x, 0, -\boldsymbol{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \boldsymbol{\Delta}_\perp}$$

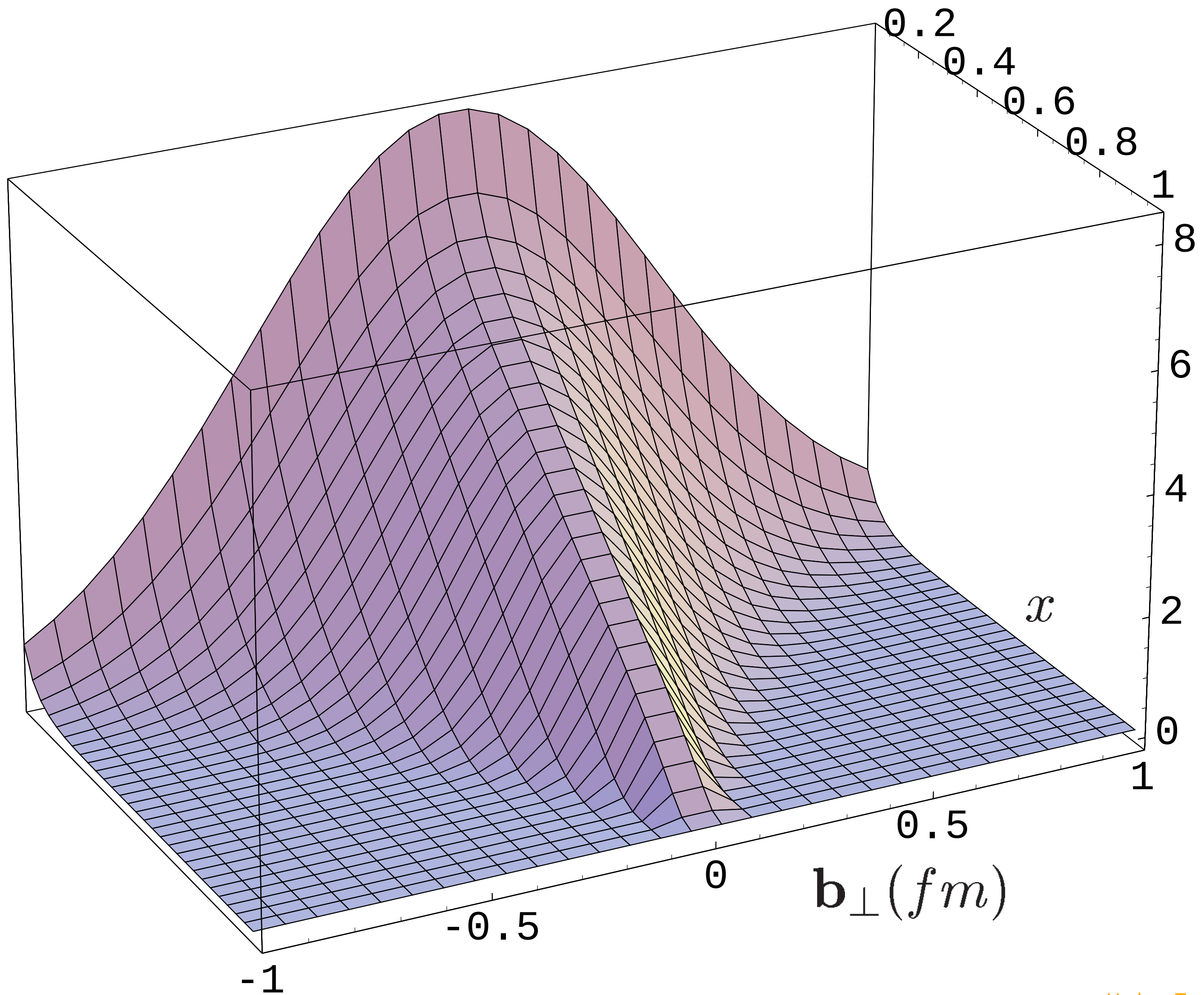
$$\Delta q(x, \mathbf{b}_\perp) = \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} \tilde{H}(x, 0, -\boldsymbol{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \boldsymbol{\Delta}_\perp}$$

- Nonrelativistically such a result not surprising!
  Absence of relativistic corrections to identification
  $H(x, 0, -\boldsymbol{\Delta}_\perp^2) \stackrel{FT}{\longrightarrow} q(x, \mathbf{b}_\perp)$ due to Galilean subgroup in IMF
- $\mathbf{b}_\perp$ distribution measured w.r.t. $\mathbf{R}_\perp^{CM} \equiv \sum_i x_i \mathbf{r}_{i,\perp}$
  $\hookrightarrow$ width of the $\mathbf{b}_\perp$ distribution should go to zero as $x \rightarrow 1$, since the active quark becomes the $\perp$ center of momentum in that limit!
  $\hookrightarrow$ $H(x, 0, -\boldsymbol{\Delta}_\perp^2)$ must become $\boldsymbol{\Delta}_\perp^2$-indep. as $x \rightarrow 1$. Confirmed by recent lattice studies (QCDSF, LHPC)

Discussion: $GPD \leftrightarrow q(x, \mathbf{b}_\perp)$

- Use intuition about nucleon structure in position space to make predictions for GPDs:
  large x: quarks from localized valence ‘core’,
  small x: contributions from larger ‘ meson cloud’
  $\hookrightarrow$ expect a gradual increase of the t-dependence ($\perp$ size) of $H(x,0,t)$ as x decreases
- very simple model: $H_q(x,0,-\boldsymbol{\Delta}_\perp^2) = q(x) e^{-a\boldsymbol{\Delta}_\perp^2 (1-x)\ln\frac{1}{x}}$.

$q(x, \mathbf{b}_\perp)$ in a simple model



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The physics of $E(x, 0, -\mathbf{\Delta}_\perp^2)$

- So far: only unpolarized (or long. polarized) nucleon

  In general, use ($\Delta^+ = 0$)

$$\begin{aligned}\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \left\langle P{+}\Delta,\uparrow \left| \bar{q}(0)\, \gamma^+ q\left(x^-\right) \right| P,\uparrow \right\rangle &= H(x,0,-\mathbf{\Delta}_\perp^2) \\ \int \frac{dx^-}{4\pi} e^{ip^+x^-x} \left\langle P{+}\Delta,\uparrow \left| \bar{q}(0)\, \gamma^+ q\left(x^-\right) \right| P,\downarrow \right\rangle &= -\frac{\Delta_x - i\Delta_y}{2M} E(x,0,-\mathbf{\Delta}_\perp^2).\end{aligned}$$

- Consider nucleon polarized in x direction (in IMF)
  $|X\rangle \equiv |p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \uparrow\rangle + |p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \downarrow\rangle$.

$\hookrightarrow$ unpolarized quark distribution for this state:

$$q_X(x,\mathbf{b}_\perp) = q(x,\mathbf{b}_\perp) - \frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} E(x, 0, -\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$$

The physics of $E(x,0,-\Delta_\perp^2)$

$q_X(x,\mathbf{b}_\perp)$ in transversely polarized nucleon is transversely distorted compared to longitudinally polarized nucleons !

- mean displacement of flavor q ($\perp$ flavor dipole moment)

$$d_y^q \equiv \int dx \int d^2\mathbf{b}_\perp q_X(x,\mathbf{b}_\perp) b_y = \frac{1}{2M}\int dx E_q(x,0,0) = \kappa_q^p$$

  with $\kappa_{u/d}^p \equiv F_2^{u/d}(0) = \mathcal{O}(1-2) \quad \Rightarrow \quad d_y^q = \mathcal{O}(0.2fm)$

- CM for flavor q shifted relative to CM for whole proton by

$$\int dx \int d^2\mathbf{b}_\perp q_X(x,\mathbf{b}_\perp) x b_y = \frac{1}{2M}\int dx\, x E_q(x,0,0)$$

$\hookrightarrow$ not surprising to find that second moment of E_q is related to angular momentum carried by flavor q

Intuitive connection with $\vec{L}_q$

- (some) DIS-kinematics (target rest frame & momentum transfer in $-\hat{z}$ direction):

$$\mathbf{p}_{e\perp} = \mathbf{p}'_{e\perp} = \mathbf{0}_\perp \qquad p_e^z \longrightarrow -\infty \qquad p_e^{z\prime} \longrightarrow -\infty$$

$\hookrightarrow$ only "$-$" component of all momenta and momentum transfers large, e.g.

$$p_e^- \equiv \frac{1}{\sqrt{2}}\left(p_e^0 - p_e^z\right) \longrightarrow \infty \qquad p_e^+ \equiv \frac{1}{\sqrt{2}}\left(p_e^0 + p_e^z\right) = \frac{m_e^2 + \mathbf{p}_{e\perp}^2}{2p_e^-} \longrightarrow 0$$

$\hookrightarrow$ only "$-$" component of the electron current $j_e^\mu = \bar{u}(p')\gamma^\mu u(p)$ large

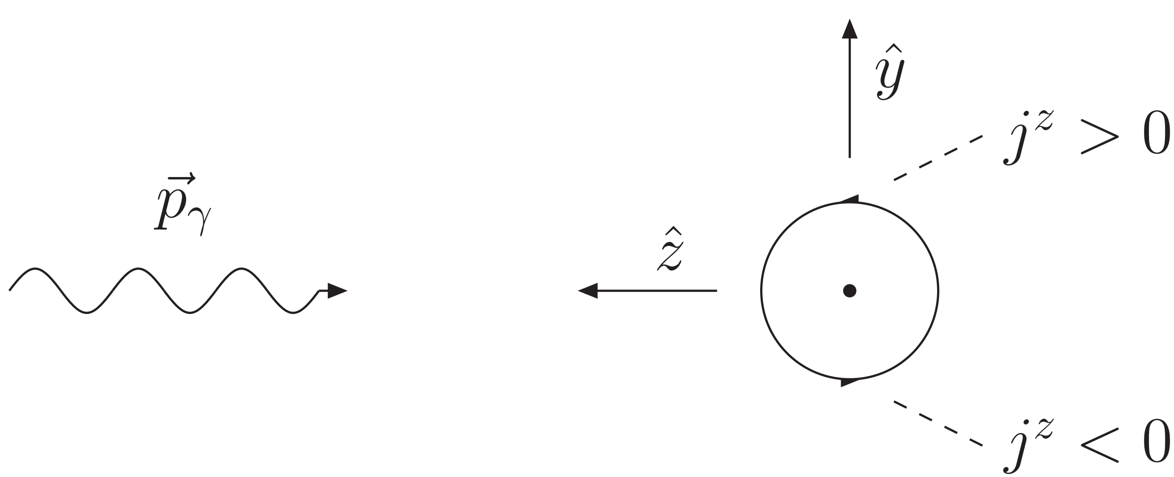
- vector-vector interaction

$$\mathcal{M} \propto j_e^\mu j_q^\nu g_{\mu\nu} = j_e^- j_q^+ + j_e^+ j_q^- - j_e^\perp j_q^\perp$$

$\hookrightarrow$ electron "sees" (for $p_e^z \rightarrow -\infty$) only j_q^+

Intuitive connection with $\vec{L}_q$

- Electromagnetic interaction couples to vector current. Due to kinematics of the DIS-reaction (and the choice of coordinates — $\hat{z}$-axis in direction of the momentum transfer) the virtual photons “see” (in the Bj-limit) only the $j^+ = j^0 + j^z$ component of the quark current
- If up-quarks have positive orbital angular momentum in the $\hat{x}$-direction, then j^z is positive on the $+\hat{y}$ side, and negative on the $-\hat{y}$ side



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- If up-quarks have positive orbital angular momentum in the $\hat{x}$-direction, then j^z is positive on the $+\hat{y}$ side, and negative on the $-\hat{y}$ side

$\hookrightarrow$ j^+ is distorted not because there are more quarks on one side than on the other but because the DIS-photons (coupling only to j^+) "see" the quarks on the $+\hat{y}$ side better than on the $-\hat{y}$ side (for $L_x > 0$).

Simple model for $E_q(x, 0, -\mathbf{\Delta}_\perp^2)$

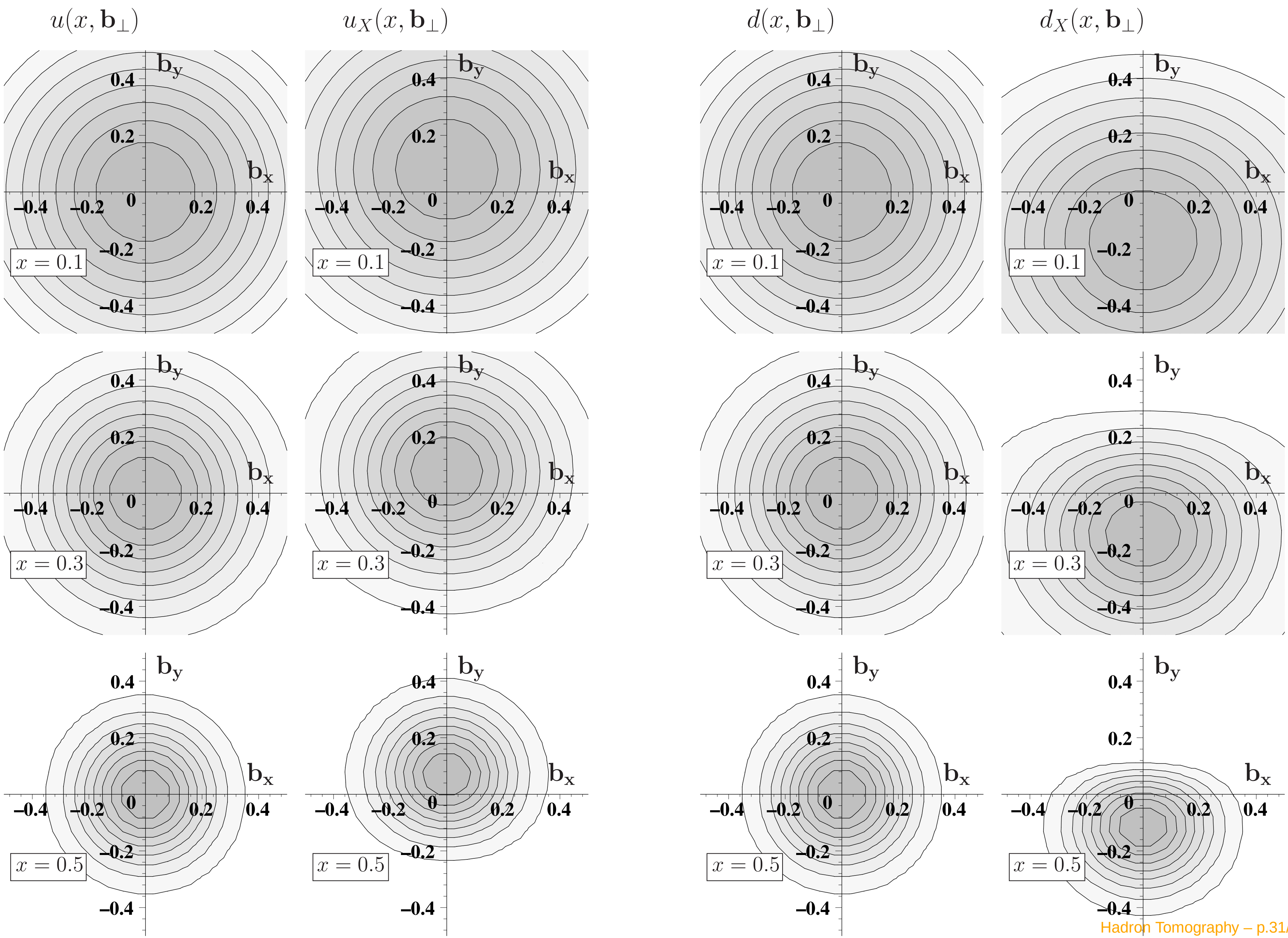
- For simplicity, make ansatz where $E_q \propto H_q$

$$
\begin{aligned}
E_u(x, 0, -\mathbf{\Delta}_\perp^2) &= \frac{\kappa_u^p}{2} H_u(x, 0, -\mathbf{\Delta}_\perp^2) \\
E_d(x, 0, -\mathbf{\Delta}_\perp^2) &= \kappa_d^p H_d(x, 0, -\mathbf{\Delta}_\perp^2)
\end{aligned}
$$

with $H_q(x, 0, -\mathbf{\Delta}_\perp^2) = q(x) e^{-a\mathbf{\Delta}_\perp^2 (1-x) \ln \frac{1}{x}}$ and

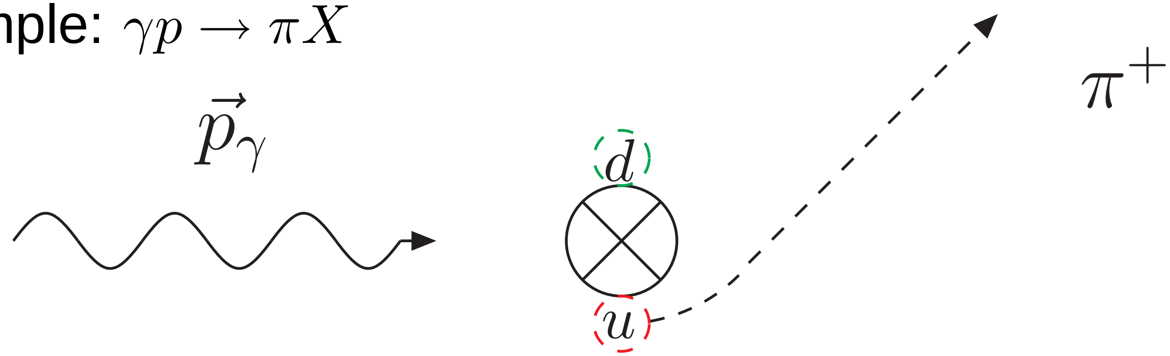
$$
\kappa_u^p = 2\kappa_p + \kappa_n = 1.673 \qquad \kappa_d^p = 2\kappa_n + \kappa_p = -2.033.
$$

- Satisfies: $\int dx E_q(x, 0, 0) = \kappa_q^P$
- Model too simple but illustrates that anticipated distortion is very significant since $\int dx E_q \sim \kappa_q$ known to be large!


$u(x, \mathbf{b}_\perp)$
$u_X(x, \mathbf{b}_\perp)$
$d(x, \mathbf{b}_\perp)$
$d_X(x, \mathbf{b}_\perp)$
$x = 0.1$
$x = 0.3$
$x = 0.5$
$\mathbf{b_x}$
$\mathbf{b_y}$

Quark Correlations $\longleftrightarrow$ SSA

- example: $\gamma p \to \pi X$



- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign determined by κ_u & κ_d

$\hookrightarrow$ attractive FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction

$\hookrightarrow$ correlation between sign of κ_q / L_q and sign of SSA

$$\langle k_u^y \rangle < 0 \qquad \text{and} \qquad \langle k_d^y \rangle > 0$$

for proton polarized in $+\hat{x}$ direction [M.B., PRD **66**, 114005 (2002)].

- signs confirmed by recent HERMES data (hep-ex/0408013).

GPDs for $x \to 1$ (or: size does matter!)

- Distance between active quark and center of momentum of all spectators

$$\mathbf{r}_\perp = \frac{1}{1-x}\mathbf{b}_\perp$$

diverges as $x \to 1$, unless $\mathbf{b}_\perp$ goes to zero!

- Assume (conjecture) no (or only few) partons outside disk of radius $1/\Lambda$ around center of momentum

$\hookrightarrow$ $q(x, \mathbf{b}_\perp) = 0$ for $\mathbf{b}_\perp^2 > \frac{(1-x)^2}{\Lambda^2}$

$\hookrightarrow$ significant contribution to form factor from quarks with $x > 1 - \frac{\Lambda}{|\boldsymbol{\Delta}_\perp|}$

$$\begin{aligned} F_{large\,x}(\boldsymbol{\Delta}_\perp^2) &\equiv \int_{1-\frac{\Lambda}{|\boldsymbol{\Delta}_\perp|}}^1 dx\, H(x,0,\boldsymbol{\Delta}_\perp^2) = \int_{1-\frac{\Lambda}{|\boldsymbol{\Delta}_\perp|}}^1 dx \int d^2\mathbf{b}_\perp e^{-i\mathbf{b}_\perp\cdot\boldsymbol{\Delta}_\perp} q(x,\mathbf{b}_\perp) \\ &= \int_{1-\frac{\Lambda}{|\boldsymbol{\Delta}_\perp|}}^1 dx q(x) = \frac{c}{4}\frac{\Lambda^4}{\boldsymbol{\Delta}_\perp^4} \qquad \text{for} \quad q(x) = c(1-x)^3. \end{aligned}$$

GPDs for $x \to 1$ (or: size does matter!)

- large-x contribution independent of detailed shape of $q(x, \mathbf{b}_\perp)$

$$F_{large\,x}(\mathbf{\Delta}_\perp^{\mathbf{2}}) = \frac{c}{4}\frac{\Lambda^4}{\mathbf{\Delta}_\perp^4} \qquad \text{for} \quad q(x) = c(1-x)^3.$$

- Different mechanism but same behavior $\frac{1}{\mathbf{\Delta}_\perp^4}$ as pQCD mechanism
- presence of this large-x contribution to the nucleon form factor may have important consequences for color transparency — or the lack thereof ...
- Need to understand large x better!

Summary

- DVCS allows probing GPDS

$$\int \frac{dx^-}{2\pi} e^{ixp^+x^-} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right) \right| p \right\rangle$$

- GPDs resemble both PDFs and form factors: defined through matrix elements of light-cone correlation, but $\Delta \equiv p' - p \neq 0$.
- t-dependence of GPDs at $\xi = 0$ (purely $\perp$ momentum transfer) $\Rightarrow$ Fourier transform of impact parameter dependent PDFs $q(x, \mathbf{b}_\perp)$

$\hookrightarrow$ knowledge of GPDs for $\xi = 0$ provides novel information about nonperturbative parton structure of nucleons: distribution of partons in $\perp$ plane

$$\boxed{\begin{aligned} q(x,\mathbf{b}_\perp) &= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} H(x,0,-\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp} \\ \Delta q(x,\mathbf{b}_\perp) &= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} \tilde{H}(x,0,-\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp} \end{aligned}}$$

- $q(x, \mathbf{b}_\perp)$, $\Delta q(x, \mathbf{b}_\perp)$ have probabilistic interpretation, e.g. $q(x, \mathbf{b}_\perp) > 0$ for $x > 0$

Summary

- $\frac{\Delta_\perp}{2M} E(x, 0, -\mathbf{\Delta}_\perp^2)$ describes how the momentum distribution of unpolarized partons in the $\perp$ plane gets transversely distorted when is nucleon polarized in $\perp$ direction.
- (attractive) final state interaction converts $\perp$ position space asymmetry into $\perp$ momentum space asymmetry

$\hookrightarrow$ simple physical explanation for sign of left-right asymmetry in semi-inclusive DIS

- Similar mechanism also applicable to many other semi-inclusive events, such as transverse polarizations in hyperon production.
- published in: M.B., PRD **62**, 71503 (2000), Int. J. Mod. Phys. **A18**, 173 (2003); see also D. Soper, PRD **15**, 1141 (1977).
- Connection to SSA in M.B., PRD **69**, 057501 (2004); NPA **735**, 185 (2004); PRD **66**, 114005 (2002).

extrapolating to $\xi = 0$

- bad news: $\xi = 0$ not directly accessible in DVCS since long. momentum transfer necessary to convert virtual γ into real γ
- good news: moments of GPDs have simple ξ-dependence (polynomials in ξ)
  $\hookrightarrow$ should be possible to extrapolate!

even moments of $H(x,\xi,t)$:

$$\begin{aligned} H_n(\xi,t) &\equiv \int_{-1}^{1} dx x^{n-1} H(x,\xi,t) = \sum_{i=0}^{\left[\frac{n-1}{2}\right]} A_{n,2i}(t)\xi^{2i} + C_n(t) \\ &= A_{n,0}(t) + A_{n,2}(t)\xi^2 + ... + A_{n,n-2}(t)\xi^{n-2} + C_n(t)\xi^n, \end{aligned}$$

i.e. for example

$$\int_{-1}^{1} dx x H(x,\xi,t) = A_{2,0}(t) + C_2(t)\xi^2.$$

- For n^{th} moment, need $\frac{n}{2}+1$ measurements of $H_n(\xi,t)$ for same t but different ξ to determine $A_{n,2i}(t)$.
- GPDs @ $\xi = 0$ obtained from $H_n(\xi = 0, t) = A_{n,0}(t)$
- similar procedure exists for moments of $\tilde{H}$

back

QCD evolution

So far ignored! Can be easily included because

- For $t \ll Q^2$, leading order evolution t-independent
- For $\xi = 0$ evolution kernel for GPDs same as DGLAP evolution kernel

likewise:

- impact parameter dependent PDFs evolve such that different $\mathbf{b}_\perp$ do not mix (as long as $\perp$ spatial resolution much smaller than Q^2)

$\hookrightarrow$ above results consistent with QCD evolution:

$$\boxed{\begin{array}{l} H(x,0,-\mathbf{\Delta}_\perp^2,Q^2) = \int d^2b_\perp q(x,\mathbf{b}_\perp,Q^2)e^{-i\mathbf{b}_\perp\mathbf{\Delta}_\perp} \\ \tilde{H}(x,0,-\mathbf{\Delta}_\perp^2,Q^2) = \int d^2b_\perp \Delta q(x,\mathbf{b}_\perp,Q^2)e^{-i\mathbf{b}_\perp\mathbf{\Delta}_\perp} \end{array}}$$

where QCD evolution of $H, \tilde{H}, q, \Delta q$ is described by DGLAP and is independent on both $\mathbf{b}_\perp$ and $\mathbf{\Delta}_\perp^2$, provided one does not look at scales in $\mathbf{b}_\perp$ that are smaller than $1/Q$.

back

Form factor vs. charge distribution (non-rel.)

- define state that is localized in position space (center of mass frame)

$$\left|\vec{R}=\vec{0}\right\rangle \equiv \mathcal{N}\int d^3\vec{p}\,|\vec{p}\rangle$$

- define charge distribution (for this localized state)

$$\rho(\vec{r}) \equiv \left\langle \vec{R}=\vec{0}\right| j^0(\vec{r}) \left|\vec{R}=\vec{0}\right\rangle$$

- use translation invariance to relate to same matrix element that appears in def. of form factor

$$
\begin{aligned}
\rho(\vec{r}) &\equiv \left\langle \vec{R}=\vec{0}\right| j^0(\vec{r}) \left|\vec{R}=\vec{0}\right\rangle \\
&= |\mathcal{N}|^2 \int d^3\vec{p} \int d^3\vec{p}\,' \, \langle \vec{p}\,'|\, j^0(\vec{r})\, |\vec{p}\rangle \\
&= |\mathcal{N}|^2 \int d^3\vec{p} \int d^3\vec{p}\,' \langle \vec{p}\,'|\, j^0(\vec{0})\, |\vec{p}\rangle e^{i\vec{r}\cdot(\vec{p}-\vec{p}\,')}, \\
&= |\mathcal{N}|^2 \int d^3\vec{p} \int d^3\vec{p}\,' \, F\left(-\left(\vec{p}\,'-\vec{p}\right)^2\right) e^{i\vec{r}\cdot(\vec{p}-\vec{p}\,')}
\end{aligned}
$$

$\hookrightarrow$ $$\boxed{\rho(\vec{r}) = \int \frac{d^3\vec{\Delta}}{(2\pi)^3} F(-\vec{\Delta}^2) e^{-i\vec{r}\cdot\vec{\Delta}}}$$

back

density interpretation of $q(x,\mathrm{b}_\perp)$

- express quark-bilinear in twist-2 GPD in terms of light-cone 'good' component $q_{(+)} \equiv \frac{1}{2}\gamma^-\gamma^+ q$

$$\bar{q}'\gamma^+ q = \bar{q}'_{(+)}\gamma^+ q_{(+)} = \sqrt{2} q'^{\dagger}_{(+)} q_{(+)}.$$

- expand $q_{(+)}$ in terms of canonical raising and lowering operators

$$q_{(+)}(x^-, \mathbf{x}_\perp) = \int_0^\infty \frac{dk^+}{\sqrt{4\pi k^+}} \int \frac{d^2\mathbf{k}_\perp}{2\pi} \sum_s$$
$$\times \left[u_{(+)}(k,s) b_s(k^+, \mathbf{k}_\perp) e^{-ikx} + v_{(+)}(k,s) d_s^\dagger(k^+, \mathbf{k}_\perp) e^{ikx} \right],$$

density interpretation of $q(x,\mathbf{b}_\perp)$

with usual (canonical) equal light-cone time x^+ anti-commutation relations, e.g.

$$\left\{b_r(k^+,\mathbf{k}_\perp),b_s^\dagger(q^+,\mathbf{q}_\perp)\right\} = \delta(k^+ - q^+)\delta(\mathbf{k}_\perp - \mathbf{q}_\perp)\delta_{rs}$$

and the normalization of the spinors is such that

$$\bar{u}_{(+)}(p,r)\gamma^+ u_{(+)}(p,s) = 2p^+\delta_{rs}.$$

Note: $\bar{u}_{(+)}(p^\prime,r)\gamma^+ u_{(+)}(p,s) = 2p^+\delta_{rs}$ for $p^+ = p^{\prime +}$, one finds for $x>0$

$$q(x,\mathbf{b}_\perp) = \mathcal{N}^\prime \sum_s \int \frac{d^2\mathbf{k}_\perp}{2\pi}\int \frac{d^2\mathbf{k}_\perp^\prime}{2\pi} \left\langle p^+,\mathbf{0}_\perp \right| b_s^\dagger(xp^+,\mathbf{k}_\perp^\prime) b_s(xp^+,\mathbf{k}_\perp) \left| p^+,\mathbf{0}_\perp \right\rangle \times e^{i\mathbf{b}_\perp\cdot(\mathbf{k}_\perp - \mathbf{k}_\perp^\prime)}.$$

density interpretation of $q(x,\mathbf{b}_\perp)$

- Switch to mixed representation:
  momentum in longitudinal direction
  position in transverse direction

$$\tilde{b}_s(k^+,\mathbf{x}_\perp) \equiv \int \frac{d^2\mathbf{k}_\perp}{2\pi} b_s(k^+,\mathbf{k}_\perp) e^{i\mathbf{k}_\perp\cdot\mathbf{x}_\perp}$$

$\hookrightarrow$

$$\begin{aligned} q(x,\mathbf{b}_\perp) &= \sum_s \left\langle p^+,\mathbf{0}_\perp \right| \tilde{b}_s^\dagger(xp^+,\mathbf{b}_\perp)\tilde{b}_s(xp^+,\mathbf{b}_\perp) \left| p^+,\mathbf{0}_\perp \right\rangle . \\ &= \sum_s \left| \tilde{b}_s(xp^+,\mathbf{b}_\perp) \left| p^+,\mathbf{0}_\perp \right\rangle \right|^2 \\ &\geq 0. \end{aligned}$$

back

Boosts in nonrelativistic QM

$$\vec{x}' = \vec{x} + \vec{v}t \qquad\qquad t' = t$$

purely kinematical (quantization surface $t = 0$ inv.)

$\hookrightarrow$ 1. boosting wavefunctions very simple

$$q\Psi_{\vec{v}}(\vec{p}_1, \vec{p}_2) = \Psi_{\vec{0}}(\vec{p}_1 - m_1\vec{v}, \vec{p}_2 - m_2\vec{v}).$$

2. dynamics of center of mass

$$\vec{R} \equiv \sum_i x_i \vec{r}_i \qquad \text{with} \qquad x_i \equiv \frac{m_i}{M}$$

decouples from the internal dynamics

Relativistic Boosts

$$t' = \gamma \left(t + \frac{v}{c^2} z \right), \qquad z' = \gamma \left(z + vt \right) \qquad \mathbf{x}'_\perp = \mathbf{x}_\perp$$

generators satisfy Poincaré algebra:

$$\begin{aligned} \left[P^\mu, P^\nu \right] &= 0 \\ \left[M^{\mu\nu}, P^\rho \right] &= i \left(g^{\nu\rho} P^\mu - g^{\mu\rho} P^\nu \right) \\ \left[M^{\mu\nu}, M^{\rho\lambda} \right] &= i \left(g^{\mu\lambda} M^{\nu\rho} + g^{\nu\rho} M^{\mu\lambda} - g^{\mu\rho} M^{\nu\lambda} - g^{\nu\lambda} M^{\mu\rho} \right) \end{aligned}$$

rotations: $M_{ij} = \varepsilon_{ijk} J_k$, boosts: $M_{i0} = K_i$.

Galilean subgroup of $\perp$ boosts

introduce generator of $\perp$ 'boosts':

$$B_x \equiv M^{+x} = \frac{K_x + J_y}{\sqrt{2}} \qquad B_y \equiv M^{+y} = \frac{K_y - J_x}{\sqrt{2}}$$

Poincaré algebra $\Longrightarrow$ commutation relations:

$$\begin{aligned} [J_3, B_k] &= i\varepsilon_{kl} B_l & \qquad [P_k, B_l] &= -i\delta_{kl} P^+ \\ \left[P^-, B_k\right] &= -iP_k & \qquad [P^+, B_k] &= 0 \end{aligned}$$

with $k, l \in \{x, y\}$, $\varepsilon_{xy} = -\varepsilon_{yx} = 1$, and $\varepsilon_{xx} = \varepsilon_{yy} = 0$.

Together with $[J_z, P_k] = i\varepsilon_{kl}P_l$, as well as

$$\begin{aligned} [P^-, P_k] &= [P^-, P^+] = [P^-, J_z] = 0 \\ [P^+, P_k] &= [P^+, B_k] = [P^+, J_z] = 0. \end{aligned}$$

Same as commutation relations among generators of nonrel. boosts, translations, and rotations in x-y plane, provided one identifies

$$\begin{aligned} P^- &\longrightarrow \text{Hamiltonian} \\ \mathbf{P}_\perp &\longrightarrow \text{momentum in the plane} \\ P^+ &\longrightarrow \text{mass} \\ L_z &\longrightarrow \text{rotations around } z\text{-axis} \\ \mathbf{B}_\perp &\longrightarrow \text{generator of boosts in the plane,} \end{aligned}$$

back to discussion

Consequences

- many results from NRQM carry over to $\perp$ boosts in IMF, e.g.
- $\perp$ boosts kinematical

$$\begin{aligned}\Psi_{\mathbf{\Delta}_\perp}(x,\mathbf{k}_\perp) &= \Psi_{\mathbf{0}_\perp}(x,\mathbf{k}_\perp - x\mathbf{\Delta}_\perp) \\ \Psi_{\mathbf{\Delta}_\perp}(x,\mathbf{k}_\perp,y,\mathbf{l}_\perp) &= \Psi_{\mathbf{0}_\perp}(x,\mathbf{k}_\perp - x\mathbf{\Delta}_\perp, y, \mathbf{l}_\perp - y\mathbf{\Delta}_\perp)\end{aligned}$$

- Transverse center of momentum $\mathbf{R}_\perp \equiv \sum_i x_i \mathbf{r}_{\perp,i}$ plays role similar to NR center of mass, e.g. $\int d^2\mathbf{p}_\perp \left|p^+,\mathbf{p}_\perp\right\rangle$ corresponds to state with $\mathbf{R}_\perp = \mathbf{0}_\perp$.

back

$\perp$ Center of Momentum

- field theoretic definition

$$p^+\mathbf{R}_\perp \equiv \int dx^- \int d^2\mathbf{x}_\perp T^{++}(x)\mathbf{x}_\perp = M^{+\perp}$$

- $M^{+\perp} = \mathbf{B}^\perp$ generator of transverse boosts
- parton representation:

$$\mathbf{R}_\perp = \sum_i x_i \mathbf{r}_{\perp,i}$$

($x_i =$ momentum fraction carried by i^{th} parton)

back

Poincaré algebra:

$$
\begin{aligned}
\left[P^\mu, P^\nu\right] &= 0 \\
\left[M^{\mu\nu}, P^\rho\right] &= i\left(g^{\nu\rho}P^\mu - g^{\mu\rho}P^\nu\right) \\
\left[M^{\mu\nu}, M^{\rho\lambda}\right] &= i\left(g^{\mu\lambda}M^{\nu\rho} + g^{\nu\rho}M^{\mu\lambda} - g^{\mu\rho}M^{\nu\lambda} - g^{\nu\lambda}M^{\mu\rho}\right)
\end{aligned}
$$

rotations: $M_{ij} = \varepsilon_{ijk}J_k$, boosts: $M_{i0} = K_i$. back

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back

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back

Consequences of Galilean subgroup

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- $\perp$ boosts kinematical

$$\begin{aligned}\Psi_{\mathbf{\Delta}_\perp}(x,\mathbf{k}_\perp) &= \Psi_{\mathbf{0}_\perp}(x,\mathbf{k}_\perp - x\mathbf{\Delta}_\perp)\\ \Psi_{\mathbf{\Delta}_\perp}(x,\mathbf{k}_\perp,y,\mathbf{l}_\perp) &= \Psi_{\mathbf{0}_\perp}(x,\mathbf{k}_\perp - x\mathbf{\Delta}_\perp, y, \mathbf{l}_\perp - y\mathbf{\Delta}_\perp)\end{aligned}$$

- Transverse center of momentum $\mathbf{R}_\perp \equiv \sum_i x_i \mathbf{r}_{\perp,i}$ plays role similar to NR center of mass, e.g. $|p^+,\mathbf{R}_\perp = \mathbf{0}_\perp\rangle \equiv \int d^2\mathbf{p}_\perp\, |p^+,\mathbf{p}_\perp\rangle$ corresponds to state with $\mathbf{R}_\perp = \mathbf{0}_\perp$.

back

Proof that $\mathbf{B}_\perp|p^+,\mathbf{R}_\perp=\mathbf{0}_\perp\rangle=0$

- Use

$$e^{-i\mathbf{v}_\perp\cdot\mathbf{B}_\perp}|p^+,\mathbf{p}_\perp,\lambda\rangle=|p^+,\mathbf{p}_\perp+p^+\mathbf{v}_\perp,\lambda\rangle$$

$\hookrightarrow$

$$e^{-i\mathbf{v}_\perp\cdot\mathbf{B}_\perp}\int d^2\mathbf{p}_\perp|p^+,\mathbf{p}_\perp,\lambda\rangle=\int d^2\mathbf{p}_\perp|p^+,\mathbf{p}_\perp,\lambda\rangle$$

$\hookrightarrow$

$$\mathbf{B}_\perp\int d^2\mathbf{p}_\perp|p^+,\mathbf{p}_\perp,\lambda\rangle=0$$

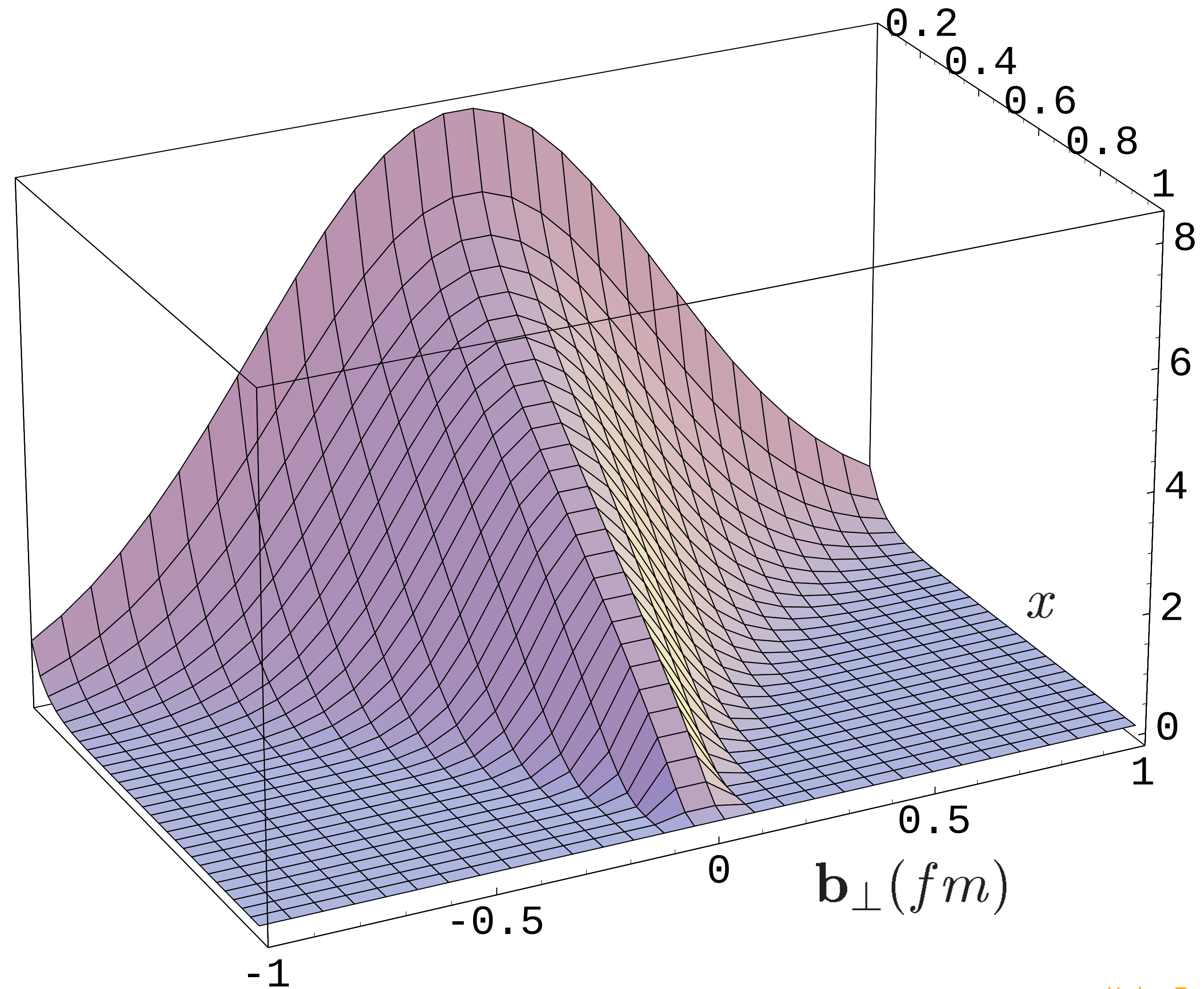
back

Example

- Ansatz: $H_q(x,0,-\mathbf{\Delta}_\perp^2) = q(x)e^{-a\mathbf{\Delta}_\perp^2(1-x)\ln\frac{1}{x}}$.

$$\hookrightarrow \quad q(x,\mathbf{b}_\perp) = q(x)\frac{1}{4\pi a(1-x)\ln\frac{1}{x}}e^{-\frac{\mathbf{b}_\perp^2}{4a(1-x)\ln\frac{1}{x}}}$$

$q(x, \mathbf{b}_\perp)$



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Lattice results for first 3 moments

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