String Thermodynamics From Scratch

- Infrared and Ultraviolet in String Theory vs Effective Field Theory.
  - Vacuum Energy Density in String Theory is normalizable. Implications.
- Equilibrium String Thermodynamics in the Canonical Ensemble.
  - Implications of Thermal Duality Transformations. NO Hagedorn Transition.
  - Holographic (2d) Growth at High T vs Field Theoretic Growth at T < String scale.
- RG Flows and g-theorem. Stability of susy type II closed oriented sector.
- Type I and Type II Unoriented Open and Closed String Ensembles.
- Type I and Type I′: The Long String Phase Transition is First Order!

J. Polchinski, Comm Math Phys 37 (1986) 104.
(pedagogical review in hep-th/0409031).
Key elements in our work were motivated from J.Atick and E. Witten (1988).

S. Chaudhuri, PRL 86 (2001) 1943. PRD 65 (2002) 066008.
PLB546 (2002) 108. hep-th/0408206 and hep-th/0409301.

Infrared & Ultraviolet in String Theory vs Field Theory

- Recall nature of UV/IR regulator dependence in the YM-Higgs theory :

$$Z = \frac{1}{\mathrm{Vol(gauge)}} \int dA\, d\Phi\;\; e^{-S_{YM}[A] - S[\Phi, A]} = \int_{\mathrm{orbit}} d\hat{A}\, d\Phi\;\; e^{-S_{YM}[\hat{A}] + S_{FP} - S[\Phi, \hat{A}]}$$

$$\langle O_1 \ldots O_N \rangle = \frac{1}{Z[A,\Phi]} \frac{1}{\mathrm{Vol(gauge)}} \int dA\, d\Phi\;\; O_1[A,\Phi] \ldots O_N[A,\Phi]\;\; e^{-S_{YM}[A] - S[\Phi,\hat{A}]}.$$

  - Only normalized correlators free of UV regulator ambiguity. Cancellation of IR divergences leaves renormalization scheme (MS) dependent finite corrections.

- 2d Quantum Gravity coupled to D Scalars = Embedded Worldsurfaces.
  Apparently: D-dim non-renormalizable EFT, in inv powers of string scale.
  In fact: String Theory is exactly renormalizable, a single Wilsonian ren'n.

$$S = \int_M d^2\xi \sqrt{g}\;\; [\lambda R(g) + \mu + m_s^2\, g^{ab} \partial_a X \partial_b X] = 4\pi\lambda(2 - 2h - b - c) + \mu A + m_s^2\, S_P[X, g].$$

$$Z = \frac{1}{\mathrm{Vol(Diff_0)}} \int dg\, dX\, e^{-\lambda S[g] - \mu A - m_s^2 S[X,g]} = \int_{\mathrm{orbit}} [d\hat{g}]\, dX e^{-\lambda S(\hat{g}) - \mu A + (d - d_{crit}) S_L[\hat{g},\varphi] - m_s^2 S[X,\hat{g}]}$$

2d Gauge Invariance and the Lack of Infrared Ambiguity

- Critical String Theory: All renormalized 2d masses vanish (Weyl Inv).

$$W_{string}[g,X] = \frac{1}{Vol(Diff_0 \times Weyl)} \int dg\, dX\ e^{-\lambda S[g] - \mu A - m_s^2 S_P[X,\hat{g}]}, \qquad d = d_{crit}.$$

$$= \int [d\tau]\ e^{-\lambda S[g]} \int dX\ e^{-m_s^2 S_P[X,\hat{g}]} = \int [d\tau] e^{-\lambda S[g]} (\det{}' \Delta)^{-(d-2)/2}. \text{ Note : } \mu_{ren} = 0!$$

  - One-loop effective potential = sum over connected vacuum graphs = an ordinary integral over worldsheet moduli. Finite, normalized, result.

$$X(a_n X_n; \lambda_n; C_n) \Rightarrow \prod_n \int \mu_n(C_n)\ da_n\ e^{-\lambda_n a_n^2} = \left[\det(\mu^{-2}\pi^{-1}\Delta)\right]^{-1/2} \Rightarrow \lim_{s\to 0} \frac{d}{ds}\left[\sum_{n=1}^{\infty} \mu_n \lambda_n^{-s}\right].$$

  - A unique gauge invariant prescription exists for the regularization of such eigenvalue sums in 2d quantum gravity. Sommerfeld-Watson Transforms.

- Correlation functions also unambiguously normalized in terms of string mass scale, dimensionless string coupling: a single Wilsonian renorm'n.
  - Refs: SC & E Novak, JHEP (2000). Pedagogical review, 0409033. Work to appear.

Free Energy of Closed Bosonic String Ensemble

J. Polchinski, CMP 37 (1986) 104.

- Helmholtz Free Energy: F = - T W(T) = V x Effective Potential.
  "Heat Bath" = background spacetime geometry, fluxes and fields.
  => Canonical, rather than Microcanonical Ensemble. Comments.

$$W_{bos} = L^{25}(4\pi^2\alpha')^{-25/2} \cdot \int_F \frac{d^2\tau}{2\tau_2^2} \cdot \tau_2^{-(d-2)/2} \cdot e^{\frac{1}{6}(d-2)\pi\tau_2} \cdot \prod_{n=1}^{\infty} |1 - e^{-2\pi i n\tau}|^{-2(d-2)} \cdot \sum_{n,w=-\infty}^{\infty} e^{\pi i n w \tau_1} \cdot e^{-\pi\tau_2\left(\frac{4\pi^2\alpha' n^2}{\beta^2} + \frac{w^2\beta^2}{4\pi^2\alpha'}\right)}.$$

- **Spatial Volume** $= V = L^{25}$. **Thermodynamic Limit :** $\alpha' \to 0, L \to \infty$.
- **Momentum Modes** $\leftrightarrow n \subset Z$. **Winding Modes** $\leftrightarrow w \subset Z$. **Orientations** $\leftrightarrow \pm n, \pm w$.
- **2d Field theory on a strip. Reparamz'n invariant sum on strips of all shape and size.**

- Thermal Duality = Euclidean timelike T-duality Transformation. In closed bosonic string theory, W(T) is thermal self-dual. Strong Holography.
- **W(T) invariant under :** $\beta \to \frac{\beta_C^2}{\beta}$, $n \leftrightarrow w$, $\beta_C^2 = 4\pi^2\alpha' \Leftrightarrow F(T) = \lim_{T\to\infty} \frac{T^2}{T_C^2} F\left(\frac{T_C^2}{T}\right) = \frac{T^2}{T_C^2}\rho_0.$
- Tachyonic thermal spectrum, starting from T=0.
- $\alpha'(\text{mass})^2 = -1 + \left(\frac{n^2T^2}{4T_C^2} + \frac{w^2T_C^2}{4T^2}\right)$, $n, w \in Z_\pm$. **Relevant** $\leftrightarrow$ **Irrelevant :** $T_{w=1} = \frac{T_C^2}{4}, T_{n=1} = 4T_C^2.$

Heterotic Ensemble: Absence of a Hagedorn Transition

S. Chaudhuri, PRL 86 (2001) 1943. hep-th/0409301 .

- Thermal duality exchanges E8xE8 and Spin(32)/Z2 theories.
- Euclidean timelike Wilson line background <=> quantization of finite temp Yang-Mills gauge theory in modified axial gauge.

$$W_{het}(\beta) \;=\; L^9(4\pi^2\alpha')^{-9/2}\cdot\int_F \frac{d^2\tau}{2\tau_2^2}\cdot\tau_2^{-4}\cdot|\eta(\tau)|^{-16}\cdot Z_{(SO(16))^2}(\beta).$$

- **Modular Invariance and (1,0) Superconformal constraints, tachyon free spectrum.**
- **Interpolates between $W_{E8xE8}(T=0)$ and $W_{Spin(32)/Z2}(\beta=0)$. Let $p'\in\Gamma_{E_8\times E_8}$, $\beta A_0=(1^8,0^8)$.**
- **$\Gamma^{(17,1)}$: $(p';\, p_L, p_R)=(p'+w\,\beta A\;\;;\;\; p_L - p'A - w\,\beta\; A^2,\quad p_R - p'A - w\,\beta\, A^2\,)$.**

- 9d spatial interpolation originally due to Ginsparg, PRD35 (1987) 648.
- Thermal effects can break susy thru thermal mode dependent phases in the string path integral, idea due to Atick Witten, NPB310 (1988) 291.

- $F(T)=\rho(T)V.\quad \lim_{T\to\infty}\rho(T)_{E8xE8}=\lim_{T\to\infty}\frac{T^2}{T_C^2}\rho\left(\frac{T_C^2}{T}\right)_{Spin(32)/Z_2}=\frac{T^2}{T_C^2}\rho(0)_{Spin(32)/Z_2}.$

$\Leftrightarrow$ **high temperature degrees of freedom grow as fast as in a 2d field theory.**

Interpol'n implements therm duality, enabling tachyon free equilib'm ensemble.

Kosterlitz-Thouless Duality PhaseTransition

S. Chaudhuri, PRD 65 (2002) 066008. hep-th/0409301 .

- Thermodynamic Potentials:

$$F(T) = -TW(T) = V\rho.\quad P=-\left(\frac{\partial F}{\partial V}\right)_T, U=T^2\left(\frac{\partial W}{\partial T}\right)_V, S=-\left(\frac{\partial F}{\partial T}\right)_V, C_V=T\left(\frac{\partial S}{\partial T}\right)_V. G=U-TS+PV.$$

- **Finite and normalizable functions of (T,V). Thermodynamic Limit : $\alpha'\to 0, L\to\infty$.**
- **Cosm Const $\Leftrightarrow$ Ideal Fluid, negative pressure. $P=-\rho$. Gibbs free energy vanishes.**
- **Thermodynamic potentials appear analytic in temperature in the vicinity of T_C.**

- Duality transition is in the Kosterlitz-Thouless Universality Class.
  (J. Kosterlitz and D. Thouless, J Phys C6 (1973) 1181.)

$$\text{Denote}:\; [d\tau] \equiv L^9(4\pi^2\alpha')^{-9/2}\left[\frac{d^2\tau}{2\tau_2^2}\tau_2^{-9/2}|\eta(\tau)|^{-16}Z_{(SO(16))^2}\right].\qquad y(\tau_2;\beta)\equiv 2\pi\tau_2\left(\frac{4\pi^2\alpha' n^2}{\beta^2}+\frac{w^2\beta^2}{4\pi^2\alpha'}\right).$$

$$\Rightarrow W_{(m)} = \int_F [d\tau d\, e^{-y}\left(-y_{(m)} - ... -(-y_{(1)})^m\right).\; F(\beta)=-\frac{1}{\beta}W_{(0)}, U=-W_{(1)},\; S=W_{(0)}-\beta W_{(1)}, C_V=\beta^2 W_{(2)}, ...$$

- **Infinite hierarchy of potentials, analytic in temperature, in the vicinity of T_C.**

Type I and Type II Open and Closed String Theories

S. Chaudhuri, hep-th/0408206 and hep-th/0409301 .

- Closed oriented string sector does not break supersymmetry at 1loop order.
- Type IIA and IIB strings have no Yang-Mills fields in the absence of a Ramond-Ramond sector => Type II thermal ensemble is unstable.

Consider Type IIB ensemble at infinitesimal temperature above $T = T_{w=1}$.

- **Tachyonic winding modes dominate W. The w = 1 mode is most relevant operator.**
- **AL (QFT) split: $F = F_{non} + F_{univ}$. String theory: 2d gauge inv picks unique UV regulator.**
- **g = bosonic vacuum degeneracy. Worldsheet RG flow is in the direction of lower "g".**

- **Thermal Phases in the Path Integral: (-1)^w allowed, (-1)^n disallowed by Modular Invariance.**
- **Mass Spectrum: $\alpha'(\mathrm{mass})^2 = -1 + \left(\frac{n^2T^2}{2T_C^2} + \frac{w^2T_C^2}{2T^2}\right)$, $n, w \in Z_\pm$.**

ONLY the pure winding modes give non-vanishing tachyonic contributions to the Free Energy.

- Infrared stable fixed point is the noncompact supersymmetric vacuum.
  Affleck and Ludwig, PRL67 (1991) 161. Harvey et al, hep-th/0111154.
  Adams et al, hep-th/0108075.

Type I Unoriented Open and Closed String Theory

S. Chaudhuri, hep-th/0408206 and hep-th/0409301 .

- Thermal Duality maps: Type IIA to Type IIB.
  Type I′ with D8branes to type IB with D9branes.
- Dbranes are half BPS states so generic type IIA and IIB are type I′ and IB.
  Closed oriented sector (oneloop = torus) does not break susy.

Consider Type IB with 32 D9branes, worldvolume Wilson line: $\oint dX^0 A_0$, $\beta A_0 = (1^8, 0^8)$.

- **Breaks gauge group to SO(16)×SO(16). Nonsupersymmetric. Note thermal phase.**
- **Oneloop vacuum energy receives contrib'ns from Annulus, Mobius strip, Klein Bottle.**
- **Cancellation of Ramond-Ramond scalar tadpole ⇒ absence of dilaton tadpole. $F_{1loop} = 0$!**

Thermal Dual Type I' with 16 D8branes on either orientifold plane. Momentum ↔ Windings.

$$F(\beta) = -\frac{1}{\beta} L^9 (4\pi^2\alpha')^{-9/2} \cdot \int_0^\infty \frac{dt}{8t} \cdot t^{-9/2} \cdot e^{-\beta^2 t/2\pi\alpha'} \cdot \eta(it)^{-8} \cdot \sum_{n=-\infty}^{\infty} [\, 2^{-8} N^2 \left(Z_{[0]} - e^{in\pi} Z_{[1]}\right)$$
$$+ \left(Z_{[0]} - e^{in\pi} Z_{[1]}\right) - 2^{-3} N \left(Z_{[0]} - e^{in\pi} Z_{[1]}\right)] \times e^{-2\pi t (4\pi^2\alpha' n^2/\beta^2)} .$$

- Nonsupersymmetric Type I vacuum with susy broken by thermal effects, but without generation of a dilaton tadpole. Comment: Higher loops?

Holography in Open and Closed String Theory

S. Chaudhuri, hep-th/0409301.

- Thermal Duality maps Type I′ with D8branes to type IB with D9branes.
- Recall: Factorization limit dominated by the asymptotics of highest mass open string modes. This gives high temperature growth, above string scale.
  Short cylinder limit dominated by lowest open string modes (gauge).

$$\lim_{\beta\to 0} F(\beta) = -\frac{1}{\beta}\cdot\sum_{n=-\infty}^{\infty}\left[\beta^{-1}\left(2a' \ \pi^3 n^2\right)^{1/2} + \beta\left(2\,\alpha' \ \pi\right)^{-1/2}\right]\cdot\rho_{high}$$

$$\lim_{\beta\to\infty} F(\beta) = \beta^{-10}\cdot\rho_{low}\,.$$

- Strong Holography, at high temperatures above the string scale: growth in degrees of freedom only as fast as that in a 2d field theory. As expected, the growth below the string scale is that of a 9+1-d quantum field theory.
  - String Theory has many fewer degrees of freedom at high temperatures!

Note: Unlike F, remaining thermodynamic potentials are non-vanishing.

Order Parameter for Long String Phase Transition

S. Chaudhuri, PRL86 (2001) 1943. hep-th/0409301 (hep-th/0409031).

- Recall expectation value of closed timelike loop = order parameter for thermal deconfinement transition in nonabelian gauge theory.

$$W_2(R,\beta) = \lim_{R\to 0}\int_0^\infty dt\,\frac{e^{-R^2 t/2\pi\alpha'}}{\eta(it)^8}\sum_{n=-\infty}^{\infty} e^{-2\pi t\ n^2(4\pi^2\alpha'/\beta^2)}\left[\left(\frac{\Theta_{00}}{\eta}\right)^4-\left(\frac{\Theta_{10}}{\eta}\right)^4-e^{in\pi}\left\{\left(\frac{\Theta_{01}}{\eta}\right)^4-\left(\frac{\Theta_{11}}{\eta}\right)^4\right\}\right].$$

- Low energy limit of the Macroscopic Loop Amplitude in String Theory => Pair Correlation Function of Polyakov-Wilson Loops.
  - Macroscopic Loop Amplitude given by the reparametrization invariant sum over worldsheets with boundaries mapped to fixed curves in embedding target space.

$$V(R,\beta) = -\left[R\left(1+\frac{16\pi^4\alpha'^2}{R^2\beta^2}\right)^{1/2}\right]^{-1},\ \beta>\beta_D.\ \ V(R,\beta) = -\left[R\left(1+\frac{\beta^2}{R^2}\right)^{1/2}\right]^{-1},\ \beta<\beta_D.\ \ T_D = 1/2\sqrt{2}\pi\alpha'^{1/2}.$$

Note: Long String Phase Transition occurs at temperature threshold for the production of the first thermal winding mode! (First Order).

- Cohen, Moore, Nelson, Polchinski, NPB 267 (1986). Point-like, off-shell propagator.
- SC, Y Chen, E Novak, PRD 62 (2000); SC, E Novak, PRD62 (2000). Macroscopic Loops.

Conclusions and Open Questions

- Vacuum Energy in String Theory vs Effective Field Theory. Dark Energy.
  - Couplings in worldvolume gauge-gravity theory on braneworld.
- String Thermodynamics in Canonical Ensemble for all six supersymmetric string theories: type I and I', type IIA and IIB, E8xE8 and Spin(32)/Z2.
  - NO Hagedorn transition. Kosterlitz-Thouless and Long String Phase Transition.
  - Strong Holographic Growth in High T (T >String scale) Degrees of Freedom.
- Instabilities, using RG Flow analysis: type I unoriented open and closed string ensemble is stable. Nonsusy, with neither tachyons, nor tadpoles.
- Order Parameter for a Deconfining Phase Transition in the Worldvolume Gauge Theory: Long String Phase Transition (First Order).
- Microcanonical Ensemble of Type I unoriented open & closed strings?
- Stat Mech of Cosmic String Gas. Intercommutation vs Splitting?

J. Polchinski, Comm Math Phys 37 (1986) 104.
(pedagogical review in ***hepth/0409031)***

S. Chaudhuri, PRL 86 (2001) 1943. PRD 65 (2002) 066008.
PLB546 (2002) 108. hep-th/0408206 and hep-th/0409301.